Exact Hessian

```python
#
#
#
#
#
#
#
```

```python
from casadi import *
from numpy import *
import casadi as c
```

We will investigate the use of an exact Hessian with the help of the Rosenbrock function

```python
x=SX.sym('x')
y=SX.sym('y')
obj = (1-x)**2+100*(y-x**2)**2
constr = x**2+y**2
nlp={'x':vertcat(x,y), 'f':obj, 'g':constr}
```

We solve the problem with an exact Hessian (default)

```python
solver = nlpsol('solver', 'ipopt', nlp)
sol = solver(lbx=-10, ubx=10, lbg=0, ubg=1)
```

```
******************************************************************************
This program contains Ipopt, a library for large-scale nonlinear
    optimization.
 Ipopt is released as open source code under the Eclipse Public License (
     EPL).
         For more information visit http://projects.coin-or.org/Ipopt
******************************************************************************


This is Ipopt version 3.12.3, running with linear solver ma57.

Number of nonzeros in equality constraint Jacobian...:        0
Number of nonzeros in inequality constraint Jacobian.:        2
Number of nonzeros in Lagrangian Hessian.............:        3

Total number of variables............................:        2
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        2
                     variables with only upper bounds:        0
Total number of equality constraints.................:        0
Total number of inequality constraints...............:        1
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        1
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du
    alpha_pr  ls
   0  1.0000000e+00 0.00e+00 2.00e+00  -1.0 0.00e+00    -  0.00e+00 0.00e
       +00   0
   1  6.8309610e+01 0.00e+00 4.32e+02  -1.0 9.09e-01    -  1.36e-02 1.00e
       +00H  1
   2  6.2418830e+00 0.00e+00 7.25e+01  -1.0 3.06e-01    -  6.76e-01 1.00e
       +00F  1
   3  6.3023184e-02 0.00e+00 5.46e-01  -1.0 2.58e-01    -  9.97e-01 1.00e
       +00f  1
   4  6.5589711e-02 0.00e+00 5.27e-03  -1.7 4.84e-02    -  1.00e+00 1.00e
       +00h  1
   5  5.0619847e-02 0.00e+00 3.07e-01  -3.8 9.95e-02    -  9.05e-01 1.00e
       +00f  1
   6  4.6180852e-02 0.00e+00 2.45e-02  -3.8 3.71e-02    -  1.00e+00 1.00e
       +00h  1
   7  4.5822797e-02 0.00e+00 1.76e-04  -3.8 3.26e-03    -  1.00e+00 1.00e
       +00h  1
   8  4.5677137e-02 0.00e+00 3.59e-05  -5.7 1.20e-03    -  1.00e+00 1.00e
       +00h  1
   9  4.5676652e-02 0.00e+00 3.22e-10  -5.7 4.39e-06    -  1.00e+00 1.00e
       +00h  1
iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du
    alpha_pr  ls
  10  4.5674810e-02 0.00e+00 5.78e-09  -8.6 1.52e-05    -  1.00e+00 1.00e
      +00h  1

Number of Iterations....: 10

                                   (scaled)                 (unscaled)
Objective...............:   4.5674810088672947e-02    4.5674810088672947e
    -02
Dual infeasibility......:   5.7761012971635439e-09    5.7761012971635439e
    -09
Constraint violation....:   0.0000000000000000e+00    0.0000000000000000e
    +00
Complementarity.........:   2.5919940506206774e-09    2.5919940506206774e
    -09
Overall NLP error.......:   5.7761012971635439e-09    5.7761012971635439e
    -09


Number of objective function evaluations             = 14
Number of objective gradient evaluations             = 11
Number of equality constraint evaluations            = 0
Number of inequality constraint evaluations          = 14
Number of equality constraint Jacobian evaluations   = 0
Number of inequality constraint Jacobian evaluations = 11
Number of Lagrangian Hessian evaluations             = 10
Total CPU secs in IPOPT (w/o function evaluations)   =      0.003
Total CPU secs in NLP function evaluations           =      0.001

EXIT: Optimal Solution Found.
                   proc           wall      num           mean
                                   mean
                   time           time     evals       proc time
           wall time
       nlp_f     0.000 [s]      0.000 [s]    14         0.00 [ms]
          0.00 [ms]
       nlp_g     0.000 [s]      0.000 [s]    14         0.00 [ms]
```

```
           0.01 [ms]
    nlp_grad_f      0.000 [s]       0.000 [s]     12        0.00 [ms]
        0.00 [ms]
     nlp_jac_g      0.000 [s]       0.000 [s]     12        0.00 [ms]
        0.00 [ms]
     nlp_hess_l      0.000 [s]       0.000 [s]     10        0.00 [ms]
        0.01 [ms]
   all previous      0.000 [s]       0.000 [s]
  callback_prep      0.000 [s]       0.000 [s]     11        0.00 [ms]
      0.00 [ms]
         solver      0.010 [s]       0.006 [s]
       mainloop      0.010 [s]       0.006 [s]
```

```
25 print 'Optimal solution (exact Hessian): %s' % sol['x']
```

```
Optimal solution (exact Hessian): [0.786415, 0.617698]
```

