PABLO

FUNDAMENTALS OF ELECTRICAL ENGINEERING.

LECTURE 3 NOTES

N.T DUAH

SINUSOIDAL ALTERNATING WAVEFORMS

- Introduction

The term alternating indicates only that the waveform alternates between two prescribed levels in a set time sequence (Fig. 3.1).

SINUSOIDAL ALTERNATING WAVEFORMS

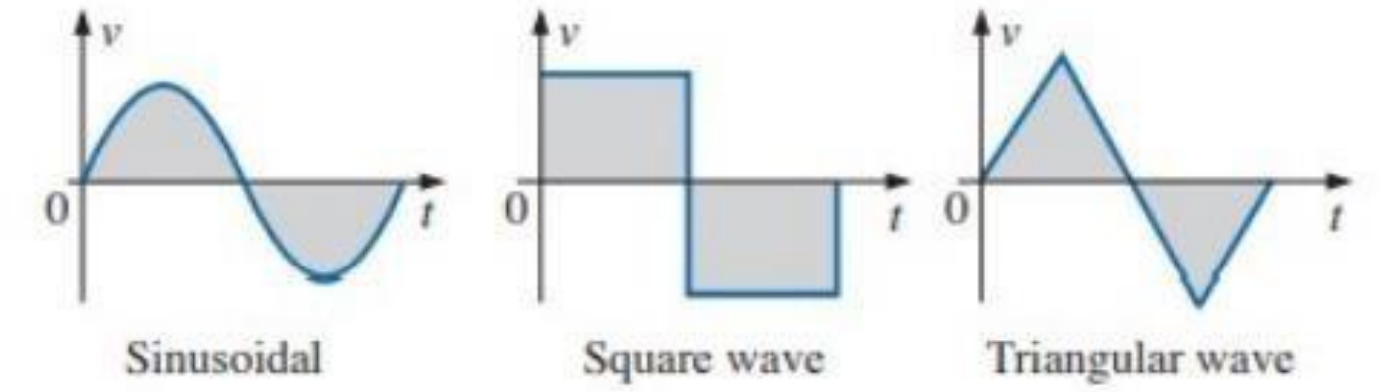


Figure 3.1: Alternating waveforms

- Sinusoidal AC Voltage

Sinusoidal ac voltages are available from a variety of sources. The most common source is the typical home outlet, which provides an ac voltage that originates at a power plant; such a power plant is most commonly fueled by water power, oil, gas, or nuclear fusion.

The sinusoidal waveform of Fig. 3.3 with its additional notation will now be used as a model in defining a few basic terms.

SINUSOIDAL ALTERNATING WAVEFORMS

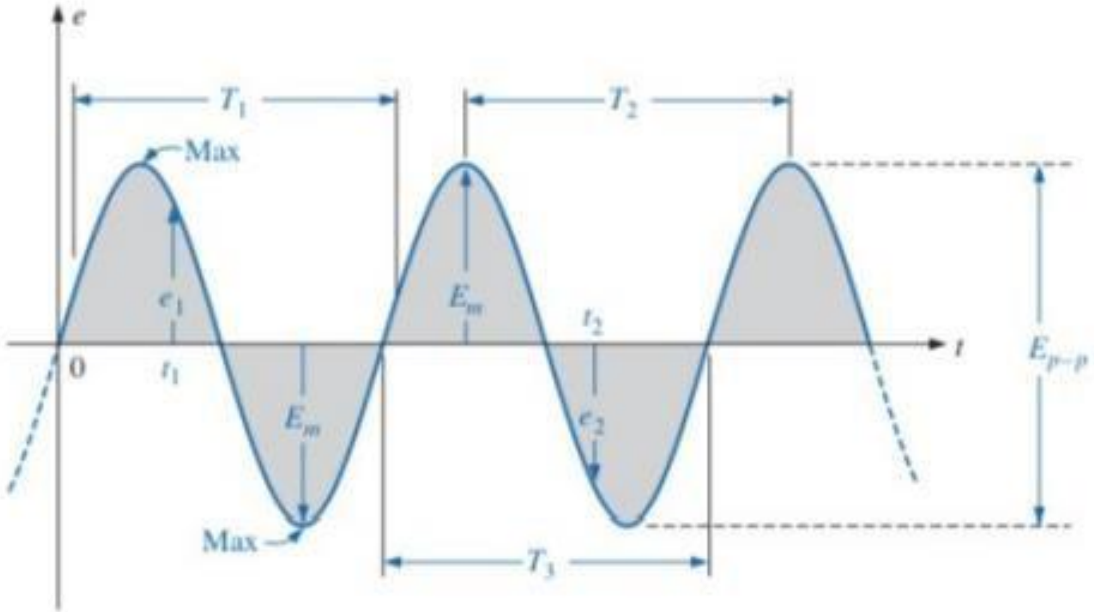


Figure 3.3: Important parameters for a sinusoidal voltage.

