PABLO

FUNDAMENTALS OF ELECTRICAL ENGINEERING.

LECTURE 5 NOTES

N.T DUAH

- **The Y-Connected Generator**

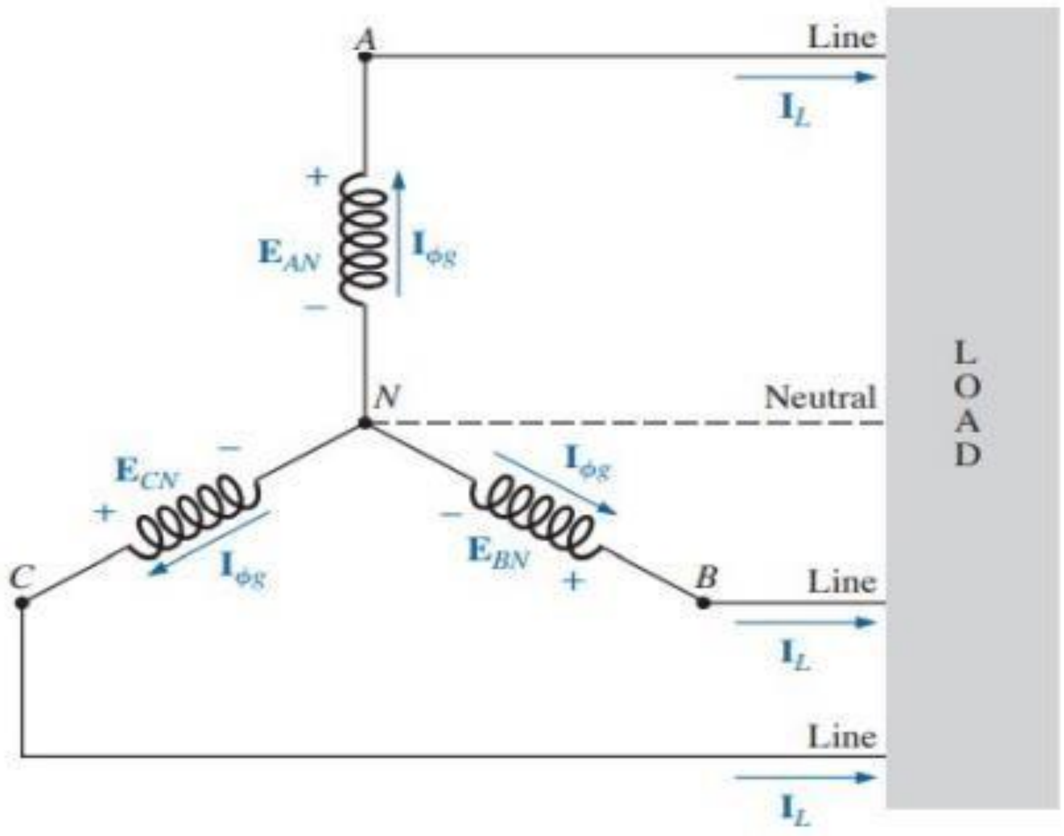


Figure 4.5: Y-connected generator

The point at which all the terminals are connected is called the neutral point. If a conductor is not attached from this point to the load, the system is called a Y-connected, threephase, three-wire generator.

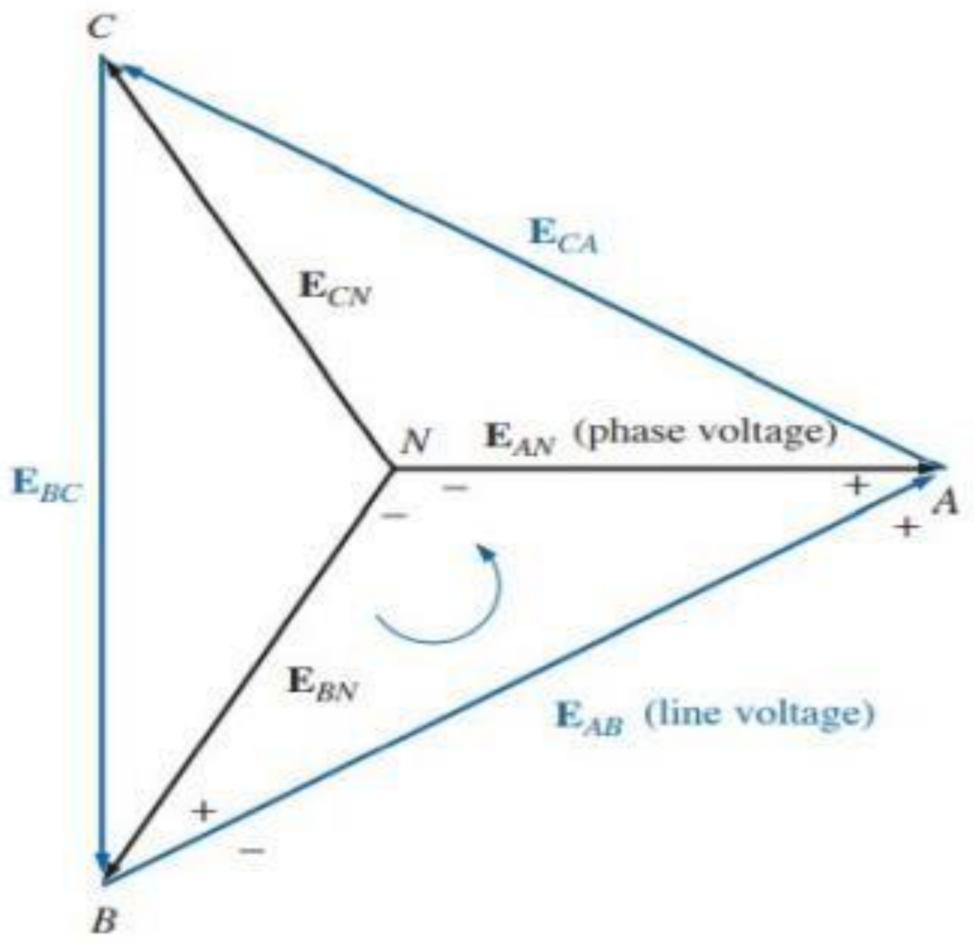
If the neutral is connected, the system is a Y-connected, threephase, four-wire generator.

For the Y-connected system, the line current equals the phase current for each phase; that is

$$I_L = I_{\phi g}$$

where ϕ is used to denote a phase quantity and g is a generator parameter.

Line and phase voltages of the Y-connected three-phase generator.


C
$\mathbf{E}_{CA}$
$\mathbf{E}_{CN}$
N
$\mathbf{E}_{AN}$ (phase voltage)
$\mathbf{E}_{BC}$
A
$\mathbf{E}_{BN}$
$\mathbf{E}_{AB}$ (line voltage)
B

The voltage from one line to another is called a line voltage. Applying Kirchoff's voltage law around the indicated loop of Fig. 4.6, we obtain

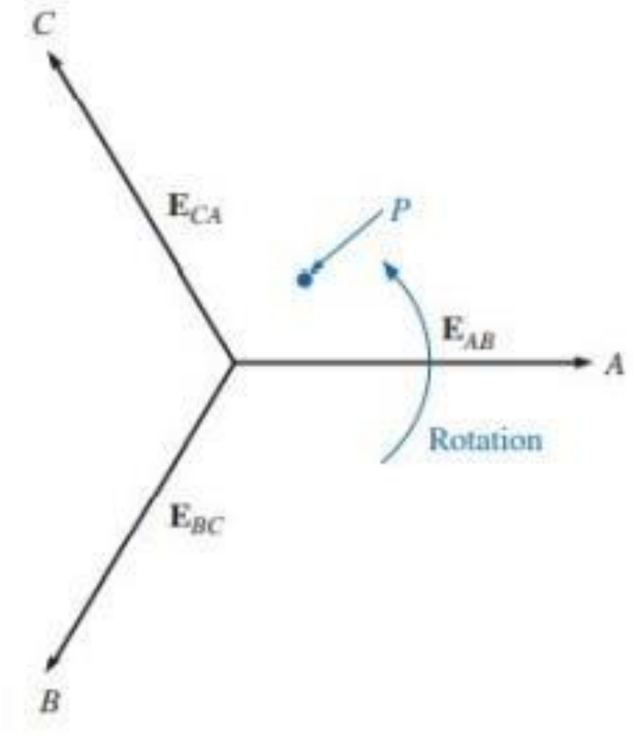
$$E_{AB} - E_{AN} + E_{BN} = 0$$

Or

$$E_{AB} = E_{AN} - E_{BN} = E_{AN} + E_{NB}$$

- **Phase Sequence (Y-Connected)**

The phase sequence can be determined by the order in which the phasors representing the phase voltages pass through a fixed point on the phasor diagram if the phasors are rotated in a counter-clockwise direction.

