Review

- Breadth-first traversal
  - Implementation using FIFO queue (deque in Python)

- Depth-first traversal
  - Implementation using recursion (or LIFO stack – also using deque in Python)
  - Three types for different purposes
    - Preorder
    - Inorder
    - Postorder

Graphs

Lecture 18

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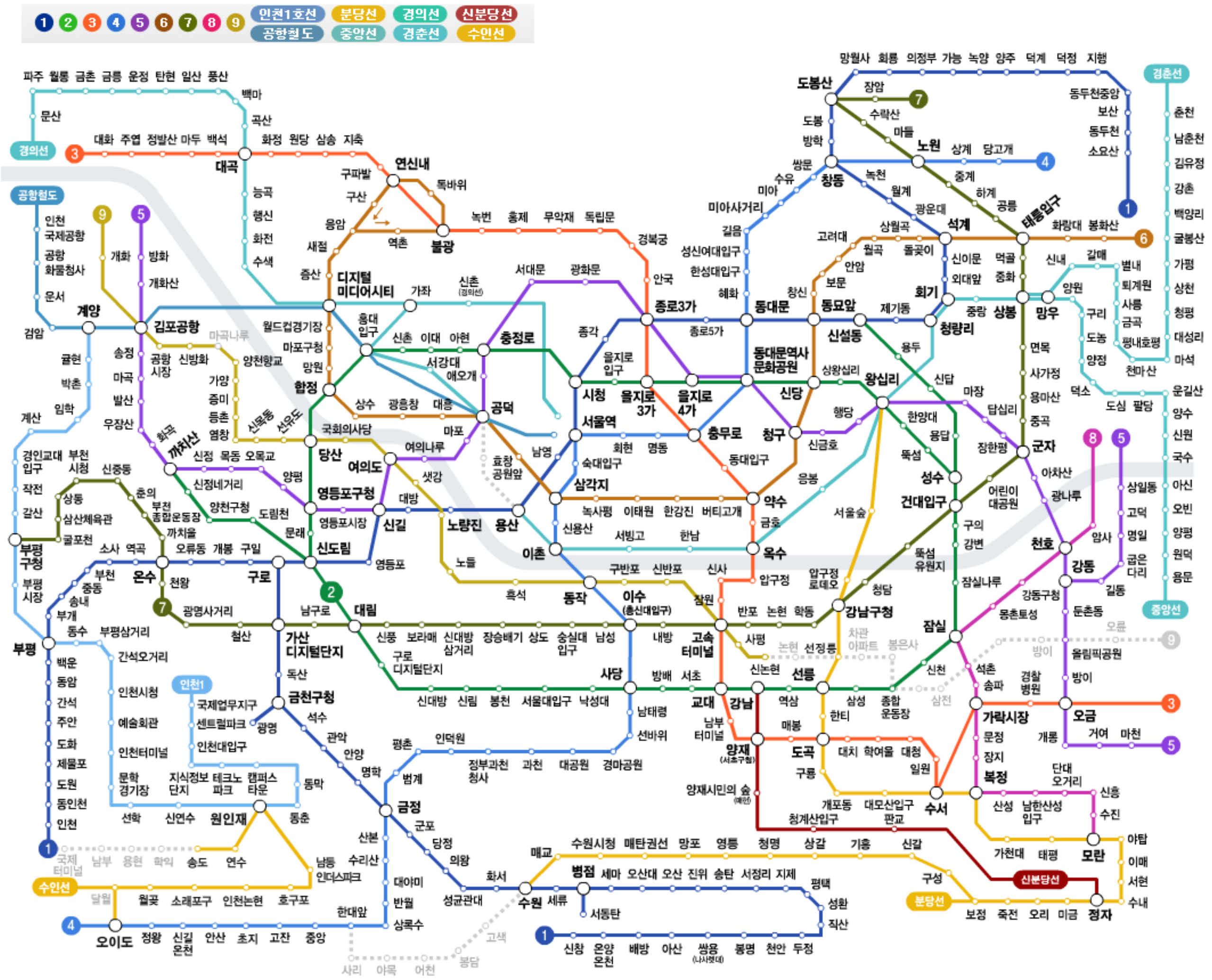
Contents

Graph

Computing Bootcamp

Trees are Great but…

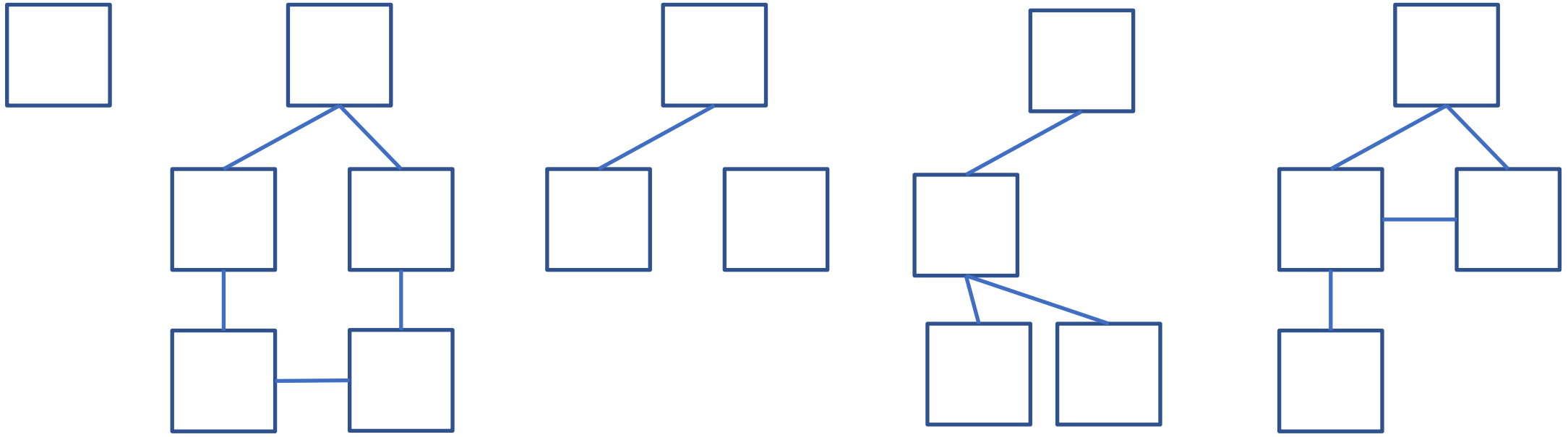
- Trees are good for representing data with hierarchy
- Is Seoul Metro map hierarchical?
- When you want to travel from station A to station B, is there usually only one path?



Trees

- A tree comprises a set of **nodes** that are **connected (linked)** to each other
- There is **only one path** between two nodes in a tree

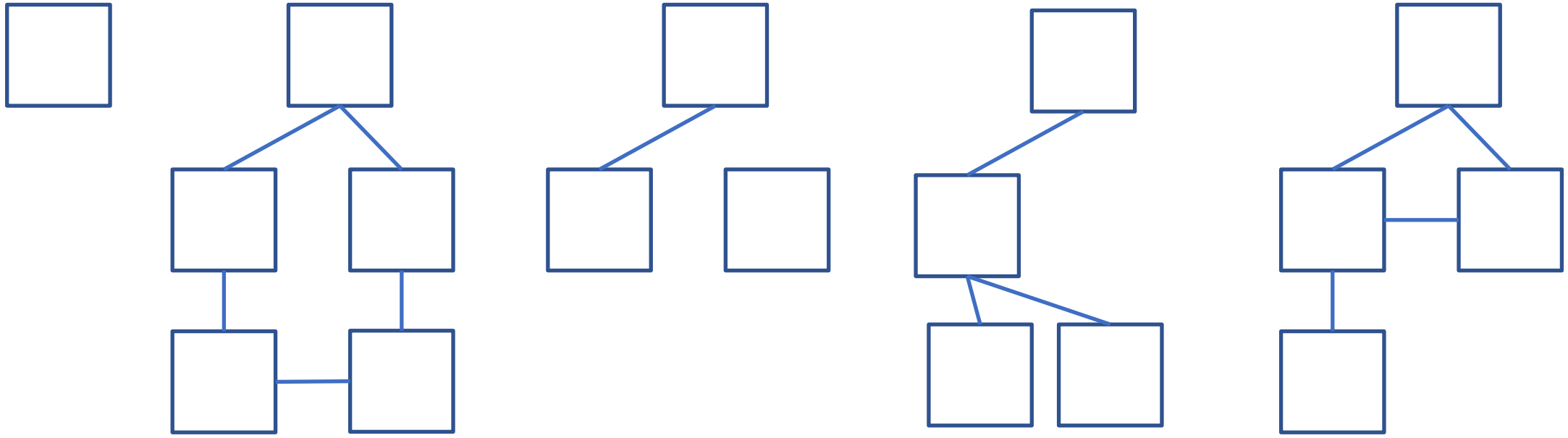
- Choose trees!



Graphs

- A graph comprises a set of **nodes** and a set of **edges** (including zero edge), each of which connects two nodes

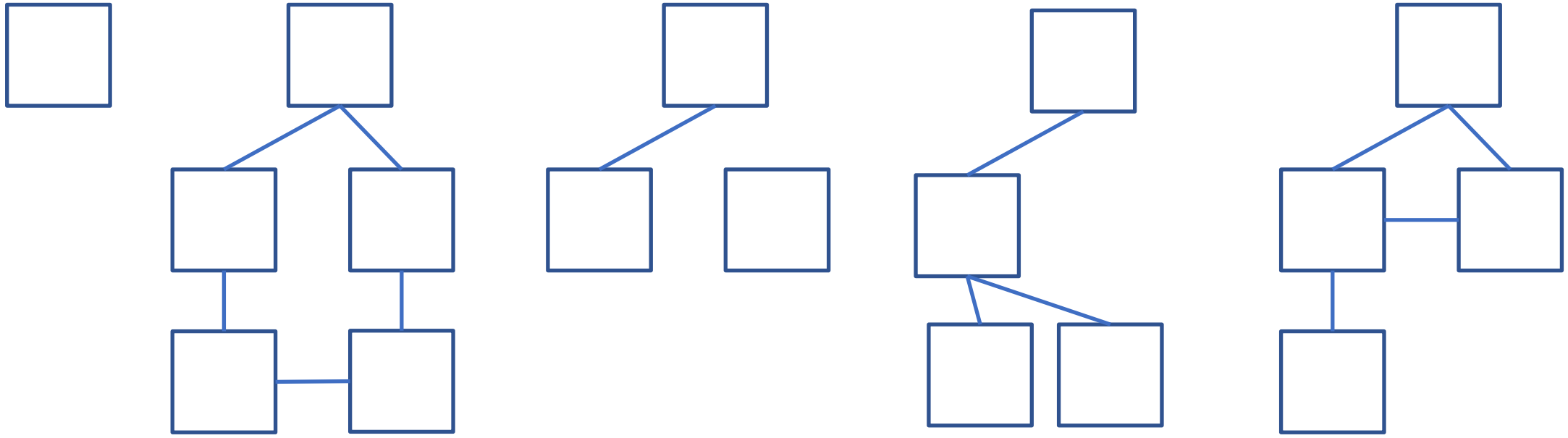
- Choose graphs!



Graphs

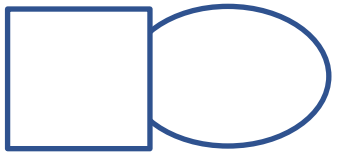
- A graph comprises a set of **nodes** and a set of **edges** (including zero edge), each of which connects two nodes

- Choose graphs!
  - Yes, graph is more general than tree (all trees are graphs)

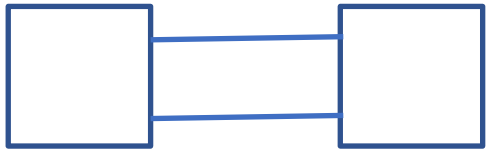


Simple Graphs

- A **simple** graph is a graph that does not have…
  - **Loop**: an edge that connects a node to itself node, such as (v,v)



  - **Parallel edge**: two edges that connects the same nodes, such as e1= (v,w) and e2 = (v,w)



- We only focus on simple graphs today ☺

Terms and Terms …

- **Vertex**: a node
  - v

- **Edge**: a pair of vertices
  - (v,w) where v and w are vertices

- Two vertices which have an edge between are **adjacent**
  - If a graph has an edge (v,w), vertices v and w are adjacent

- A **path** is a sequence of vertices connected by edges
  - If there are three edges in a graph, (v,w), (w,y), and (y,z), the graph has a path (v,w,y,z)

Terms and Terms …

- A path with unique (non-repeated) vertices is a **simple** path
  - (v,w,y,v,z) is not simple, but (v,w,y,z) is simple

- A **cycle** is a path where the first and last vertices are the same
  - A path (v,w,y,v) is a cycle. Another path (v,w,y,z) is not a cycle
  - A graph having a cycle is a **cyclic** graph
  - A graph without a cycle is an **acyclic** graph

- If there is a path between two vertices, the two vertices are **connected**
  - If there is a path (v,w,y,z), vertices v and z are connected

Terms and Terms …

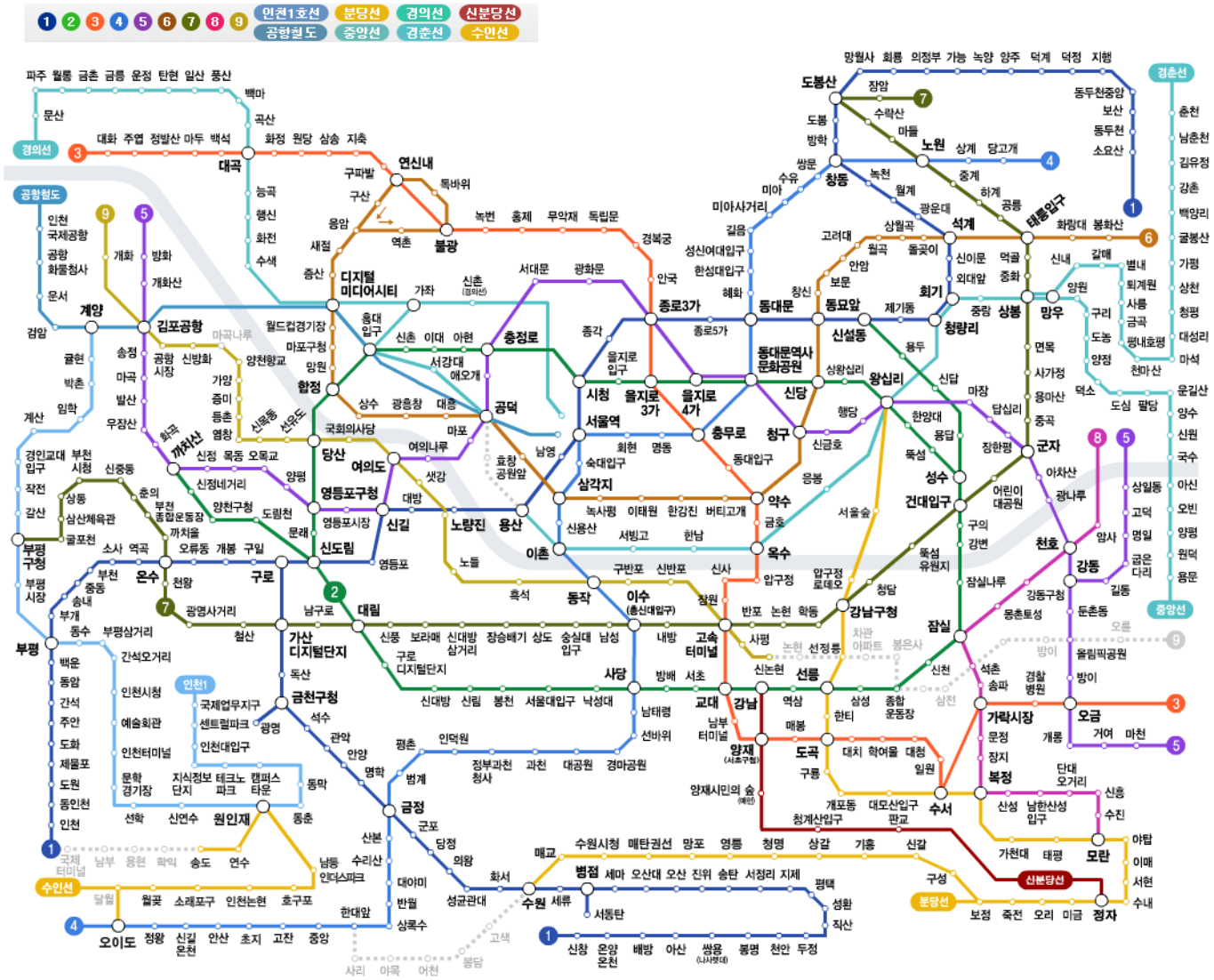
- An edge is either **undirected** (like a line) or **directed** (like an arrow)
  - If an edge (v,w) is undirected, v and w can reach each other
  - If an edge (v,w) is directed, v can go to w, not vice versa
    - We need another edge (w,v) so that **w** can go to **v**

- A graph with undirected edges is an **undirected graph**

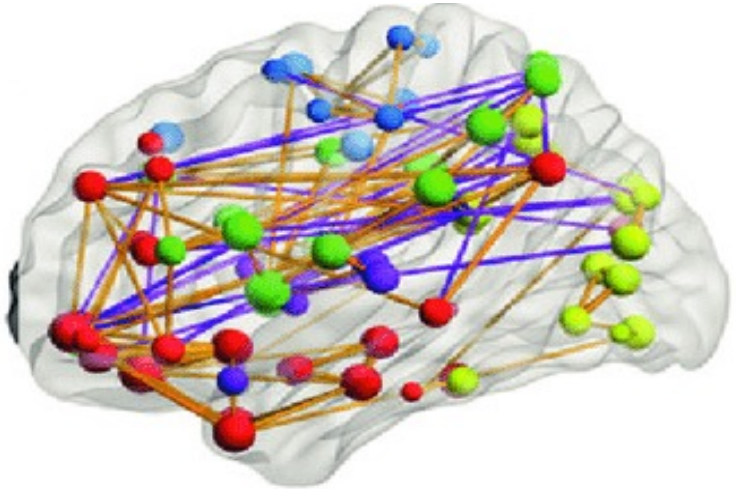
- A graph with directed edges is a **directed graph**

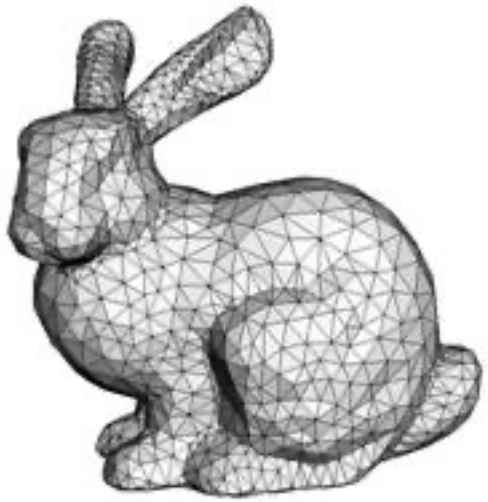
So… What is this?