- Important Parameters for Sinusoidal Voltage

Waveform: The path traced by a quantity plotted as a function of some variable such as time, position, degrees, radians, temperature, and so on.

Instantaneous value: The magnitude of a waveform at any instant of time; denoted by lowercase letters (e1, e2).

Peak amplitude: The maximum value of a waveform as measured from its average, or mean, value, denoted by uppercase letters (such as E_m for sources of voltage and V_m for the voltage drop across a load). For the waveform of the average value is zero volts, and E_m is as defined by the figure.

Peak value: The maximum instantaneous value of a function as measured from the zero-volt level. For the waveform of the peak amplitude and peak value are the same, since the average value of the function is zero volts.

Peak-to-peak value: Denoted by E_{p-p} or V_{p-p}, the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks.

Periodic waveform: A waveform that continually repeats itself after the same time interval.

Period (T): the periodic function is the time of one complete cycle or the number of seconds per cycle.

Frequency (f): The number of cycles that occur in 1s. The reciprocal of period is frequency.

- Sinusoidal Wave

A sinusoid is a signal that has the form of the sine or cosine function.

The unit of measurement for the horizontal axis is the degree. A second unit of measurement frequently used is the radian (rad). The conversion equations between the two are the following:

$$\text{Radians} = \left(\frac{\pi}{180^\circ}\right) \times (\text{degrees})$$

$$\text{Degrees} = \left(\frac{180^\circ}{\pi}\right) \times (\text{radians})$$

SINUSOIDAL ALTERNATING WAVEFORMS

The velocity with which the radius vector rotates about the center, called the angular velocity, can be determined from the following equation:

$$\text{Angular velocity} = \frac{\text{distance (degrees or radians)}}{\text{time (seconds)}} \qquad \omega = \frac{\alpha}{t}$$

Since ω is typically provided in radians per second, the angle obtained is usually in radians. The time required to complete one revolution is equal to the period (T) of the sinusoidal waveform.

The radians subtended in this time interval are 2Π. Substituting, we have

$$\boxed{\omega = \frac{2\pi}{T}}$$

If f = 1/T. Thus

$$\boxed{\omega = 2\pi f} \qquad \text{(rad/s)}$$

- General Format for Sinusoidal Voltage or Current.

The basic mathematical format for the sinusoidal waveform is

$A_m \sin\alpha$

where A_m is the peak value of the waveform and a is the unit of measure for the horizontal axis, as shown in Fig. 3.6.
The general format of a sine wave can also be written

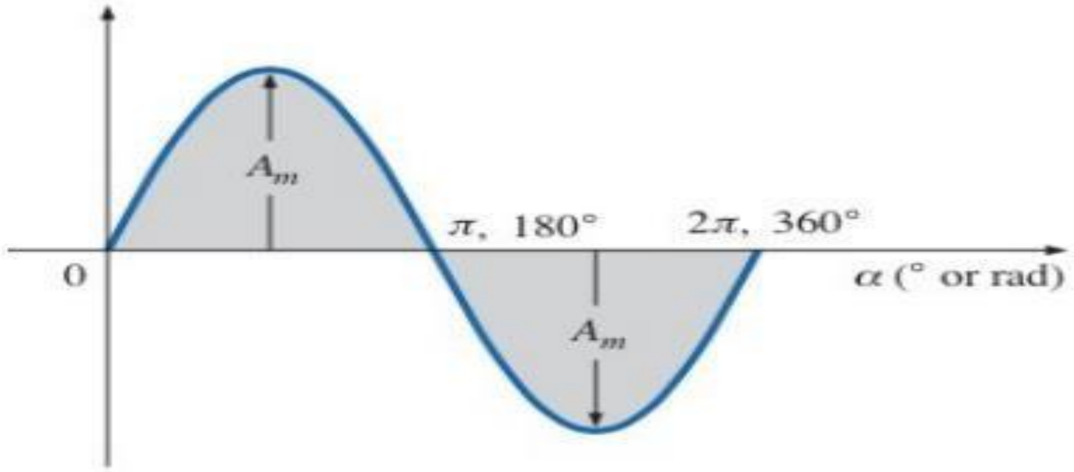


Figure 3.6:

SINUSOIDAL ALTERNATING WAVEFORMS

$$A_m \sin\omega t$$

For electrical quantities such as current and voltage, the general format is

$$i = I_m \sin\omega t = I_m \sin\alpha \quad e = E_m \sin\omega t = E_m = E_m \sin\alpha$$

- Example

Find the amplitude, phase, period, and frequency of the sinusoid

$$v(t) = 12\cos(50t + 10^\circ)$$

Solution:

The amplitude is $V_m = 12$ V.

The phase is $\phi = 10^\circ$.

The angular frequency is $\omega = 50$ rad/s.

The period $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{50} = 0.1257$ s.

The frequency is $f = \dfrac{1}{T} = 7.958$ Hz.

- Exercise

Given the sinusoid $5\sin(4\pi t - 60^\circ)$, calculate its amplitude, phase, angular frequency, period, and frequency.

