The Tube Map

A: Algebra

C: Calculus

CN: Complex Numbers

DE: Differential Equations

E: Equations and Inequalities

F: Functions

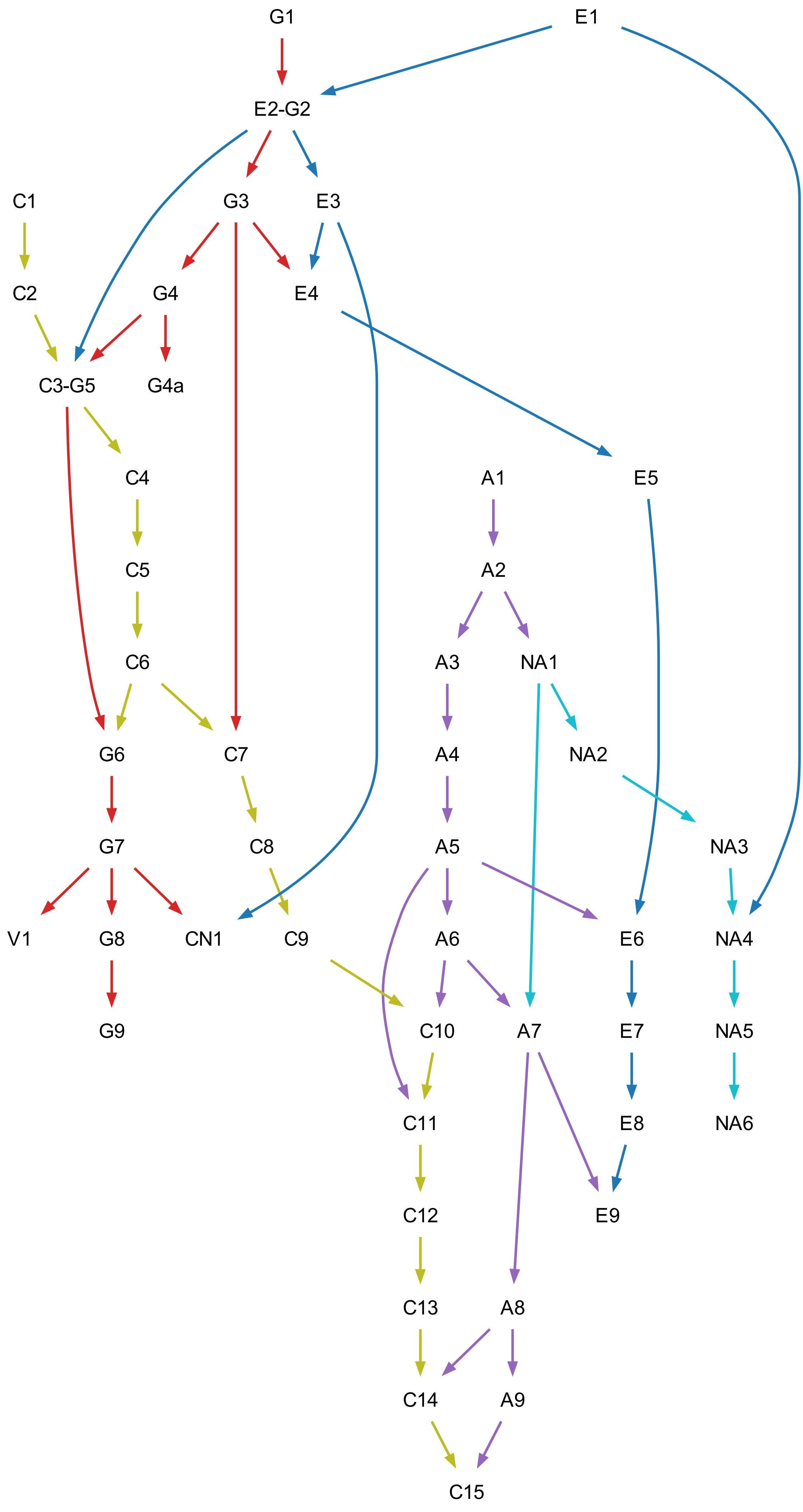
G: Geometry

M: Mechanics

NA: Numbers and Algorithms

P: Probability

V: Vectors


G1
E1
E2-G2
C1
G3
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C2
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C5
A2
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A3
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C7
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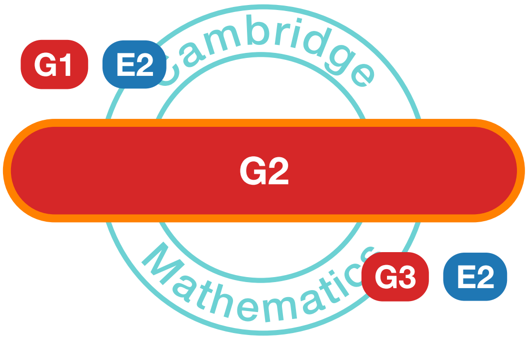


What is the connection between algebra and geometry, and how can we exploit it?