- Tree or graph?
- Connected or not?
- Cyclic or acyclic?
- Undirected or directed?

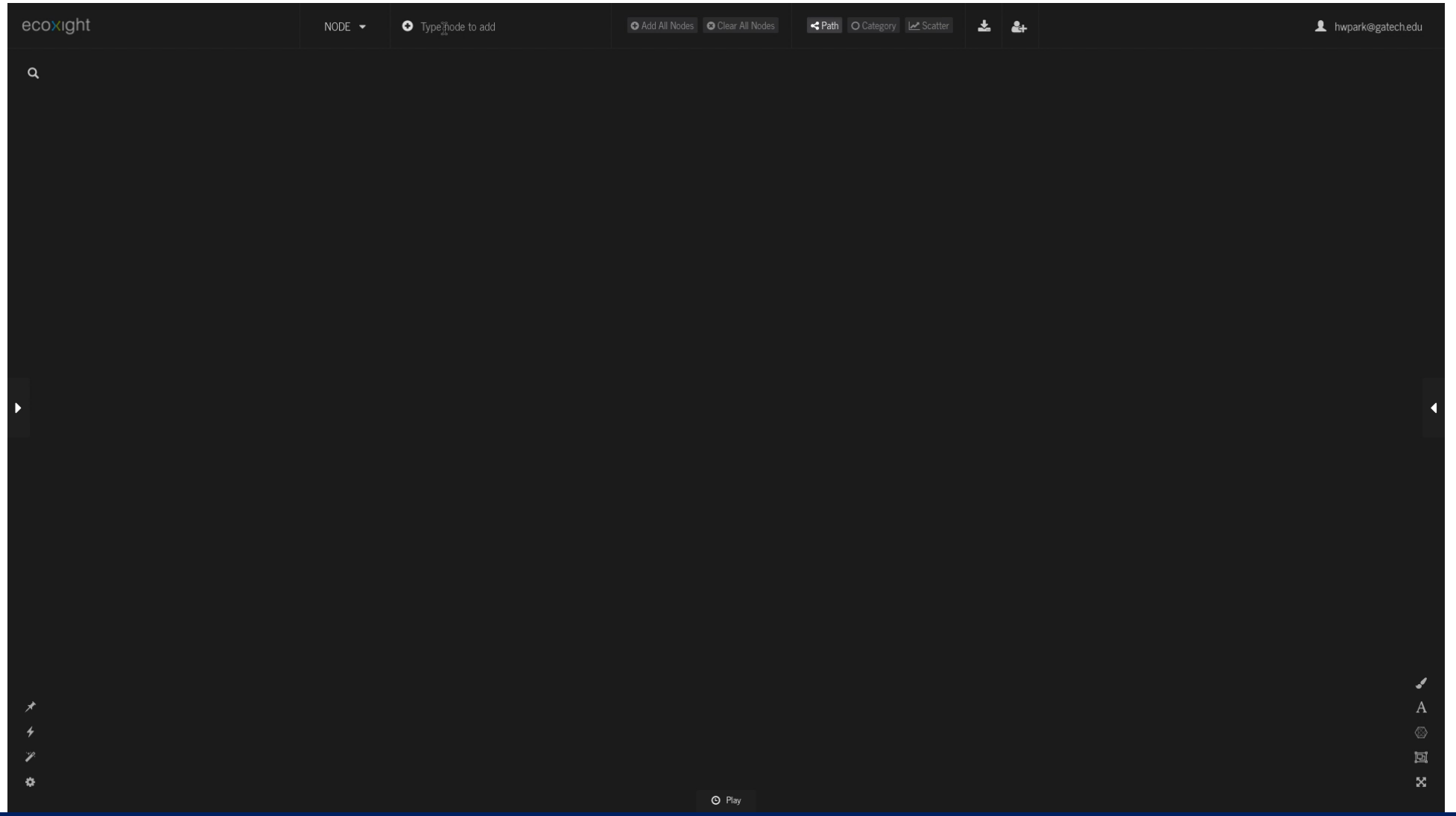


Graphs in Practice





Graph Visualization (Prof. Hyunwoo Park)



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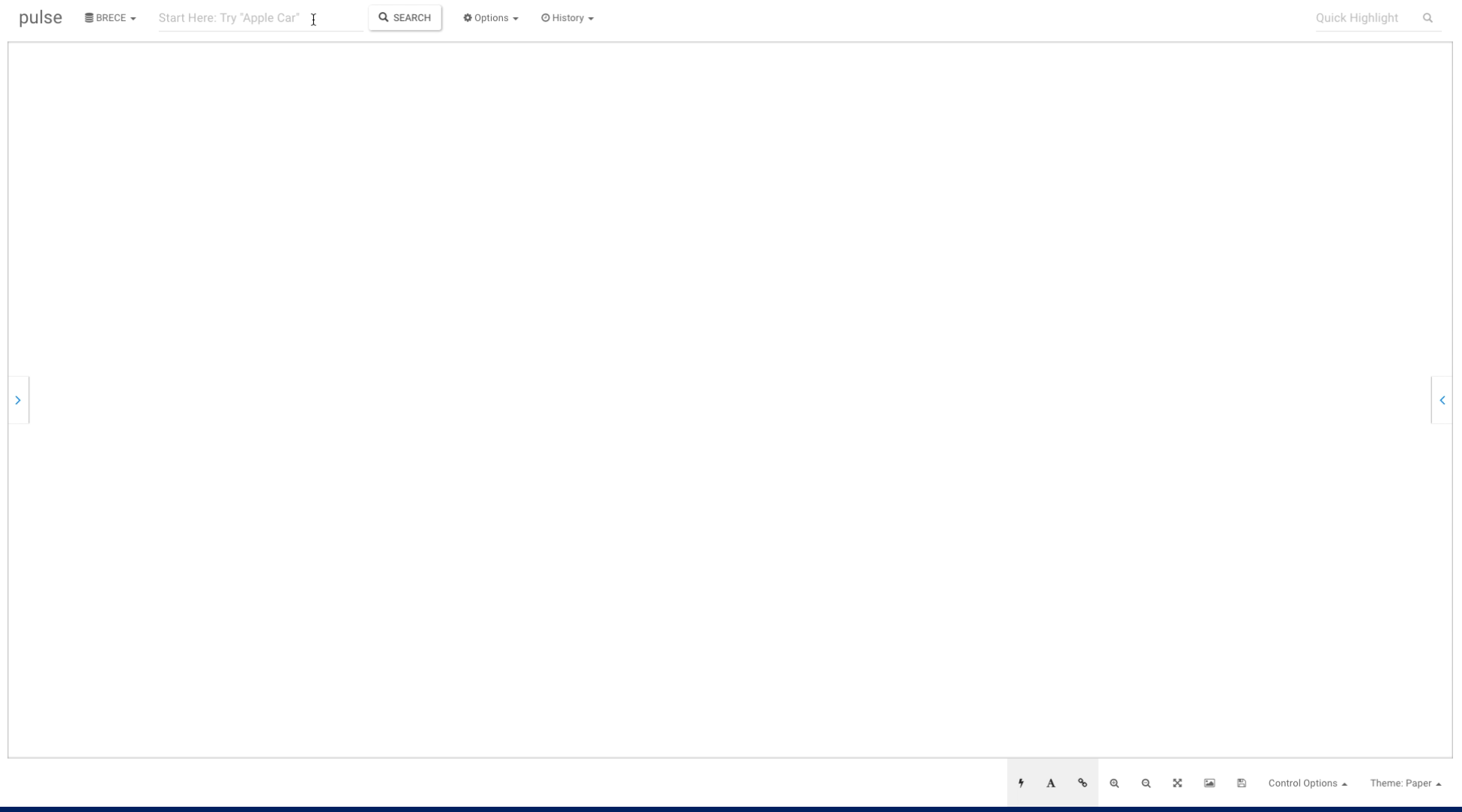


Graph Construction

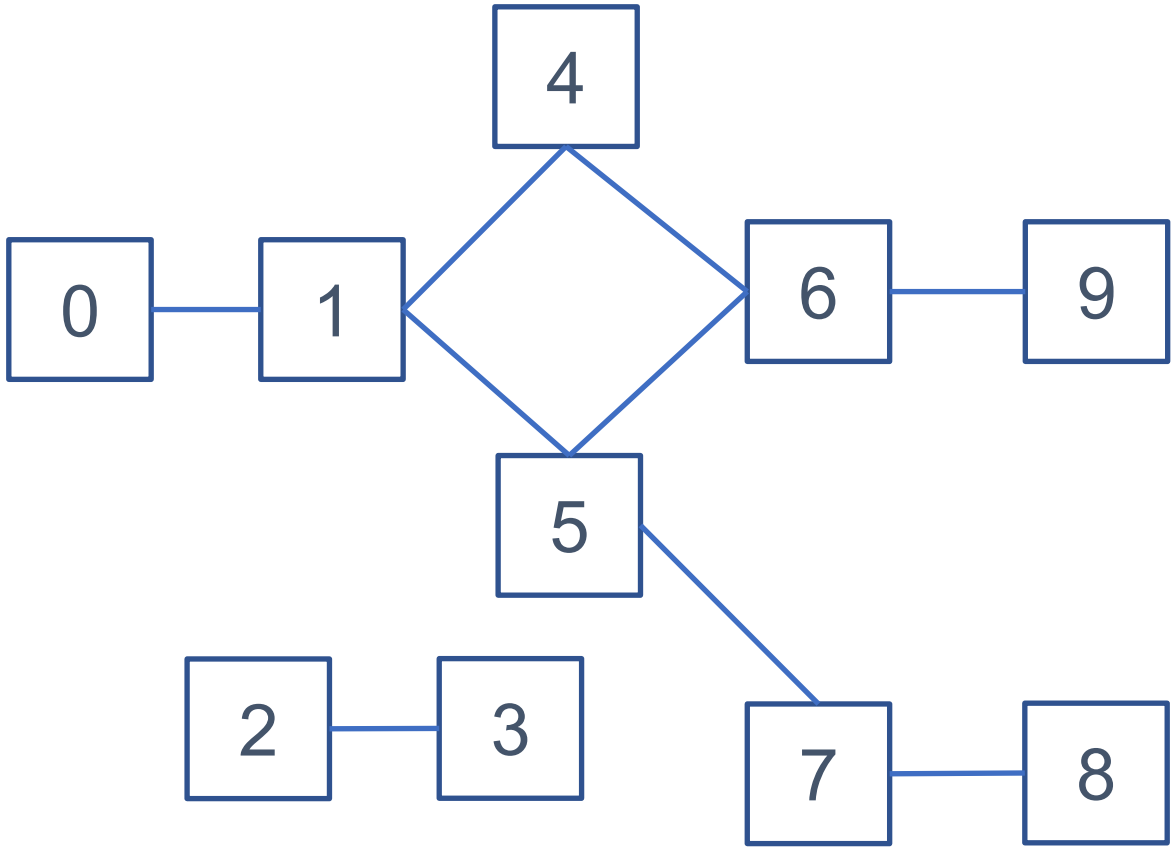
- Given vertices **V** and edges **E**, build relationship between the vertices

```
class undi_graph():
        def __init__(self, V: list, E: list) -> None:
            self.V = V[:]
            self.neighbor = {}
            for v in V:
                self.neighbor[v] = []
            for (v, w) in E:
                self.neighbor[v].append(w)
                self.neighbor[w].append(v)
```

Graph Traversals

- Graphs have **cycles** and can be **disconnected**

- We still should visit **all nodes**



Good news!

We can still use depth-first traversals and bread-first traversals,
but a little bit differently ☺

Graph Traversal

- DFT - preorder

Depth-First Traversal – Preorder

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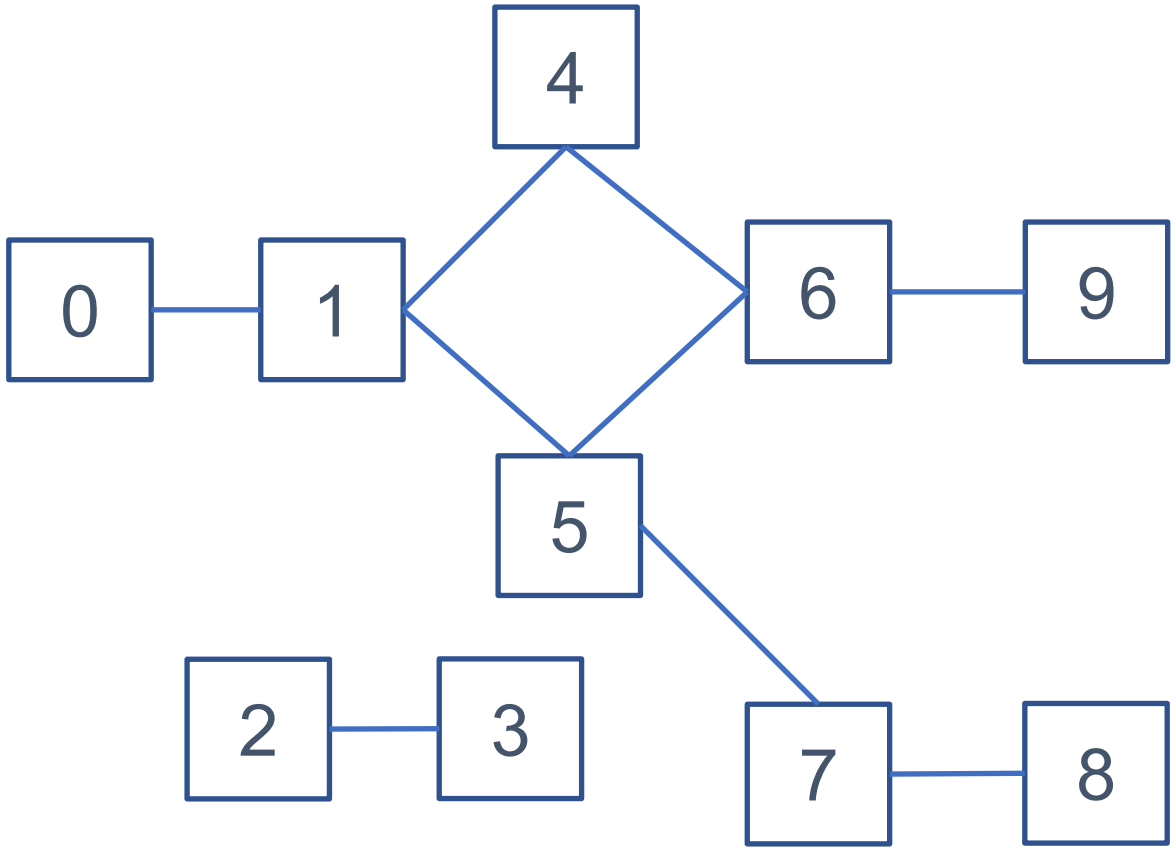

1. Now we should **mark** the nodes we already visited to handle **cycles**
2. Now one __DFTHelp call is **not enough** to traversal a **disconnected** graph

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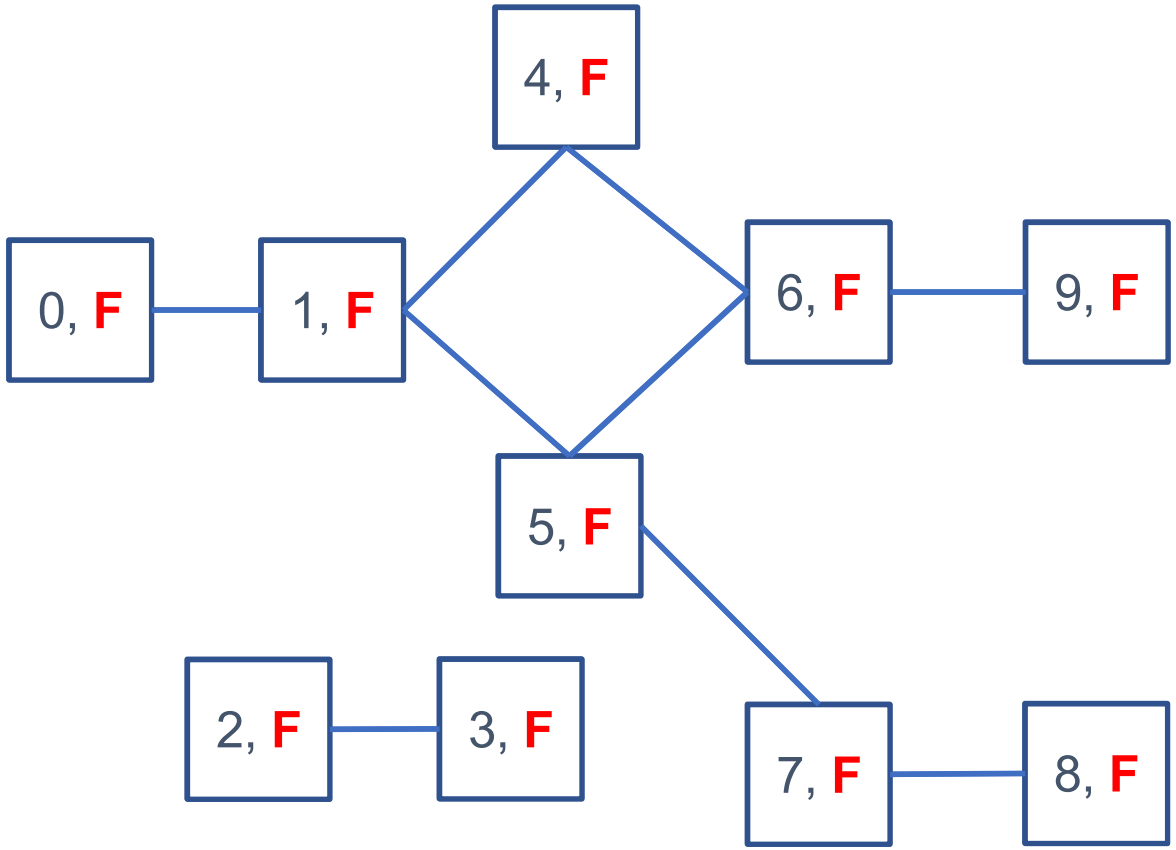