Answer: 5, -60°, 12.57 rad/s, 0.5 s, 2 Hz.

- Introduction to Phasors

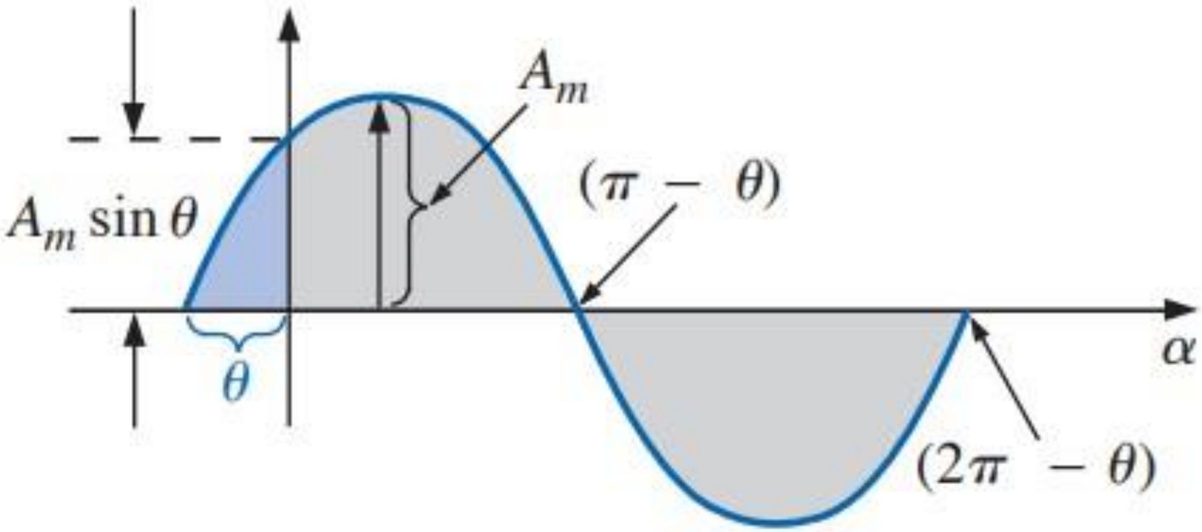
Thus far, we have considered only sine waves that have maxima at $\pi/2$ and $3\pi/2$, with a zero value at 0, π, and 2π. If the waveform is shifted to the right or left of 0°, the expression becomes

$$\boxed{A_m \sin(\omega t \pm \theta)}$$

where θ is the angle in degrees or radians that the waveform has been shifted.

If the waveform passes through the horizontal axis with a positive-going (increasing with time) slope before 0°,

SINUSOIDAL ALTERNATING WAVEFORMS

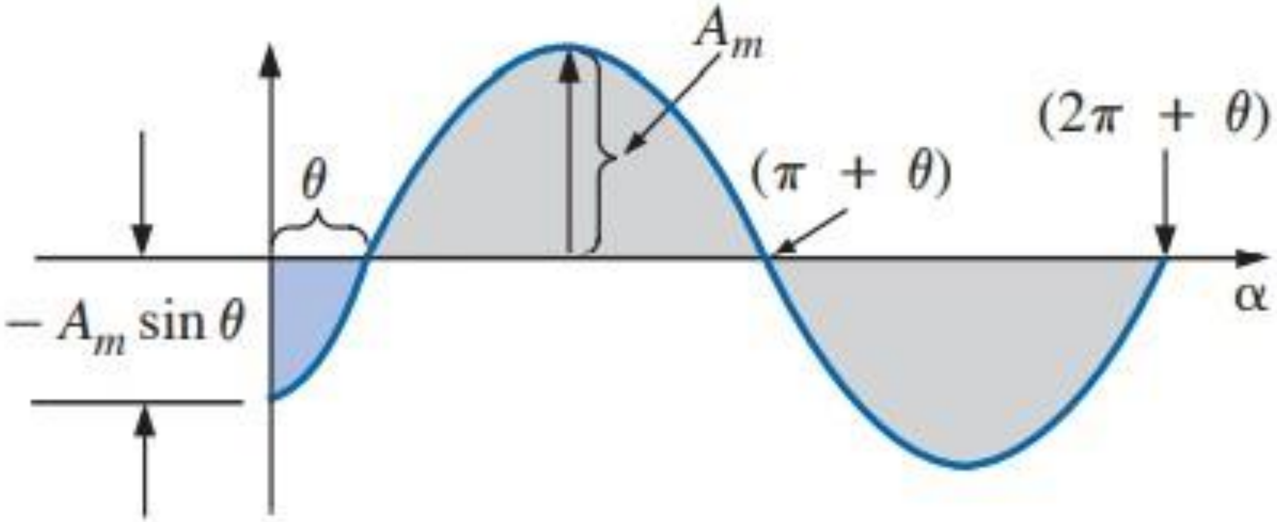


$$\boxed{A_m \sin(\omega t + \theta)}$$

At ωt = α = 0°, the magnitude is determined by $A_m$sinθ. If the waveform passes through the horizontal axis with a positivegoing slope after 0°, the expression is

