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FUNDAMENTALS OF ELECTRICAL ENGINEERING

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January 14, 2022

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Contents

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Chapter 1

Basic Electrical Quantities

At the end of the chapter students should be able to:
1. State the basic SI units.
2. Recognize derived SI units.
3. Understand prefixes denoting multiplication and division.
4. State the units for basic electrical parameters and perform simple calculations involving these units.
5. Understand the functions of capacitors and inductors and perform simple calculations involving workdonein capacitors and inductors.
6. Understand the current and voltage sources and conversions of these sources.

1.1 Units of measurement

In any technical field it is important to understand the basic concepts and the impact they will have on certain parameters. However, the application of these rules and laws will be successful only if the mathematical operations involved are applied correctly. In particular, it is vital that the importance of applying the proper unit of measurement to a quantity is understood and appreciated. Students often generate a numerical solution but decide not to apply a unit of measurement to the result because they are somewhat unsure of which unit should be applied.
Consider, for example, the following fundamental physics equation:

$$v = \frac{d}{t} \tag{1.1}$$

where v= velocity, d=distance, t=time.

Assume, for the moment, that the following data are obtained for a moving object: d= 4000ft, t = 1min. If v is desired in miles per hour. Often, without a second thought or consideration, the numerical values are simply substituted into the equation, with the result here that

$$v = \frac{d}{t} = \frac{400ft}{1\,\mathrm{min}} = 4000mi/h$$

As indicated above, the solution is totally incorrect. If the result is desired in miles per hour, the unit of measurement for distance must be miles, and that for time, hours. In a moment, when the problem is analyzed

properly, the extent of the error will demonstrate the importance of ensuring that the numerical value substituted into an equation must have the unit of measurement specified by the equation.
Therefore there is the need to convert the distance and time to the proper unit of measurement.
1mi = 5280 ft, therefore 4000ft = 0.7576

$$1\,\mathrm{min} = \frac{1}{60}h = 0.0167h$$

Substituting into Eq. (1.1)

$$v = \frac{d}{t} = \frac{0.7576mi}{0.0167h} = 45.37mi/h$$

In review, before substituting numerical values into an equation, be absolutely sure of the following:
Each quantity has the proper unit of measurement as defined by the equation.
The proper magnitude of each quantity as determined by the defining equation is substituted.
Each quantity is in the same system of units (or as defined by the equation).
The magnitude of the result is of a reasonable nature when compared to the level of the substituted quantities.
The proper unit of measurement is applied to the result.

1.2 Systems of units

The systems of units most commonly use are the English and metric, as outlines in Table 1.1. the metric is subdivided into two interrelated standards: the (Meter- kilogram-second) MKS and the (centimeter – gramsecond) CGS.. General Conference adopted a system called International System of Units, which has the international abbreviation SI. For comparison, the SI units of measurement and their abbreviations appear in Table 1.1. In this book the SI units will be employed
Table 1, Comparison of the English and metric systems of units

English	Metric		
	MKS	CGS	SI
Length: Yard (yd) (0.914 m)	Meter (m) (39.37 in.) (100 cm)	Centimeter (cm) (2.54 cm = 1 in.)	**Meter (m)**
Mass: **Slug** (14.6 kg)	Kilogram (kg) (1000 g)	Gram (g)	**Kilogram (kg)**
Force: **Pound** (lb) (4.45 N)	Newton (N) (100,000 dynes)	Dyne	**Newton (N)**
Temperature: Fahrenheit (°F) $\left(=\frac{9}{5}{}^\circ C + 32\right)$	Celsius or Centigrade (°C) $\left(=\frac{5}{9}({}^\circ F - 32)\right)$	Centigrade (°C)	**Kelvin (K)** K = 273.15 + °C
Energy: Foot-pound (ft-lb) (1.356 joules)	Newton-meter (N•m) or joule (J) (0.7376 ft-lb)	Dyne-centimeter or erg (1 joule = 10^7 ergs)	**Joule (J)**
Time: Second (s)	Second (s)	Second (s)	**Second (s)**

1.2.1 Prefixes

It is apparent that large and small numbers will frequently be encountered in engineering. To ease the difficulty of mathematical operations with numbers of such varying size, powers of ten are usually employed. Specific powers of ten in engineering notation have been assigned prefixes and symbols, as appearing in Table 1.2

1.3 Electrical quantities

1.3.1 Current

Consider a short length of copper wire cut with an imaginary perpendicular plane, producing the circular cross section shown in Figure. 1.1. At room temperature with no external forces applied, there exists within the copper wire the random motion of free electrons created by the thermal energy that the electrons gain from the surrounding medium. When atoms lose their free electrons, they acquire a net positive charge and are referred to as positive ions. The free electrons are able to move within these positive ions and leave the general area of the parent atom, while the positive ions only oscillate in a mean fixed position. For this reason, the free electron is the charge carrier in a copper wire or any other solid conductor of electricity. Free electrons continually gain or lose energy by virtue of their changing direction and velocity. The causes of the

Multiplication Factors	SI Prefix	SI Symbol
$1\ 000\ 000\ 000\ 000 = 10^{12}$	tera	T
$1\ 000\ 000\ 000 = 10^{9}$	giga	G
$1\ 000\ 000 = 10^{6}$	mega	M
$1\ 000 = 10^{3}$	kilo	k
$0.001 = 10^{-3}$	milli	m
$0.000\ 001 = 10^{-6}$	micro	μ
$0.000\ 000\ 001 = 10^{-9}$	nano	n
$0.000\ 000\ 000\ 001 = 10^{-12}$	pico	p

1,000,000 ohms	1×10^{6} ohms	1 megaohms (MΩ)
100,000 meters	100×10^{3} meters	100 kilometers (km)
0.0001 second	0.1×10^{-3} second	0.1 millisecond (ms)
0.000001 farad	1×10^{-6} farad	1 microfarad (µF)

motion is by collisions with positive ions and other electrons.
attractive forces for the positive ions. force of repulsion that exists between electrons.

This random motion of free electrons is such that over a period of time, the number of electrons moving to the right across the circular cross section is exactly equal to the number passing over to the left. With no external forces applied, the net flow of charge in a conductor in any one direction is zero.

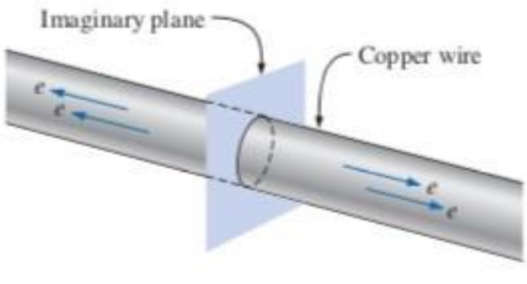


Figure 1.1: Random motion of electrons in a copper wire with no external “pressure” (voltage) applied

Let us connect copper wire between two battery terminals and a light bulb, as shown in Figure. 1.2, to create the simplest of electric circuits. The battery places a net positive charge at one terminal and a net negative charge

on the other. The instant the final connection is made, the free electrons (of negative charge) will drift toward the positive terminal, while the positive ions left behind in the copper wire will simply oscillate in a mean fixed position. The negative terminal is a "supply" of electrons to be drawn from when the electrons of the copper wire drift toward the positive terminal. The flow of charge (electrons) through the bulb will cause the bulb to glow. If electrons drift at uniform velocity through the imaginary circular cross section of Figure. 1.2 in 1 second, the flow of charge, or current, is said to be 1 ampere (A). The current associated with only a few electrons per second would be of little practical value.
To establish numerical values that permit immediate comparisons between levels, a coulomb (C) of charge is defined as the total charge associated with 6.242 $\times 10^{18}$ electrons. The current level in a conductor is measured with ammeter. The current in amperes can now be calculated using the following equation:

$$I = \frac{Q}{t}, I = Amperes(A), Q = coulombs(C), t = seconds(s) \quad (1.2)$$

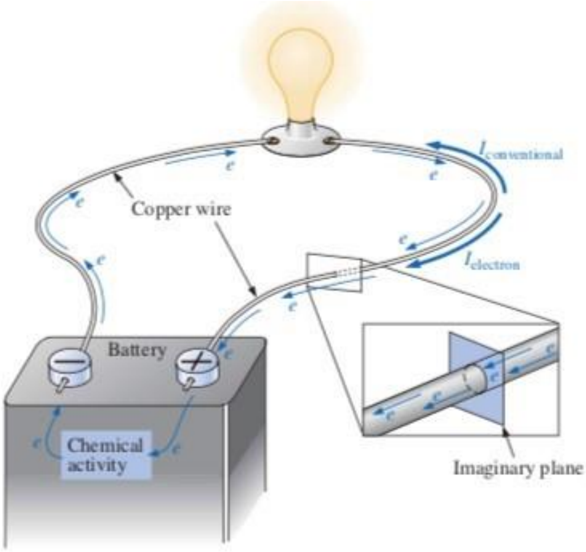


Figure 1.2: Basic electric circuit

Example 1.1
The charge flowing through the imaginary surface of Figure. 1.2 is 0.16 C every 64 ms. Determine the current in amperes.

