USC Marshall
School of Business
Full-Time MBA Program

MBA

Welcome Class of 2027 Orientation

JumpStart Sessions

Business Math

July 23, 2025

Business Math Topics

- Order of Operations & Basic Algebra Techniques
- Linear Equations
- Non-linear Equations
  - Quadratic, Polynomial, Exponential
- Exponential & Logarithmic Functions
- Descriptive Statistics

Order of Operations

- Parentheses
- Exponents
- Multiplication
- Division
- Addition
- Subtraction

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Order of Operations: Examples

$$3 \times 7^2 + 2^3 =$$

$$10 \div 2 + 3 =$$

$$(2 \times 3)^2 \times 5 =$$

$$6 + 8/2 + 2^3 =$$

$$2 \times 3^2 \times 5 =$$

$$3 \times (7^2 + 2^3) =$$

Other Mathematical Rules

Distributive law: $a(b+c) = ab + ac$

Multiplying fractions: $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

Dividing fractions: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$

Adding Fractions

Common denominator: $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$

No common denominator: $\frac{a}{b} + \frac{c}{d} = \frac{a}{b}\frac{d}{d} + \frac{c}{d}\frac{b}{b} = \frac{ad+cb}{bc}$

Algebra: Application

- A used car salesman makes \$500 per month plus \$100 for every car he sells.
- Let Y be his total monthly income and X be the number of cars he sells.
- How can we write Y as a function of X?

Linear Equations

- *X* and *Y* are *variables*, and the function relating them is an example of a *linear equation*.
- A straight line is a linear equation, but planes (or hyperplanes) in multiple dimensions can also be linear.
- In general, linear equations are the easiest functions to work with, so we often try to convert other forms into something linear.

Linear Equations: Application

Total Cost = Fixed Cost + Variable Cost

- Each additional pair of shoes produced by a shoe factory costs $7.
- The shoe factory has *fixed costs* totaling $100,000.
- What is the linear function for the *total cost* of shoes produced?
- What is the *actual total cost* of producing 5000 pairs of shoes?

Linear Equations: Slopes

Since each extra pair costs \$7, we can write:

$$\frac{Cost\ of\ Producing\ Extra\ Shoes}{Number\ of\ Extra\ Pairs\ Produced} = \$7$$

- The slope of an equation tells us how much *Y* changes when *X* changes by one unit.
- The slope formula is: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Linear Equations: Basics

A (two-dimensional) line is determined by two pieces of information:

- Two points on the line, which are used to determine the slope
- One point on the line and the line's slope

Popular ways of writing a line's equation:

- Point-slope formula: $y - y_1 = m(x - x_1)$
- Slope-intercept form:

(1) Algebra: $y = mx + b$ (2) Statistics: $y = b_0 + b_1 x$

Simultaneous Linear Equations

Suppose *demand* and *supply* for a product are

(1) $D = 49 - P$ and (2) $S = 1 + 2P$

where D is demand, S is supply, and P is price.

- What price should be set so that *demand = supply*?
- What number of products are expected to be sold?

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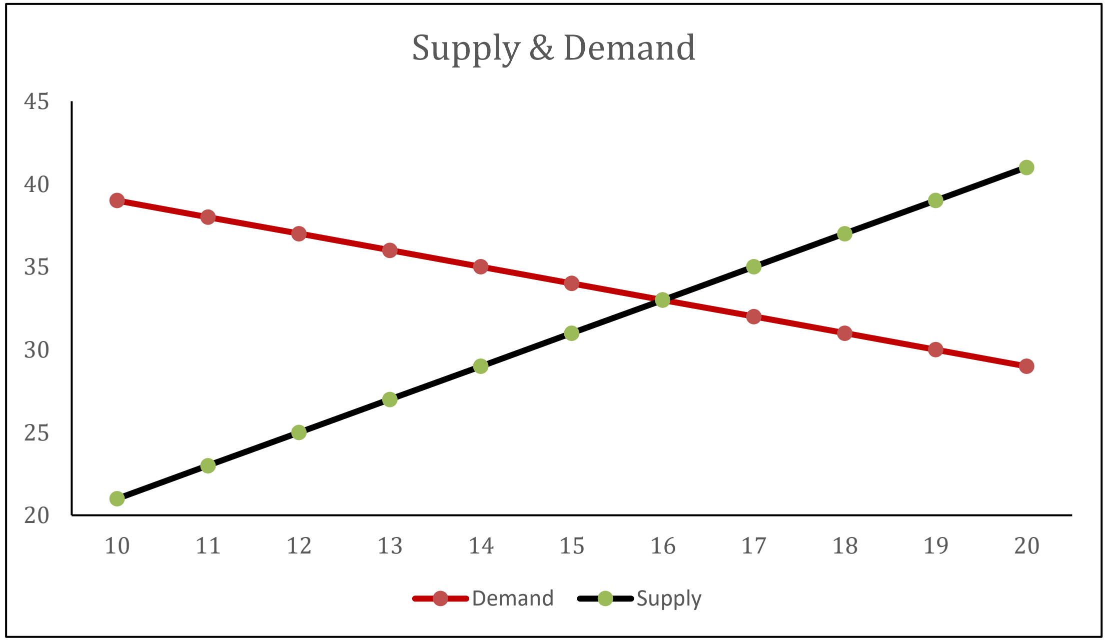
Simultaneous Eqns: Supply & Demand

Economics suggests choosing *P* so that *supply* = *demand*.



Solving for P:

$$49 - P = 1 + 2P$$
$$\rightarrow 48 = 3P$$
$$\rightarrow P = 16$$

Plugging in to get D:

$$D = 49 - P$$
$$= 49 - 16$$
$$= 33$$

Non-Linear Equations

Often, situations are more complicated than what can be represented through linear functions.

For example, linear functions don’t explain:

- Value of an investment over time
- Relationship between profit and price
- Pace of technological development over time

We need more complex equations to explain these.

Non-Linear Equations: Example

Assume Q is the number of units (in thousands) a firm expects to sell at price P.

- Suppose $Q = 5 - P$.
- Also let R be the firm's revenue.
- How can we write R as a function of price, P?

Quadratic Functions

These functions take the form of

$$y = ax^2 + bx + c$$

where *a*, *b*, and *c* are just numbers.

- The graph of a quadratic function is curved.
- Unlike simple linear equations, quadratic functions have optimal, or best, values at either *minimum* or *maximum* points.

Quadratic Functions: Application

Suppose the firm from the previous slide has costs of $4000 (or just "4" if written in thousands).

- What function should be used to write *profit* in terms of *price*?
- What price should be charged to break even?

The Quadratic Equation

There are two common ways to solve a quadratic function.