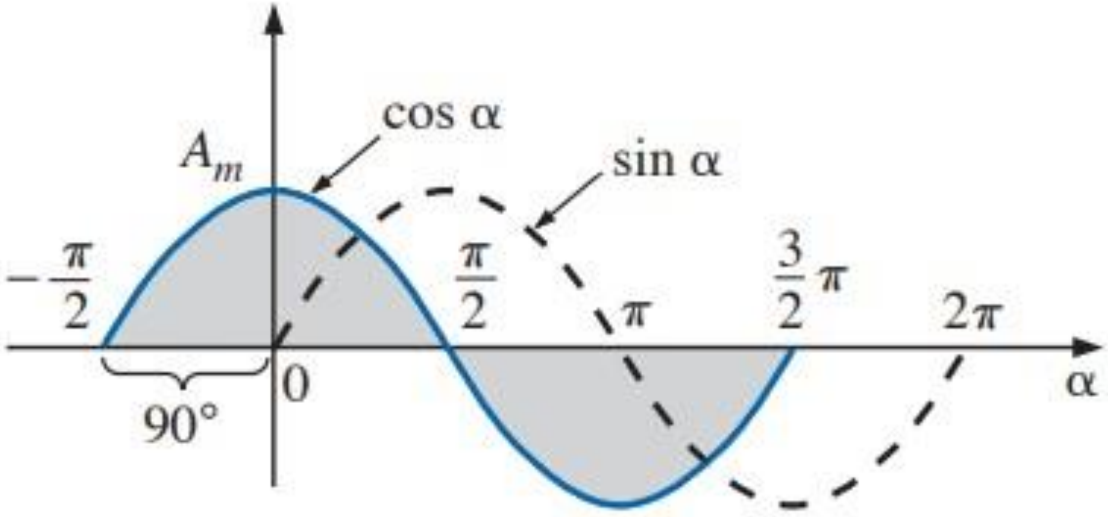
SINUSOIDAL ALTERNATING WAVEFORMS

$$\boxed{A_m \sin(\omega t - \theta)}$$



If the waveform crosses the horizontal axis with a positivegoing slope 90° ($\pi/2$) sooner, it is called a cosine wave; that is,

SINUSOIDAL ALTERNATING WAVEFORMS



$$\sin(\omega t + 90°) = \sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t$$

- Average value

The average or mean value of a symmetrical alternating quantity, (such as a sine wave), is the average value measured over a half cycle, (since over a complete cycle the average value is zero).

Average or mean value = (area under the curve)/(length of base)

The procedure of calculus that gives the exact solution is known as integration.

SINUSOIDAL ALTERNATING WAVEFORMS

$$\text{Area} = \int_0^{\pi} A_m \sin\alpha \, d\alpha$$

The average of a sinusoidal signal is usually take for a half-cycle, thus $T=\pi$. Thus the average of $x(t) = X_m \sin\theta d\theta$

$$
\begin{aligned}
X_0 &= \frac{1}{\pi}\int_0^{\pi} X_m \sin\theta d\theta \\
&= \frac{1}{\pi}X_m(-\cos\theta)_0^{\pi} \\
&= \frac{1}{\pi}X_m(\cos\theta)_{\pi}^{0} \\
&= \frac{1}{\pi}X_m(\cos 0 - cos\pi) \\
&= \frac{2}{\pi}X_m \\
&= 0.637X_m
\end{aligned}
$$

- Effective(rms) values

The effective value of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current. The effective value of a periodic signal is its root mean square (rms) value.

Thus the rms of x(t) = $X_m \sin\theta d\theta$

$$
\begin{aligned}
X_{rms} = &= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} X_m^2 \sin^2\theta d\theta} \\
&= \sqrt{\frac{X_m^2}{2\pi}\int_0^{2\pi}\left(\frac{1}{2}(1-\cos 2\theta d\theta\right)} \\
&= \sqrt{\frac{X_m^2}{4\pi}\left(\theta-\frac{\sin 2\theta}{2}\right)_0^{2\pi}} \\
&= \sqrt{\frac{X_m^2}{4\pi}\left(2\pi-0-\frac{(\sin 4\pi-\sin 0)}{2}\right)} \\
&= \sqrt{\frac{X_m^2}{4\pi}(2\pi)} = \sqrt{\frac{X_m^2}{2}} \\
&= \frac{X_m}{\sqrt{2}} = 0.707 X_m
\end{aligned}
$$

- Response of Basic R,L and C Elements

- **Resistor**

For power-line frequencies and frequencies up to a few hundred kilohertz, resistance is, for all practical purposes, unaffected by the frequency of the applied sinusoidal voltage or current.

