Projected Gradient Waveform Design for Fully Adaptive Radar STAP

Pawan Setlur
Wright State Research Institute
Beavercreek, OH 45431

Muralidhar Rangaswamy
US Air Force Research Laboratory
Sensors Directorate
WPAFB, OH 45433

***Abstract*—We consider waveform design for radar space time adaptive processing (STAP), accounting for the waveform dependence of the clutter correlation matrix. It was shown previously in [1], that the joint problem of receiver weight vector optimization and radar waveform design is an intractable optimization problem, and constrained alternating minimization was proposed. In this paper, we propose projected gradient minimization. This minimization algorithm affords a numerically stable, lower computational complexity solution than the previously proposed alternating minimization. However, as a trade-off it results in larger iteration counts to achieve similar error variances of the STAP filter. Step size rules and the optimal step size for descent are derived analytically and validated via numerical simulations.**

I. Introduction

In this paper we addresses waveform design for radar STAP [2]–[4] considering the important problem of the dependence of the waveform on the clutter correlation matrix. The noise and interference are also considered but unlike the clutter, are not dependent on the waveform. For convenience, an airborne calibrated uniform linear array is assumed in the modeling. Other common assumptions made in this paper are identical to those made in [1], [5] and can be seen there. A minimum variance distortionless response (MVDR) optimization objective is formulated and is a function of the traditional STAP weight vector as well as the waveform.

It was shown previously that the joint receive filter and waveform design for radar STAP does not have a closed form solution [1], [5]. The STAP objective is individually convex in either the waveform or in the filter design, but not both [1]. This lent a strong motivation to analyze the alternating minimization technique, and was the focus in [1]. Practical waveform constraints pertinent to the radar STAP problem were considered and closed form solutions to the resulting constrained alternating minimization were subsequently derived in [1].

As such, classical radar airborne STAP is computationally heavy and practical real time implementation is always challenging [2]–[4]. Waveform adaptive STAP was shown to increase the computational complexity even further [1], [5]. The solutions in [1] involved large matrix inversions for each iteration in the waveform and filter design stage, and therefore are computationally heavy on the already stretched STAP processor. In this paper, we revisit the waveform adaptive STAP problem while restricting the solutions to simple and computationally efficient algorithms. Toward this end, we consider constrained gradient projection alternating minimization for the filter and waveform design stages. Like the constrained alternating minimization, the projected gradient version is iterative, but differs from the former by employing a gradient descent along with a projection onto the constraint sets. In essence, the former uses the exact minimization for the objective at each stage while the latter uses an approximated objective in the filter design and waveform design stages. The projected gradient descent solutions do not require matrix inversion but only require simple matrix vector multiplication.

Step size rules for the projected gradient are stated and are numerically validated via simulations. The optimal line search as in the classical gradient descent is extended to the projected gradient minimization for the radar STAP waveform design problem.

Literature: Clutter dependence on the waveform was recognized early on in [6] and references therein. However the iterative solution proposed in [6] is for a single sensor radar and is not readily extended to a multi-sensor radar framework such as STAP. In [5], STAP radar waveform design ignoring the signal dependence of clutter was treated, giving rise to the well known minimum eigenvector solution. Other iterative solutions similar to the constrained alternating alternating minimization were proposed but not for STAP in [7], [8],[9]–[11], for the joint optimization of the receive filter and the waveform design. Waveform design for radar using metrics other than MVDR have been considered in the past. Signal dependent clutter waveform design with the objective of maximizing detection was formulated in [12] and gave rise to a waterfilling like solution [13]. Waveforms designed from maximizing the one step, the two step mutual information, and considering the signal independent / dependent clutter were the subject of [13], [14] and [15], respectively.

Organization: The paper is organized as follows, in Section II, the model is presented and is similar to the one derived in [5]. The waveform design is considered in Section III, first for the general unconstrained waveform, and next for waveform constrained to a constant modulus design. Supporting simulations are presented in section IV, and conclusions are drawn in Section V.

978-1-4799-8232-5/151$31.00@2015IEEE

II. STAP MODEL

The radar consists of an air-borne calibrated uniform linear array comprising M sensor elements, which transmits a burst of L pulses in a coherent processing interval (CPI). The waveform transmitted is assumed to be discretized and consists of N samples.

