Development of Cyclic Macroeconomic Instabilities in Competitive Market Economies

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ABSTRACT

This paper demonstrates emergence of cyclic instabilities (business cycle) in the basic settings of competitive single commodity market with several producers. The cycles develop from "Tragedy of the commons"-style competition of producers for market shares, resulting in commodity overproduction followed by market depression due to accumulation of overproduced commodity. The cycles only emerge for durable goods and do not affect nondurable goods, a well-known property of real business cycles. The cycles also produce characteristic commodity inventory patterns present in real economies. These findings present the problem of business cycles in a new light as an emergent phenomenon of open markets.

Economic or business cycles are some of the most noted features of market economies also ranked among the most serious of economic problems (Keynes 2006; Schumpeter 1954). Economic cycles can be traced historically for over two hundred years spanning over various social systems and economic organizations (McConnel and Brue 2008; Schumpeter 1954). A number of theories had been proposed in the past to explain the economic cycles, including the theories of inventory cycles (Kitchin 1923; Schumpeter 1954), politically based cycles (Nordhaus 1975; Nordhaus, Alesina, and Schultze 1989), credit/debt cycles (Fisher 1933; Eckstein and Sinai 1990), the real business cycles (Long and Plosser 1983; Plosser 1989; Lucas 1977; Kydland and Prescott 1982), and many other (Schumpeter 1954; Mattick 1969; Grossman and Kennedy 1992; George 1997; Goodwin 1967). In mainstream economics, economic cycles are typically associated with the volatility of investment and the Keynesian multiplier (Burns and Mitchell 1946; Mitchell 1951; Lee 1955; Keynes 2006; McConnel and Brue 2008). The framework for description of these effects is provided by the multiplier-accelerator model (Samuelson 1939). In this model, aggregate demand Y_t as a function of time is defined as

$$Y_t = C_t + I_t = C_0 + cY_{t-1} + I_0 + b(C_{t-1} - C_{t-2}),$$

where consumption C_t is modeled using the simple Keynesian consumption function $C_t = C_0 + cY_{t-1}$ and investment I_t is affected by past consumption via the so called accelerator effect $I_t = I_0 + b(C_{t-1} - C_{t-2})$. The driver of the oscillations in this model can be identified with the accelerator effect. In particular, the accelerator term assumes its maximal value when Y_t passes through the equilibrium point $Y^* = (C_0 + I_0)/(1-c)$, which causes the economy to pass the equilibrium and continue towards either the upward or the downward swing of the cycle.

One can observe that in all existing theories of economic cycles the cycles are associated with conditions that are macroeconomically suboptimal. For example, in multiplier-accelerator model the accelerator mechanism describes a macroeconomical situation in which investments continue to rise even as the economy passes the equilibrium point and moves towards a recession. The theory of inventory cycles (Kitchin 1923) ascribes economic oscillations to time-lags in business decision making. Political business cycle theory (Nordhaus 1975; Nordhaus, Alesina, and Schultze 1989) associates economic oscillations with distortions in the market caused by improper government interventions. The theory of the real business cycle (Kydland and Prescott 1982; Lucas 1977; Plosser 1989) attributes economic oscillations to impact on the market of real exogenous shocks such as disruptive technologies, wars, etc. In general, it is not clear how or whether the macroeconomically inefficient conditions associated with economic cycles can arise microeconomically, especially as the rational microeconomic behaviors are commonly assumed to result in optimal and efficient market systems.

In this paper we demonstrate how macroeconomic oscillations can arise from the behavior of completely rational microeconomic agents, framed as the classical public goods dilemma of the “Tragedy of the commons” (Harding 1968). We consider an extremely simple and first-principled model of a single commodity market shared by several competitive producers. We show that microeconomic competition in these settings can lead to situations reminiscent of the tragedy of the commons, which result in overproduction of the commodity and subsequent market oversaturation and crash. In the tragedy of the commons, several selfish agents are allowed to exploit a common resource (Harding 1968). It had been shown that in these settings the collective outcome of the agents’ individually rational decisions leads to degradation of the resource and even its complete destruction (Harding 1968; Ostrom et al. 2002). In the case of the market model above, we show below that the market itself can play the role of such a common "resource" shared by the producers, and that "overexploitation" of this resource is manifested as commodity overproduction and oversaturation in the market. We subsequently show that accumulation of the overproduced commodity on the market for durable commodities can cause subsequent market depression and market crash, triggering an economic cycle.

