

Pseudocódigo:

Función main

Input:

- file containing sudoku board
- n for board size (ex. 9 for 9x9)

Output:

- txt containing solution

```
function main(file, n)
    initialize sudokuBoard as matrix of n*n size

    sudokuBoard = board from file
    if not isValidBoard(sudokuBoard, n)
        return error: "Board not valid"
    else
        if solveSudoku(sudokuBoard, n)
            output to file from sudokuBoard
        else
            return error: "Solution not found"
```

Función isSafe

Inputs:

- T for sudoku table
- row for row the number is in
- col for column the number is in
- num for the number we're validating

Output: boolean value

```
function isSafe(T, row, col, n, num)
    // n = 9 because our grid is 9x9

    for i to n-1
        if (T[row][i] == num)
            return false

    for i to n-1
        if (T[i][col] == num)
            return false
```

```

// Check the 3x3 subgrid (of our 9x9 sudoku grid)
startRow = row - (row % 3)
startCol = col - (col % 3)
for i from 0 to 2
    for j from 0 to 2
        if T[startRow + i][startCol + j] == num
            return false

// if all checks pass, the number is safe to place
return true

```

Función solveSudoku

Inputs:

- T for the sudoku table
- n for the size of table (9 for out 9x9 table)

Output: boolean if the solution has been found

```

function solveSudoku(T, n)
    row, column = 0
    isEmpty = true

    // Find the next empty cell (value 0)
    for row from 0 to n-1
        for col from 0 to n-1
            if T[row][col] == 0
                isEmpty = false
                Break from inner loop
        if isEmpty == false
            Break from outer loop

    // if no empty cell is found, the board is solved
    if isEmpty
        return true

    // Try placing numbers from 1 to n
    for num from 1 to n
        if isSafe(T, row, col, n, num)
            T[row][col] = num

            // Recursively try to solve the rest of the board

```

```

        if solveSudoku(T, n)
            return true

        // if placing num didn't work, backtrack
        T[row][col] = 0

    // if no number can be placed, the puzzle is unsolvable
    return false

```

isValidBoard function

Inputs:

- T for sudoku table
- n for number for size of board (9, for a 9x9 board)

Output: boolean if board is valid

```

function isValidBoard(T, n)
    // Check rows and columns
    for i from 0 to n-1
        Create an empty set for rowSet and colSet
        for j from 0 to n-1
            // Check row uniqueness
            if T[i][j] is not 0
                if T[i][j] exists in rowSet
                    return false
                add T[i][j] to rowSet

            // Check column uniqueness
            if T[j][i] is not 0
                if T[j][i] exists in colSet
                    return false
                add T[j][i] to colSet

    // Check 3x3 subgrids
    for row from 0 to n-1, incrementing by 3
        for col from 0 to n-1, incrementing by 3
            Create an empty set gridSet

            // check the subgrid

```

```

        for i from 0 to 2
            for j from 0 to 2
                num = T[row + i][col + j]
                if num is not 0
                    if num exists in gridSet
                        return false
                    add num to gridSet

// if all checks pass, the board is valid
return true

```

Análisis de complejidad:

Función isValidBoard

for i from 0 to n-1	O(n)
Create an empty set for rowSet and colSet	O(1)
for j from 0 to n-1	O(n), por anidación: O(n ²)
if T[i][j] is not 0	O(1)
if T[i][j] exists in rowSet	O(1)
return false	O(1)
add T[i][j] to rowSet	O(1)
if T[j][i] is not 0	O(1)
if T[j][i] exists in colSet	O(1)
return false	O(1)
add T[j][i] to colSet	O(1)
for row from 0 to n-1, incrementing by 3	$O(\frac{n}{\sqrt{n}}) \Rightarrow O\sqrt{n}$ La complejidad computacional es raíz cuadrada ya que se incrementa por 3 el for loop, esto porque hay 3 subcuadrados en el tablero sudoku 9x9, si fuese 16x16 el número sería 4 (raíz de 16).