Key Questions

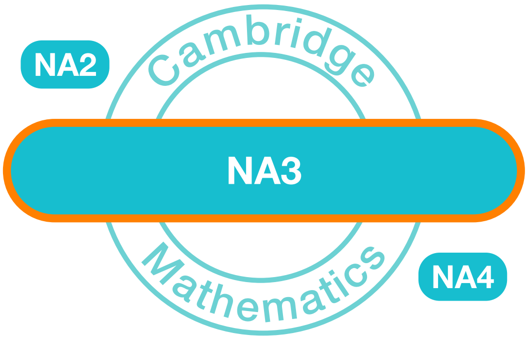
1. What do we mean by *Cartesian coordinates*?
2. How can we describe circles and lines (in two dimensions) algebraically?
3. How can we consider parallel and perpendicular lines using algebra?
4. How can we use algebra to find the intersections of geometric objects?

Resources

- Introductory problem - Straight lines
- Introductory problem - Parallel and Perpendicular Lines
- Exposition - The equation of a circle
- Problem inviting multiple approaches or representations - Finding circles
- Carefully designed set of problems - Equations of circles
- Carefully designed set of problems - Matching circles and equations
- Problem requiring decisions - Olympic rings
- Fluency exercises - Diamond collector
- Lucky dip - Parabella
- The bigger picture - Cartesian coordinates

Pervasive Ideas

- Multiple representations
- Linearity
- History of maths



What are highest common factors and why do they matter?

Key Questions

1. What do we mean by *highest common factor* and *lowest common multiple*, and how are they related?
2. How can we find the highest common factor of two numbers?
3. What is Euclid's algorithm all about?
4. What is the *Fundamental theorem of arithmetic*, and why does it matter?

Resources

Introductory investigation - Picture this!
Worked examples - Euclid's algorithm
Scaffolded task - Factorial fun
Scaffolded task - Division game
Scaffolded task - The Fundamental Theorem of Arithmetic
Open-ended investigation - Buckets and ponds
Lucky dip - LCM sudoku
Lucky dip - A Diophantine equation
Lucky dip - There's always one, isn't there?
Lucky dip - $n^5 - n$
Go and think about it... - S-prime numbers
Go and think about it... - A BMO2 question

Pervasive Ideas

- Proof
- Conjecturing
- Visualising
- Iteration
- Generalising and specialising

Pervasive Idea Families

Concept

- Linearity (./pervasiveIdeas/PI2.html)
- Axioms and axiomatic systems (./pervasiveIdeas/PI6.html)
- Symmetry (./pervasiveIdeas/PI7.html)
- Iteration (./pervasiveIdeas/PI9.html)
- Averages (./pervasiveIdeas/PI11.html)

Context

- History of maths (./pervasiveIdeas/PI8.html)
- Group theory (./pervasiveIdeas/PI10.html)

Process

- Multiple representations (./pervasiveIdeas/PI1.html)
- Proof (./pervasiveIdeas/PI3.html)
- Conjecturing (./pervasiveIdeas/PI4.html)
- Visualising (./pervasiveIdeas/PI5.html)
- Generalising and specialising (./pervasiveIdeas/PI12.html)

Pervasive Ideas

Our tube map shows the connections between many stations, and each station contains some mathematical content. There are, however, many important mathematical ideas that do not fit naturally at a station, because they somehow span a number of stations. These are our *pervasive ideas*.

We have grouped these ideas loosely into three families.

Concepts are mathematical ideas that occur across content strands.

Context is all about how mathematical ideas fit together in a bigger picture. This family includes applications of mathematical ideas to the real world, indications of how mathematical ideas come together in more advanced mathematical topics, and historical background, for example.

Processes are about the way in which we do mathematics.

We believe that it is important and useful for students and teachers to be aware of these pervasive ideas as they are studying mathematics, and so we have highlighted occurrences of pervasive ideas at stations as well as giving information and resources for each pervasive idea at its own page. We hope that you will explore these as you travel through the CMEP tube map.

