Multi linear regression

β_0	x_1 cgpa	x_2 iq	y placement
	$8^{(\beta_1)}$	$80^{(\beta_2)}$	8
	7	70	7
	5	120	15

$y_1 = 8 \quad y_2 = 7 \quad y_3 = 15$

$\hat{y}_1 = 7 \quad \hat{y}_2 = 2 \quad \hat{y}_3 = 2$

Predict

$\hat{y}_1 = \beta_0 + \beta_1 x_{11} + \beta_2 x_{12} + \cdots \beta_m x_m$

$\hat{y}_2 = \beta_0 + \beta_1 x_{21} + \beta_2 x_{22} + \cdots \beta_m x_{2m}$

$\hat{y}_3 = \beta_0 + \beta_1 x_{31} + \beta_2 x_{32} + \cdots \beta_m x_{3m}$

$\vdots$

$\hat{y}_n = \beta_0 + \beta_1 x_{n1} + \beta_2 x_{n2} + \cdots \beta_m x_{nm}$

$$\begin{bmatrix} \hat{y}_1 = \beta_0 + \beta_1 8 + \beta_2 80 \\ \hat{y}_2 = \beta_0 + \beta_1 7 + \beta_2 70 \\ \hat{y}_3 = \beta_0 + \beta_1 5 + \beta_2 120 \end{bmatrix}$$

$$\hat{Y} = \underbrace{\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_n \end{bmatrix}}_{\boxed{n \times 1}} = \underbrace{\begin{bmatrix} \beta_0 + \beta_1 x_{11} + \cdots \beta_m x_{1m} \\ \beta_0 + \beta_1 x_{21} + \cdots \beta_m x_{2m} \\ \beta_0 + \beta_1 x_{31} + \cdots \beta_m x_{3m} \\ \vdots \\ \beta_0 + \beta_1 x_{n1} + \cdots \beta_m x_{nm} \end{bmatrix}}_{\boxed{n \times 1}}$$

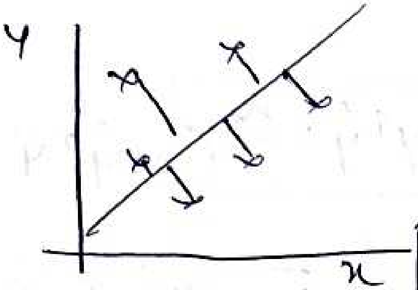
$$= \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1m} \\ 1 & x_{21} & x_{22} & \cdots & x_{2m} \\ 1 & x_{31} & x_{32} & \cdots & x_{3m} \\ \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_m \end{bmatrix}$$

$$\boxed{\hat{Y} = X\beta}$$

$n \times (m+1) \longleftrightarrow (m+1) \times 1$

Shape $\rightarrow \boxed{n \times 1} \rightarrow$ prooved

derived and prooved



$$\boxed{d_1^2 + d_2^2 + d_3^2 + \dots d_n^2}$$

min

$$\boxed{E = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2} \rightarrow \text{minimize}$$

→ matrix form

$$Y_i = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{bmatrix} \qquad \hat{y}_i = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \\ \vdots \\ \hat{y}_n \end{bmatrix}$$

$$e = y - \hat{y} = \begin{bmatrix} y_i - \hat{y}_1 \\ y_2 - \hat{y}_2 \\ \vdots \\ y_n - \hat{y}_n \end{bmatrix} \rightarrow \boxed{n \times 1}$$

$$e^T e = \underbrace{\begin{bmatrix} y_1 - \hat{y}_1 & y_2 - \hat{y}_2 & \dots & y_n \hat{y}_n \end{bmatrix}}_{\boxed{1 \times n}} \underbrace{\begin{bmatrix} y_1 - \hat{y}_1 \\ y_2 - \hat{y}_2 \\ \vdots \\ y_n - \hat{y}_n \end{bmatrix}}_{\boxed{n \times 1}}$$

$$e^T e = (y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2 + \dots (y_n + \hat{y}_n)^2$$

$$= \sum_{i=1}^{n} (y_1 - \hat{y}_i)^2$$

$$\boxed{E = e^T e} \rightarrow \text{II equation}$$

↑ minimize

$E = e^T e$

$E = (y-\hat{y})^T (y-\hat{y}) = y^T y - \boxed{y^T \hat{y} = \hat{y}^T y} + \hat{y}^T \hat{y}$

