Pressure Distribution in a Convergent-Divergent Nozzle

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1 Introduction

The purpose of this experiment is to analyze the pressure distribution along a convergent-divergent nozzle at different exit pressure conditions. In this experiment, the pressure within the nozzle was measured using a movable probe at various positions from the inlet to the outlet while maintaining different exit pressures (p_e). The exit pressures studied were 4.4 bar, 4.1 bar, 2.9 bar, 2.0 bar, and 0.5 bar. The pressure readings were normalized by the inlet pressure (p_0) to obtain the dimensionless pressure ratio (p/p_0). By analyzing the pressure distribution for each exit pressure condition, we aim to understand the different flow regimes, including subsonic flow, choking, and shock wave formation within the nozzle.

2 Experimental Data and Observations

The experimental setup involved measuring the pressure at various positions within the nozzle using a probe that moves from the inlet to the outlet. The probe pressure (p) was recorded at different positions for each exit pressure condition (p_e). The inlet pressure (p_0) was kept relatively constant, and the pressure ratio (p/p_0) was calculated for analysis.

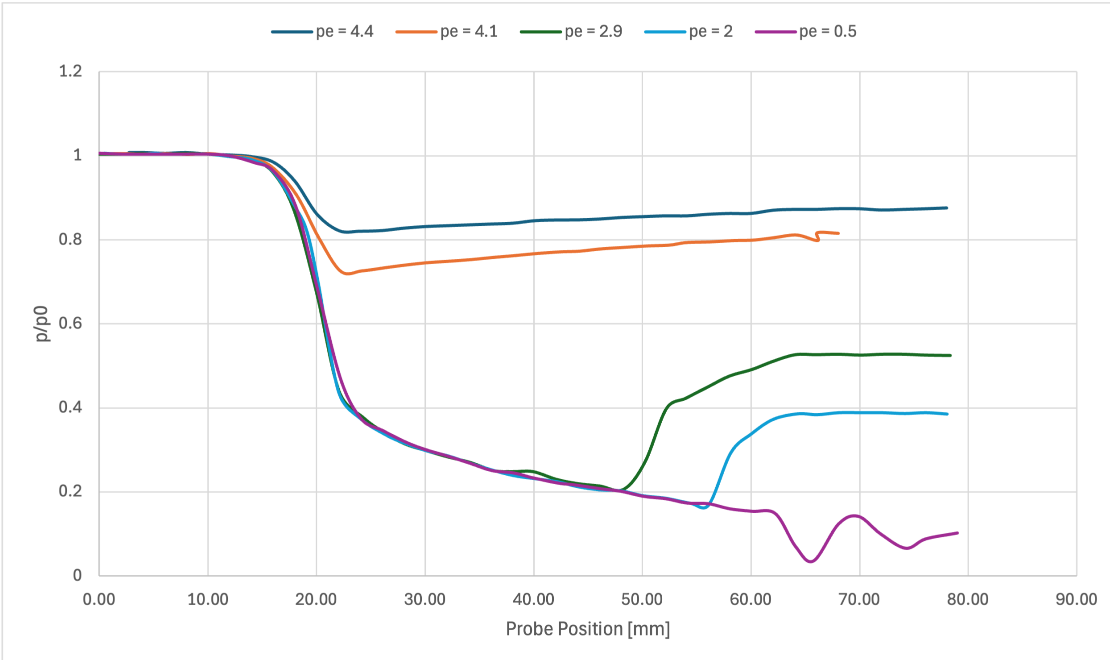

Figure 1: Pressure ratio (p/p_0) along the nozzle length for different exit pressures (p_e).

2.1 Exit Pressure $p_e = 4.4$ bar and $p_e = 4.1$ bar

At the highest exit pressures of 4.4 bar and 4.1 bar, the pressure ratio (p/p_0) remained close to constant throughout the nozzle. The pressure decreased slightly along the nozzle, but the flow remained subsonic at all positions. No shock waves were observed in these conditions.

2.2 Exit Pressure $p_e = 2.9$ bar, $p_e = 2.0$ bar, and $p_e = 0.5$ bar

At lower exit pressures of 2.9 bar, 2.0 bar, and 0.5 bar, significant changes in the pressure distribution were observed. The pressure ratio decreased more substantially along the nozzle, and abrupt changes in pressure were noted at certain positions, indicating the presence of shock waves. The flow achieved sonic conditions at the throat and became supersonic in the divergent section before being decelerated by shock waves.