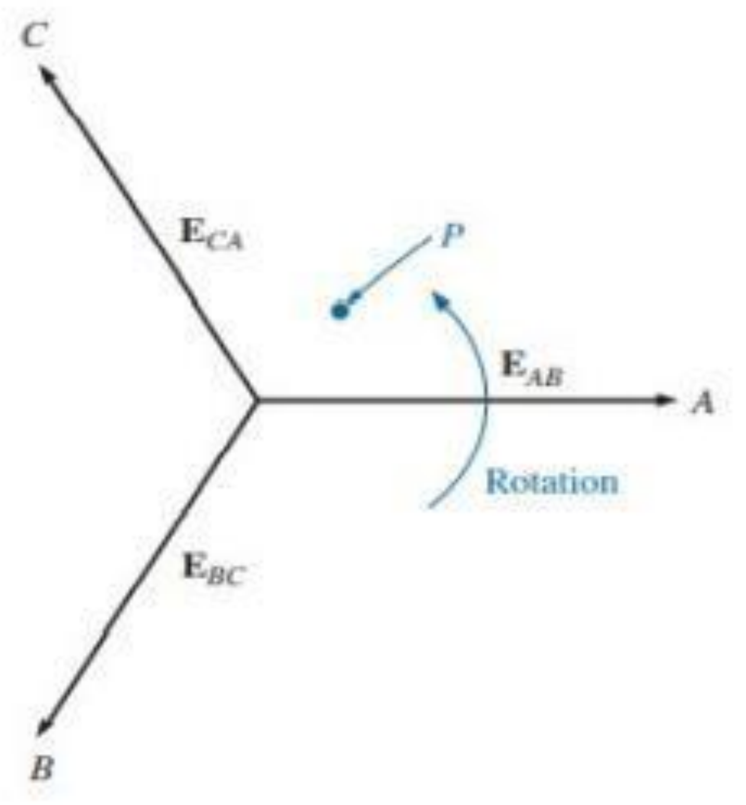


Determining the phase sequence from the phase voltages of a three-phase generator

The phase sequence is ABC. However, since the fixed point can be chosen anywhere on the phasor diagram, the sequence can also be written as BCA or CAB.

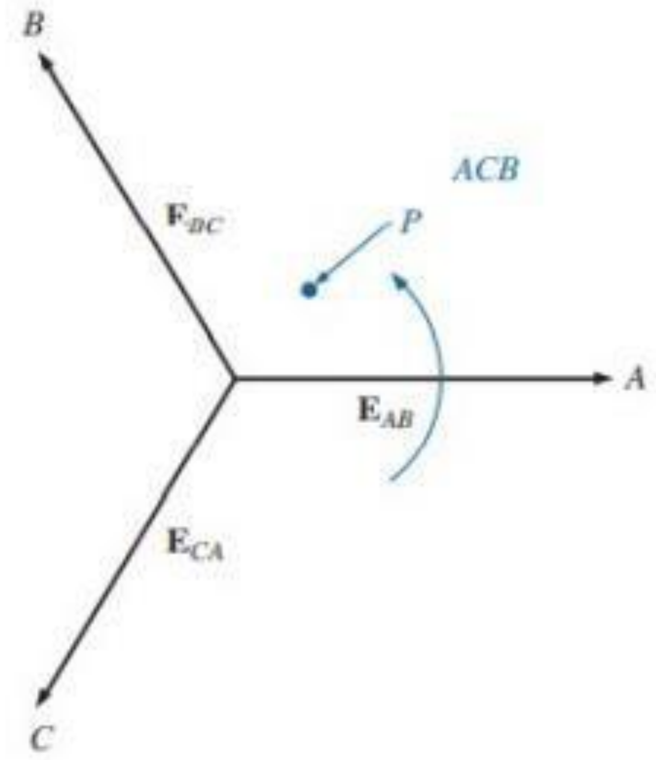
Determining the phase sequence from the line voltages of a three-phase generator

In phasor notation,



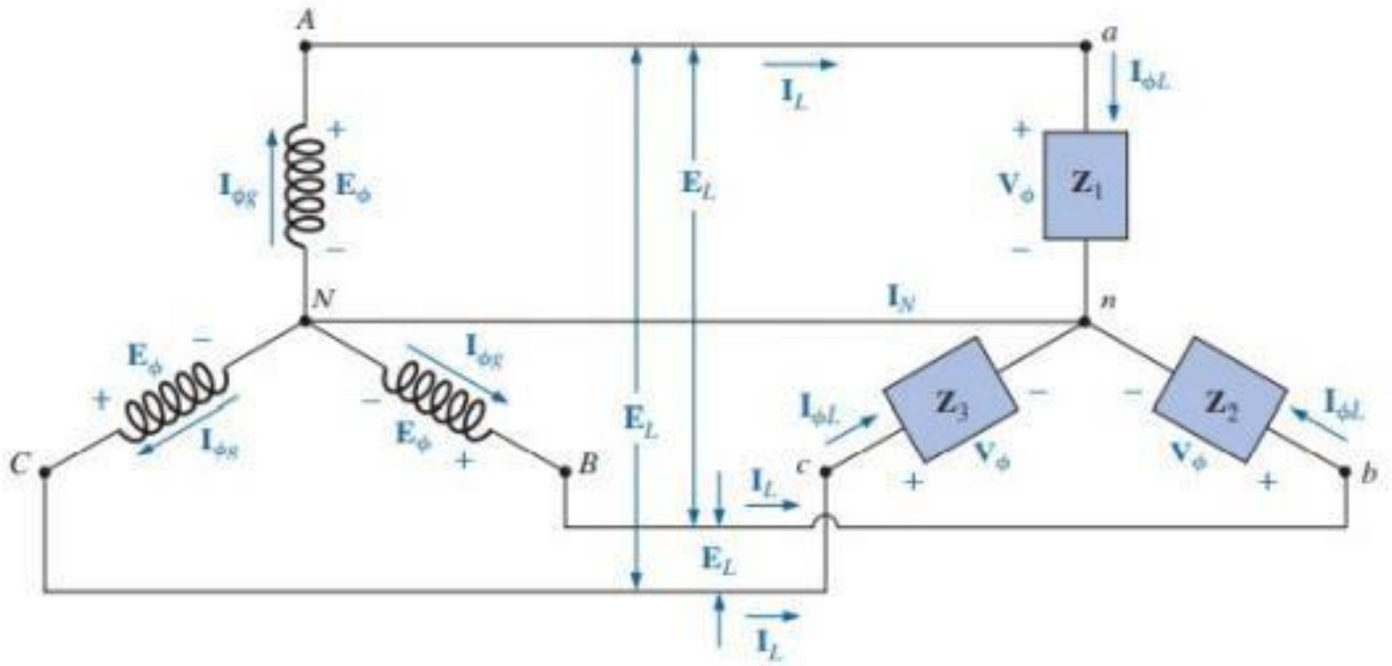
Line voltages

$$E_{AB} = E_{AB}\angle 0^\circ (reference)$$
$$E_{CA} = E_{CA}\angle -120^\circ$$
$$E_{BC} = E_{BC}\angle +120^\circ$$



Drawing the phasor diagram from the phase sequence.

- **The Y-Connected Generator With A Y-Generator**



Load

Figure 4.11: Y-connected generator with a Y-connected load.

If the load is balanced, the neutral connection can be removed without affecting the circuit in any manner; that is, if

$$\mathbf{Z_1 = Z_2 = Z_3}$$

then I_N will be zero.

$$\mathbf{I_{\phi g} = I_L = I_{\phi L}}$$

For a balanced or an unbalanced load, since the generator and load have a common neutral point, then

$$V_{\phi} = E_{\phi}$$

$$E_L = \sqrt{3}V_{\phi}$$

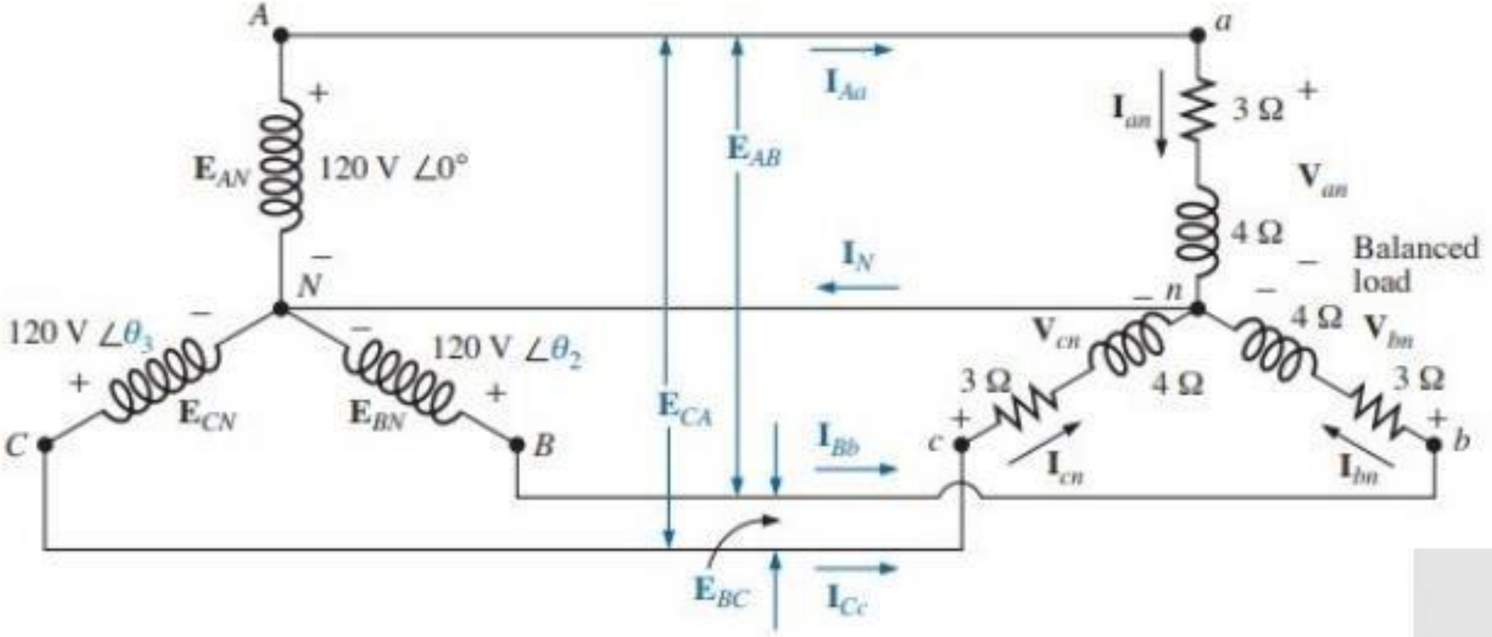
The phase sequence of the Y-connected generator in Fig. 4.12 is ABC.

a. Find the phase angles θ_2 and θ_3.

b. Find the magnitude of the line voltages.

c. Find the line currents.

d. Verify that, since the load is balanced, I_N=0.



