MATH 4939 March 10, 2021

Quiz 6

Suppose $\hat{\phi}_1 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$, $\hat{\phi}_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $E(\hat{\phi}_1) = E(\hat{\phi}_2) = \phi$

$Var(\hat{\phi}_1) = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}$, $Var(\hat{\phi}_2) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

1) Find the minimum variance unbiased estimator of ϕ using a linear combination of $\hat{\phi}_1$ and $\hat{\phi}_2$.

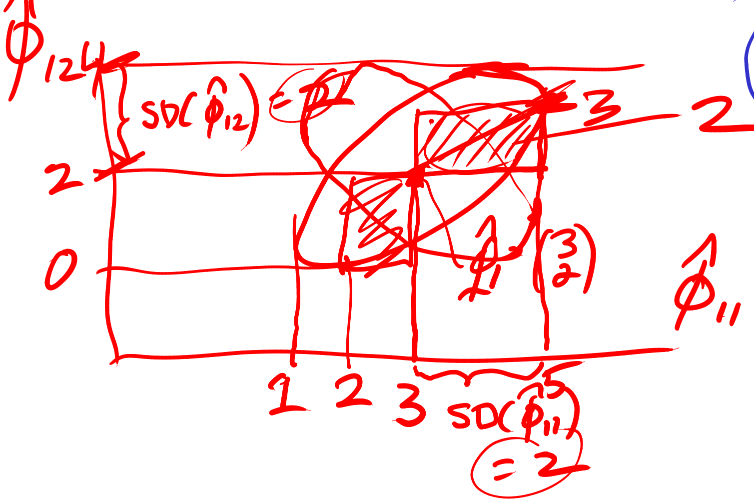
2) Draw a sketch showing $\hat{\phi}_1$, $\hat{\phi}_2$ and the combined estimator together with the variance ellipses around $\hat{\phi}_1$ and $\hat{\phi}_2$. Make the ellipses as accurate as you can using what you know about variance ellipses.

$$\hat{\phi}_c = \left(\begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}^{-1} + I^{-1} \right)^{-1} \left(\begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}^{-1} \hat{\phi}_1 + I^{-1} \hat{\phi}_2 \right)$$

$$\{ \phi : (\phi - \hat{\phi})^T Var^{-1} (\phi - \hat{\phi}) = 1 \}$$

$$\hat{\phi}_1 = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \qquad Var(\hat{\phi}_1) = \begin{pmatrix} 4 & -2 \\ -2 & 4 \end{pmatrix}$$

