

2 ellipses centered at 0

$$E_1 = x^T \Sigma_1^{-1} x = 1$$

$$E_2 = x^T \Sigma_2^{-1} x = 1$$

$$\Sigma_1 = A A^T$$

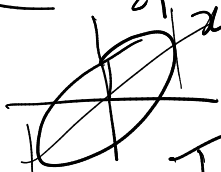
$$x^T A^{-T} A^{-1} x = 1$$

$$\text{Let } u = A^{-1} x$$

$$x = A u$$

$$E_2 = u^T [A^T \Sigma_2^{-1} A] u = u^T A^T \Gamma A (A^T A)^{-1} u$$

$$E_1 = x^T \Sigma_1^{-1} x = 1$$



$$y = T_1 x$$

$$V(y) = T_1 \Sigma_1 T_1^{-1}$$

$$\text{want } T_1 \Sigma_1 T_1^{-1} = I$$

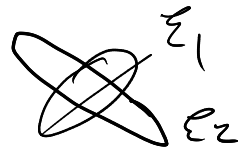
$$A^{-1} A A^T A^{-1} = I$$

$$T_1 = A^{-1} \text{ when } \Sigma_1 = A A^T$$

and new E_2 has var

$$T_1 \Sigma_2 T_1^{-1}$$

$$T_2 \Lambda T_2^{-1}$$



1) Transform E_1 to a circle with T_1

$$x \rightarrow u$$

$$(T_1^T \Sigma_1 T_1) = I$$

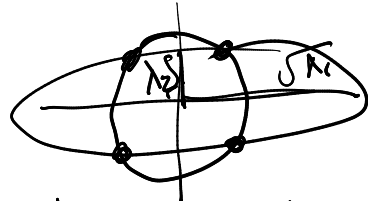
$$\Sigma_1 = A A^T$$

$$\Sigma_1^{-1} = A^{-T} A^{-1}$$

$$x^T A^{-T} A^{-1} x = 1$$

$$T_1 = A^{-1}$$

Transformation $T_2 T_1 = T^T A = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$



$$x^2 = \frac{\lambda_1 \lambda_2 - \lambda_2}{\lambda_1 - \lambda_2}$$

if $\lambda_1 \geq 1 \geq \lambda_2$
or otherwise

$$y^2 = 1 - x^2$$

Original

$$(T_2 T_1)^{-1} \begin{pmatrix} x \\ y \end{pmatrix} = T_1^{-1} T_2^{-1} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\text{original pts} = A^{-1} T^T \begin{pmatrix} x \\ y \end{pmatrix}$$