O Examp

✦Solution

a. For an ABC phase sequence,

θ_2 = -120° and θ_3=+120°

b. $E_L = \sqrt{3}\ E_\phi = (1.73)(120V) = 208V$. Therefore, $E_{AB} = E_{BC} = E_{CA} = 208V$

c. $V_\phi = E_\phi$. Therefore,

$V_{an} = E_{AN}$ $V_{bn} = E_{BN}$ $V_{cn} = E_{CN}$

$$I_{\phi L} = I_{an} = \frac{V_{an}}{Z_{an}} = \frac{120V\angle 0^\circ}{3\Omega + j4\Omega} = \frac{120V\angle 0^\circ}{5\Omega\angle 53.13^\circ} = 24A\angle -53.13^\circ$$

$$I_{bn} = \frac{V_{bn}}{Z_{bn}} = \frac{120V\angle -120^\circ}{5\Omega\angle 53.13^\circ} = 24A\angle -173.13^\circ$$

$$I_{cn} = \frac{V_{cn}}{Z_{cn}} = \frac{120V\angle +120^\circ}{5\Omega\angle 53.13^\circ} = 24A\angle 66.87^\circ$$

and, since I_L=I_ϕ,

$$I_{Aa} - I_{an} - 24A\angle -53.13^\circ$$
$$I_{Bb} = I_{bn} = 24A\angle -173.13^\circ$$
$$I_{Cc} = I_{cn} = 24A\angle 66.87^\circ$$

d. Applying Kirchhoff's current law, we have

$$I_N = I_{Aa} + I_{Bb} + I_{cc}$$

In rectangular form,

$$I_{Aa} = 24A\angle -53.13° = 14.40A - j19.20A$$
$$I_{Bb} = 24A\angle -173.13° = -22.83A - j2.87A$$
$$I_{Cc} = 24A\angle 66.87° = 9.43A + j22.07A$$
$$\Sigma(I_{Aa} + I_{Bb} + I_{Cc}) = 0 + j0$$

and I_N is in fact equal to zero, as required for a balanced load.

- **The Y-Δ System**

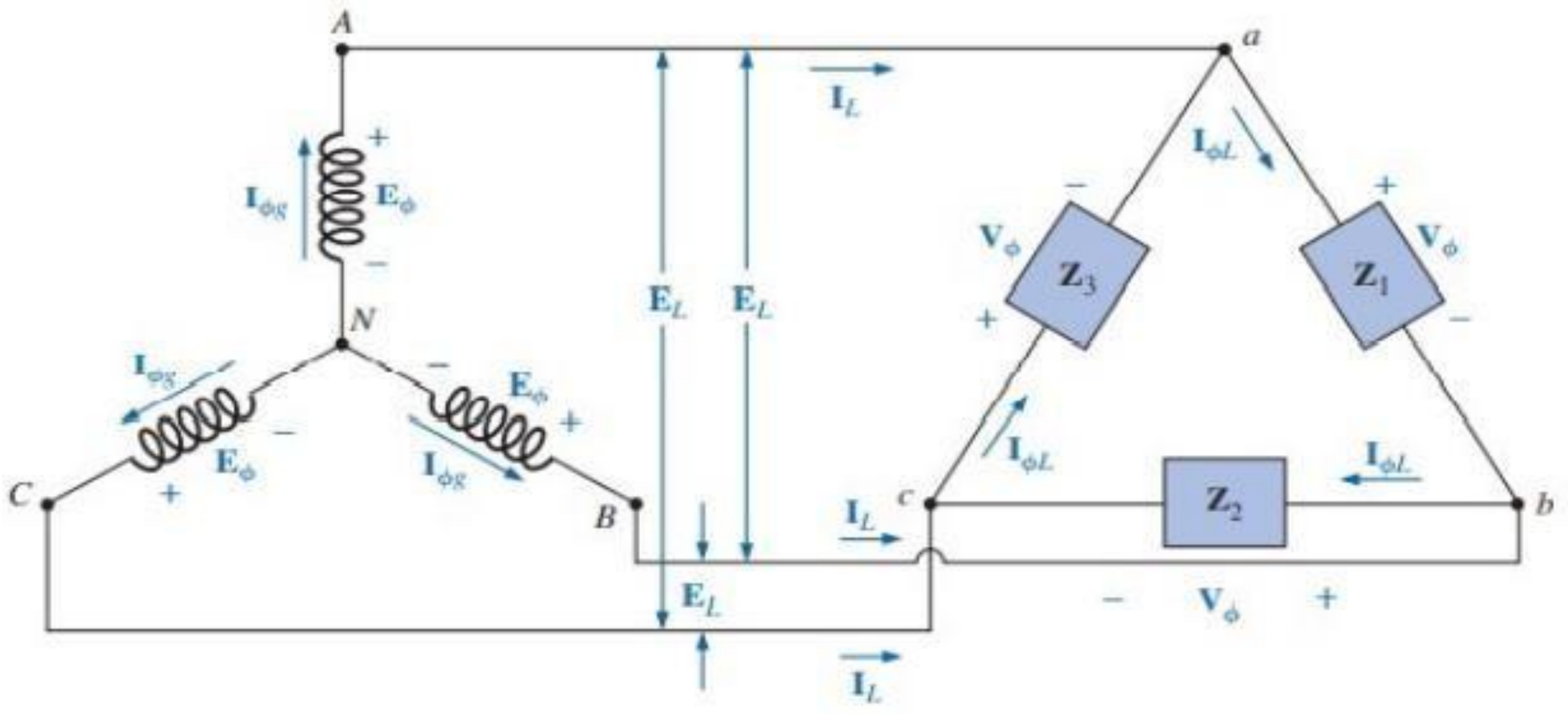


Figure 4.13: Y-connected generator with a Δ-connected load

For a balanced load

$$\mathbf{Z_1 = Z_2 = Z_3}$$

The voltage across each phase of the load is equal to the line voltage of the generator for a balanced or an unbalanced load:

$$\mathbf{V_\phi = E_L}$$

$$\mathbf{I_L = \sqrt{3}I_\phi}$$

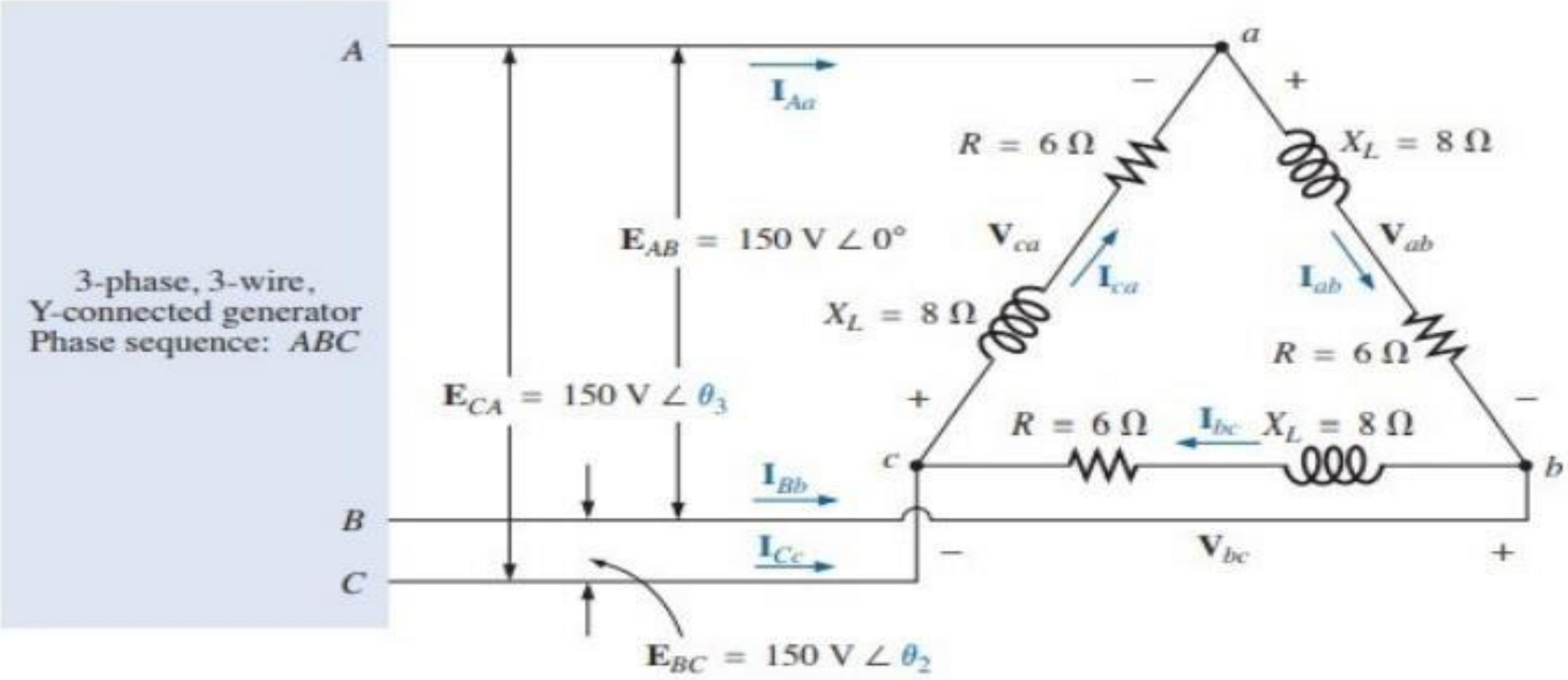
OExample

For the three-phase system of Fig. 4.14:

a. Find the phase angles θ_2 and θ_3.

b. Find the current in each phase of the load.

c. Find the magnitude of the line currents.