Pervasive Idea Families

Concept

- Linearity (./pervasiveIdeas/PI2.html)
- Axioms and axiomatic systems (./pervasiveIdeas/PI6.html)
- Symmetry (./pervasiveIdeas/PI7.html)
- Iteration (./pervasiveIdeas/PI9.html)
- Averages (./pervasiveIdeas/PI11.html)

Context

- History of maths (./pervasiveIdeas/PI8.html)
- Group theory (./pervasiveIdeas/PI10.html)

Process

- Multiple representations (./pervasiveIdeas/PI1.html)
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Symmetry

Summary

Humans are very good at detecting symmetry. It seems that we have evolved to look for certain underlying structure, which includes symmetry. Mathematicians have taken this further, and describe various sorts of structure as being examples of symmetry. Finding symmetry is often satisfying in itself, but can also lead mathematicians to be able to explain something better, or to understand an idea in more depth.

Resources

- The bigger picture (../resources/PI7_RT15/index.html)

Appearances

Resource Types

Introductory investigation

An exploratory task, designed to be presented with little preamble. Students rapidly get engaged with the task, which then gives them an opportunity to discover the relevant ideas or concepts for themselves, perhaps by working collaboratively with others. This might well be followed by some sort of summary, or exposition, or worked examples, to formalise what has been found.

Introductory problem

A task that gets students started on the sort of thinking or mathematics in a section of work, but that is perhaps less open and exploratory than an introductory investigation.

Exposition

A presentation of some mathematics, perhaps in the form of an article or a video. It might, but need not, include questions for the student to consider while reading/watching the presentation.

Worked examples

An illustration of a standard technique, or a standard way of presenting an argument, for example, through carefully chosen examples. Crucially the worked examples are followed by problems for students to try themselves, and while these problems are related to the worked examples, they require more than just 'templating' (changing a few numbers in the worked examples). Another sort of resource in this category could be an invitation to the student to come up with their own examples, as a way of getting a feel for the scope of an idea or technique.

Scaffolded task

A task that gives students an opportunity to engage with material that would otherwise be out of their grasp. For example, it might be a 'proof sorter' activity that enables students to understand the components of a proof and their ordering, although creating the proof from scratch would be beyond the students. Or it might be a 'hide and reveal' task, which has a number of sections that can be revealed one at a time, to offer some structure (e.g. subtasks leading to a big task) that helps students to make progress on something that would otherwise be too big or demanding.

Problem inviting multiple approaches or representations

Students sometimes seize the first idea that comes to mind, when it might not necessarily be the most effective. These problems draw students' attention to the thought that there might be several ways to tackle a problem, or to represent an idea. They might do this not only by setting a problem that can be tackled in several ways, but also by illustrating some of these ways, or suggesting several starting points.

Carefully designed set of problems

This is a set of problems that have been designed and that should be thought of as a single entity. It might be that the problems build on each other, so that by the end they are requiring more sophistication but in a way that is accessible to students who have worked through the whole set. Or it might be that by working on all of the problems, the student is naturally prompted to explore some underlying structure or to make a generalisation.

Problem requiring decisions

Students are often used to problems being posed in such a way that they have all the information that they require for a solution, and no more. Problems (especially from the real world) are very often not like this, and so resources of this type will give students the opportunity to develop the skills needed to deal with this. Some problems might not contain enough information to answer a question, and so students will have to find some additional information (perhaps by making assumptions, perhaps by asking an expert (teacher), perhaps by carrying out an experiment, perhaps by researching online). And some problems might contain too much data, so that part of the challenge is to identify the useful information.

Open-ended investigation

Something for students to get stuck into. It might start with just an interesting context, or with some initial questions to explore, but the emphasis is on students posing their own questions, and pursuing the avenues that interest them. This is good for developing various mathematical skills, and gives students the opportunity to work on things that really inspire them. These investigations might be good for after-school clubs, or individual projects.

Fluency exercises

A bank of routine exercises (perhaps randomly generated) that students can use to develop fluency. Students can be encouraged to do as many as they need to do to feel confident and fluent with a particular idea or technique.

Lucky dip

A mixed collection of problems for further practice. Students (or teachers) might pick a problem at random, or might rummage through the bag to find a problem that appeals to them. Working on these problems is not necessary for understanding the topic, but it is useful to have these problems available for additional practice and for variety. Many of them might be fairly short.

Review questions

Students will regularly want to test their understanding, both of a particular topic and of their ability to draw together ideas from a number of topics. Review questions might include some past examination questions, alongside other fairly closed problems and other problems designed to get students to look back over what they have learned. They might be pitched at different levels, and might be flagged to show whether they incorporate ideas from other topics.

Go and think about it...

A problem for which students (probably) have the required mathematical knowledge, but where the challenge is identifying how to get started, what tools might help, and how to apply the relevant mathematical knowledge. Students might tackle such a problem at home, for a challenge, as these problems are often more suited for individual consideration than classroom collaboration.

