

Package ‘zernike’

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Author M.L. Peck <mpeck1@ix.netcom.com>

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Description Routines for Manipulation of Zernike polynomials and
Interferogram fringe analysis

Maintainer M.L. Peck <mlpeck54@gmail.com>

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addfit	<i>Add zernike coefficients to a matrix.</i>
--------	--

Description

Add zernike coefficients to a matrix.

Usage

```
addfit(..., th = 0, zcm = NULL, theta = numeric(0))
```

Arguments

...	One or more fits as from psifit, etc.
th	Rotation angles, in degrees
zcm	The matrix to be added to (defaults to NULL)
theta	The vector of rotation angles to be added to

Author(s)

M.L. Peck

aiapsi	<i>Iterative algorithms for PSI with unknown phase shifts</i>
--------	---

Description

Three iterative algorithms for PSI with unknown phase shifts.

Usage

```
aiapsi(im.mat, phases, ptol = 0.001, maxiter=20, trace=1)
aiapsiC(im.mat, phases_init, ptol, maxiter, trace)
hkpsi(im.mat, phases, maxiter = 20, ptol = 0.001,
      trace = 1, plotprogress = TRUE)
tiltpsi(im.mat, phases, coords, ptol = 0.01, maxiter = 20, trace = 1)
tiltpsiC(im.mat, phases, coords, ptol, maxiter, trace)
```

Arguments

im.mat	a <i>matrix</i> of interferogram values
phases	Starting guess for phase shifts
ptol	Convergence criterion for phase shifts
maxiter	Maximum number of iterations
trace	Boolean: Print some summary data at each iteration.
plotprogress	Plot some summary data for each iteration? Also, for tiltpsi and tiltpsiC
coords	Low order Zernike polynomial matrix

Details

The “variable tilt” algorithm now allows an indefinite number of low order Zernike terms to be variable between phase steps. coords can be created with [zpm](#) setting maxorder to a small value, say 4, discarding the first (dc) column and retaining as many as desired. There must be at least two columns for tilts. The third will be defocus, the next two astigmatism, the next two primary coma, ...

aiapsi and tiltpsi are wrappers for the calls to the C++ code in aiapsiC and tiltpsiC with sensible defaults for ptol, maxiter, and trace.

Value

A list containing the following elements:

phi	The wrapped phase estimate. This is a vector as long as the number of rows in im.mat.
mod	Modulation estimate.
phases	Phase shift estimates.
iter	Number of iterations.
sse	Sum squared error at each iteration.

Also, for tiltpsi

zcs	Matrix of Zernike coefficients, with one row for each column in coords and number of columns = number of columns of im.mat.
-----	---

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

References

- Zhaoyang Wang and Bongtae Han, “Advanced iterative algorithm for phase extraction of randomly phase-shifted interferograms,” *Opt. Lett.* 29, 1671-1673 (2004).
- Han, G-S and Kim, S-W,, “Numerical correction of reference phases in phase-shifting interferometry by iterative least squares fitting,” *Applied Optics* 33, 7321-7325 (1994),
- Lin, B-J et al., “An iterative tilt-immune phase-shifting algorithm,” OSA conference Optical Fabrication and Testing 2010.

See Also[psifit](#)

`astig.bath`*Zernike coefficients for astigmatism due to Bath geometry.*

Description

Calculates Bath astigmatism coefficients with optional rotation of phi degrees.

Usage

```
astig.bath(D, rc, s, lambda = 632.8, phi = 0)
```

Arguments

D	Diameter
rc	Radius of curvature
s	separation of reference and test beams
lambda	Wavelength
phi	angle of image horizontal relative to interferometer axis, in degrees

Details

D, rc, s, must have the same units. If those units are mm the source wavelength lambda should be in nm.

Value

The Zernike coefficients for primary astigmatism terms.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

brcutpuw

*Branch cut algorithm for phase unwrapping***Description**

Solves a modification of the assignment problem to minimize the total length of branch cuts.

Usage

```
brcutpuw(phase, pen = 0, details = FALSE)
```

Arguments

phase	Matrix containing the wrapped phase map
pen	Optional penalty value for connecting a residue to an edge
details	boolean: if TRUE return some extra details for diagnostics

Value

A matrix containing the unwrapped wavefront. If details==TRUE a named list starting with the unwrapped wavefront in puw.

Author(s)

M.L. Peck

See Also

[qpuw idiffpuw](#)

Examples

```
set.seed(1234)
PW <- wrap(matrix((0:100)*pi/10,101,101))
## need a border of NA's
PW <- cbind(rep(NA,101), PW, rep(NA,101))
PW <- rbind(rep(NA,103), PW, rep(NA,103))
PW <- PW + rnorm(103^2)
mtext(rmap(PW, plot=TRUE))
PU <- brcutpuw(PW, details=TRUE)
image(1:103, 1:103, PU$bcuts, col="blue", add=TRUE)
X11()
image(PU$puw, col=grey256, asp=1, useRaster=TRUE)
```

circle.hough

*Estimate parameters of a circle using Hough Circle Transform***Description**

Uses a portion of the canny algorithm to find candidate edge points and the direction of the gradient at those points, then uses Hough Circle Transform to estimate circle parameters.

Usage

```
circle.hough(
  im,
  fw = 2,
  qt = 0.995,
  excl = 5,
  rmin = min(dim(im))/4,
  rmax = min(dim(im))/2,
  rstep = 1,
  dtheta_max = 0.5,
  dtheta_step = 0.05,
  nn = 7,
  plots = TRUE,
  details = FALSE
)
```

Arguments

im	The image to find a circle in (a modulation estimate is best)
fw	Size of Gaussian blur to smooth image
qt	Threshold to accept strong edge candidate
excl	Number of pixels to exclude around edge of frame as candidates
rmin	Minimum circle radius
rmax	Maximum circle radius
rstep	step size in constructing lookup table
dtheta_max	maximum assumed error in gradient direction
dtheta_step	increment for dtheta
plots	plot?
details	Return extra details?
number	of nearest neighbors for alternate calculation

Details

The Hough transform section first creates a lookup table of candidate radii and center points, then for each candidate edge point calculates potential centers along a fan of rays near the gradient direction. An inner join then finds matches in the lookup table and increments an accumulator vector. Highest vote at the end wins.

Value

If details is FALSE a named list with the circle parameters

Note

This is experimental and can be very slow. A good guess for the radius is very helpful. Experimental feature: find the nn nearest neighbors of the selected trio of parameters and calculate a vote weighted mean. This is returned as rxy_alt if details is TRUE.

See Also

`circle.pars()`, `pupil.pars()`

Examples

```
example("psifit", package="zernike", ask=FALSE)
X11()
cp2 <- circle.hough(tfit$mod, rmin=round(tfit$cp$rx)-10, rmax=round(tfit$cp$rx)+10)
```

circle.pars	<i>Pupil parameters</i>
-------------	-------------------------

Description

Automatically determine the center and radius of a circular interferogram image.

Usage

```
circle.pars(im, fw=2, qt=0.995, excl=5,
  plots=TRUE, details=FALSE)
```

Arguments

im	A matrix containing an image of a circular disk
fw	Amount to smooth image
qt	Threshold to accept an edge point, expressed as a quantile
excl	number of pixels around border of frame to exclude
plots	Plot edge candidates and fit?
obstructed	Logical: is there a central obstruction?

Details

This routine partially implements the Canny algorithm for edge detection. After optionally smoothing the input image the gradient is calculated using a Sobel filter, and edge pixels are identified by locating local maxima in the magnitude of the gradient.

The edge pixels with qt percentile largest gradients are passed to nlsrob in package robustbase lqs in package MASS to determine robustly the best fit circle.

Value

A list with the following components:

xc	X coordinate of the center of the pupil
yc	Y coordinate of the center of the pupil
rx	Horizontal radius of the pupil
ry	Vertical radius of the pupil = rx
obstruct	Obstruction fraction (always = 0)

Note

This routine is only effective on modulation estimates, and will almost certainly fail on interferogram images. Since data quality varies widely considerable experimentation may be needed on any given image. Increasing the smoothing parameter `fw` helps to suppress artifacts. Depending on how strong the actual edge is compared to artifacts `qt` may need to be either increased or decreased from the default value.

if `details==TRUE` several more pieces of data are returned. This is mostly for debugging purposes and may be eliminated in the future.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

See Also

Many routines require the pupil parameters in the form returned by `circle.pars`. For example [psifit](#), [fftfit](#), [pupil](#), etc.

col3d

OpenGL plot

Description

Returns a vector of colors similar to `image()` display.