✦Solution

a. For an ABC sequence,

θ_2=-120° and θ_3=+120° b. V_ϕ=E_L. Therefore,

V_{ab}=E_{AB} V_{ab}=E_{CA} V_{ab}=E_{BC}

The phase currents are

$$I_{ab} = \frac{V_{ab}}{Z_{ab}} = \frac{150V\angle 0^\circ}{6\Omega + j8\Omega} = \frac{150V\angle 0^\circ}{10\Omega\angle 53.13^\circ} = 15A\angle -53.13^\circ$$

$$I_{bc} = \frac{V_{bc}}{Z_{bc}} == \frac{150V\angle -120^\circ}{10\Omega\angle 53.13^\circ} = 15A\angle -173.13^\circ$$

$$I_{ca} = \frac{V_{ca}}{Z_{ca}} == \frac{150V\angle +120^\circ}{10\Omega\angle 53.13^\circ} = 15A\angle 66.87^\circ$$

c. $I_L = \sqrt{3} I_\phi = (1.73A)(15A) = 25.95A$. Therefore,

$I_{Aa} = I_{Bb} = I_{Cc} = 25.95A$

- **The Δ-Connected Generator.**

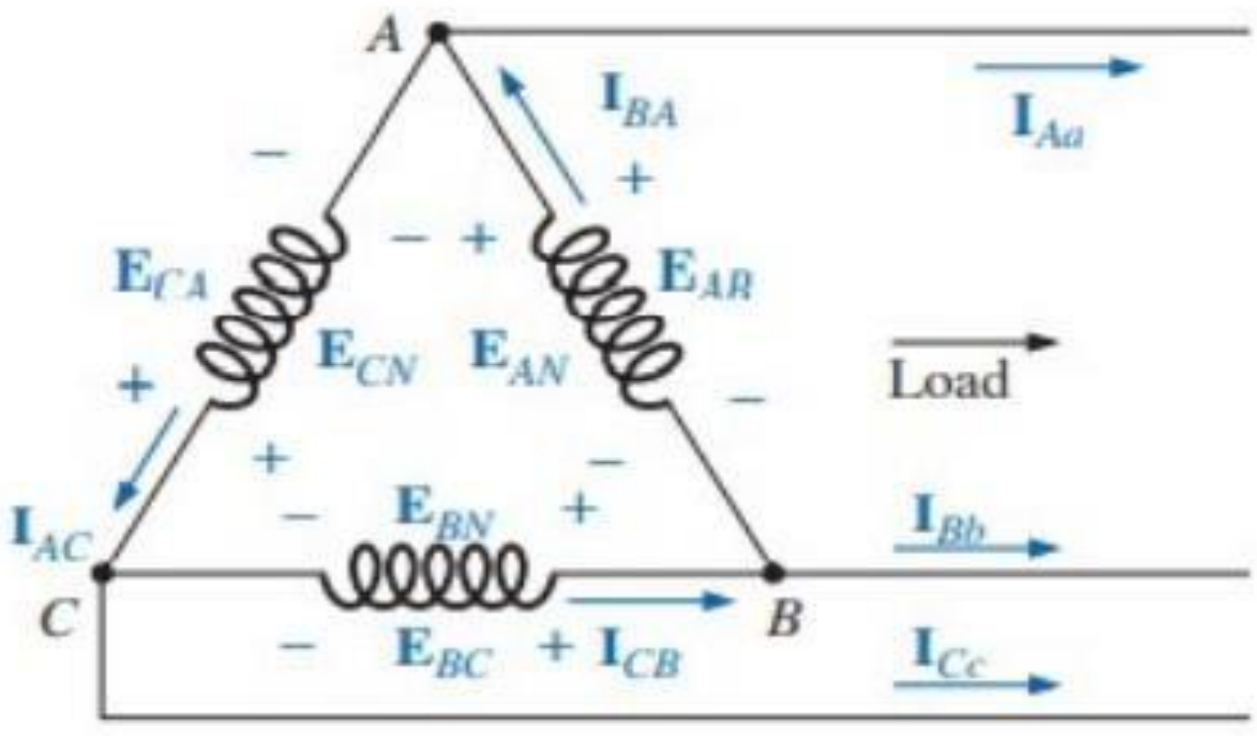


Figure 4.15: Δ-connected generator

Δ-connected ac generator. In this system, the phase and line voltages are equivalent and equal to the voltage induced across each coil of the generator; that is,

Phase sequence ABC;

$E_{AB}=E_{AN}$ and $e_{AN}=\sqrt{2}E_{AN}\sin\omega t$

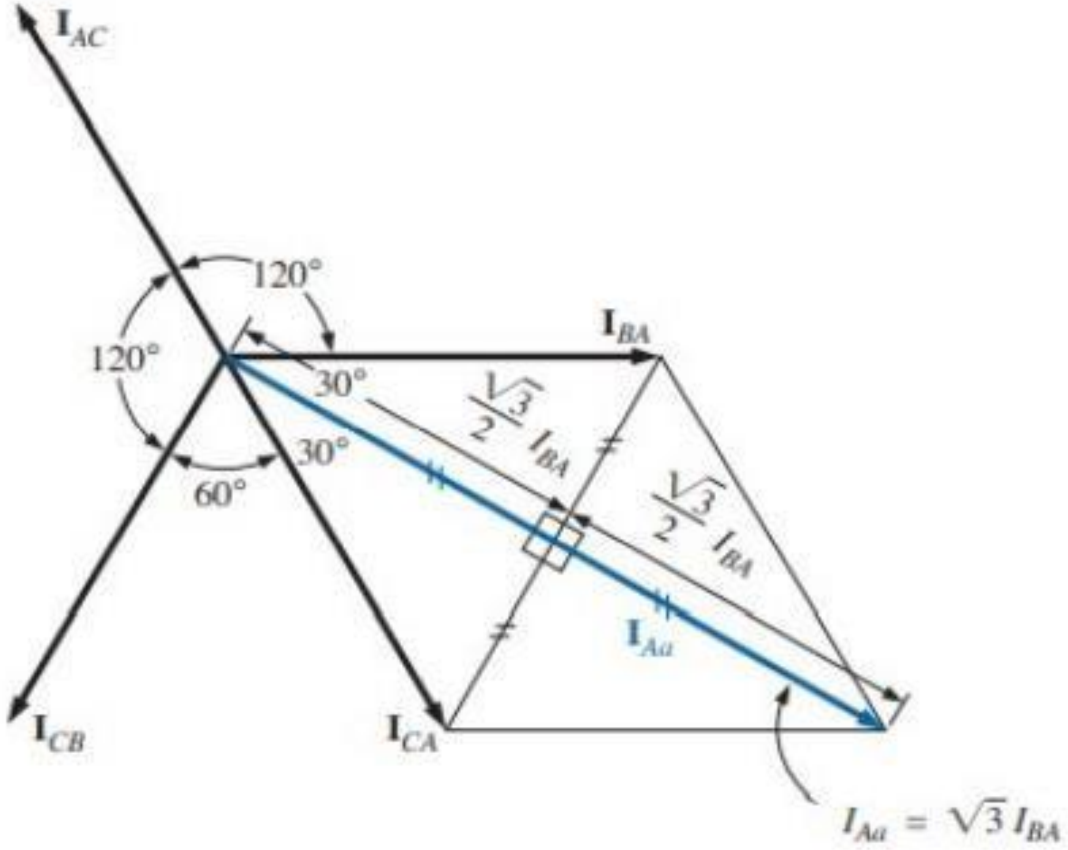
$E_{BC}=E_{BN}$ and $e_{BN}=\sqrt{2}E_{BN}\sin(\omega t\text{-}120°)$

$E_{CA}=E_{CN}$ and $e_{CN}=\sqrt{2}E_{CN}\sin(\omega t+120°)$

Or $E_L = E_{\phi g}$

Unlike the line current for the Y-connected generator, the line current for the Δ-connected system is not equal to the phase current.

Determining a line current from the phase currents of a Δ-connected, three-phase generator.



The relationship between the two can be found by applying Kirchoff's current law at one of the nodes and solving for the line current in terms of the phase currents; that is, at node A.