3 Results

The experimental data demonstrate how varying the exit pressure (p_e) affects the flow within the convergent-divergent nozzle. The key observations and their explanations are discussed below.

3.1 High Exit Pressure Conditions ($p_e = 4.4$ bar and $p_e = 4.1$ bar)

At the high exit pressures, the pressure ratio (p/p_0) remained close to 1 throughout the nozzle, indicating that the flow remained subsonic from the inlet to the outlet. This is because the pressure difference between the inlet and the exit is insufficient to accelerate the flow to sonic conditions at the throat.

According to the isentropic flow relations, the critical pressure ratio required for the flow to reach Mach 1 at the throat is given by:

$$\left(\frac{p_e}{p_0}\right)_{\text{critical}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \tag{1}$$

For air, with $\gamma = 1.4$, the critical pressure ratio is approximately 0.528. Since the exit pressures are much higher than this critical ratio relative to the inlet pressure, the flow does not choke, and no shock waves are formed in the nozzle. The flow remains subsonic, and the pressure decreases gradually due to friction and area changes.

3.2 Intermediate and Low Exit Pressure Conditions ($p_e = 2.9$ bar, $p_e = 2.0$ bar, and $p_e = 0.5$ bar)

As the exit pressure is reduced below the critical pressure ratio, the flow reaches sonic conditions at the throat (choking occurs). Beyond the throat, in the divergent section, the flow accelerates to supersonic speeds. However, depending on the exit pressure, shock waves may form in the divergent section.

3.2.1 Exit Pressure $p_e = 2.9$ bar

At $p_e = 2.9$ bar, the flow chokes at the throat and becomes supersonic in the divergent section. A normal shock wave forms at a certain position in the divergent section, causing a sudden increase in pressure and a reduction in Mach number back to subsonic flow. This is evident from the abrupt increase in the pressure ratio (p/p_0) observed in the data.

3.2.2 Exit Pressure $p_e = 2.0$ bar

At $p_e = 2.0$ bar, similar behavior is observed, but the shock wave occurs further downstream in the divergent section compared to the $p_e = 2.9$ bar case. The lower exit pressure allows the supersonic flow to extend further before being decelerated by the shock wave.

3.2.3 Exit Pressure $p_e = 0.5$ bar

At the lowest exit pressure of $p_e = 0.5$ bar, the flow remains supersonic throughout the divergent section, and no shock wave forms within the nozzle. This is because the exit pressure is sufficiently low to allow for full expansion of the supersonic flow without the need for a shock to adjust the pressure back to subsonic levels.

4 Governing Equations

4.1 Isentropic Flow Relations

The flow through the nozzle can be analyzed using isentropic flow relations, which describe the relationship between pressure, temperature, and Mach number in an isentropic (frictionless and adiabatic) process:

$$\frac{p}{p_0} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{\gamma}{\gamma - 1}} \tag{2}$$

$$\frac{T}{T_0} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1} \tag{3}$$

$$\frac{\rho}{\rho_0} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{1}{\gamma - 1}} \tag{4}$$

where p, T, and ρ are the static pressure, temperature, and density, respectively, p_0, T_0, and ρ_0 are the total (stagnation) pressure, temperature, and density, M is the Mach number, and γ is the ratio of specific heats (for air, $\gamma = 1.4$).

4.2 Normal Shock Relations

When a normal shock wave forms, the flow properties change abruptly. The relations across a normal shock are given by:

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2 - (\gamma - 1)}{\gamma + 1} \tag{5}$$

$$M_2 = \sqrt{\frac{(\gamma - 1)M_1^2 + 2}{2\gamma M_1^2 - (\gamma - 1)}} \tag{6}$$

where subscript 1 refers to the conditions before the shock and subscript 2 refers to the conditions after the shock.

4.3 Choked Flow and Critical Pressure Ratio

Choked flow occurs when the flow reaches Mach 1 at the throat of the nozzle. The critical pressure ratio for choked flow is:

$$\left(\frac{p_e}{p_0}\right)_{\text{critical}} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}} \tag{7}$$

For air with $\gamma = 1.4$, this critical pressure ratio is approximately 0.528.