Quiz 2

Binary Search

Input: $A = \langle a_1, a_2, a_3 \dots \rangle$

- Ordenado
- Ascendiente
- P = inicio / primer índice
- r = fin / último índice
- t = valor

if $p \leq r$	$O(1)$
mid = $\lceil (p+r)/2 \rceil$	$O(1)$
if A[mid] == t	$O(1)$
return true	$O(1)$
else if t < A[mid]	$O(1)$
BinarySearch(A, p, mid-1, t)	$O(\frac{n}{2})$
else	$O(1)$
BinarySearch(A, mid+1, r, t)	$O(n/2)$
else	$O(1)$
return false	$O(1)$

1. ecuación de recurrencia

$$T(n) = \begin{cases} T(n/2) + 1 & , n > 1 \\ 1 & , n = 1 \end{cases}$$

2. Solución Por sostitución

$$T\left(\tfrac{n}{2}\right) + 1 \;\rightarrow\; \left[T\left(\tfrac{n}{2^2}\right) + 1\right] + 1$$

$$\left[T\left(\tfrac{n}{2^2}\right)\right] + 1(2) \;\rightarrow\; \left[T\left(\tfrac{n}{2^3}\right) + 1\right] + 1(2)$$

$$T\left(\tfrac{n}{2^3}\right) + 1(3) \;\rightarrow\; \underline{T\left(n/2^k\right) + 1(k)}$$

$T(n/2^k) + k$

Caso base

$\frac{n}{2^k} = 1 \Rightarrow k = \log_2 n$

$\boxed{T(n) = 1 + \log_2 n}$

Binary Search es $O(\lg(n))$

3. Vía teorema maestro

$T(n) = T\left(\frac{n}{2}\right) + 1$

$a = 1$ $\quad b = 2$ $\quad f(n)$

1. $f(n) = O(n^{\log_b a - \varepsilon})$, $\varepsilon > 0$

$1 = O(n^{\log_2 1 - \varepsilon}) \Rightarrow 1 \neq O(n^{-\varepsilon})$

2. $f(n) = \Theta(n^{\log_b a})$

$1 = \Theta(n^{\log_2 1}) \Rightarrow 1 = \Theta(1)$

$T(n) = \Theta(n^{\log_2 1} \lg n) \Rightarrow \boxed{T(n) = \Theta(\log_2 n)}$

$n^0 \to 1$

4 Sustitución

$$T(n) = 2T\left(\frac{n}{2}\right) + n^4 \quad , \quad n > 1$$
$$T(1) = 1 \quad , \quad n = 1$$

$$2T\left(\frac{n}{2}\right) + n^4$$

$$2\left[2T\left(\frac{n}{2^2}\right) + \left(\frac{n}{2}\right)^4\right] + n^4 \qquad 2^2\left[T\left(\frac{n}{2^3}\right) + \left(\frac{n}{2^2}\right)^4\right] \cancel{+\left(\frac{n}{2}\right)^4} + n^4$$
$$+ 2\left(\frac{n}{2}\right)^4$$

$$2^3 T\left(\frac{n}{2^3}\right) + \cancel{n^4}\ 2^2\left(\frac{n}{2^2}\right)^4 + 2\left(\frac{n}{2}\right)^4 + n^4$$

$$2^3 T\left(\frac{n}{2^3}\right) + 2^2\left(\frac{n^4}{2^8}\right) + 2\left(\frac{n^4}{2^4}\right) + n^4$$

$$2^3 T\left(\frac{n}{2^3}\right) + n^4\underbrace{\left(\frac{1}{2^6} + \frac{1}{2^3} + \frac{1}{1}\right)} \rightarrow \sum \overset{\text{aprox}}{\approx} 1$$

$$2^k T\left(\frac{n}{2^k}\right) + n^4 (1)$$

Caso Base

$$\frac{n}{2^k} = 1 \qquad k = \lg(n)$$

$$2^{\lg(n)} T(1) + n^4$$

$$n + n^4$$

$$\boxed{O(n^4)}$$

5 Teorema Maestro

$$T(n) = 2T\left(\frac{n}{2}\right) + n^4$$

$a=2 \quad b=2 \quad f(n)$

1.

$$n^4 = O\left(n^{\log_2 2 - \varepsilon}\right)$$

$<$

No válido

2.

$$n^4 = \Theta\left(n^{\lg 2}\right) \rightarrow n^4 = \Theta(n^1)$$

$=$

No válido

3.

I. $n^4 \geq \Omega\left(n^{\lg 2 + \varepsilon}\right), \varepsilon > 0$

$$n^4 = \Omega\left(n^{1+1}\right), \varepsilon = 1$$

válido

II. $af\left(\frac{n}{b}\right) \leq cf(n), \quad c < 1$

$$2f\left(\left[\frac{n}{2}\right]^4\right) \leq cf(n^4)$$

$$2f\left(\frac{n^4}{2^4}\right) \leq cf(n^4)$$

$$\boxed{\Theta(n^4)}$$

$$2f\left(\frac{n^4}{2^4}\right) \leq cf(n^4), \quad c = \frac{1}{8}$$

$$f\left(\frac{n^4}{16}\right) \leq \frac{1}{16} f(n^4)$$

válido