1. *Factoring.* Break down the equation into non-quadratic elements and solve.
2. *Using the quadratic equation*. Plug appropriate values into:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where the original equation is written as:

$$ax^2 + bx + c = 0$$

Quadratic Functions: Application

The earlier firm most likely *doesn't* just want to break even but wants to make a profit, though.

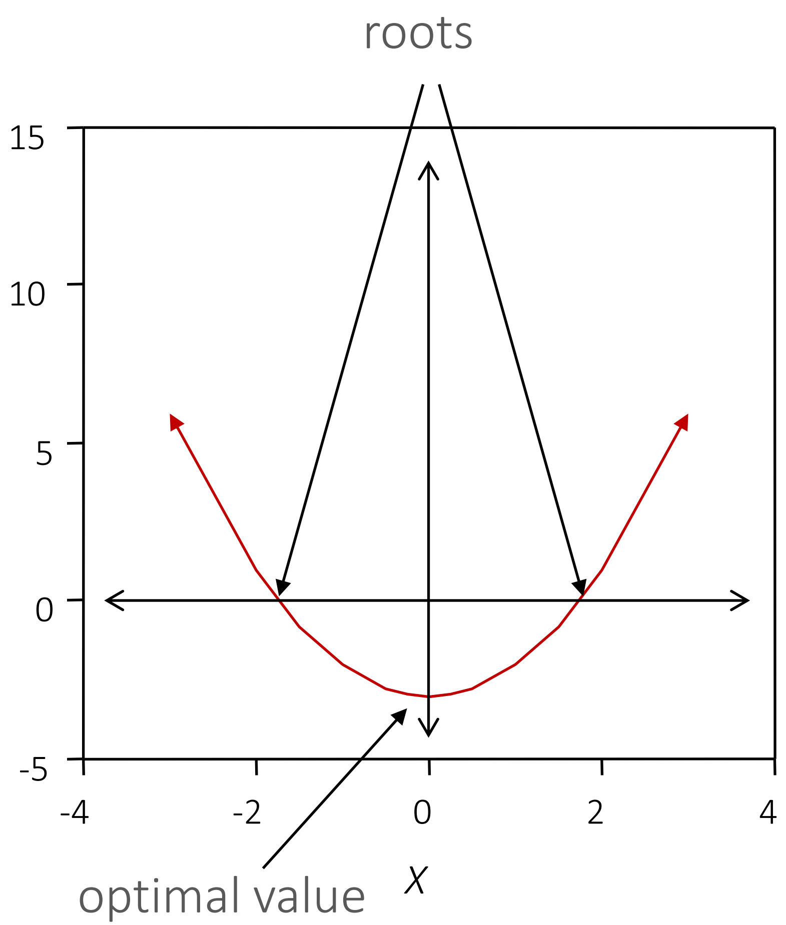
- What price should it charge to *maximize* profit?
- Recall $Q = 5 - P$ and costs are \$4000 (or "4" in thousands).

Quadratic Functions: Optimality

A quadratic function can have up to 2 roots (solutions), but only one optimal value.

Using the Quadratic Equation, the optimal point is $x = -b/2a$.

- If $a > 0, x$ is a *minimum*
- If $a < 0, x$ is a *maximum*



Quadratic Functions: Application

Assume your average cost for producing something is $0.2Q + 4 + (400/Q)$.

If you produce Q items, you must price them at $P = 400 - 2Q$ to sell them all.

- Write revenue as a function of Q.
- Write total cost as a function of Q.
- Write profit as a function of Q.

How much should you produce to *maximize* profit?

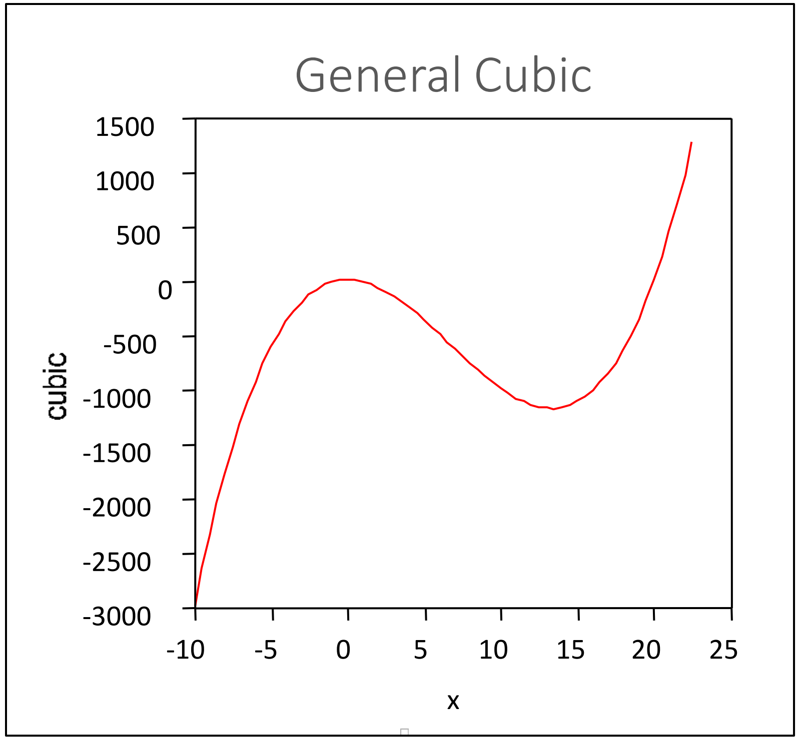
Higher–Order Polynomials

An n^{th} order polynomial is written as:

$$y = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

A cubic function ($n = 3$) is:

$$y = a_3 x^3 + a_2 x^2 + a_1 x + a_0$$



In general, higher powers add more *bumps* to the polynomial.

Exponential Functions

There is quite a large difference between x^2 and 2^x.

When $x = 10,\ x^2 = 100$ but $2^x = 1024$.

Functions like 2^x are called *exponential functions*.

