Wing Flutter Analysis

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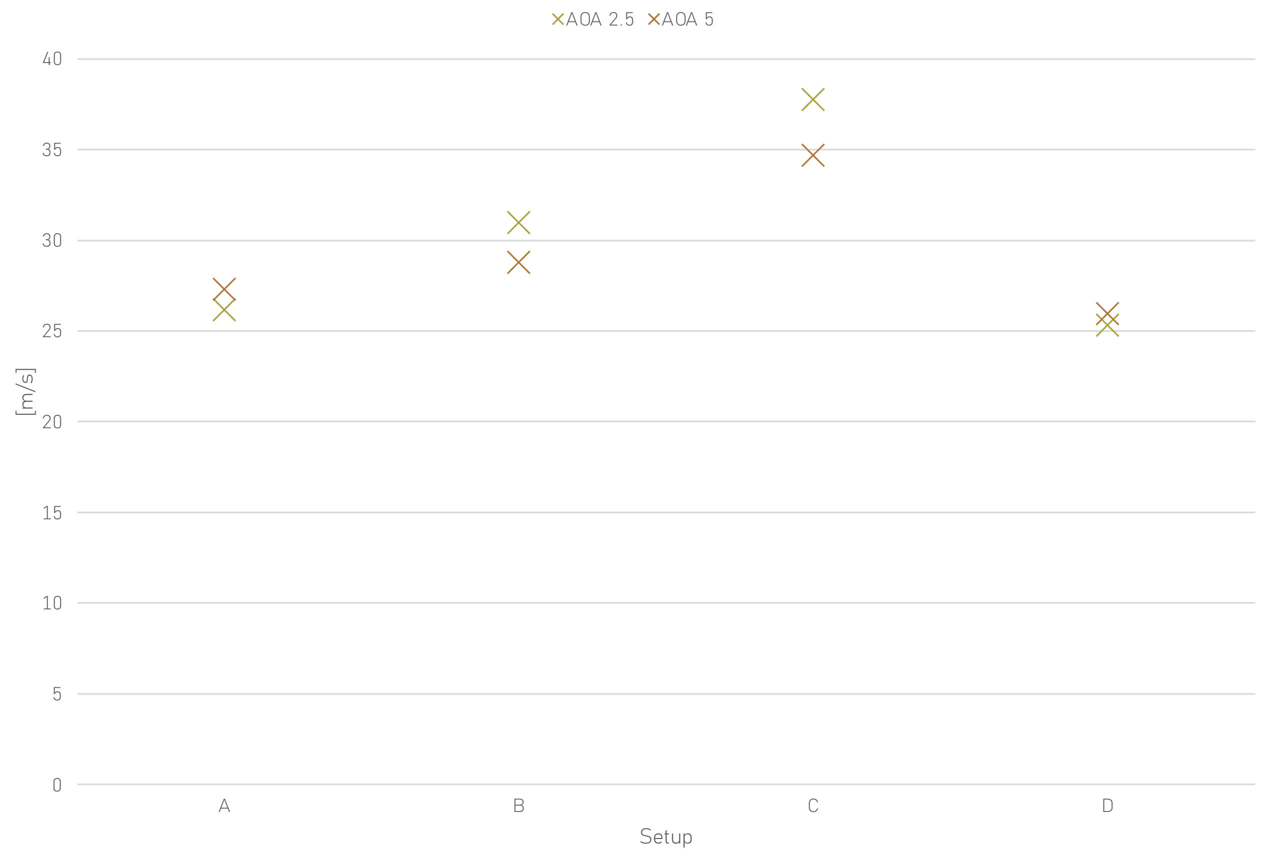
Data

Table 1: Flutter Test Bench Data

Test	Setup	AOA	Flutter Velocity
1	C	2.5	37.8
2	D	2.5	25.34
3	B	2.5	31
4	B	5	28.81
5	A	5	27.33
6	A	2.5	26.18
7	D	5	25.97
8	C	5	34.72

*Velocity Measured in m/s

Data Cont.