Usage

```
col3d(surf, surf.col=topo.colors(256), zlim = NULL, eqa=FALSE)
```

Arguments

surf	A matrix of surface values
surf.col	Color palette for surface
zlim	Range of values to display
eqa	Equal area per color?

Value

A vector of color values the same length as surf.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

References

The **rgl** package is described at <http://rgl.neoscientists.org/about.shtml>, and available from CRAN.

See Also

[plot.pupil](#)

convolve2d

2D convolution

Description

General 2D convolution using FFTs

Usage

```
convolve2d(im, kern)
```

Arguments

im	A matrix representing an image
kern	the convolution kernel

Value

The filtered matrix im.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

See Also

[gblur](#). Called by [circle.pars](#).

crop	<i>Crop an array</i>
------	----------------------

Description

Crop a matrix or 3D array. Main application is to trim excess pixels from an image array, wavefront, etc.

Usage

```
crop(img, cp, npad = 20)
```

Arguments

img	Array to be cropped.
cp	A list describing the pupil boundary.
npad	Amount of padding to leave around the edge.

Details

cp is the list provided by [circle.pars](#).

Value

im	The cropped array
cp	Revised value of cp

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

fftfit	<i>Fourier transform interferogram analysis</i>
--------	---

Description

High level routines for FFT analysis of interferograms.

Usage

```
fftfit(imagedata, cp = NULL,
      sl = c(1, 1), filter = NULL, taper = 2,
      options = psfit_options())
```

Arguments

imagedata	A matrix containing the interferogram
cp	A list describing the pupil boundary, as returned by pupil.pars
sl	Position of sidelobe in the form c(x,y)
filter	Size of background filter around DC
taper	Size of taper applied to edge of half plane cut
options	a list of parameters passed to other functions. See psfit_options .

Details

If `is.null(filter)` (the default), [pick.sidelobe](#) will be called to select a Fourier domain sidelobe and background filter size.

If `is.null(cp)` [circle.pars](#) is applied to the modulation to estimate the pupil parameters.

See [wf_net](#) for details of the process of creating net and smoothed wavefronts from raw unwrapped wavefront maps.

Value

A list with the following components:

phase	Wrapped phase map
mod	The estimated modulation
cp	A list describing the pupil boundary
cp.orig	The precropped value of cp
wf.net	Net unsmoothed wavefront; a matrix of class " pupil "
wf.smooth	Net smoothed wavefront
wf.residual	Difference between net wavefront and polynomial fit
fit	Return value from fitzernikes
zcoef.net	Net Zernike coefficients from fit

Note

These functions are based largely on the work of Roddier and Roddier (1987).

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

References

Roddier, C. and Roddier, F. 1987, **Interferogram analysis using Fourier transform techniques**, *Applied Optics*, vol. 26, pp. 1668-1673.

See Also

[wf_net](#), [pupil.pars](#), [pick.sidelobe](#).

Description

Miscellaneous utilities for working with 2D images in the Fourier domain.

Usage

```
wftophase(X, lambda=1)
padmatrix(X, npad, fill = mean(X, na.rm=TRUE))
submatrix(X, size = 255)
fftshift(X)
.up2(nr, nc=nr)
```

Arguments

X	A matrix
lambda	Value of the wavelength, in the same units as X
npad	Size of padded matrix
fill	Values to be assigned to padded matrix elements
size	Size of returned matrix
nr	A number
nc	A number

Details

wftophase computes the complex phase from wavefront values.

padmatrix pads a matrix to size npad x npad, placing the original matrix in the lower left hand corner of the padded matrix.

submatrix extracts a size x size matrix from the center of a larger matrix.

fftshift shuffles the quadrants of a matrix around to put the DC element (1,1) in the center of the transformed matrix, with spatial frequencies increasing to the right and up.

Value

A matrix transformation of the input matrix X.

.up2 returns the next higher power of 2 than max(nr, nc).

Note

These low level routines are used by several higher level functions that operate in the Fourier domain.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

See Also

[startest](#), [fftfit](#).

fitzernikes

Least Squares fit to Zernike polynomials

Description

Performs a least squares fit of a specified set of Zernike polynomials to a vector of wavefront measurements.

Usage

```
fitzernikes(wf, rho, theta, phi = 0, maxorder = 14, uselm = FALSE, isoseq = FALSE)
```

Arguments

wf	A vector of wavefront values
rho	A vector of radial coordinates.
theta	A vector of angular coordinates, in radians.
phi	Orientation of the image, in degrees
maxorder	Maximum Zernike polynomial order
uselm	Boolean: use <code>lm()</code> for least squares fit
isoseq	Boolean: use ISO/ANSI sequencing

Details

wf, rho, and theta must be the same length.

As of version 3.7.0 Zernike polynomials in ISO/ANSI sequence can be used through a call to [zpm_cart](#).

Value

The model fit as returned by [lm](#), or the coefficients of the least squares fit if uselm is FALSE.

Note

The model fit is of the form $wf \sim Z_0 + Z_1 + Z_2 + \dots$. With the standard ordering of Zernikes Z_0 is the piston term, Z_1 and Z_2 are x and y tilts, Z_3 is defocus, etc.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

See Also

[zpm](#), [zpm_cart](#), [psifit](#), [fftfit](#), [vortexfit](#), [wf_net](#).

foucogram

Simulate a Foucaultgram

Description

Simulates the appearance of a wavefront under the Foucault test.

Usage

```
foucogram(wf, edgex = 0, phradius = 0, slit = FALSE,
           pad = 4, gamma = 1, map = FALSE, lev = 0.5)
```

Arguments

wf	An object of class pupil containing wavefront values
edgex	lateral position of knife edge
phradius	radius of light source
slit	Logical: Is source a slit or pinhole?
pad	pad factor for FFT
gamma	Gamma value for graphics display
map	Logical: Overlay contours from wavefront map?
lev	Increment for contour levels, if used

Details

The default value of 0 for phradius simulates a monochromatic point source. Try values in the range 10-30 to suppress diffraction effects.

Value

A matrix of intensity levels in the simulated image.

Note

The key approximations here are treating the light source as monochromatic and spatially coherent, which is usually not the case for an extended source. Also, Fraunhofer diffraction theory is used.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

References

See http://home.netcom.com/~mpeck1/astro/foucault/ext_foucault.pdf for an outline of the mathematical treatment of an extended source.

See Also

[pupil](#)

gblur

Gaussian blur

Description

Blur an image by fw pixels

Usage

```
gblur(X, fw=0, details=FALSE)
```

Arguments

X	A matrix representing an image
fw	Width of the Gaussian convolution kernel, in pixels
details	Return convolution kernel?

Details

fw is the standard deviation of the Gaussian.

Value

The filtered matrix X.

Note

the details option is mostly for debugging purposes and may go away.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

See Also

[convolve2d](#)

gpcapsi	<i>Generalized Principal components algorithm for phase shifting interferometry</i>
---------	---

Description

A generalized principal components algorithm for phase shifting interferometry developed by the author. This is the “low level” implementation.

Usage

```
gpcapsi(im.mat, ptol = 0.001, maxiter = 20, trace = 1)
gpcapsiC(im.mat, ptol, maxiter, trace)
```

Arguments

im.mat	Matrix containing the unmasked pixels from a set of interferograms.
ptol	Convergence tolerance for phase shifts
maxiter	Maximum number of iterations
trace	Print progress of nonlinear solver every trace iterations. Use trace=0 for silent operation.

Details

gpcapsi is a wrapper to the C++ call in gpcapsiC.

Value

A list with the following items:

phi	Estimated wrapped phase.
mod	Estimated modulation.
phases	Estimated phase shifts.
snr	An estimate of the S/N of the interferograms.
eigen	Eigenvalues of the crossproduct matrix

Note

This is the low level interface to the algorithm. The matrix im.mat should contain the unmasked pixel values from the input interferogram array. No checks are made for valid data. This should normally be called through the high level function [psifit](#).