The bigger picture

A resource that puts the mathematics into context, perhaps by offering a historical perspective, or describing the mathematicians who worked on it, or linking it to areas of current research, or illustrating how it leads on to further topics (e.g. at university level). They would not all need to be accessible to all students: sometimes it might be appropriate to have something that is designed for a student with a particular interest (e.g. for a student with an interest in Physics, who is studying A-level Physics alongside Maths). This sort of resource might be an article or video, for example. It might, but need not, contain questions for the student to consider while reading or watching.

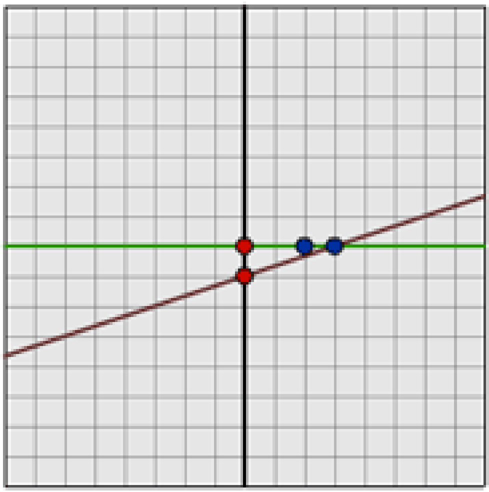
Parallel and Perpendicular Lines

G2_RT2_2

In this interactivity you can move the dots to determine the positions of two lines.

Move the dots until the lines are parallel. What do you notice about the equations of the lines?

Experiment with the lines in several positions. What can you say about the equations of parallel lines?



Can you **explain** your findings?

You can now use the interactivity to explore equations of perpendicular lines. Move the dots until the lines are perpendicular. How do you know that they are perpendicular? What do you notice about the equations of the lines?

Experiment with the lines in several positions. What can you say about the equations of perpendicular lines?

Can you **explain** your findings?

How can we tell whether two lines intersect? Can we tell this just from their equations? Can we find the number of points of intersection without actually finding the points themselves? You may like to use the interactivity to explore.

How can we use the equations of the lines to find all their points of intersection?

For each of the following questions, try to think geometrically and also algebraically (working just from the equations of the lines, with no picture).

Can you find two lines that do not intersect?

Can you find two lines that intersect at exactly one point?

Can you find two lines that intersect at exactly two points?

Can you find two lines that intersect at exactly n points, for any $n \geq 3$?

Can you find two lines that intersect at infinitely many points?

How many points of intersection might there be amongst three lines? What if there are even more lines?

Relevance

E2 How is the solution of equations related to problems in geometry?

G2 What is the connection between algebra and geometry, and how can we exploit it?