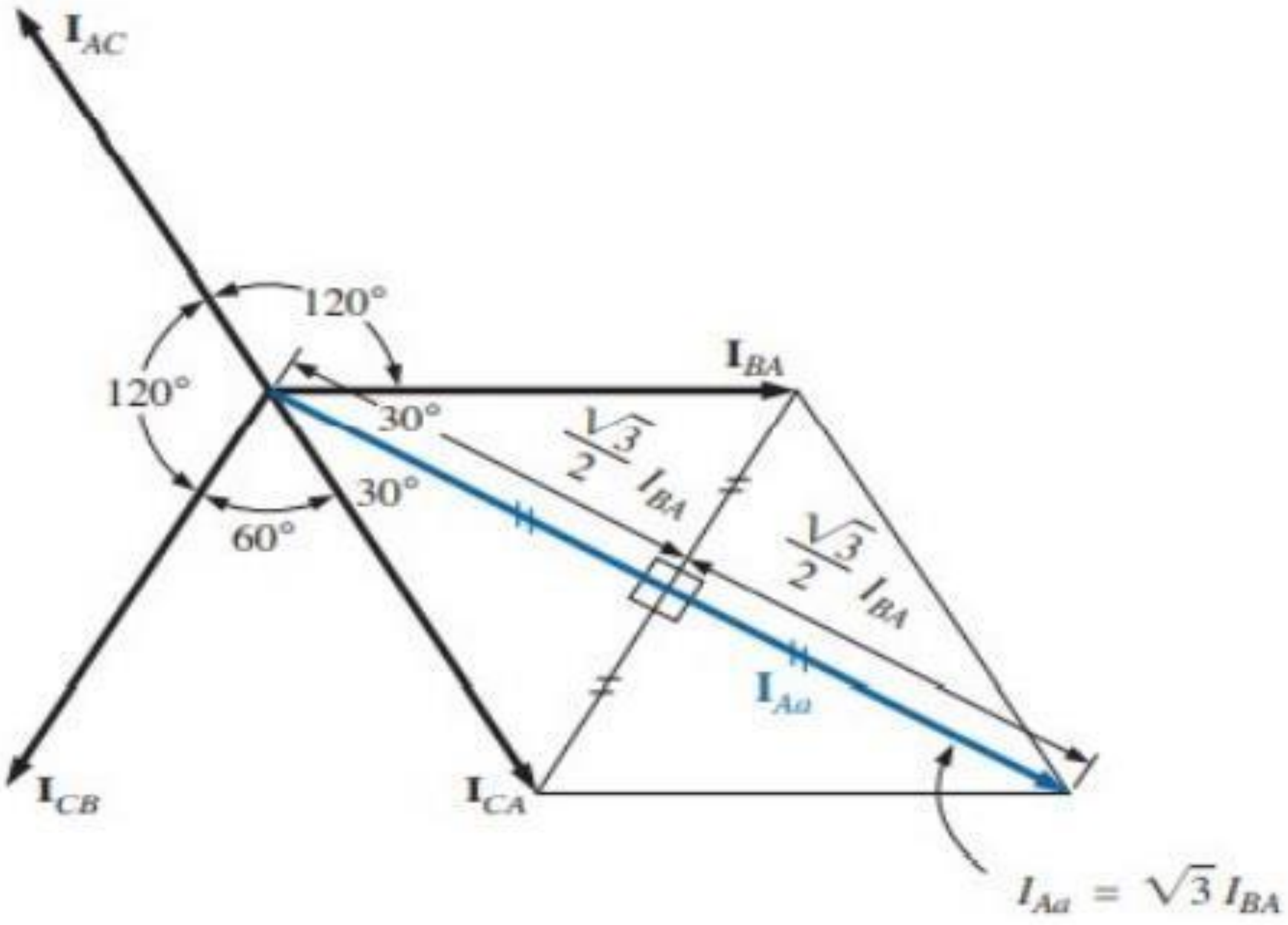
$$I_{BA}=I_{Aa}+I_{AC}$$

$$\text{or } I_{Aa}=I_{BA}\text{-}I_{AC}=I_{BA}+I_{CA}$$

$$I_{Aa} = \sqrt{3}I_{BA}\angle -30^\circ$$
$$I_{Bb} = \sqrt{3}I_{CB}\angle -150^\circ$$
$$I_{Cc} = \sqrt{3}I_{AC}\angle 90^\circ$$
$$I_L = \sqrt{3}I_{\phi g}$$

with the phase angle between a line current and the nearest phase current at 30◦ .

✦Phase Sequence (Δ-connected Generator)



$$E_{AB} = E_{AB}\angle 0^\circ$$
$$E_{BC} = E_{BC}\angle -120^\circ$$
$$E_{CA} = E_{CA}\angle 120^\circ$$

- **The Δ-Δ,Δ-Y Three-phase Systems.**

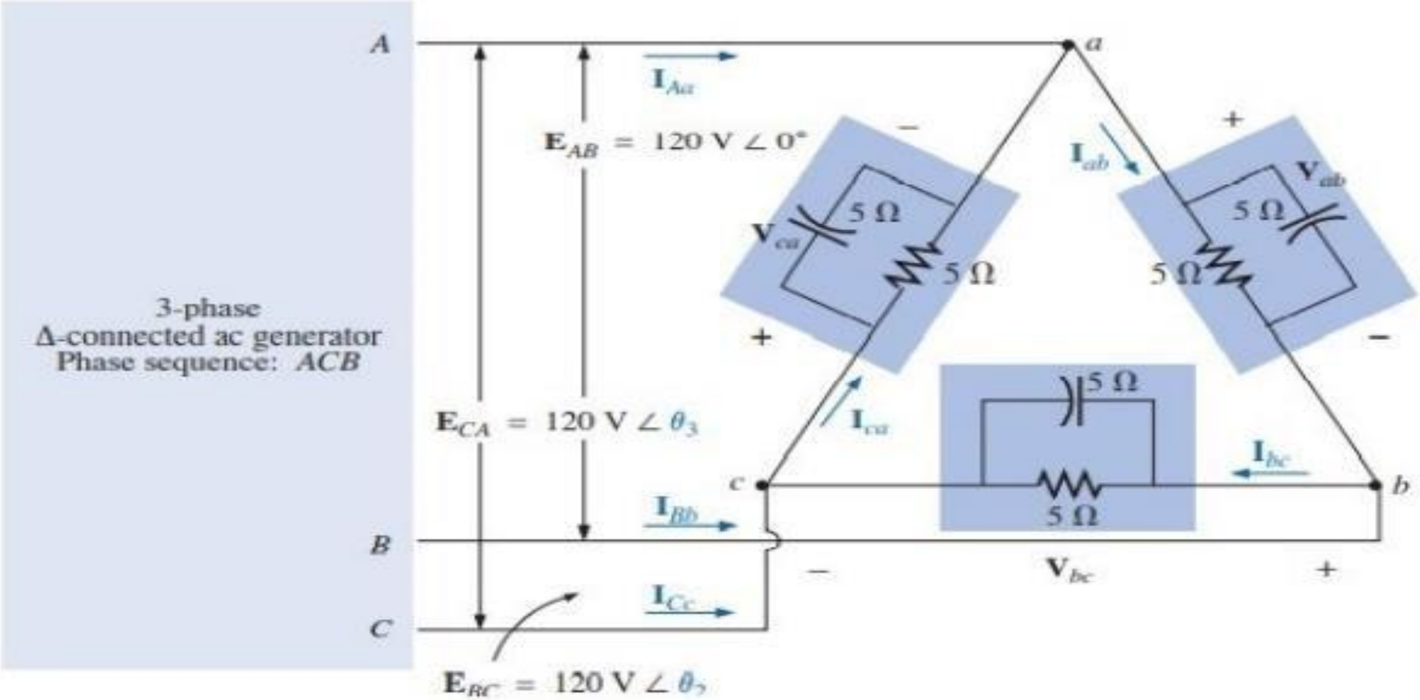
Example

For the Δ-Δ system shown in Fig. (4.18):

a. Find the phase angles θ_2 and θ_3 for the specified phase sequence.

b. Find the current in each phase of the load.

c. Find the magnitude of the line currents.



✦Solution

a. For an ABC phase sequence,
θ_2=120° and θ_3=-120°

b. V_ϕ=E_L. Therefore,

$V_{ab}=E_{AB}$ $V_{ca}=E_{CA}$ $V_{bc}=E_{BC}$ The phase currents are

$$I_{ab} = \frac{V_{ab}}{Z_{ab}} = \frac{120V\angle 0^\circ}{\frac{(5\Omega\angle 0^\circ)(5\Omega\angle -90^\circ)}{5\Omega - j5\Omega}} = \frac{120V\angle 0^\circ}{\frac{25\Omega\angle -90^\circ}{7.071\angle -45^\circ}}$$

$$= \frac{120V\angle 0^\circ}{3.45\Omega\angle -45^\circ} = 33.9A\angle 45^\circ$$

$$I_{bc} = \frac{V_{bc}}{Z_{bc}} = \frac{120V\angle 120^\circ}{3.45\Omega\angle -45^\circ} = 33.9A\angle 165^\circ$$

