Chapter 2

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Theorem 14. It is not true that $(AXB) \cup (CXD) = (A \cup C)X(B \cup D)$.

Proof. Let

$$\begin{aligned} A &= 1 \\ B &= 2 \\ C &= 3 \\ D &= 4 \end{aligned}$$

Then,

$$\begin{aligned} A \times B &= \{(1,2)\} \\ C \times D &= \{(3,4)\} \\ A \cup C &= \{1,3\} \\ B \cup D &= \{2,4\} \\ (A \times B) \cup (C \times D) &= \{(1,2),(3,4)\} \\ (A \cup B) \times (B \cup D) & \\ &= \{(1,2),(1,4),(3,2),(3,4)\} \end{aligned}$$

$\square$

result. As $(A \times B) \cup (C \times D)$ and $(A \cup B) \times (B \cup D)$ produce different sets, they cannot be said to be equal.

Exercise 18. Write this in English:

$$\forall k \in 3\mathbb{Z}, \exists S \subseteq \mathbb{N}, |S| = k$$

(Is it true?) What is the negation of this statement? (Is the negation true?)

result. For all integers k that are multiples of 3, there exists a subset S of the natural numbers, so that the cardinality is equal to k.
The statement is not true, as there cannot be a set with a negative number of elements.
Negation in logic notation:

$$\exists k \in 3\mathbb{Z}, \forall S \subseteq \mathbb{N}, |S| \neq K$$

Negation in English: There exists an integer k that is a multiple of 3, for all subsets S of the natural numbers, so that the cardinality of S is not equal to k.
The statement is true, as there cannot be a susbet with a negative number of elements.

Theorem 20. $\mathbb{Z} = \{3k|k \in \mathbb{Z}\} \cup \{3k+1|k \in \mathbb{Z}\} \cup \{3k+2|k \in \mathbb{Z}\}$

Proof. Recall the definition of a union between sets to include all the elements from each set.
Recall the closure property to state that addition and multiplication between integers results in an integer.
Let $S = \{3k|k \in \mathbb{Z}\} \cup \{3k+1|k \in \mathbb{Z}\} \cup \{3k+2|k \in \mathbb{Z}\}$.
$3k$ is an integer for $k \in \mathbb{Z}$. Therefore, $3k+1$, $3k+2 \in \mathbb{Z}$ through closure property.
Thus, $S \subseteq \mathbb{Z}$.

Using the division algorithm, we get

$$n = 3q + r \text{ where } 0 \leq r < 3$$

If r = 0, then n = 3q, which belongs to $3k|k \in \mathbb{Z}$
If r = 1, then n = 3q + 1, which belongs to $3k+1|k \in \mathbb{Z}$
If r = 2, then n = 3q + 2, which belongs to $3k+2|k \in \mathbb{Z}$
This shows that every integer n fits into one of the three sets. Thus, $\mathbb{Z} \subseteq S$ □

result. Hence, since $\mathbb{Z}$ and S both contain the same elements, it can be said that

$$Z = \{3k|k \in \mathbb{Z}\} \cup \{3k+1|k \in \mathbb{Z}\} \cup \{3k+2|k \in \mathbb{Z}\}$$

Exercise 29. Write the negation of x is prime or $x < 52$. (Don't say, "It's not true that ...").

negation. x is not prime and $x \geq 52$.

Exercise 32. Write each of the following statements using formal logic notation.
(a) Even numbers are never prime
(b) Triangles never have four sides.
(c) There are no integers a, b such that $a^2/b^2 = 2$
(d) No square number immediately follows a prime number.

translations. Below are the translations.
(a)
$$\forall n \in \mathbb{Z}, \forall p \in \mathbb{P}, \neg(2n = p)$$

(b) Let $T =$ Triangle and $S =$ Number of sides a shape has.
Then,
$$\forall T, \neg(S = 4)$$

(c)
$$\exists a, b \in \mathbb{Z} | a^2/b^2 = 2$$

(d)
$$\neg(\exists p \in \mathbb{P}, \exists n \in \mathbb{Z} | n^2 = p + 1)$$

Theorem 40. $A \cap B = A \setminus B$

disproof. Recall the definition of an intersection between sets to contain elements common to each set.
Recall subtraction between sets to mean creating a set containing only elements from the first set that are not present in the second set.
Let $A = \{1, 2\}$ and $B = \{2, 3\}$.
Then,
$$A \cap B = \{2\}$$
$$A \setminus B = \{1\}$$

Thus, since $2 \neq 1$, then $A \cap B \neq A \setminus B$. □

Condition for Truth. Let $A = \emptyset$. For any set B, we have
$$A \cap B = \emptyset$$
$$A \setminus B = \emptyset$$

Thus, $A \cap B = A \setminus B$ when $A = \emptyset$

result. Thus, the only case in which the statement is true is when $A = \emptyset$ for any set B.