Matching circles and equations

Problem

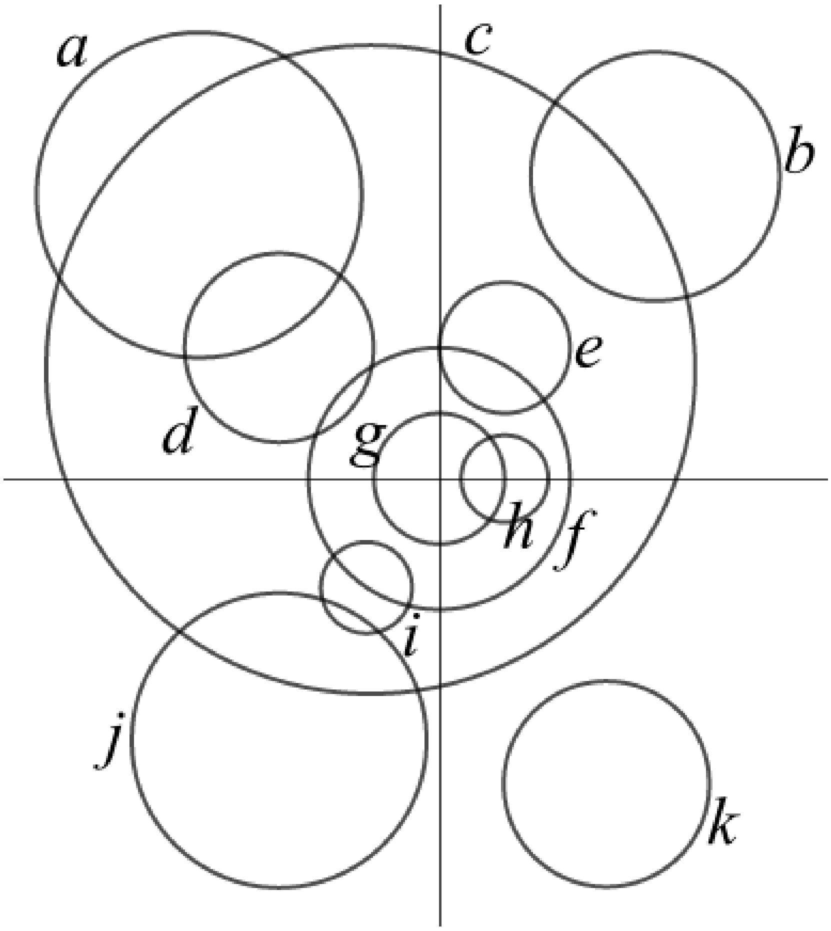


Figure G2_RT7_2.1: Circles

We have thought of 13 circles. Some of them are represented in the above diagram, with the labels of the x-axis and y-axis removed. Here are equations for some of the circles.

1. $(x+10)^2+(y+15)^2=4\pi^2$
2. $x^2+y^2=324$
3. $(x+22)^2+(y+36)^2=411$
4. $(x+3\pi)^2+(y-15)^2=1990$
5. $\left(x-21\sqrt{2}\right)^2+\left(y-24\sqrt{3}\right)^2=131\sqrt{5}$
6. $(x+33)^2+(y-39)^2=500$
7. $x^2+y^2=9$
8. $(x-23)^2+(y+42)^2=200$
9. $x^2+y^2=81$
10. $(x-18)^2+(y+36)^2=1990$
11. $(x-9)^2+y^2=36$

Can you match them up, find the missing equations and construct the remaining circles on the diagram?

Relevance

E2 How is the solution of equations related to problems in geometry?

G2 What is the connection between algebra and geometry, and how can we exploit it?

- Multiple representations

Matching circles and equations

Solution

Here's a summary of the matching between equations and circles.

Circle	Equation
a	6
b	5
c	4
d	$(x+22)^2+(y-18)^2=169$
e	$(x-9)^2+(y-18)^2=81$
f	2
g	9
h	11
i	1
j	3
k	8
l	7
m	10

Here's a description of one way in which we could have found that matching.

Equation 11 describes a circle with centre on the x-axis, and we might notice that circle h is such a circle. So we hope that these two match up. (In fact, one can show that we would end up with more than 13 circles if equation 11 and circle h didn't match up.)

As circle g passes through the centre of circle h, equation 9 belongs to circle g.

Moreover, circle f corresponds to equation 2 and we can draw the circle corresponding to equation 7. It is circle l in the diagram below.

From the information we worked out so far, we deduce that circle e has centre $(9, 18)$ and radius 9. So its equation is $(x-9)^2+(y-18)^2=81$ (which isn't on the given list of equations).

By further considering the centres of the circles described by the remaining equations (e.g. by looking at which quadrant they lie in), we can match up circle a and equation 6, circle b and equation 5, circle c and equation 4, circle i and equation 1, circle j and equation 3, and circle k and equation 8.

To find the equation of circle d, we first work out its centre $(-22, 18)$, e.g. by considering the centres of circles e and j. Moreover, to obtain its radius we note that the point on circle d which has the largest x-value and the point on circle g which has the smallest x-value form a straight line parallel to the y-axis. Thus, the radius is 13 and the equation of circle d is $(x+22)^2+(y-18)^2=169$.

Finally, to plot the circle with equation 10, we first construct its centre by using circle j and circle f. Then, we could use the bisector method to find the centre of circle c and take its radius to construct circle m.

