



Fig. 1. The movement of x_i in case 2.

experiment 1, the comparison of CNPSO and other five PSOs is done to test the quality of CNPSO. To further verify the performance of the proposed algorithm, CNPSO is compared with some improved ABC algorithms and some DEs, CAs in experiment 2 and experiment 3 respectively. In experiment 4, CNPSO and other algorithms are used to optimize a practical problem to test the capability of proposed algorithm.

4.1. Experiment 1: Compared with five PSOs

In this subsection, CNPSO is compared with basic PSO [3] and other four improved algorithms: AIWPSO [37], PSOGSA [38], PSOd [39] and H-PSO-SCAC [40] on 27 test functions. The minimum value(min), mean value (mean) and standard deviation (std) of the obtained results is used as the performance testing indicator. Besides, all algorithms are run in Matlab 9.2.0 (Win 64) of personal computer with Inter(R) Core(TM) i5-4258U CPU @2.40 GHz under Windows 7 system.

4.1.1. Benchmark function

In experiment 1, 27 test functions are employed, which include unimodal functions, multi-modal functions and rotational functions. $f_1 - f_{12}$ are unimodal functions, $f_{13} - f_{24}$ are multi-modal functions, $f_{25} - f_{27}$ are multi-modal rotational functions. These functions are tested on 30 and 100 dimensions respectively. The range of variable and the optimal value of each function are given in Table 1.

Table 1
22 benchmark functions in experiment 1.

Functions	Range	Optimal value
$f_1 = 0.26(x_1^2 + x_2^2) - 0.48x_1x_2$	$[-10, 10]$	0
$f_2 = 4x_1^2 - 2.1x_1^4 + (x_1^6)/3 + x_1x_2 - 4x_2^2 + 4x_2^4$	$[-5, 5]$	-1.0316
$f_3 = \sum_{i=1}^D x_i^2$	$[-100, 100]$	0
$f_4 = \sum_{i=1}^D x_i + \prod_{i=1}^D x_i $	$[-10, 10]$	0
$f_5 = \sum_{i=1}^D (\sum_{j=1}^i x_j)^2$	$[-100, 100]$	0
$f_6 = \max_i \{ x_i , 1 \leq i \leq D\}$	$[-100, 100]$	0
$f_7 = \sum_{i=1}^D ix_i^2$	$[-10, 10]$	0
$f_8 = \sum_{i=1}^D ix_i^4$	$[-1.28, 1.28]$	0
$f_9 = \sum_{i=1}^D x_i ^{(i+1)}$	$[-1, 1]$	0
$f_{10} = \sum_{i=1}^D (10^6)^{\frac{i-1}{D-1}} x_i^2$	$[-100, 100]$	0
$f_{11} = \sum_{i=1}^D (x_i + 0.5)^2$	$[-1.28, 1.28]$	0
$f_{12} = \sum_{i=1}^D ix_i^4 + \text{random}[0, 1]$	$[-1.28, 1.28]$	0
$f_{13} = \sum_{i=1}^D (x_i^2 - 10 \cos(2\pi x_i) + 10)$	$[-5.12, 5.12]$	0
$f_{14} = -20 \exp(-0.2 * \sqrt{\sum_{i=1}^D x_i^2 / D}) - \exp(\sum_{i=1}^D \cos(2\pi x_i / D)) + 20 + e$	$[-32, 32]$	0
$f_{15} = \frac{1}{4000} \sum_{i=1}^D x_i^2 - \prod_{i=1}^D \cos(\frac{x_i}{\sqrt{i}}) + 1$	$[-600, 600]$	0
$f_{16} = 0.5 + \frac{\sin(\sqrt{\sum_{i=1}^D x_i^2}) - 0.5}{(1 + 0.001 \sum_{i=1}^D x_i^2)^2}$	$[-100, 100]$	0
$f_{17} = \sum_{i=1}^D \frac{(x_i^4 - 16x_i^2 + 5x_i)}{D}$	$[-5, 5]$	-78.3323
$f_{18} = \sum_{i=1}^D x_i \sin(x_i) + 0.1x_i $	$[-10, 10]$	0
$f_{19} = \begin{cases} \sum_{i=1}^D (x_i^2 - 10 \cos(2\pi x_i) + 10), & x_i < 0.5 \\ \sum_{i=1}^D ((\frac{\text{random}(2x_i)}{2})^2 - 10 \cos(\Pi \text{random}(2x_i)) + 10), & x_i \geq 0.5 \end{cases}$	$[-5.12, 5.12]$	0
$f_{20} = \frac{\Pi}{10} 10 \sin^2(\Pi y_1) + \frac{\Pi}{10} \sum_{i=1}^{D-1} (y_i - 1)^2 [1 + 10 \sin^2(\Pi y_{i+1})] + \frac{\Pi}{10} (y_D - 1)^2 + \sum_{i=1}^D u(x_i, 10, 100, 4)$ $y_i = 1 + \frac{x_i + 1}{4}$		
$u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m, & x_i > a \\ 0, & -a \leq x_i \leq a \\ k(-x_i - a)^m, & x_i < -a \end{cases}$	$[-50, 50]$	0
$f_{21} = 418.98288727243369 * D - \sum_{i=1}^D x_i \sin(\sqrt{ x_i })$	$[-500, 500]$	0
$f_{22} = \sum_{i=1}^D (10^{\frac{i-1}{D-1}} x_i)^2 - 10 \cos(2\Pi 10^{\frac{i-1}{D-1}} x_i) + 10$	$[-5.12, 5.12]$	0
$f_{23} = 0.1 \{ \sin^2(3\Pi x_1) + \sum_{i=1}^D (x_i - 1)^2 [1 + \sin^2(3\Pi x_i + 1)] \} + 0.1 \{ (x_D - 1)^2 [1 + \sin^2(2\pi x_D)] \} + \sum_{i=1}^D u(x_i, 5, 100, 4)$		
$u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m, & x_i > a \\ 0, & -a \leq x_i \leq a \\ k(-x_i - a)^m, & x_i < -a \end{cases}$	$[-50, 50]$	0
$f_{24} = \sin(\pi \omega_1)^2 + \sum_{i=1}^{D-1} (w_i - 1)^2 [1 + 10 \sin(\pi \omega_i + 1)^2] + (\omega_D - 1)^2 [1 + \sin(2\pi \omega_D)^2], \omega_i = 1 + \frac{x_i}{4}$	$[-10, 10]$	0
$f_{25} = \sum_{i=1}^D (20^{\frac{i-1}{D-1}} * z_i)^2, z = x * M$	$[-100, 100]$	0
$f_{26} = (1000x_1)^2 + \sum_{i=2}^D z_i^2, z = x * M$	$[-100, 100]$	0
$f_{27} = \frac{1}{4000} \sum_{i=1}^D z_i^2 - \prod_{i=1}^D \cos(\frac{z_i}{\sqrt{i}}) + 1, z = x * M$	$[-600, 600]$	0