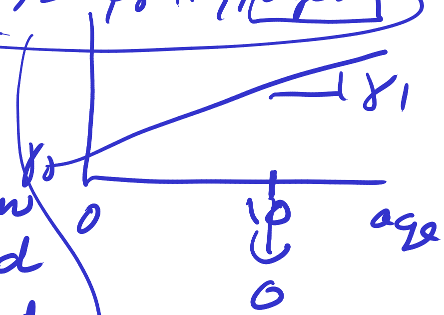
MATH 4939 Quiz 8 March 24, 2021

```
> dd <- as.data.frame(subset(Orthodont, Sex == 'Female'))
> head(dd,2)
   distance age Subject    Sex
65       21   8      F01 Female
66       20  10      F01 Female
> fit <- lme(distance ~ age, dd, random = ~ 1 + age | Subject,
+           correlation = corAR1(form = ~ 1| Subject))
> summary(fit)

Random effects:
 Formula: ~1 + age | Subject
 Structure: General positive-definite, Log-Cholesky parametrization
            StdDev    Corr
(Intercept) 1.9370273 (Intr)
age         0.1681860 -0.397
Residual    0.6479416
```

Quiz 8

$E(Y)=\psi_0+\psi_1 age2$

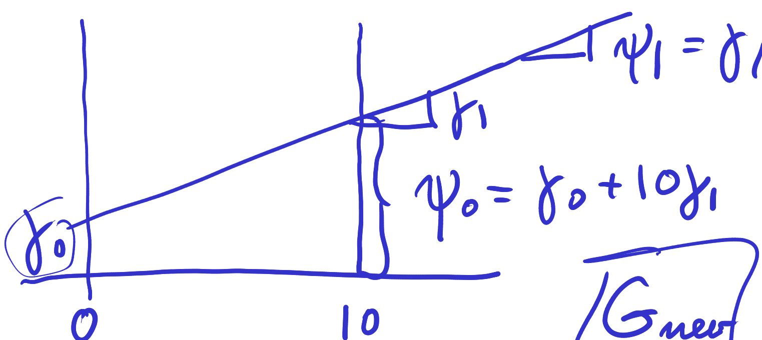


1. Calculate the estimated G matrix.
2. Find the G matrix if age had been centered at 10, i.e. if you had used age2 = age - 10 in the model instead of age.

$-0.397\times 1.937\times 0.168 \rightarrow \rho_{01}$

$$G=\begin{bmatrix} g_{00} & g_{01} \\ g_{10} & g_{11}\end{bmatrix}=\begin{bmatrix} 1.937^2 & \boxed{} \\ & 0.168^2\end{bmatrix}$$

$g_{01}=\rho_{01}\times\sqrt{g_{11}}\times\sqrt{g_{00}}$



$$\begin{pmatrix}\psi_0\\ \psi_1\end{pmatrix}=\begin{bmatrix}1 & 10\\ 0 & 1\end{bmatrix}\begin{pmatrix}\gamma_0\\ \gamma_1\end{pmatrix}$$

$$G_{new}=\begin{bmatrix}1 & 10\\ 0 & 1\end{bmatrix} G_{old} \begin{bmatrix}1 & 0\\ 10 & 1\end{bmatrix}$$

$E(Y)=\gamma_0+\gamma_1 age$ $\qquad age2 = age-10$

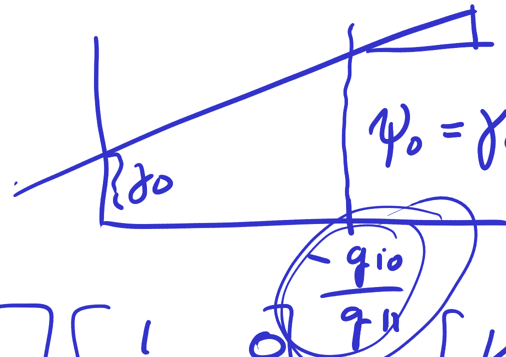
$=\gamma_0+\gamma_1(age_2+10)$

$=\underbrace{\gamma_0+\gamma_1 10}_{\psi_0}+\gamma_1 age_2$

$\psi_0 + \psi_1 age_2$

$\psi_0=\gamma_0+\gamma_1 10$
$\psi_1=\gamma_1$
$=\begin{bmatrix}1 & 10\\ 0 & 1\end{bmatrix}\begin{pmatrix}\gamma_0\\ \gamma_1\end{pmatrix}$

Centering x at $-\frac{g_{10}}{g_{11}}$



$\psi_0 = \gamma_0 - \frac{g_{10}}{g_{11}} \times \gamma_1$

$$\begin{bmatrix}1 & -g_{10}/g_{11}\\ 0 & 1\end{bmatrix}\begin{bmatrix}g_{00} & g_{10}\\ g_{10} & g_{11}\end{bmatrix}\begin{bmatrix}1 & 0\\ -\frac{g_{10}}{g_{11}} & 1\end{bmatrix} = \begin{bmatrix}\checkmark & 0\\ 0 & \checkmark\end{bmatrix}$$

$$V_i = Z_i G Z_i' + R_i$$

$R_i = \sigma^2 C$

I

Guess G & R_i

$$\Sigma = Z\begin{bmatrix}G & & \\ & G & \\ & & \ddots G\end{bmatrix}Z' + \begin{pmatrix}R_1 & & \\ & R_2 & \\ 0 & & \ddots R_n\end{pmatrix}$$

σ^2, ρ, ϕ

$$\hat{\beta} = (X'\Sigma^{-1}X)^{-1}(X'\Sigma^{-1}y)$$

Invert $G = \Gamma \Lambda \Gamma'$

$$\boxed{G^{-1} = \Gamma \Lambda^{-1} \Gamma'}$$

X

dvar(X, id)