Ohm's law can be applied as follows. For $v = V_m \sin \omega t$,

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

where

$$\boxed{I_m = \frac{V_m}{R}}$$

In addition, for a given i,

$$v = iR = (I_m \sin \omega t)R = I_m R \sin \omega t = V_m \sin \omega t$$

where $$\boxed{V_m = I_m R}$$

for a purely resistive element, the voltage across and the current through the element are in phase, with their peak values related by Ohm's law.

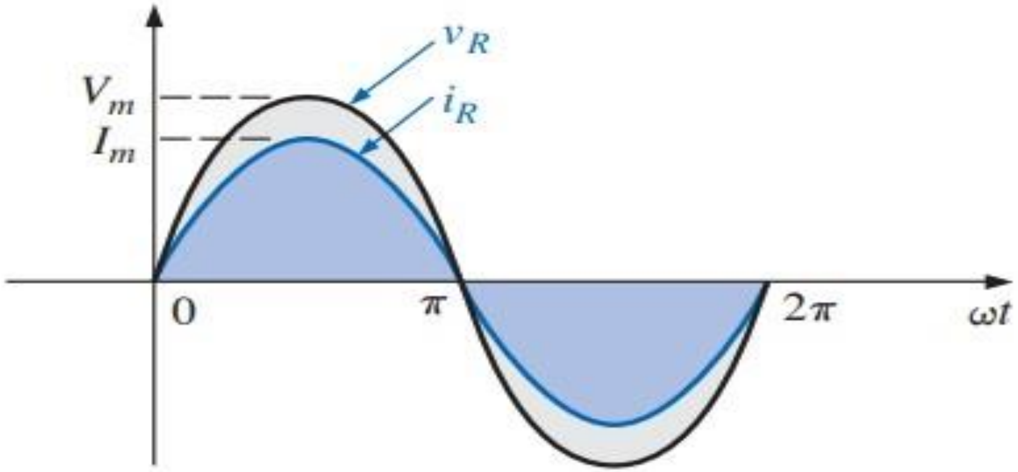


FIG. 14.5
The voltage and current of a resistive element are in phase.

○ Inductor

- The voltage across an inductor is directly related to the rate of change of current through the coil.
- The higher the inductance, the greater the rate of change of the flux linkages, and the greater the resulting voltage across the coil.
- The inductive voltage, therefore, is directly related to the frequency (or, more specifically, the angular velocity of the

sinusoidal ac current through the coil) and the inductance of the coil.

For the inductor,

$$v_L = L\,\frac{di_L}{dt}$$

and, applying differentiation,

$$\frac{di_L}{dt} = \frac{d}{dt}(I_m \sin \omega t) = \omega I_m \cos \omega t$$

Therefore, $\quad v_L = L\,\dfrac{di_L}{dt} = L(\omega I_m \cos \omega t) = \omega L I_m \cos \omega t$

or $\quad v_L = V_m \sin(\omega t + 90°)$

where $\quad V_m = \omega L I_m$

SINUSOIDAL ALTERNATING WAVEFORMS

For an inductor, v_L leads i_L by 90°, or i_L lags v_L by 90°.

SINUSOIDAL ALTERNATING WAVEFORMS

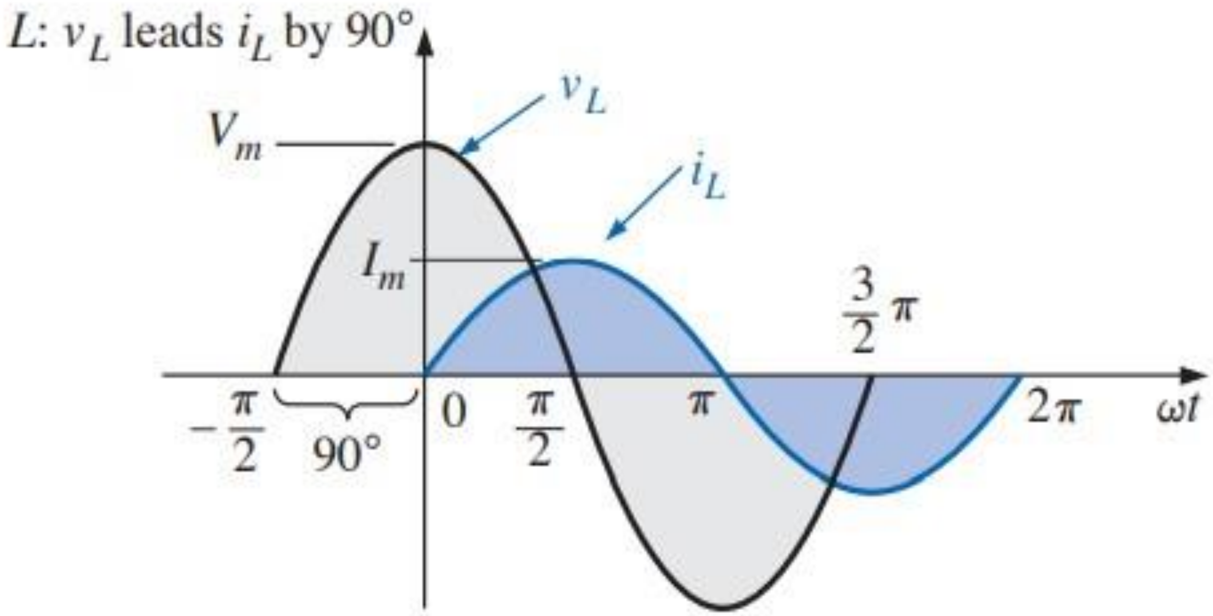


FIG. 14.9

For a pure inductor, the voltage across the coil leads the current through the coil by 90°.