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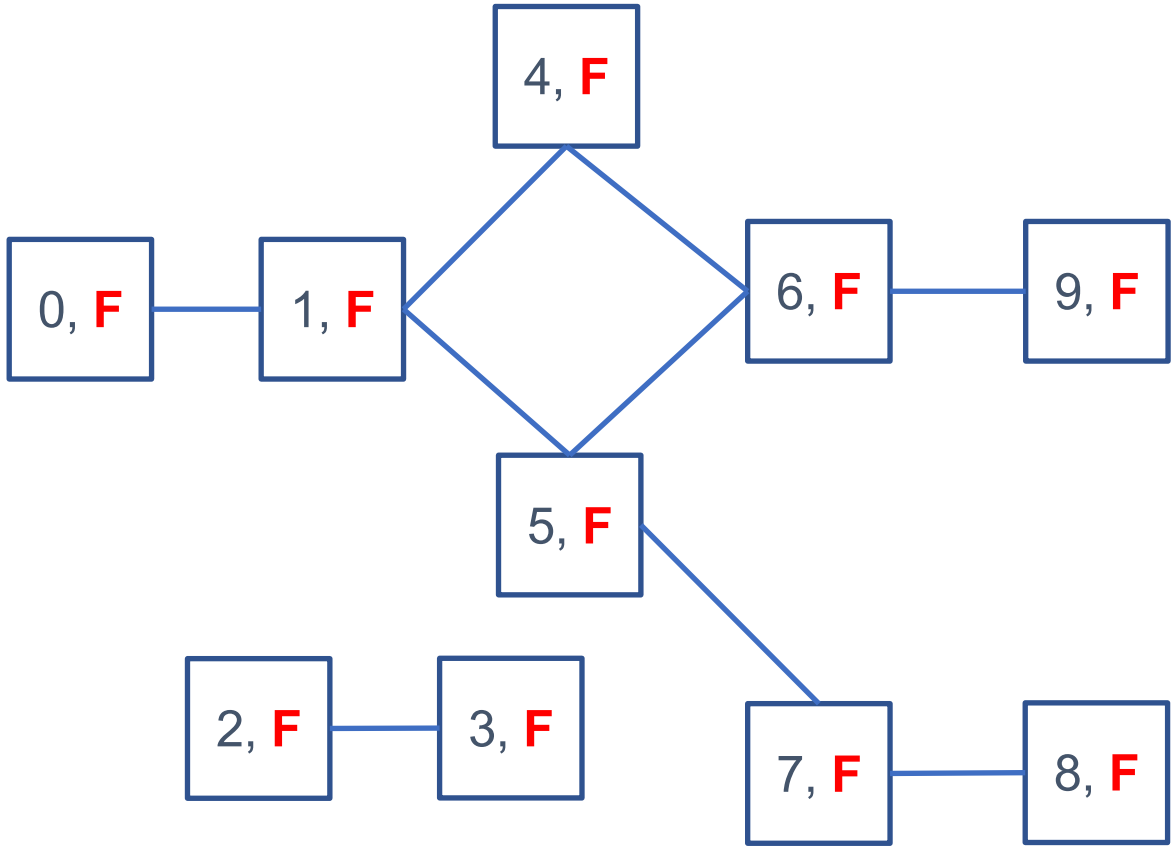

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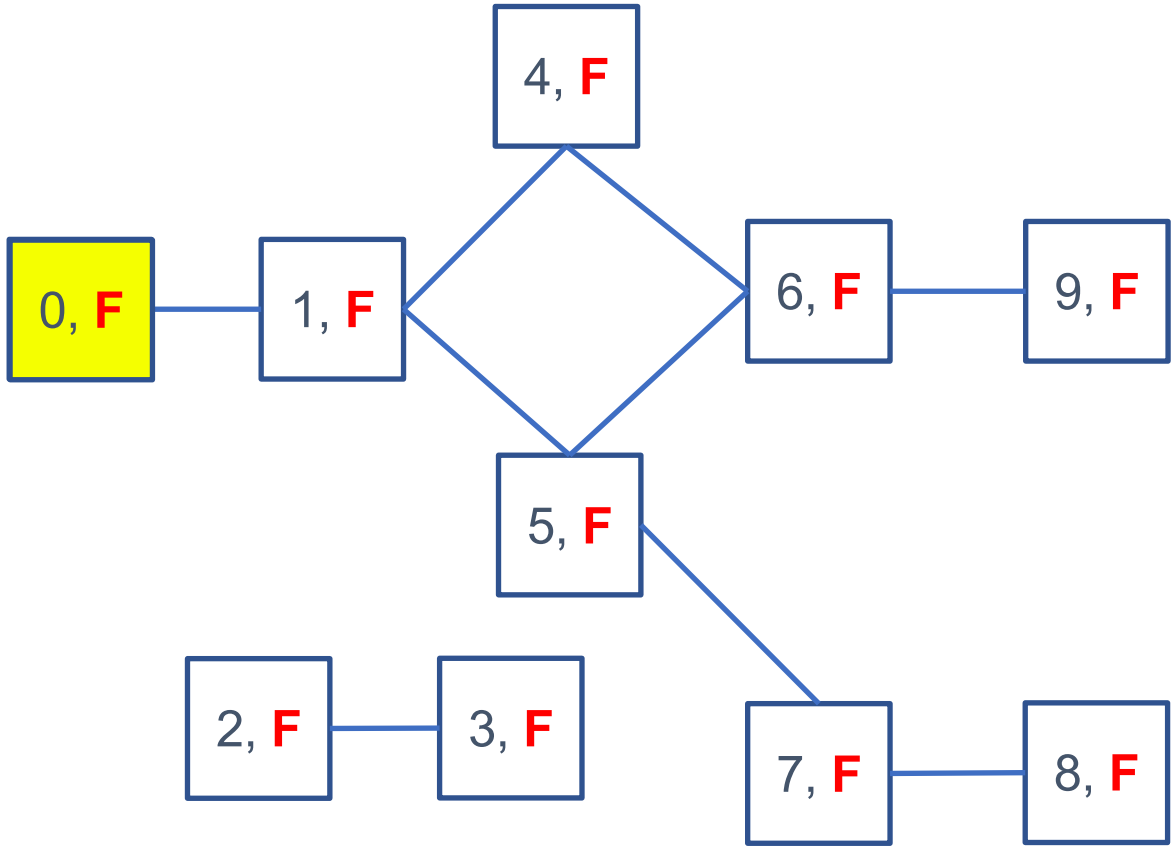

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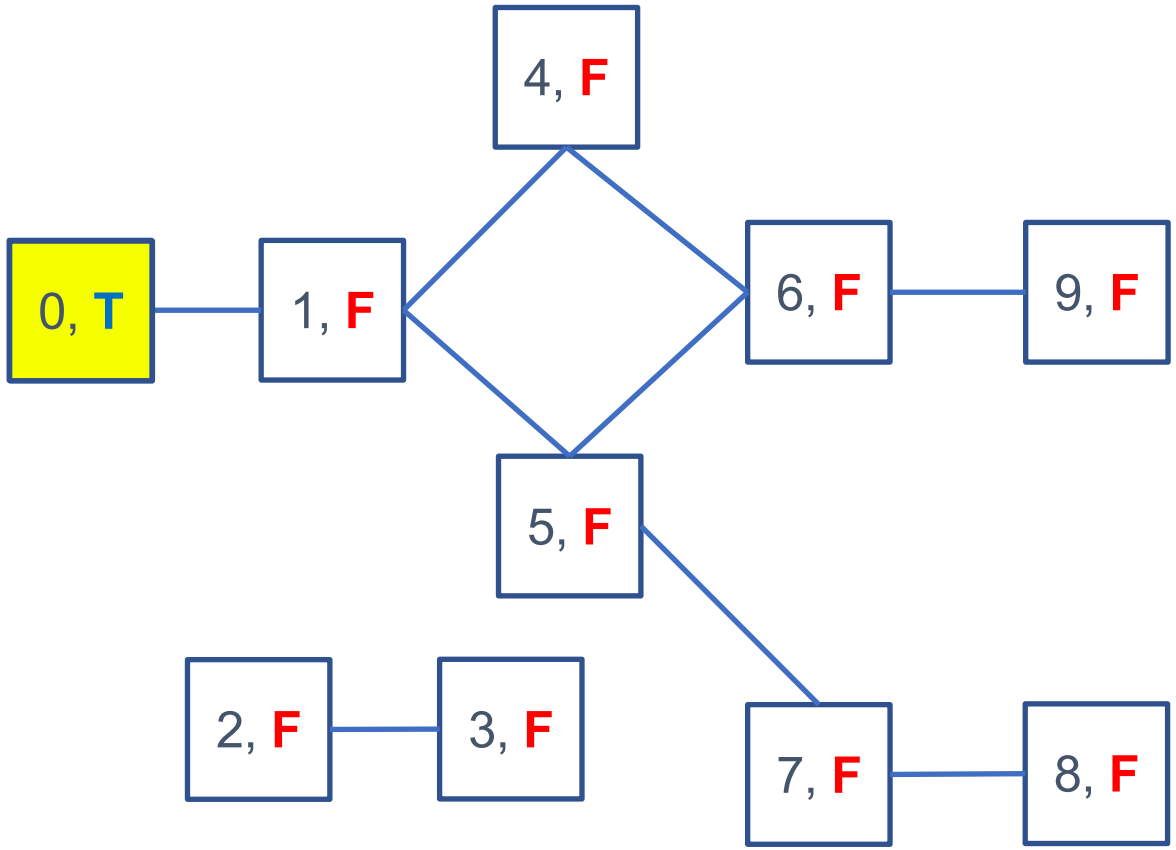

Out: 0

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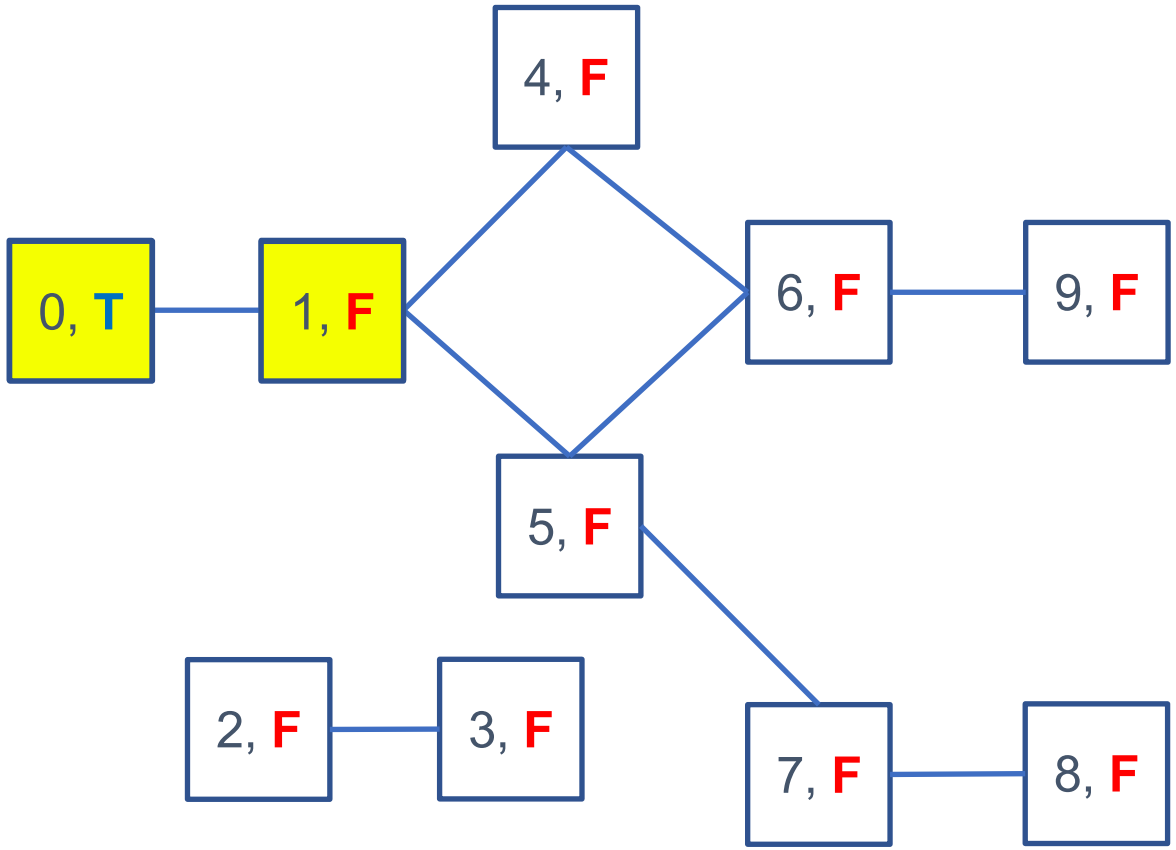

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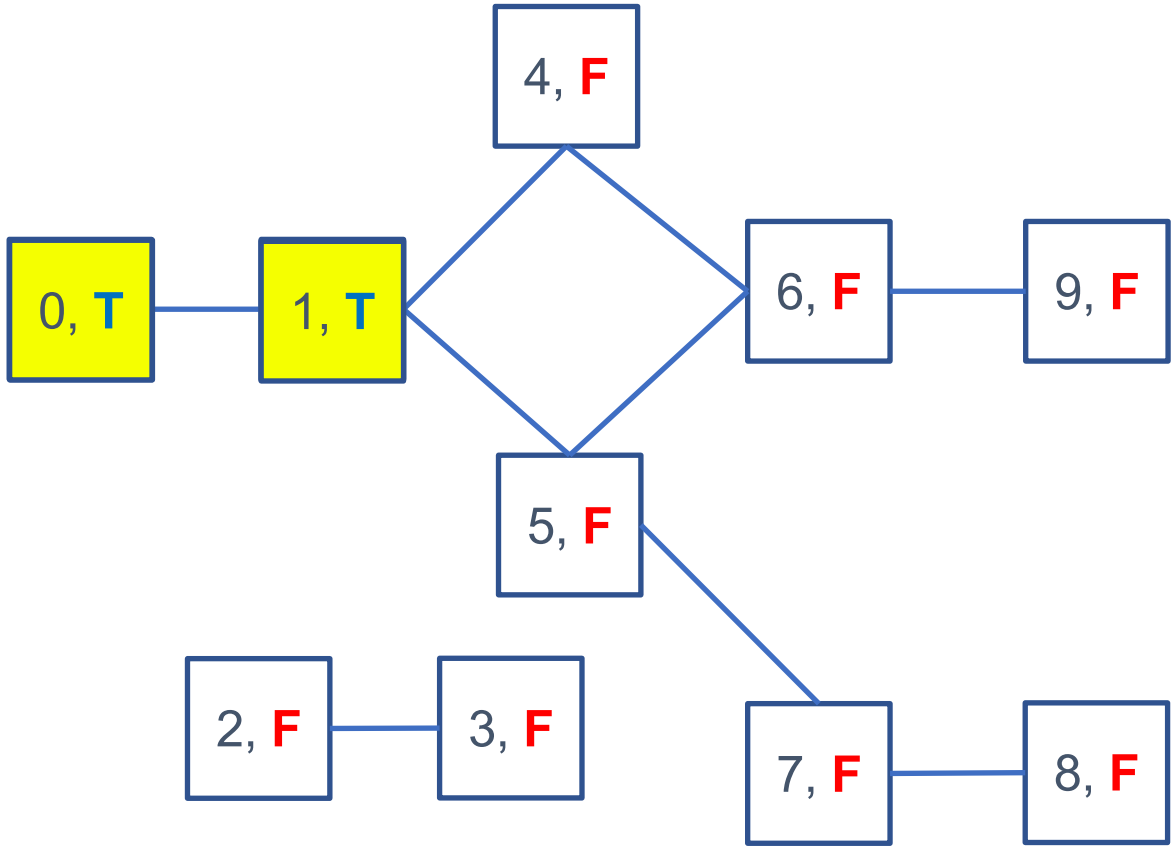