- Most common bases for exponential functions are 2, 10, and $e = 2.718\ldots$

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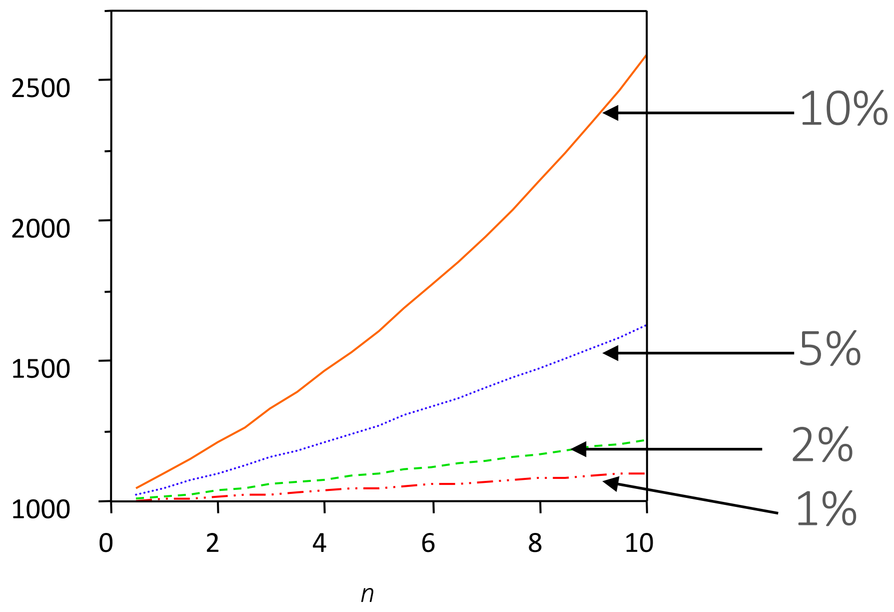
Exponential Functions: Application

Exponentials are at the heart of *compound interest*.

- If you invest \$*P* at an interest rate of $100 \times i\%$ compounded annually, after *n* years you will have $\$P \times (1+i)^n$.



Exponential Functions: Future Value

The standard *single-sum equation* for *future value* is:

$$FV = PV \times (1 + i)^n$$

where *FV = Future value*
PV = Present value
i = Interest rate
n = Number of time periods

Single-Sum Future Value: Application

Suppose $120 is invested at an annual interest rate of 6% for 7 years. What is its future value?

$$FV = PV \times (1+i)^n$$
$$FV = 120 \times (1+0.06)^7$$
$$FV = \$180.44$$

Compound Interest: Application

- How much will you have in 10 years if you invest $1000 at 8% interest compounded annually?

- How much will you have in 20 years? In 30 years?

- What if the compounding were done *monthly*?

Rules for Exponents & Powers

- $a^{x+y} = a^x a^y$
- a is called the base and x, y are the exponents

Examples:

$$2^{3+2} = 2^3 2^2 = 8 \times 4 = 32$$

$$a^{xy} = (a^x)^y$$

$$2^{2\times 3} = (2^2)^3 = 4^3 = 64$$

Logarithms & Rules

Logarithms undo exponentials.

- $log_b(x)$ *asks* "to what power must you raise b to get x?"

For any $b > 0$:

- $log_b(b^x) = x$
- $b^{log_b(x)} = x$

Other rules:

- $log(ab) = log(a) + log(b)$
- $log(a/b) = log(a) - log(b)$
- $log(a^b) = b\, log(a)$

Logarithms: Calculating Periods

Using the previous rules, how long will it take to *double* a portfolio if it grows at a compound annual rate of 8%?

Solving for *n*: $FV = PV \times (1+i)^n \rightarrow \frac{FV}{PV} = (1+i)^n$

$$log\left(\frac{FV}{PV}\right) = log(1+i)^n = nlog(1+i)$$

$$n = \frac{log\frac{FV}{PV}}{log(1+i)}$$

$$n = \frac{log2}{log(1.08)} \approx 9.$$

Logarithms: Examples

$log_2 8 =$

$log_{10} 1000 =$

$ln\, e^{23} =$

$2^{log_2 8} =$

Just simplify these!

$2log5 - 3log3$

$6\, lnx + 4\, lny$

Logarithms & Radicals

- Logarithms are useful for solving equations like $a^x = b$ because $x\, log(a) = log(b)$.
- We can also now solve equations of the form $x^n = b$ using the fact that $(x^n)^{1/n} = x$, so that $x = b^{1/n}$.
- $b^{1/n}$ is the "n^{th} root of b." For example, $b^{1/2} = \sqrt{b}$.

Logarithms & Radicals: Application

A classic application here is calculating interest, or growth, rates. Assume you began with \$125, and after 4 years it had grown to \$265. What was the compound growth rate? Solve for **i**.

$$i = \left(\frac{FV}{PV}\right)^{1/n} - 1$$

$$i = \left(\frac{265}{125}\right)^{1/4} - 1 = 0.2067 = 20.67\%$$

Logarithms & Radicals: Application

- If your money doubles in value after 10 years, what annual interest rate must you be earning?

- How long must you leave $1000 at 10% interest before you have $2000?

- How long before you have $1,000,000?

Logarithms: Cobb-Douglas Function

In Economics, the Cobb-Douglas Production Function assumes three inputs affect the amount of output one can produce: capital (*K*), labor (*L*), and materials (*M*).

$$Q = \beta_0 K^{\beta_1} L^{\beta_2} M^{\beta_3}$$

- This looks complicated, but *logarithms* help a lot here.

Logarithms: Cobb-Douglas Function

Some suggestions:

- Take *logs* of both sides of the equation
- Treat *log*(*K*), for example, as a new variable
- Now the Cobb-Douglas equation becomes linear

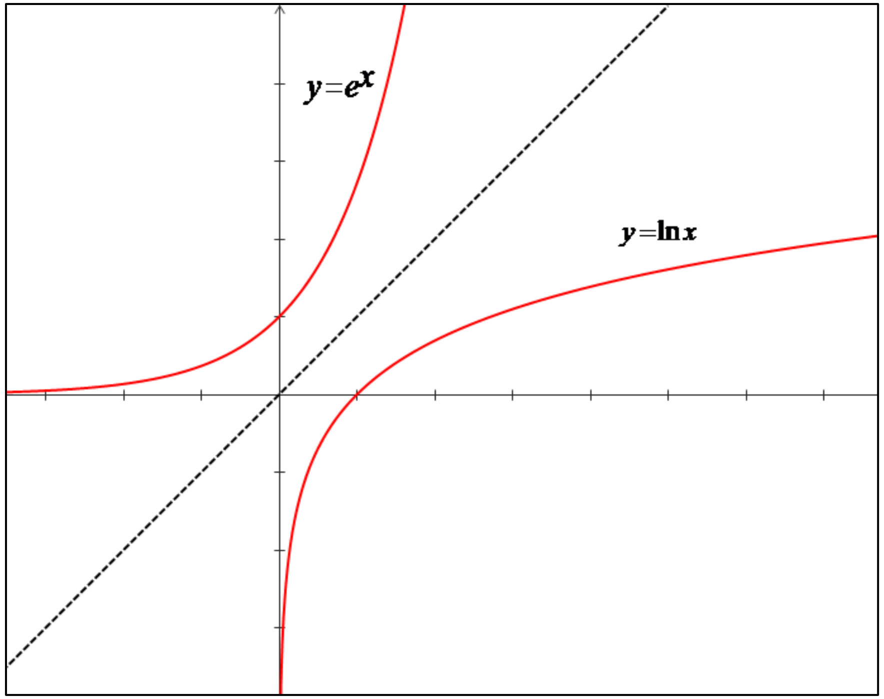
$$log(Q) = log(\beta_0) + \beta_1 log(K) + \beta_2 log(L) + \beta_3 log(M)$$

Natural Logs: Reversing

- The most common version of logarithms is the natural log. When using this, we will need to "back out" of our results to get the calculation we need.

