## Derivatives (turev) ($f(t) \to f'(t)$ or $f(t) \to \dot{f}(t)$ or $f(t) \to df(t)/dt$)

**Table: Derivatives Rules**

| Rule | Formula | Example |
|---|---|---|
| | $(af(t))' = af(t)'$ | $(at^2)' = 2at$ |
| | $(f(at))' = af(at)'$ | $\cos(at)' = a\cos(at)$ |
| Sum rule | $(f_1(t) + f_2(t))' = f_1'(t) + f_2'(t)$ | $(t^2 + t^4)' = (t^2)' + (t^4)' = 2t + 4t^3$ |
| Product rule | $(f_1(t)f_2(t))' = f_1'(t)f_2(t) + f_1(t)f_2'(t)$ | $(t^2 \sin t)' = (2t)\sin t + t^2(\cos t)$ |
| Chain rule | $(f_1[f_2(t)])' = f_1'[f_2(t)]f_2'(t)$ | $(\sin[t^2])' = \cos[t^2] \cdot (2t)$ |
| Division rule | $(\frac{f_1(t)}{f_2(t)})' = \frac{f_1'(t)f_2(t) - f_1(t)f_2'(t)}{f_2(t)^2}$ | $(\frac{\sin t}{t})' = \frac{t(\cos t) - (1)\cdot \sin t}{t^2}$ |

For vectors, the derivative just means taking derivatives of every component, $\vec{r}' = (r_x', r_y', r_z')$.

**Table: Derivatives of Simple Functions**

| name | $f(t)$ | $f'(t)$ | name | $f(t)$ | $f'(t)$ |
|---|---|---|---|---|---|
| constant | 1 | 0 | exponential | $e^t$ | $e^t$ |
| linear | $t$ | 1 | nat. logarithm | $\ln t$ | $1/t$ |
| quadratic | $t^2$ | $2t$ | | $a^t = e^{\ln(a)t}$ | $\ln(a)a^t$ |
| cubic | $t^3$ | $3t^2$ | logarithm | $\log_a t = \ln t / \ln a$ | $1/(\ln(a)t)$ |
| power | $t^n$ | $nt^{n-1}$ | inverse power | $t^{-n}$ | $-nt^{-n-1}$ |
| sinus | $\sin(t)$ | $a\cos(t)$ | | | |
| cosinus | $\cos(t)$ | $-\sin(t)$ | | | |
| tangent | $\tan(t)$ | $1/cos(t)^2$ | | | |

## Derivatives practice

**Example:** $(4t^5 - \sin t)' = (4t^5)' - (\sin t)' = 4(t^5)' - (\sin t)' = 4 * 5t^4 - \cos t = 20t^4 - \cos t$.
**Example:** (vector) $(t^3, t^2 + 2\cos t, 5)' = ((t^3)', (t^2 + 2\cos t)', 5') = (3t^2, 2t - 2\sin t, 0)$.
**Example:** (composite function) $(\sin t^4)'$;
For composite functions it is most important to first understand what $f_1(t)$ and $f_2(t)$ are. In this example, $f_1(t) = \sin t$ and $f_2(t) = t^4$, so that you have $\sin t^4 = \sin[(t^4)]$.
Then, $f_1(t)' = \cos t$, $f_2'(t) = 4t^3$, then the full derivative is written as
$f_1'(f_2(t))f_2'(t) = \{f_1' = \cos\}(f_2(t) = t^4) \cdot (f_2'(t) = 4t^3) = \cos(t^4) \cdot 4t^3$.

Find derivatives of the following functions:

1. $t^6$
2. $t^3 + t^6$
3. $\frac{1}{3}t^3 + \frac{1}{4}t^4$
4. $t + 1/t$ (use the fact that $1/t = t^{-1}$)
5. $t^3 + 1/t^2$ (use the fact that $1/t = t^{-1}$)
6. $4\cos(3t) + 3\sin(3t)$
7. $2\sin(4t) + \tan(4t)$
8. $\cos(3t + 1)$
9. $t\cos(3t)$
10. $5\sin(3t)\cos(3t)$
11. $\cos(5t)\tan(5t)$
12. $t^2\sin(3t) + 2t\cos(3t)$
13. $(\sin(3t))^3$ (use chain rule, $f_1(t) = t^3$ and $f_2(t) = \sin(3t)$
14. $\cos(t^4)$ (use chain rule ...)
15. $\sin(t^4 + 4t^3) + \cos(t^4 + 2t^2)$
16. $(\cos(t))^2 + \cos(t^2)$
17. $(2 + t - t^2/2, 4t^3 + \sin(t), cos(\pi t))$ (THIS IS A VECTOR)
18. $(\cos(\pi t), -\sin(\pi t), \pi t^3)$
19. $4 \cdot (\cos(\pi t), 3t^2, 3)$
20. $(12t^2, 8t^3, 6t^4)$