Kirchhoff's rules

Objective

Become familiar with the principles and applications of Kirchhoff's rules in electric circuits.

Theory

Kirchhoff's rules are the foundation of many practical methods for calculating complex electric circuits such as Norton's theorem. There are two Kirchhoff's rules.

First (loop) Kirchhoff's rule

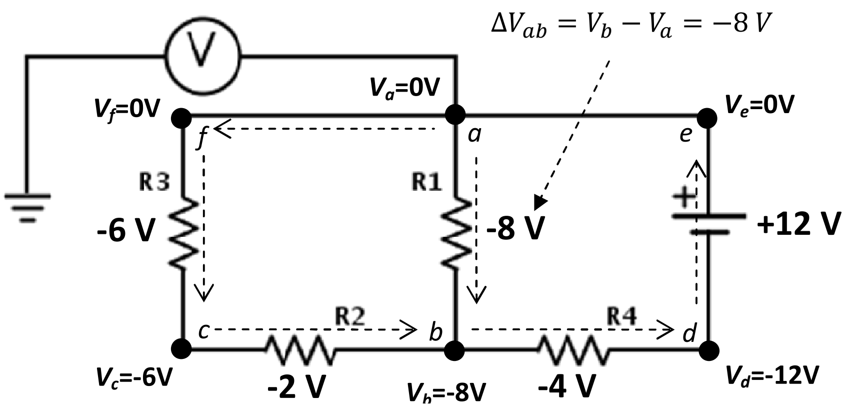

Figure 1 Voltage in electric circuit is a function of position in the circuit: voltage may change from point to point, but at every point there is a definite value of the voltage, V_a. The voltage difference between any two points in the circuit is the difference of these voltage values at these points, $\Delta V_{ab} = V_b - V_a$.

The first Kirchhoff's rule states that the sum of all voltage differences in any closed loop in a circuit is zero,

$$\sum \Delta V = 0$$

For example, the sum of voltage difference in the loops *abdea* or *afcba* in Figure 1 is zero. For *abdea*, this is +12 V – 8 V – 4 V = 0, and for *afcba* this is -6 V – 2 V + 8 V = 0. *This is always the case for every loop in every electric circuit.*

It is easy to understand this rule if we observe that the voltage in electric circuit is a function of point in the circuit. That is, the voltage may change from point to point in electric circuit, but at every point voltage has definite value (Figure 1). For any two points, the voltage difference between these two points in a circuit, such as measured by a voltmeter, is the difference of the *voltage values* at these points, for example, $\Delta V_{ab} = V_b - V_a = -8V - 0V = -8\,V$ in Figure 1.

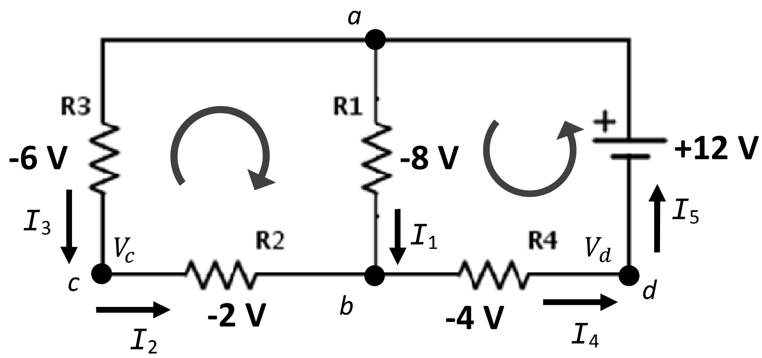

Figure 2 First Kirchhoff's rule observes, that if one moves in electric circuit so that he comes back to the point where he started (for example in the loop *abca* or *abda*), one comes to the point having the same voltage as in the start, and, therefore, the the sum of voltage differences ΔV_{ab} over all passed circuit elements should be zero.

If we now pick any loop, for example, such as the right loop in Figure 2, starting from the point *a* and coming back to the point *a* passing through the points *b* and *d*, the sum of the voltage differences in such a loop should add up to ΔV_{aa}, since you started and ended at the same point, but $\Delta V_{aa} = V_a - V_a = 0$.

$$0 = V_a - V_a = V_a - V_b + V_b - V_a = V_a - V_b + V_b - V_d + V_d - V_a = -\Delta V_{ab} - \Delta V_{bd} - \Delta V_{da} = 0$$

or

$$\Delta V_{ab} + \Delta V_{bd} + \Delta V_{da} = -8V - 4V + 12V = 0$$

Similarly, for loop *acb*a you can write

$$0 = V_a - V_a = V_a - V_c + V_c - V_a = V_a - V_c + V_c - V_b + V_b - V_a = -\Delta V_{ac} - \Delta V_{cb} - \Delta V_{ba} = 0$$

or

$$\Delta V_{ac} + \Delta V_{cb} + \Delta V_{ba} = -6V - 2V - (-8V) = 0$$

Note how the last term here is ΔV_{ba} instead of ΔV_{ab}. This is because the resistor R_1 was passed in the left loop in the *opposite* direction, therefore, $\Delta V_{ba} = V_a - V_b = -(V_b - V_a) = -\Delta V_{ab} = -(-8V)$.

Now this kind of logic can be applied to any loop in electrical circuit of any complexity.

For every resistor passed in such a loop, the voltage difference over that resistor is defined by the *Ohm's law* $\Delta V = \pm IR$. Note the "±" sign here – if the resistor is passed in the same direction with the direction of the electric current in that resistor, then $\Delta V = -IR$, otherwise $\Delta V = +IR$. In other words, the current in the resistor flows from high to low voltage and, therefore, the voltage in resistors drops in the direction of the electric current.

