Equations of circles

Solution

1. A circle of centre (a, b) and radius r has equation:

$$(x-a)^2+(y-b)^2=r^2.$$

- Centre $(a+1, b)$, radius r: $(x-a-1)^2+(y-b)^2=r^2$
- Centre $(a-1, b)$, radius r: $(x-a+1)^2+(y-b)^2=r^2$
- Centre $(a, b+1)$, radius r: $(x-a)^2+(y-b-1)^2=r^2$
- Centre $(a, b-1)$, radius r: $(x-a)^2+(y-b+1)^2=r^2$
- Centre $(-a, b)$, radius r: $(x+a)^2+(y-b)^2=r^2$
- Centre (b, a), radius r: $(x-b)^2+(y-a)^2=r^2$
- Centre $(a-b, 0)$, radius r: $(x-a+b)^2+y^2=r^2$
- Centre $(2a, b)$, radius r: $(x-2a)^2+(y-b)^2=r^2$
- Centre (a, b), radius $2r$: $(x-a)^2+(y-b)^2=4r^2$
- Centre (a, b), radius $\frac{1}{3}r$: $(x-a)^2+(y-b)^2=\frac{1}{9}r^2$
- Centre (a, b), radius $r+1$: $(x-a)^2+(y-b)^2=(r+1)^2$

2.
- $(x-\pi)^2+(y+2)^2=3$ is the equation of a circle with centre $(\pi, -2)$ and radius $\sqrt{3}$.
- There are no real values of x and y that satisfy the equation $(x+1)^2+(y-4)^2=-1$, so we can't plot this in the x–y plane.
- $x^2+2x+y=2$ can be rearranged to give $y=-x^2-2x+2$, which is the standard form of a quadratic equation, and so describes a parabola:
- $x^2+y^2=4$ is the equation of a circle centred at the origin with radius 2.
- $(x-1)^2+y^2=4$ is the equation of a circle with centre $(1,0)$ and radius 2.
- $(x-1)^2-y^2=4$ is the equation of a hyperbola:
- $x^2+y^2-3x-y=-1.5$ can be rearranged to give $\left(x-\frac{3}{2}\right)^2+\left(y-\frac{1}{2}\right)^2=1$, so this is the equation of a circle with centre $(\frac{3}{2}, \frac{1}{2})$ and radius 1.
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  This equation looks rather similar to the previous one. Maybe we can use that to save ourselves some effort.

  $x^2+y^2+3x+y=-1.5$ can be rearranged to give $\left(x+\frac{3}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=1$, so this is the equation of a circle with centre $(-\frac{3}{2}, -\frac{1}{2})$ and radius 1.

3. The centre of the circle must be the midpoint of the line segment from $(-5, 2)$ to $(3, -1)$, so the centre of the circle is at $\left(-1, \frac{1}{2}\right)$. The radius is then the distance from the centre to either of the points $(-5, 2)$ or $(3, -1)$, say $(3, -1)$:

$$r=\sqrt{(-1-3)^2+\left(\frac{1}{2}-(-1)\right)^2}=\sqrt{16+\frac{9}{4}}=\frac{\sqrt{73}}{2}.$$

This is equivalent to working out the diameter, d, of the circle as the distance between the points $(5, 2)$ and $(3, -1)$, and then saying $r=\frac{1}{2}d$.

