

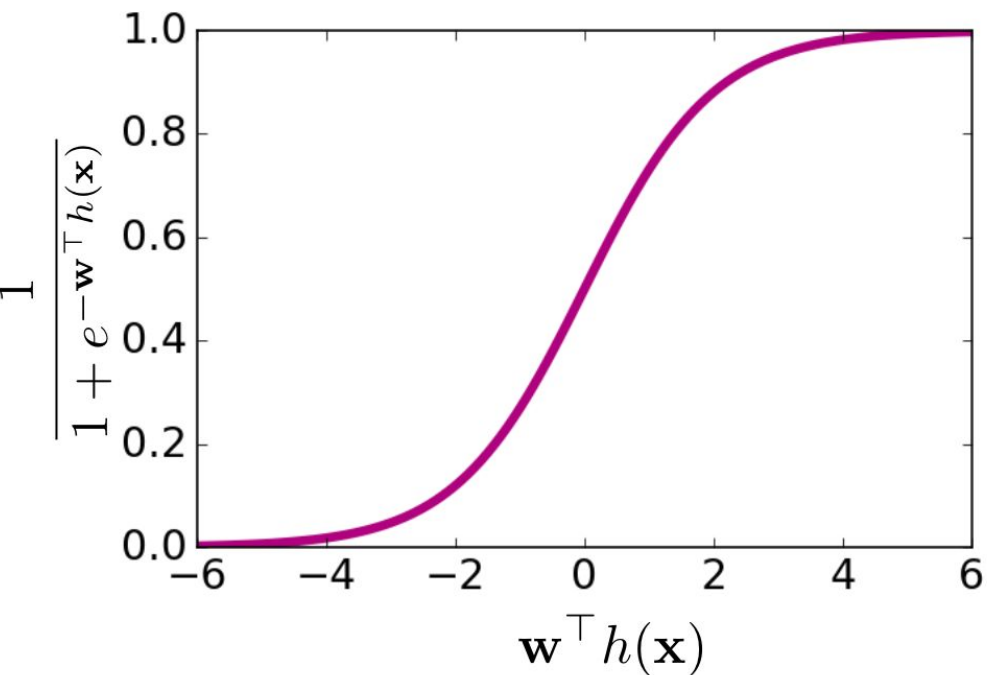
Regularizations in logistic regression

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A special thank you!

Lots of plots and ideas of this talk are taken from excellent notes from Emily Fox @ U Washington

With a logistic link, we can apply our knowledge on linear regression to classification problems

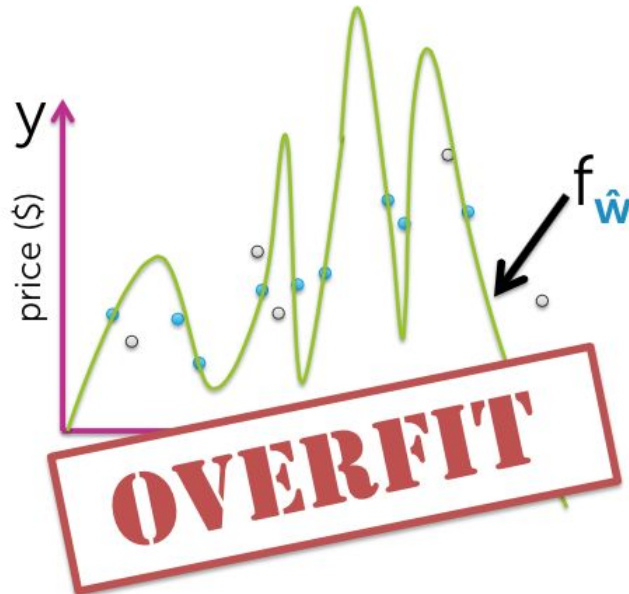
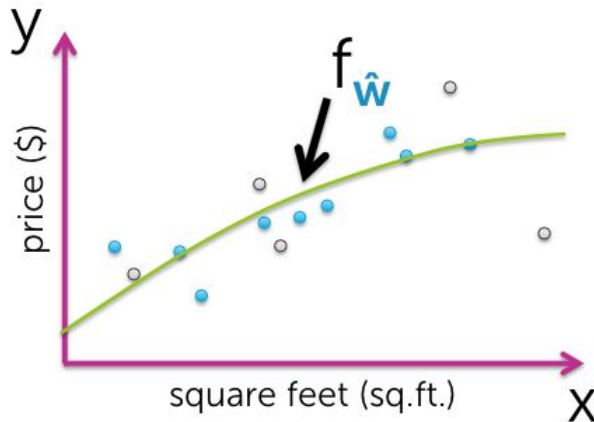