As such, the development of this cycle is related to the ability of the overproduced commodity to accumulate on the market. That is, the cycle can develop in the markets of goods that are durable but cannot appear in the markets of nondurable goods or services. Interestingly, this is an otherwise well-known property of real economic cycles, see for example (McConnel and Brue 2008) or Fig. 1A. Furthermore, the cycle is found to produce characteristic patterns in commodity's inventories, which we also succeed in finding in real economies (Fig. 1B). These findings present the problem of business cycles in a new light as an emergent property of market economies, existing in competitive market systems at a very fundamental level.

More specifically, we consider a model of a single commodity competitive market with several producers. The producers choose their production outputs individually and rationally, so as to maximize their individual profits defined conventionally as,

(1) $\pi_i = Y_i \cdot (g(Y;X) - c)$.

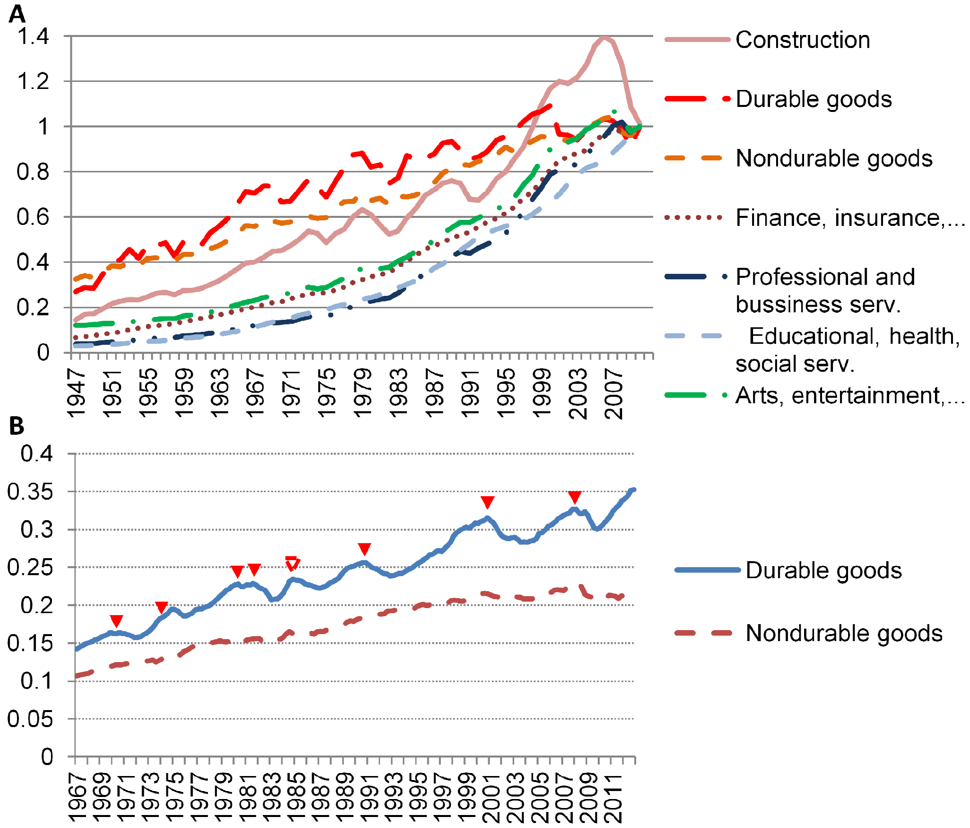

Fig. 1: Business cycles appear prominently throughout economic history and display certain notable patterns. A) Business cycles are known to affect primarily durable goods manufacturing and construction while nondurable goods and services remain essentially unaffected. Graph A shows the value added by different industries in the U.S. economy since 1947, normalized to the year 2010; the difference between durable goods and construction and nondurable goods and services is clearly visible. B) Business cycle exhibits the pattern of inventories accumulation prior to and reduction during the recession part of the cycle. Graph B shows the changes in durable and nondurable goods inventories in the U.S. economy (in trillions of chained 2005 US dollars) during the last 7 recessions. The beginning of each recession is marked with a triangle. The pattern is clearly visible in durable goods but not nondurable goods inventories. Dashed triangle shows one case of the pattern appearing without an official recession. (Source: U.S. Bureau for Economic Analysis.)

Here, Y_i is the production of i^{th} producer, c is the production cost, and $g(Y;X)$ is market's payoff function. We assume that all producers are equal, for simplicity. The payoff function $g(Y;X)$ depends on the total production output $Y = \sum Y_i$ and the total market demand *X*; it is a non-increasing function of supply *Y* and a non-decreasing function of demand *X*. We shall also assume that the market has a finite size, that is, there is a limit on how much commodity can be consumed on the market. This will be identified with the symbol *X*. Note that the assumption of finite market size effectively implies that the consumer utility function is capped at certain value, which is a feature violating the common assumption of monotonicity of consumer preferences typically made for the general equilibrium (Mas-Colell, Whinston, and Green 1995).