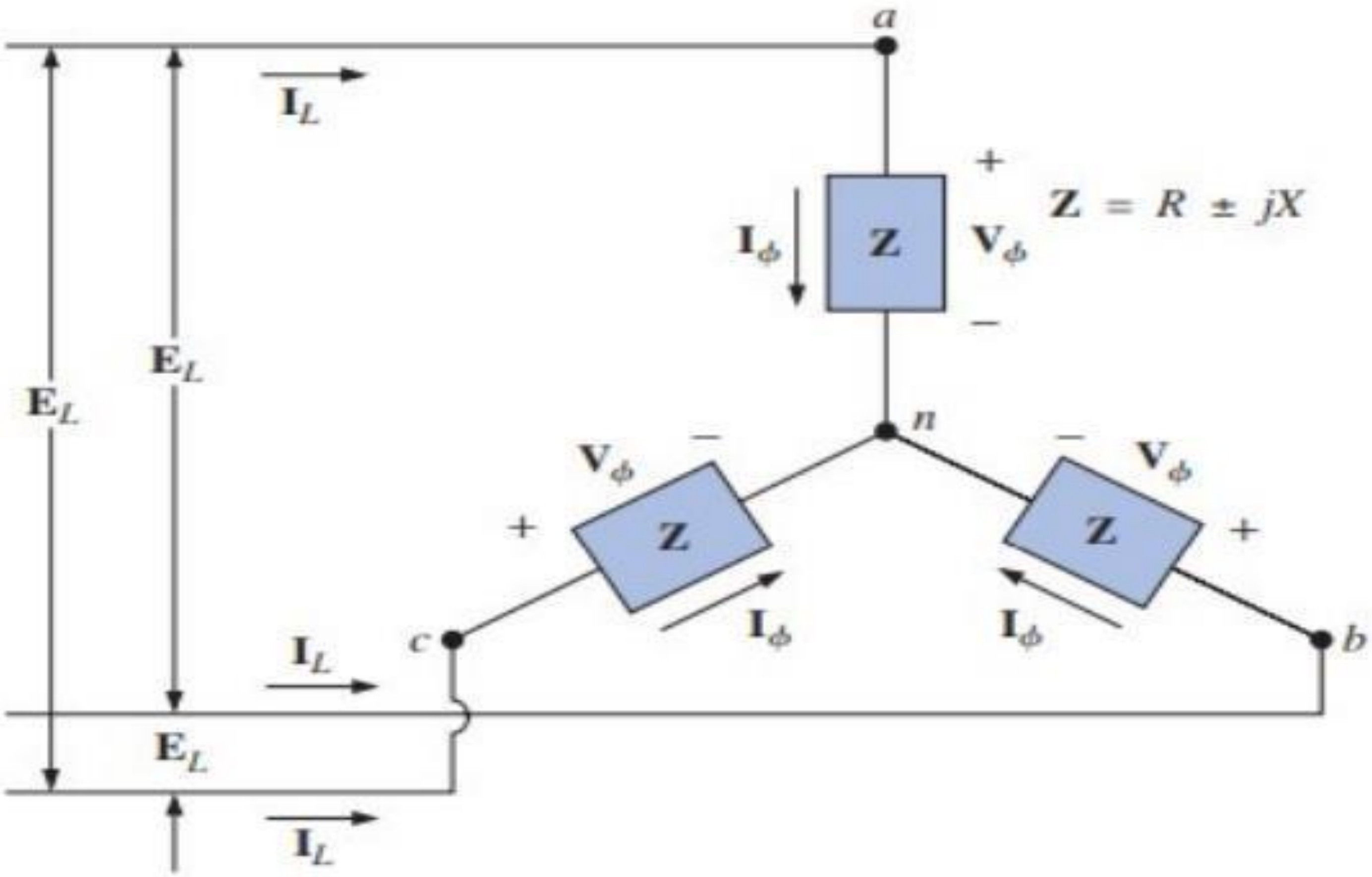
$$I_{ca} = \frac{V_{ca}}{Z_{ca}} = \frac{120V\angle -120^\circ}{3.45\Omega\angle -45^\circ} = 33.9A\angle -75^\circ$$

c. $I_L = \sqrt{3} I_\phi = (1.73)(34A) = 58.82A$. Therefore,

$I_{Aa} = I_{Bb} = I_{Cc} = 58.82A$

- **Power In Three-phase Systems**

OY-Connected Balanced Load



✦Average Power

$$P_\phi = V_\phi I_\phi \cos\theta_{I_\theta}^{V_\phi} = I_\phi^2 R_\phi = \frac{V_R^2}{R_\phi}(watts, W)$$

where $\theta^{V\phi}_{I\theta}$ indicates that θ is the phase angle between V_ϕ and I_ϕ. The total power to the balanced load is $P_T = 3P_\phi (W)$

$$V_\phi = \frac{E_L}{\sqrt{3}} \text{ and } I_\phi = I_L$$

Or, since

$$\text{then } P_T = 3\frac{E_L}{\sqrt{3}} I_L \cos\theta_{I_\phi}^{V_\phi}$$

But

$$\left(\frac{3}{\sqrt{3}}\right)(1) = \left(\frac{3}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

Therefore,

$$P_T = \sqrt{3}E_L I_L \cos\theta_{I_\phi}^{V_\phi} = 3I_L^2 R_\phi (W)$$

✦Reactive Power

$$P_T = 3\frac{E_L}{\sqrt{3}} I_L \cos\theta_{I_\phi}^{V_\phi}$$

But

$$\left(\frac{3}{\sqrt{3}}\right)(1) = \left(\frac{3}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

Therefore,

$$P_T = \sqrt{3} E_L I_L \cos\theta_{I_\phi}^{V_\phi} = 3 I_L^2 R_\phi (W)$$

The total reactive power of the load is

$Q_T = 3Q_\phi$ (VAR)

or, proceeding in the same manner as above, we have

$$Q_T = \sqrt{3} E_L I_L \sin\theta_{I_\phi}^{V_\phi} = 3 I_L^2 X_\phi \text{ (VAR)}$$

✦Apparent Power

The apparent power of each phase is

$S_\phi = V_\phi I_\phi$ (VA)

The total apparent power of the load is

$$F_p = \frac{P_T}{S_T} = \cos\theta_{I_\phi}^{V_\phi} \text{ (leading or lagging)}$$

$S_T = 3S_\phi$ (VA)

or, as before,

$S_T = \sqrt{3} E_L I_L$ (VA)

✦ **Power Factor**

O Example

For the Y-connected load of Fig. (4.21):

a. Find the average power to each phase and the total load.
b. Determine the reactive power to each phase and the total reactive power.

c. Find the apparent power to each phase and the total apparent power.
d. Find the power factor of the load.

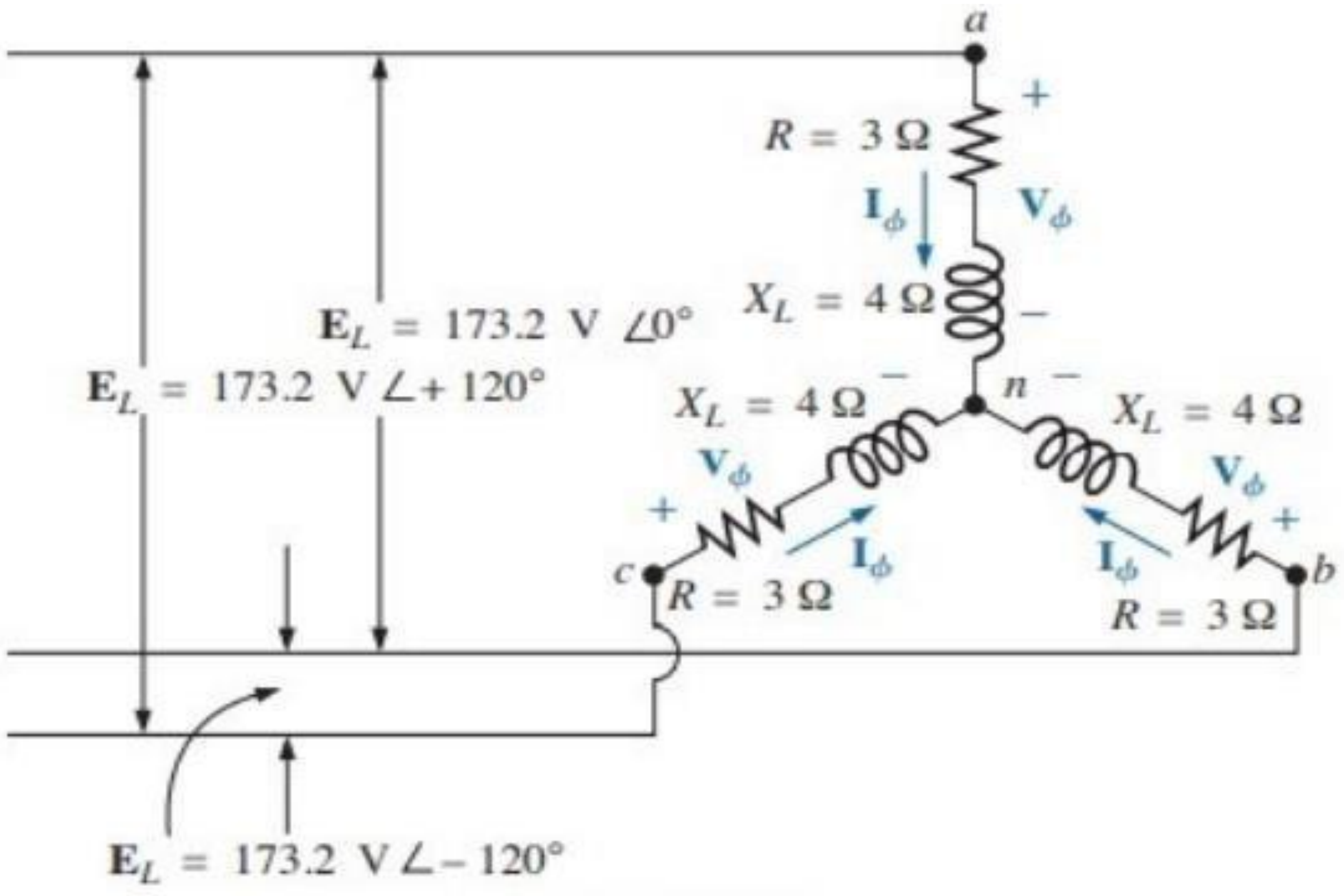
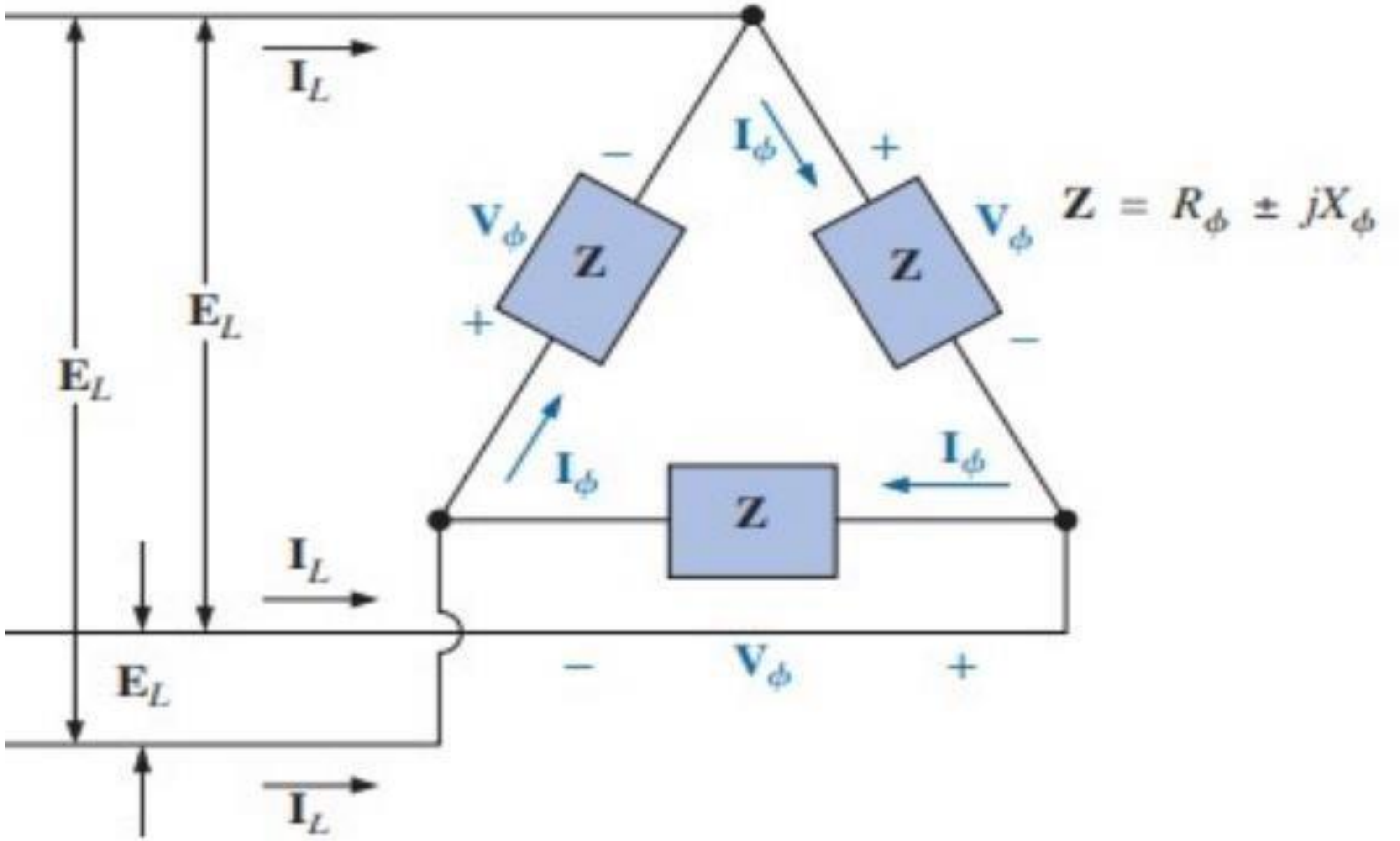