Author(s)

M. L. Peck

See Also

[pcapsi psifit](#)

gradzpm_cart	<i>Calculate Zernike polynomial values and Cartesian gradients in ISO/ANSI sequence for a set of Cartesian coordinates.</i>
--------------	---

Description

Calculate Zernike polynomial values and Cartesian gradients in ISO/ANSI sequence for a set of Cartesian coordinates.

Usage

```
gradzpm_cart(x, y, maxorder = 12L, unit_variance = FALSE, return_zpm = TRUE)
```

Arguments

x	a vector of x coordinates for points on a unit disk.
y	a vector of y coordinates.
maxorder	the maximum radial polynomial order (defaults to 12).
unit_variance	logical: return with orthonormal scaling? (default false)
return_zpm	logical: return Zernike polynomial matrix? (default true)

Details

Zernike polynomial values and their directional derivatives in Cartesian coordinates. The recurrence relations used are known to be both efficient and numerically stable (see the reference).

Columns are in ISO/ANSI sequence: for each radial order $n \geq 0$ the azimuthal orders m are sequenced $m = -n, -(n-2), \dots, (n-2), n$, with sine components for negative m and cosine for positive m . Note this is the opposite ordering from the extended Fringe set and the ordering of aberrations is quite different. For example the two components of trefoil are in the 7th and 10th column while coma is in columns 8 and 9 (or 7 and 8 with 0-indexing). Note also that except for tilt and coma-like aberrations ($m=1$) non-axisymmetric aberrations will be separated.

All three matrices will have the same dimensions on return. Columns 0 and 1 of $dzdx$ will be all 0, while columns 0 and 2 of $dzdy$ are 0.

Value

a named list with the matrices zm (optional but returned by default), $dzdx$, $dzdy$.

References

Anderson, T.B. (2018) Optics Express 26, #5, 18878 <https://doi.org/10.1364/OE.26.018878> (open access)

See Also

`zpm()` uses the same recurrence relations for polar coordinates and extended Fringe set ordering, which is the more common indexing scheme for optical design/testing software.

`zpm_cart()` calculates and returns the Zernike polynomial values only.

Examples

```
rho <- seq(0.2, 1, length=5)
theta <- seq(0, 1.6*pi, length=5)
rt <- expand.grid(theta, rho)
x <- c(0, rt[,2]*cos(rt[,1]))
y <- c(0, rt[,2]*sin(rt[,1]))
gzpm <- gradzpm_cart(x, y)
```

gray256	8 bit Grayscale
---------	-----------------

Description

A vector of gray scale levels

Usage

```
gray256
grey256
```

Value

Defined as `gray256 <- grey(seq(0, 1, length=256))`

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>.

hypot	Hypotenuse
-------	------------

Description

The Euclidean length of a vector

Usage

```
hypot(x)
```

Arguments

x a vector

Value

the length of the vector

Author(s)

M.L. Peck

Examples

```
hypot(c(1,2))
```

idiffpuw

Phase unwrapping by Integrating DIFFerences

Description

Simple path following algorithm for two dimensional phase unwrapping.

Usage

```
idiffpuw(phase, mask = phase, ucall = TRUE, dx = NULL, dy = NULL)
```

Arguments

phase	A matrix of wrapped phase values
mask	Matrix the same size as phase indicating masked pixels
ucall	Boolean: User call?
dx	Matrix of x differences
dy	Matrix of y differences

Details

mask indicates pixels that shouldn't be unwrapped. In the simplest (default) case these are just pixels where phase is undefined.

Value

if(ucall), a matrix of class "[pupil](#)" with unwrapped wavefront values, otherwise a list with items:

puw	Unwrapped phase
uw	Matrix indicating pixels that have been unwrapped.

Note

`brcutpuw` calls `rmap` first to check for the presence of residues. If there are none `idiffpuw` is guaranteed to work and is called to do the phase unwrapping.

If there *are* residues `brcutpuw` creates a mask then calls `idiffpuw` to unwrap unmasked portions of the phase map.

This function is user callable as well; use a call of the form `idiffpuw(phase)`.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>. Thanks to Steve Koehler for programming ideas to considerably speed up the algorithm.

References

Ghiglia, D.C., and Pritt, M.D., 1998, **Two-Dimensional Phase Unwrapping**, New York: Wiley & Sons, Inc., ISBN 0-471-24935-1.

See Also

`rmap`, `brcutpuw`

id_dxy_uw

Compiled code via Rcpp for Itoh's method of phase unwrapping

Description

Called by `brcutpuw()` for fast phase unwrapping

Usage

```
id_dxy_uw(nr, nc, phase, mask, dx, dy, uw)
```

Arguments

nr	number of rows in phase matrix
nc	number of columns in phase matrix
phase	phase matrix converted to vector
mask	matrix of mask values converted to vector
dx	wrapped phase differences in x direction
dy	wrapped phase differences in y direction

Details

This is called by `brcutpuw()` through `idiffpuw()` but is also user callable. Wrapped phase values and differences are divided by 2π before input making the input values in the range $[-1/2, 1/2)$. In `brcutpuw()` the mask indicates areas outside the interferogram area and lines of branch cuts

Value

a vector with the unwrapped phase

Author(s)

M.L. Peck (mlpeck54 -at- gmail.com)

See Also

[brcutpuw\(\)](#), [idiffpuw\(\)](#)

id_uw

Compiled code via Rcpp for Itoh's method of phase unwrapping

Description

Called by [idiffpuw\(\)](#) for fast phase unwrapping

Usage

```
id_uw(nr, nc, phase)
```

Arguments

nr	number of rows in phase matrix
nc	number of columns in phase matrix
phase	phase matrix converted to vector

Details

This is called by [idiffpuw\(\)](#) but is also user callable. Wrapped phase values are divided by 2π before input making the input values in the range $[-1/2, 1/2)$. In [brcutpuw\(\)](#) the mask indicates areas outside the interferogram area and lines of branch cuts

Value

a vector with the unwrapped phase

Author(s)

M.L. Peck (mlpeck54 -at- gmail.com)

See Also

[brcutpuw\(\)](#), [idiffpuw\(\)](#)

load.images	<i>Read images</i>
-------------	--------------------

Description

Loads image files in jpeg, tiff or raw format. load.pgm provides legacy support for reading files in pgm format.

Usage

```
load.images(files, channels=c(1,0,0), scale=1, FLIP=FALSE)
load.pgm(files, imdiff=NULL)
```

Arguments

files	A vector of character strings with file names
channels	channel weights
scale	scale factor for image resize
FLIP	flip image left for right?

Details

set FLIP=TRUE to reverse mirror imaged interferograms.

Any file extension other than jpg, jpeg, tif, tiff is assumed to be in RAW format. Supported raw formats are determined by libraw and may not be up to date

Value

An array containing the contents of the image files.

Note

load.pgm is the original load.images included for legacy support of greyscale portable anymap files.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

`lspsi`*Phase Shifting Interferometry*

Description

Least squares fitting of phase shifted interferograms.

Usage

```
lspsi(images, phases, wt = rep(1, length(phases)))  
lspsiC(images, phases, wt)
```

Arguments

<code>images</code>	An array containing the interferogram images
<code>phases</code>	A vector of phase shifts
<code>wt</code>	A vector of weights

Details

`images` is a 3 dimensional array with dimensions `nrow` x `ncol` x `length(phases)`, where `nrow` and `ncol` are the number of rows and columns in the individual interferogram images.

`lspsi` reshapes the image array into a matrix and calls `lspsiC` which in turn calls the compiled C++ routine.

Value

A list containing the following components:

<code>phi</code>	Estimated wrapped wavefront phase.
<code>mod</code>	Estimated modulation

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

See Also

[psifit](#)

makezlist.iso	<i>Construct list of ZP indexes in ISO/ANSI sequence with sine terms first</i>
---------------	--

Description

Construct list of ZP indexes in ISO/ANSI sequence with sine terms first

Usage

```
makezlist.iso(maxorder = 12)
```

Arguments

maxorder	maximum radial and azimuthal order
----------	------------------------------------

Value

a named list with n=radial indexes, m=azimuthal, and t indicating trig function to apply

Examples

```
zlist.iso <- makezlist.iso(maxorder=6)
zlist <- makezlist(0, 6)
```

mpinv	<i>Moore-Penrose generalized inverse</i>
-------	--

Description

Computes the Moore-Penrose generalized inverse of a matrix using singular value decomposition.

