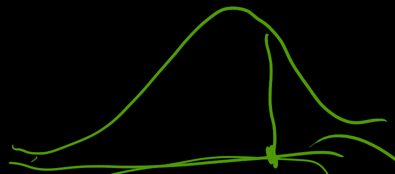
3.5.2 Continuous Conditional Densities

Given a joint density $f_{XY}(x,y)$

We can get $f_X(x) = \int f_{XY}(x,y)\,dy$

Conditional density for Y given $X = x$

$$f_{Y|X}(y|x) = \begin{cases} \dfrac{f_{XY}(x,y)}{f_X(x)} & \text{if } f_X(x) > 0 \\ 0 & \text{otherwise} \end{cases}$$



$$\int_a^b f(x)\,dx = 0$$

$P(X \in [a,b])$

Note: If X is continuous then $P(X=x)=0$

so we are conditioning on a set of probability 0

But that's okay because $f_{Y|X}(y|x)$

is NOT the probability of $Y=y$ given $X=x$,

It's just a density that gives you

conditional probabilities when you integrate.

The Law of Total Probability

$$f_X(x) = \int \boxed{f(x,y)}\, dy$$

$$= \int \boxed{f(x|y) f_Y(y)}\, dy$$

other

$= f(x|y)$ averaged over Y

Marginal Probability = Mean Conditional probability

mean density = mean cond'l density

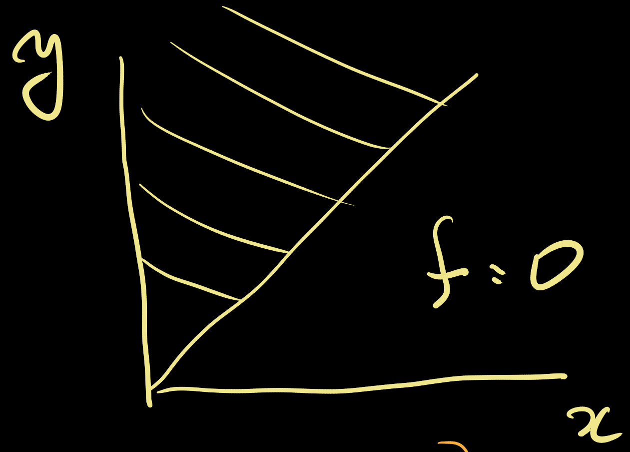
$$E(g(z)) = \int g(z) f_Z(z)\, dz$$

$$P(A \cap B) = P(B) P(A|B)$$

$$P(A|B) = P(A \cap B) / P(B)$$

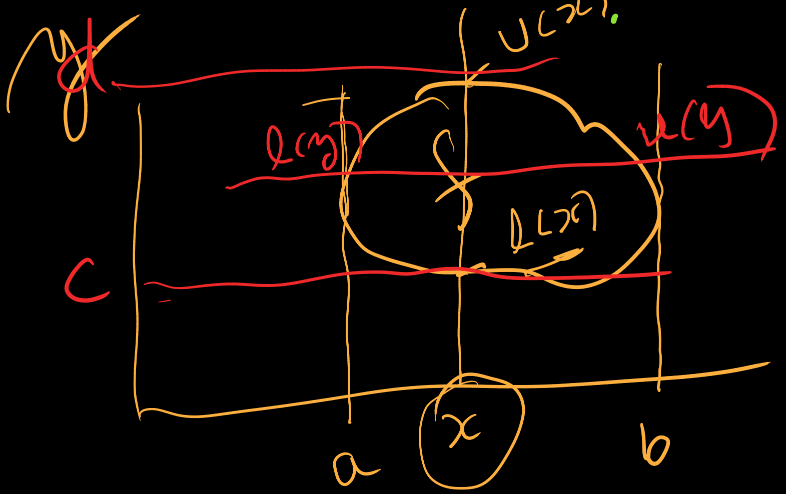
"Example D"

$$f_{XY}(x,y) = \begin{cases} \lambda^2 e^{-\lambda y} & 0<x<y \\ 0 & \text{otherwise} \end{cases}$$



Note: How to integrate f

- Choose 1 variable for marginal bound
- Other for conditional



$$a \le x \le b$$

$$l(x) < y \le u(x)$$

$$\int_a^b \left[\int_{l(x)}^{u(x)} f(x,y)\, dy \right] dx$$

$0 < x < y$

y-first

$0 < y < \infty$

$0 < x < y$

outer integral

inner

$$\int_0^{\infty} \left[\int_0^{y} f(x,y)\,dx \right] dy$$

x-first

$0 < x < \infty$

$x < y < \infty$

$$\int_0^{\infty} \left[\int_x^{\infty} f(x,y)\,dy \right] dx$$

Define: outer then inner

Evaluate: inner then outer.

$$f_{XY}(x,y) = \begin{cases} \lambda^2 e^{-\lambda y} & 0 < x < y \\ 0 & \text{otherwise} \end{cases}$$

Find $f_{X|Y}(x|y)$

Step 1 : Find marginal for conditioning variable, Y.

$$f_Y(y) = \int_0^y \lambda^2 e^{-\lambda y}\, dx = \left[x \lambda^2 e^{-\lambda y} \right]_0^y$$

$$= y \lambda^2 e^{-\lambda y}, \quad y > 0$$

$$f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)} = \frac{\cancel{\lambda^2 e^{-\lambda y}}}{y\,\cancel{\lambda^2 e^{-\lambda y}}}, \quad 0 < x < y$$

$$= \frac{1}{y} \qquad 0 < x < y$$

$$\therefore \; x \mid y \sim U\left(0, \frac{1}{y}\right)$$