

Chapter 1

Michael Garcia
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Theorem 4. The sum of two odd numbers is even.

Proof. Recall the definition of an odd number to be a number, n , that can be written as $2k-1$ where k is an integer. Recall also that the definition of an even number is a number, m , that can be written as $2j$ where j is an integer. Let $n_1 = |2k-1|$ and $n_2 = |2j-1|$. By adding these two we get $2k-1+2j-1$ which turns into $2k+2j-2$. By factoring out the 2 we receive $2(k+j-1)$. \square

Result. Thus, since k and j are integers, subtracting 1 still results in an integer. Therefore we can rewrite $2k+j-1$ as m . Therefore, we are left with the definition of an even number, ergo $2m$.

Theorem 7. Every binary number n that ends in 0 is even.

Proof. The definition of a binary number is a number that can be written as

$$n = a_{n-1} \cdot 2^{n-1} + \dots + a_1 \cdot 2^1 + a_0 \cdot 2^0$$

Each number except the last one can have a 2 factored out. Therefore let $m = a_{n-1} \cdot 2^{n-1} + \dots + a_1 \cdot 2^1$. As such, the formula can be written as

$$n = 2m + a_0 \cdot 2^0$$

Recall that the definition of an even number to be a number that can be written as $2m$. Since $2^0 = 1$, and since the last number is 0, we have

$$n = 2m + 0 \cdot 1 = 2m$$

\square

Result. Thus, through the definition of an even number, any integer that can be written as $2m$, where m is an integer is even. Therefore, since any binary number n ending in 0 can be written as $2m$, it follows that n is even.

Theorem 10. The difference of two consecutive perfect squares is odd.

Proof. The definition of a perfect square is any number that can be written n^2 , while a consecutive squared number is any number that can be written as $(n + 1)^2$. Finding the difference, we get $n^2 + n + n + 1 - n^2$. This results in $n + n + 1$ which can be written as $2n + 1$. Recall the definition of an odd number to be any number that can be written $2n + 1$. \square

Result. Therefore, through the definition of an odd number, any integer that can be written that can be written as $2n + 1$, where n is an integer is even. Thus, as any difference between consecutive perfect squares can be written as $2n + 1$, it must therefore be odd.

Theorem 17. At least two flights at Chicago O'Hare International Airport must take off within 90 seconds of each other

Proof. \square

Result.

Theorem 22a.

Proof. \square

Result.

Theorem 22b.

Proof. \square

Result.

Theorem 23. If n is any integer, then $3n^3 + n + 5$ is odd.

Proof. \square

Result.