Math 4939 Quiz 10

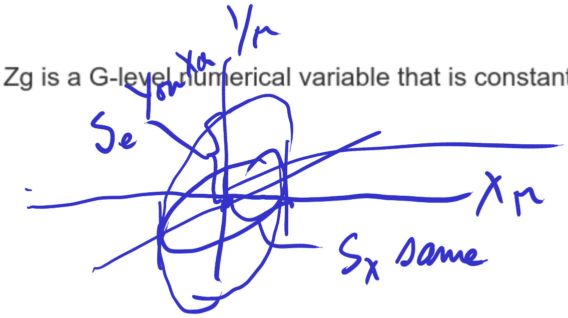
$$\underset{\sim}{e}^T x = \underset{\sim}{0}$$

$\hat{\epsilon} \perp \mathcal{L}(x)$ Quiz 10 β_z β_x β_x f_x

Let Y and X be numerical variables and let G be a factor.
Let Yr and Xr be the residuals of Y and X regressed on G.
Let Xh be the least-squares predictor of X using G. Note that `X = Xh + Xr`.
All the following models except one must produce the same least-squares coefficient for the effect of X or Xr.

1. Which model may produce a different coefficient? Explain why briefly. d
2. In the list below, choose 6 pairs of models that produce the same coefficient for X or Xr and state which model in each pair would result in the smaller SE for the effect of X or Xr, or whether the SEs would be equal. Justify briefly. A rough sketch is acceptable.

a. `Y ~ X + G` — AVP $Y_r \sim X_r$ $SE(a) = SE(b)$
b. `Yr ~ Xr`
c. `Y ~ Xr + G` → Y reg on G → Y_r X_r reg on G $X_{rr} = X_r$ $SE(c) = SE(b)$
d. `Y ~ X`
e. `Y ~ Xr`
f. `Y ~ X + Xh`
g. `Y ~ X + Xh + Zg` where Zg is a G-level numerical variable that is constant within levels of G.



Se for $Y \sim X + G$

$SE(e) > SE(a)$

doubly robust

(f) X_r resid of reg. X on X_h → X_r from (a)

for a) Y_r is resid of reg of Y on G

f) " " " " " " Y on X_h $X_h \subset span(G)$

$Y_{r(f)}$ has $\geq S_e$ than $Y_{r(a)}$

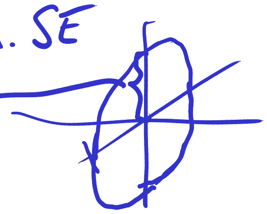
$$\boxed{SE(\hat{\beta}) = \frac{1}{\sqrt{n}} \frac{Se}{S_{X_r}}}$$

$SE(f) \geq SE(a)$

(g) $X_{r(g)}$ resid of reg of X on X_h and G $X_{r(g)} = X_{r(a)}$

$Y_{r(g)}$ resid of reg of Y on X_h an Z_g

se - resid. SE



$se(g)$ = SD of resid of Y on $X_h + Z_g$ G

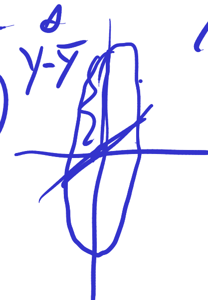
$se(g) \leq se(f)$

$se(a) \leq se(g)$

(e) vs (f)

$Y \sim X_r$ $Y \sim X + X_h$

Both have a $se(e)$



$X_{r(e)} = X_{r(f)} = X_{r(a)}$

$se(f)$ reg of Y on X_h

$se(f) \leq se(e)$

$SE(\hat{\beta})(f) \leq SE(\hat{\beta})(e)$