$$I = \frac{Q}{t} = \frac{0.16C}{64 \times 10^{-3}s} = \frac{160 \times 10^{-3}}{64 \times 10^{-3}} = 2.5A$$

1.3.2 Voltage

The flow of charge (current) is established by an external "pressure" derived from the energy that a mass has by virtue of its position potential energy. The energy is caused by the diffence in height. A "positioning" of the charges are established that will result in a potential difference between the terminals. If a conductor is connected between the terminals of the battery, the electrons at the negative terminal have sufficient potential energy to overcome collisions with other particles in the conductor and the repulsion from similar charges to reach the positive terminal to which they are attracted. The potential difference between two point is measured by Voltmeter.
Charge can be raised to a higher potential level through the expenditure of energy from an external source, or it can lose potential energy as it travels through an electrical system. A potential difference of 1 volt (V) exists between two points if 1 joule (J) of energy is exchanged in moving 1 coulomb (C) of charge between the two points. The unit of measurement is volt (honor Alessandro Volta). Voltage is therefore an indication of how much energy is involved in moving a charge between two points in an electrical system. The potential difference between two points is determined by

$$v = \frac{W}{Q}, W = Energy, Q = Charge \quad (1.3)$$

Example 1.2

Find the potential difference between two points in an electrical system if 60 J of energy are expended by a charge of 20 C between these two points.

$$v = \frac{W}{Q} = \frac{60J}{20C} = 3V$$

An occasional source of confusion is the terminology applied to this subject matter. Terms commonly encountered include potential, potential difference, voltage, voltage difference (drop or rise), and electromotive force. Some are used inter- changeably.

Potential: The voltage at a point with respect to another point in the electrical system. Typically the reference point is ground, which is at zero potential.
Potential difference: The algebraic difference in potential (or voltage) between two points of a network.
Voltage: When isolated, like potential, the voltage at a point with respect to some reference such as ground (0 V).
Voltage difference: The algebraic difference in voltage (or potential) between two points of the system. A voltage drop or rise is as the terminology would suggest.
Electromotive force (emf): The force that establishes the flow of charge (or current) in a system due to the application of a difference in potential. It is often associated primarily with sources of energy.

1.3.3 Resistance

The flow of charge through any material encounters an opposing force similar to mechanical friction. This opposition, due to the collisions between electrons and between electrons and other atoms in the material, which converts electrical energy into another form of energy such as heat, is called the resistance of the material. The unit of measurement of resistance is ohm(Ω). The circuit symbol for resistance appears in Fig. 1.3 with the graphic abbreviation for resistance (R). The resistance of any material with a uniform cross-sectional area is determined by Material, Length, Cross-sectional area and Temperature. Resistance is directly proportional to length and inversely proportional to area. As the temperature of most conductors increases, the increased motion of the particles within the molecular structure makes it increasingly difficult for the "free" carriers to pass through, and the resistance level increases. At a fixed temperature of 20 C (room temperature), the resistance is related to the other three factors by

$$R = \rho\frac{l}{A}(ohms, \Omega) \quad (1.4)$$

where ρ is a characteristic of the material called the resistivity, l is the length of the sample, and A is the cross-sectional area of the sample. Resistors are made in many forms, but all belong in either of two groups: fixed or variable. The most common of the low-wattage, fixed-type resistors is the molded carbon composition resistor. The relative sizes of all fixed and variable resistors change with the wattage (power) rating, increasing in size for increased wattage ratings in order to withstand the higher currents and dissipation losses. Variable resistors have a terminal resistance that can be varied by turning a dial, knob, screw, or whatever seems appropriate for the application. The ohmmeter is an instrument use in measuring resistance.
By finding the reciprocal of the resistance of a material, we have a measure of how well the material will conduct electricity. The quantity is called conductance, has the symbol G, and is measured in siemens (S). In equation form, conductance is

$$G = \frac{1}{R} = \frac{A}{\rho l}(siemens, S) \quad (1.5)$$

Figure 1.3: Resistance

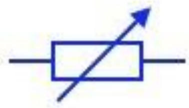


Figure 1.4: Variable Resistance

1.4 Capacitors

A capacitor is a passive element designed to store energy in its electric field. A capacitor consists of two conducting plates separated by an insulator (or dielectric). In many practical applications, the plates may be aluminium foil while the dielectric may be air, ceramic, paper, or mica. A capacitor is typically constructed as depicted in figure 1.5. When a voltage source v is connected to the capacitor, as in Figure 1.6, the source deposits a positive charge q on one plate and a negative charge -q on the other. The capacitor is said to store the electric charge. The amount of charge stored, represented by q, is directly proportional to the applied voltage v so that;

$$q = CV \tag{1.6}$$

where C, the constant of proportionality, is known as the capacitance of the capacitor. The unit of capacitance is the farad (F), (English physicist Michael Faraday). Capacitance is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads (F). The capacitance C of a capacitor is the ratio of the charge q per plate to the applied voltage v, it does not depend on q or v. It depends on the physical dimensions of the capacitor. The capacitance is given by

$$C = \frac{\varepsilon A}{d} \tag{1.7}$$

where A is the surface area of each plate, d is the distance between the plates, and is the permittivity of the dielectric material between the plates. when the voltage across a capacitor is not changing with time (i.e., dc voltage), the current through the capacitor is zero. A capacitor is an open circuit to dc. However, if a battery (dc voltage) is connected across a capacitor, the capacitor charges. The voltage on the capacitor must be continuous. the voltage across a capacitor may take the form shown in Figure 1.7 (a), whereas it is not physically possible for the capacitor voltage to take the form shown in Figure 1.7 (b) because of the abrupt change. Figure 1.8 shows the circuit symbols for fixed and variable capacitors. The energy stored in the capacitor is therefore; $W = \frac{1}{2}CV^2$ (1.8)

Example 1.3

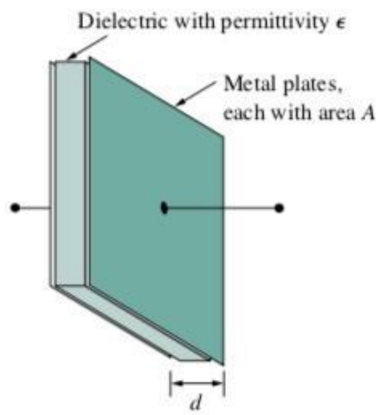


Figure 1.5: Typical capacitor

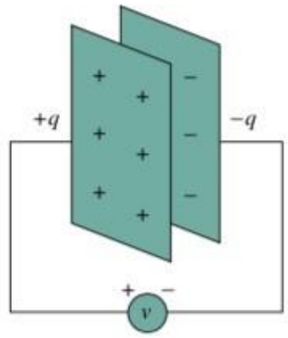


Figure 1.6: A capacitor with applied voltage V

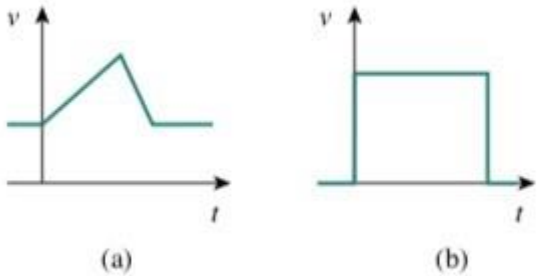


Figure 1.7: Voltage across a capacitor: (a)allowed, (b)not allowable; an abrupt change is not possible

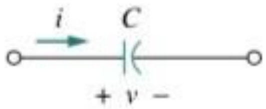


Figure 1.8: Circuit symbols for capacitors

(a) Calculate the charge stored on a 3-pF capacitor with 20 V across it. (b) Find the energy stored in the capacitor.
Solution a.
q = cv

$$q = 3 \times 10^{-12} = 60pC$$

b. The stored energy is

$$W = \frac{1}{2}CV^2 = \frac{1}{2} \times 3 \times 10^{-12} \times 400$$
$$= 600pJ$$

1.5 Inductors

An inductor is a passive element designed to store energy in its magnetic field. Inductors find numerous applications in electronic and power systems. They are used in power supplies, transformers, radios, TVs, radars, and electric motors. An inductor consists of a coil of conducting wire. If current is allowed to pass through an inductor, it is found that the voltage across the inductor is directly proportional to the time rate of change of the current

$$v = L\frac{di}{dt} \tag{1.9}$$

where L is the constant of proportionality called the inductance of the inductor. The unit of inductance is the henry (H), Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it measured in henrys (H). The power delivered to the inductor is;

$$W = \frac{1}{2}Li^2 \tag{1.10}$$

The voltage across an inductor is zero when the current is constant. Thus, an inductor acts like a short circuit to dc. The current through an inductor cannot change instantaneously. For example, the current through an inductor may take the form shown in Figure 1.9(a), whereas the inductor current cannot take the form shown in Figure 1.9 (b) in real-life situations due to the discontinuities. The circuit symbols for inductors are shown in figure 1.10.

