Kirchhoff's rules in AC circuits

Objective

Become familiar with the basic properties of AC electric circuits.

Theory

As you learned in one of the previous lab, Kirchhoff's rules describe how voltage and current change in electric circuits.

The Kirchhoff's "loop" or "voltage" rule states that adding together voltage drops over different electric elements in any closed loop in a circuit should always add up to zero, as the loop starts and begins at the same point in the circuit and, therefore, one should have the same voltage in the beginning and the end of the loop. The Kirchhoff's "junction" or "current" rule states that, as charge cannot accumulate at any point in the circuit, the balance of current requires that any incoming current at any point is exactly balanced by the outgoing current.

AC (or alternating current) circuits differ from DC circuits in that the power supplied by the power source is not constant but changes with time as

$$V(t) = V_0 \sin(\omega t + \varphi)$$

$$I(t) = I_0 \sin(\omega t + \varphi)$$

In these formulas, V_0 and I_0 are called *peak voltage/current*, ω is called *frequency* of alternating current, and φ is the *phase* of alternating current. The AC power is important because it is the power produced by all electric power generators, and used in all modern power stations. Most of the electric power coming to our households, therefore, is in AC form.

If in DC circuits voltage is a constant, in AC circuits voltage is a sin-function and changes with time. Accordingly, if in DC circuits we could simply add voltages on different circuit elements, in AC circuits such voltages have to be added using sin-functions.

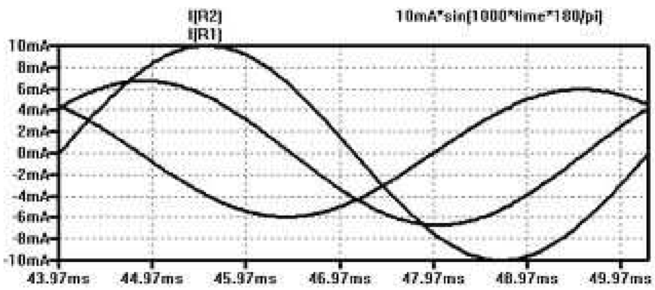


Figure 1 Since in AC circuits voltage and current change with time as sin-function, when voltages on two elements add together they should be added as sinusoidal functions, not simple numbers. This makes analysis of voltages addition in AC circuits complicated.

In order to simplify this process, one typically uses the method of "*phasors*". The idea behind phasors is to notice that sinusoidal AC voltage, $V(t) = V_0 \sin(\omega t + \varphi)$, can be also viewed as x-projection of a certain vector of length V_0 that rotates with speed ω, that is $V(t) = V_0 \sin(\omega t + \varphi) = \vec{V}_{0,x}$, see Figure 1.

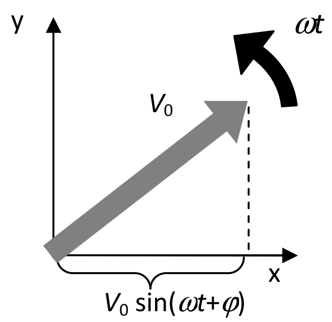


Figure 2 Sinusoidal voltage in AC circuits can be viewed as a x-projection of a certain rotating vector, a phasor.

When two or more AC voltages need to be added together, $V_1(t) + V_2(t) = V_1 \sin(\omega t + \varphi_1) + V_2 \sin(\omega t + \varphi_2)$ can be represented as a sum of the projections of the corresponding vector phasors, that is $V_1(t) + V_2(t) = (\vec{V}_1 + \vec{V}_2)_x = V_1 \sin(\omega t + \varphi_1) + V_2 \sin(\omega t + \varphi_2)$.

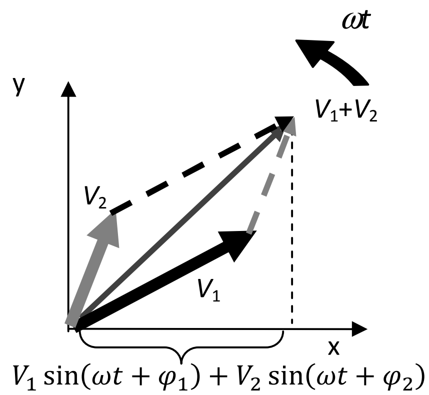


Figure 3 The sum of two sinusoidal voltages in AC circuit can be viewed as the x-projection of the vector sum of their corresponding phasors.

It is in this sense that we say that the voltage in AC circuits is represented by vector "phasors" and not numbers, and that the voltages in AC circuits add as *vectors*, not numbers.