- Luckily, the exponential function is the inverse of the natural log, so we simply need to apply that concept.

$$e^{ln\,x} = x \text{ and } ln(e^x) = x$$

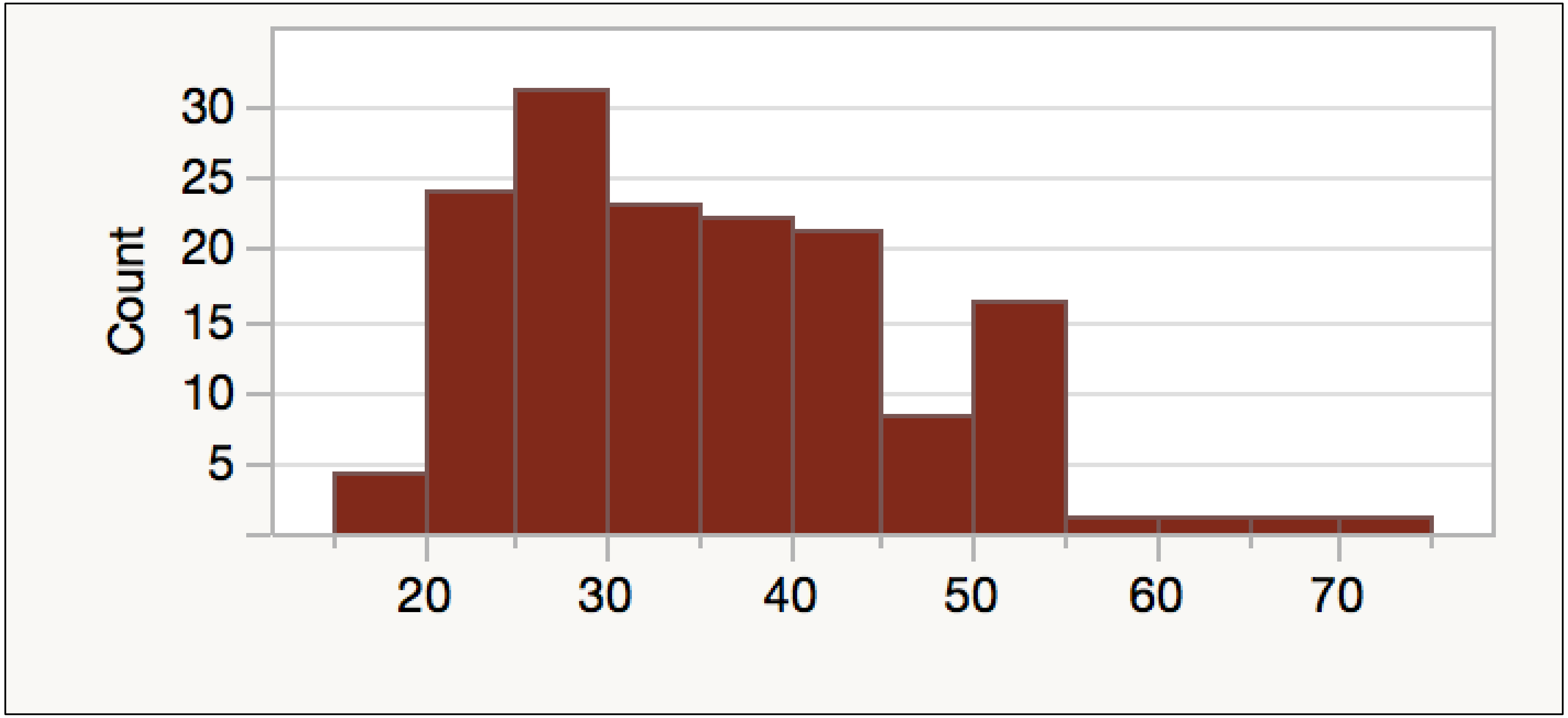


Descriptive Statistics Topics

- Basic Terminology
- Scales of Measurement
- Numerical Measures
  - Central tendency
  - Dispersion
- Skewness: Shape of Distributions

Population vs Sample

MPG for 153 Hybrid Cars



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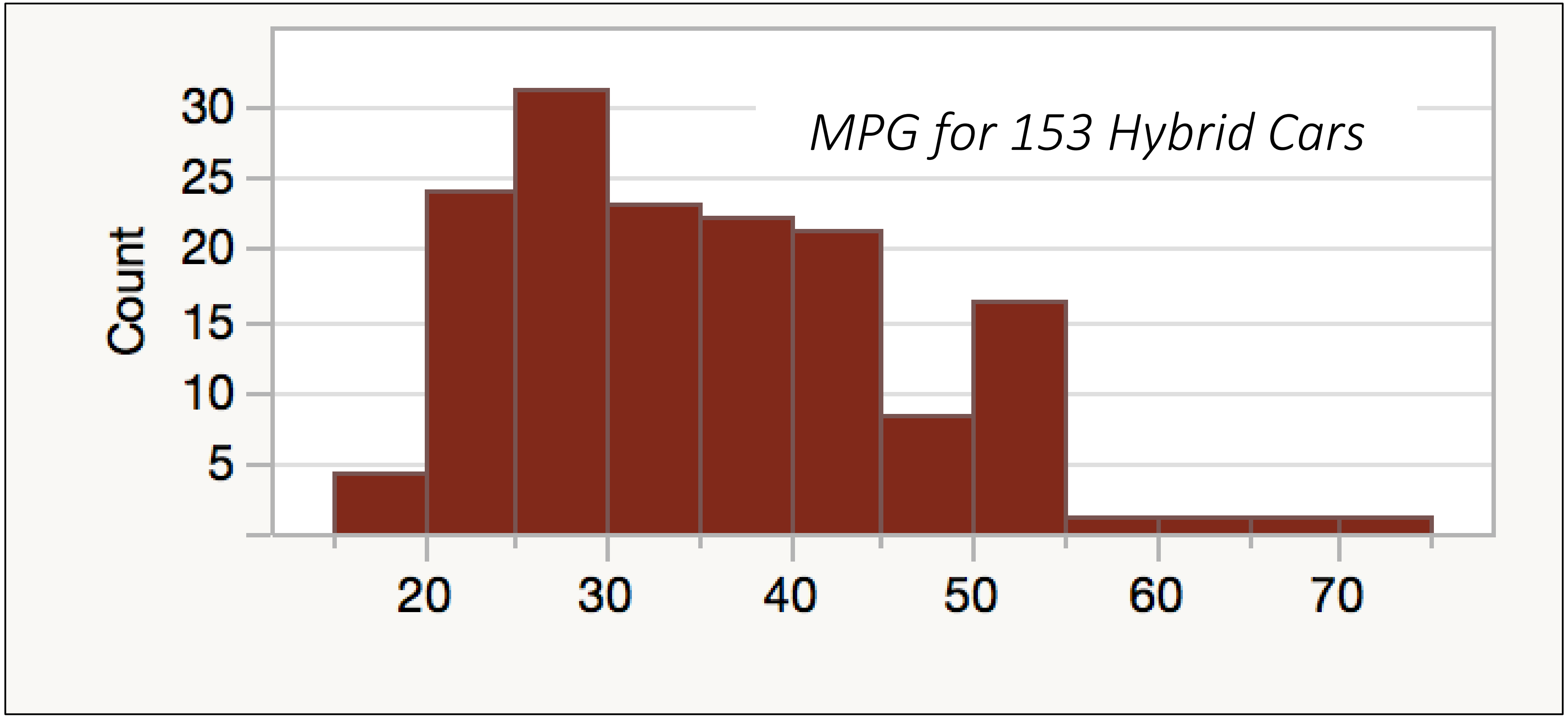
Population vs Sample