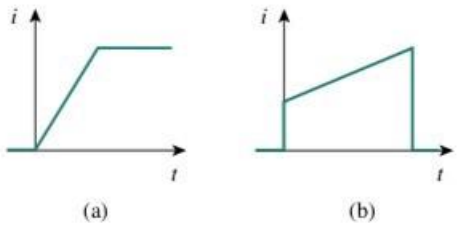


Figure 1.9: Current through an inductor: (a)allowed, (b)not allowable; an abrupt change is not possible.

Figure 1.10: Circuit symbol for inductors

Example 1.4

The current through a 0.1-H inductor is

$$I(t) = 10te^{-5t}A$$

Find the voltage across the inductor and the energy stored in it. **Solution**

$$v = L\frac{di}{dt}$$

and

$$L = 0.1H$$

$$v = 0.1\frac{dy}{dx}(10te^{-5t}) = e^{-5t} + t(-5)e^{-5t} = e^{-5t}(1-5t)V$$

The energy stored is

$$W = \frac{1}{2}Li^2 = \frac{1}{2}(0.1)100t^2e^{-10t} = 5t^2e^{-10t}J$$

1.6 DC supplies -dc Voltage sources and dc current sources

Two types of current are readily available to the consumer today. One is direct current (dc), in which ideally the flow of charge (current) does not change in magnitude (or direction) with time. Direct current encompasses the various electrical systems in which there is a unidirectional ("one direction") flow of charge The other is sinusoidal alternating current (ac), in which the flow of charge is continually changing in magnitude (and direction) with time. The symbol used for all dc voltage supplies is shown in Figure 1.11. Dc voltage sources can be divided into three broad categories: batteries (chemical action), generators (electromechanical), and power supplies (rectification). Ideally a dc voltage source will provide a fixed terminal voltage, even though the current demand from the electrical/electronic system may vary. A dc current source is the dual of the voltage source; that is, the current source will supply, ideally, a fixed current to an electrical/electronic system, even though there may be variations in the terminal voltage as determined by the system. The symbol for dc current source is shown in figure 1.12 A current source determines the current in the branch in which it is located. The magnitude and polarity of the voltage across a current source are a function of the network to which it is applied.

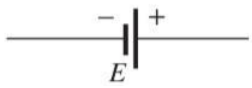


Figure 1.11: DC voltage.

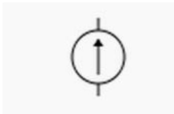


Figure 1.12: DC current source

1.6.1 Source conversations

The current source described is called an ideal source due to the absence of any internal resistance. In reality, all sources—whether they are voltage or current—have some internal resistance in the relative positions shown in Figs. 1.13 and 1.14. For the voltage source, if Rs = 0 Ω or is so small compared to any series resistor that it can be ignored, then we have an "ideal" voltage source. For the current source, if Rs =∞ Ω or is large enough compared to other parallel ele- ments that it can be ignored, then we have an "ideal" current source. We want the equivalence to ensure that the applied load of Figs. 1.13 and 1.14 will receive the same current, voltage, and power from each source and in effect not know, or care, which source is present. In Fig. 1.13 if we solve for the load current I_L, we obtain

$$I_L = \frac{E}{R_S + R_L} \tag{1.11}$$

If we multiply this by a factor of 1,

$$I_L = \frac{(1)E}{R_S + R_L} = \frac{(R_S/R_S)E}{R_S + R_L} = \frac{R_S(E/R_S)}{R_S + R_L} = \frac{R_S I}{R_S + R_L} \tag{1.12}$$

If we define I = E/Rs, Equation (1.12) is the same as that obtained by applying the current divider rule to the network of Fig. 1.14. The result is an equivalence between the networks of Figs. 1.13 and 1.14 that simply requires that I = E/R_s and the series resistor R_s of Fig. 1.13 be placed in parallel, as in Fig. 1.14.

For clarity, the equivalent sources, as far as terminals a and b are concerned, are repeated in Fig. 1.15 with

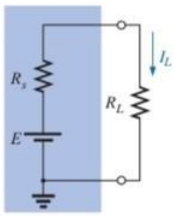


Figure 1.13: Practical voltage source.

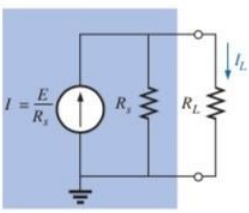


Figure 1.14: Practical current source

the equations for converting in either direction. Note, as just indicated, that the resistor R_s is the same in each source; only its position changes. The current of the current source or the voltage of the voltage source is determined using Ohm's law and the parameters of the other configuration

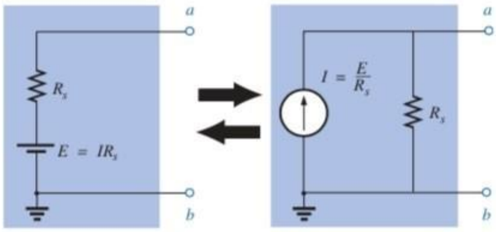


Figure 1.15: Source conversion

Example 1.5

1. Convert the voltage source of Fig.1.16(a) to a current source, and calculate the current through the 4-Ω load for each source.
2. Replace the 4-Ω load with a 1-kΩ load, and calculate the current IL for the voltage source.
3. Repeat the calculation of part (b) assuming that the voltage source is ideal (Rs = 0 Ω) because RL is somuch larger than Rs. Is this one of those situations where assuming that the source is ideal is an appropriate approximation?

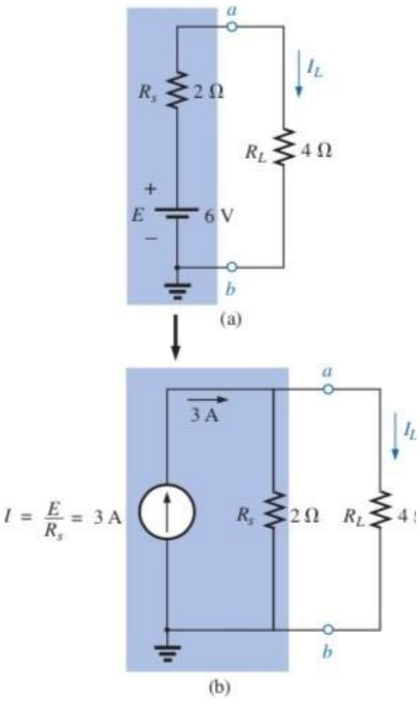


Figure 1.16: Example

Solution

a. See Fig 1.16

$$Fig.1.16(a) : I_L = \frac{E}{R_S + R_L} = \frac{6V}{2\Omega + 4\Omega} = 1$$

$$Fig.1.16(b) : I_L = \frac{R_S I}{R_S + R_L} = \frac{(2\Omega)(3A)}{2\Omega + 4\Omega} = 1A$$

b.

$$I_L = \frac{E}{R_S + R_L} = \frac{6V}{2\Omega + 1k} = 5.99mA$$

$$I_L = \frac{E}{R_L} = \frac{6V}{1k} = 6mA \cong 5.99mA$$

c.

$$Yes, R_L \gg R_S (voltagesource)$$

Chapter 2

DC Circuit Theory

At the end of the chapter students should be able to:

1. State and apply Ohm's law.
2. Define electrical power and energy, and perform simple calculations involving power and energy.
3. Understand basics terminologies such as branch, node and loop as used in electric circuits.
4. State Kirchoff's Voltage and current laws and perform calculations using the law.
5. Understand Voltage and current divider rule.
6. Understand open-circuits and short-circuits.
7. Understand Wye-delta transformations and its application.
8. Perform basic electrical circuit calculations using mesh and nodal analysis.
9. Apply Superposition and Reciprocity theorem in dc electrical circuits.

2.1 Ohms Law

An excellent analogy for the simplest of electrical circuits is the water in a hose connected to a pressure valve. Think of the electrons in the copper wire as the water in the hose, the pressure valve as the applied voltage, and the size of the hose as the factor that determines the resistance. If the pressure valve is closed, the water simply sits in the hose without motion, much like the electrons in a conductor without an applied voltage. When the pressure valve is open, water flows through the hose much like the electrons in a copper wire when the voltage is applied.

