Chapter 1

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Theorem 4. The sum of two odd numbers is even.

Proof. Recall the definition of an odd number to be a number, n, that can be written as 2k-1 where k is an integer. Recall also that the definition of an even number is a number that can be written as 2m where m is an integer.
Let n_1 and n_2 both be odd numbers. Then,

$$\begin{aligned} n_1 + n_2 &= (2k-1) + (2j-1) \\ &= 2k - 1 + 2j - 1 \\ &= 2k + 2j - 2 \\ &= 2(k + j - 1) \\ &= 2m \end{aligned}$$

where $m = k + j - 1$. As integers are closed under addition and subtraction m itself must be an integer. □

Result. Therefore, $k + j - 1$ can be written as m, and we are left with the definition of an even number, 2m.

Theorem 7. Every binary number n that ends in 0 is even

Proof. Recall the definition of an even number to be a number that can be written as 2m. The definition of a binary number is a number that can be written as

$$n = a_{n-1} \cdot 2^{n-1} + ... + a_1 \cdot 2^1 + a_0 \cdot 2^0$$

Then,

$$m = a_{n-1} \cdot 2^{n-1} + ... + a_1 \cdot 2^1 n = 2m + a_0 \cdot 2^0$$

Since $2^0 = 1$, and since the last number is 0, we have

$$n = 2m + 0 \cdot 1 = 2m$$

. □

Result. Thus, through the definition of an even number, any integer that can be written as 2m, where m is an integer is even. Therefore, since any binary number n ending in 0 can be written as 2m, it follows that n is even.

Theorem 10. The difference of two consecutive perfect squares is odd.

Proof. Recall the definition of an odd number to be any number that can be written $2n + 1$. The definition of a perfect square is any number that can be written n^2, while a consecutive squared number is any number that can be written as $(n + 1)^2$. Then,

$$n^2 + n + n + 1 - n^2 n + n + 12n + 1$$

□

Result. Thus, as any difference between consecutive perfect squares can be written as $2n + 1$, it must therefore be odd.

Theorem 17. At least two flights at Chicago O'Hare International Airport must take off within 90 seconds of each other

Proof. □

Result.

Theorem 22a. There is not a property in the list of five distinct numbers of length 20 that says there must be two subsets of the list with the same sum

Proof. □

Result.

Theorem 22b.

Proof. □

Result.

Theorem 23. If n is any integer, then $3n^3 + n + 5$ is odd.

Proof. □

Result.