Out: 0 1

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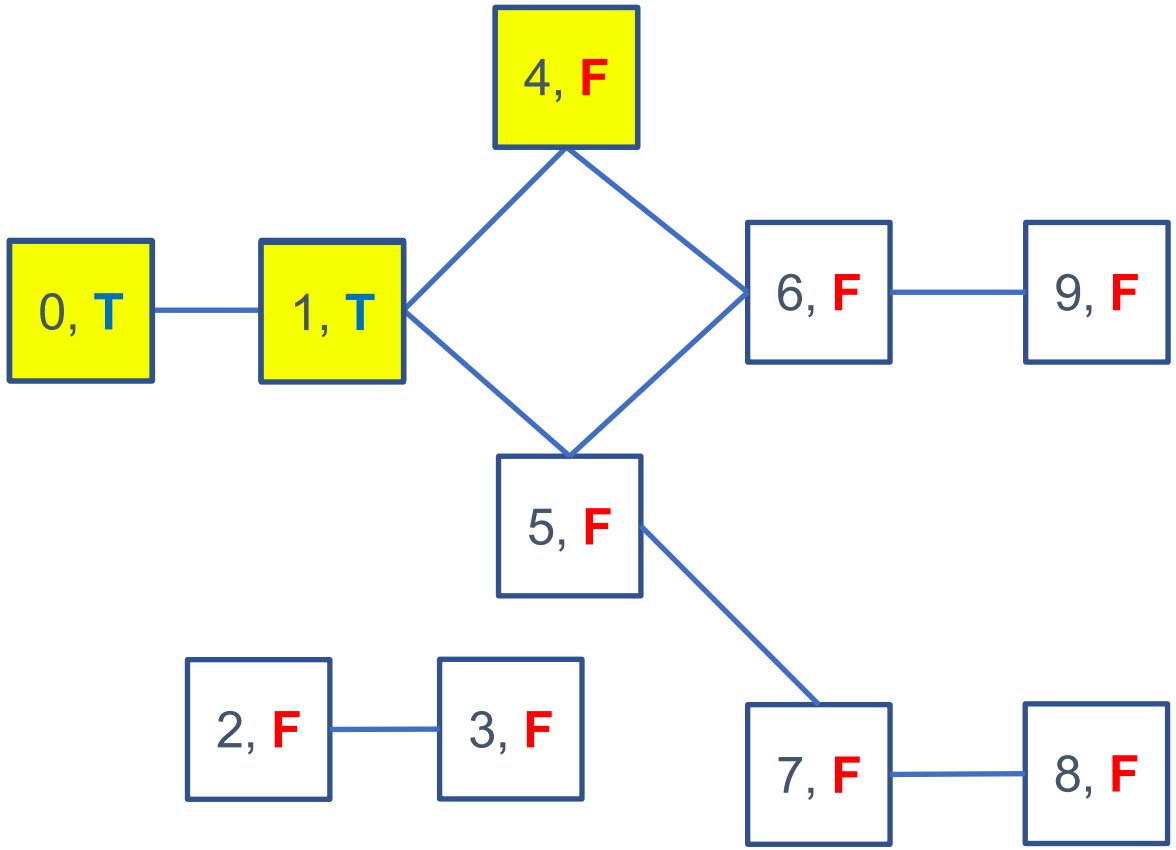

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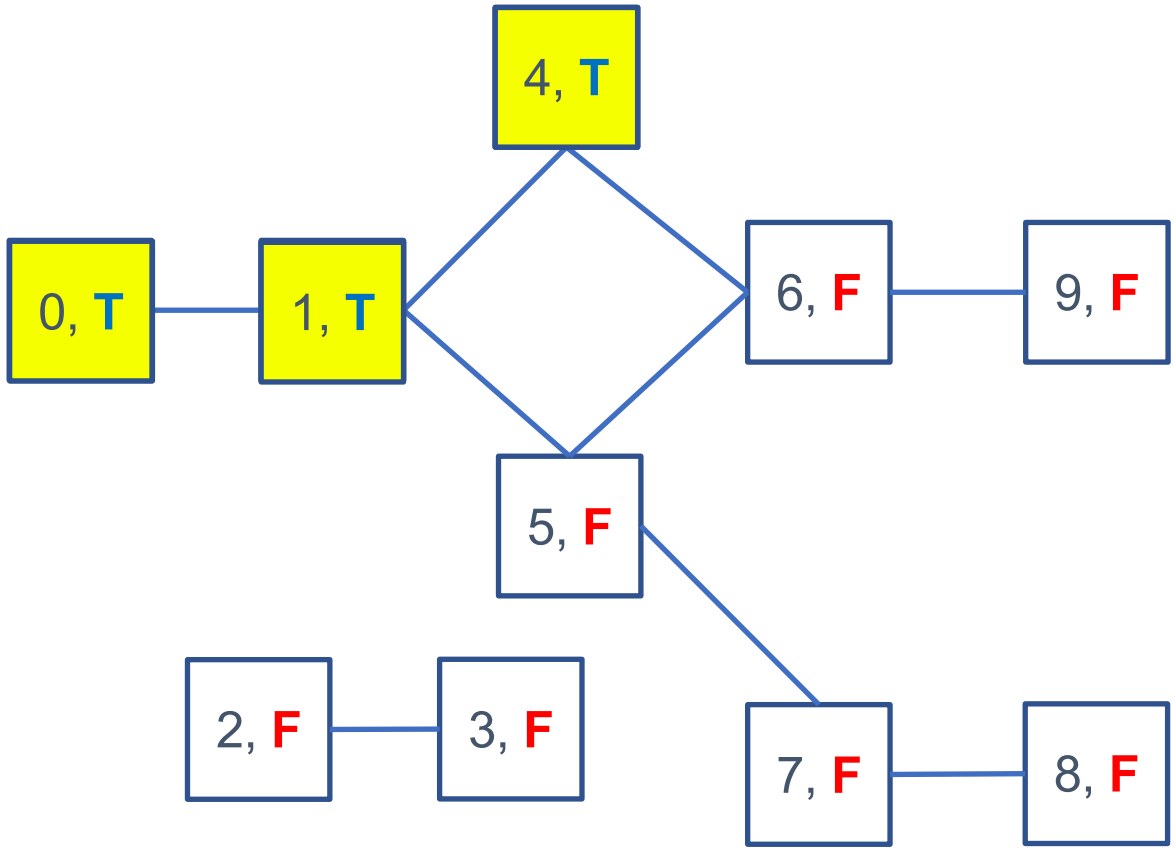

Out: 0 1 4

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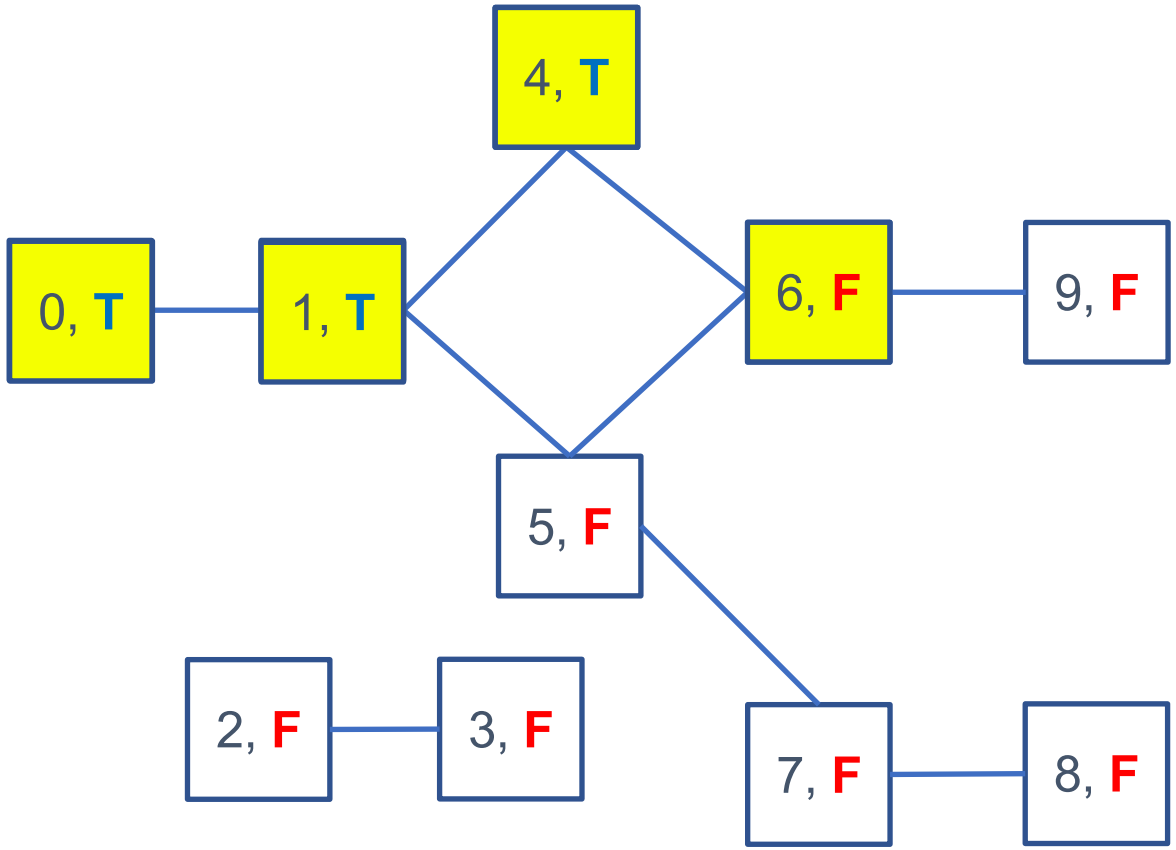

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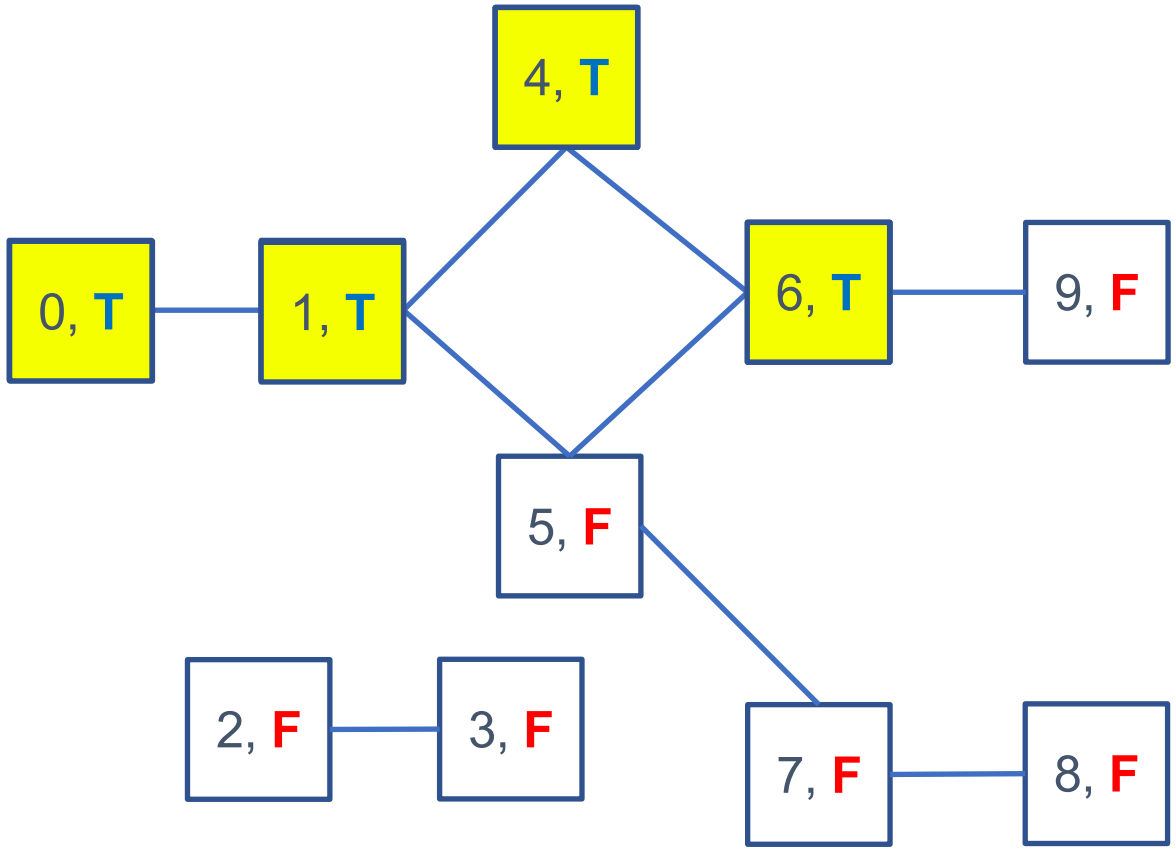

Out: 0 1 4 6

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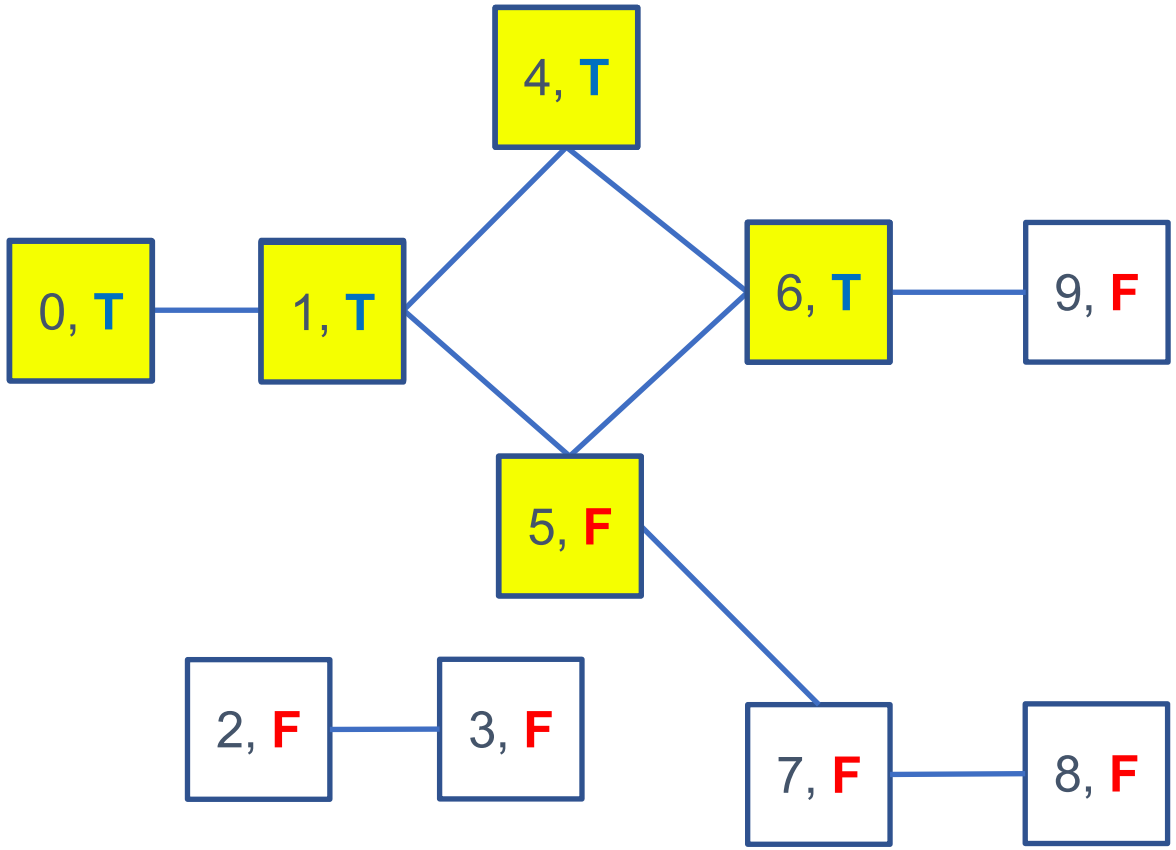

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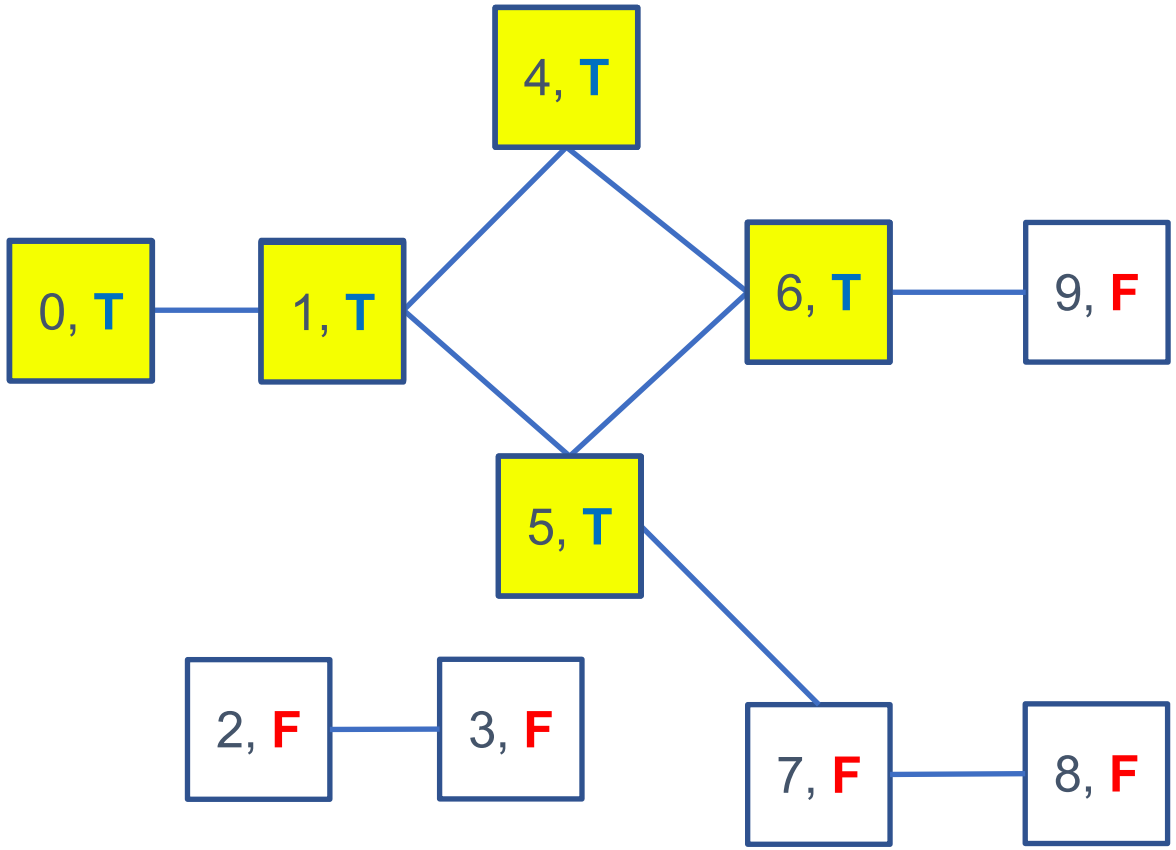