In other words, the absence of the "pressure" in one case and the voltage in the other will simply result in a system without motion or reaction. The rate at which the water will flow in the hose is a function of the size of the hose. A hose with a very small diameter will limit the rate at which water can flow through the hose, just as a copper wire with a small diameter will have a high resistance and will limit the current. This relationship between current and voltage for a resistor is known as Ohm's law. Ohm's law states that the voltage v across a resistor is directly proportional to the current i flowing through the resistor. The law clearly reveals that for a fixed resistance, the greater the voltage (or pressure) across a resistor, the more the current, and the more the resistance for the same voltage, the less the current. In other words, the current is proportional to the applied voltage and inversely proportional to the resistance.

$$i = \frac{v}{R}(ampere, A) \tag{2.1}$$

Example 2.1

Determine the current resulting from the application of a 9-V battery across a network with a resistance of 2.2 Ω.

$$I = \frac{V}{R} = \frac{9V}{2.2\Omega} = 4.09A$$

Example 2.2
Calculate the resistance of a 60-W bulb if a current of 500 mA results from an applied voltage of 120 V.

$$R = \frac{V}{I} = \frac{120V}{500 \times 10^{-3}A} = 240\Omega$$

2.2 Power

Power is an indication of how much work (the conversion of energy from one form to another) can be done in a specified amount of time, that is, a rate of doing work. For instance, a large motor has more power than a small motor because it can convert more electrical energy into mechanical energy in the same period of time. Since converted energy is measured in joules (J) and time in seconds (s), power is measured in joules/second (J/s). The electrical unit of measurement for power is the watt (W), defined by 1 joule/second (J/s). In equation form, power is determined by ;

$$P = \frac{W}{t}(watts, W, or J/s) \tag{2.2}$$

horsepower (hp) is a measure of the average power of a strong dray horse over a full working day. The horsepower and watt are related in the following manner: 1 hp=746 watts. The power delivered to, or absorbed by, an electrical device or system can be found in terms of the current and voltage by first substituting equation 1.3 into 2.2.

$$P = \frac{W}{t} = \frac{QV}{t} = V\frac{Q}{t}$$

But

$$I = \frac{Q}{t}$$

Therefore

$$P = VI \tag{2.3}$$

By direct substitution of Ohm's law, the equation for power can be obtained in two other forms:

$$P = VI = V(\frac{V}{R})(watts)$$

And

$$P = VI = \frac{V^2}{R}(watts$$

$$P = VI = (IR)I\ ) \tag{2.4}$$

And

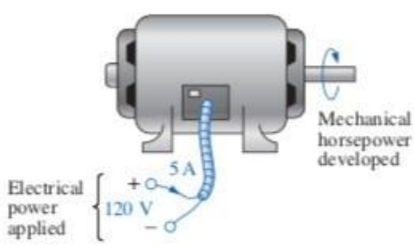
$$P = VI = I^2R(watts) \tag{2.5}$$

2.3 Energy

Energy is the capacity to do work, measured in Joules (J). For power, which is the rate of doing work, to produce an energy conversion of any form, it must be used over a period of time. For example, a motor may have the power to run a heavy load, but unless the motor is used over a period of time, there will be no energy conversion. Therefore, the longer the motor is used to drive the load, the greater will be the energy expended. The energy (W) lost or gained by any system is therefore determined by

$$W = Pt \tag{2.6}$$

Example 2.3
Find the power delivered to the dc motor of Figure. 2.1.



PABLO

Figure 2.1: Example

$$P = VI = (120 \times 5) = 600W = 0.6kW$$

Example 2.4

What is the power dissipated by a 5 Ω resistor if the current is 4 A?

$$P = I^2R = 4^2 \times 5\Omega = 80W$$

Example 2.5

How much energy (in kilowatthours) is required to light a 60-W bulb continuously for 1 year (365 days)?

$$W = \frac{Pt}{1000} = \frac{60W \times (24\frac{h}{day})(365days)}{1000} = \frac{525,600Wh}{1000} = 525.60kWh$$

2.4 Definition of Basic terms

The elements of an electric circit can be interconnected in several ways . Some basic terminologies used are branches, nodes, loops.

A branch represents a single element such as a voltage source or a resistor. The circuit in Fig. 2.2 has five branches, namely, the 10-V voltage source, the 2-A current source, and the three resistors.

A node is the point of connection between two or more branches. A node is usually indicated by a dot in a circuit. The circuit in Fig. 2.2 has three nodes a, b, and c. Notice that the three points that form node b are connected by perfectly conducting wires and therefore constitute a single point.

A loop is any closed path in a circuit. A loop is a closed path formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once. A loop is said to be independent if it contains a branch which is not in any other loop.

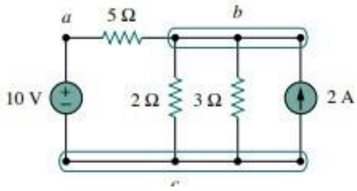


Figure 2.2:

2.5 Voltage sources in series

Voltage sources can be connected in series, as shown in Fig. 2.3, to increase or decrease the total voltage applied to a system. The net voltage is determined simply by summing the sources with the same polarity and subtracting the total of the sources with the opposite “pressure.” The net polarity is the polarity of the larger sum.

In Fig. 2.3 (a), for example, the sources are all “pressuring” current to the right, so the net voltage is

$$E_T = E_1 + E_2 + E_3 = 10V + 6V + 2V = 18V$$

as shown in the figure. In Fig. 2.3(b), however, the greater “pressure” is to the left, with a net voltage of

$$E_T = E_2 + E_3 - E_1 = 9V + 3V - 4V = 8V$$

2.6 Kirchoff's Law

2.6.1 Kirchoff's Voltage Law

Kirchhoff's voltage law (KVL) states that the algebraic sum of the potential rises and drops around a closed loop (or path) is zero. i.e. the applied voltage of a series circuit equals the sum of the voltage drops across

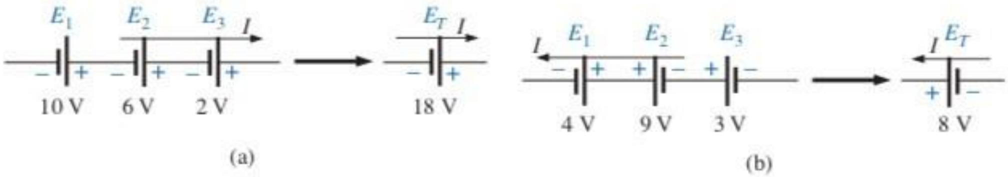


Figure 2.3:

the series elements. A closed loop is any continuous path that leaves a point in one direction and returns to that same point from another direction without leaving the circuit. In Fig. 2.4, by following the current, we can trace a continuous path that leaves point a through R_1 and returns through E without leaving the circuit. Therefore, abcda is a closed loop. For us to be able to apply Kirchhoff's voltage law, the summation of potential rises and drops must be made in one direction around the closed loop. A plus sign is assigned to a potential rise (- to +), and a minus sign to a potential drop (+ to -). In symbolic form, where Σ represents summation, and V the potential drops and rises, we have

$$\Sigma V = 0 \tag{2.7}$$

Applying KVL (clockwise direction, following the current I and starting at point d)

$$+E - V_1 - V_2 = 0$$

or

$$E = V_1 + V_2$$

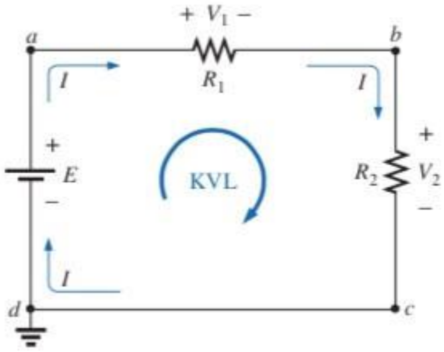


Figure 2.4:

Example 2.6

For the circuit of Fig. 2.5 : a. Find RT

b. Find I.

c. Find V1 and V2

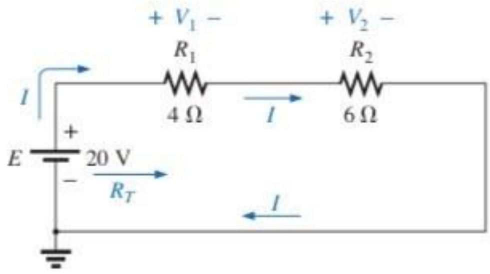


Figure 2.5:

a.

$$R_T = R_1 + R_2 = 4\Omega + 6\Omega = 10\Omega$$ b.

$$I = \frac{E}{R_T} = \frac{20V}{10V} = 2A$$

c.

$$V_1 = IR_1 = (2A)(4\Omega) = 8V$$
$$V_2 = IR_2 = (2A)(6\Omega) = 12V$$

2.6.2 Kirchoff's Current Law

Kirchhoff's current law (KCL) states that the algebraic sum of the currents entering and leaving an area, system, or junction is zero. In other words, the sum of the currents entering an area, system, or junction must equal the sum of the currents leaving the area, system, or junction. In equation form:

$$\Sigma I_{entering} = \Sigma I_{leaving} \tag{2.8}$$

In Fig. 2.6, for instance, the shaded area can enclose an entire system, a complex network, or simply a junction of two or more paths. In each case the current entering must equal that leaving, as witnessed by the fact that

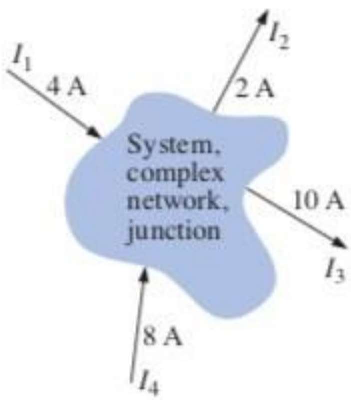


Figure 2.6:

$$I_1 + I_4 = I_2 + I_3$$
$$4A + 8A = 2A + 10A$$

2.7 Series Circuits

A circuit consists of any number of elements joined at terminal points, providing at least one closed path through which charge can flow. The circuit of Figure 2.7 has three elements joined at three terminal points (a, b, and c) to provide a closed path for the current I.