At the considered range gate, the contaminated snapshot from the STAP data cube is modeled as:

$$\begin{aligned}\bar{\mathbf{y}} &= \mathbf{y} + \mathbf{y_i} + \mathbf{y_c} + \mathbf{y_n} \\ &= \mathbf{y} + \mathbf{y_u}\end{aligned} \tag{1}$$

where $\mathbf{y_i}, \mathbf{y_c}, \mathbf{y_n}$ are the contributions from the interference, clutter and noise, respectively, and are assumed to be statistically uncorrelated with one another. The target response is denoted as $\mathbf{y} \in \mathbb{C}^{NML}$ and is given by

$$\mathbf{y} = \rho_t \mathbf{v}(f_d) \otimes \mathbf{s} \otimes \mathbf{a}(\theta_t, \phi_t) \tag{2}$$

where ρ_t is the target reflectivity assumed to be random but identical for the entire CPI. In (2), the discretized waveform vector is $\mathbf{s} = [s(0), s(1), \ldots, s(N-1)]^T \in \mathbb{C}^N$. Similarly, the targets spatial and temporal steering vectors are denoted as $\mathbf{a}(\theta_t, \phi_t)$ and $\mathbf{v}(f_d)$, respectively, where (θ_t, ϕ_t) are the targets azimuth and elevation angles, and f_d is the Doppler frequency of the target. More details are readily seen in [5]. The contribution of the undesired returns are treated next, starting with the noise as it is the simplest.

Noise: The noise is assumed to be zero mean, identically distributed across the sensors, across pulses, and in the fast time samples. The correlation matrix of $\mathbf{y_n}$ is denoted as $\mathbf{R_n} \in \mathbb{C}^{NML \times NML}$.

Interference: The interference consists of jammers. Let us assume that there are K interference sources,the correlation matrix for the interference is modeled as [5]:

$$\mathbf{R_i} = \mathbf{A}(\theta, \phi)\mathbf{R}_{\boldsymbol{\alpha}}\mathbf{A}(\theta, \phi)^H \tag{3}$$

where

$$\begin{aligned}&\mathbf{R}_{\boldsymbol{\alpha}} := \mathrm{Diag}\{\mathbf{R}_{\boldsymbol{\alpha}}^1, \mathbf{R}_{\boldsymbol{\alpha}}^2, \ldots, \mathbf{R}_{\boldsymbol{\alpha}}^K\} \in \mathbb{C}^{NMLK \times NMLK} \\ &\mathbf{A}(\theta, \phi) \in \mathbb{C}^{NML \times NLMK} \\ &:= [\mathbf{I}_{NL} \otimes \mathbf{a}(\theta_1, \phi_1), \mathbf{I}_{NL} \otimes \mathbf{a}(\theta_2, \phi_2), \ldots, \mathbf{I}_{NL} \otimes \mathbf{a}(\theta_K, \phi_K)],\end{aligned}$$

for $\mathbf{I}_{NL}$ the identity matrix of size $NL \times NL$, and $\mathrm{Diag}\{\cdot, \cdot, \ldots, \cdot\}$ the matrix diagonal operator which converts the matrix arguments into a bigger diagonal matrix. For example, $\mathrm{Diag}\{\mathbf{A}, \mathbf{B}, \mathbf{C}\} = \begin{bmatrix} \mathbf{A} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{C} \end{bmatrix}$.

Clutter: The ground is a major source of clutter in air-borne radar applications and is present in all range gates upto the gate corresponding to the platform horizon. As in [5], other sources of clutter are ignored. Let us assume that there are Q ground clutter patches indexed by parameter q comprising of P scatterers in each patch, indexed by p. The radar return from the p-th scatterer in the q-th clutter patch is given by

$$\gamma_{pq} \otimes \mathbf{v}(fc_q) \otimes \mathbf{s} \otimes \mathbf{a}(\theta_q, \phi_q)$$