a
+
R = 3 Ω
I_ϕ
V_ϕ
X_L = 4 Ω
−
E_L = 173.2 V ∠0°
E_L = 173.2 V ∠+ 120°
X_L = 4 Ω
−
n
−
X_L = 4 Ω
V_ϕ
V_ϕ
+
+
I_ϕ
I_ϕ
c
R = 3 Ω
b
R = 3 Ω
E_L = 173.2 V ∠− 120°

Figure 4.21:

- **Δ-Balanced Load**



$\mathbf{I}_L$
$\mathbf{E}_L$
$\mathbf{E}_L$
$\mathbf{I}_\phi$
$\mathbf{V}_\phi$
Z
Z
Z
$\mathbf{V}_\phi$
$\mathbf{Z} = R_\phi \pm jX_\phi$
$\mathbf{I}_\phi$
$\mathbf{I}_\phi$
$\mathbf{I}_L$
$\mathbf{E}_L$
$\mathbf{V}_\phi$
$\mathbf{I}_L$

✦Average Power

$$P_\phi = V_\phi I_\phi \cos\theta_{I_\phi}^{V_\phi} = I_\phi^2 R_\phi = \frac{V_R^\phi}{R_\phi}(W)$$
$$P_T = 3P_\phi(W)$$

✦Reactive Power

$$Q_\phi = V_\phi I_\phi \sin\theta_{I_\phi}^{V_\phi} = I_\phi^2 X_\phi = \frac{V_x^2}{X_\phi}(VAR)$$
$$Q_T = 3Q_\phi(VAR)$$

✦Apparent Power

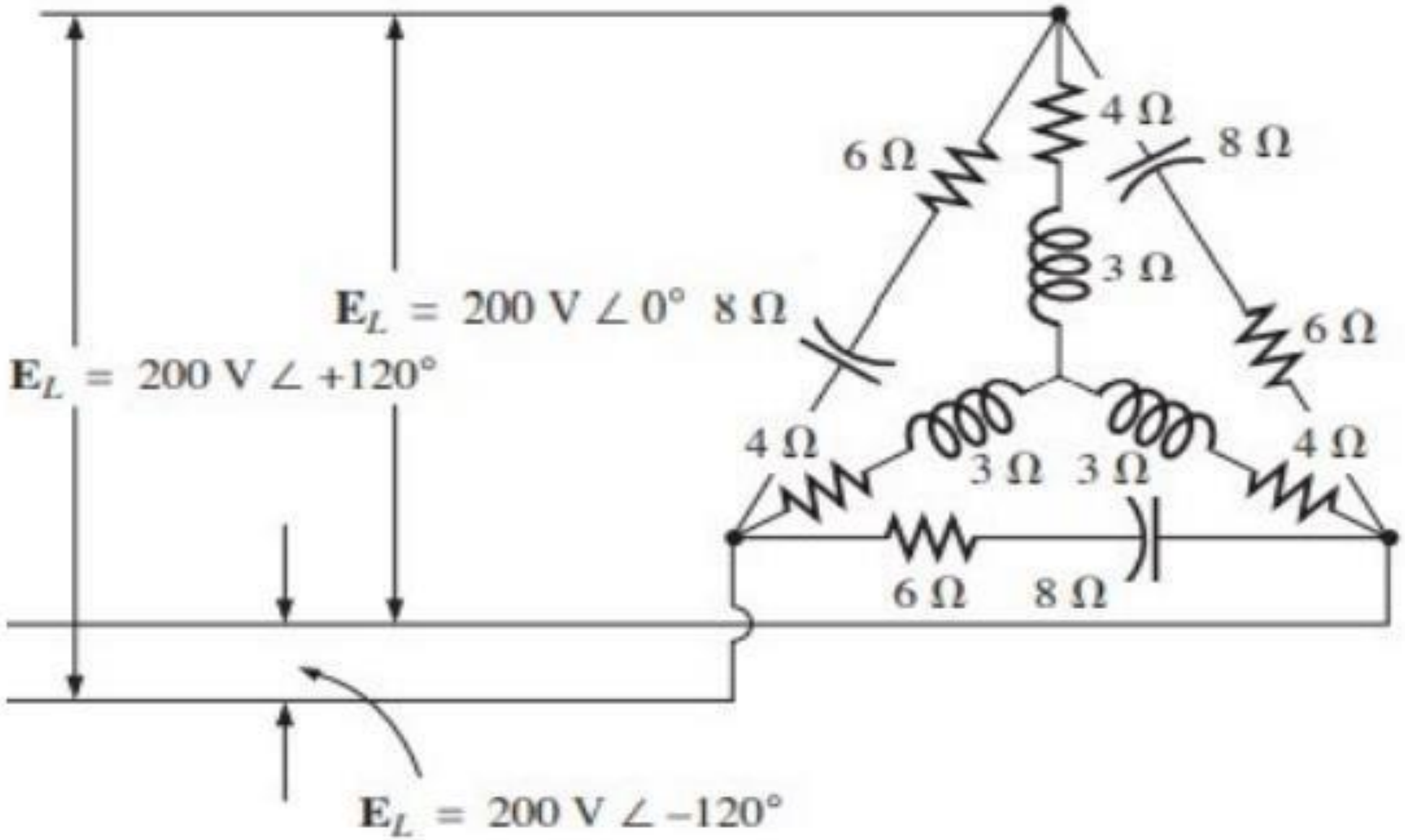
$$S_\phi = V_\phi I_\phi (VA)$$
$$S_T = 3S_\phi = \sqrt{3} E_L I_L (VA)$$

✦Power Factor

$$F_p = \frac{P_T}{S_T}$$

○ Example

For the Δ-Y connected load of Fig. (4.23), find the total average, reactive, and apparent power. In addition, find the power factor of the load.



$E_L = 200\ V \angle 0°$
$E_L = 200\ V \angle +120°$
$E_L = 200\ V \angle -120°$
4 Ω
8 Ω
6 Ω
3 Ω
8 Ω
6 Ω
4 Ω
3 Ω
3 Ω
4 Ω
6 Ω
8 Ω

Figure 4.23:

✦Solution

Consider the Δ and Y separately.