Usage

```
mpinv(X)
```

Arguments

X	A matrix
---	----------

Value

Matrix containing the generalized inverse. If X is an n x m matrix the return will have dimension m x n.

Note

The threshold for determining if a matrix is rank deficient is `eps <- .Machine$double.eps * max(dim(X)) * S$d[1]`

Author(s)

M. L. Peck

Examples

```
X <- matrix(rnorm(18), 6, 3) ## this should be full rank almost always
mpinv(X) %*% X

X <- matrix(1:18, 6, 3) ## this is not
mpinv(X) %*% X
```

norm_zpm

Normalize matrix of Zernike polynomial values.

Description

Convert a matrix of Zernike polynomial values from unit scaled to unit variance aka orthonormal form.

Usage

```
norm_zpm(uzpm, maxorder = 12L)
```

Arguments

uzpm	matrix of Zernike polynomial values
maxorder	the maximum radial order.

Details

This is intended only for ISO/ANSI ordered matrices. The only check performed is that the number of columns in the matrix matches the expected number given by the argument `maxorder`. This is called by `gradzpm_cart()` and `zpm_cart()` if `unit_variance` is set to `true` in the respective function calls.

Value

matrix in orthonormal form.

pcapsi*Vargas et al.'s Principal Components method for PSI*

Description

Compute the phase using the Principal components algorithm.

Usage

```
pcapsi(im.mat, bgsub = TRUE, group_diag = "v")
```

Arguments

im.mat	A <i>matrix</i> of interferogram values
bgsub	Boolean - subtract the pixelwise mean as background estimate?
group_diag	controls treatment of singular values of the data matrix

Details

Images are input into an array by [load.images](#). This must be reshaped into a matrix for this function. Also, a mask should be applied if available prior to the call.

Value

A list containing the following elements:

phi	The wrapped phase estimate. This is a vector as long as the number of rows in im.mat.
mod	Modulation estimate.
phases	Phase shift estimates.
snr	An estimate of the signal to noise ratio in the input data.
eigen	Singular values of the crossproduct matrix.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

References

- J. Vargas, J. Antonio Quiroga, and T. Belenguer, "Phase-shifting interferometry based on principal component analysis," *Opt. Lett.* **36**, 1326-1328 (2011) <http://www.opticsinfobase.org/ol/abstract.cfm?URI=ol-36-8-1326>
- J. Vargas, J. Antonio Quiroga, and T. Belenguer, "Analysis of the principal component algorithm in phase-shifting interferometry," *Opt. Lett.* **36**, 2215-2217 (2011) <http://www.opticsinfobase.org/ol/abstract.cfm?URI=ol-36-12-2215>

See Also[psifit](#),

`pick.sidelobe`*Select an interferogram sidelobe in the Fourier domain*

Description

Interactively locate the center of a first order sidelobe in the FFT of an interferogram, and mark the width of the background filter.

Usage

```
pick.sidelobe(imagedata, logm=FALSE, gamma=3)
```

Arguments

<code>imagedata</code>	A matrix containing an interferogram image
<code>logm</code>	Logical: pass <code>fn="logMod"</code> to plot.cmat ?
<code>gamma</code>	gamma value for display

Details

Uses the basic graphics utility [locator](#).

Value

A list with the following components:

<code>sl</code>	The coordinates $c(x,y)$ of the selected sidelobe
<code>filter</code>	Estimated size of background filter

Note

The high level FFT interferogram analysis routine [fftfit](#) requires the approximate location of the intended first order interferogram sidelobe to be specified.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

See Also[fftfit](#),

plot.cmat	<i>Plot a complex matrix</i>
-----------	------------------------------

Description

Plot a real valued function of a complex matrix

Usage

```
plot.cmat(X, fn = "Mod", col = grey256,  
cp=NULL, zoom=1, gamma=1, ...)
```

Arguments

X	A complex valued matrix
fn	A function returning a real value
col	Color palette for graph
cp	pupil parameters as returned by pupil.pars
zoom	zoom factor for display
gamma	gamma value for display
...	Other parameters to pass to image.default

Details

In addition to the functions described in [complex](#) fn can be assigned the values "logMod", which will call an internally defined function returning the value $\log(1+\text{Mod}(X))$, "Mod2" to plot the power spectrum, and "logMod2" to plot the logarithm of the power spectrum.

If the parameter cp is passed axes will display spatial frequencies in cycles per pupil radius.

Value

none

Note

This is used primarily for displaying FFT's of interferograms. In the case of an interferogram in which the background has not been removed use fn="logMod" to make the first order sidelobes visible.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

See Also

[pick.sidelobe](#), [fftfit](#).

plot.pupil

*Pupils and wavefronts***Description**

Plot and summary methods for objects of class "pupil".

Usage

```
plot.pupil(wf, cp=NULL, col = topo.colors(256), addContours = TRUE, cscale = FALSE,
           eqa=FALSE, zlim=NULL, ...)
summary.pupil(wf)
```

Arguments

wf	An object of class "pupil"
cp	Pupil parameters; a list as returned by pupil.pars
col	Color palette for plot
addContours	Logical: add contour lines?
cscale	Add a color scale legend?
eqa	Perform an "equal area" plot?
zlim	z limits to pass to image
...	Additional parameters to pass to image.default

Details

These give simple plot and summary methods for objects of class [pupil](#).

If eqa is TRUE, each color in the palette will be used for an equal number of pixels (as opposed to representing an equal interval). Note: the color scale (when cscale == TRUE) may be inaccurate if a very small number of colors are used.

Value

none

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

See Also

[pupil](#), [pupilrms](#), [pupilpv](#), [strehlratio](#), [pupil.pars](#).

`plotn`*Wavefront comparison plots*

Description

Plot an arbitrary number of wavefronts and all differences.

Usage

```
plotn(..., labels = NULL, addContours=FALSE, wftype = "net",  
col = rygcb(400), qt = c(0.01, 0.99))
```

Arguments

<code>...</code>	List of wavefront estimates as returned by wf_net .
<code>labels</code>	Labels to identify the wavefronts.
<code>addContours</code>	Boolean to add contours to top row plots
<code>wftype</code>	If the inputs are from <code>wf_net</code> , one of "net", "smooth", "residual".
<code>col</code>	Color palette for top row of plot
<code>qt</code>	Quantiles of differences to plot in comparisons.

Details

`...` can be any number of objects containing wavefront estimates as returned for example by [wf_net](#).

Wavefronts are displayed on the top row, and differences of all pairs on subsequent rows. Grayscale is used to render the difference plots, and the color palette given in `col` is used for the wavefronts.

Value

none

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

See Also

[plot.pupil wf_net](#)

plotxs

Plot cross-sections (profiles) through a wavefront map.

Description

Plots an arbitrary number of cross-sections through a wavefront map, with one highlighted.

Usage

```
plotxs(wf, cp, theta0 = 0, ylim = NULL, N = 4, n = 101,
col0 = "black", col = "gray", lty = 2)
```

Arguments

wf	A matrix of wavefront values.
cp	List of pupil parameters as returned by pupil.pars .
theta0	Angle of highlighted profile, in degrees.
ylim	range of heights to plot.
N	Number of cross sections.
n	Number of points for each cross section.
col0	Highlight color.
col	Cross section color.
lty	Line type for plots.

Details

The cross sections are equally spaced in angle from 0 to $\pi \cdot (N-1)/N$. Any angle can be specified for the highlighted profile at theta0.

Value

none

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

See Also

[plot.pupil](#) is the main wavefront plotting routine.

psfit_options
Options for PSI and FFT based fitting routines

Description

Get and optionally set parameters controlling various aspects of PSI algorithms, Zernike polynomial fitting, and data display

Usage

```
psfit_options(...)
```

Arguments

colors	topo.colors(256)
refine	TRUE
puw_alg	"qual"
fringescale	1
wt	NULL
bgsub	TRUE
maxiter	20
ptol	1e-04
trace	1
nzcs	2
zc0	6:7
satarget	c(0, 0)
astig.bath	c(0, 0)
maxorder	14
uselm	FALSE
isoseq	FALSE
sgs	1
nthreads	parallel::detectCores()/2
plots	TRUE
crop	FALSE

Details

Calling `psifit_options` with an empty argument list returns the default values of the options used in `psifit` and `wf_net` as itemized above. The list can be modified directly or by passing argument value pairs to the function call.