for col from 0 to n-1, incrementing by 3	$O(\sqrt{n})$, por anidación es $O(n)$
Create an empty set gridSet	$O(1)$
for i from 0 to 2	$O(\sqrt{n})$, por anidación es $O(n^{\frac{3}{2}})$ Raíz cuadrada ya que cada subcuadrado tiene el tamaño de $O(\sqrt{n})$ por $O(\sqrt{n})$.
for j from 0 to 2	$O(\sqrt{n})$, por anidación es $O(n^2)$
num = T[row + i][col + j]	$O(1)$
if num is not 0	$O(1)$
if num exists in gridSet	$O(1)$
return false	$O(1)$
add num to gridSet	$O(1)$
return true	$O(1)$

Complejidad: $O(n^2) + O(n^2) = \mathbf{O(n^2)}$

Función solveSudoku:

row, column = 0	$O(1)$
isEmpty = true	$O(1)$
for row from 0 to n-1	$O(n)$
for col from 0 to n-1	$O(n)$, por anidación: $O(n^2)$
if T[row][col] == 0	$O(1)$
isEmpty = false	$O(1)$
Break	$O(1)$
if isEmpty == false	$O(1)$
Break	$O(1)$
if isEmpty	$O(1)$

return true	$O(1)$
for num from 1 to n	$O(n)$
if isSafe(T, row, col, n, num)	$O(n)$ [llamada externa], por anidación: $O(n^2)$
T[row][col] = num	$O(1)$
if solveSudoku(T, n)	$O(k^n)$ [más explicación al final]
return true	$O(1)$
T[row][col] = 0	$O(1)$
return false	$O(1)$

De acuerdo al artículo *Prune-and-Search | A Complexity Analysis Overview* en *GeeksForGeeks*, los algoritmos recursivos que descartan posibilidades (como lo hacemos con la función isSafe) tienen en el peor caso una complejidad $O(k^n)$. En la cual k es una constante de los valores posibles, 9 en el peor caso. Luego, n es el número de celdas por llenar, en el peor caso 81. Dándonos una complejidad en el peor caso de $O(9^{81})$. Pero, debido a pruning (eliminar valores imposibles) la complejidad se reduce conforme el tiempo avanza.

$O(k^m) \mid k = n, m = n * n, n = \text{longitud (o ancho) del tablero sudoku}$
También expresado como: $O(n^{n*n})$

Complejidad: **$O(n^{n*n})$**

Función isSafe:

for i to n-1	$O(n)$
if (T[row][i] == num)	$O(1)$
return false	$O(1)$
for i to n-1	$O(n)$
if (T[i][col] == num)	$O(1)$
return false	$O(1)$
startRow = row - (row % 3)	$O(1)$
startCol = col - (col % 3)	$O(1)$
for i from 0 to 2	$O(\sqrt{n})$

for j from 0 to 2	$O(\sqrt{n})$, por anidación es $O(n)$
if T[startRow + i][startCol + j] == num	$O(1)$
return false	$O(1)$
return true	$O(1)$

Complejidad: $O(n) + O(n) + O(n) = \mathbf{O(n)}$

Función main:

initialize sudokuBoard as matrix of n*n size	$O(1)$
sudokuBoard = board from file	$O(1)$
if NOT isValidBoard(sudokuBoard, n)	$O(n^2)$
return error: "Board not valid"	$O(1)$
else	$O(1)$
if solveSudoku(sudokuBoard, n)	$O(k^n) = O(n^{n^n})$
output to file from sudokuBoard	$O(1)$
else	$O(1)$
return error: "Solution not found"	$O(1)$

$n = |\text{row}|$ o $|\text{column}|$ (9 para sudoku 9x9)

Complejidad total: $\mathbf{O(n^{n^n})}$

Recursos:

<https://www.geeksforgeeks.org/prune-and-search-a-complexity-analysis-overview/>

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600198000
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- Buscar un lugar vacío (0)

530...

- Probar valores

1 (0)

isSafe(1)

↓

5,3,1,0,7,0,0,0,0

1 2
isSafe(1) isSafe(2)

X
ya hay en fila

5,3,1,2,7,0,0,0,0

1 2 3 4
isSafe(1) isSafe(2) isSafe(3) isSafe(4)
X X X
fila y subcadena X fila

5,3,1,2,7,4,0,0,0