We now describe in what sense the above setting creates the tragedy of the commons. As was briefly outlined above, the tragedy of the commons is a public goods dilemma in which a group of players is allowed to exploit a shared resource (a "commons"), which is typically exemplified with a common pasture, fishery, or forest. Each player can choose the level of resource exploitation, independently and rationally according to one's self-interest. The payoff of each such decision is defined by a condition similar to Eq. (1), where Y_i is understood as the resource exploitation by player *i*, c is the cost associated with exploiting the resource, and $g(Y;X)$ is the resource's payoff. An essential property of the tragedy of the commons is that $g(Y;X)$ is assumed to decline with the overall exploitation level *Y*. This is a common situation for many shared resources: increased exploitation of a pasture leads to its degradation, overexploited fishery leads to lower fish reproduction rates, etc. With that assumption, the following property of the tragedy of the commons can be shown: in its Nash equilibrium the resource is necessarily overexploited in the sense that the output extracted by the players is always lower than that maximally attainable (Faysse 2005; Ostrom 1999; Bierman and Fernández 1998; Ostrom et al. 2002; Osborne 1994). (For greater details see Appendix A).

The market model described above is mathematically equivalent to the tragedy of the commons: the producers' gains Y_i are defined by the same Eq. (1) and the payoff $g(P;X)$ is similarly decreasing with total production *Y*. Then, it can be similarly shown that in the Nash equilibrium necessarily $Y > X$, that is, the commodity is overproduced and the market is oversaturated. Mathematically, this result follows from the fact that in the "optimal" configuration $Y = X$ a unilateral increase in the production of any one of the producers Y_i can still lead to an increase in that producer's profits due to increase in the market share $x_i = Y_i / Y$. Clearly, such an increase would come at the cost of the market share and the profits of all the other producers. Therefore, simply in order to maintain parity in the market, all producers are forced to increase their production beyond the level $Y = X$.

A following qualitative example helps to illustrate this point. Let us consider a company A that manufactures a certain product and has an option of selling it on different markets *X* and *X'*. Let us assume that the company is currently dominating market *X* and that market *X'* is currently dominated by company B. Should company A attempt an expansion to market *X'*? The rational answer to this question is always a "yes" as long as the losses from the product to be left unsold on market *X'* can be offset by additional revenue gained from capturing a share of the market of company B. Note that such a decision of company A directly corresponds to oversaturation of market *X'* and overproduction of the product beyond existing demand.

Overproduction is frequently associated with economic recessions (Beaudreau 2004; Grossman and Kennedy 1992; Mattick 1969; McConnel and Brue 2008;

Keynes 2006). It is therefore interesting to find that we already have the situation of overproduction without invoking any additional preconditions. Still, we do not observe that that overproduction here results in a recession; in fact, the production outputs in our model can be observed to converge towards the Nash equilibrium monotonically and no recession occurs. If a recession is to follow, therefore, a different mechanism is needed to trigger the drop in production outputs.

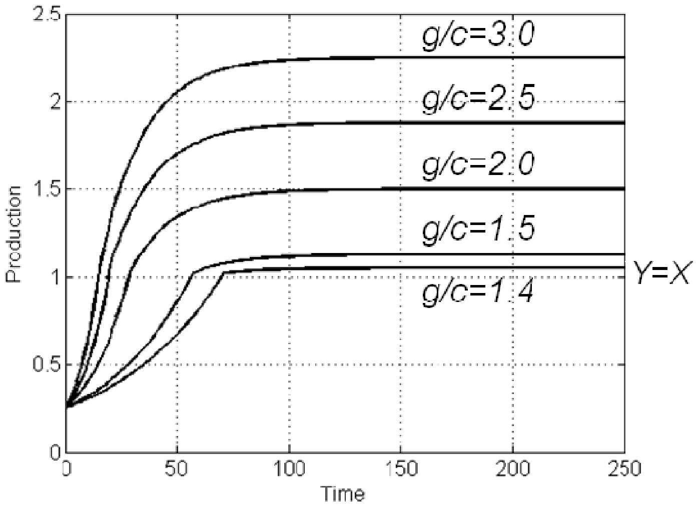

Fig. 2: In a competitive finite market, competition of producers for market shares results in a situation in which commodity gets overproduced and the market becomes oversaturated. Overproduction by itself, however, does not necessarily result in an economic recession, as the model's production outputs can be seen to achieve the equilibrium monotonically. g/c here represents the profit margins with g being the commodity's market price and c its production cost.