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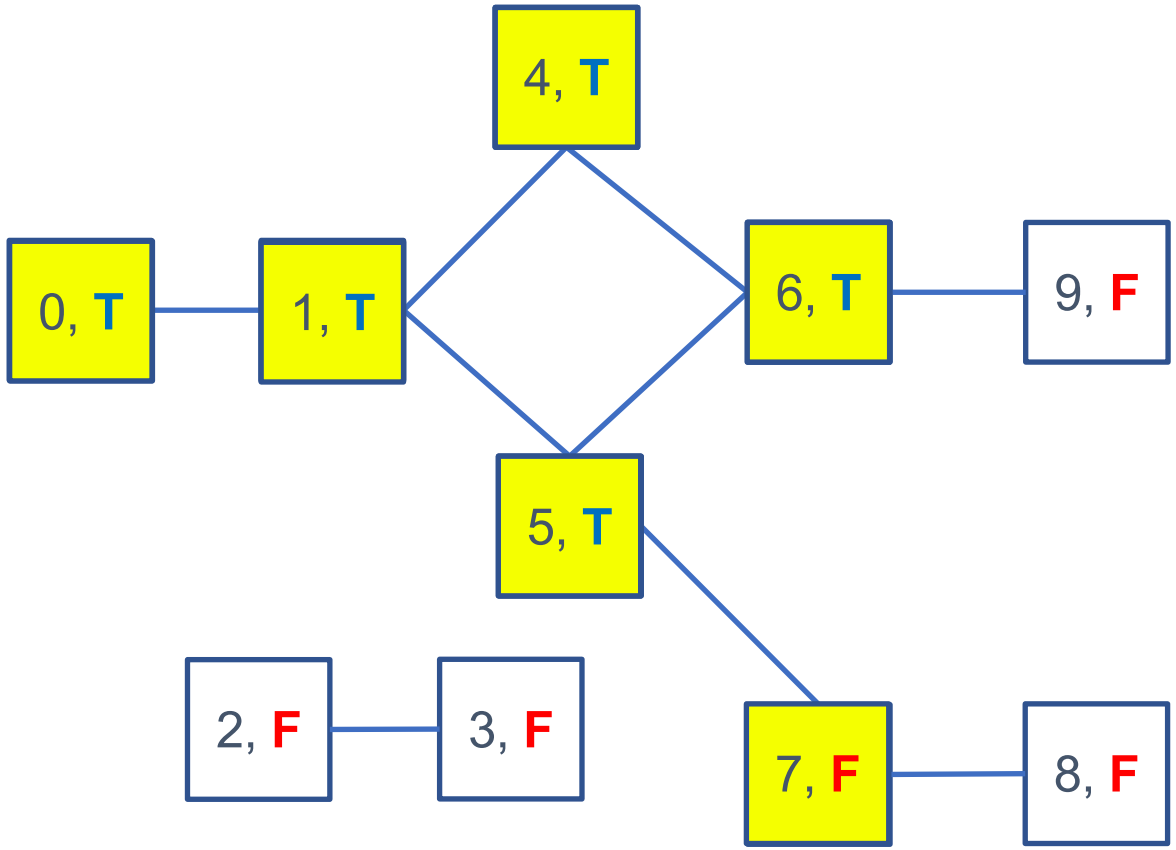

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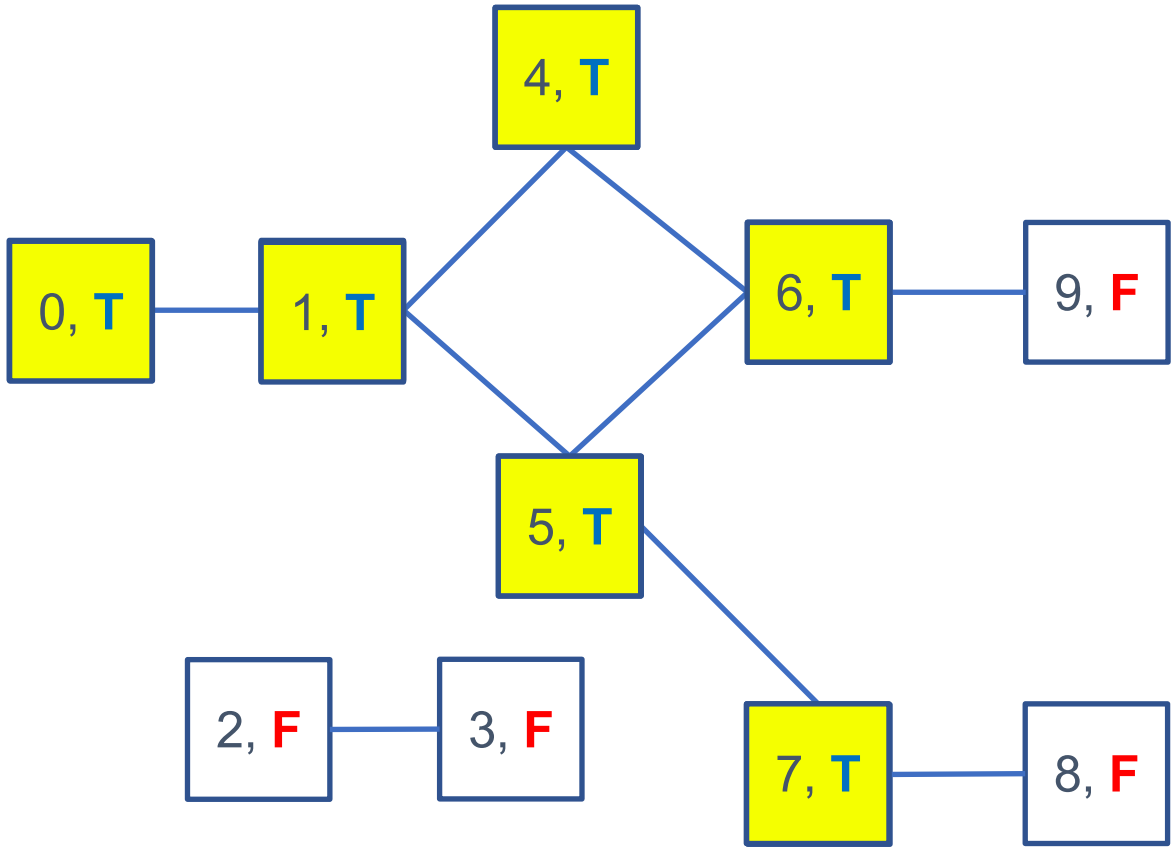

Out: 0 1 4 6 5 7

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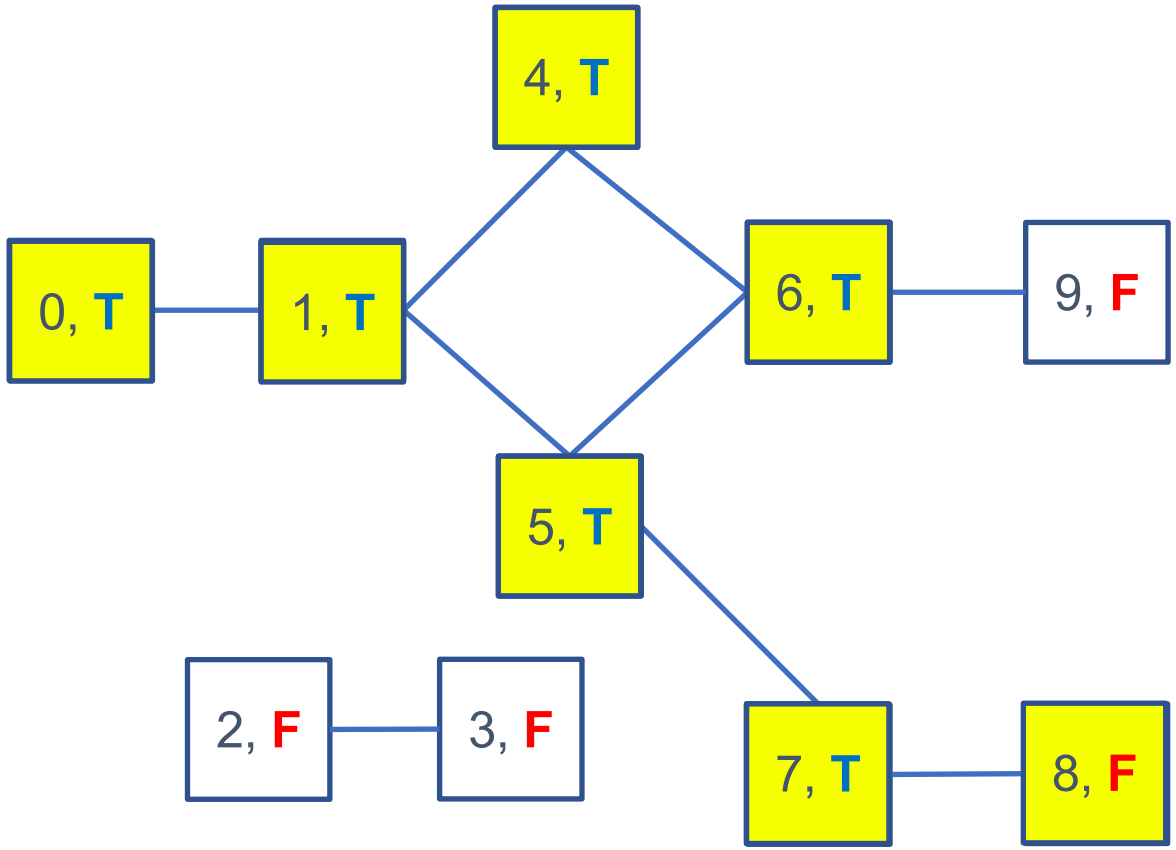

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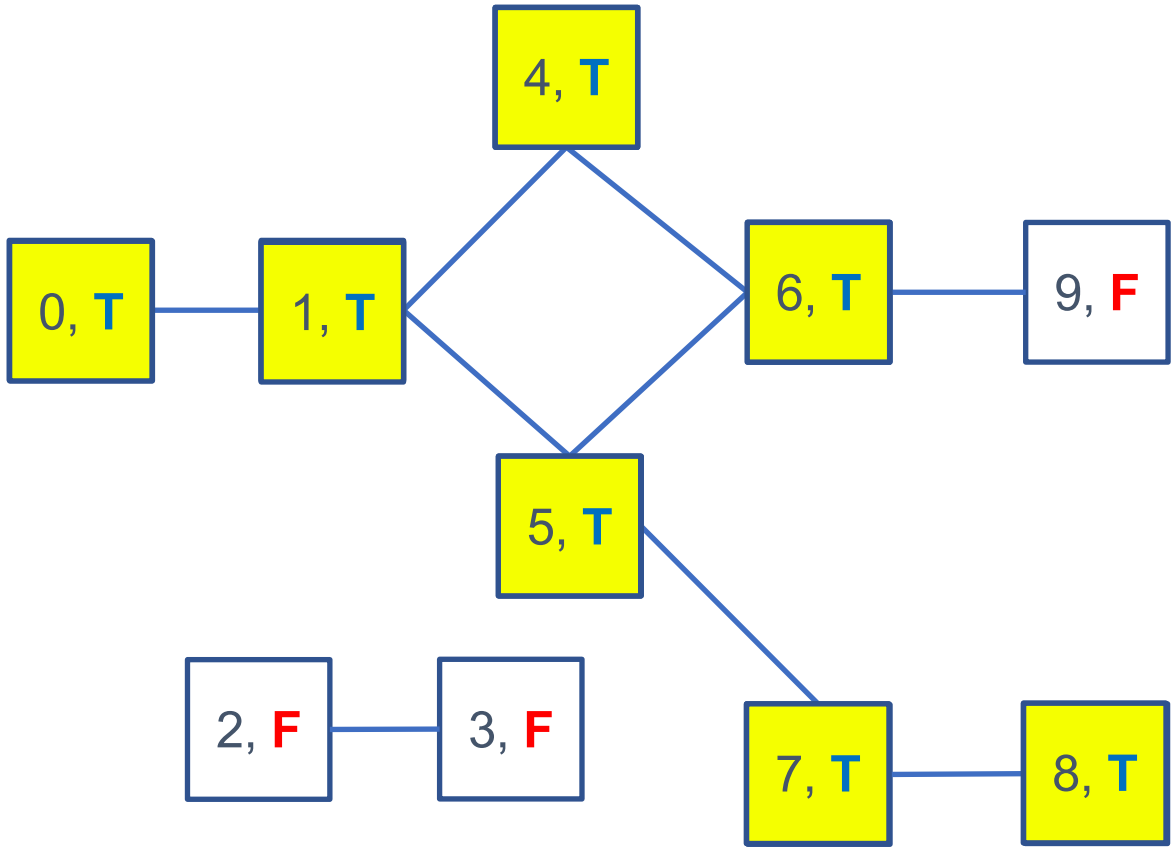

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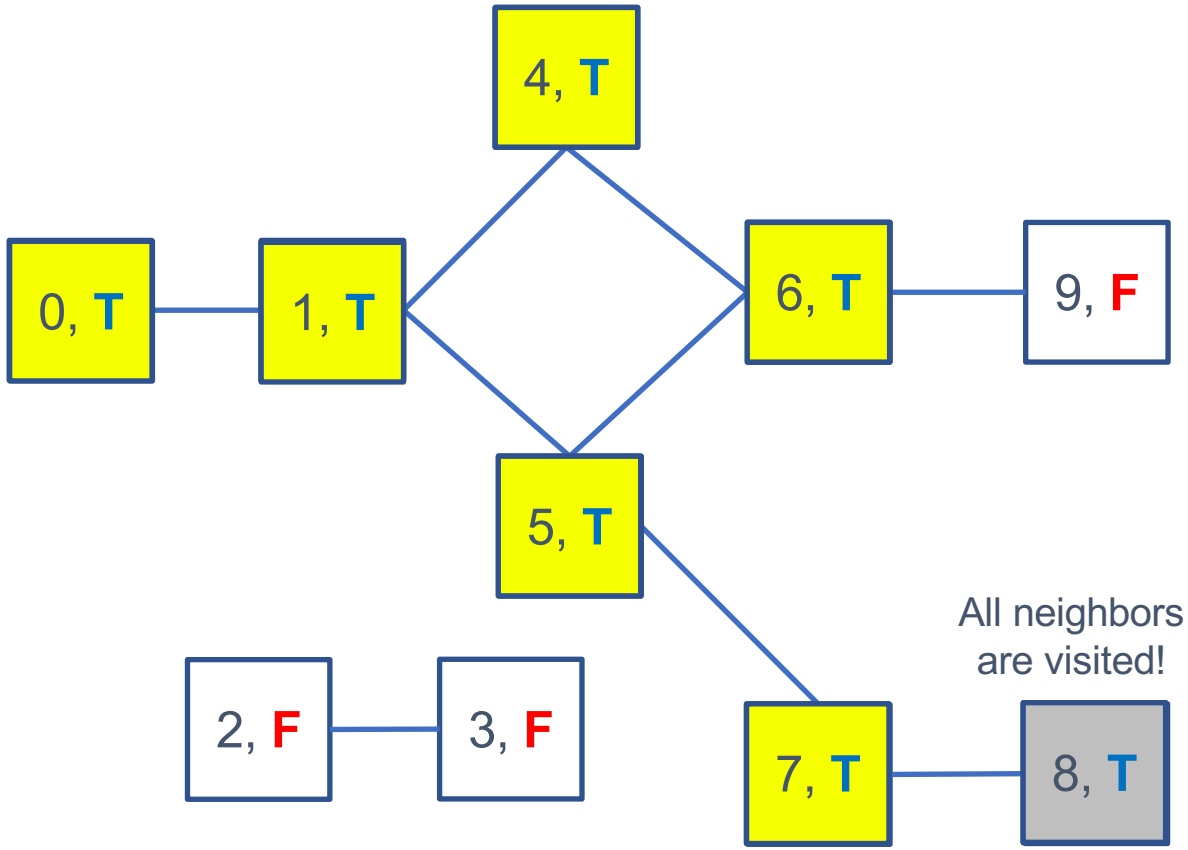

Out: 0 1 4 6 5 7 8

Depth-First Traversal – Preorder

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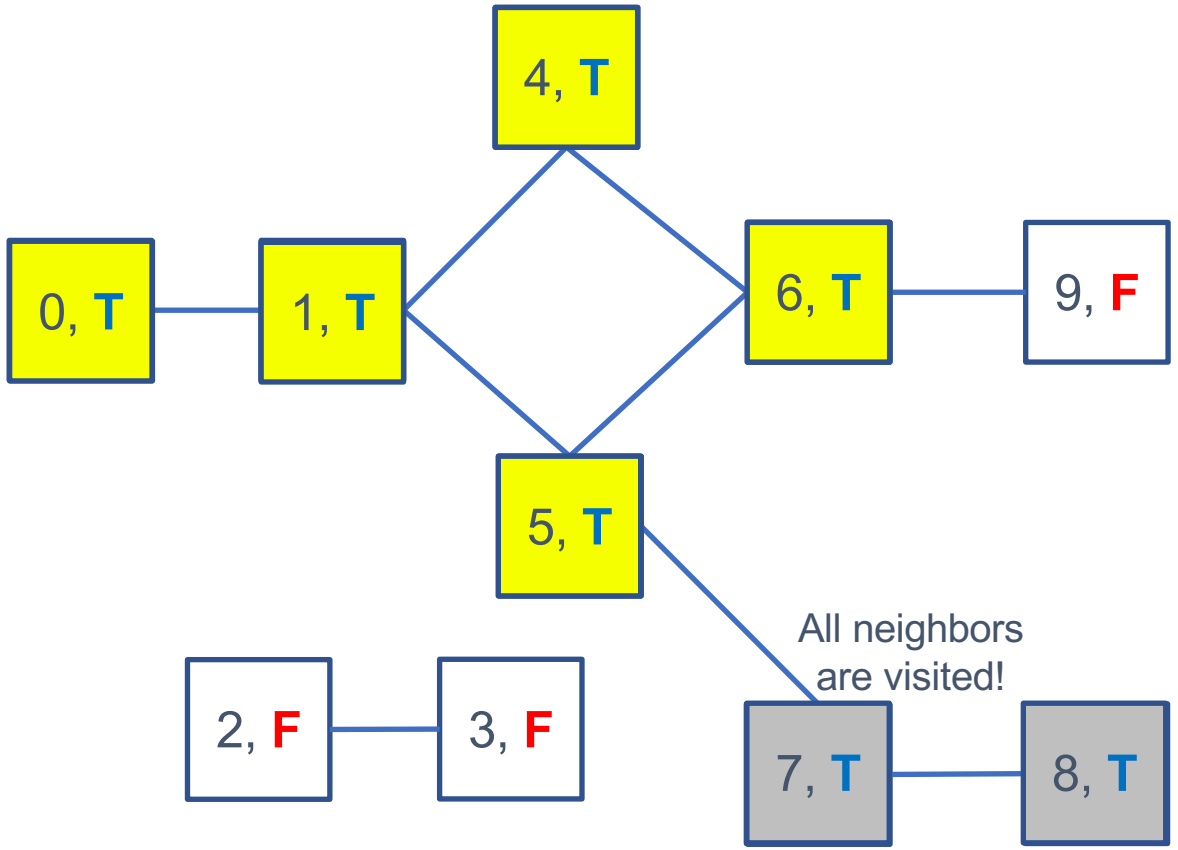