Same problem but with limited memory BFSG

```
28 solver = nlpsol('solver', 'ipopt', nlp, {'ipopt.hessian_approximation':'
       limited-memory'})
29 sol = solver(lbx=-10, ubx=10, lbg=0, ubg=1)
```

```
This is Ipopt version 3.12.3, running with linear solver ma57.

Number of nonzeros in equality constraint Jacobian...:        0
Number of nonzeros in inequality constraint Jacobian.:        2
Number of nonzeros in Lagrangian Hessian.............:        0

Total number of variables............................:        2
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        2
                     variables with only upper bounds:        0
Total number of equality constraints.................:        0
Total number of inequality constraints...............:        1
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        1
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du
    alpha_pr  ls
   0  1.0000000e+00 0.00e+00 2.00e+00   0.0 0.00e+00    -  0.00e+00 0.00e
       +00   0
   1  8.1099664e-01 0.00e+00 8.51e+00  -5.2 1.67e+00    -  8.49e-01 1.24e
       -01f  4
   2  7.8913241e-01 0.00e+00 7.56e+00  -0.7 3.79e-01    -  2.45e-01 1.00e
       +00F  1
   3  5.1038399e-01 0.00e+00 2.00e+00  -1.6 1.47e-01    -  1.00e+00 1.00e
       +00h  1
   4  9.4636921e-01 0.00e+00 1.66e+01  -1.7 2.10e-01    -  6.36e-01 1.00e
       +00H  1
   5  4.3659026e-01 0.00e+00 8.99e-01  -2.6 1.70e-01    -  1.00e+00 1.00e
       +00f  1
   6  3.9428196e-01 0.00e+00 8.08e-01  -3.2 5.30e-02    -  1.00e+00 1.00e
       +00h  1
   7  1.5277780e+00 0.00e+00 3.32e+01  -4.2 3.31e-01    -  4.87e-01 1.00e
       +00H  1
   8  3.5006028e-01 0.00e+00 1.56e+00  -3.3 5.71e-01    -  1.00e+00 1.00e
       +00f  1
   9  3.1926755e-01 0.00e+00 1.63e+00  -3.9 1.60e-01    -  1.00e+00 1.00e
       +00h  1
iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du
    alpha_pr  ls
  10  2.7958742e-01 0.00e+00 4.47e+00  -4.1 8.31e+00    -  1.00e+00 1.15e
       -02f  4
  11  4.1529404e-01 0.00e+00 1.74e+01  -4.6 3.79e-01    -  7.49e-01 1.00e
       +00H  1
  12  1.9552435e-01 0.00e+00 7.94e+00  -3.4 1.93e-01    -  1.00e+00 1.00e
       +00f  1
  13  1.0284176e-01 0.00e+00 1.20e+00  -4.6 5.10e-02    -  1.00e+00 1.00e
       +00f  1
  14  7.9168874e-02 0.00e+00 1.74e+00  -6.0 1.24e-01    -  1.00e+00 1.00e
       +00f  1
  15  6.4711554e-02 0.00e+00 3.50e+00  -7.3 1.52e-01    -  1.00e+00 1.00e
       +00h  1
  16  5.5234143e-02 4.93e-03 3.01e+00  -4.6 1.34e-01    -  1.00e+00 3.44e
       -01h  1
  17  4.7148548e-02 0.00e+00 9.32e-01  -4.3 7.45e-03    -  1.00e+00 1.00e
       +00h  1
  18  4.5698648e-02 0.00e+00 1.46e-01  -6.0 6.27e-03    -  1.00e+00 8.55e
       -01h  1
  19  4.5674859e-02 0.00e+00 1.07e-03  -6.7 1.78e-04    -  1.00e+00 9.95e
       -01h  1
iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du
    alpha_pr  ls
  20  4.5674809e-02 0.00e+00 1.79e-07  -8.8 1.52e-06    -  1.00e+00 1.00e
       +00h  1
  21  4.5674808e-02 0.00e+00 2.24e-09 -11.0 1.39e-08    -  1.00e+00 1.00e
       +00h  1

Number of Iterations....: 21

                                   (scaled)                 (unscaled)
Objective...............:  4.5674807514535586e-02    4.5674807514535586e
    -02
Dual infeasibility......:  2.2446746822391006e-09    2.2446746822391006e
    -09
Constraint violation....:  0.0000000000000000e+00    0.0000000000000000e
    +00
Complementarity.........:  1.0000913016783289e-11    1.0000913016783289e
    -11
Overall NLP error.......:  2.2446746822391006e-09    2.2446746822391006e
    -09


Number of objective function evaluations             = 36
Number of objective gradient evaluations             = 22
Number of equality constraint evaluations            = 0
Number of inequality constraint evaluations          = 36
Number of equality constraint Jacobian evaluations   = 0
Number of inequality constraint Jacobian evaluations = 22
Number of Lagrangian Hessian evaluations             = 0
Total CPU secs in IPOPT (w/o function evaluations)   =      0.007
Total CPU secs in NLP function evaluations           =      0.000
```

```
EXIT: Optimal Solution Found.
                    proc             wall       num             mean
                                      mean
                    time             time      evals        proc time
                        wall time
         nlp_f     0.000 [s]       0.000 [s]     36        0.00 [ms]
             0.00 [ms]
         nlp_g     0.000 [s]       0.000 [s]     36        0.00 [ms]
             0.01 [ms]
    nlp_grad_f     0.000 [s]       0.000 [s]     23        0.00 [ms]
        0.00 [ms]
     nlp_jac_g     0.000 [s]       0.000 [s]     23        0.00 [ms]
         0.00 [ms]
  all previous     0.000 [s]       0.000 [s]
 callback_prep     0.000 [s]       0.000 [s]     22        0.00 [ms]
     0.01 [ms]
        solver     0.020 [s]       0.015 [s]
      mainloop     0.020 [s]       0.015 [s]
```

```python
print 'Optimal solution (BFGS): %s' % sol['x']
```

```
Optimal solution (BFGS): [0.786415, 0.617698]
```