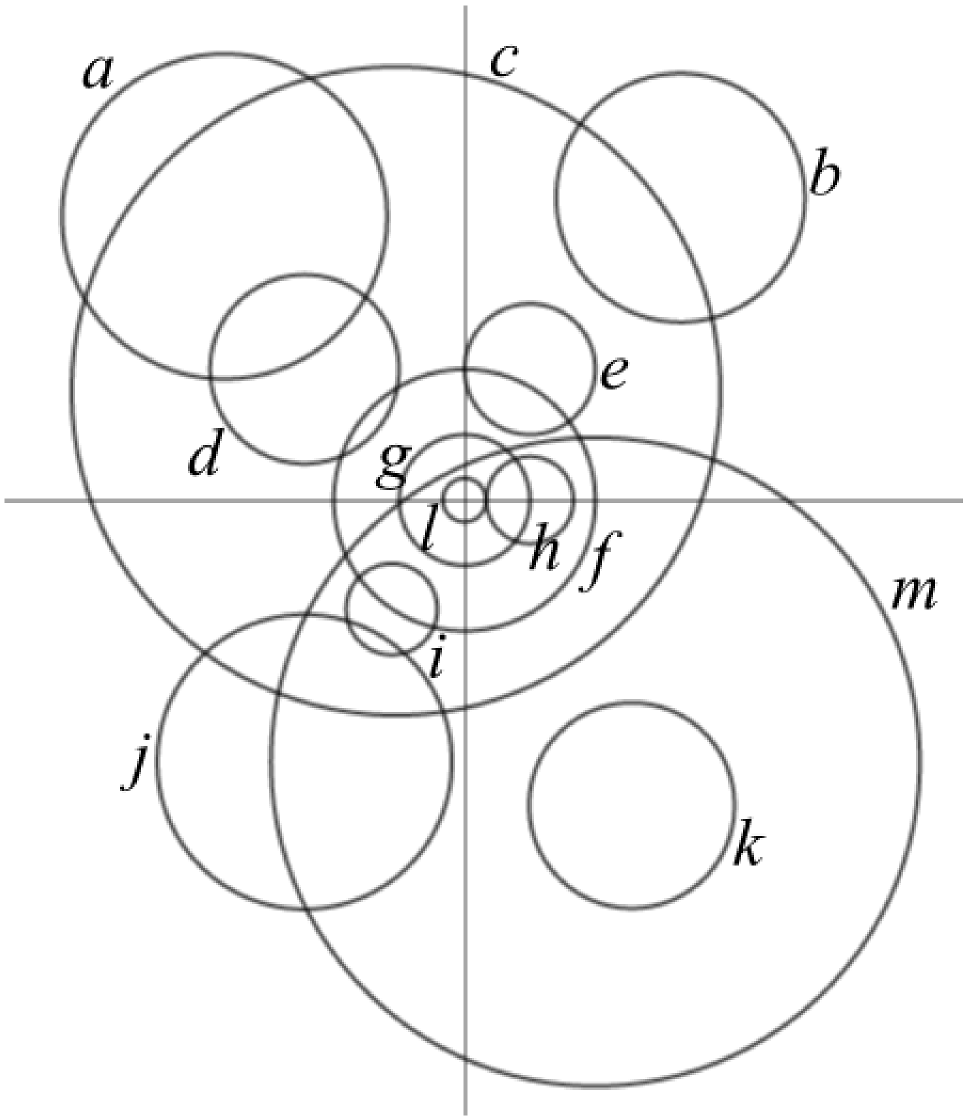


Figure G2_RT7_2.1: Complete set of circles

Relevance

E2 How is the solution of equations related to problems in geometry?

G2 What is the connection between algebra and geometry, and how can we exploit it?

- Multiple representations

Diamond collector

G2_RT11

This is a game for two players (you can choose to play against the computer if you wish). Each player gets to specify three lines, with the aim of collecting as many diamonds as possible.

Eventually there will be a version of this with levels that allow people to collect diamonds by specifying circles as well as lines.

Relevance

E2 How is the solution of equations related to problems in geometry?

G2 What is the connection between algebra and geometry, and how can we exploit it?

Review questions

Q1 Hint Solution

Prove that the points whose coordinates satisfy the equation

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

lie on a circle. State the coordinates of the centre of the circle and the length of its radius.

Prove that the circles

$$x^2 + y^2 - 20x - 16y + 128 = 0$$

and

$$4x^2 + 4y^2 + 16x - 24y - 29 = 0$$

lie entirely outside each other, and find the length of the shortest distance from a point on one circle to a point on the other.

UCLES A level Maths, QP 185/2, June 1953, Q2.
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Q2 Hint Solution

Show that the four points $(3,0)$, $(0,4)$, $(3,4)$, $(4,2)$ lie on a circle.

Find the centre and radius of this circle.

UCLES A level Maths 1, Syllabus A, Pure Mathematics 1, 9200/1, 9208/1, 1987, Q6.
Reproduced by permission of Cambridge Assessment Group Archives.

Q3 Hint Solution

Find the equation of the circle through the points $A\,(2,0)$, $B\,(8,0)$, and $C\;(10,4)$, and prove that it touches the y-axis.

Without the use of tables [or calculators] or measurement, find the equation of the the other tangent to this circle from the origin.

UCLES A level Maths, QP 417/2 and 447/2, June 1963, Q2.
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Q4 Hint Solution

Prove that the line $y = mx + c$ will touch the circle $x^2 + y^2 = 25$ if $c^2 = 25(1 + m^2)$. Hence or otherwise find the equations of the two tangents to this circle from the point $(2, 11)$.

UCLES A level Maths 2, QP 417/2 and 447/2, June 1964, Q2.