where γ_{pq} is its random complex reflectivity, and fc_q is the Doppler shift observed from the q-th clutter patch, and (θ_q, ϕ_q) are the azimuth and elevation angles of the q-th clutter patch. It is implicitly assumed that the scatterers in a particular clutter patch have identical Doppler as they are in the same range gate. It is assumed that the spatial responses of scatterers in the same clutter patch are identical to one another, and thus the clutter correlation matrix for the q-th patch is written as: $\mathbf{R}_{\boldsymbol{\gamma}}^q := \mathbf{B_q}\mathbf{R}_{\boldsymbol{\gamma}}^{pq}\mathbf{B_q}^H$ where, $\mathbf{B_q} = [\mathbf{v}(fc_q) \otimes \mathbf{s} \otimes \mathbf{a}(\theta_q, \phi_q), \ldots, \mathbf{v}(fc_q) \otimes \mathbf{s} \otimes \mathbf{a}(\theta_q, \phi_q)] \in \mathbb{C}^{NML \times P}$ and $\mathbf{R}_{\boldsymbol{\gamma}}^{pq} \in \mathbb{C}^{P \times P}$ is the correlation matrix of the random vector, $[\gamma_{1q}, \gamma_{2q}, \ldots, \gamma_{Pq}]^T$. The matrix $\mathbf{B_q}$ can further be simplified as $\mathbf{B_q} := \breve{\mathbf{B}}_{\mathbf{q}}(\mathbf{I}_P \otimes \mathbf{s})$, where $\breve{\mathbf{B}}_{\mathbf{q}} := [\mathbf{v}(fc_q) \otimes \mathbf{A_q}, \ldots, \mathbf{v}(fc_q) \otimes \mathbf{A_q}] \in \mathbb{C}^{NML \times PN}$, and the structure of the matrix $\mathbf{A_q} \in \mathbb{C}^{NM \times N}$ (not shown here) is defined such that $\mathbf{s} \otimes \mathbf{a}(\theta_q, \phi_q) = \mathbf{A_q}\mathbf{s}$. Therefore, using these simplifications, the clutter correlation matrix for the q-th patch is expressed as:

$$\mathbf{R}_{\boldsymbol{\gamma}}^q = \breve{\mathbf{B}}_{\mathbf{q}}(\mathbf{I}_P \otimes \mathbf{s})\mathbf{R}_{\boldsymbol{\gamma}}^{pq}(\mathbf{I}_P \otimes \mathbf{s}^H)\breve{\mathbf{B}}_{\mathbf{q}}^H. \tag{4}$$

Assuming that a particular scatterer from one clutter patch is uncorrelated to any other scatterer belonging to any other clutter patch, the resulting clutter correlation matrix is given by

$$\mathbf{R_c} = \sum_{q=1}^{Q} \mathbf{R}_{\boldsymbol{\gamma}}^q. \tag{5}$$

III. WAVEFORM DESIGN

The joint receiver filter and transmit waveform design optimization is formulated.

A. Preliminaries

The radar return at the considered range gate is processed by a filter characterized by a weight vector, $\mathbf{w}$, whose output is given by $\mathbf{w}^H\bar{\mathbf{y}}$. Since the vector $\mathbf{s}$ prominently figures in the steering vectors, the objective is to jointly obtain the desired weight vector, $\mathbf{w}$, and waveform vector, $\mathbf{s}$. Mathematically, we may formulate this problem as:

$$\begin{aligned}\min_{\mathbf{w},\mathbf{s}} \quad & \mathbb{E}\{|\mathbf{w}^H\mathbf{y_u}|^2\} \\ \text{s. t} \quad & \mathbf{w}^H(\mathbf{v}(f_d) \otimes \mathbf{s} \otimes \mathbf{a}(\theta_t, \phi_t)) = \kappa \\ & \mathbf{s}^H\mathbf{s} \le P_o\end{aligned} \tag{6}$$

where, we may rewrite $\mathbb{E}\{|\mathbf{w}^H\mathbf{y_u}|^2\} = \mathbf{w}^H\mathbf{R_u}(\mathbf{s})\mathbf{w}$. In (6), a power constraint is enforced via the second constraint to addresses hardware limitations. Optimizing (6) w.r.t $\mathbf{w}$ first, the solution to (6) is well known, and expressed as

$$\mathbf{w}_o = \frac{\kappa\mathbf{R_u}^{-1}(\mathbf{s})(\mathbf{v}(f_d) \otimes \mathbf{s} \otimes \mathbf{a}(\theta_t, \phi_t))}{(\mathbf{v}(f_d) \otimes \mathbf{s} \otimes \mathbf{a}(\theta_t, \phi_t))^H\mathbf{R_u}^{-1}(\mathbf{s})(\mathbf{v}(f_d) \otimes \mathbf{s} \otimes \mathbf{a}(\theta_t, \phi_t))} \tag{7}$$

where $\mathbf{R_u}(\mathbf{s}) = \mathbf{R_i} + \mathbf{R_c}(\mathbf{s}) + \mathbf{R_n}$. We further emphasize that the the weight vector is an explicit function of the waveform. Now substituting $\mathbf{w}_o$ back into the cost function in (6), the minimization is purely w.r.t $\mathbf{s}$, and cast as,

$$\min_{\mathbf{s}} \frac{\kappa^2}{(\mathbf{v}(f_d)\otimes\mathbf{s}\otimes\mathbf{a}(\theta_t,\phi_t))^H\mathbf{R_u}^{-1}(\mathbf{s})(\mathbf{v}(f_d)\otimes\mathbf{s}\otimes\mathbf{a}(\theta_t,\phi_t))}$$
$$\text{s. t. } \mathbf{s}^H\mathbf{s} \leq P_o \tag{8}$$

A solution to (8) is not immediate, given the dependence of $\mathbf{R_u}$ on the waveform vector $\mathbf{s}$. It was shown in [5] that ignoring the waveform dependence, the solution to (8) is a simple Rayleigh-Ritz optimization leading to the minimum eigenvector solution, with appropriate scaling in order to satisfy the power constraint.