The quantity ωL, called the reactance (from the word reaction) of an inductor, is symbolically represented by X_L and is measured in ohms; that is,

$$\boxed{X_L = \omega L} \qquad \text{(ohms, } \Omega\text{)}$$

In an Ohm's law format, its magnitude can be determined from

$$\boxed{X_L = \frac{V_m}{I_m}} \qquad \text{(ohms, } \Omega\text{)}$$

Capacitor

- For capacitive networks, the voltage across the capacitor is limited by the rate at which charge can be deposited on, or released by, the plates of the capacitor during the charging and discharging phases, respectively.

Also, for a particular capacitance, the greater the rate of change of voltage across the capacitor, the greater the capacitive current.

- ✦The current of a capacitor is therefore directly related to the frequency (or, again more specifically, the angular velocity) and the capacitance of the capacitor.

For the Capacitor,

$$i_C = C\,\frac{dv_C}{dt}$$

and, applying differentiation,

$$\frac{dv_C}{dt} = \frac{d}{dt}(V_m \sin \omega t) = \omega V_m \cos \omega t$$

Therefore,

$$i_C = C\,\frac{dv_C}{dt} = C(\omega V_m \cos \omega t) = \omega C V_m \cos \omega t$$

or $$i_C = I_m \sin(\omega t + 90°)$$

where $$I_m = \omega C V_m$$

For a capacitor, i_C leads v_C by 90°, or v_C lags i_C by 90°.

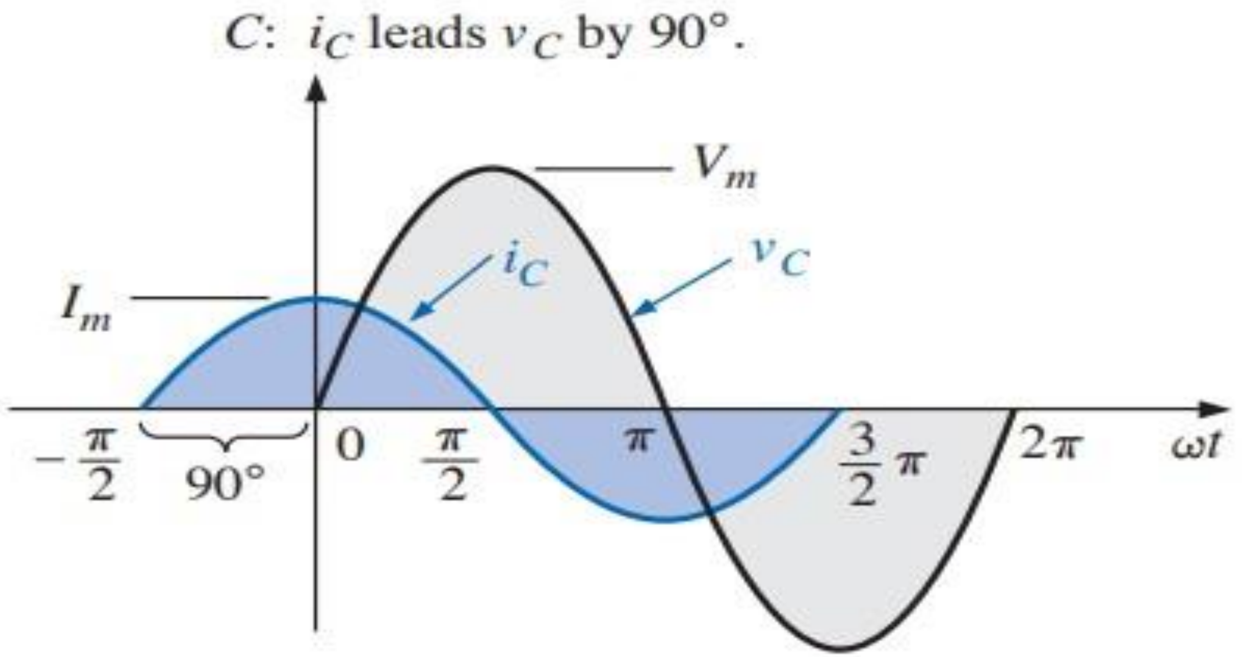


FIG. 14.12

The current of a purely capacitive element leads the voltage across the element by 90°.