Population



Population vs Sample

MPG for 153 Hybrid Cars

Population: Set of all items of interest in a statistical problem.

Parameter: Descriptive measure of population

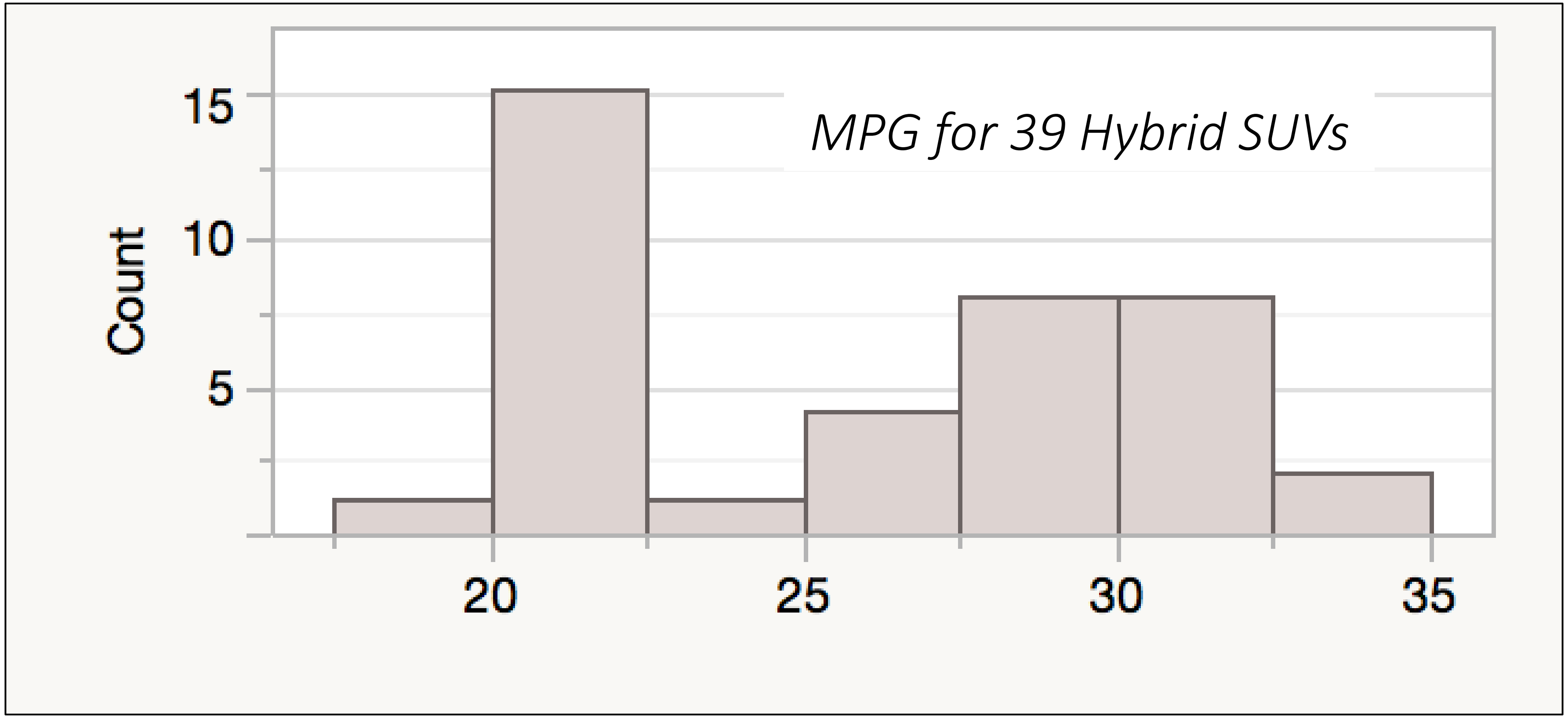
- N = population size
- μ = population average
- σ = population standard deviation

Population: 153 Hybrid Cars

- N = 153
- μ = mean = average = 34.80 mpg
- σ = standard deviation = typical fluctuation = 10.97 mpg

Population vs Sample

Sample



Population vs Sample

MPG for 39 Hybrid SUV Cars

Sample: Set of data drawn from the population

Statistic: Descriptive measure of sample

- n = sample size
- $\bar{x}$ = sample average
- s = sample standard deviation

Sample: 39 Hybrid SUV Cars

- n = 39
- $\bar{x}$ = mean = average = 26.01 mpg
- s = standard deviation = typical fluctuation = 4.60 mpg

Types of Data & Related Graphs

Numerical (quantitative)

- Natural measurement system
- Ratios and comparisons make sense

→

Histograms
Boxplots
Scatterplots

TERM 2, Week 1

Categorical (qualitative)

- Nominal: no inherent ordering
- Ordinal: ordered, but distance between classes may vary

→

Bar Charts
Pie Charts
Side-by-side Boxplots

TERM 2, Week 1

Scales of Measurement

Discrete: Possible number of values is countable

- Number of Hybrid SUV Cars
- Number of Comedy films released in 2017
- Number of games in any given World Series

Continuous: Possible number of values is uncountably infinite

- MPG of Hybrid Cars
- Height, weight, distance

Cross-sectional: Snapshot of data at a specific point in time

- Economic indicators for several countries in 2019

Time Series: Result of tracking one or more variables over time

- Economic indicators for only the US from 1990-2023

Measures of Central Tendency

How do we describe a dataset, especially if it is rather large, without having to present a table of meaningless numbers?