B. Constrained Projected Gradient Alternating Minimization

Consider for the k-th iteration the projected gradient descent algorithm, for the filter design, expressed as

$$\begin{aligned}\mathbf{w}_k =& \text{Proj}\,(\hat{\mathbf{w}}_k) \\ &\text{s.t. } \mathbf{w}_k \in \mathcal{C}_{1k} \\ \hat{\mathbf{w}}_k =& \mathbf{w}_{k-1} - \alpha_k\mathbf{R}_u(\mathbf{s}_{k-1})\mathbf{w}_{k-1}\end{aligned} \tag{9}$$

where Proj$(\cdot)$ is the projection of a vector onto a convex set. The constraint set varies from one iteration to another and is defined as, $\mathcal{C}_{1k} := \{\mathbf{w} \mid \mathbf{w}^H\mathbf{G}\mathbf{s}_{k-1} = \kappa\}$. The matrix $\mathbf{G}$ can be inferred from the linear relation, $(\mathbf{v}(f_d)\otimes\mathbf{s}_{k-1}\otimes\mathbf{a}(\theta_t,\phi_t)) = \mathbf{G}\mathbf{s}_{k-1}$. In (9) the varying step size controlling descent is α_k, assumed to be greater than zero. A solution to (9) may be readily shown, and is given by

$$\mathbf{w}_k = \hat{\mathbf{w}}_k - \left(\frac{\hat{\mathbf{w}}_k^H\mathbf{G}\mathbf{s}_{k-1}-\kappa}{||\mathbf{G}\mathbf{s}_{k-1}||^2}\right)^*\mathbf{G}\mathbf{s}_{k-1} \tag{10}$$

where $(\cdot)^*$ is the complex conjugate of $(\cdot)$.

Remark 1. At the k-th iteration, $\mathbf{w}_k = \hat{\mathbf{w}}_k$ iff $\hat{\mathbf{w}}_k^H\mathbf{G}\mathbf{s}_{k-1} = \kappa$.

This is readily seen from (10), but more importantly, the gradient projection is the classical gradient descent when the constraints are satisfied by the gradient descent iteration.

For the the waveform design update, at the k-th iteration the gradient projection is formulated as

$$\begin{aligned}\mathbf{s}_k =& \text{Proj}\,(\hat{\mathbf{s}}_k) \\ &\text{s.t. } \mathbf{s}_k \in \mathcal{C}_{2k} \\ \hat{\mathbf{s}}_k =& \mathbf{s}_{k-1} - \beta_k\mathbf{Z}(\mathbf{w}_k)\mathbf{s}_{k-1}\end{aligned} \tag{11}$$

where $\mathbf{Z}(\mathbf{w}_k) = \sum_{q=1}^{Q}\mathbf{H}^T(\mathbf{R}_\gamma^{pq}\otimes\mathbf{x_q}\mathbf{x_q}^H)\mathbf{H}$, $\mathbf{x_q} \in \mathbb{C}^{NP} := \breve{\mathbf{B}}_{\mathbf{q}}^H\mathbf{w}_k$. The matrix $\mathbf{H} \in \mathbb{R}^{P^2N\times N}$ can be inferred from the linear equation, $\text{vec}(\mathbf{I}_P\otimes\mathbf{s}_k) = \mathbf{H}\mathbf{s}_k$ (also see the discussion immediately after (18)). The constraint set, $\mathcal{C}_{2k} := \{\mathbf{s} \mid \mathbf{w}_k^H\mathbf{G}\mathbf{s} = \kappa, \mathbf{s}^H\mathbf{s} = P_o\}$. In (11) the varying step size controlling descent is β_k, assumed to be greater than zero. A solution to (9) may be readily shown, and is given by

$$\mathbf{s}_k = \hat{\mathbf{s}}_k - \left(\frac{\mathbf{w}_k^H\mathbf{G}\hat{\mathbf{s}}_{\mathbf{k}}-\kappa}{||\mathbf{w}_k^H\mathbf{G}||^2}\right)(\mathbf{w}_k^H\mathbf{G})^H. \tag{12}$$

Remark 2. At the k-th iteration, $\mathbf{s}_k = \hat{\mathbf{s}}_k$ iff $\mathbf{w}_k^H\mathbf{G}\hat{\mathbf{s}}_k = \kappa$.