The quantity $1/\omega C$, called the reactance of a capacitor, is symbolically represented by X_C and is measured in ohms; that is,

$$\boxed{X_C = \frac{1}{\omega C}} \qquad \text{(ohms, } \Omega\text{)}$$

In an Ohm's law format, its magnitude can be determined from

$$\boxed{X_C = \frac{V_m}{I_m}} \qquad \text{(ohms, } \Omega\text{)}$$

TABLE 9.2 Summary of voltage-current relationships.

Element	Time domain	Frequency domain
R	$v = Ri$	$\mathbf{V} = R\mathbf{I}$
L	$v = L\dfrac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$
C	$i = C\dfrac{dv}{dt}$	$\mathbf{V} = \dfrac{\mathbf{I}}{j\omega C}$

If the source current leads the applied voltage, the network is predominantly capacitive, and if the applied voltage leads the source current, it is predominantly inductive.

- Example

The voltage across a resistor is indicated. Find the sinusoidal expression for the current if the resistor is 10Ω. Sketch the curves for v and i.

a. $v = 100 \sin 377t$

b. $v = 25 \sin(377t + 60°)$

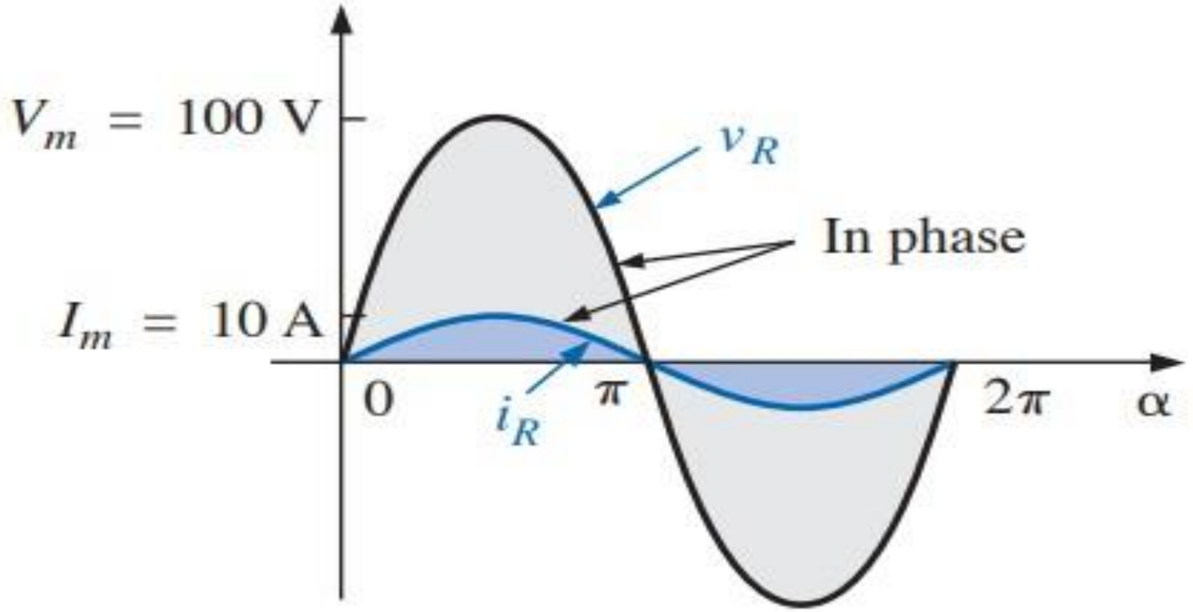
✦Solution

(a) $$I_m = \frac{V_m}{R} = \frac{100\ \text{V}}{10\ \Omega} = 10\ \text{A}$$

(v and i are in phase), resulting in

$$\mathbf{i = 10 \sin 377t}$$

SINUSOIDAL ALTERNATING WAVEFORMS



- Exercise

The current through a 0.1-H coil is provided. Find the sinusoidal expression for the voltage across the coil. Sketch the v and i curves.

a. $i = 10 \sin 377t$

b. $i = 7 \sin(377t - 70°)$

- Introduction to Complex Numbers

A complex number z can be written in rectangular form as

$$z = x + jy$$

where $j = \sqrt{-1}$; x is the real part of z; y is the imaginary part of z.

The complex number z can also be written in polar

or exponential form $z = r\angle\phi = re^{j\phi}$ as where r is

the magnitude of z, and ϕ is the phase of z.