Parameters you might want to change include:

`satarget` sets the target SA for “numerical nulling.” This is a vector of length 2 setting the target values of primary and 5th order SA.

`ptol` sets convergence tolerances for iterative PSI algorithms. These have different definitions and different values may be suitable for different algorithms. A value around 0.01 is appropriate for `tiltpsi`.

The number of variable Zernike terms in the algorithm `tiltpsi` is controlled by `nzcs`. Set it to 3 to include defocus, 5 to include primary astigmatism, 7 to include coma.

`maxorder` sets the maximum Zernike polynomial order for wavefront fitting. It must be even and at least 6. The default generally produces a good wavefront representation but you may want to experiment with higher order fits.

A new and somewhat experimental feature from version 3.6.0 of the package is threaded computation of Zernike polynomial matrices. The number of threads used is set with the option `nthreads`. The default is to use half the number of cores detected because on CPUs that support multi- or hyperthreading the number of cores reported by `detectCores()` is double the number of physical cores. Setting `nthreads = 1` will turn off threading. Different C++ routines are used for the matrix fill in the unthreaded (`zpmC`) and threaded (`zpmCP`) cases. Speed improvements, if any, may vary.

If you don’t like the default color palette there are many other choices. If you like rainbows `rygcb` defined in this package produces a relatively perceptually uniform version that’s well suited for display on an RGB monitor.

Value

A named list with the current values of the arguments.

Author(s)

M.L. Peck <mlpeck54@gmail.com>

psifit

Phase Shifting Interferometry

Description

High level function for Least squares analysis of phase shifted interferograms.

Usage

```
psifit(images, phases, cp = NULL, satarget = NULL, psialg = "ls", options = psifit_options())
```

Arguments

<code>images</code>	An array containing the interferogram images
<code>phases</code>	A vector of phase shifts
<code>cp</code>	A list describing the pupil boundary, as returned by pupil.pars
<code>satarget</code>	Target 4th and 6th order SA coefficients in non-null tests of aspheres
<code>psialg</code>	String identifying the PSI algorithm to use
<code>options</code>	a list of options

Details

`images` is a 3 dimensional array with dimensions `nrow` x `ncol` x `length(phases)`, where `nrow` and `ncol` are the number of rows and columns in the individual interferogram images.

The current values recognized for `psialg` are

ls least squares with known phase shifts

aia the “advanced iterative algorithm” [aiapsi](#)

pc1 pca with `group_diag = "v"`

pc2 pca with `group_diag = "u"`

gpc my generalized PC algorithm in [gpcapsi](#)

gpcthentilt first [gpcapsi](#) the [tiltpsi](#)

tilt [tiltpsi](#)

Value

A list with the following components

<code>phi</code>	wrapped phase estimate
<code>mod</code>	modulation estimate
<code>phases</code>	phase shifts
<code>cp</code>	the interferogram boundary
<code>wf.net</code>	net, unfiltered wavefront (see wf_net)
<code>wf.smooth</code>	Zernike fit wavefront
<code>wf.residual</code>	the difference
<code>fit</code>	Coefficients of Zernike fit to wavefront
<code>zcoef.net</code>	Net Zernike coefficients
<code>extras</code>	any extra data returned by low level functions

Author(s)

M.L. Peck <mlpeck54@gmail.com>

See Also

[lspsi](#), [aiapsi](#), [tiltpsi](#), [gpcapsi](#), [pcapsi](#)

Examples

```
## reuse the files from the demo for an example of two stage fitting
## using gpca then tiltpsi
require(zernike)
fpath <- file.path(find.package(package="zernike"), "psidata")
files <- scan(file.path(fpath, "files.txt"), what="character")
for (i in 1:length(files)) files[i] <- file.path(fpath, files[i])

# load the images into an array

images <- load.images(files)

# parameters for this run

source(file.path(fpath, "parameters.txt"))

# phase shifts

phases <- wrap((0:(dim(images)[3]-1))/frames.per.cycle*2*pi)
phases <- switch(ps.dir, ccw = -phases, cw = phases, phases)

# target SA coefficients for numerical null.

sa.t <- sconic(diam,roc,lambda=wavelength)
zopt <- psfit_options()
zopt$satarget <- sa.t
zopt$ptol <- 0.01
tfit <- psifit(images, phases, psialg="gpcthentilt", options=zopt)
```

pupil

Pupils and wavefronts

Description

Create a pupil object and optionally fill it with a wavefront. For our purposes a “pupil” is defined to be a matrix representation of a circular or annular aperture. Simple plot and summary methods are also provided.

Usage

```
nrow.default <- 640
ncol.default <- nrow.default
cp.default <- list(xc=320.5, yc=320.5, rx=319.5, ry=319.5, obstruct=0)

pupil(zcoef=NULL, maxorder=14L, isoseq=FALSE,
      phi=0, piston=0,
      nrow=nrow.default, ncol=ncol.default,
      cp=cp.default)
pupil.arb(zcoef=NULL, zlist=makezlist(),
```

```
phi=0, piston=0,
nrow=640, ncol=nrow,
Cp=list(xc=320.5,yc=320.5,rx=319.5,ry=319.5,obstruct=0))
```

Arguments

zcoef	A vector of Zernike coefficients
maxorder	Maximum Zernike polynomial order
isoseq	ZPs in ISO/ANSI sequence
zlist	List of indexes the same length as zcoef
phi	Amount to rotate image, in degrees
piston	Constant to add to wavefront values
nrow	Number of rows in output matrix
ncol	Number of columns in output matrix
cp	A list with items xc - x coordinate of central pixel, yc - y coordinate of central pixel, rx - x radius in pixels, ry - y radius in pixels, obstruct - central obstruction fraction.

Details

[plot.pupil](#) and [summary.pupil](#) provide simple plot and summary methods for objects of class "pupil".

`pupil.arb` will accept an arbitrary list of Zernike indexes.

`pupil` requires a complete set of Zernikes as returned by [zpm](#) or [zpm_cart](#).

Default values `nrow.default`, `ncol.default` and `cp.default` are now (as of version 3.7.0) stored in the package environment. These can be used or overridden by setting values in the user's global environment.

Value

A matrix of size `nrow` x `ncol`. The matrix is assigned to the class "pupil". NAs are used to fill the matrix outside the defined area of the pupil.

Note

The parameter `cp` is used to define the dimensions of the pupil. See [pupil.pars](#) for details.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

See Also

[Zernike](#), [makezlist](#), [pupilrms](#), [pupilpv](#), [strehlratio](#), [pupil.pars](#), [circle.pars](#).

Examples

```
wf <- pupil(zcoef=rnorm(length(makezlist(0, 14)$n), 0, 0.01))
plot(wf, addContours=FALSE)
summary(wf)
```

pupil.pars

*Pupil parameters***Description**

Interactively determine the center, radius, and obstruction fraction of a circular or annular interferogram image.

Usage

```
pupil.pars(im = NULL, obstructed = FALSE)
```

Arguments

im	A matrix containing an interferogram image
obstructed	Logical: is there a central obstruction?

Details

In `pupil.pars`, if the image has already been plotted `im` can be `NULL`, which is the default.

Value

A list with the following components:

xc	X coordinate of the center of the pupil
yc	Y coordinate of the center of the pupil
rx	Horizontal radius of the pupil
ry	Vertical radius of the pupil
obstruct	Obstruction fraction

Note

`pupil.pars` uses the basic graphics library routine [locator](#) to interactively mark the edge of the pupil, and optionally the edge of the obstruction. After right clicking to terminate `locator()` a least squares fit is performed to the marked points to determine the center and radius of the pupil.