This is readily seen from (12), and is similar to Rem.1. Therefore, the conclusions drawn immediately after Rem.1 also apply here, albeit for the waveform design nonetheless.

The iterative nature of the constrained projected gradient alternating minimization is highlighted in Table I.

TABLE I
CONSTRAINED PROJECTED GRADIENT ALTERNATING MINIMIZATION FOR JOINT FILTER AND WAVEFORM ADAPTIVE RADAR STAP

1) *Initialize*: Start with an initial filter and waveform design, defined as $\mathbf{w}_0, \mathbf{s}_0$, set counter $k = 1$.
2) *Filter design*: Using (10) obtain *updated* filter $\mathbf{w}_k$.
3) *Waveform design*: Using (12) obtain *updated* waveform, $\mathbf{s}_k$.
4) *Check:* If convergence is achieved, exit, else $k = k+1$, go back to step-2 and repeat.

C. Step Size Rules: Gradient Projection

Some definitions, and facts are useful for future discussions, expressed below.

Definition 1. (*Lipschitz continuous gradient*) A function $f(\bar{\mathbf{x}}) : \mathbb{R}^N \to \mathbb{R}$ has a Lipschitz constant (and trivially real positive), L, when $||\nabla_{\bar{\mathbf{x}}}f(\bar{\mathbf{x}}) - \nabla_{\bar{\mathbf{y}}}f(\bar{\mathbf{y}})|| \leq \mathtt{L}||\bar{\mathbf{x}}-\bar{\mathbf{y}}||$, and $\forall\bar{\mathbf{x}}, \bar{\mathbf{y}} \in \mathbb{R}^N$.

Fact 1. *The Lipschitz constant for the complex quadratic form $f(\mathbf{x}) : \mathbb{C}^N \to \mathbb{R} = \mathbf{x}^H\mathbf{B}\mathbf{x}$ is the maximum eigenvalue of $\mathbf{B}$, i.e. $\lambda_{\max}(\mathbf{B})$, where $\mathbf{B} \in \mathbb{C}^{N\times N}, \mathbf{B} = \mathbf{B}^H$ $\mathbf{x} \in \mathbb{C}^N$.*

The following may be derived analytically but are stated here as facts, without proof. Define $f(\mathbf{w},\mathbf{s}) := \mathbf{w}^H\mathbf{R}_u(\mathbf{s})\mathbf{w}$. Then, we have the following.

Fact 2. *To guarantee descent at the k-th iteration for the filter design, i.e. $f(\mathbf{w}_k,\mathbf{s}_{k-1}) \leq f(\mathbf{w}_{k-1},\mathbf{s}_{k-1})$, we have $\alpha_k \leq \frac{2}{L_{1k}}$*

Fact 3. *To guarantee descent at the k-th iteration for the waveform design, i.e. $f(\mathbf{w}_k,\mathbf{s}_k) \leq f(\mathbf{w}_k,\mathbf{s}_{k-1})$, we have $\beta_k \leq \frac{2}{L_{2k}}$*

In the above Fact 2 and Fact 3, L_{1k}, L_{2k} denote the Lipschitz constants, associated with the k-th iteration of the filter design and waveform design steps, respectively. From, Fact 1, it may be readily seen that $L_{1k} = \lambda_{\max}(\mathbf{R}_u(\mathbf{s}_{k-1}), L_{2k} = \lambda_{\max}(\mathbf{Z}(\mathbf{w}_k))$. Monotonic decrease in the objective function for each iteration is guaranteed if the step sizes are chosen to satisfy the relations in Fact 2 and Fact 3.

Instead of using the step sizes as defined by Fact 2 and Fact 3, we may analytically derive optimal step sizes. For the filter design and for the k-th iteration, this optimization is cast as

$$\min_{\alpha_k} \quad \mathbf{w}_k^H\mathbf{R}_u(\mathbf{s}_{k-1})\mathbf{w}_k \tag{13}$$

It may be shown readily that a solution to (13), denoted as

α_{ko} is

$$\alpha_{ko} = \frac{\mathrm{Re}\{f_1(\mathbf{w}_{k-1};\mathbf{s}_{k-1})\}}{\mathrm{Re}\{f_2(\mathbf{w}_{k-1};\mathbf{s}_{k-1})\}}. \tag{14}$$

where after straightforward simplifications, $f_i(\cdot,\cdot), i = 1, 2$ are functions of $\mathbf{w}_{k-1}, \mathbf{s}_{k-1}$, readily derived by equating the gradient of (14) to zero.