Two elements are in series if

1. They have only one terminal in common (i.e., one lead of one is connected to only one lead of the other).
2. The common point between the two elements is not connected to another current-carrying element.

2.7.1 Resistors In Series

In Fig. 2.7, the resistors R_1 and R_2 are in series because they have only point b in common. The other ends of the resistors are connected elsewhere in the circuit. For the same reason, the battery E and resistor R_1 are in series (terminal a in common), and the resistor R_2 and the battery E are in series (terminal c in common).

Since all the elements are in series, the network is called a series circuit. For series circuit, the current is the same through series elements.

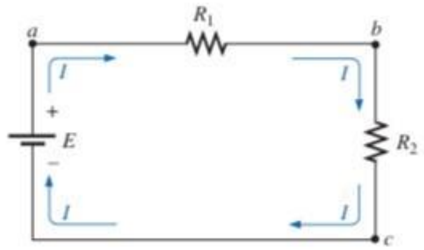


Figure 2.7: Series circuit

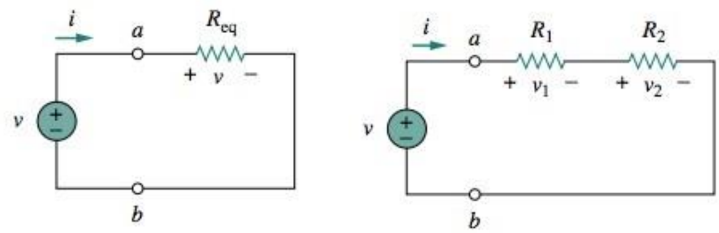


Figure 2.8: Applying

Ohm's law to each of the resistors, we obtain

$$v_1 = iR_1, v_2 = iR_2 \tag{2.9}$$

If we apply KVL to the loop (moving in the clockwise direction), we have

$$-v + v_1 + v_2 = 0 \tag{2.10}$$

$$v = v_1 + v_2 = i(R_1 + R_2) \tag{2.11}$$

or

$$i = \frac{v}{R_1 + R_2}$$

equation 2.11 can be written as.

$$v = iR_{eq}$$

implying that the two resistors can be replaced by an equivalent resistor Req; that is,

$$R_{eq} = R_1 + R_2$$

Thus, Fig. 2.7 can be replaced by the equivalent circuit in Fig. 2.8. The two circuits in Figs. 2.7 and 2.8 are equivalent because they exhibit the same voltage-current relationships at the terminals a-b. The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances. For N resistors in series then,

$$[R_{eq} = R_1 + R_2 + ... + R_N = \sum_{n=1}^{N} R_n \qquad (2.12)$$

2.8 Voltage Divider Rule

In a series circuit, the voltage across the resistive elements will divide as the magnitude of the resistance levels. Voltage divider rule permits the voltage levels across reisistors in series without first finding the current. the voltage divider rule states that the voltage across a resistor in a series circuit is equal to the value of that resistor times the total impressed voltage across the series elements divided by the total resistance of the series elements. In general, if a voltage divider has N resistors ($R_1, R_2, \dots, R_N$) in series with the source voltage v, the nth resistor (Rn) will have a voltage drop of

$$v_n = \frac{R_n}{R_1 + R_2 + ... + R_N} v \qquad (2.13)$$

Example 2.7

Using the voltage divider rule, determine the voltages V_1 and V_3 for the series circuit of Fig. 2.9.

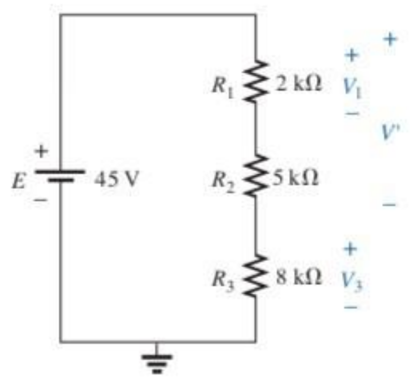


Figure 2.9:

Solution

$$v_1 = \frac{R_1 E}{R_T} = \frac{(2k\Omega)(45V)}{2k\Omega + 5k\Omega + 8k\Omega} = \frac{(2k\Omega)(45V)}{15k\Omega}$$
$$= \frac{(2 \times 10^3 \Omega)(45V)}{15 \times 10^3 \Omega} = \frac{90V}{15} = 6V$$

$$v_3 = \frac{R_3 E}{R_T} = \frac{(8k\Omega)(45V)}{15k\Omega} = \frac{(8 \times 10^3 \Omega)(45V)}{15 \times 10^3 \Omega}$$
$$= \frac{360V}{15} = 24V$$

2.9 Parallel Circuits

Two network configurations, series and parallel, form the framework for some of the most complex network structures. Two elements, branches, or networks are in parallel if they have two points in common.

2.9.1 Resistor In Parallel

In Fig. 2.10, for example, elements 1 and 2 have terminals a and b in common; they are therefore in parallel. The total resistance of parallel resistors is always less than the value of the smallest resistor.

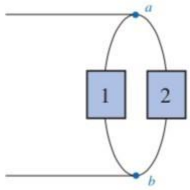


Figure 2.10:

Consider the circuit in Fig. 2.11, where two resistors are connected in parallel and therefore have the same voltage across them. From Ohm's law,

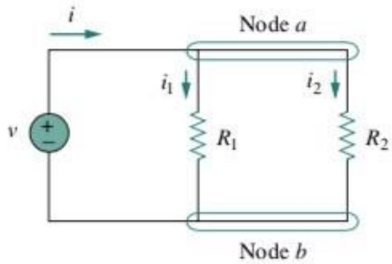


Figure 2.11:

$$v = i_1 R_1 = i_2 R_2 \tag{2.14}$$

$$i_1 = \frac{v}{R_1}, i_2 = \frac{v}{R_2} \tag{2.15}$$

Applying KCL at node a gives the total current i as

$$i = i_1 + i_2 \tag{2.16}$$

Substituting eq. (2.15) into (2.16)

$$i = \frac{v}{R_1} + \frac{v}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V}{R_{eq}}$$

where Req is the equivalent resistance of the resistors in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \tag{2.17}$$

The equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum. We can extend the result in Eq. (2.17) to the general case of a circuit with N resistors in parallel. The equivalent resistance is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N} \tag{2.18}$$

Example 2.8

Find the currents and voltages in the circuit.

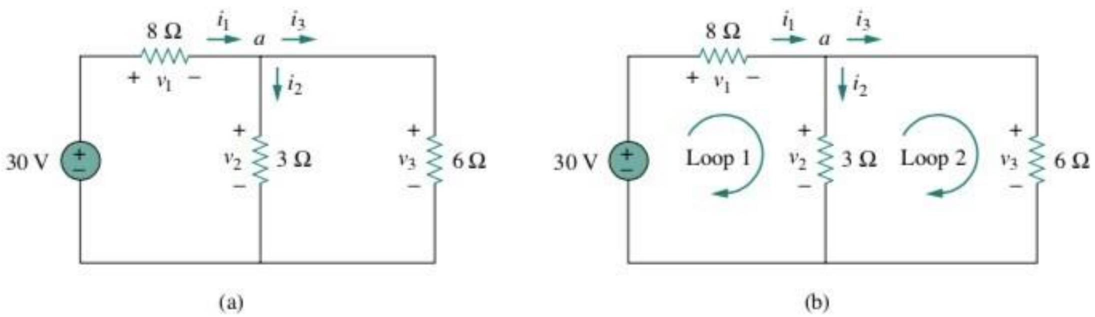


Figure 2.12:

Solution

We apply Ohm's law and Kirchoff's laws. By Ohm's law,

$$v = 8i_1, v = 3i_2, v = 6i_3$$

Since the voltage and current of each resistor are related by Ohm's law as shown, we are really looking for three things: (v_1,v_2,v_3) or (i_1,i_2,i_3). At node a, KCL gives

$$i_1 - i_2 - i_3 = 0$$

Applying KVL to loop 1 as in Fig. 2.11(b).

$$-30 + v_1 + v_2 = 0$$

We express this in terms of i_1 and i_2

$$-30 + 8i_1 + 3i_2 = 0$$

or

$$i_1 = \frac{(30 - 3i_2)}{8}$$

Applying KVL to loop 2.

$$-v_2 + v_3 = 0 \Rightarrow v_3 + v_2$$

as expected since the two resistors are in parallel. We express v_1 and v_2 in terms of i_1 and i_2.

$$6i_3 = 3i_2 \Rightarrow i_3 = \frac{i_2}{2}$$

$$\frac{(30 - 3i_2)}{8} - i_2 - \frac{i_2}{2} = 0$$

$$i_2 = 2A$$

From the value of i_2;

$$i_1 = 3A, i_3 = 1A, v_1 = 24V, v_2 = 6V, v_3 = 6V$$

Example 2.9

Find R_{eq} for the circuit shown in Fig. 2.13.