Generally, just two numbers are needed:

1. Measure of central tendency (i.e. typical value, or location),
2. Measure of dispersion (fluctuation).

Common measures of **central tendency:**

Mean (μ):	Average or expected value
Median (M_d):	Middle point of ordered observations
Mode (M_o):	Most frequent value

Central Tendency: Population Mean

The **mean** of a **population** of N measurements $x_1, \cdots, x_N$:

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i = \frac{1}{N}(x_1 + x_2 + \cdots + x_N)$$

Eg: If our *Hybrid Car MPG* data as a **population**, the population mean is

$$\mu = \frac{1}{153}\sum_{i=1}^{153} x_i = \frac{1}{153}(41.26 + 54.1 + \cdots + 37) = \boxed{34.7975 \text{ mpg}}$$

Central Tendency: Sample Mean

The mean of a sample of *n* measurements $x_1, \cdots, x_n$:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{n}(x_1 + x_2 + \cdots + x_n)$$

Eg: Assessing only the SUV cars from the *Hybrid Car MPG* dataset, of which there are 39 rows, the sample mean is

$$\bar{x} = \frac{1}{39}\sum_{\text{i}=1}^{39} x_i = \frac{1}{39}(18.82 + 21 \ldots + 33.64) = \boxed{26.0077 \text{ mpg}}$$

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Central Tendency: Drawbacks

We can use $\bar{x}$ as an estimate of μ, but we then need to assess the *accuracy* of this and draw conclusions, or *make inferences*, about μ.

Problem: $\bar{x}$ is extremely sensitive to outliers.

- Outliers may be due to errors in recording data
- May be real (but exceptional) observations
- Usually set aside outliers before computing
- Can also use *median*

Whenever a dataset has extreme values, the **median** is the preferred measure of central location.

Central Tendency: Median

Given *n* measurements arranged in order of magnitude,

Median = Middle value if *n* is odd, or
Average of two middle values if *n* is even.

Eg: CEO compensation for 5 food processing firms:

Pillsbury	$698,000
Borden	$1,200,000
Campbell Soup	$646,000
Hershey Foods	$573,000
Ralston Purina	$750,000

Central Tendency: Median

Converting to multiples of $1,000 and arranging in order:

573, 646, 698, 750, 1200

Median compensation is? $698,000
Mean compensation is? $773,400

- *Mean* > *median* because of outlier, Borden.

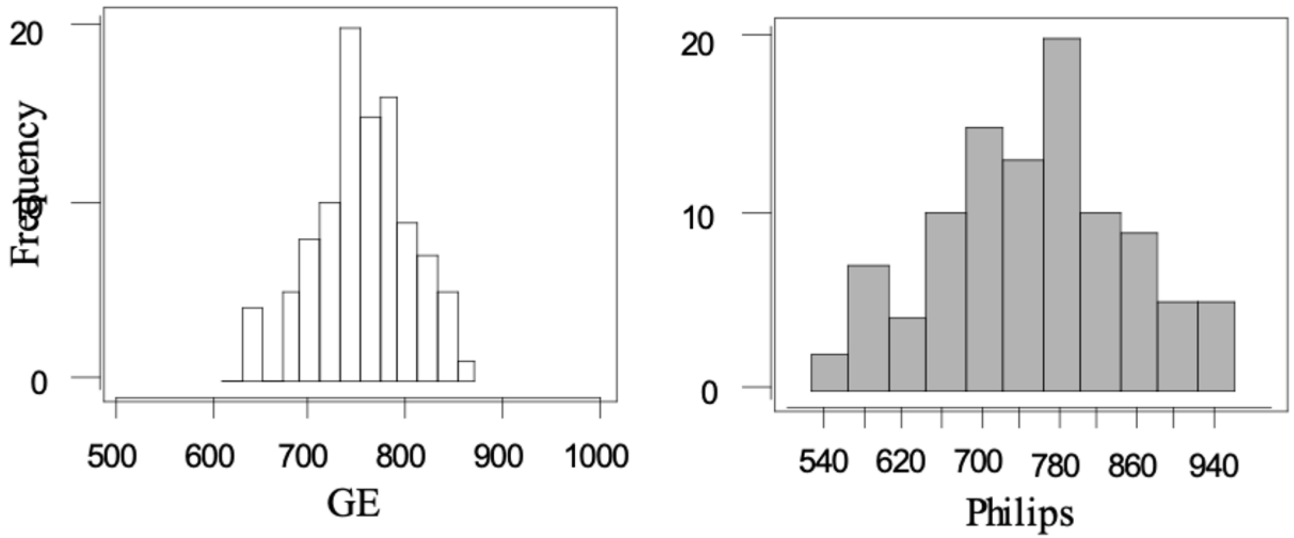
Removing Borden, *mean* = $666,750 < $672,000 = *median*

- Divides data set into two equal parts
- Half of data lies below median, half lies above it
- Resistant to outliers

Measures of Dispersion

Mean and median do not completely summarize a dataset… we also need to know how spread out the data is.

Lightbulb Lifetimes (hrs): GE vs Philips



- GE exhibits better quality control: not much variation
- Philips has more fluctuation although average is same as GE

Measures of Dispersion: Range

Range: Largest minus smallest measurement

- Crude measure with little info about dispersion of values
- No resistance to outliers

Eg: **Range** of Hybrid Car MPG dataset

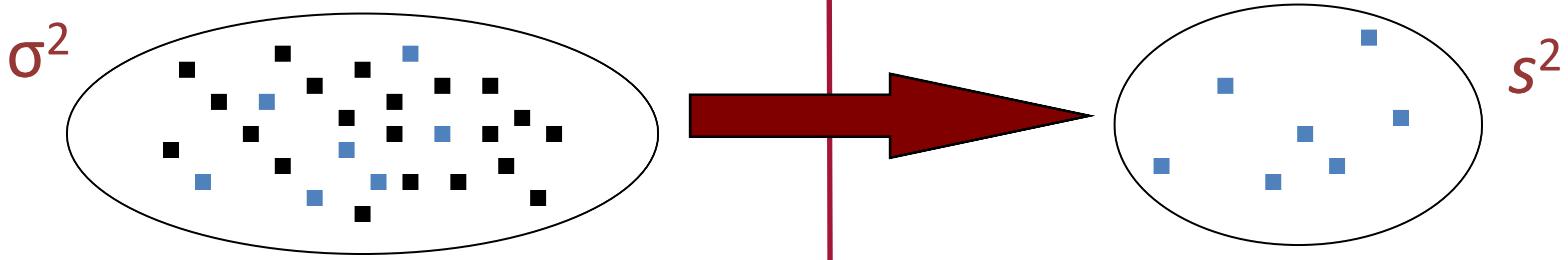
- Highest value: 72.92 mpg (Prius Alpha V)
- Lowest value: 17 mpg (Silverado 2WD)