We find such a mechanism by observing that simply allowing overproduced commodity to remain and accumulate on the market over an extended period of time suffices to trigger cyclic growth-recession patterns in production outputs.

More specifically, we define the dynamic behaviour of the producers in these settings by the following relations,

$$(2)\quad \begin{aligned} &\frac{\Delta Y_i}{Y_i} = a\cdot(g(Y;\tilde{X}) - c + Y_i \cdot g'(Y;\tilde{X})),\\ &\Delta S = -\alpha\cdot S + (Y - X). \end{aligned}$$

Namely, the relative change in the producers' outputs $\Delta Y_i / Y_i$ is associated with the expected change in producers' profits $d\pi_i / dY_i = g(Y;\tilde{X}) - c + Y_i \cdot g'(Y;\tilde{X})$, where we used the notation $g'(Y;\tilde{X}) = dg(Y;\tilde{X})/dY$. The market size, however, is taken in the form $\tilde{X} = X - S$, where S is the amount of the commodity overproduced and accumulated on the market. The latter point aims to reflect the fact that previously produced and now persisting on the market commodity depresses demand for newly produced products. The second equation is simply the condition of the commodity's accumulation on the market. There, $Y - X$ describes excess in commodity's production and $-\alpha \cdot S$ models commodity's persistence on the market, with α being the fraction of overstocked commodity naturally lost in one time-period.

Different solutions of system (2) are shown in Fig. 3. (For a more detailed analysis interested reader can refer to Appendix B). Depending on the value of the parameter α, the model exhibits three different types of behavior. For large α, the production approach the Nash equilibrium monotonically without oscillations (α=1 in Fig. 3). In this case, the commodity does not accumulate on the market and the Nash equilibrium is directly achieved, as we observed in Fig. 2. This situation can be seen to correspond to the case of nondurable goods or services (no persistence on the market). For α below a certain threshold, however, damped oscillations begin to develop (α=0.1 in Fig. 3) and for yet smaller α the model becomes unstable. In the latter case, cyclic instabilities develop in macroeconomic outputs from any small deviation from the exact equilibrium and persistent nonlinear oscillation ensue (α=0.01 in Fig. 3). Small values of α correspond most evidently to the situation with durable goods or construction (high persistence on the market).

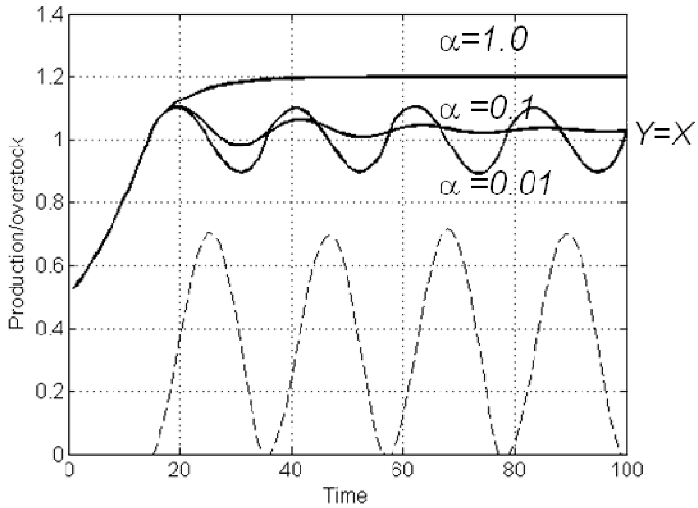

Fig. 3: Cyclic boom-recession patterns in the production outputs develop if overproduced commodity is allowed to accumulate on the market. The cycle is directly related to the ability of the overproduced commodity to accumulate on the market, that is, the cycle develops for the markets of durable goods (small α) but not for nondurable goods (large α). Dashed line shows the evolution of the commodity inventories during the cycle. Note the co-cyclic pattern similar to that in real economies, Fig. 1B.

Therefore, here we demonstrate that macroeconomic cyclic instabilities in activity can emerge already in such a basic setting as a single commodity market shared by several rational competitive producers. The cycle develops from initial overproduction caused by "tragedy of the commons"-style competition of the producers for market shares, followed by market depression due to accumulation of overproduced commodity on the market. The possibility and the severity of such cycle is directly related to the ability of the overproduced commodity to accumulate on the market, that is, the cycle appears for durable goods but does not appear for non-durable goods, otherwise a well-known property of real business cycles. The cycle also produces a specific pattern in commodity inventories, with excess accumulation of commodity immediately before and drop during and after the recession segment of the cycle, a pattern that can be also observed in real economies. Respectively, these findings present the problem of economic cycles in an interesting new light as an emergent property of fully rational and efficient market systems.