Q5 Hint Solution

Show that the circles having equations $x^2 + y^2 = 25$ and $x^2 + y^2 - 24x - 18y + 125 = 0$ touch each other. Calculate the coordinates of the point at which they touch.

UCLES A level Maths, QP 9200/1 and 9208/1, Summer 1986, Q7.

Printables

PDF version (./index.pdf)

Relevance

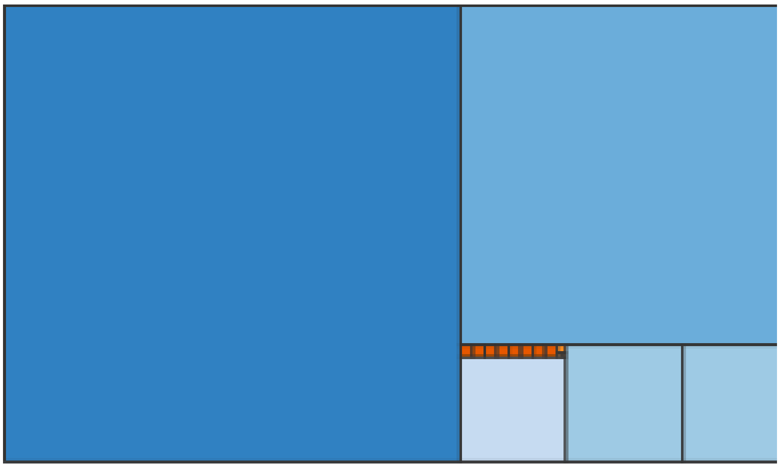
G2 (../../stations/G2.html) What is the connection between algebra and geometry, and how can we exploit it?

Picture this!

NA3_RT1

In this interactivity, you can specify two positive integers (whole numbers) and the computer will draw a corresponding diagram.

Experiment with a few examples, and then consider the questions that follow.



- Given two positive integers, can you draw the corresponding diagram?
- Given a diagram, can you determine the corresponding pair of positive integers?

You should now have a good understanding of the relationship between the diagram and the pair of positive integers. There are many questions that you might now ask yourself.

Investigate the questions that interest you. You might want to make some conjectures and then try to prove (justify) them or to disprove them by finding counterexamples.

Then you can look at our list of questions below.

What is the relationship between the side length of the smallest square in the diagram and the pair of positive integers?

How many steps (different colours) can we have? Which pairs of integers give many steps and which give few?

How can we record the information from the diagram in the form of equations?

When might the diagram be more convenient? When might the equations be more convenient?

What is the point of the process captured by this diagram? What is it useful for? When might it be more or less useful than our existing techniques?

Relevance

NA3 What are highest common factors and why do they matter?

LCM sudoku

NA3_RT12

			*	*				
	*			*		*		
*	*							*
*		*						
	*	*				*		
	*				*			
*		*			*		*	
			*				*	
				*	*			*

Like a standard sudoku, this sudoku variant has the basic rule that every row, every column and every 3×3 box in the grid contains the digits 1 to 9.

The first step to solving this puzzle is to find the values of the unknown digits (all indicated by asterisks) in the cells of the 9×9 grid. At the bottom and right side of the 9×9 grid are numbers, each of which is the least common multiple (LCM) of all the starred numbers in the row or column preceding it.

We define the least common multiple of a set of numbers as the smallest number which is divisible by all of them. For example, the least common multiple of 3, 4 and 8 is 24.

In total 18 least common multiples are given as clues for solving the puzzle—one for each row and each column of the grid.

After finding the values of all the unknown digits, the puzzle is solved as a traditional sudoku with the starred numbers as a starting point.

Relevance

NA3 What are highest common factors and why do they matter?

Review questions

Q1 Solution

A number of the form $1/N$, where N is an integer greater than 1, is called a *unit fraction*.

Noting that

$$\frac{1}{2}=\frac{1}{3}+\frac{1}{6} \quad \text{and} \quad \frac{1}{3}=\frac{1}{4}+\frac{1}{12},$$

guess a general result of the form

$$\frac{1}{N}=\frac{1}{a}+\frac{1}{b}$$

and hence prove that any unit fraction can be expressed as the sum of two distinct unit fractions.

By writing the previous equation in the form

$$(a-N)(b-N)=N^2$$

and by considering the factors of N^2, show that if N is prime, then there is only one way of expressing $1/N$ as the sum of two distinct unit fractions.

Prove similarly that any fraction of the form $2/N$, where N is prime number greater than 2, can be expressed uniquely as the sum of two distinct unit fractions.

UCLES STEP Maths II, 2000, Q1.