In the same spirit, for the waveform design and for the k-th iteration, this optimization for the optimal step size is cast as

$$\min_{\beta_k} \quad \mathbf{w}_k^H \mathbf{R}_u(\mathbf{s}_k)\mathbf{w}_k. \tag{15}$$

A solution to (15), denoted as β_{ko} is expressed as,

$$\beta_{ko} = \frac{\mathrm{Re}\{h_1(\mathbf{w}_k;\mathbf{s}_{k-1})\}}{\mathrm{Re}\{h_2(\mathbf{w}_k;\mathbf{s}_{k-1})\}} \tag{16}$$

where after straightforward simplifications, $h_i(\cdot,\cdot), i = 1, 2$ are functions of $\mathbf{w}_k, \mathbf{s}_{k-1}$, readily derived by equating the gradient of (15) to zero.

D. Alternating Minimization Solutions

For the radar STAP adaptive waveform design problem, the alternating minimization technique proposed in [1] is highlighted for comparison (see also [16] for solutions in closed form). This algorithm is iterative, with the solutions for the k-th iteration, as expressed below [16].

$$\mathbf{w}_k = \frac{\kappa \mathbf{R_u}^{-1}(\mathbf{s}_{k-1})(\mathbf{v}(f_d)\otimes \mathbf{s}_{k-1}\otimes \mathbf{a}(\theta_t,\phi_t))}{(\mathbf{v}(f_d)\otimes \mathbf{s}_{k-1}\otimes \mathbf{a}(\theta_t,\phi_t))^H \mathbf{R_u}^{-1}(\mathbf{s}_{k-1})(\mathbf{v}(f_d)\otimes \mathbf{s}_{k-1}\otimes \mathbf{a}(\theta_t,\phi_t))} \tag{17}$$

$$\mathbf{s}_k = \frac{\kappa\left(\sum_{q=1}^{Q}\mathbf{Z_q}(\mathbf{w}_k)\right)^{-1}\mathbf{G}^H\mathbf{w}_k}{\mathbf{w}_k^H\mathbf{G}\left(\sum_{q=1}^{Q}\mathbf{Z_q}(\mathbf{w}_k)\right)^{-1}\mathbf{G}^H\mathbf{w}_k}. \tag{18}$$

where $\mathbf{Z_q}(\mathbf{w}_k) = \mathbf{H}^T(\mathbf{R}_\gamma^{pq}\otimes \mathbf{x_q}\mathbf{x_q}^H)\mathbf{H}$, and $\mathbf{x_q} \in \mathbb{C}^{NP} := \breve{\mathbf{B}}_\mathbf{q}^H\mathbf{w}_k$. Further, here we also have $\mathrm{vec}(\mathbf{I}_P\otimes \mathbf{s}) = \mathbf{Hs}$, with $\mathbf{H}\in\mathbb{R}^{P^2N\times N} = [\mathbf{H_1}^T, \mathbf{H_2}^T, \ldots, \mathbf{H_P}^T]^T$. The matrix $\mathbf{H_k} \in \mathbb{R}^{PN\times N}, k = 1, 2, \ldots, P$ is further decomposed into P, $N\times N$ matrices, and is defined such that the k-th $N\times N$ matrix is $\mathbf{I}_N$ and the other $(N-1)$, $N\times N$ matrices are all equal to zero matrices.

IV. Simulations

We considered $M = 5, L = 32, N = 5$. The element spacing i.e. $d = \lambda_o/2$. The noise correlation matrix was assumed to have a correlation function given by $\exp(-|0.005n|), n = 0, 1, \ldots, NML$. The carrier frequency was chosen to be 1GHz, and the radar bandwidth was 50MHz. Two interference sources were considered at $(\theta = 0.3941, \phi = 0.3)$ and at $(-0.4941, 0.3)$. Both these interference sources had identical discrete correlation functions given by $0.2^{|n|}, n = \pm 0, \pm 1, \ldots$. To simulate clutter we considered two clutter patches, consisting of five scatters each. The clutter correlation functions corresponding to the two patches were

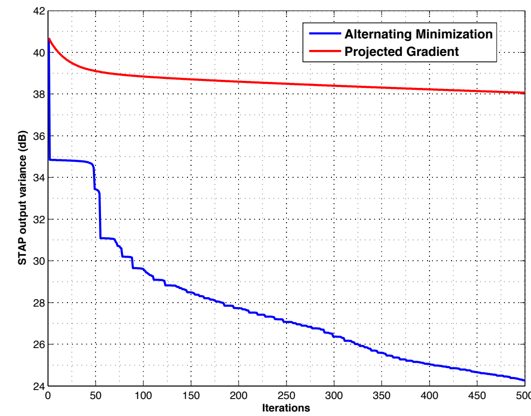

Fig. 1. Alternating minimization vs. projected gradient for the joint STAP filter and STAP waveform design.