Acknowledgements

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Appendix A

The Nash equilibrium in the "Tragedy of the commons"

In the tragedy of the commons, a group of individual players is allowed to exploit a common shared resource, the "commons", with its essential characteristic being that the resource's quality declines with increasing exploitation. The gain of each player from exploiting the resource is defined as $G_i = p_i \cdot (g(P;X) - c)$, where p_i is the exploitation level for player *i*, c is the cost associated with exploiting the resource, and $g(P;X)$ is the resource's payoff function. $P = \sum P_i$ is the total exploitation level of the resource by all agents. The key condition of the tragedy of the commons – the degradation of the resource with increasing exploitation *P* – can be mathematically represented as $g'(P;X) = dg(P;X)/dP < 0$ for all *P*.

The Nash equilibrium in these settings is defined by the condition that neither of the players can increase their individual gain by any unilateral action. This corresponds to

(A1) $dG_i / dp_i = g(P;X) - c + p_i \cdot g'(P;X) = 0$,

where we used the notation $g'(P;X) = dg(P;X)/dP$. On the other hand, the "collectively" optimal configuration corresponding to the maximal total output $G_{tot} = \sum G_i = P \cdot (g(P;X) - c)$ is defined by

(A2) $dG_{tot} / dP = g(P;X) - c + P \cdot g'(P;X) = 0$.

Noting that $P = \sum P_i$, we can then show trivially that in the Nash equilibrium $dG_{tot} / dP = dG_i / dp_i + (P - p_i) g'(P;X) < 0$ as long as $g'(P;X) < 0$. This means that in the Nash equilibrium the total output is decreasing and the resource is overexploited.

Appendix B

Solutions of competitive single commodity market model

We begin this analysis by examining the simplest case of dynamical system (2) in which the commodity's price is assumed to be a constant *g*. Note that even as the commodity's price remains constant the market payoff function $g(Y;X)$ need not. In particular, in the market oversaturation regime $Y \geq X$ the amount of commodity sold on the market saturates resulting in $g(Y;X)Y = gX$ and $g(Y;X) = gX/Y$.

With these assumptions, Eqs. (2) can be then rewritten as the following set of 1st order difference equations,

$$\text{(B1)}\quad \begin{aligned} &\begin{bmatrix} Y_{t+1} \\ S_{t+1} \end{bmatrix} = \begin{bmatrix} 1+a(g-c) & 0 \\ 1 & 1-\alpha \end{bmatrix} \begin{bmatrix} Y_t \\ S_t \end{bmatrix} + \begin{bmatrix} 0 \\ -X \end{bmatrix},\ Y_t < \tilde{X} \\ &\begin{bmatrix} Y_{t+1} \\ S_{t+1} \end{bmatrix} = \begin{bmatrix} 1-ac & -a\tilde{g} \\ 1 & 1-\alpha \end{bmatrix} \begin{bmatrix} Y_t \\ S_t \end{bmatrix} + \begin{bmatrix} a\tilde{g}X \\ -X \end{bmatrix},\ Y_t \geq \tilde{X} \\ &Y_t \geq 0,\ S_t \geq 0 \end{aligned},$$

where $\tilde{g} = (N-1)g/N$.

The behaviour of Eqs. (B1) consists of two regimes $Y < \tilde{X}$ and $Y > \tilde{X}$. The regime $Y < \tilde{X}$ is particularly simple and describes a simple exponential growth. The regime $Y > \tilde{X}$ is more complex and is controlled by the transition matrix

$$\text{(B2)}\quad A = \begin{bmatrix} 1-ac & -a\tilde{g} \\ 1 & 1-\alpha \end{bmatrix},$$

whose properties are defined by the eigenvalues λ determined by the characteristic equation

$$\text{(B3)}\quad \det\begin{vmatrix} 1-\lambda-ac & -a\tilde{g} \\ 1 & 1-\lambda-\alpha \end{vmatrix} = (1-\lambda)^2 - (ac+\alpha)(1-\lambda) + (ac\alpha + a\tilde{g}) = 0\,.$$

Eq. (B3) is a quadratic equation with the solutions

(B4) $\lambda_{1,2} = 1 - \frac{\alpha + ac}{2} \pm \sqrt{\Delta}, \; \Delta = \left(\frac{\alpha - ac}{2}\right)^2 - a\tilde{g}$.