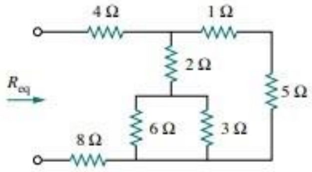


Figure 2.13:

Solution

To get R_{eq}, we combine resistors in series and in parallel. The 6-Ω and 3-Ω resistors are in parallel, so their equivalent resistance is

$$6\Omega \left\| 3\Omega = \frac{6 \times 3}{6+3} = 2\Omega \right.$$

Also, the 1-Ω and 5-Ω resistors are in series; hence their equivalent resistance is

$$1\Omega + 5\Omega = 6\Omega$$

In Fig. 2.14(a), we notice that the two 2-Ω resistors are in series, so the equivalent resistance is

$$2\Omega + 2\Omega = 4\Omega$$

This 4-Ω resistor is now in parallel with the 6-Ω resistor in Fig. 2.14(a); their equivalent resistance is

$$4\Omega \left\| 6\Omega = \frac{4 \times 6}{4+6} = 2.4\Omega \right.$$

The circuit in Fig. 2.14(a) is now replaced with that in Fig. 2.14 (b). In Fig. 2.14(b), the three resistors are

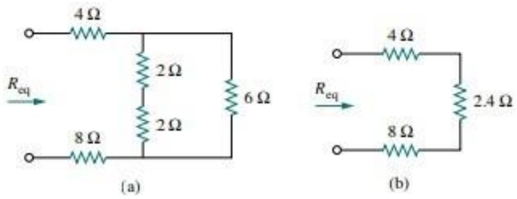


Figure 2.14:

in series. Hence, the equivalent resistance for the circuit is

$$R_{eq} = 4\Omega + 2.4\Omega + 8\Omega = 14.4\Omega$$

Try.

1. Calculate the equivalent resistance R_{ab} in the circuit in Fig. 2.15. **Answer:** 11.2Ω

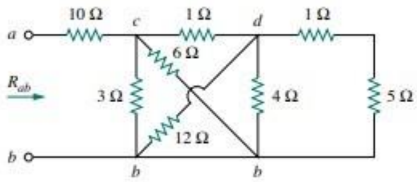


Figure 2.15:

2. Find R_{ab} for the circuit in Fig. 2.16.

Answer: 11Ω

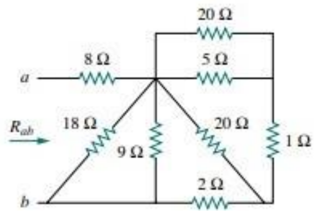


Figure 2.16:

2.10 Current Divider Rule

The current divider rule (CDR) will determine how the current entering a set of parallel branches will split between the elements. For two parallel elements of equal value, the current will divide equally. For parallel elements with different values, the smaller the resistance, the greater the share of input current. For parallel elements of different values, the current will split with a ratio equal to the inverse of their resistor values. For the particular case of two parallel resistors, as shown in Fig. 2.17,

$$I_1 = \frac{R_2 I}{R_1 + R_2} \tag{2.19}$$

$$I_2 = \frac{R_1 I}{R_1 + R_2} \tag{2.20}$$

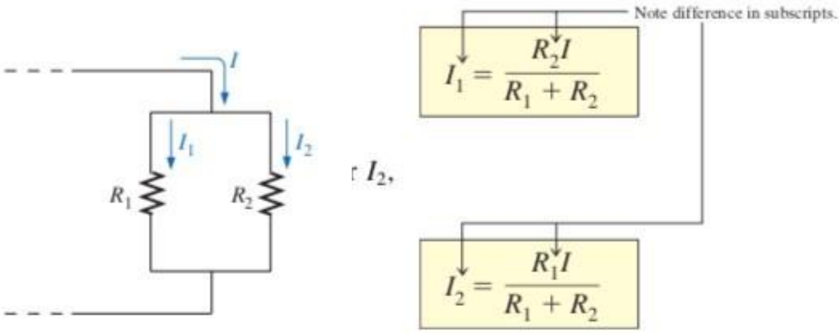


Figure 2.17:

Example 3.0

Determine the magnitude of the currents I_1, I2, and I_3 for the network of Fig. 2.18.

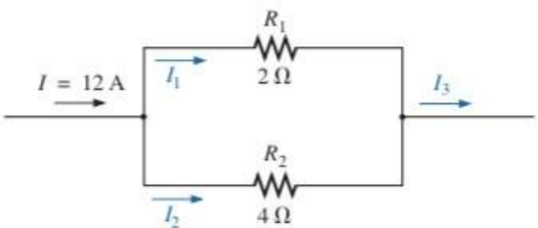


Figure 2.18:

Solution

By Eq. 2.19, the current divider rule,

$$I_1 = \frac{R_2 I}{R_1 + R_2} = \frac{(4\Omega)(12A)}{2\Omega + 4\Omega} = 8A$$

Applying Kirchoff's current law

$$I = I_1 + I_2$$

$$I_2 = I - I_1 = 12A - 8A = 4A$$

or using the current divider rule again,

$$I_2 = \frac{R_1 I}{R_1 + R_2} = \frac{(2\Omega)(12A)}{2\Omega + 4\Omega} = 4A$$

The total current entering the parallel branches must equal that leaving. Therefore,

$$I_3 = I = 12A$$

or

$$I_3 = I_1 + I_2 = 8A + 4A = 12A$$

2.11 Voltage Sources In Parallel

Voltage sources are placed in parallel as shown in Fig. 2.19 only if they have the same voltage rating. The primary reason for placing two or more batteries in parallel of the same terminal voltage would be to increase the current rating (and, therefore, the power rating) of the source. As shown in Fig. 2.19, the current rating of the combination is determined by $I_s = I_1 + I_2$ at the same terminal voltage. The resulting power rating is twice that available with one supply.

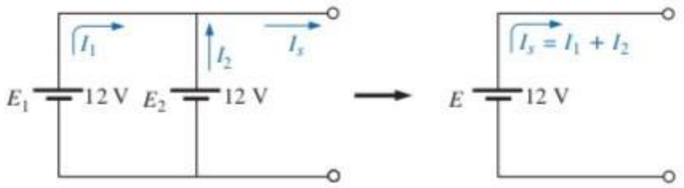


Figure 2.19:

2.12 Voltage Sources In Series

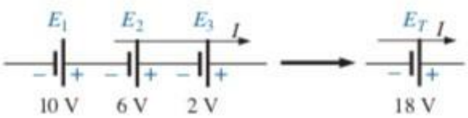
Voltage sources can be connected in series, as shown in Fig. 2.20, to increase or decrease the total voltage applied to a system. The net voltage is determined simply by summing the sources with the same polarity and subtracting the total of the sources with the opposite "pressure." The net polarity is the polarity of the larger sum.

In Fig. 2.20(a), for example, the sources are all "pressuring" current to the right, so the net voltage is

$$E_T = E_1 + E_2 + E_3 = 10V + 6V + 2V = 18V$$

as shown in the figure. In Fig. 2.20(b), however, the greater "pressure" is to the left, with a net voltage of

$$E_T = E_2 + E_3 - E_1 = 9V + 3V - 4V = 8V$$



(a)

2.13 Current Sources In Parallel

If two or more current sources are in parallel, they may all be replaced by one current source having the magnitude and direction of the resultant, which can be found by summing the currents in one direction and subtracting the sum of the currents in the opposite direction.

2.14 Current Sources In Series

The current through any branch of a network can be only single-valued. For the situation indicated at point a in Fig. 2.21, we find by application of Kirchhoff's current law that the current leaving that point is greater than that entering an impossible situation. Therefore, current sources of different current ratings are not connected in series.

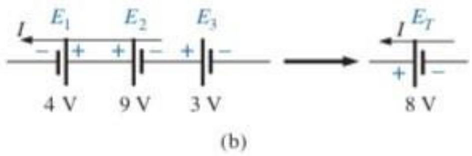


Figure 2.20: Reducing series dc voltage sources to a single source

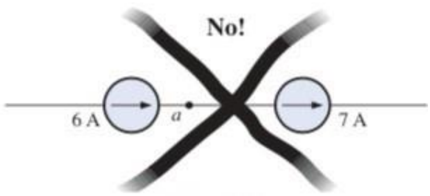


Figure 2.21: Invalid situation

Example 2.9
Reduce the parallel current sources of Fig. 2.22 to a single current source.