Range = 72.92 mpg – 17 mpg = $\boxed{\text{55.92 mpg}}$

Dispersion: Variance & Std Deviation

Population $X_1, X_2, \ldots, X_N$

Sample $x_1, x_2, \ldots, x_n$



Population Variance:

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2$$

Population Standard Deviation:

$$\sigma = \sqrt{\sigma^2}$$

Sample Variance:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Sample Standard Deviation:

$$s = \sqrt{s^2}$$

Dispersion: Variance & Std Deviation

MPG of Population of 153 Hybrid Cars:

Mean: $\mu = \frac{1}{N}\sum_{i=1}^{N} x_i = \frac{1}{153}(41.26 + 54.1 + \cdots + 37)$ = $\boxed{34.8 \text{ mpg}}$

Variance: $\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2$
$= \frac{1}{153}[(41.26 - 34.8)^2 + \cdots + (37 - 34.8)^2]$ = $\boxed{120.3958 \text{ mpg}^2}$

Standard Deviation: $\sigma = \sqrt{\sigma^2}$ = $\boxed{10.9725 \text{ mpg}}$

Dispersion: Variance & Std Deviation

MPG of Sample of 39 SUV Hybrid Cars:

Mean: $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_{\mathrm{i}} = \frac{1}{39}(18.82 + 21 + \cdots + 33.64)$ = $\boxed{\text{26 mpg}}$

Variance: $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_{\mathrm{i}} - \bar{x})^2$

$= \frac{1}{38}[(18.82 - 26)^2 + \cdots + (33.64 - 26)^2]$ = $\boxed{\text{21.149 mpg}^2}$

Standard Deviation: $s = \sqrt{s^2}$= $\boxed{\text{4.599 mpg}}$

Distribution Shapes

Histograms and boxplots help uncover distribution shape:

Symmetrical (roughly equal tails)

- Bell-Shaped Distribution.

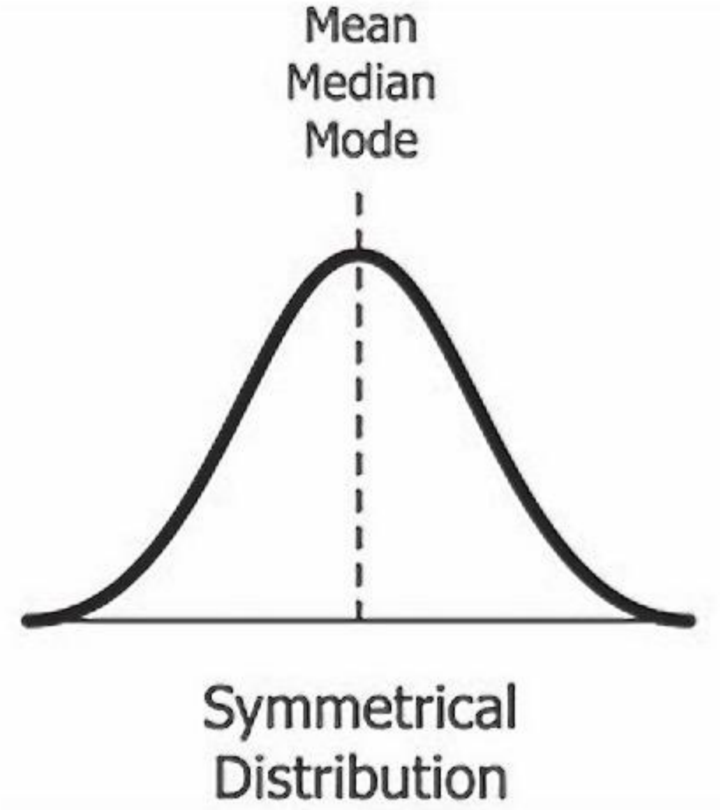
Positively Skewed – skewed right (long tail on right)

- Income Distributions.

Negatively Skewed – skewed left (long tail on left)

- Scores on an easy exam.

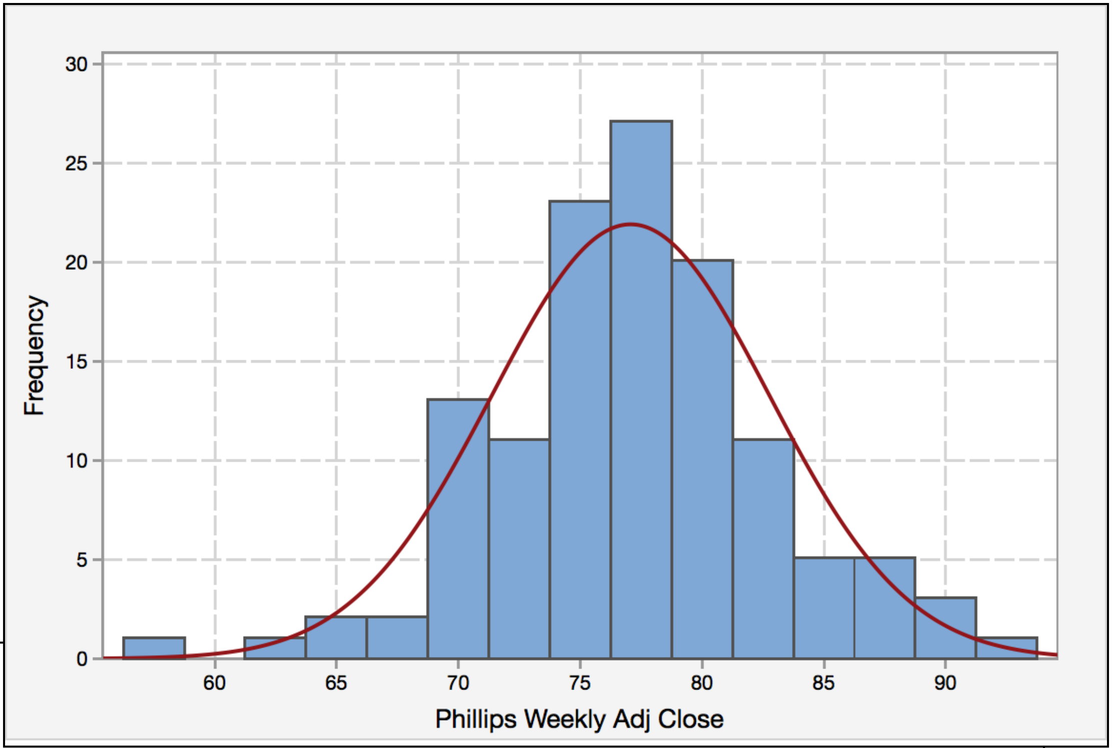
Distribution Shapes



Symmetric Distribution

Phillips Stock Prices:*

Mean ≈ Median and Median is *somewhat* close to being about halfway between 25th and 75th percentiles.