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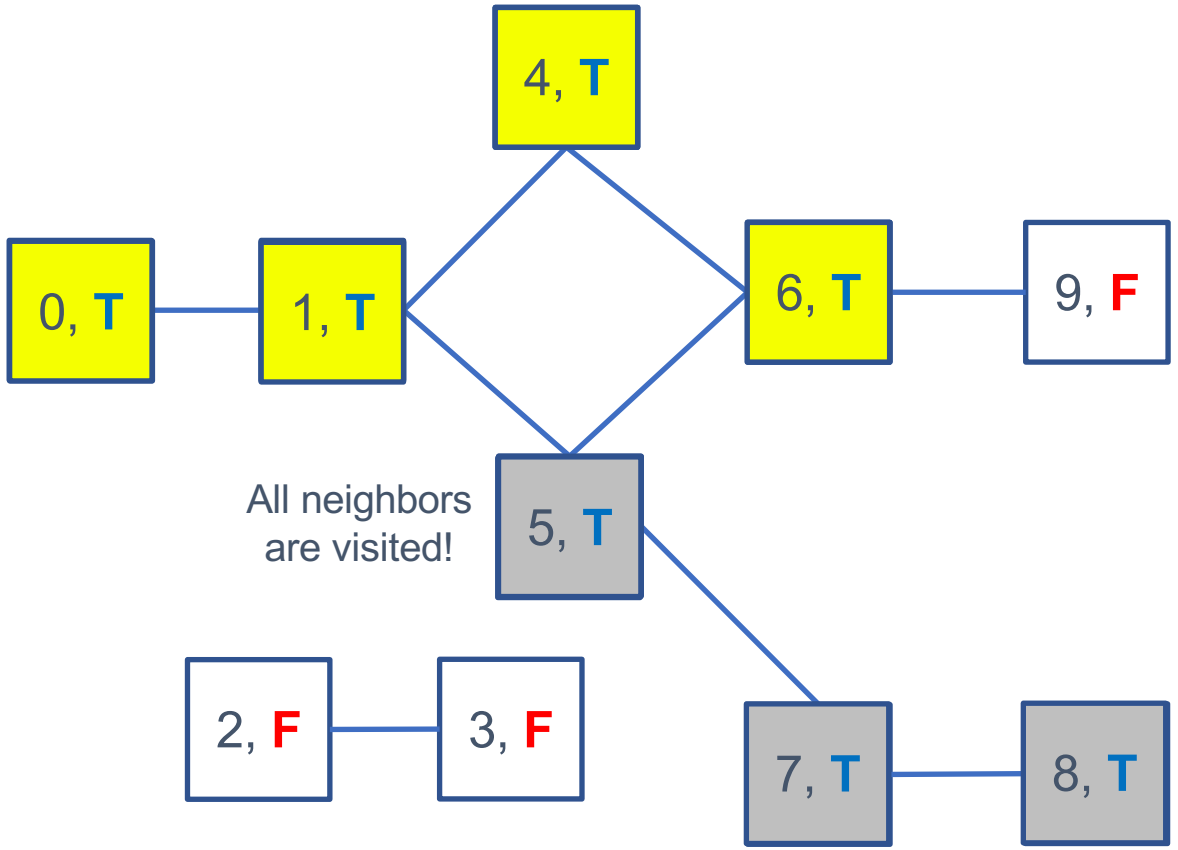

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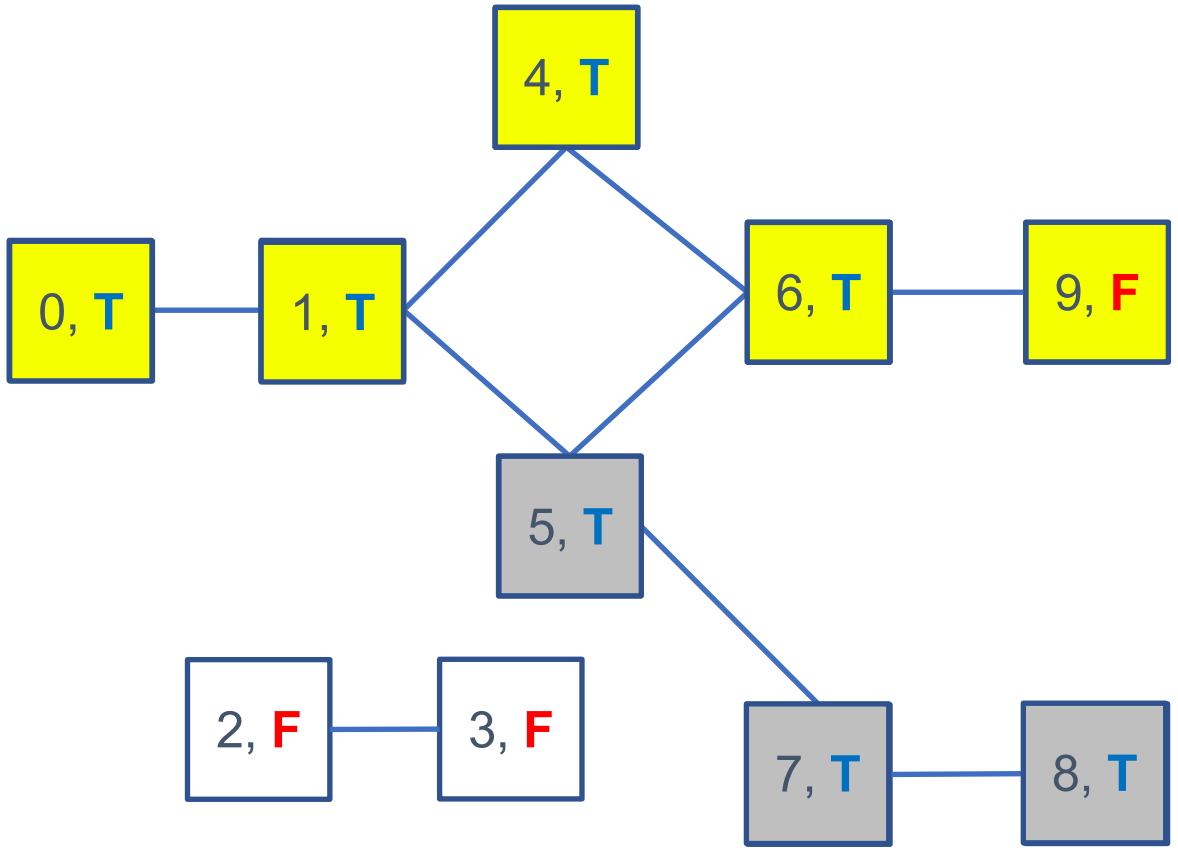

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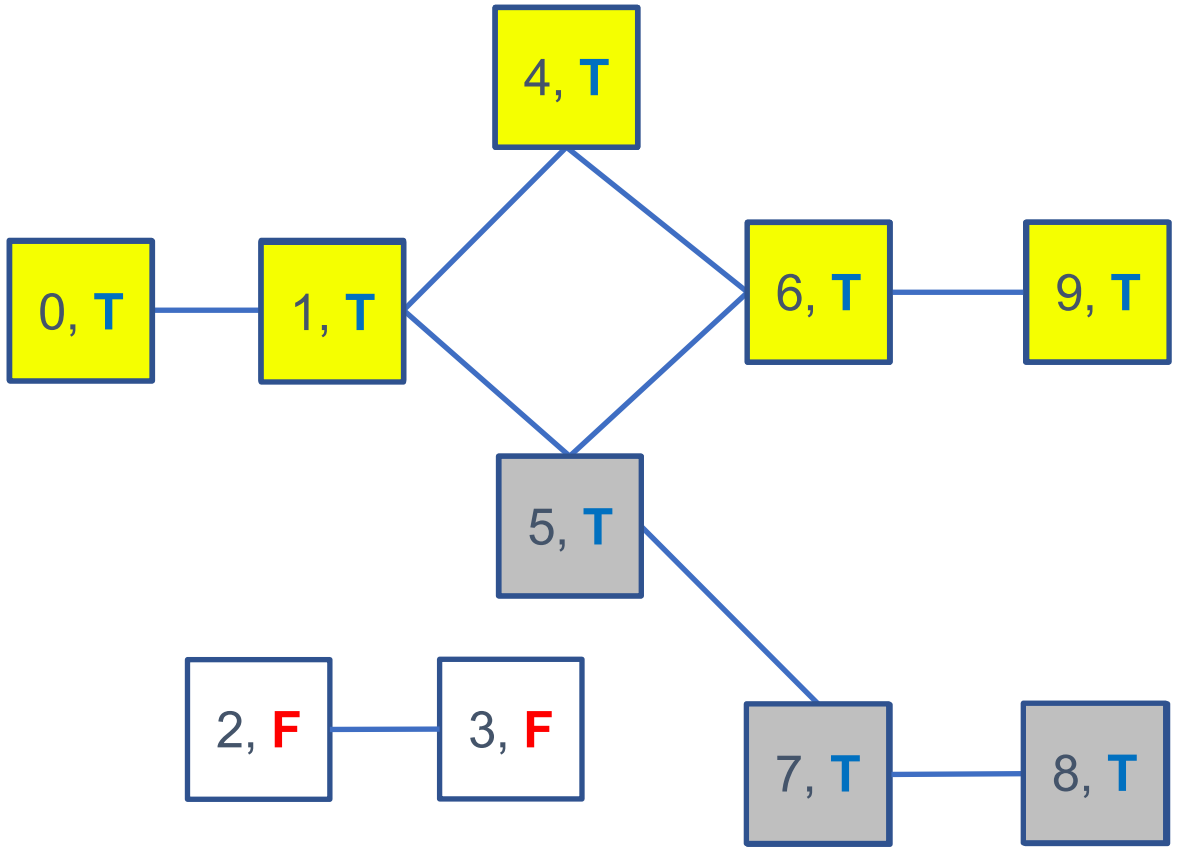

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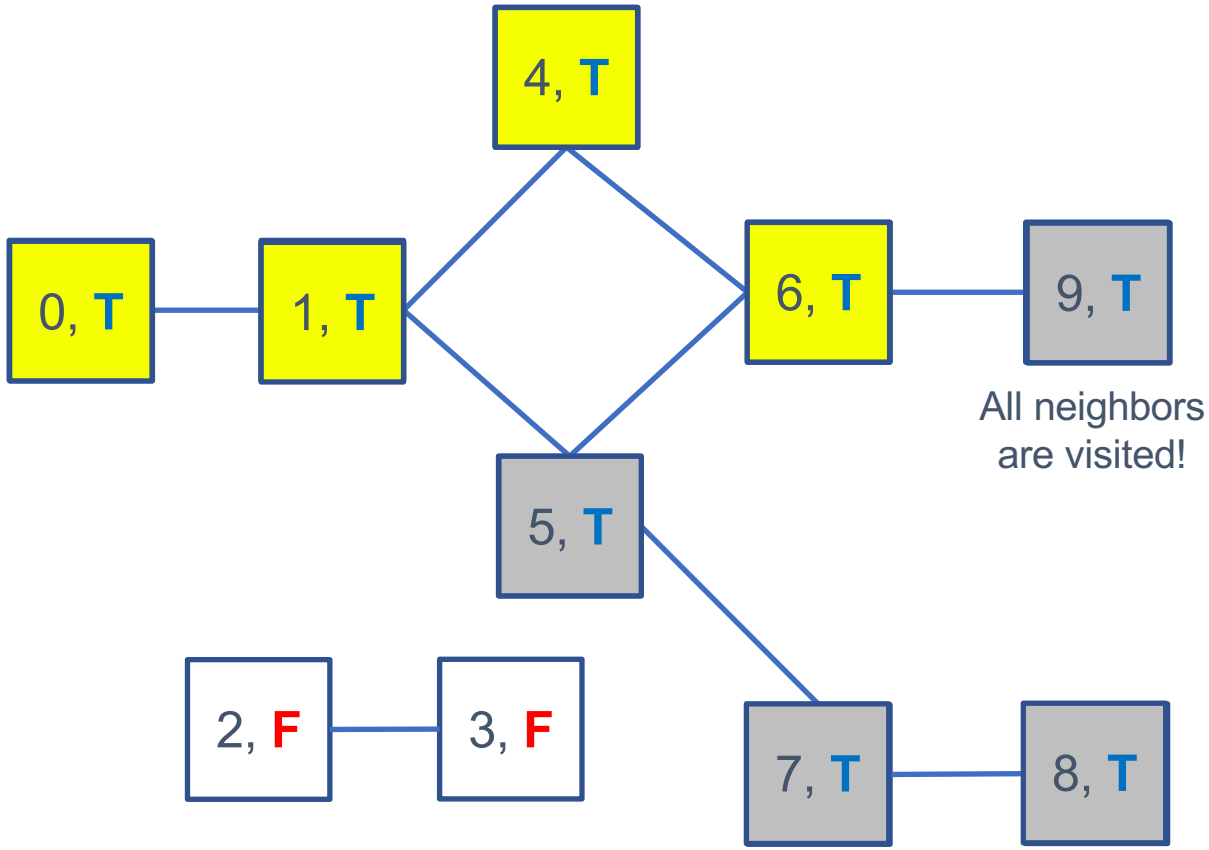

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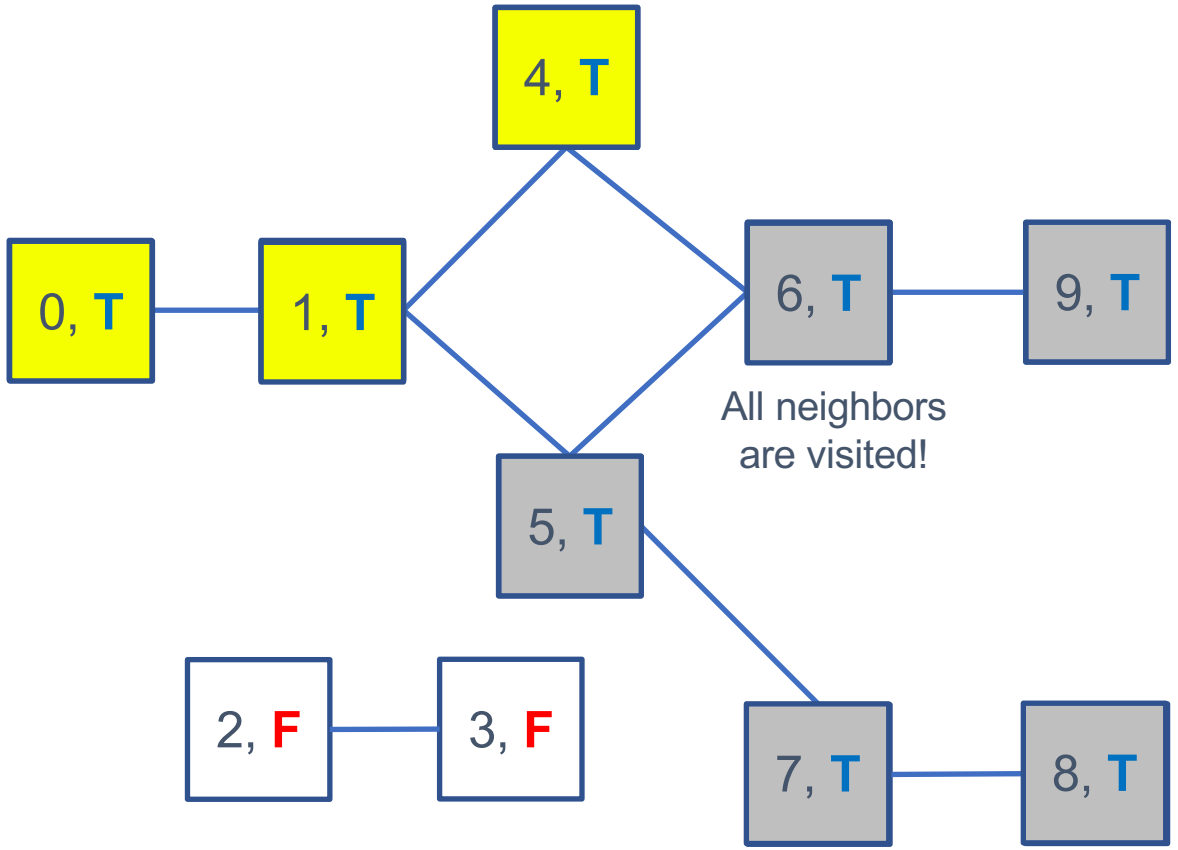