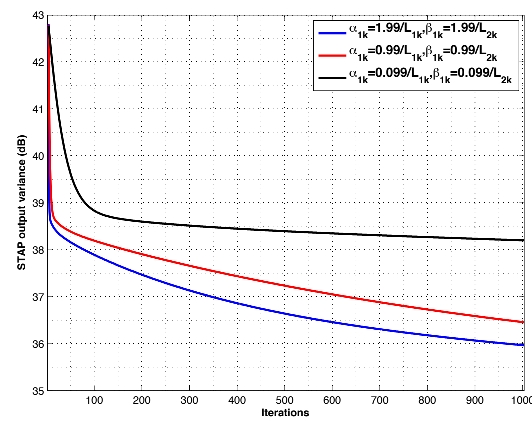

Fig. 2. Projected gradient with step sizes related to the Lipschitz constants.

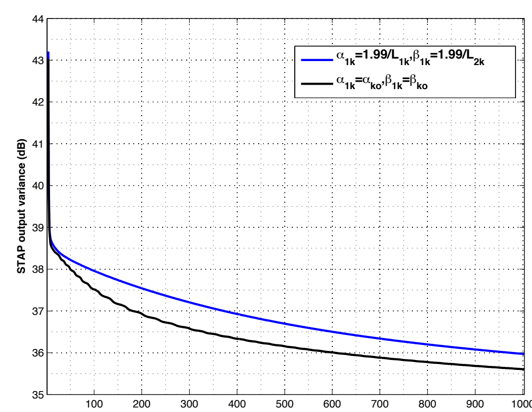

Fig. 3. Projected gradient comparison with the upper bound Lipschitz step size and the optimal step size.

$\exp(-0.2|p|)$ and $\exp(-0.1|p|), p = \pm 0, \pm 1, \ldots, \pm P$. The rest of the parameters are identical to those used in [1].

A. Comparison with Alternating minimization

In Fig. 1, the constrained projected gradient derived here vs. the constrained alternating minimization derived in [1] are compared. Both the algorithms were initialized identically and therefore start with the same objective value. One one hand, the projected gradient algorithm, per iteration is computationally cheap and and therefore fast, but overall takes a larger iteration count to achieve a similar objective value as the constrained alternating minimization. On the other hand, the constrained alternating minimization is computationally heavy and involves large matrix inversions, especially in the filter design step and for each iteration. The peculiar behavior of these algorithms may be explained as follows. From a simulation perspective, the constrained alternating minimization inherently scales the waveform energy from one iteration to another, resulting in larger decreases in the STAP objective from one iteration to another. This is not the case with the projected gradient as it is permitted to move only slightly for each iteration along the negative gradient. Another straightforward explanation for larger iteration counts of the projected gradient is that a quadratic approximation is used for each iteration. Whereas for the constrained alternating minimization the exact objective is minimized during the filter design stage as well as the waveform design stage.

B. Step size rules: validation

In Fig. 2, the projected gradient algorithm was tested with different step sizes but were nonetheless still related to the Lipschitz constants. Three step sizes rules were considered, namely, $\alpha_{1k} = \frac{1.99}{L_{1k}}, \beta_{1k} = \frac{1.99}{L_{2k}}$, $\alpha_{1k} = \frac{0.99}{L_{1k}}, \beta_{1k} = \frac{0.99}{L_{2k}}$, and $\alpha_{1k} = \frac{0.099}{L_{1k}}, \beta_{1k} = \frac{0.099}{L_{2k}}$. The results are shown in Fig.2. As expected the first step size rule has the largest displacement along the gradient direction and therefore performs better than the other two step size rules. Surprisingly however, the middle curve (red) in Fig. 2 is only approximately 0.5 dB away from the lower curve (blue). The middle and lower curves in Fig.2 were generated by using their respective step sizes which differ only by one half.

C. Optimal step size comparison

In Fig. 3, the results comparing the optimal step size as derived in (14) and (16) to $\alpha_{1k} = \frac{1.99}{L_{1k}}, \beta_{1k} = \frac{1.99}{L_{2k}}$ are shown. In the worst case over the 1000 iterations, the two curves differ by 0.7 dB. It is noted that since the Lipschitz constants are related to the maximum eigenvalue, an eigen-decomposition may be used to compute them. This is computationally expensive. However, the optimal step sizes are straightforward to compute and without requiring any computationally heavy eigen-decomposition.

