Change Title

Michael Garcia, Ayden Barclay, Ty Turoczy, Harlow Sharp
MAT 203 - Discrete Mathematics

February 20, 2026

Theorem. The function $f : \mathbb{Z} \to \mathbb{N}$ defined by $f(n) = \begin{cases} 2n & \text{if } n > 0 \\ -2n+1 & \text{if } n \leq 0 \end{cases}$ is an injection.

Proof. Recall that to prove that a function is an injection, we must prove that whenever $f(a) = f(b)$, then it must be true that $a = b$.
In other words, we must show that the preimage of every element in the range is associated with exactly one element of the domain.
Let $a, b \in \mathbb{Z}$ and assume that $f(a) = f(b)$.

Notice that whenever the function input, n, is greater than zero, then the output, $f(n) = 2n$, is even.

Similarly, whenever the function input, n, is less than or equal to zero, then the output, $f(n) = -2n + 1$, is odd.

An even number cannot equal an odd number, so there are two possible cases for $f(a) = f(b)$; either both a and b are greater than zero, so that $f(a)$ and $f(b)$ are both even, or both a and b are less than or equal to zero, so that $f(a)$ and $f(b)$ are both odd.

Case 1:
Assume both a and b are greater than zero.
Then $f(a) = 2a$ and $f(b) = 2b$
By our assumption that $f(a) = f(b)$, we get:

$$\begin{aligned} f(a) &= f(b) \\ 2(a) &= 2(b) \\ a &= b \end{aligned}$$

Case 2:
Assume both a and b are less than or equal to zero.

Then $f(a) = -2a + 1$ and $f(b) = -2b + 1$
By our assumption that $f(a) = f(b)$, we get:

$$\begin{aligned} f(a) &= f(b) \\ -2(a) + 1 &= -2(b) + 1 \\ -2(a) &= -2(b) \\ a &= b \end{aligned}$$

In all possible cases, whenever $f(a) = f(b)$, we get $a = b$.

This is what we needed to show

Thus, the function is an injection. $\square$