For the Δ:

$$I_\phi = \frac{E_L}{Z_\Delta} = \frac{200V}{10\Omega} = 20A$$
$$P_{T_\Delta} = 3I_\phi^2 R_\phi = (3)(20A)^2\,(6\Omega) = 7200W$$
$$Q_{T_\Delta} = 3I_\phi^2 X_\phi = (3)(20A)^2(8\Omega)9600VAR(C)$$
$$S_{T_\Delta} = 3V_\phi I_\phi = (3)(200V)(20A) = 12,000VA$$

For the Y:

$$Z_Y = 4\Omega + j3\Omega = 5\Omega\angle 36.87°$$
$$I_\phi = \frac{E_L/\sqrt{3}}{Z_Y} = \frac{200V/\sqrt{3}}{5\Omega} = \frac{116V}{5\Omega} = 23.12A$$
$$P_{T_Y} = 3I_\phi^2 R_\phi = (3)(23.12)^2(4\Omega) = 6414.41W$$
$$Q_{T_Y} = 3I_\phi^2 X_\phi = (3)(23.12)^2(3\Omega) = 4810.81VAR(L)$$
$$S_{T_Y} = 3V_\phi I_\phi = (3)(116V)(23.12A) = 8045.76VA$$

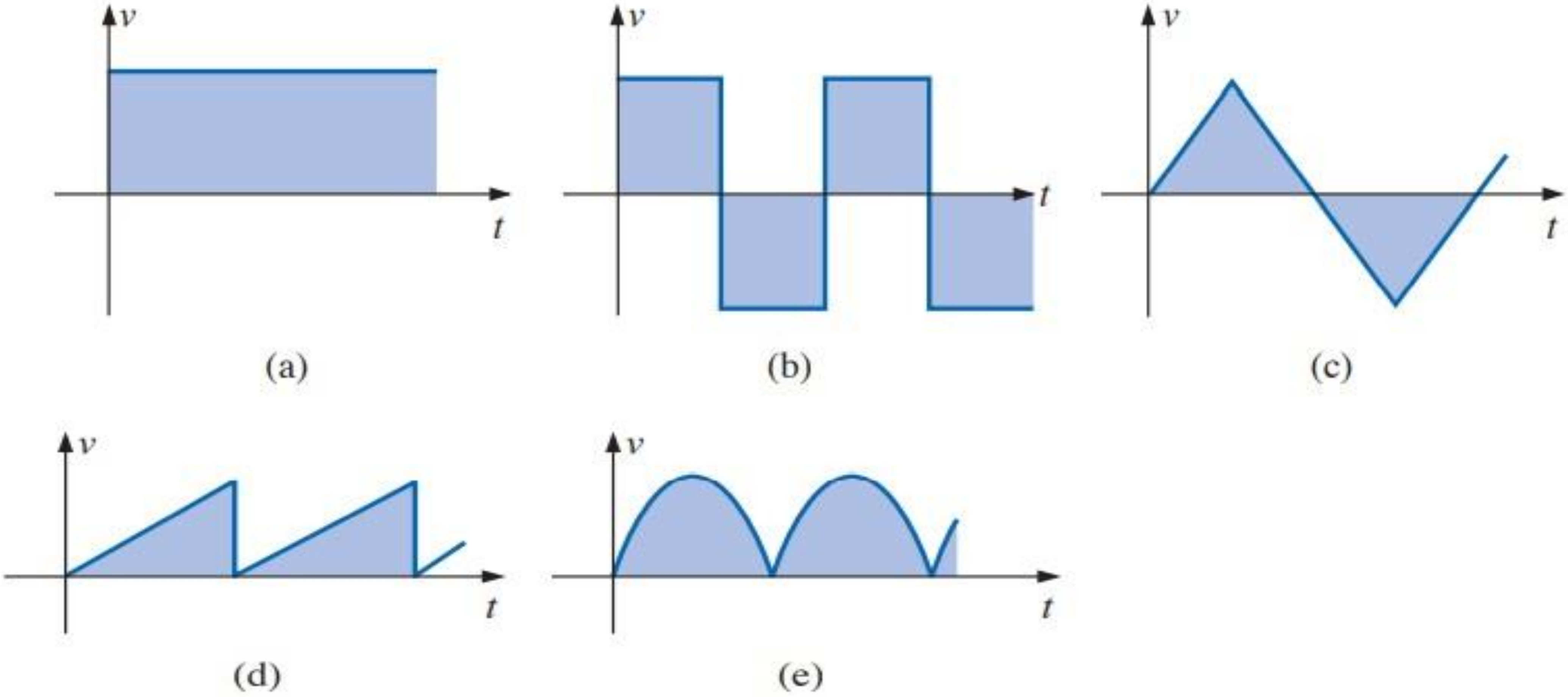
For the total load:

$$P_T = P_{T_\Delta} + P_{T_Y} = 7200W + 6414.41W = 13,614.41W$$
$$Q_T = Q_{T_\Delta} - Q_{T_Y} = 9600VAR(C) - 4810.81VAR(I) = 4789.19VAR(C)$$
$$S_T = \sqrt{P_T^2 + Q_T^2} = \sqrt{(13,614.41W)^2 + (4789.19VAR)^2} = 14,432.2VA$$
$$F_p = \frac{P_T}{S_T} = \frac{13{,}614.41W}{14{,}432.20VA} = 0.943 leading$$

- **Non-Sinusoidal Circuits**

Any waveform that differs from the basic description of the sinusoidal waveform is referred to as non-sinusoidal.

The most obvious and familiar are the dc, square-wave, triangular, sawtooth, and rectified waveforms.


v
t
(a)
v
t
(b)
v
t
(c)
v
t
(d)
v
t
(e)

Common non-sinusoidal waveforms: (a) dc; (b) square-wave; (c) triangular; (d) sawtooth; (e) rectified.

The output of many electrical and electronic devices will be non-sinusoidal, even though the applied signal may be purely sinusoidal.

For example, the network of Fig. 25.2 employs a diode to clip off the negative portion of the applied signal in a process called half-wave rectification, which is used in the development of dc levels from a sinusoidal input.

The output waveform is definitely non-sinusoidal, but note that it has the same period as the applied signal and matches the input for half the period.

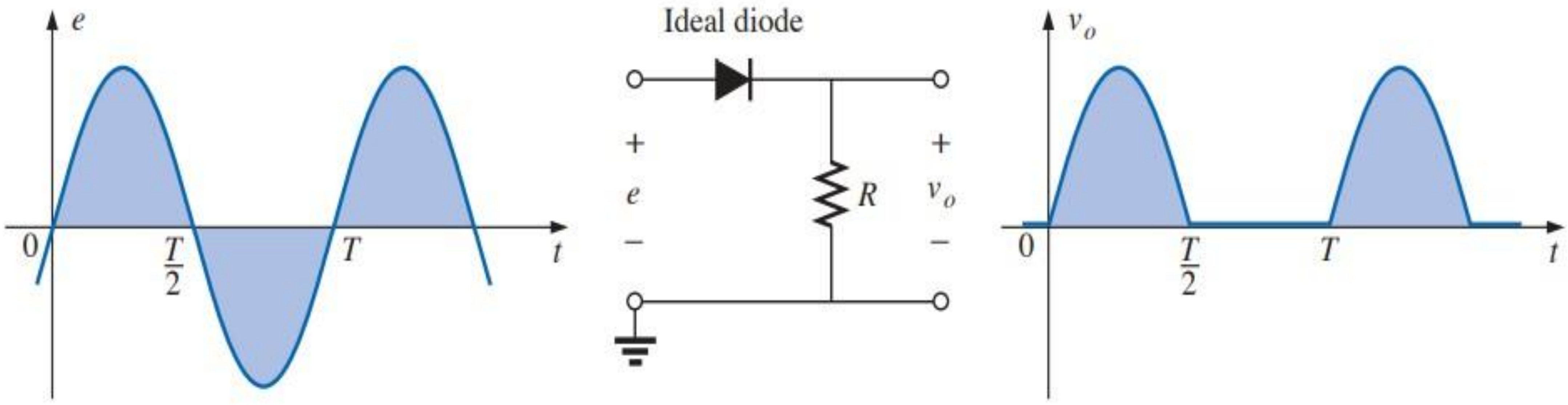


FIG. 25.2
Half-wave rectifier producing a nonsinusoidal waveform.

- **Fourier Series**

Fourier series refers to a series of terms, developed in 1826 by Baron Jean Fourier, that can be used to represent a nonsinusoidal periodic waveform. In the analysis of these waveforms, we solve for each term in the Fourier series:

$$f(t) = \underbrace{A_0}_{\substack{\text{dc or} \\ \text{average value}}} + \underbrace{A_1 \sin \omega t + A_2 \sin 2\omega t + A_3 \sin 3\omega t + \cdots + A_n \sin n\omega t}_{\text{sine terms}}$$
$$+ \underbrace{B_1 \cos \omega t + B_2 \cos 2\omega t + B_3 \cos 3\omega t + \cdots + B_n \cos n\omega t}_{\text{cosine terms}}$$

- ✦The Fourier series has three basic parts. The first is the dc term A_0, which is the average value of the waveform over one full cycle.
- ✦The second is a series of sine terms.
- ✦The third part is a series of cosine terms.
- ✦The first term of the sine and cosine series is called the fundamental component.
- ✦The other terms with higher-order frequencies (integer multiples of the fundamental) are called the harmonic terms. A term that has a frequency equal to twice the fundamental

is the second harmonic; three times, the third harmonic; and so on.