Note that all routines that make use of Zernikes implicitly assume a circular pupil, or an annular one with small obstruction. We allow $rx \neq ry$ for imaging sensors with non-square aspect ratios.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

See Also

Many routines require the pupil parameters in the form returned by `pupil.pars`. For example `psifit`, `fftfit`, `pupil`, etc.

pupil.rhotheta	<i>Polar coordinates</i>
----------------	--------------------------

Description

Calculate matrixes of polar coordinates for `pupil`'s.

Usage

```
pupil.rhotheta(nrow, ncol, cp)
```

Arguments

nrow	Number of rows in interferogram images
ncol	Number of columns in interferogram images
cp	A list describing the pupil boundary, as returned by <code>pupil.pars</code>

Value

A list with the following components:

rho	A matrix of radial coordinates
theta	A matrix of angular coordinates

Note

My Zernike polynomial routines work in polar coordinates, which this function provides. Also, NA's are used to fill the matrix outside the pupil boundary, making the returned values convenient for selecting pixels inside interferograms.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

See Also

`Zernike`, `pupil`.

pupilrms

*Wavefront statistics***Description**

Compute basic statistics of wavefronts stored in "pupil" objects.

Usage

```
pupilrms(pupil)
pupilpv(pupil)
strehlratio(rms)
```

Arguments

pupil	A matrix of class "pupil"
rms	An rms wavefront error

Value

Estimates of the RMS and P-V wavefront errors. strehlratio calculates Mahajan's approximation to the Strehl ratio.

Note

pupilrms simply calculates the standard deviation of finite values in the matrix pupil. This is a crude, but usually accurate enough estimate of the true RMS wavefront error.

[summary.pupil](#) calls these functions.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

References

Schroeder, D.J. 2000, *Astronomical Optics, 2nd Edition*, Academic Press, chapter 10.

See Also

[summary.pupil](#).

Examples

```
zcoef <- rnorm(length(makezlist())$n), 0, 0.01)
wf <- pupil(zcoef=zcoef)
plot(wf)
summary(wf)
sqrt(crossprod(zcoef)) # A more accurate estimate of RMS
```

PVr	<i>Zygo's "robust" PV</i>
-----	---------------------------

Description

A peak to valley error estimate that reduces the effect of noise and artifacts

Usage

```
PVr(wf.zfit, wf.residual)
```

Arguments

wf.zfit	matrix containing the smoothed Zernike fit wavefront
wf.residual	matrix of the difference between the raw wavefront and the Zernike fit. These values are returned by <code>wf_net()</code>

Details

no check is performed on the wavefronts, so it's the user's responsibility to make sure these come from the same source

Value

the estimated PVr

References

Evans, C. (2009) Optical Engineering 48(4), 43605. <https://doi.org/10.1117/1.3119307>

qpuw	<i>Quality guided algorithm for phase unwrapping</i>
------	--

Description

Quality guided algorithm for two dimensional phase unwrapping.

Usage

```
qpuw(phase, qual)
```

Arguments

phase	A matrix of wrapped phase values
qual	A matrix of quality values the same size as phase.

Value

puw A matrix of class "[pupil](#)" with the unwrapped wavefront.

Note

This is a straightforward implementation of the quality guided algorithm of G&P.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

References

Ghiglia, D.C., and Pritt, M.D., 1998, **Two-Dimensional Phase Unwrapping**, New York: Wiley & Sons, Inc., ISBN 0-471-24935-1.

See Also

[idiffpuw](#), [brcutpuw](#)

q_uw

Compiled code via Rcpp for quality guided phase unwrapping

Description

Called by [qpuw\(\)](#) for fast quality guided phase unwrapping

Usage

```
q_uw(nr, nc, phase, qual)
```

Arguments

nr	number of rows in phase matrix
nc	number of columns in phase matrix
phase	phase matrix converted to vector
qual	quality matrix converted to vector

Details

This is called by [qpuw\(\)](#) but is also user callable. Wrapped phase values are divided by 2π before input making the input values in the range $[-1/2, 1/2)$.

Value

a vector with the unwrapped phase

Author(s)

M.L. Peck (mlpeck54 -at- gmail.com) with valuable programming advice from Steve Koehler

See Also

[qpuw\(\)](#), [idiffpuw\(\)](#)

readjpeg

Read a jpeg or tiff file

Description

Reads a jpeg or tiff file and combines the channels to produce a monochrome image in a matrix.

Usage

```
readjpeg(filename, channels)
readtiff(filename, channels)
```

Arguments

filename	File name
channels	A vector of length 3 with the channel weights

Details

Values in channels should be non-negative, but need not add to one.

Value

A double precision matrix with the image data.

Note

The matrix must have rows reversed and transposed to display properly with `image()`.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

rescale	<i>Rescale an image.</i>
---------	--------------------------

Description

Rescale a matrix containing a bitmapped image using bilinear interpolation.

Usage

```
rescale(im, scale)
```

Arguments

im	A matrix with image data.
scale	Scale factor.

Details

A value <1 will shrink the image.

Value

A matrix containing the rescaled image data.

Note

NA's are OK.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

See Also

Called by [load.images](#) if necessary.

`rmap`*Utilities for phase unwrapping*

Description

Utility functions for use in 2D phase unwrapping.

Usage

```
rmap(phase, dx = NULL, dy = NULL, plot = FALSE, ...)  
wrap(phase)
```

Arguments

<code>phase</code>	Matrix of wrapped phase values
<code>dx</code>	Matrix of x differences
<code>dy</code>	Matrix of y differences
<code>plot</code>	Boolean: plot residue positions?
<code>...</code>	additional arguments for image

Details

`dx` and `dy` must have the same dimension as `phase`.

Value

In `rmap` if `plot == TRUE`

<code>nr</code>	the number of residues identified in the map
-----------------	--

otherwise

<code>phase</code>	wrapped phase returned by <code>wrap</code>
--------------------	---

<code>residues</code>	Matrix the same size as <code>phase</code> with residues marked as + or - 1.
-----------------------	--

Note

These are primarily intended for internal use but can be used interactively. Calling `rmap(phase, plot=TRUE)` will plot the positions of residues and return nothing. If `(plot==FALSE)` in the call to `rmap` a matrix the same size as `phase` is returned with residues identified with values of +1 or -1.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>. Steve Koehler is responsible for the efficient implementation of the `wrap` function.

References

Ghiglia, D.C., and Pritt, M.D., 1998, **Two-Dimensional Phase Unwrapping**, New York: Wiley & Sons, Inc., ISBN 0-471-24935-1.

See Also

Called by [brcutpuw](#).

rygcb

A better rainbow.

Description

Produces a rainbow color palette with colors ranging from "red" to "blue" or "magenta". Perceptual uniformity should be superior to R's rainbow.

Usage

```
rygcb(n)  
rygcbm(n)
```

Arguments

n	Number of color levels
---	------------------------

Details

The palette is created using colorRampPalette.

Value

A vector of colors.

Note

The call to colorRampPalette sets space="Lab" and interpolate="spline" with the intent of creating a more perceptually uniform rainbow.

Author(s)

M.L. Peck

See Also

[grey256](#)

Examples

```
plotsp <- function(spectrum) {
  sl <- length(spectrum)
  rgbv <- col2rgb(spectrum)
  plot((0:(sl-1))+0.5, rgbv[1,], type="l", col="red", xlim=c(0,sl),ylim=c(0,300),xlab="Index",ylab="Channel value")
  points((0:(sl-1))+0.5, rgbv[2,], type="l", col="green")
  points((0:(sl-1))+0.5, rgbv[3,], type="l", col="blue")
  grid()
  rect(0:(sl-1), 260, 1:sl, 300, col=spectrum, density=NA)
}
plotsp(rygcb(400))
X11()
plotsp(rygcbm(500))
```

sconic	<i>Sconic</i>
--------	---------------

Description

twice the radial height difference between a sphere and conic surface

Usage

```
sconic(D, rc, b = -1, lambda = 632.8, nmax = 6)
```

Arguments

D	Diameter (mm)
rc	Radius of curvature (mm)
b	conic constant
lambda	source or test wavelength (nm)
nmax	maximum polynomial order

Value

Zernike polynomial coefficients

Note

This estimates twice the radial distance between a sphere and conic surface with same paraxial radius of curvature, and returns Zernike coefficients of polynomial expansion. Intended for “numerical nulling” when testing an asphere at center of curvature, and should be more accurate than the vertical height difference calculated by [zconic](#) for that purpose.