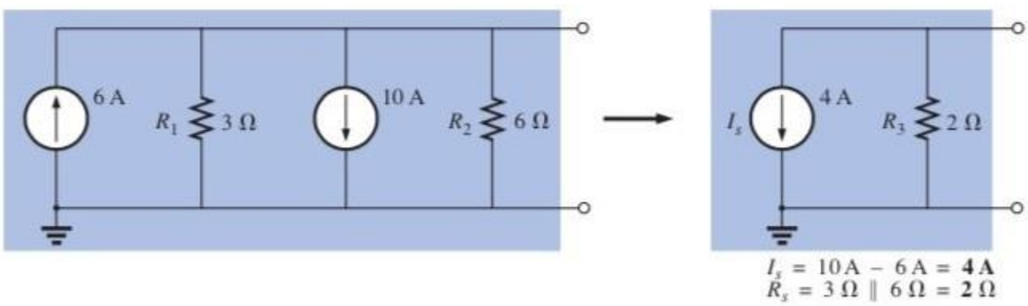


Figure 2.22:

2.15 Open and Short Circuits

An open circuit is simply two isolated terminals not connected by an element of any kind as shown in Fig. 2.23(a). Since a path for conduction does not exist, the current associated with an open circuit must always be zero. The voltage across the open circuit, however, can be any value, as determined by the system it is connected to. Therefore, an open circuit can have a potential difference (voltage) across its terminals, but the current is always zero amperes.

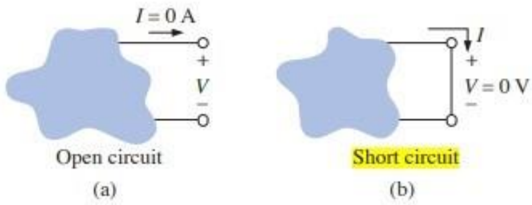


Figure 2.23: Two special network configurations.

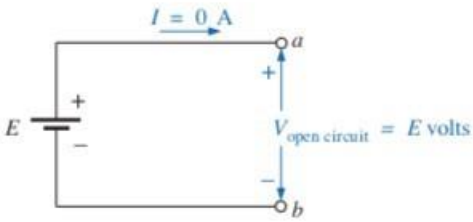


Figure 2.24:

A short circuit is a very low resistance, direct connection between two terminals of a network, as shown in Fig. 2.23(b) . The current through the short circuit can be any value, as determined by the system it is connected to, but the voltage across the short circuit will always be zero volts because the resistance of the short circuit is assumed to be essentially zero ohms and V = IR = I(0) = 0 V. In summary, therefore, a short circuit can carry a current of a level determined by the external circuit, but the potential difference (voltage) across its terminals is always zero volts.

Example 2.9

Determine the voltage V_{ab} for the network of Fig. 2.25.

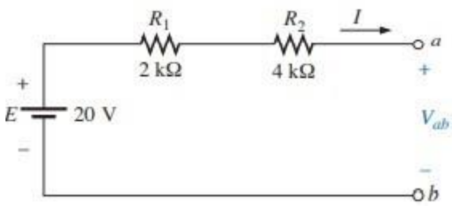


Figure 2.25:

Solution

The open circuit requires that I be zero amperes. The voltage drop across both resistors is therefore zero volts since V = IR = (0)R = 0 V. Applying Kirchhoff's voltage law around the closed loop,

$$V_{ab} = E = 20V$$

2.16 Wye-Delta Transformations

Situations often arise in circuit analysis when the resistors are neither in parallel nor in series. For example, consider the bridge circuit in Fig. 2.26. How do we combine resistors R_1 through R_6 when the resistors are neither in series nor in parallel? Many circuits of the type shown in Fig. 2.26. can be simplified by using three-terminal equivalent networks. These are the wye (Y) or tee (T) network shown in Fig. 2.27 and the (Δ) or (Π) network shown in Fig. 2.28. These networks occur by themselves or as part of a larger network.

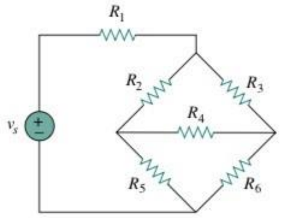


Figure 2.26:

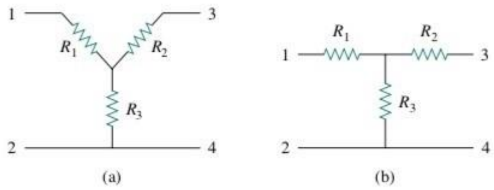


Figure 2.27: Two forms of the same network; (a)Y, (b)T

2.16.1 Delta to Wye Conversion

Suppose it is more convenient to work with a wye network in a place where the circuit contains a delta configuration. We superimpose a wye network on the existing delta network and find the equivalent resistances in the wye network. To obtain the equivalent resistances in the wye network, we compare the two networks and make sure that the resistance between each pair of nodes in the Δ (or Π) network is the same as the resistance between the same pair of nodes in the Y (or T) network. For terminals 1 and 2 in Figs. 2.47 and 2.48, for example,

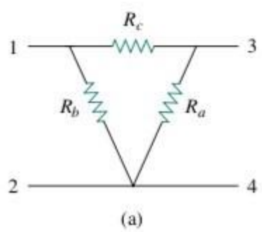


Figure 2.28:

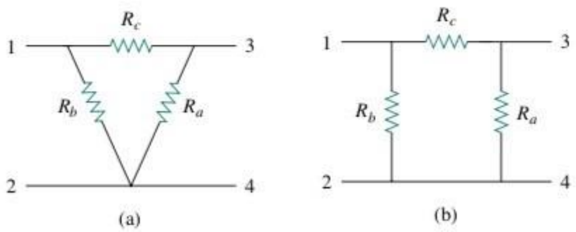


Figure 2.29: Two forms of the same network; (a)Δ, (b)Π

$$R_{12}(Y) = R_1 + R_3 \tag{2.21}$$

$$R_{12}(\Delta) = R_b \| (R_a + R_c)$$

Setting

$$R_{12}(Y) = R_{12}(\Delta)$$

gives

$$R_{12} = R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} \tag{2.22}$$

Similarly,

$$R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} \tag{2.23}$$

$$R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \tag{2.24}$$

Subtracting Eq. (2.24) from Eq. (2.22), we get

$$R_1 - R_2 = \frac{R_c(R_b - R_a)}{R_a + R_b + R_c} \tag{2.25}$$

Adding Eqs.(2.23) and (2.25), we gives

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \tag{2.26}$$

and subtracting Eq.(2.25) from Eq.(2.23) yields

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} \tag{2.27}$$

Subtracting Eq. (2.26) from Eq. (2.22), we obtain

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} \tag{2.28}$$

To transform a Δ network to Y, we create an extra node n as shown in Fig. 2.30 and follow this conversion rule:

Each resistor in the Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors.

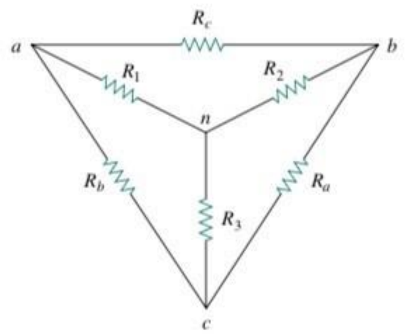


Figure 2.30: Superposition of Y and Δ networks as an aid in transforming one to other network

2.16.2 Wye to Delta Conversion

To obtain the conversion formulas for transforming a wye network to an equivalent delta network, we note from Eqs. (2.26) to (2.28) that

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_aR_bR_c(R_a+R_b+R_c)}{(R_a+R_b+R_c)^2} = \frac{R_aR_bR_c}{R_a+R_b+R_c} \tag{2.29}$$

Dividing Eq. (2.30) by each of Eqs. (2.26) to (2.28) leads to the following equations

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \tag{2.30}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \tag{2.31}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \tag{2.32}$$

From Eqs. (2.30) to (2.32) and Fig. 2.30, the conversion rule for Y to Δ is as follows:

Each resistor in the Δnetwork is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor.

The Y and Δ networks are said to be balanced when

$$R_1 = R_2 = R_3 = R_Y, R_a = R_b = R_c = R_\Delta$$

Under these conditions, conversion formulas become

$$R_Y = \frac{R_\Delta}{3}$$

or

$$R_\Delta = 3R_Y$$

Example 3.0

Obtain the equivalent resistance R_{ab} for the circuit in Fig. 2.31 and use it to find current i.