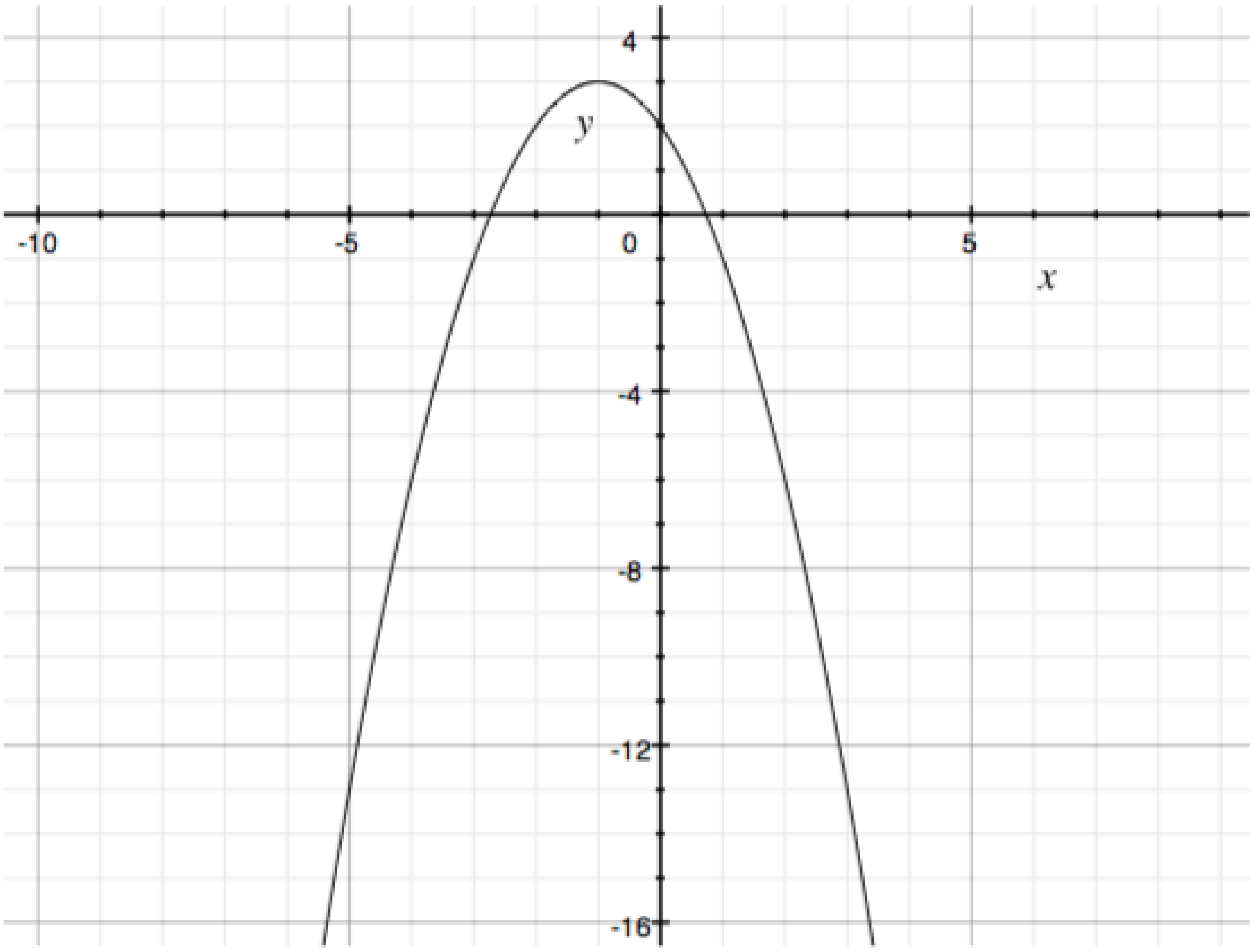

Figure G2_RT7.1: Figure 1

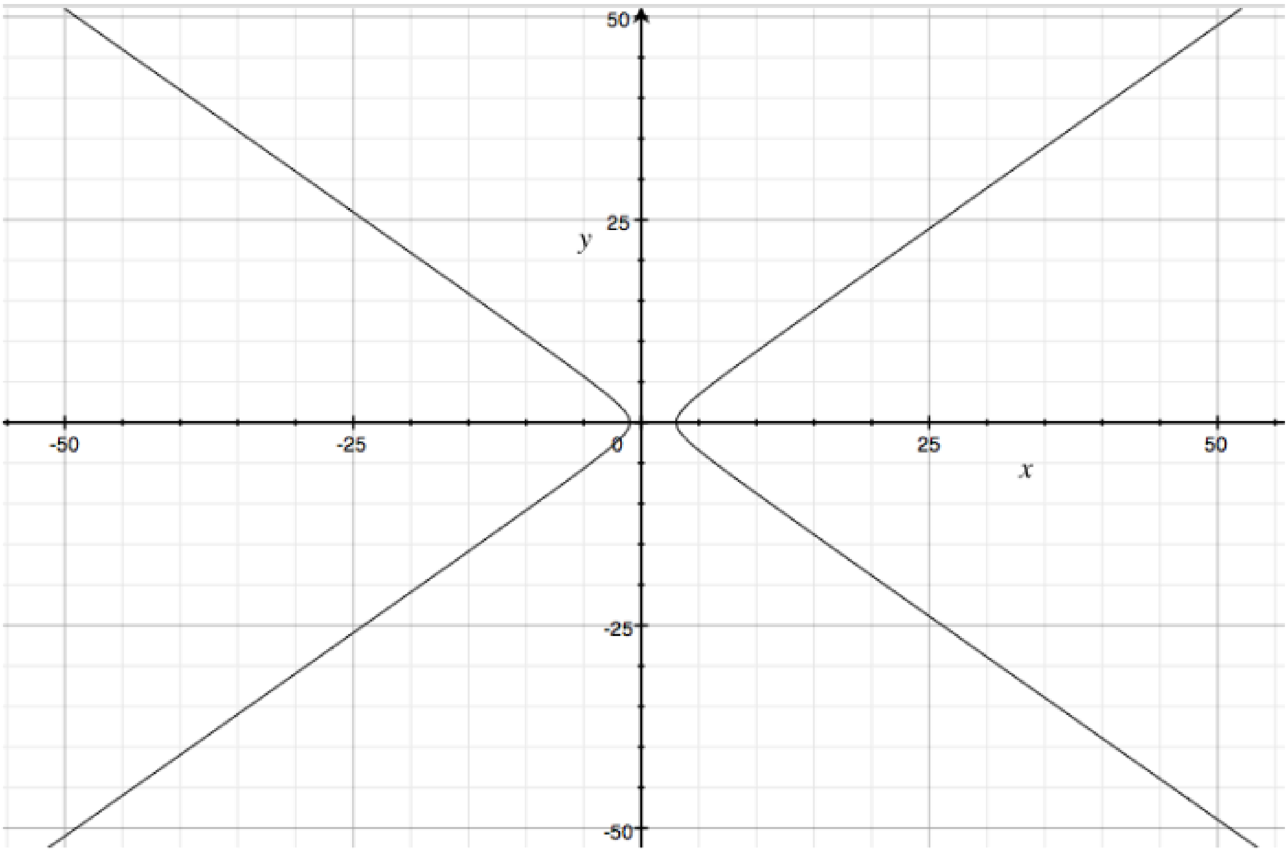

Figure G2_RT7.2: Figure 2

So the equation of the circle of which the line segment from $(-5, 2)$ to $(3, -1)$ is a diameter is

$$(x+1)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{73}{4}.$$