Phasors are not real physical objects, they are just mathematical tools used to describe and add sinusoidal voltages in AC circuits in a simpler way. If in DC circuits voltage was constant and could be represented as number V, in AC circuits voltage is a sin-function and one has to take care about its phase φ. For this reason, in AC circuits we represent voltage as phasor having length V and direction φ. Addition of sinusoidal voltages in AC circuits, then, can be correctly performed by adding such vector phasors.

In AC circuits, the same logic as in Kirchhoff's rules can be applied to state that the sum of voltages over any loop should be zero. However, such sum now is a vector sum of the voltage phasors and zero is a zero vector.

Such addition of voltages in AC circuits is usually represented using *phasor diagrams*. In a phasor diagram, the voltage phasors are drawn as vectors relative to a phasor representing the current in the circuit, which is drawn at a fixed position (Figure 4). It is imagined that, as the current phasor "rotates" with time, the entire diagram should "rotate" together with it. The relative positions of all phasors, however, would remain the same, so it is sufficient to make single drawing to understand how voltages should add together.

There are three types of phasors that can be present in such a diagram – resistance phasors and two so called reactance phasors Resistance phasors (for resistors) are always drawn in the same direction with the current and have length defined by normal Ohm's law, $V_R = IR$. Reactance phasors correspond to the voltages on the elements such as capacitors and inductances. For capacitors, the voltage phasors are drawn in the direction of 90° back from the current phasor, and have length $V_C = IX_C$, and for inductances the voltage phasors are drawn in the direction of 90° forward from the current phasor, and have length $V_L = IX_L$. The constants X_C and X_L are called *reactances* and can be calculated for different capacitors and inductances using the table below.

Note that when there is more than one resistor, capacitor, or inductance in the circuit, V_R, V_C,V_L are the total voltages on all these elements, since phasors from different resistors, capacitors, or inductances will all add in the same direction in the phasor diagram.

The total sum of all such phasors should produce and be equal to the total voltage provided to the AC circuit by the AC power source, Figure 4.

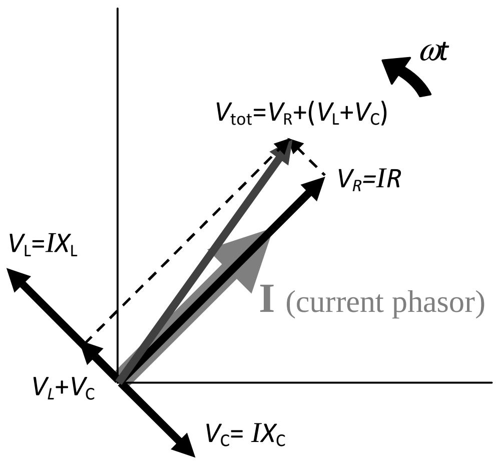


Figure 4 Addition of voltages in AC circuits is commonly represented using phasor diagrams. In phasor diagrams, all voltages are drawn as vectors relative to a phasor representing the current in the circuit. It is imagined that, as the current phasor "rotates" with time, the entire diagram should "rotate" together with it. There are three types of phasors in AC circuits – resistance phasors, which are in the same direction with the current, capacitance phasor V_C, which is 90° behind the current, and inductance phasor V_L, which is 90° ahead of the current. The total sum of these phasors should equal as a vector the total voltage provided to the circuit by AC power source.

Element	Voltage (phasor length)	Resistance/reactance (in Ohm's law)	Voltage phase (phasor direction)
Resistor, *R*	$V_R = IR$	R	φ=0° (same with current)
Capacitor, *C*	$V_C = IX_C = I \cdot (\frac{1}{\omega C})$	$X_C = \frac{1}{\omega C}$	φ=-90° (90° back from the current)
Inductance, *L*	$V_L = IX_L = I \cdot (\omega L)$	$X_L = \omega L$	φ=+90° (90° forward from the current)

Figure 5 Ohm's law, resistances and reactances, and corresponding voltages and phases in AC circuits for resistor, capacitor, and inductance elements.

The voltage addition in AC circuits, therefore, can be represented and performed using such phasor diagrams.

As can be seen from Figure 4, the voltage in AC circuits, because of its “vector” nature, can behave in rather interesting and unexpected ways. For example, the total voltage drop in the circuit is not $V_{sum} = V_R + V_C + V_L$, but $V_{tot} = \sqrt{V_R^2 + (V_L - V_C)^2}$. The current in the circuit is not $I = \frac{V_{tot}}{R+X_L+X_C}$, but $I = \frac{V_{tot}}{\sqrt{R^2+(X_L-X_C)^2}}$. Furthermore, adding new elements to the circuit can *reduce* its resistivity and *increase* the current! In fact, it can be easily seen from the formula for I that the maximal current is obtained when $X_L = X_C$. Furthermore, when $X_L = X_C$ also $V_{tot} = V_R$ regardless of V_L and V_C!