Logistic regression model

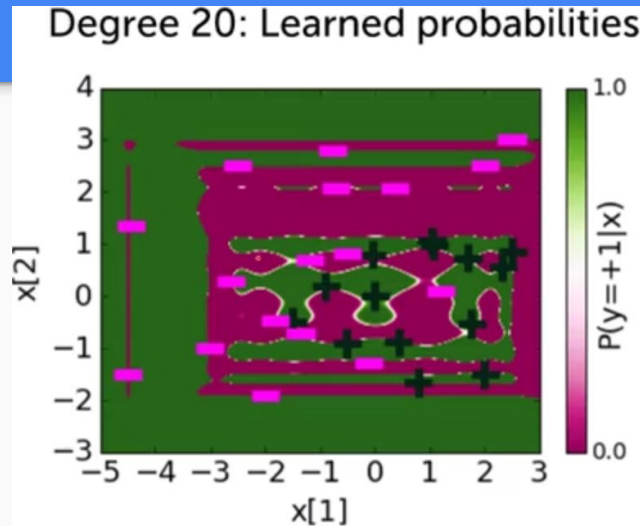
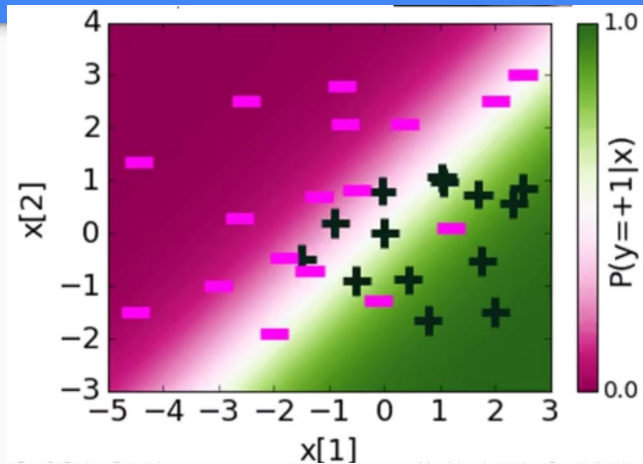
$$\begin{array}{c} \text{Score}(\mathbf{x}_i) = \hat{\mathbf{w}}^\top h(\mathbf{x}_i) \\ \begin{array}{ccc} -\infty & \xleftarrow{\hspace{1cm}} & \xrightarrow{\hspace{1cm}} +\infty \\ & \downarrow 0.0 & \\ & 0.5 & \\ & \downarrow & \\ 0.0 & \xleftarrow{\hspace{1cm}} & \xrightarrow{\hspace{1cm}} 1.0 \end{array} \\ \hat{P}(y=+1|\mathbf{x}, \hat{\mathbf{w}}) = \frac{1}{1 + e^{-\hat{\mathbf{w}}^\top h(\mathbf{x})}} \end{array}$$

Remember in linear regression we have problem with overfitting?

$$y_i = w_0 + w_1 x_i + w_2 x_i^2 + \dots + w_p x_i^p + \varepsilon_i$$



Signs of overfitting in logistic regression



- Overly complex decision boundary
- Overly large coefficients
- Side effect: “overconfidence” due to large coefficients

Regularizations to the rescue

Instead of maximizing the likelihood function, we maximize

Total quality = quality of fit + measure of model complexity

L1 penalty: $\ell(\mathbf{w}) - \lambda \|\mathbf{w}\|_1$

L2 penalty: $\ell(\mathbf{w}) - \lambda \|\mathbf{w}\|_2^2$

(Note that L1 regularization leads to sparsity)

How much to penalize?

Tuning factor λ controls the model complexity

Optimized through validation/cross-validation

(note that in sklearn, $C = 1/\lambda$)

Demo time!