

Chapter 1

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Theorem 4. The sum of two odd numbers is even.

Proof. Recall the definition of an odd number to be a number, n , that can be written as $2k-1$ where k is an integer. Recall also that the definition of an even number is a number, m , that can be written as $2j$ where j is an integer. Let $n_1 = |2k - 1|$ and $n_2 = |2j - 1|$. By adding these two we get $2k - 1 + 2j - 1$ which turns into $2k + 2j - 2$. By factoring out the 2 we receive $2(k + j - 1)$. \square

Result. Thus, since k and j are integers, subtracting 1 still results in an integer. Therefore we can rewrite $2k + j - 1$ as m . Therefore, we are left with the definition of an even number, ergo $2m$.

Theorem 7. Every binary number n that ends in 0 is even.

Proof. The definition of a binary number is a number that can be written as

$$n = a_{n-1} \cdot 2^{n-1} + \dots + a_1 \cdot 2^1 + a_0 \cdot 2^0$$

Each number except the last one can have a 2 factored out. Therefore let $m = a_{n-1} \cdot 2^{n-1} + \dots + a_1 \cdot 2^1$. As such, the formula can be written as

$$n = 2m + a_0 \cdot 2^0$$

.

Recall that the definition of an even number to be a number that can be written as $2m$. Since $2^0 = 1$, and since the last number is 0, we have

$$n = 2m + 0 \cdot 1 = 2m$$

.

\square

Result. Thus, through the definition of an even number, any integer that can be written as $2m$, where m is an integer is even. Therefore, since any binary number n ending in 0 can be written as $2m$, it follows that n is even.

Theorem 10. The difference of two consecutive perfect squares is odd.

Proof. The definition of a perfect square is any number that can be written n^2 , while a consecutive squared number is any number that can be written as $(n + 1)^2$. Finding the difference, we get $n^2 + n + n + 1 - n^2$. This results in $n + n + 1$ which can be written as $2n + 1$. Recall the definition of an odd number to be any number that can be written $2n + 1$. □

Result. Therefore, through the definition of an odd number, any integer that can be written that can be written as $2n + 1$, where n is an integer is even. Thus, as any difference between consecutive perfect squares can be written as $2n + 1$, it must therefore be odd.

Theorem 17. At least two flights at Chicago O'Hare International Airport must take off within 90 seconds of each other

Proof. □

Result.

Theorem 22a.

Proof. □

Result.

Theorem 22b.

Proof. □

Result.

Theorem 23. If n is any integer, then $3n^3 + n + 5$ is odd.

Proof. □

Result.