There are infinitely many circles that pass through both of the points $(-5, 2)$ and $(3, -1)$. Some examples are shown in figure 3.

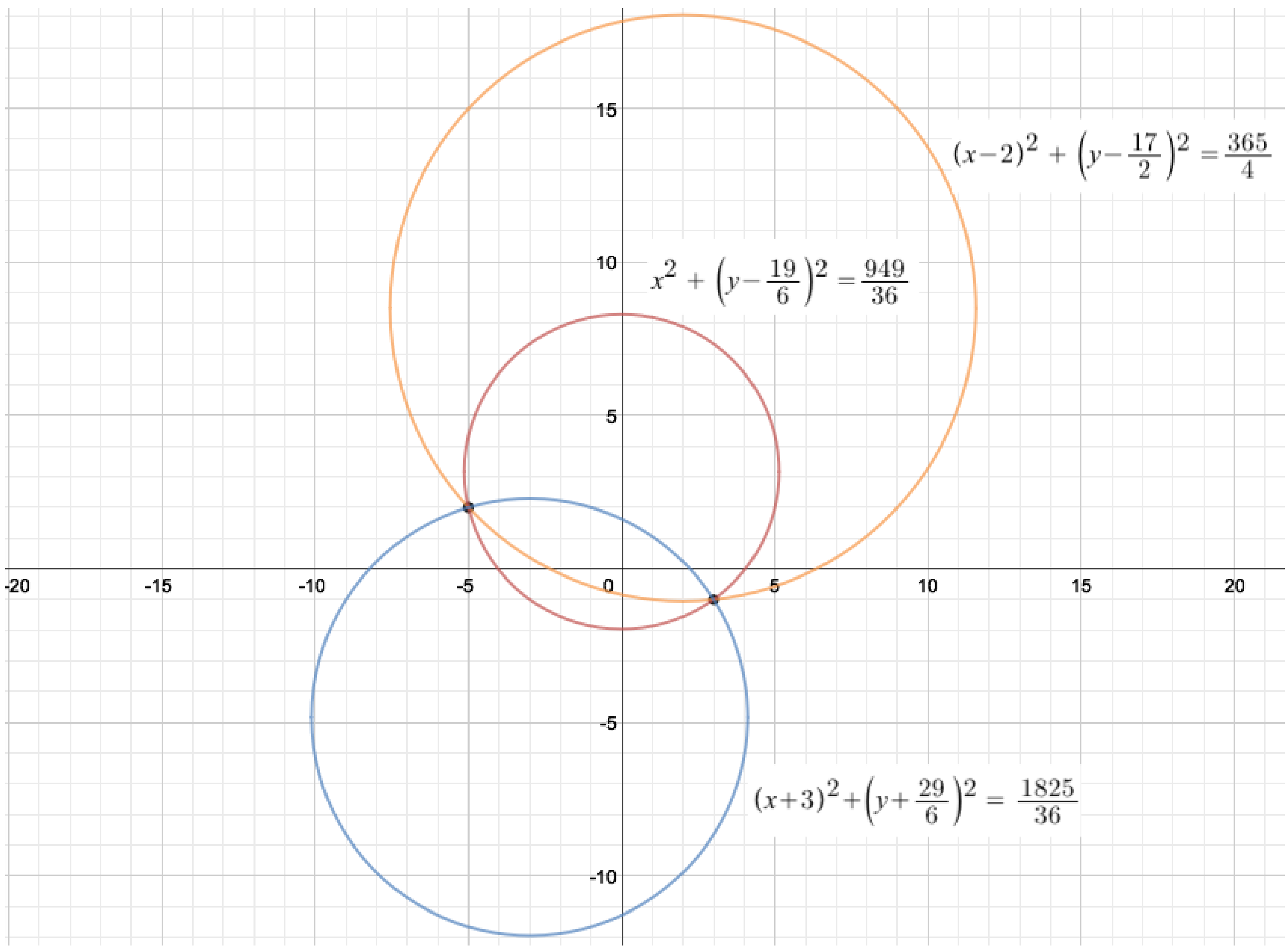

Figure G2_RT7.3: Figure 3

Relevance

E2 How is the solution of equations related to problems in geometry?

G2 What is the connection between algebra and geometry, and how can we exploit it?