Author(s)

M.L. Peck

See Also

[zconic](#)

Examples

```
2.*zconic(1000,5000)
sconic(1000,5000)
```

separate.wf	<i>Separate wavefronts</i>
-------------	----------------------------

Description

Separate “polished in” from “instrumental” aberrations if possible

Usage

```
separate.wf(zcm, theta, maxorder = 14)
```

Arguments

zcm	Matrix of observed Zernike coefficients
theta	Vector of rotation angles (in radians)
maxorder	Maximum Zernike order to extract

Value

zcb	Table of extracted coefficients and standard errors
-----	---

Author(s)

M.L. Peck

startest	<i>Star test simulator</i>
----------	----------------------------

Description

Simulates a star test.

Usage

```
startest(wf=NULL, zcoef=NULL, maxorder=14L, phi=0,
lambda = 1, defocus=5, cp=NULL,
obstruct=NULL, npad = 4,
gamma=2, psfmag=2, displaymtf=TRUE, displaywf=FALSE)
```


Arguments

wf	A matrix of class pupil containing wavefront values
zcoef	Vector of Zernike coefficients
maxorder	maximum Zernike polynomial order
phi	Angle to rotate wavefront
lambda	Wavelength, in same units as coefficients
defocus	Amount of defocus in waves
cp	pupil parameters
obstruct	Obstruction fraction
npad	Pad factor for FFT
gamma	Gamma value for graphics display
psfmag	Magnification factor for in focus PSF display
displaymtf	Logical: Display MTF?
displaywf	Logical: Display calculated wavefront?

Details

If wf is NULL the wavefront is calculated from the the Zernike coefficients (which should be non-NULL).

Value

A list with the following components:

psf	The in focus point spread function.
otf	The complex optical transfer function, a complex matrix of size pupilsize.
mtf	The modulation transfer function, a real matrix of size pupilsize.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

References

- Born, M. and Wolf, E. 1999, *Principles of Optics, 7th Edition*, Cambridge University Press.
Suiter, H. R., 1994, *Star Testing Astronomical Telescopes*, Willman-Bell, Inc.

See Also

[Zernike](#), [pupil](#).

summary.wf_fitted	<i>Methods for class "wf_fitted"</i>
-------------------	--------------------------------------

Description

Summary, print, and plot methods for the returned list of values from [psifit\(\)](#), [fftfit\(\)](#), or [vortexfit\(\)](#)

Usage

```
## S3 method for class 'wf_fitted'
summary(wffit, digits = 3)
```

Arguments

wffit	the return values from one of the fringe analysis routines
digits	number of digits to display in print and summary methods
...	values passed to plot.pupil()

Value

print method returns data frame with Zernike coefficients

synth.interferogram	<i>Synthetic interferogram</i>
---------------------	--------------------------------

Description

Compute and display a synthetic interferogram.

Usage

```
synth.interferogram(wf = NULL, zcoef = NULL, maxorder = NULL,
  nr = nrow(wf), nc = ncol(wf), cp = NULL,
  phi = 0, addzc = rep(0, 4), fringescale = 1,
  plots = TRUE)
```

Arguments

wf	A matrix of wavefront values
zcoef	A vector of Zernike coefficients
maxorder	Maximum Zernike polynomial order
nr	Number of rows in the output matrix
nc	Number of columns in the output matrix

cp	A list describing the pupil boundaries, as created by pupil.pars
phi	Amount to rotate the wavefront, in degrees
addzc	A 4-vector with piston, tilt, and defocus terms to be added
fringescale	Fringe scale. Should be 1 for single pass, 0.5 for double, etc.
plots	Logical: Plot the interferogram?

Details

Either wf or zcoef must be non-null, but not both. If zcoef is specified maxorder must be as well. Additional piston, tilt, and defocus terms can be added to the calculated wavefront using addzc.

Value

A matrix of intensity levels in the calculated interferogram, assigned class "[pupil](#)".

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

See Also

[pupil.](#)

Examples

```
# create a list of zernikes
zcoef <- rnorm(length(zlist.fr$n), mean=0, sd=0.01)

iwf <- synth.interferogram(zcoef=zcoef, maxorder=12)

X11()

# show again with some tilt
iwf <- synth.interferogram(zcoef=zcoef, maxorder=12, addzc=c(0,5,5,0))
```

turbwf

Kolmogorov Turbulence

Description

Simulates the optical effects of atmospheric turbulence using Noll's (1976) calculation of the covariance matrix of Zernike polynomials under Kolmogorov turbulence.

Usage

```
turbwf(friedratio = 1, zlist = makezlist(2, 40), reps = 1)
```

Arguments

friedratio	Ratio of pupil diameter to Fried parameter
zlist	A list of Zernikes, as returned for example by makezlist
reps	Number of draws to simulate

Details

The default value of zlist has 440 elements, which may be more than necessary for a reasonable representation of an “atmospheric” wavefront.

Value

A list with the following components:

zcoef.turb	A reps x length(zlist\$n matrix of simulated draws of Zernike coefficients.
V	Covariance matrix of the indexed Zernikes.

Note

Typos in the original source material have been corrected in the code. Note that scintillation is not modelled.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

References

Noll, R.J. 1976, **Zernike polynomials and atmospheric turbulence**, *J. Opt. Soc. Am.*, Vol. 66, No. 3, p. 207.

See Also

[Zernike](#), [pupil](#).

Examples

```
# Simulate a single draw from a turbulent atmosphere
zcoef.turb <- turbwf(friedratio=5, zlist=makezlist(2,30), reps=1)$zcoef.turb
# Warning: this can take a while
wf <- pupil(zcoef=zcoef.turb, zlist=makezlist(2,30))
plot(wf)
summary(wf)
```

vortexfit	<i>Vortex transform.</i>
-----------	--------------------------

Description

Fringe analysis by Vortex aka Spiral Quadrature transform.

Usage

```
vortexfit(  
  imagedata,  
  cp = NULL,  
  filter = NULL,  
  fw.o = 10,  
  options = psfit_options()  
)
```

Arguments

imagedata	matrix containing the interferogram data
cp	list with circle parameters describing interferogram location. Defaults to NULL
filter	size of filter to remove background
fw.o	size of gaussian blur to smooth orientation estimate
options	A list with general fitting and display options. See psfit_options() .

Details

Implements the Vortex or spiral phase quadrature transform method of Larkin et al. (2001) <https://doi.org/10.1364/JOSAA.18> including the fringe orientation estimation approach in Larkin (2005) <https://doi.org/10.1364/OPEX.13.008097>. Thanks to Steve Koehler for ideas on implementation details.

Value

a list with wavefront estimates, wrapped phase, modulation, etc.

Warning

This routine is offered as is with no license, as it may be in violation of one or more US and international patents.

See Also

This is one of two routines provided for analysis of single interferograms, along with [fftfit\(\)](#). This *may* be suitable for interferograms with closed fringes.

Examples

```
require(zernike)
fpath <- file.path(find.package(package="zernike"), "psidata")
fname <- "Image197.jpg"
img <- load.images(file.path(fpath, fname))

# parameters for this run

source(file.path(fpath, "parameters.txt"))

# target SA coefficients for numerical null.

sa.t <- sconic(diam,roc,lambda=wavelength)
zopt <- psfit_options()
zopt$satarget <- sa.t

# display an interferogram

if (tolower(.Platform$OS.type) == "windows") windows() else x11()
image(1:nrow(img), 1:ncol(img), img, col=grey256, asp=1,
      xlab="X", ylab="Y", useRaster=TRUE)
mtext("Sample Interferogram")

if (tolower(.Platform$OS.type) == "windows") windows() else x11()
vfit <- vortexfit(img, filter=15, fw.o=10, options=zopt)
```

wf3d.pupil

OpenGL wavefront plot

Description

Interactive plot of a wavefront using the OpenGL package **rgl**. This is a 3D plotting method for objects of class "[pupil](#)".

Usage

```
wf3d.pupil(wf, cp=NULL, zoom.wf = 1, surf.col = topo.colors(256), bg.col = "black",
           eqa=FALSE)
```

Arguments

wf	A matrix of wavefront values
cp	A list describing the pupil boundary
zoom.wf	Zoom factor for heights
surf.col	Color palette for surface
bg.col	Background color
eqa	Equal area per color?