$$\beta_{y \cdot x} = \frac{\sigma_{xy}}{\sigma_x^2} = \frac{2}{4} = 1/2$$



$V = AA^T$

$A = [\underset{\sim}{a_1}\ \underset{\sim}{a_2}]$

$\|\underset{\sim}{a_i}\| = 1$

$\langle \underset{\sim}{a_1}, \underset{\sim}{a_2} \rangle = 0$

↑

conjugate to $\underset{\sim}{a_1}$

Euclidean norm

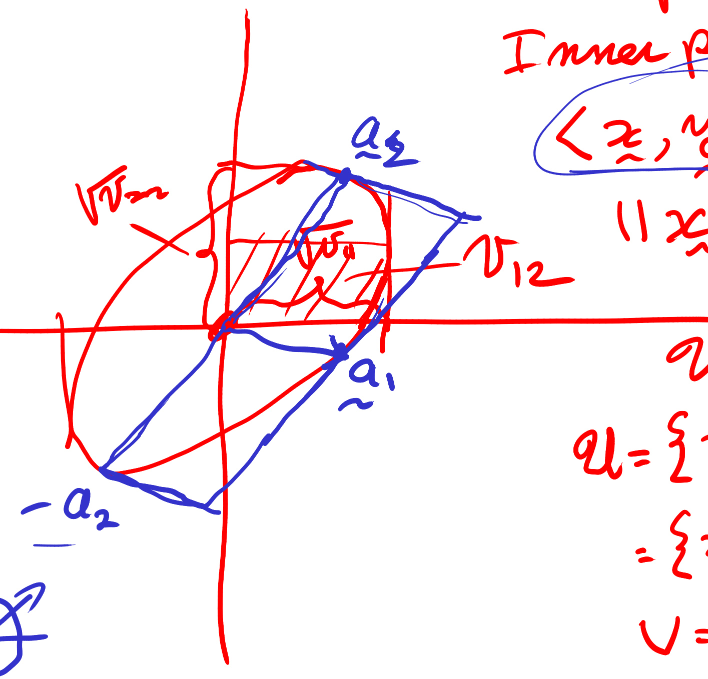
$V = I$

V is pos. def. matrix

Inner product

$\langle \underset{\sim}{x}, \underset{\sim}{y} \rangle = \underset{\sim}{x}' V^{-1} \underset{\sim}{y}$

$\|\underset{\sim}{x}\|^2 = \langle \underset{\sim}{x}, \underset{\sim}{x} \rangle$

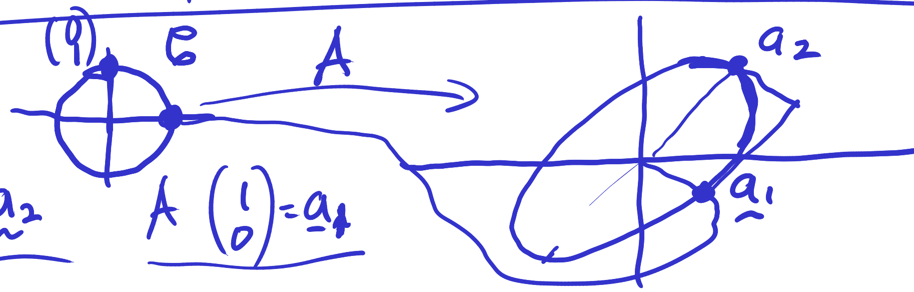


Unit sphere

$\mathcal{U} = \{\underset{\sim}{x} : \|\underset{\sim}{x}\|^2 = 1\}$

$= \{x : x'V^{-1}x = 1\}$

$V = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}$



$A\mathcal{C} = \mathcal{U}_V$

$A\binom{0}{1} = \underset{\sim}{a_2}$ $\quad A\binom{1}{0} = \underset{\sim}{a_1}$

$Y \sim X_1 + X_2 + X_3 + X_2 * X_3$

Level	1	2	1	1x2

random = ~ $1 + X_1 + X_3$ | id

gicc

gicc(hs, ~school) $G = \begin{bmatrix} g_{00} & 0 & 0 \\ & g_{11} & 0 \\ & & g_{22} \end{bmatrix}$ 6 parameter

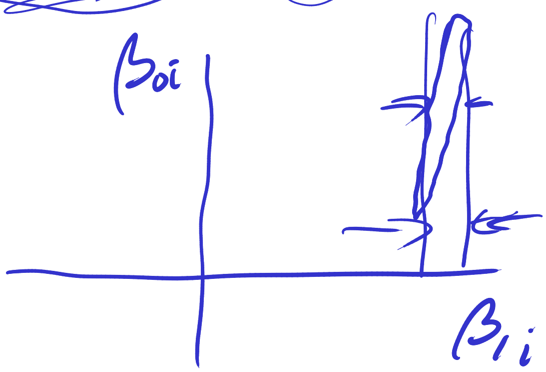
$0 \to 1$

covar

$Y_i = X_i \gamma + Z_i u_i + \underset{\sim}{\varepsilon_i}$

$$Var(Y_i) = Z_i G Z_i' + \sigma^2 I$$

$$Var(\hat{\beta}_i) = \boxed{G + \sigma^2 (X_i' X_i)^{-1}}$$



G almost singular

lack of convergence