$\boxed{y^T \hat{y} = \hat{y}^T y}$ Proove

$y = A \quad \hat{y} = B$

$\boxed{(AB)^T = B^T A^T}$

$(A^T)^T = A$

$A^T B = B^T A$

Took this and transpose eqn

$A^T B \quad (A^T B)^T = B^T A$

Now want want to prove $A^T B$ is equal to $(A^T B)^T$

$A^T B = (A^T B)^T$

Let $A^T B = C \rightarrow$ symmetric

then Proove

$C = C^T$

$A = y \quad \cancel{X} B = \hat{y}$ we kno that

$y^T \hat{y} = y^T X B \rightarrow (m+1) \times 1$

$1 \times n \quad n \times (m+1)$

$n \times 1$

$1 \times 1 =$ Scaler $[1] \quad [2]$

Proved $A^T B$ is symetric

So, $\boxed{y^T \hat{y} = \hat{y}^T y}$

$\boxed{E = Y^TY - 2Y^T\hat{Y} + \hat{Y}^T\hat{Y}}$ → III eqn

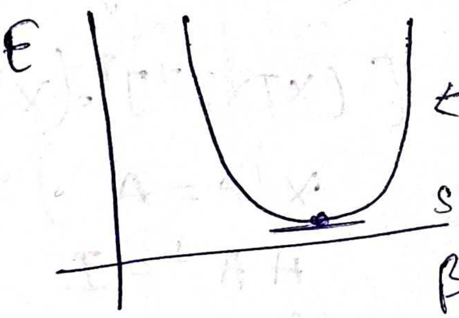
$\hat{Y} = X\beta$

$E = Y^TY - 2Y^TX\beta + (X\beta)^T(X\beta)$

$\boxed{E = Y^TY - 2Y^TX\beta + \beta^TX^TX\beta}$

$Y = f(x)$ change, change, change

$E(\beta)$ find such value of β matrix for which E is min



← loss function

$\frac{dE}{d\beta} = 0$

$\boxed{E = Y^TY - 2Y^TX\beta + (X\beta)^T(X\beta)}$ → differentiate with respect to β

$\frac{dE}{d\beta} = 0 - 2Y^TX + 2\beta^TX^TX$

$\frac{d\,\beta^TX^TX\beta}{d\beta} = x^TAx$

if A is symmetric

$\cancel{2}\beta^TX^TX = \cancel{2}Y^TY$

$\boxed{\beta^TX^TX = Y^TY}$

product both side $(X^TX)^{-1}$

$\beta^TX^TX(X^TX)^{-1} = Y^TX(X^TX)^{-1}$

$$\beta^T I = Y^T X (X^T X)^{-1}$$

$$\beta^T = Y^T X (X^T X)^{-1}$$

$$(\beta^T)^T = [\underbrace{Y^T X}_{A} \underbrace{(X^T X)^{-1}}_{B}]^T$$

$$\beta = [(X^T \cancel{X})^{-1}]^T (Y^T X)^T$$

$$\beta = [[X^T X]^{-1}]^T X^T Y$$

$$\boxed{\beta = (X^T X)^{-1} X^T Y} \to \cancel{IV} \text{ eqn}$$

β = Values

$((m+1) \times 1)$

$[X^T X]^{-1}$

$m+1 \times n$ $\quad n \times (m+1)$

$(m+1) \times (m+1)$ $\quad (m+1) \times n$

$(m+1) \times n$ $\quad n \times 1$

$$\boxed{(m+1) \times 1}$$

$(X^T X)^{-1}$ symmetric

$[(X^T X)^{-1}]^T = (X^T X)^{-1}$

$X^T X = A$

$A A^{-1} = I$

$(A A^{-1})^T = I^T$

$(A^{-1})^T A^T = I$

Proved $(A^{-1})^T I = A^{-1}$

$$\boxed{(A^{-1})^T = A^{-1}}$$