Statistics

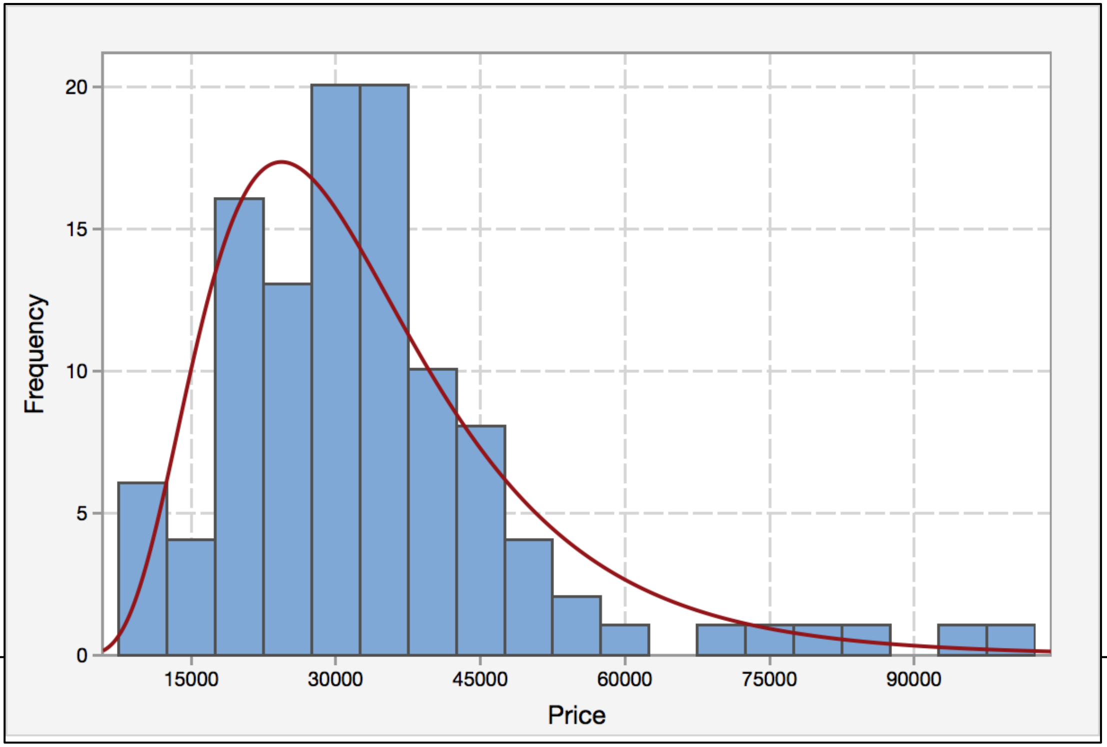
Variable	N	Mean	StDev	Minimum	Q1	Median	Q3	Maximum
Adj Close	125	77.0677	5.6917	58.3264	73.9521	77.2600	79.8241	91.3296

*Weekly closing prices, 1/6/14 – 5/23/16

Right-Skewed Distribution

LA Used Car Prices:

Mean > Median and Mean is closer to the 75th percentile than to the 25th percentile.



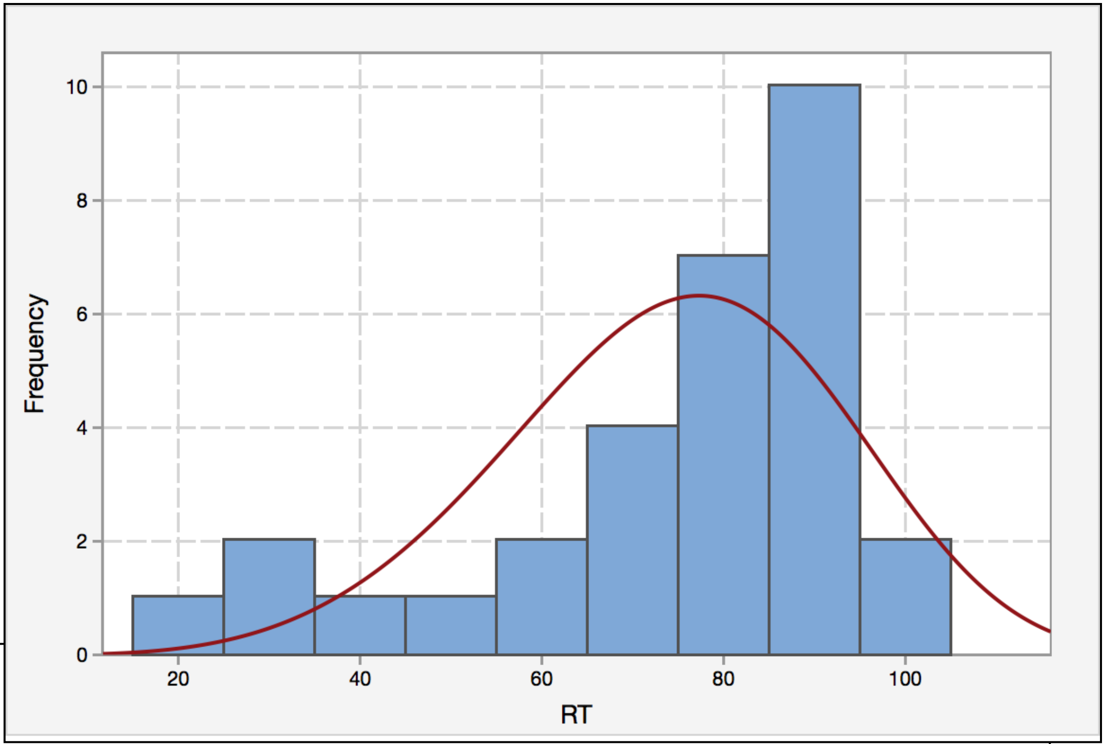
Statistics

Variable	N	Mean	StDev	Minimum	Q1	Median	Q3	Maximum
Price	110	33598	16314	9950	22991	31210	39220	99999

Left-Skewed Distribution

Top Movies in China, Rotten Tomatoes Score:

Mean < Median and Median is closer to 25th than to 75th percentile.



Statistics

Variable	N	Mean	StDev	Minimum	Q1	Median	Q3	Maximum
RT	30	74.433	21.716	18.000	68.500	81.000	91.000	98.000