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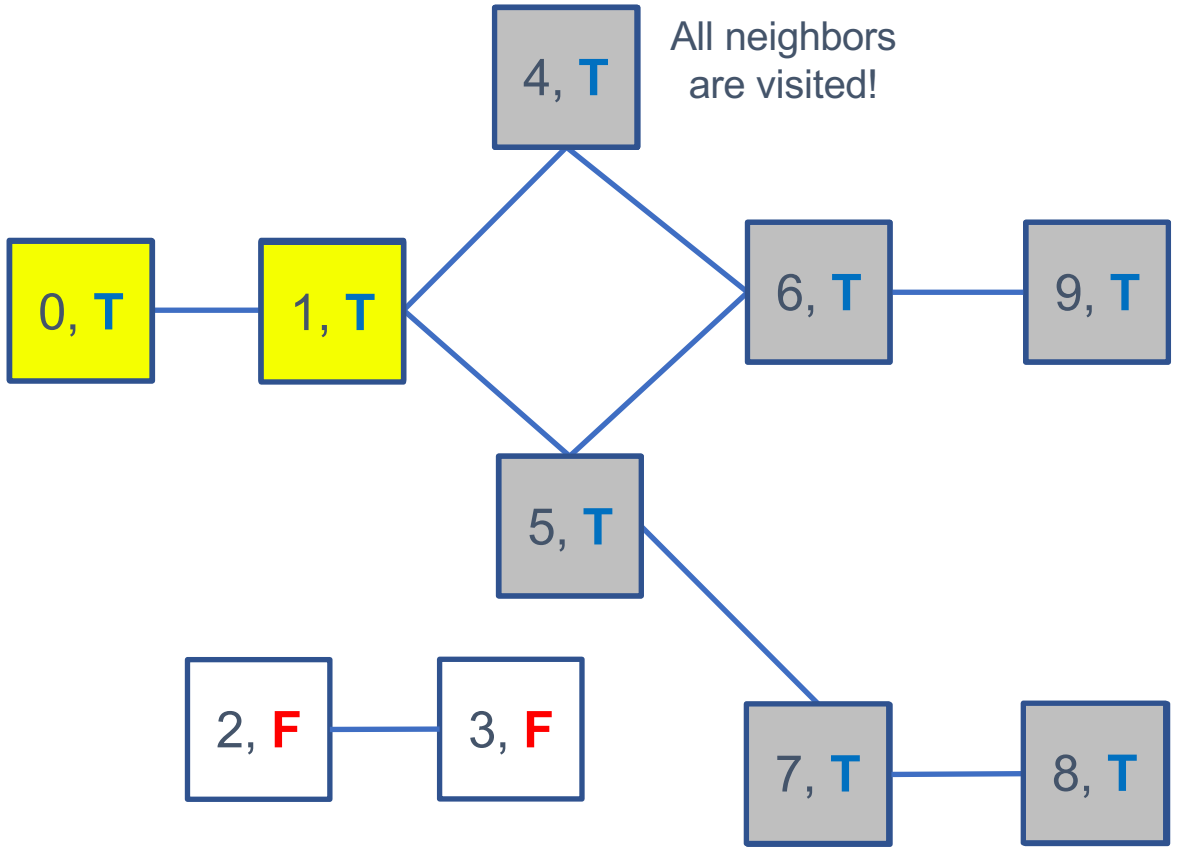

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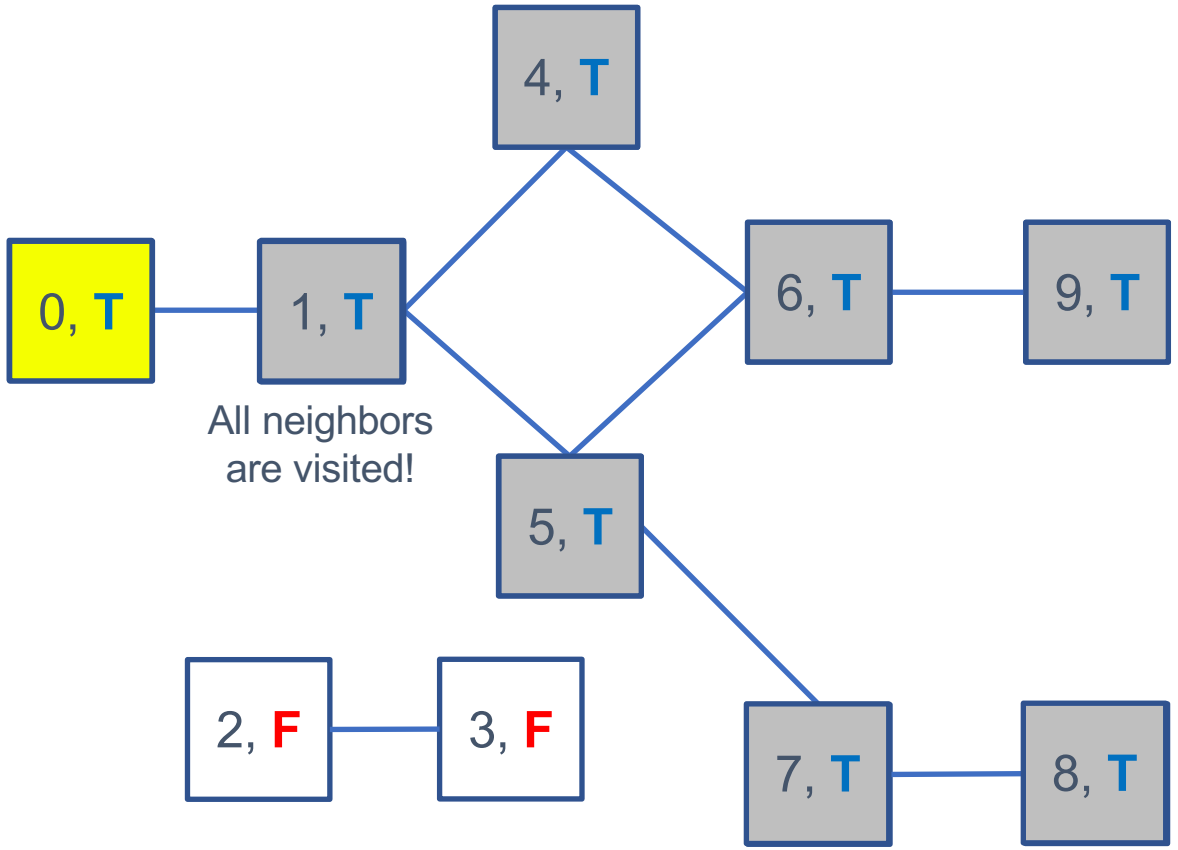

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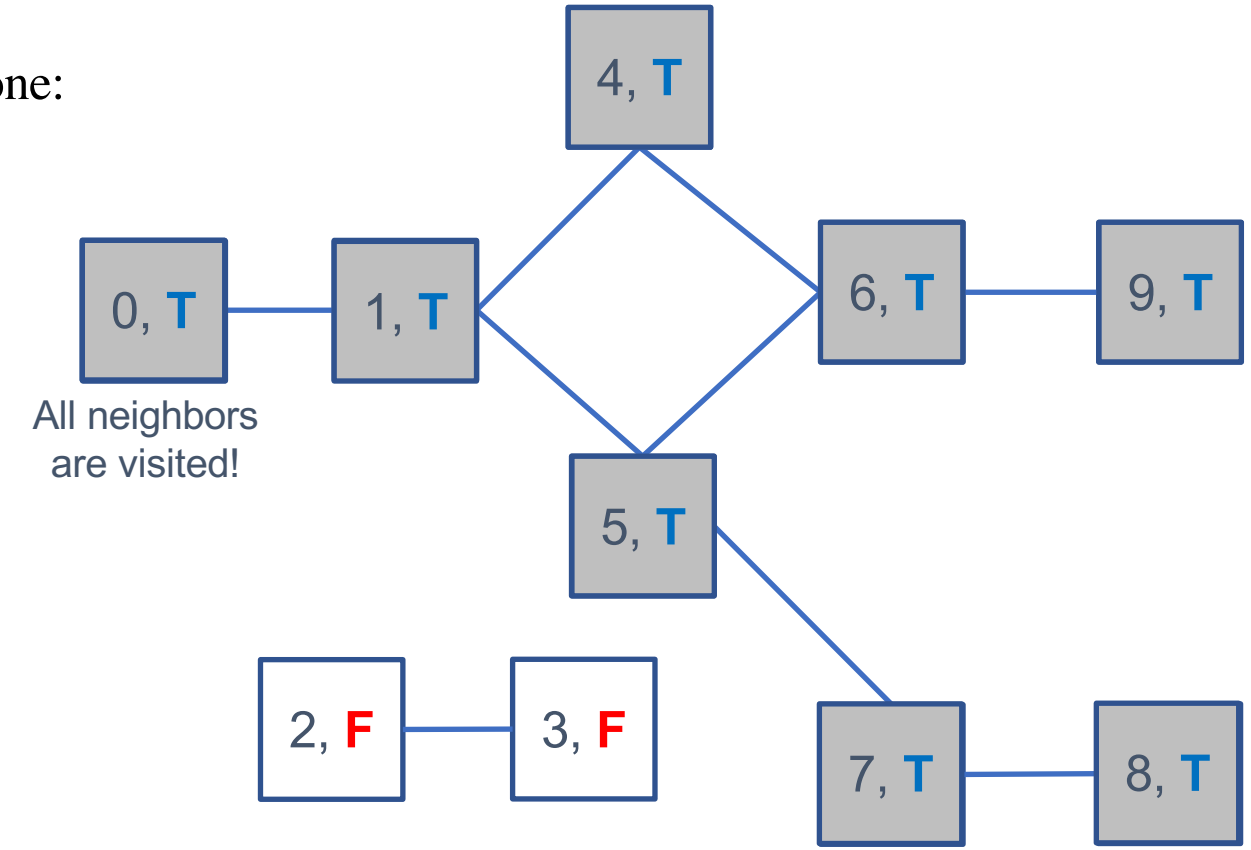

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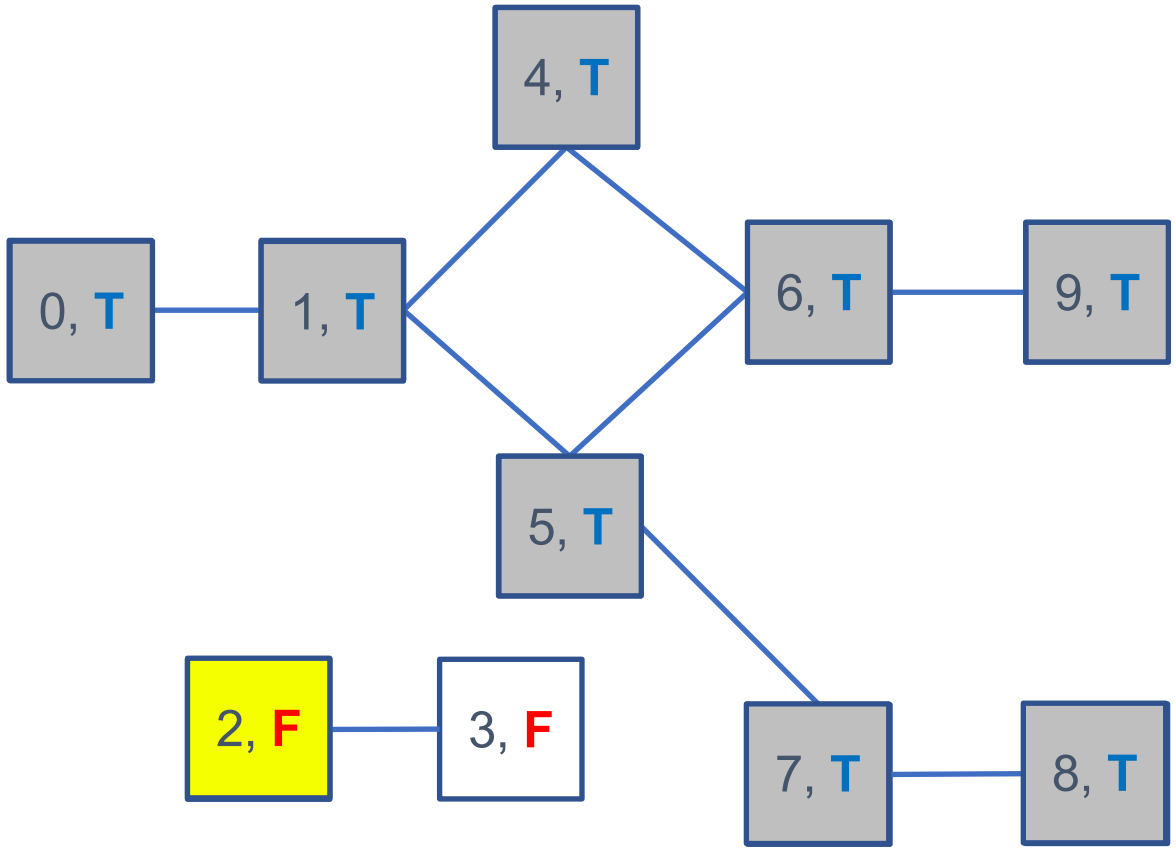

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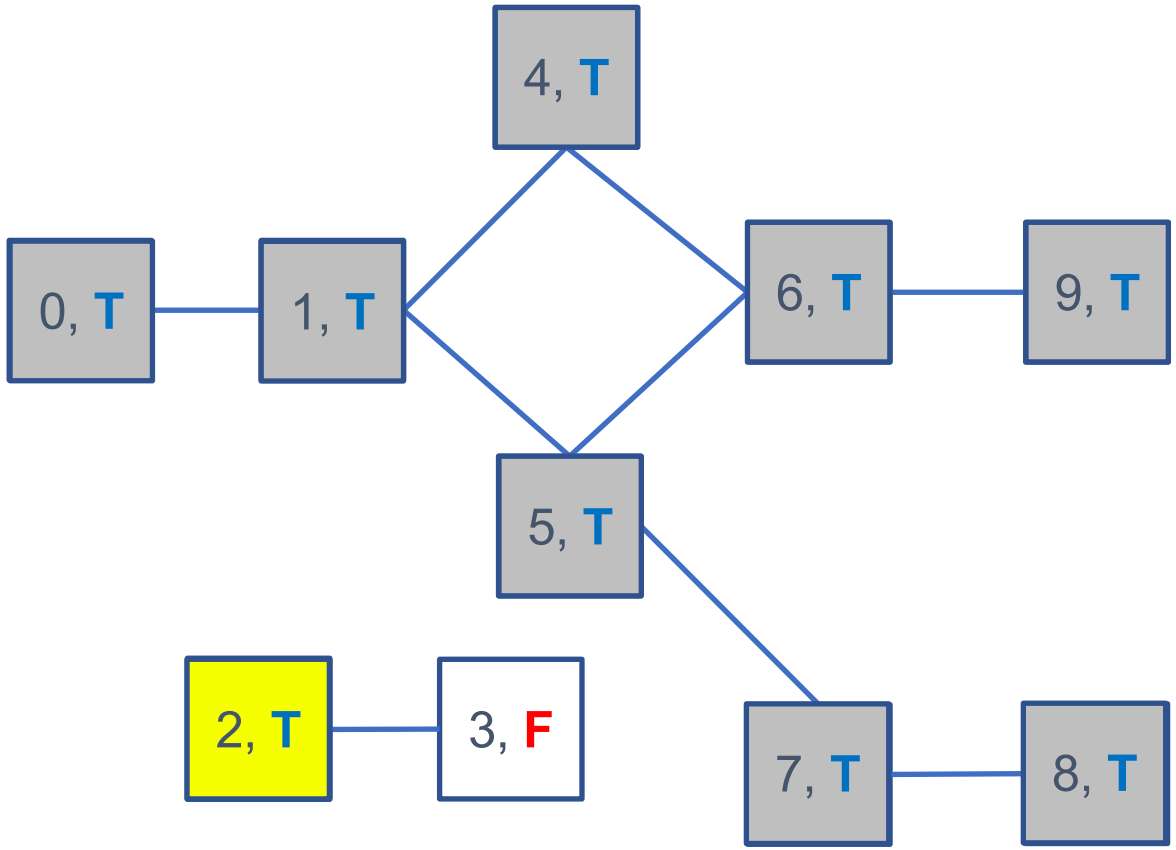

Out: 0 1 4 6 5 7 8 9 2

Depth-First Traversal – Preorder

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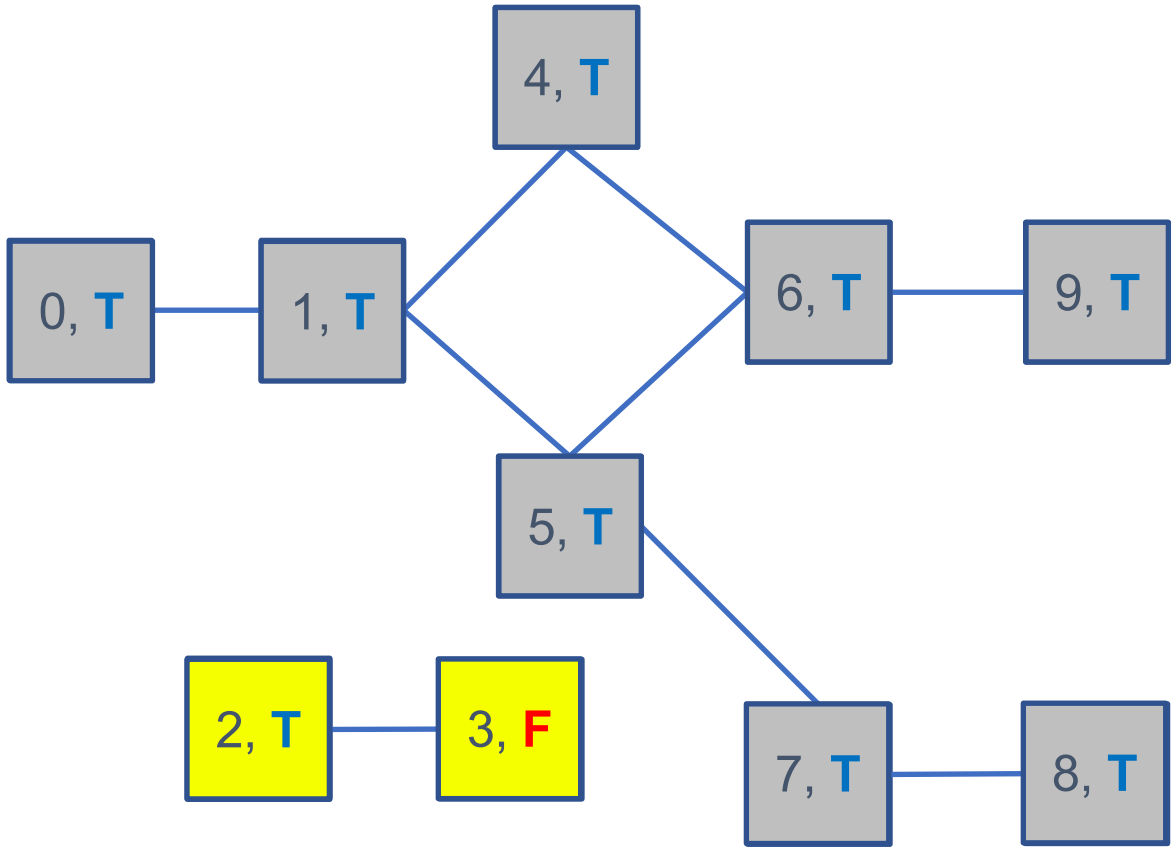

