Review questions

Q1 Solution

A number of the form $1/N$, where N is an integer greater than 1, is called a *unit fraction*.

Noting that

$$\frac{1}{2} = \frac{1}{3} + \frac{1}{6} \quad \text{and} \quad \frac{1}{3} = \frac{1}{4} + \frac{1}{12},$$

guess a general result of the form

$$\frac{1}{N} = \frac{1}{a} + \frac{1}{b}$$

and hence prove that any unit fraction can be expressed as the sum of two distinct unit fractions.

By writing the previous equation in the form

$$(a - N)(b - N) = N^2$$

and by considering the factors of N^2, show that if N is prime, then there is only one way of expressing $1/N$ as the sum of two distinct unit fractions.

Prove similarly that any fraction of the form $2/N$, where N is prime number greater than 2, can be expressed uniquely as the sum of two distinct unit fractions.

UCLES STEP Maths II, 2000, Q1.

Q2 Solution

Consider the arithmetic sequences $1998, 2005, 2012, \ldots$ and $1996, 2005, 2014, \ldots$. What is the next number after 2005 that appears in both sequences?

SMC, 2005, Q7.

Q3 Solution

For how many integers n is $\frac{n}{100-n}$ also an integer?

SMC, 2009, Q15.

Q4 Solution

Four positive integers a, b, c and d are such that

$$abcd + abc + bcd + cda + dab + ab + bc + cd + da + ac + bd + a + b + c + d = 2009.$$

What is the value of $a + b + c + d$?

SMC, 2009, Q25.

Q5 Solution

The *primorial* of a number is the product of all the prime numbers less than or equal to that number. For example, the primorial of 6 is $2 \times 3 \times 5 = 30$.

How many different whole numbers have a primorial of 210?

SMC, 2011, Q12.

Printables

PDF version (./index.pdf)

Relevance

NA3 (../../stations/NA3.html) What are highest common factors and why do they matter?