For every battery or power source passed in such a loop, ΔV is positive if the battery is passed from negative to positive terminal and negative otherwise. That is, the voltage in battery is raised from negative to positive terminal. The value of *ΔV* is the rating of that battery *E* (1.5V, 3.0V, 4.5V, etc.).

In general, therefore, for any closed loop in an electric circuit we get this Kirchhoff's rule,

$$\sum \pm IR + \sum \pm E = 0.$$

For every resistor ($\pm IR$) the sign is "-" if the current flows in the same direction as the resistor is passed in the loop and "+" otherwise. For every battery ($\pm E$) the sign is "+" if the battery is passed from negative to positive terminal and "-" otherwise. To remember these important sign-rules just remember that the *electric current flows from high voltage to low in resistors* and *the positive terminal in batteries is at higher voltage than the negative*.

Using these rules, for example, for loop *abca* in Figure 2 we can obtain this Kirchhoff's rule,

$$0 = -I_1R_1 + I_2R_2 + I_3R_3$$

and for loop *abda*,

$$0 = -I_1R_1 - I_4R_4 + E$$

Second (junction) Kirchhoff's rule

Second Kirchhoff's rule describes how electric current changes from point to point in an electric circuit. It is based on simple observation that whatever electric current enters a point in an electric circuit should also exit that point. That is, no electric charge accumulates anywhere in the circuit, so whatever current comes in has to go out.

If for a single wire this observation is trivial – whatever current enters one end of the wire should exit at the other end, where three or more wires come together (such points are called *junctions*) this rule becomes very important. It states that the sum of incoming currents in such a junction should always equal the sum of outgoing currents, for example in Figure 3,

$$I_1 = I_2 + I_3$$

Figure 3 Kirchhoff's second rule is that simple observation that whatever current enters any given point in a circuit should also exit that point. This observation is trivial for single wires (left) but is very important when applied to junctions (right).

This logic applied to junctions of general electric circuits gives the second Kirchhoff's rule,

$$\sum_{incoming} I_i = \sum_{outgoing} I_j$$

Application of Kirchhoff's rules

The first and the second Kirchhoff's rules, when written for all loops and all junctions in an electric circuit, produce a system of linear equations that can be solved to give electric currents and voltages at every point in the circuit. For example, for the circuit in Figure 2 the system of Kirchhoff's equations is,

$0 = -I_1R_1 + I_2R_2 + I_3R_3$	(left loop)
$0 = -I_1R_1 - I_4R_4 + E$	(right loop)
$0 = -I_3R_3 - I_2R_2 - I_4R_4 + E$	(outside loop)
$I_5 = I_1 + I_3$	(junction *a*)
$I_1 + I_2 = I_4$	(junction *b*)
$I_4 = I_5$	(junction *d*)
$I_3 = I_2$	(junction *c*)

Equipment

- Electric circuit experiments set
- Resistors R_1= , R_2= , R_3= .

Procedures

1. Use provided resistors and cables to implement the *circuit A* shown below, connected to 5V DC power source and the batteries pack. **Note:** Once complete, verify your circuit with the instructor and obtain permission to proceed. DO NOT TURN ON THE POWER BEFORE TALKING WITH THE INSTRUCTOR.
2. Use voltmeter and amperemeter instruments to measure voltage differences $\Delta V_{ab}, \Delta V_{bd}, \Delta V_{da}, \Delta V_{bc}, \Delta V_{ca}$ and the currents $I_{a\to b}, I_{d\to a}, I_{c\to a}$. (You will have to connect the amperemeter in series and the voltmeter in parallel to do these measurements; if you are not sure how to do this, ask your instructor.)
   Note: Your voltmeter and amperemeter can only measure positive voltages and currents. Positive here means that the red terminal is at *higher voltage* and the black terminal is at *lower voltage* – the voltage drops and the current flows *from red to black* terminals. If voltmeter or amperemeter appears to show negative reading, switch the red and the black terminals so that you now measure the voltage and the current in the opposite direction.
3. ***When writing down your measurements be sure to note between which points and in what direction the voltage and the current had been measured.*** If you connect red terminal to point *A* and black terminal to point *B*, the voltmeter will show $\Delta V_{BA} = V_A - V_B$ and the amperemeter will show $I_{A\to B}$. Because $\Delta V_{AB} = V_B - V_A = -(V_A - V_B) = -\Delta V_{BA}$ and $I_{A\to B} = -I_{B\to A}$, it is very easy to mix up the signs in your Kirchhoff's equations if you are not careful.

ANALYSIS (TO BE PERFORMED IN THE REPORT)

4. Use your measurements of the voltage differences ΔV and the currents I in (2) to verify separately for each resistor R_1, R_2, and R_3 the Ohm's law ΔV=IR.
5. Write down the first Kirchhoff's rule for the loops in the circuit *abda*, *abca*, and *acbda* using *ΔV,* and use your experimental measurements of the voltage differences ΔV in (2) to show that these are indeed correct (that is, show for example that indeed $\Delta V_{ab} + \Delta V_{bc} + \Delta V_{ca} = 0$, and so on for all other loops). **Be sure to use correct signs for your measured ΔV when writing down the Kirchhoff's equations.**
6. Write down the second Kirchhoff's rule for the junctions *a* and *b* and use your measurements of the currents I in (2) to show that in each junction the sums of incoming and outgoing currents are indeed equal. **Be sure to use correct signs for your measured I when writing down the Kirchhoff's equations.**

ELECTRIC CIRCUIT DIAGRAMS

Circuit A;