Depending on the value of the commodity's persistence constant α, the solutions of Eqs. (B1) can exhibit three types of behavior. For $\alpha > \alpha_{crit,1} = 2\sqrt{\tilde{g}a} + ca$, both eigenvalues are real and below one, and the corresponding solution is stable and non-periodic (Fig. B1).

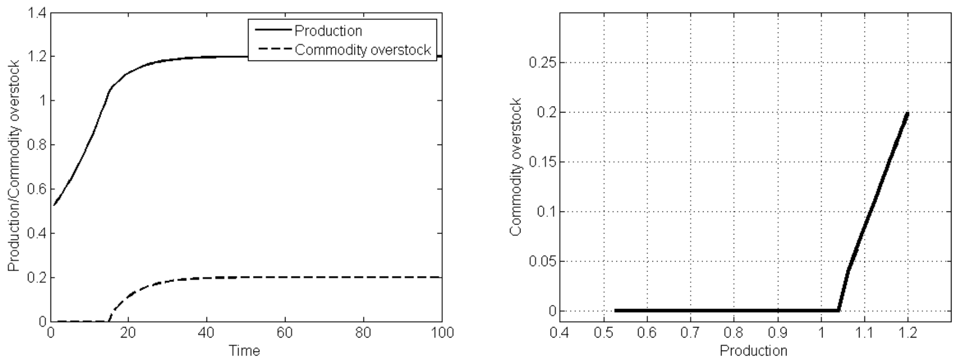

Fig. B1 Example of a non-periodic solution of Eqs. (B1) for $\alpha > \alpha_{crit,1}$. Model parameters a=1/20, g/c=2, market size X=1, N=4 producers, and α=1. Left panel shows the evolution of the production and commodity overstock as a function of time. Right panel shows the same evolution in production-commodity overstock plain.

For $\alpha < \alpha_{crit,1}$, the eigenvalues (B4) become complex and the solution becomes periodic. Depending on the magnitude of $|\lambda_{1,2}|^2 = (1 - \frac{\alpha + ac}{2})^2 + a\tilde{g} - (\frac{\alpha - ac}{2})^2 = 1 - \alpha(1 - ac) + a(\tilde{g} - c)$, however, one of two cases can realize. For $\alpha_{crit,1} > \alpha > \alpha_{crit,2} = a(\tilde{g} - c)/(1 - ac)$, the eigenvalues are smaller than one and the corresponding solution is a damped oscillation. Its decay time-constant can be found as $T_{decay} \approx 2/(\alpha(1-ac)-a(\tilde{g}-c))$ and period constant as $T_{period} \approx 2\pi/\sqrt{-\Delta}$ (Fig. B2).

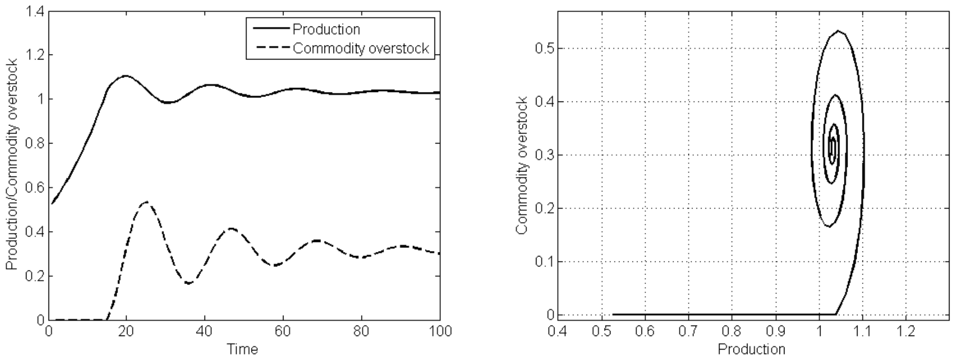

Fig. B2 Example of a damped cycle solution of Eqs. (B1) for $\alpha_{crit,1} > \alpha > \alpha_{crit,2}$. Model parameters are the same as in Fig. B1 and α=0.1.

For $\alpha < \alpha_{crit,2}$, $|\lambda_{1,2}|^2 = 1 + a(\tilde{g}-c) - \alpha(1-ac) > 1$ and the dynamic system (B1) becomes unstable. The consequences of this instability are two-fold. Firstly, the corresponding market model ceases to have a stable equilibrium, that is, the cycles develop from any however small deviations from exact equilibrium. Secondly, the oscillations become nonlinear – the excess commodity is always reduced to zero at some point during the cycle – and the cycle becomes self-sustained, that is, the cycle does not decay out, Fig. B3. The market also assumes mildly chaotic character as can be seen from the cycle in the production-commodity overstock plane in the right panel of Fig. B3.