Equipment

- Electric circuit experiment set.
- Cables.
- Multimeter.
- Lamp element (to be used as resistor), *R*=____________________.
- Capacitor element, *C*=____________________.
- Coil element, *R*=____________________ and *L*=____________________.

Procedures

1. Use provided circuit elements and cables to implement *circuit A* below.
   NOTE: Once complete, verify your circuit with the instructor and obtain permission to proceed.
2. Turn the power on and use the multimeter in “AC voltage” mode to measure the voltage on the lamp and the capacitor, V_{lamp}, V_C, and the total voltage from the power source V_{tot}.
3. Turn the power off. Use provided circuit elements and cables to implement *circuit B*.
   NOTE: Once complete, verify your circuit with the instructor and obtain permission to proceed.
4. Turn the power on and use the multimeter in “AC voltage” mode to measure the voltage on the lamp and the coil, V_{lamp}, V_{coil}, and the total voltage from the power source V_{tot}.
   NOTE: The coil is actually two electric elements – one resistance and one inductance. Note that what you measure with multimeter is the *combined* voltage on these two elements (the voltage *outside* the box on the diagram)!
5. Turn the power off. Use provided circuit elements and cables to implement *circuit C*.
   NOTE: Once complete, verify your circuit with the instructor and obtain permission to proceed.
6. Turn the power on and use the multimeter in “AC voltage” mode to measure the voltage on each circuit element, V_C, V_{lamp}, V_{coil}, and the total voltage from the power source V_{tot}.

ANALYSIS (TO BE PERFORMED IN THE REPORT)

7. Using the table in the Theory section, calculate the reactances X_L and X_C for the conductance in the coil and the capacitor. Use alternating frequency ω=314 Hz. **NOTE:** Be sure to use proper units so that your results are in Ohms – you should use Henry for inductance *L* and Farad for capacitance *C*.
8. For your measurements in 2):
   a. Verify that $V_{tot} = \sqrt{V_R^2 + {V_C}^2}$. Calculate $V_{sum} = V_R + V_C$ and show that $V_{sum} > V_{tot}$.
   b. Use measurement for V_{lamp} to find the current in the circuit, I, from Ohm’s law, *V*=*IR*.
   c. Use X_C from 7) and I from 8b) to verify Ohm’s law for the capacitor in circuit A (that is $V_C=IX_C$).
9. For your measurements in 4):
   a. Use measurement for V_{lamp} to find the current in the circuit, I, from Ohm’s law, *V*=*IR*.

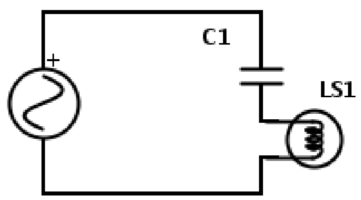
b. Use X_L from 7), I from 9a), and Ohm's law for resistor and inductance (that is $V_R=IR$ and $V_L=IX_L$) to calculate V_R and V_L on the coil's resistance and inductance.
NOTE: The coil is two electric elements – one resistance and one inductance. Their properties are given in the *Equipment* section. The total voltage that you measured on the coil is the sum of V_R and V_L; note that these are phasors and have different phases, and so should be added as vectors!

c. Use 9b) to calculate the total voltage drop on the coil (that is both from coil's resistance *and* inductance) and compare it with the experimental value V_{coil} you measured in 4).

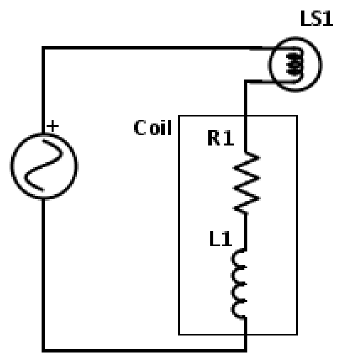
10. For your measurements in 6):
    a. Use measurement for V_{lamp} to find the current in the circuit, I, from Ohm's law, $V=IR$.
    b. Use X_L from 7), I from 9a), and Ohm's law for resistor and inductance (that is $V_R=IR$ and $V_L=IX_L$) to calculate V_R and V_L on the coil's resistance and inductance.
    c. Verify that $V_{tot} = \sqrt{V_{tot,R}^2 + (V_L - V_C)^2}$. Calculate $V_{sum} = V_{tot,R} + V_C + V_L$ and show that $V_{sum} > V_{tot}$. Note that in this case there are two resistors in the circuit – the lamp and the coil's resistance, so $V_{tot,R}$ should contain both voltages from the lamp and the coil's resistance!

ELECTRIC CIRCUIT DIAGRAMS

Circuit A;



Circuit B;



Circuit C;