Q2 Solution

Consider the arithmetic sequences $1998, 2005, 2012, \ldots$ and $1996, 2005, 2014, \ldots$. What is the next number after 2005 that appears in both sequences?

SMC, 2005, Q7.

Q3 Solution

For how many integers n is $\frac{n}{100-n}$ also an integer?

SMC, 2009, Q15.

Q4 Solution

Four positive integers a, b, c and d are such that

$$abcd + abc + bcd + cda + dab + ab + bc + cd + da + ac + bd + a + b + c + d = 2009.$$

What is the value of $a + b + c + d$?

SMC, 2009, Q25.
Reproduced by kind permission of the United Kingdom Mathematics Trust (http://ukmt.org.uk/).

Q5 Solution

The *primorial* of a number is the product of all the prime numbers less than or equal to that number. For example, the primorial of 6 is $2 \times 3 \times 5 = 30$.

How many different whole numbers have a primorial of 210?

SMC, 2011, Q12.
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Printables

PDF version (./index.pdf)

Relevance

NA3 (../../stations/NA3.html) What are highest common factors and why do they matter?

A BMO2 question

NA3_RT14_2

Suppose x, y, z are positive integers satisfying the equation

$$\frac{1}{x} - \frac{1}{y} = \frac{1}{z},$$

and let h be the highest common factor of x, y, z.

Prove that $hxyz$ is a perfect square.

Prove also that $h(y-x)$ is a perfect square.

BMO2 1998 Q3.

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Relevance

NA3 What are highest common factors and why do they matter?

Cartesian coordinates

G2_RT15

How do you remember the spot on a desert island where you buried your treasure? You pick a landmark, say a palm tree, and measure how many steps North/South and how many steps East-/West you have to go from that landmark to get to the treasure. It's a simple idea but, surprisingly, mathematicians took centuries to develop its full potential in their own field. When they finally did, it revolutionised mathematics by bringing together two areas that on the face of it have little to do with each other: algebra and geometry.

When we start learning geometry we usually think of simple shapes like lines, triangles and circles in the two-dimensional plane. You can construct these and more complicated shapes using a ruler, compasses and protractor. The ancient Greeks were masters at this type of geometry: using just compasses and a straight-edge (an unmarked ruler) they were able to construct a range of shapes and they could even prove mathematical results, such as Pythagoras' theorem, using these simple tools.

There are certain things, however, that you cannot do using these basic methods. Two shapes, say a line and a circle, may or may not intersect, and they may intersect in different ways: perhaps the line just touches the circle, perhaps it shaves a little arc off it, or perhaps it cuts it in half. To record this information you need a way of describing the locations of the shapes.

This is where the treasure island idea comes in useful. It illustrates what is called the *Cartesian coordinate system*. Choose a point in the plane, called the *origin*, and draw two perpendicular axes through it, one horizontal and one vertical. Any point in the plane can be reached from the origin by travelling a certain distance x along the horizontal axis and a certain distance y along the vertical axis. The numbers (x, y) are the coordinates of the point. The origin itself has coordinates $(0, 0)$. The part of the horizontal axis (also called the x-axis) that lies to the left of the origin and the part of the vertical axis (the y-axis) below the origin are described by negative numbers.

Cartesian coordinates are named after the 17th century French philosopher and mathematician René Descartes. There is a (probably untrue) story that Descartes invented these coordinates while lying in bed watching a fly on the ceiling and wondering how to describe its location. Descartes' penchant for lying in bed until noon may actually have been the cause of his demise, which occurred in Stockholm in 1650. Descartes was in Sweden to act as maths tutor to Queen Christina, who unfortunately preferred to work early in the morning. According to some reports it was these early hours and the Scandinavian temperatures which caused the pneumonia that eventually killed him. Others have suggested that he was poisoned by a Catholic priest worried about Descartes' radical theology.

Either way, the Cartesian coordinate system is one of Descartes' most important legacies (although he was not the only person to have the idea). It allows us to answer geometric problems using algebra and to visualise algebraic relationships that would otherwise remain quite abstract. Take for example the equation

$$y = 2x - 1.$$

We can plot the graph of this function in a Cartesian coordinate system by plotting all points whose coordinates are of the form $(x, 2x - 1)$: points such as $(0, -1)$, $(1, 1)$, $(2, 3)$, $(-1, -3)$, $(-2, -5)$, $(-\frac{1}{2}, -2)$, and $(1.73, 2.46)$. In this case the graph is a straight line that meets the y-axis at the point $(0, -1)$ and has a slope of 2.

More generally every straight line is given by an equation of the form

$$y = mx + b,$$