V. Conclusions

Waveform design in STAP was the focus of this paper assuming the dependence of the clutter response on the transmitted waveform. A projected gradient algorithm was derived for the joint receive filter and waveform design optimization for radar STAP. This algorithm was compared to the constrained alternating minimization as derived in [1]. Numerical simulations demonstrate a slower convergence rate for the projected gradient algorithm than the constrained alternating minimization. Nevertheless, the projected gradient is computationally cheap unlike the constrained alternating minimization which inverts large matrices for each iteration in the waveform design and the filter design steps. The projected gradient solutions does not require full rank matrices for computations. Therefore as a next step, it may be suitable to compare its performance to the proximal constrained algorithms for radar STAP [17]. Step size rules guaranteeing descent were stated and simulations were shown to validate them numerically. Expressions for the optimal step size in the filter design stage as well as, the waveform design stage were derived.

Acknowledgment

This work was sponsored by US AFOSR under project 13RY10COR. All views and opinions expressed here are the authors own and does not constitute endorsement from the Department of Defense.

References

[1] P. Setlur and M. Rangaswamy, "Signal dependent clutter waveform design for radar STAP," in *In Proc. IEEE Radar Conference*, 2014.

[2] R. Klemm, *Principles of Space-Time Adaptive Processing*. Institution of Electrical Engineers, 2002.

[3] J. Ward, *Space-time Adaptive Processing for Airborne Radar*, ser. Technical report (Lincoln Laboratory). Massachusetts Institute of Technology, Lincoln Laboratory, 1994.

[4] J. Guerci, *Space-Time Adaptive Processing for Radar*. Artech House, 2003.

[5] P. Setlur, N. Devroye, and M. Rangaswamy, "Waveform design and scheduling in space-time adaptive radar," in *In proc. IEEE Radar Conference*, 2013.

[6] D. DeLong and E. Hofstetter, "On the design of optimum radar waveforms for clutter rejection," *IEEE Trans. Inf. Theory*, vol. 13, no. 3, pp. 454–463, 1967.

[7] S. Pillai, H. Oh, D. Youla, and J. Guerci, "Optimal transmit-receiver design in the presence of signal-dependent interference and channel noise," *IEEE Trans. Inf. Theory*, vol. 46, no. 2, pp. 577–584, 2000.

[8] C.-Y. Chen and P. Vaidyanathan, "MIMO radar waveform optimization with prior information of the extended target and clutter," *TransSP*, vol. 57, no. 9, pp. 3533–3544, 2009.

[9] A. Aubry, A. DeMaio, A. Farina, and M. Wicks, "Knowledge-aided (potentially cognitive) transmit signal and receive filter design in signal-dependent clutter," *IEEE Trans. Aerospace and Electronic Systems*, vol. 49, no. 1, pp. 93–117, Jan 2013.

[10] G. Cui, H. Li, and M. Rangaswamy, "MIMO radar waveform design with constant modulus and similarity constraints," *IEEE Trans. Signal Processing*, vol. 62, no. 2, pp. 343–353, Jan 2014.

[11] P. Stoica, H. He, and J. Li, "Optimization of the receive filter and transmit sequence for active sensing," *IEEE Trans. Signal Processing*, vol. 60, no. 4, pp. 1730–1740, April 2012.

[12] S. Kay, "Optimal signal design for detection of gaussian point targets in stationary gaussian clutter/reverberation," *IEEE Jour. Sel. Top. Signal Proc.*, vol. 1, no. 1, pp. 31–41, 2007.

[13] M. Bell, "Information theory and radar waveform design," *IEEE Trans. Inf. Theory*, vol. 39, no. 5, pp. 1578–1597, 1993.

[14] P. Setlur, D. N., and M. Rangaswamy, "Radar waveform design with the two step mutual information," in *In Proc. IEEE Radar Conference*, 2014.

[15] R. Romero and N. Goodman, "Information-theoretic matched waveform in signal dependent interference," in *IEEE Radar Conference*, 2008, pp. 1–6.

[16] P. Setlur and M. Rangaswamy, "Joint filter and waveform design for radar STAP in signal dependent interference," US Air Force Res. Lab., Sensors Directorate, WPAFB, Dayton, OH, Tech. Rep. *Submitted*, 2014.

[17] ——, "Proximal constrained waveform design algorithms for cognitive radar STAP," in *Proc. Asilomar Conf. Signals, Systems and Computers*, 2014.