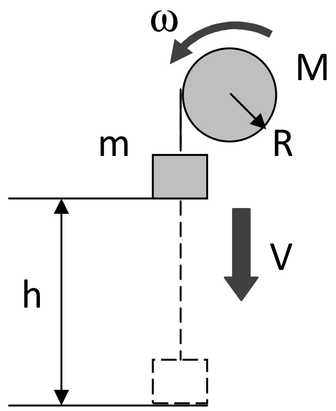
FINAL PROBLEM PRACTICE SET

This is the problem practice set for the topics after the 2nd midterm exam. These include rotation and rolling motion, oscillatory motion, and waves. Also use the **practice sets I and II** to prepare for earlier topics: *the final exam will cover all topics in this semester*. Be sure also to review all of the example problems solved in the class; any of these can appear in the final exam.

Problem 1: A box of mass *m* falls from height *h* while being connected by a rope to a heavy pulley. The pulley is a solid disk of radius *R* and mass *M*. Assuming there is *no slipping* between the rope and the pulley, what will be the speed of the box at the end of the fall? (Use the energy conservation to solve this problem; note that while the box is falling, the pulley also has to rotate at specific speed to make sure that the rope is not slipping, which will require kinetic energy!)



Problem 2: Consider a car that is accelerating from rest. The engine of the car applies torque τ *to each of the four wheels* of the car. What is the acceleration of the car? Assume the wheels' radius is *R*, the car's mass is *M* and the moment of inertia of each wheel is I.

To answer this question:

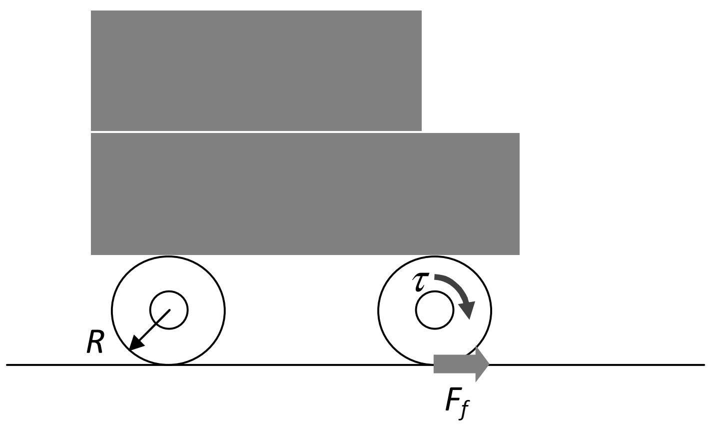
(a) Write down the total torque for each of the wheels from the engine and the friction force F_f.

(b) Write down the total force applied to the car as a whole (in the horizontal direction).

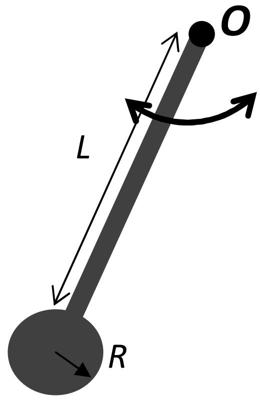
(c) Write down the 2nd Newton's law for the car's linear acceleration and for the angular acceleration of the wheels. Note that the rotational motion of all wheels is the same, so you only have one equation to describe rotation of all four wheels.

(d) Write down the no-slipping condition for the car's linear speed and acceleration and the wheels' rotational speed and accelerations, respectively

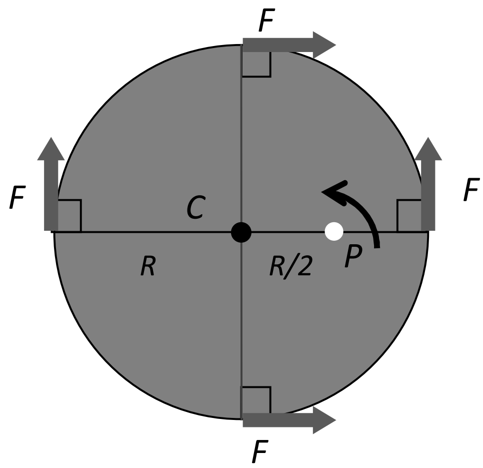
(e) Solve obtained system of equations to find car's acceleration.



Problem 3: A pendulum of one clock is made of a heavy thin steel rod of mass *m* and length *L* connected to a heavy steel ball of mass *M* and radius *R*. What is the oscillation period of this pendulum?