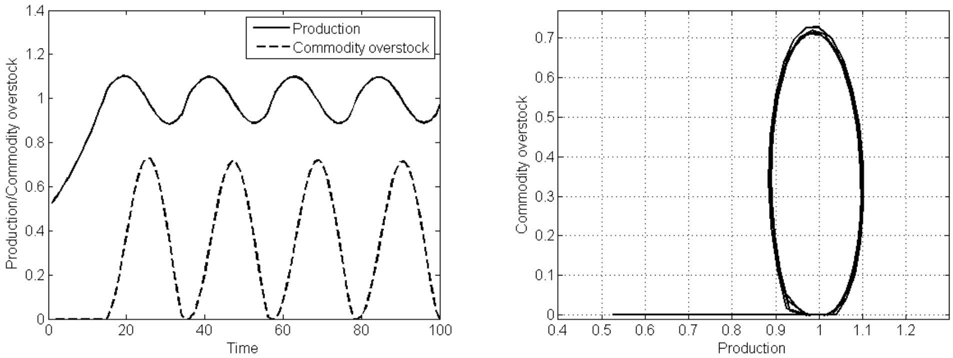

Fig. B3 Example of a nonlinear sustained cycle solution of Eqs. (B1) for $\alpha < \alpha_{crit,2}$. Model parameters are the same as in Fig. B1 and α=0.001.

Oscillations in the markets with flexible price

If the price is not fixed, the above conclusions still hold. In particular, the market payoff function $g(Y;X)$ still is decreasing with *Y*, in fact decreasing now for all *Y*, and the settings necessary for the tragedy of the commons are still realized. Furthermore, in a rational economy we can conclude that the decrease in the price should not be such as to prevent producers from extract greater profits from higher volume of sales as long as demand is not fully satisfied. With this assumption, it can be shown immediately that the Nash equilibrium is necessarily in the market oversaturation regime $Y > X$.

As an example, let us consider the model (2) with a power-law price function $p(Y;X) = g \cdot (X/Y)^{\beta}$. The condition on the growing producers' profits can be mathematically rewritten as $d\pi_{tot}/dY = (1-\beta)g(X/Y)^{\beta} - c \geq 0$, from which we obtain $(1-\beta) \cdot g \geq c$. Eqs. (2) then become

(B5) $$\begin{cases} \begin{bmatrix} Y_{t+1} \\ S_{t+1} \end{bmatrix} = \begin{bmatrix} 1+a(g_\beta \cdot (\tilde{X}/Y_t)^\beta - c) & 0 \\ 1 & 1-\alpha \end{bmatrix} \cdot \begin{bmatrix} Y_t \\ S_t \end{bmatrix} + \begin{bmatrix} 0 \\ -X \end{bmatrix}, \; Y_t < \tilde{X} \\ \begin{bmatrix} Y_{t+1} \\ S_{t+1} \end{bmatrix} = \begin{bmatrix} -ac & -a\tilde{g}_\beta (\tilde{X}/Y_t)^\beta \\ 1 & -\alpha \end{bmatrix} \cdot \begin{bmatrix} Y_t \\ S_t \end{bmatrix} + \begin{bmatrix} a\tilde{g}_\beta (\tilde{X}/Y_t)^\beta X \\ -X \end{bmatrix}, \; Y_t \geq \tilde{X} \end{cases},$$

where $g_\beta = (1-\beta/N)\cdot g$ and $\tilde{g}_\beta = (1-(\beta+1)/N)\cdot g$. Using the above inequality $(1-\beta)\cdot g \geq c$, we show from Eqs. (B5) that ΔY is strictly positive for $\tilde{X}/Y \geq 1$, implying that the equilibrium can only be achieved in the oversaturation region $Y > \tilde{X}$, as expected. The details of the cycle, however, do depend on the specific choice of the price function *p*.