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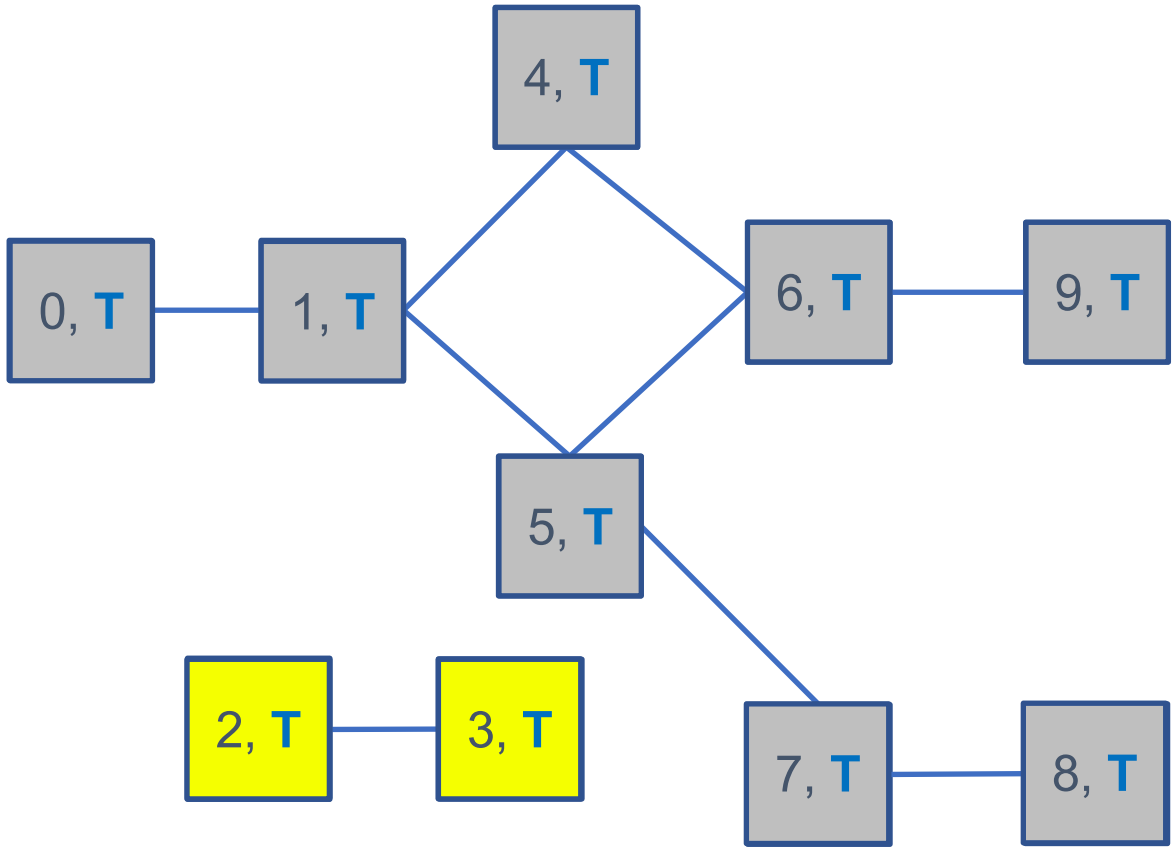

Out: 0 1 4 6 5 7 8 9 2 3

Depth-First Traversal – Preorder

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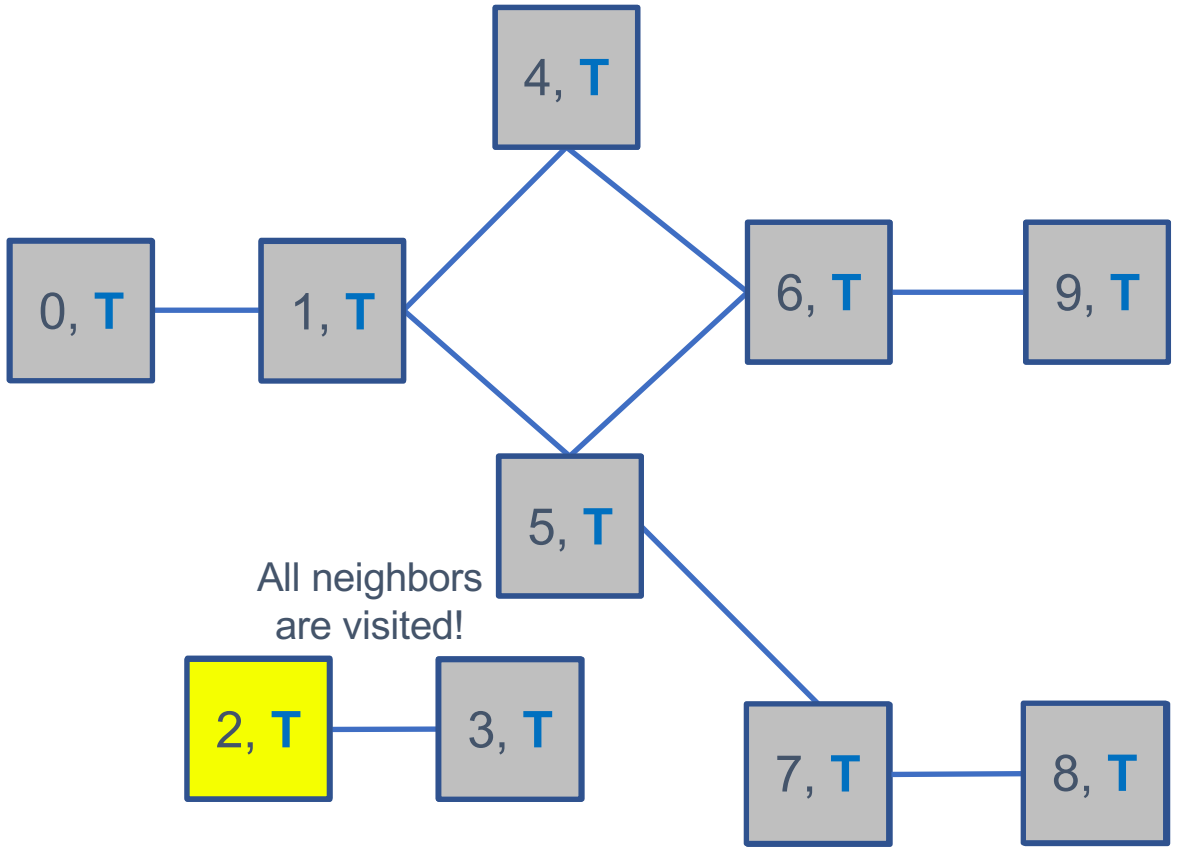

Out: 0 1 4 6 5 7 8 9 2 3

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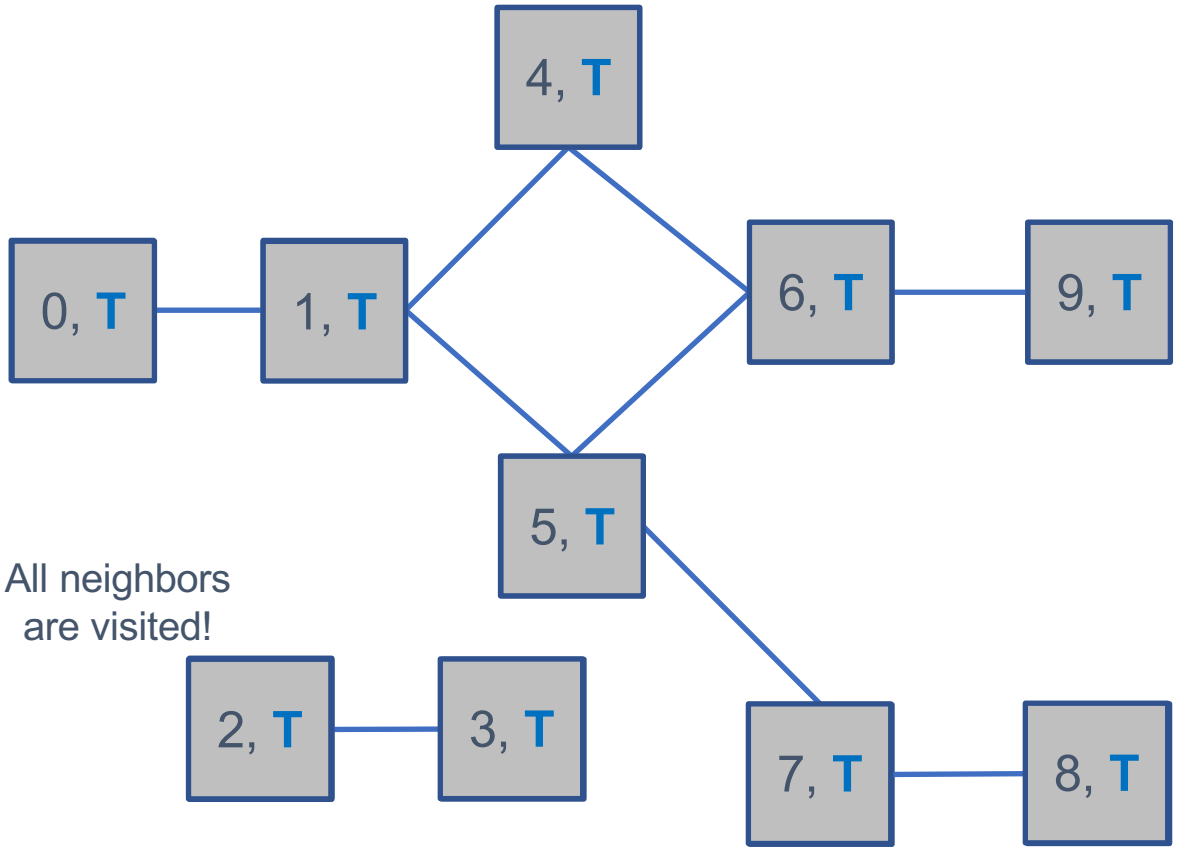

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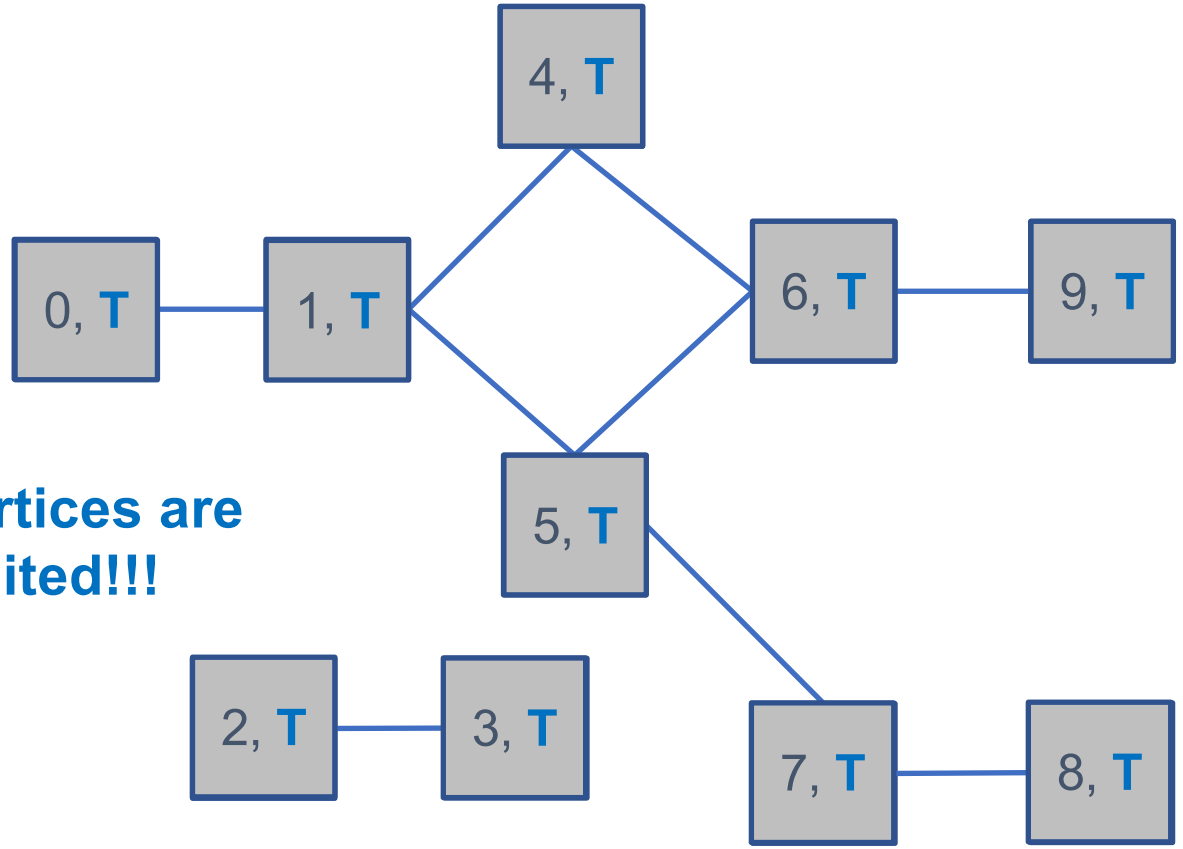


All vertices are visited!!!

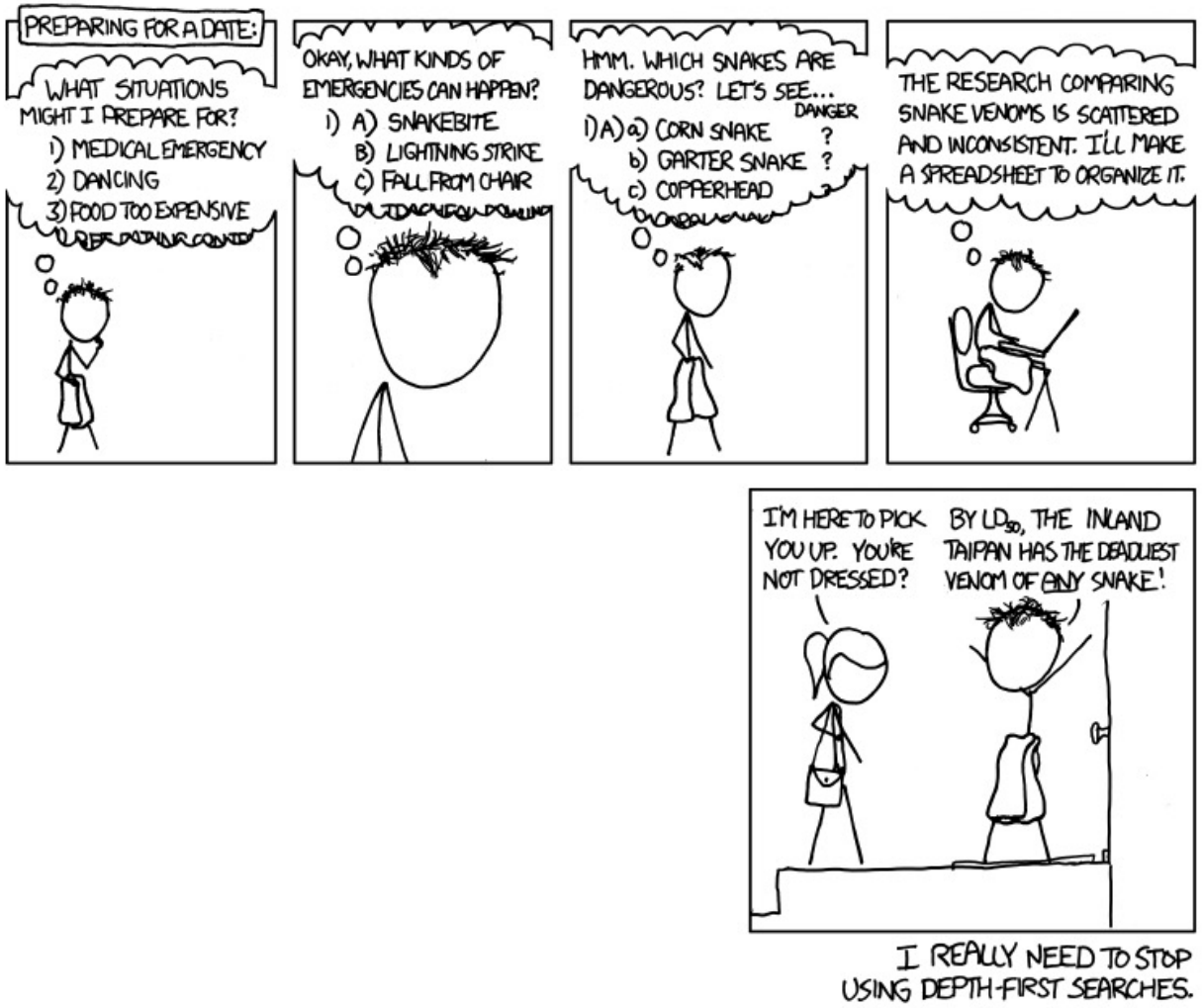
Out: 0 1 4 6 5 7 8 9 2 3

Depth-First Traversal

- When you want to know if two vertices are **connected**
- When you want to know if all vertices in a graph are **connected**
- When you want to know how many **disjoint islands** are in a graph
- When you want to know if a graph has a **cycle**

Depth-First Traversal

- https://xkcd.com/761/



Summary

- Graph is a more general concept than tree
  - Cycle and disconnection

- Graph traversals are a bit different from tree traversals but their core ideas are the same
  - Depth first traversals
  - Breadth first traversals

- Graph traversals are the most common tools for solving graph problems

Thanks!