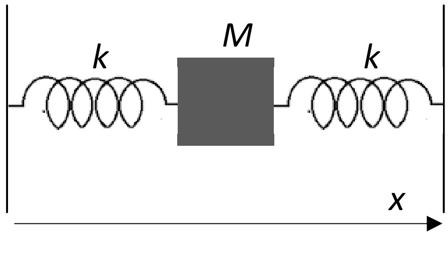


Problem 4: Four forces are applied to a disk of radius *R* and mass *M* rotating around a pivot point *P* at distance *R*/2 from the disk's center *C*, as shown. All forces have equal magnitude *F* and are at 90° to disk's radii. What is the angular acceleration of the disk?



5. A heavy ball of mass *M* moving at speed *V* hits a massless spring of spring constant *k*, and sticks to it. What is the magnitude of the resulting oscillations of the spring?

6. A block of mass *M* is attached to two identical springs of spring constant *k*. Write down the 2^{nd} Newton's law for the block at different horizontal positions *x*. Use this and the equation of motion for harmonic oscillations, as shown in the class, to find the frequency of the oscillations of this block.

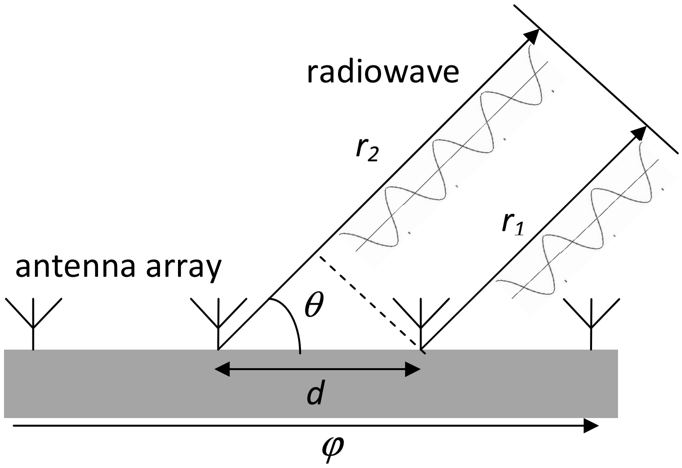


7. Five lamps hang from a ceiling in a store. All lamps can be treated as pendulums with oscillation periods T=1 sec, 2 sec, 4 sec, 6 sec and 10 sec. During an earthquake the ceiling shakes with *linear* frequency *f*=0.24 Hz. If the amplitude of driven oscillations of the first lamp was 1 cm, what were the amplitudes of driven oscillations of all other lamps? Which of the lamps will break down and fall during this earthquake?

8. In a certain material the speed of sound is V=2 km/s. For a sound wave with *linear* frequency f=1000Hz, find all of the following (especially pay attention here to which formulas you used and which of the wave quantities they bind).

(a) Oscillation period of this wave;
(b) Wavelength of this wave;
(c) Angular frequency of this wave;
(d) Wave number of this wave.

9. A phased array is an antenna for creating directed radio beams which can be rotated rapidly without having to rotate the antenna itself; it is made of an array of point-like antennas separated by distance d, producing radio waves shifted by a phase shift φ. Accordingly, the radio waves in direction θ from any two such antennas can be described by formulas $y_1 = A\sin(kr_1 - \omega t)$, $y_2 = A\sin(kr_2 - \omega t + \varphi)$, where φ is the phase shift in the phased array. Prove that the interference of these two waves results in the largest radio signal in the direction for which $2\pi d/\lambda \cos\theta = \varphi$, where λ is the wavelength of the wave. If you want to produce radio signal in direction θ=45° what phase shift φ should you use? Same question for direction θ=90°?



10. You see different colors in the light reflected from oil on water because of waves' interference. In this case, the interfering waves are that reflected from the upper and the lower surface of the oil layer (figure below). These two waves can be described using $y_1 = A\sin(kr - \omega t - \pi)$ and $y_2 = A\sin(kr - \omega t + 2\pi n\left(d + \frac{d}{\sin\theta}\right)/\lambda)$. Here d is the thickness of the oil layer, n≈1.5 is the index of refraction for oil. The phase shift in the second wave is because this wave has to travel more up and down through the oil. For the light of wavelength λ find the first angle θ at which the reflected waves result in positive (constructive) interference. Light of different colors has different wavelengths, red has larger wavelength λ and violet has smaller λ. Using this fact and your answer above, why do you think you see the rainbow pattern from such an oil layer?