

CasADi tutorial 2

```
0 #
1 #
2 #
3 #
4 #
5 #
6 #
```

This tutorial file explains the use of CasADi's Function in a python context. We assume you have read through the SX tutorial.

Introduction

Let's start with creating a simple expression tree z.

```
17 from casadi import *
18 from numpy import *
19 x = SX.sym("x")
20 y = x**2
21 z = sin(y) + y
22 print z
```

```
@1=sq(x), (sin(@1)+@1)
```

The printout value of z may give you the false impression that the evaluation of z will involve two multiplications of x. This is not the case. This is what's going on under the hood:

The expression tree of z does not contain two subexpressions $x*x$, rather it contains two pointers to a single subexpression $x*x$. In fact, in the C++ implementation, an S object is really not more than a collection of pointers. It are SXNode objects which really contain the data associated with subexpressions.

CasADi generates SXnodes at a very fine-grained level. Even 'sin(y)' is an SXNode, even though we have not ourselves declared a variable to point to it.

When evaluating z for a particular numerical value of x, the product is computed only once.

Functions

CasADi's Function has powerful input/output behaviour. The following input/output primitives are supported: A function that uses one primitive as input/output is said to be 'single input'/'single output'.

In general, an SXfunction can map from-and-to list/tuples of these primitives.

Functions with scalar valued input

The following code creates and evaluates a single input (scalar valued), single output (scalar valued) function.

```
45 f = Function('f', [x], [z]) # z = f(x)
46
47 print "%d -> %d" % (f.n_in(), f.n_out())
```

```
1 -> 1
```

```
46 f_in = f.sx_in()
47 print f_in, type(f_in)
```

```
[SX(x)] <type 'list'>
```

```
48 f_out = f(*f_in)
49 print f_out, type(f_out)
```

```
@1=sq(x), (sin(@1)+@1) <class 'casadi.casadi.SX'>
```

```
50 z0 = f(2)
51 print z0
```

```
3.2432
```

```
52 print type(z0)
```

```
<class 'casadi.casadi.DM'>
```

```
53 z0 = f(3)
54 print z0
```

```
9.41212
```

We can evaluate symbolically, too:

```
56 print f(y)
```

```
@1=sq(sq(x)), (sin(@1)+@1)
```

Since numbers get cast to SXConstant object, you can also write the following non-efficient code:

```
58 print f(2)
```

```
3.2432
```

We can do symbolic derivatives: $f' = dz/dx$.

```
60 print SX.grad(f)
```

```
((x+x)*(1+cos(sq(x))))
```

The following code creates and evaluates a multi input (scalar valued), multi output (scalar valued) function.

```
63 x = SX.sym("x") # 1 by 1 matrix serves as scalar
64
65 y = SX.sym("y") # 1 by 1 matrix serves as scalar
66
67 f = Function('f', [x, y], [x*y, x+y])
68 print "%d -> %d" % (f.n_in(), f.n_out())
```

```
2 -> 2
```

```
66 r = f(2, 3)
67
68 print [r[i] for i in range(2)]
```

```
[DM(6), DM(5)]
```

```
69 print [[SX.grad(f,i,j) for i in range(2)] for j in range(2)]
```

```
[[SX(y), SX(x)], [SX(1), SX(1)]]
```

Symbolic function manipulation

```
73 x=SX.sym("x")
74 a=SX.sym("a")
75 b=SX.sym("b")
```

```

76 f = Function('f', [x, vertcat(a,b)], [a*x + b])
77
78 print f(x, vertcat(a,b))

```

```

    ((a*x)+b)
79 print f(SX(1.0), vertcat(a,b))

```

```

    (a+b)
80 print f(x, vertcat(SX.sym("c"), SX.sym("d")))

```

```

    ((c*x)+d)
81 print f(SX(), vertcat(SX.sym("c"), SX.sym("d")))

```

```

    d
84
85
86 k = SX(a)
87 print f(x, vertcat(k[0], b))

```

```

    ((a*x)+b)
86 print f(x, vertcat(SX.sym("c"), SX.sym("d")))
    ((c*x)+d)

```

Functions with vector valued input

The following code creates and evaluates a single input (vector valued), single output (vector valued) function.

```

92
93 x = SX.sym("x")
94 y = SX.sym("y")
95 f = Function('f', [vertcat(x, y)], [vertcat(x*y, x+y)])
96 print "%d -> %d" % (f.n_in(), f.n_out())

```

```

    1 -> 1
96 r = f([2, 3])
97 print z
    @1=sq(x), (sin(@1)+@1)

```

```

98 G=SX.jac(f).T
99 print G

```

```

    @1=1,
    [[y, @1],
    [x, @1]]

```

The evaluation of v can be efficiently achieved by automatic differentiation as follows:

```

102 df = f.derivative(1,0)
103 res = df([2,3], [7,6])
104 print res[1] # v
    [33, 13]

```

Functions with matrix valued input

```

108 x = SX.sym("x", 2,2)
109 y = SX.sym("y", 2,2)
110 print x*y # Not a dot product

```

```

    [[(x_0*y_0), (x_2*y_2)],
    [(x_1*y_1), (x_3*y_3)]]
111 f = Function('f', [x,y], [x*y])
112 print "%d -> %d" % (f.n_in(), f.n_out())

```

```

    2 -> 1
113 print f(x,y)

```

```

    [[(x_0*y_0), (x_2*y_2)],
    [(x_1*y_1), (x_3*y_3)]]
114 r = f(DM([[1,2],[3,4]]), DM([[4,5],[6,7]]))
115 print r

```

```

    [[4, 10],
    [18, 28]]
116 print SX.jac(f,0).T

```

```

    [[y_0, 00, 00, 00],
    [00, y_1, 00, 00],
    [00, 00, y_2, 00],
    [00, 00, 00, y_3]]
117 print SX.jac(f,1).T

```

```

    [[x_0, 00, 00, 00],
    [00, x_1, 00, 00],
    [00, 00, x_2, 00],
    [00, 00, 00, x_3]]

```

```

119
120 print 12

```

```

    12
121
122 f = Function('f', [x,y], [x*y,x+y])
123 print type(x)

```

```

    <class 'casadi.casadi.SX'>
123 print f(x,y)
    (SX(
    [[(x_0*y_0), (x_2*y_2)],
    [(x_1*y_1), (x_3*y_3)]]), SX(

```

```
    [(x_0+y_0), (x_2+y_2)],  
    [(x_1+y_1), (x_3+y_3)])])
```

```
124 print type(f(x,y))
```

```
<type 'tuple'>
```

```
125 print type(f(x,y)[0])
```

```
<class 'casadi.casadi.SX'>
```

```
126 print type(f(x,y)[0][0,0])
```

```
<class 'casadi.casadi.SX'>
```

```
128  
129 f = Function('f', [x], [x+y])  
130 print type(x)
```

```
<class 'casadi.casadi.SX'>
```

```
130 print f(x)
```

```
    [(x_0+y_0), (x_2+y_2)],  
    [(x_1+y_1), (x_3+y_3)])])
```

```
131 print type(f(x))
```

```
<class 'casadi.casadi.SX'>
```

A current limitation is that matrix valued input/output is handled through flattened vectors. Note the peculiar form of the gradient.

Conclusion

This tutorial showed how Function allows for symbolic or numeric evaluation and differentiation.