Questions

WHAT PREDICTIONS CAN BE MADE OF THE DIVERGENCE VELOCITY OF THE AIRFOIL FOR THE CONDITIONS WHERE THE SPRINGS WERE AT THE LEADING AND TRAILING EDGES (OPTION B & C).

Analysis

Step 1: Predicting the Divergence Velocity

Given Parameters

- **Airfoil chord,** $c = 0.1\,\mathrm{m}$
- **Spring stiffness per spring,** $k = 0.03\,\mathrm{N/mm} = 30\,\mathrm{N/m}$
- **Effective span,** $s = 266.25\,\mathrm{mm} = 0.26625\,\mathrm{m}$
- **Number of springs,** 4 (two at leading edge, two at trailing edge)
- **Air density,** $\rho_\infty = 1.225\,\mathrm{kg/m^3}$
- **Aerodynamic center,** $x_{ac} = 0.25c = 0.025\,\mathrm{m}$
- **Lift-curve slope,** $C_{L_\alpha} = 2\pi\,\mathrm{rad}^{-1}$

Calculations

Wing Area

$$S = c \times s = 0.1\,\mathrm{m} \times 0.26625\,\mathrm{m} = 0.026625\,\mathrm{m}^2$$

Equivalent Torsional Stiffness

$$k_{\text{edge}} = 2k = 2 \times 30\,\mathrm{N/m} = 60\,\mathrm{N/m}$$
$$b = \frac{c}{2} = 0.05\,\mathrm{m}$$
$$k_T = 2k_{\text{edge}} b^2 = 2 \times 60\,\mathrm{N/m} \times (0.05\,\mathrm{m})^2 = 0.3\,\mathrm{N\cdot m/rad}$$

Pivot Point Location

$$x_0 = \frac{c}{2} = 0.05\,\mathrm{m}$$

Divergence Dynamic Pressure

$$q_D = \frac{k_T}{S C_{L_\alpha}(x_0 - x_{ac})} = \frac{0.3}{0.026625 \times 2\pi \times 0.025} \approx 71.658\,\mathrm{N/m^2}$$

Analysis

Divergence Velocity

$$U_D = \sqrt{\frac{2q_D}{\rho_\infty}} = \sqrt{\frac{2 \times 71.658}{1.225}} \approx 10.8\,\mathrm{m/s}$$

The predicted divergence velocity for the configuration with springs at the leading and trailing edges is approximately 10.8 m/s. However, experimental observations indicated flutter occurring at 37.8 m/s, suggesting possible discrepancies due to factors such as additional structural stiffness and/or simplifications in the theoretical model.

Question

- WHAT DO YOU THINK THE EFFECT OF MOVING THE SPRINGS TO THE OTHER TWO POSITIONS (A & D) WILL BE?
  - BASED ON THE PRELAB, A & D SHOULD DECREASE THE VELOCITY REQUIRED TO REACH FLUTTER

Analysis

- How do your observations match the predictions?
  - The data and observations match the prediction made as both Option A & D required less velocity to cause flutter for AOA: 2.5 and AOA: 5.0 as shown in Slides 2 and 3 [Data & Data Cont.]

Question

WHAT IS THE PREDICTED DIVERGENCE VELOCITY FOR THE DIFFERENT SPRING POSITIONS BY TREATING THE SPRING ARRANGEMENT AS A TORSIONAL SPRING?

Answer

Option A Calculations

1. Calculate x_0 and b

$$x_0 = \frac{x_{\text{front}} + x_{\text{rear}}}{2} = \frac{0 + 0.1}{2} = 0.05\,\text{m}$$

$$l = x_{\text{rear}} - x_{\text{front}} = 0.1 - 0 = 0.1\,\text{m}$$

$$b = \frac{l}{2} = \frac{0.1}{2} = 0.05\,\text{m}$$

2. Calculate Equivalent Torsional Stiffness k_T

$$k_T = 2kb^2 = 2 \times 30\,\text{N/m} \times (0.05\,\text{m})^2 = 0.15\,\text{N} \cdot \text{m/rad}$$