We notice that z can be represented in three ways:

$z = x + jy$ Rectangular form

$z = r\angle\phi$ Polar form

$z = re^{j\phi}$ Exponential form

Given x and y, we can get r and ϕ as

$$r = \sqrt{x^2 + y^2}, \qquad \phi = \tan^{-1}\frac{y}{x}$$

On the other hand, if we know r and ϕ, we can obtain x and y as

$$x = r\cos\phi, \qquad y = r\sin\phi$$

Thus, z may be written as

$$\boxed{z = x + jy = r\angle\phi = r(\cos\phi + j\sin\phi)}$$

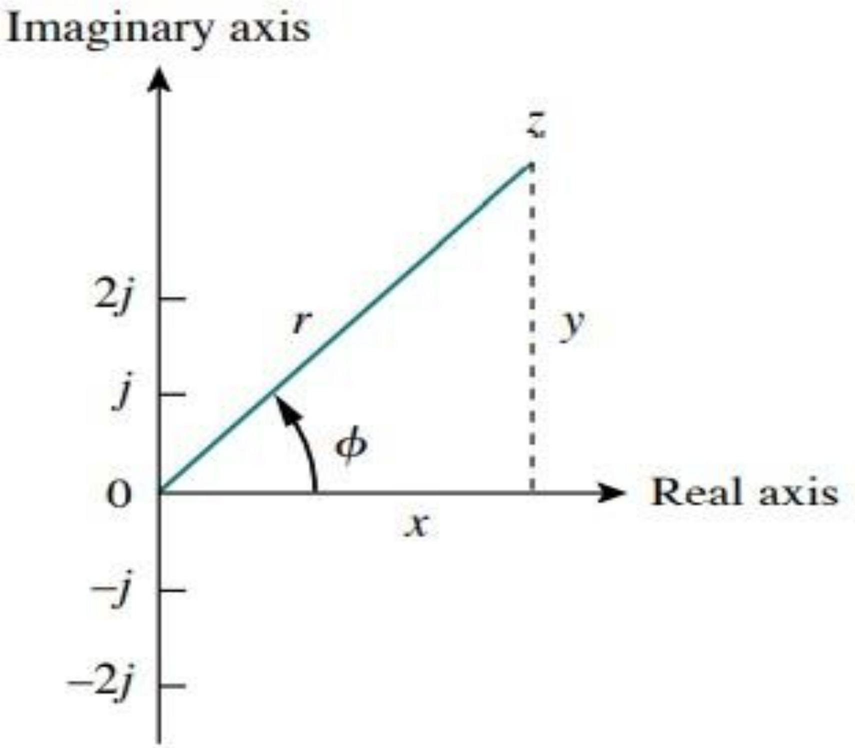


Figure 9.6 Representation of a complex number $z = x + jy = r\angle\phi$.

Addition and subtraction of complex numbers are better performed in rectangular form; multiplication and division are better done in polar form. Given the complex numbers

$$z = x + jy = r\angle\phi, \qquad z_1 = x_1 + jy_1 = r_1\angle\phi_1$$
$$z_2 = x_2 + jy_2 = r_2\angle\phi_2$$

the following operations are important.

Addition:

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

Subtraction:

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

Multiplication:

$$z_1 z_2 = r_1 r_2 \underline{/\phi_1 + \phi_2}$$

Division:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \underline{/\phi_1 - \phi_2}$$

Reciprocal:

$$\frac{1}{z} = \frac{1}{r} \underline{/- \phi}$$

Square Root:

$$\sqrt{z} = \sqrt{r} \underline{/\phi/2}$$

Complex Conjugate:

$$z^* = x - jy = r\angle -\phi = re^{-j\phi}$$

The idea of phasor representation is based on Euler's identity. In general,

$$\boxed{e^{\pm j\phi} = \cos\phi \pm j\sin\phi}$$

which shows that we may regard cos ϕ and sin ϕ as the real and imaginary parts of $e^{j\phi}$; we may write

$$\cos\phi = \mathrm{Re}(e^{j\phi})$$
$$\sin\phi = \mathrm{Im}(e^{j\phi})$$

where R_e and I_m stand for the real part of and the imaginary part of. Given a sinusoid $v(t) = V_m \cos(\omega t + \phi)$,

$$v(t) = V_m \cos(\omega t + \phi) = \mathrm{Re}(V_m e^{j(\omega t+\phi)})$$

or

$$v(t) = \mathrm{Re}(V_m e^{j\phi} e^{j\omega t})$$

Thus,

$$\boxed{v(t) = \mathrm{Re}(\mathbf{V}e^{j\omega t})}$$

where

$$\mathbf{V} = V_m e^{j\phi} = V_m \angle \phi$$

Given a phasor, we obtain the time-domain representation as the cosine function with the same magnitude as the phasor and the argument as ωt plus the phase of the phasor.

TABLE 9.1 Sinusoid-phasor transformation.

Time-domain representation	Phasor-domain representation
$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	$V_m \angle \phi - 90^\circ$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle \theta - 90^\circ$

- Example

Transform these sinusoids to phasors:

(a) $v = -4\sin(30t + 50^\circ)$

(b) $i = 6\cos(50t - 40^\circ)$

Solution:

(a) Since $-\sin A = \cos(A + 90^\circ)$,

$$v = -4\sin(30t + 50^\circ) = 4\cos(30t + 50^\circ + 90^\circ)$$
$$= 4\cos(30t + 140^\circ)$$

The phasor form of v is

$$\mathbf{V} = 4\angle 140^\circ$$

(b) $i = 6\cos(50t - 40^\circ)$ has the phasor

$$\mathbf{I} = 6\angle -40^\circ$$