Oscillations in the markets with elastic size

We now consider the above market model where the market size *X* is not constant but is related to past consumers' income that is in turn affected by the past production output. To incorporate this effect, we take the market size *X* in the form $X = X + g^{-1}\gamma c Y$, where $\gamma c Y$ is the consumers' income corresponding to production *Y*, γ is a constant related to the marginal propensity to consume as well as the wages contribution to the production cost *c*, and g^{-1} describes the consumer's income in the units of the commodity of price *g*. Eqs. (B1) then become,

(B6) $$\begin{cases} \begin{bmatrix} Y_{t+1} \\ S_{t+1} \end{bmatrix} = \begin{bmatrix} 1+a(g-c) & 0 \\ 1-\gamma c/g & 1-\alpha \end{bmatrix} \cdot \begin{bmatrix} Y_t \\ S_t \end{bmatrix} + \begin{bmatrix} 0 \\ -X \end{bmatrix}, \; Y_t < \tilde{X} \\ \begin{bmatrix} Y_{t+1} \\ S_{t+1} \end{bmatrix} = \begin{bmatrix} 1+ac\tilde{\gamma}-ac & -a\tilde{g} \\ 1-\gamma c/g & 1-\alpha \end{bmatrix} \begin{bmatrix} Y_t \\ S_t \end{bmatrix} + \begin{bmatrix} a\tilde{g}X \\ -X \end{bmatrix}, \; Y_t \geq \tilde{X} \end{cases},$$

where $\tilde{\gamma} = (N-1)/N\gamma$. The characteristic equation (B3) then becomes

(B7) $$(1-\lambda)^2 - (ac(1-\tilde{\gamma})+\alpha)(1-\lambda) + ac(1-\tilde{\gamma}) + a\tilde{g} = 0\,.$$

Comparing Eq. (B7) with Eq. (B3), it can be observed that the effect of the elasticity in the market size is to aggravate the oscillations by reducing $ac \to ac(1-\tilde{\gamma})$, Fig. B4, as is also well known from the studies of the effect of Keynesian multiplier on the economic recessions.

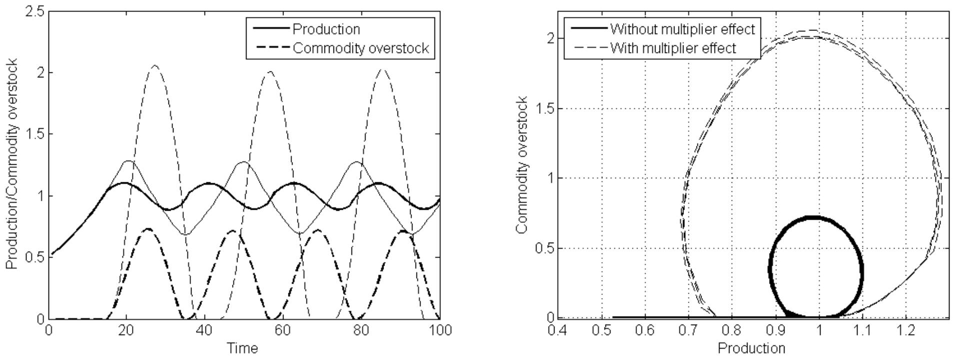

Fig. B4 Aggravation of the cycle due to elasticity in the market demand. Original model, Eqs. (B1), is shown with thick lines and the model with the multiplier effect, Eqs. (B7), is shown with thin lines. Model parameters are the same as in Fig. B3 and γ=0.9.

Identification of the constant a

Constant *a* in the difference equations (B1) is an important constant representing the flexibility of businesses in expanding or shrinking their production. For simplicity, we assumed here this constant to be the same for both expansion and shrinking, although this needs not be the case in reality. This constant also may depend on the production size *Y* as well as other factors, in principle.

To get a rough sense of the nature and the magnitude of this constant, we can consider the model (B1) in the regime $Y << X$, where the equation $\Delta Y = a(g-c)Y$ can be reinterpreted as the condition of complete reinvestment of profits by businesses in a rapidly growing economy, $\Delta Y = \frac{(g-c)Y}{\kappa}$, where κ is the fixed capital necessary to increase commodity's production by one unit. By comparing the two equations we readily identify $a \approx \kappa^{-1}$. The key quantities ag and ac then are identified with the ratios $ag \approx g/\kappa$ and $ac \approx c/\kappa$, and the period of the cycles, decay time-constant, and critical values can be also written as

$$
\text{(B8)} \quad
\begin{aligned}
T_{period} &= 4\pi / \sqrt{4\tilde{g}/\kappa - (\alpha - c/\kappa)^2} \\
T_{decay} &= 2/(\alpha(1 - c/\kappa) - (\tilde{g}/\kappa - c/\kappa)) \\
\alpha_{crit,1} &= c/\kappa + 2\sqrt{\tilde{g}/\kappa} \\
\alpha_{crit,2} &= (\tilde{g}/\kappa - c/\kappa)/(1 - c/\kappa)
\end{aligned}
.
$$