Details

The default color palette will match the colors in the default version of [plot.pupil](#).

Value

none

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

References

The **rgl** package is described at <http://rgl.neoscientists.org/about.shtml>, and available from CRAN.

See Also

[plot.pupil](#)

Examples

```
# create a random wavefront

wf <- pupil(zcoef=rnorm(length(makezlist())$n), mean=0, sd=0.01))
# the default method

plot(wf)

#this is more fun

wf3d(wf)
```

wf_net

Wavefront smoothing

Description

Calculate net and smoothed wavefronts from a raw wavefront containing low order nuisance aberrations.

Usage

```
wf_net(wf.raw, cp, options)
```

Arguments

wf.raw	Raw wavefront to be processed
cp	a list describing the pupil boundary
options	a list of options. See psfit_options

Details

Called by [psifit](#)

Value

A list with the following components:

wf.net	Net unsmoothed wavefront; a matrix of class " pupil "
wf.smooth	Net smoothed wavefront
wf.residual	Difference between net wavefront and polynomial fit
fit	Return value from fitzernikes
zcoef.net	Net Zernike coefficients from fit

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

zconic	<i>Zernike coefficients for a conic surface</i>
--------	---

Description

Calculates the radially symmetric Zernike coefficient values up to order nmax for a conic surface relative to a sphere of the same paraxial radius of curvature.

Usage

```
zconic(D, rc, b = -1, lambda = 1e-06, nmax = 6)
```

Arguments

D	Diameter
rc	Radius of curvature
b	Conic constant
lambda	Wavelength – defaults to 1 nm.
nmax	Maximum radial polynomial order

Details

D, rc, and lambda must have the same units.

Value

A vector of length $n_{\max}/2-1$ of coefficient values, in increasing radial order, $n=c(4,6, \dots)$.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

See Also

[Zernike](#)

Examples

```
zconic(200,2000)
zconic(10, 20, b=-1.05, lambda=632.8E-9, nmax=12)
```

Zernike

Zernike Polynomials

Description

Routines for creating and manipulating Zernike polynomials.

Usage

```
Zernike(rho, theta, n, m, t)
rzernike(rho, n, m)
drzernike(rho, n, m)
```

Arguments

rho	normalized radius, $0 \leq \rho \leq 1$
theta	angular coordinate
n	radial polynomial order
m	azimuthal order
t	character for trig function: one of c("n", "c", "s")

Note

These functions return Zernikes scaled such that they form an orthonormal basis set for the space of functions defined on the unit circle. Note that this is not the most commonly used definition (as given e.g. in *Born and Wolf*). The definition I use is often associated with *Noll (1976)*.

The function `zmult` can be used to convert between normalized and conventionally defined vectors of Zernike coefficients.

The basic low level functions `rzernike` and `drzernike` use numerically stable recurrence relationships for the radial Zernikes.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

References

Born, M. and Wolf, E. 1999, *Principles of Optics, 7th Edition*, Cambridge University Press, chapter 9 and appendix VII.

Noll, R.J. 1976, **Zernike polynomials and atmospheric turbulence**, *J. Opt. Soc. Am.*, Vol. 66, No. 3, p. 207.

<http://wyant.opt-sci.arizona.edu/zernikes/zernikes.htm>

<http://mathworld.wolfram.com/ZernikePolynomial.html>

See Also

`makezlist`, `zlist.fr`, `zmult`, `zpm`, `pupil`, `pupilrms`, `pupilpv`, `strehlratio`.

Examples

```
Zernike(1, 0, 4, 0, "n") # == sqrt(5)

# A slightly more complex example

rho <- seq(0, 1, length = 101)
theta <- rep(0, 101)

plot(rho, Zernike(rho, theta, 6, 0, "n"), type="l",
     ylim=c(-3.5,3.5), main="Some 6th order Zernike Polynomials")
lines(rho, Zernike(rho, theta, 5, 1, "c"), lty=2)
lines(rho, Zernike(rho, theta, 4, 2, "c"), lty=3)
lines(rho, Zernike(rho, theta, 3, 3, "c"), lty=4)
```

zlist	<i>Lists of Zernike polynomial indexes</i>
-------	--

Description

Ordered lists of Zernike polynomial indexes.

Usage

```
makezlist(minorder = 2, maxorder = 14)
zlist.fr
zmult(zlist = makezlist())
```

Arguments

minorder	minimum value of $n+m$
maxorder	maximum value of $n+m$
zlist	a list of the form returned by makezlist

Details

Zernike polynomials are indexed by a radial index n , an azimuthal index m , and include cosine, sine, and radial terms. These routines return lists of indexes using a popular ordering scheme for Zernike polynomials.

Value

makezlist and zlist.fr return lists with the following components:

n	radial order
m	azimuthal order
t	one of c("c", "s", "n")

zmult returns a vector the same length as the components of zlist.

Note

zlist.fr is an augmented “Fringe” set of Zernike polynomials equivalent to makezlist(2,12).

makezlist returns a complete list of indexes for all orders from minorder through maxorder, where “order” is the value of $n+m$.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

See Also

Virtually all high level functions that work with Zernike polynomials use these lists. See for example [pupil](#), [psifit](#), [fftfit](#).

Examples

```
zlist <- makezlist(2,12)
zcoef <- rnorm(length(zlist))
zcoef # a vector of normalized Zernike coefficients
zcoef*zmult(zlist) # Coefficients in conventional representation
sqrt(crossprod(zcoef)) # This is the RMS error of the wavefront
# constructed from these Zernikes
```

zmoments	<i>Zernike moments</i>
----------	------------------------

Description

Calculate Zernike moments from a vector of coefficients

Usage

```
zmoments(zcoef, maxorder = 14)
```

Arguments

zcoef	Zernike coefficients
maxorder	Maximum order to return

Value

A table of the moments along with radial and azimuthal orders

References

M.L. Peck

zpm

*Matrixes of Zernike polynomials***Description**

Create a matrix of Zernike polynomial values.

Usage

```
zpm(rho, theta, phi= 0 , maxorder = 14, nthreads=parallel::detectCores()/2)
zpmC(rho, theta, maxorder)
zpmCP(rho, theta, maxorder)
zpm.arb(rho, theta, phi = 0, zlist = makezlist())
```

Arguments

rho	A vector of radial coordinates.
theta	A vector of angular coordinates, in radians.
phi	Orientation of the image, in degrees
zlist	A list of indexes, as returned by makezlist
maxorder	The maximum Zernike polynomial order
nthreads	Number of threads for threaded function call

Details

rho and theta must be the same length.

Value

zpm.arb returns a matrix of size $\text{length}(\text{rho}) \times \text{length}(\text{zlist}\$n)$ with values of Zernike polynomials evaluated at the polar coordinates $(\text{rho}, \text{theta} - \pi \cdot \text{phi} / 180)$.

zpm, zpmC, and zpmCP return a matrix of size $\text{length}(\text{rho}) \times (\text{maxorder}/2+1)^2$ of Zernike polynomial values including a piston term.

Note

These are used by various routines to make least squares fits of sets of Zernike polynomials to measured wavefront values.

zpmC is the C++ routine that does the computations for zpm. No column names are returned.

Threaded computation of the matrix is now available using zpmCP.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

See Also

[Zernike](#), [makezlist](#), [zlist.fr](#), [fitzernikes](#)

zpm_cart	<i>Calculate Zernike polynomial values in ISO/ANSI sequence for a set of Cartesian coordinates.</i>
----------	---

Description

Calculate Zernike polynomial values in ISO/ANSI sequence for a set of Cartesian coordinates.

Usage

```
zpm_cart(x, y, maxorder = 12L, unit_variance = TRUE)
```

Arguments

x	a vector of x coordinates for points on a unit disk.
y	a vector of y coordinates.
maxorder	the maximum radial polynomial order (defaults to 12).
unit_variance	logical: return with orthonormal scaling? (default false)

Details

This is the same algorithm and essentially the same code as [gradzpm_cart\(\)](#) except directional derivatives aren't calculated.

Value

a matrix of Zernike polynomial values evaluated at the input Cartesian coordinates and all radial and azimuthal orders from 0 through maxorder.

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