Chapter 1

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Theorem 4. The sum of two odd numbers is even.

Proof. Recall the definition of an odd number to be a number, n, that can be written as 2k-1 where k is an integer. Recall also that the definition of an even number is a number, m, that can be written as 2j where j is an integer. Let $n_1 = |2k - 1|$ and $n_2 = |2j - 1|$. By adding these two we get $2k - 1 + 2j - 1$ which turns into $2k + 2j - 2$. By factoring out the 2 we receive $2(k + j - 1)$. □

Result. Thus, since k and j are integers, subtracting 1 still results in an integer. Therefore we can rewrite $2k + j - 1$ as m. Therefore, we are left with the definition of an even number, ergo 2m.

Theorem 7. Every binary number n that ends in 0 is even

Proof. The definition of a binary number is a number that can be written as

$$n = a_{n-1} \cdot 2^{n-1} + ... + a_1 \cdot 2^1 + a_0 \cdot 2^0$$

Each number except the last one can have a 2 factored out. Therefore let $m = a_{n-1} \cdot 2^{n-1} + ... + a_1 \cdot 2^1$. As such, the formula can be written as

$$n = 2m + a_0 \cdot 2^0$$

.

Recall that the definition of an even number to be a number that can be written as 2m. Since $2^0 = 1$, and since the last number is 0, we have

$$n = 2m + 0 \cdot 1 = 2m$$

. □

Result. Thus, through the definition of an even number, any integer that can be written as 2m, where m is an integer is even. Therefore since any binary number n ending in 0 can be written as 2m, it follows that n is even.

Theorem 10. The difference of two consecutive perfect squares is odd

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Proof. □

 Result.

Theorem 17. At least two flights at Chicaog O'Hare International AIrport must take off within 90 seconds of each other

Proof. □

 Result.

Theorem 22a.

Proof. □

 Result.

Theorem 22b.

Proof. □

 Result.

Theorem 23. If n is any integer, then $3n^3 + n + 5$ is odd.

Proof. □

 Result.