3. Calculate Dynamic Pressure q_D

$$q_D = \frac{k_T}{SC_{L_\alpha}(x_0 - x_{ac})} = \frac{0.15}{0.026625 \times 2\pi \times (0.05 - 0.025)}$$

$$q_D = \frac{0.15}{0.026625 \times 6.28319 \times 0.025} = \frac{0.15}{0.004186} \approx 35.836\,\text{N/m}^2$$

4. Calculate Divergence Velocity U_D

$$U_D = \sqrt{\frac{2q_D}{\rho_\infty}} = \sqrt{\frac{2 \times 35.836}{1.225}} = \sqrt{58.513} \approx 7.65\,\text{m/s}$$

Option B Calculations

1. Calculate x_0 and b

$$x_0 = \frac{0 + 0.12}{2} = 0.06\,\text{m}$$

$$l = 0.12 - 0 = 0.12\,\text{m}$$

$$b = \frac{0.12}{2} = 0.06\,\text{m}$$

2. Calculate k_T

$$k_T = 2kb^2 = 2 \times 30 \times (0.06)^2 = 0.216\,\text{N} \cdot \text{m/rad}$$

3. Calculate q_D

$$q_D = \frac{0.216}{0.026625 \times 6.28319 \times (0.06 - 0.025)} = \frac{0.216}{0.026625 \times 6.28319 \times 0.035}$$

$$q_D = \frac{0.216}{0.005861} \approx 36.853\,\text{N/m}^2$$

4. Calculate U_D

$$U_D = \sqrt{\frac{2 \times 36.853}{1.225}} = \sqrt{60.201} \approx 7.76\,\text{m/s}$$

Answer Cont.

Option C Calculations

1. Calculate x_0 and b

$$x_0 = \frac{-0.02 + 0.12}{2} = 0.05\,\mathrm{m}$$

$$l = 0.12 - (-0.02) = 0.14\,\mathrm{m}$$

$$b = \frac{0.14}{2} = 0.07\,\mathrm{m}$$

2. Calculate k_T

$$k_T = 2kb^2 = 2 \times 30 \times (0.07)^2 = 0.294\,\mathrm{N \cdot m/rad}$$

3. Calculate q_D

$$q_D = \frac{0.294}{0.026625 \times 6.28319 \times (0.05 - 0.025)} = \frac{0.294}{0.004186} \approx 70.23\,\mathrm{N/m^2}$$

4. Calculate U_D

$$U_D = \sqrt{\frac{2 \times 70.23}{1.225}} = \sqrt{114.703} \approx 10.72\,\mathrm{m/s}$$

Option D Calculations

1. Calculate x_0 and b

$$x_0 = \frac{-0.02 + 0.1}{2} = 0.04\,\mathrm{m}$$

$$l = 0.1 - (-0.02) = 0.12\,\mathrm{m}$$

$$b = \frac{0.12}{2} = 0.06\,\mathrm{m}$$

2. Calculate k_T

$$k_T = 2kb^2 = 2 \times 30 \times (0.06)^2 = 0.216\,\mathrm{N \cdot m/rad}$$

3. Calculate q_D

$$q_D = \frac{0.216}{0.026625 \times 6.28319 \times (0.04 - 0.025)} = \frac{0.216}{0.026625 \times 6.28319 \times 0.015}$$

$$q_D = \frac{0.216}{0.002513} \approx 85.946\,\mathrm{N/m^2}$$

4. Calculate U_D

$$U_D = \sqrt{\frac{2 \times 85.946}{1.225}} = \sqrt{140.395} \approx 11.85\,\mathrm{m/s}$$

Analysis

- How do your experiment compare to predictions?
  - Option C has the highest divergence velocity (UD = 10.72 m/s), due to the increased distance between springs, leading to higher torsional stiffness kT .
  - Option A has the lowest divergence velocity (UD = 7.65 m/s), with springs at the LE and TE.

  - Both predictions align with the predictions in the prelab and the data where C required the most velocity to reach flutter and A (or D) required the least velocity to induce flutter but the experimental observations showed flutter occurring at 37.8 m/s, significantly higher than the predicted velocities, suggesting there may be errors in our assumptions.