In this circuit, there are two Y networks and one Δ network. Transforming just one of these will simplify the circuit. If we convert the Y network comprising the 5-Ω, 10-Ω, and 20-Ω resistors, we may select

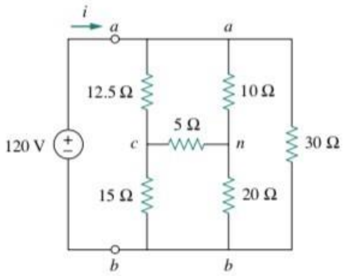


Figure 2.31:

$$R_1 = 10\Omega, R_2 = 20\Omega, R_3 = 5\Omega$$

Thus from Eqs. (2.30) to (2.32) we have

$$R_a = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1} = \frac{10\times20+20\times5+5\times10}{10}$$
$$= \frac{350}{10} = 35\Omega$$
$$R_b = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1} = \frac{350}{20} = 17.5\Omega$$
$$R_c = \frac{R_1R_2+R_2R_3+R_3R_1}{R_3} = \frac{350}{50} = 70\Omega$$

$$70 \Big\| 30 = \frac{70\times30}{70+30} = 21\Omega$$
$$12.5 \Big\| 17.5 = \frac{12.5\times17.5}{12.5+17.5} = 7.2917\Omega$$
$$15 \Big\| 35 = \frac{15\times35}{15+35} = 10.5\Omega$$

so that the equivalent circuit is shown in Fig. 2.32(b). Hence, we find

$$292 + 10.5) \Big\| 21 = \frac{17.792\times21}{17.792+21} = 9.632\Omega$$

Then

$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.632} = 12.458A$$

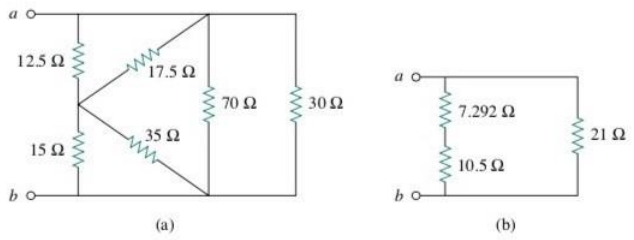


Figure 2.32:

2.17 Methods of Analysis for DC Circuit

2.17.1 Mesh Analysis

Using mesh currents instead of element currents as circuit variables is convenient and reduces the number of equations that must be solved simultaneously. A mesh is a loop that does not contain any other loop within it. mesh analysis applies KVL to find unknown currents. The systematic approach outlined below should be followed when applying this method.

1. Assign a distinct current in the clockwise (anti clockwise) direction to each independent, closed loop of thenetwork.
2. Indicate the polarities within each loop for each resistor as determined by the assumed direction of loopcurrent for that loop.
3. Apply Kirchhoff's voltage law around each closed loop in the clockwise (anticlockwise) direction.

a. If a resistor has two or more assumed currents through it, the total current through the resistor is theassumed current of the loop in which Kirchhoff's voltage law is being applied, plus the assumed currents of the other loops passing through in the same direction, minus the assumed currents through in the opposite direction.

b. The polarity of a voltage source is unaffected by the direction of the assigned loop currents.

4. Solve the resulting simultaneous linear equations for the assumed loop currents.

Example 3.1

For the circuit in Fig. 2.33, find the branch currents I_1, I_2, and I_3 using mesh analysis.

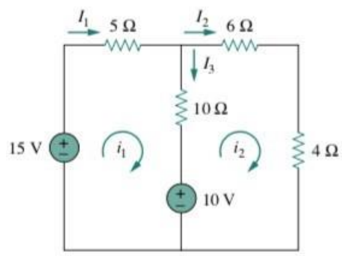


Figure 2.33:

Solution

We first obtain the mesh currents using KVL. For mesh I.

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

or

$$3i_1 - 2i_2 = 1 \tag{2.33}$$

For mesh 2.

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

or

$$i_1 = 2i_2 - 1 \quad (2.34)$$

Using the substitution method, we substitute Eq. (2.34) into Eq. (2.33), and write

$$6i_2 - 3 - 2i_2 = 1 \Rightarrow i_2 = 1A$$

From Eq. (2.34),

$$i_1 = 2i_2 - 1 = 2 - 1 = 1A$$

. Thus,

$$I_1 = i_1 = A, I_2 = i_2 = 1A, I_3 = i_1 - i_2 = 0$$

2.17.2 Mesh Analysis With Current Sources

Applying mesh analysis to circuits containing current sources may appear complicated. But it is actually much easier than what we encountered in the previous section, because the presence of the current sources reduces the number of equations. Consider the following two possible cases.

Case 1

When a current source exists only in one mesh: Consider the circuit in Fig. 2.34, for example. We set $i_2 =$ -5 A and write a mesh equation for the other mesh in the usual way, that is,

$-10 + 4i + 6(i_1 - i_2) = 0, i_1 = -2A$

Case 2

When a current source exists between two meshes: Consider the circuit in Fig. 2.35(a), for example. We create a supermesh by excluding the current source and any elements connected in series with it, as shown in Fig. 2.35(b). Thus, A supermesh results when two meshes have a current source in common.

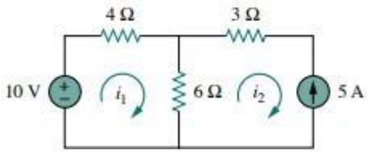


Figure 2.34: A circuit with a current source.

The scope of this manual does not include supermesh.

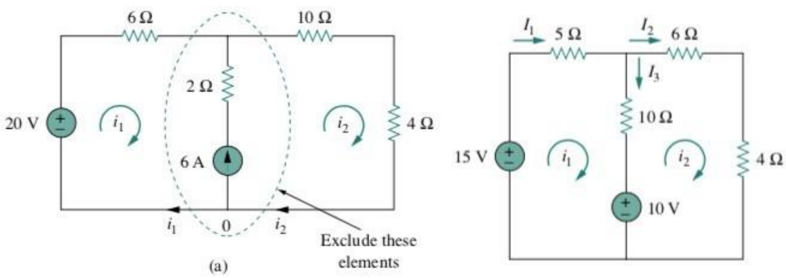


Figure 2.35:

Example 3.2

Find the branch currents of the network of Fig. 2.36.

Solution

Steps 1 and 2 are as indicated in the circuit.

Step 3: Kirchhoff's voltage law is applied around each closed loop:

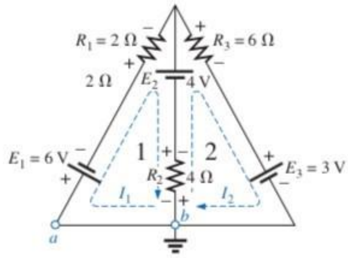


Figure 2.36:

loop 1: $-E_1 - I_1R_1 - E_2 - V_2 = 0(clockswise\_from\_point\_a)$
$-6V - (2\Omega)I_1 - 4V - (4\Omega)(I_1 - I_2) = 0$

loop 2: $-V_2 + E_2 - V_3 - E_3 = 0(clockwise\_from\_point\_b)$
$-(4\Omega)(I_2 - I_1) + 4V - (6\Omega)(I_2) - 3V = 0$ which are written as

$-10 - 4I_1 - 2I_1 + 4I_2 = 0, -6I_1 + 4I_2 = +10$

$+1 + 4I_1 - 4I_2 - 6I_2 = 0, +4I_1 - 10I_2 = -1$

or by multiplying the top equation by -1, we obtain

$$6I_1 - 4I_2 = -10$$
$$4I_1 - 10I_2 = -1$$

$$I_1 = \frac{\begin{vmatrix} -10 & -4 \\ -1 & -10 \end{vmatrix}}{\begin{vmatrix} 6 & -4 \\ 4 & -10 \end{vmatrix}} = \frac{100 - 4}{-60 + 16} = \frac{96}{-44} = \mathbf{-2.182\ A} \qquad I_2 = \frac{\begin{vmatrix} 6 & -10 \\ 4 & -1 \end{vmatrix}}{-44} = \frac{-6 + 40}{-44} = \frac{34}{-44} = \mathbf{-0.773\ A}$$

The current in the 4Ω resistor and 4V source for loop 1 is

$$I_1 - I_2 = -2.282A - (-0.773A) = -2.182A + 0.773 = -1.409A$$

revealing that it is 1.409A in a direction opposite(due to the minus sign) to I_1 in loop 1.

2.17.3 Nodal Analysis

Kirchhoff's current law is employed to develop a method referred to as nodal analysis. A node is defined as a junction of two or more branches. If we now define one node of any network as a reference (that is, a point of zero potential or ground), the remaining nodes of the network will all have a fixed potential relative to this reference. For a network of N nodes, therefore, there will exist (N-1) nodes with a fixed potential relative to the assigned reference node. The nodal analysis method is applied as follows:

1. Determine the number of nodes within the network.
2. Pick a reference node, and label each remaining node with a subscripted value of voltage: V_1, V_2, and so on.
3. Apply Kirchhoff's current law at each node except the reference.

Assume that all unknown currents leave the node for each application of Kirchhoff's current law.

4. Solve the resulting equations for the nodal voltages.

Example 3.3

Apply nodal analysis to the network of Fig. 2.37

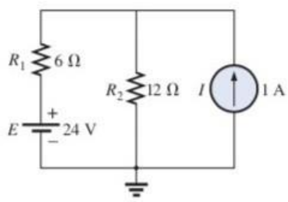


Figure 2.37:

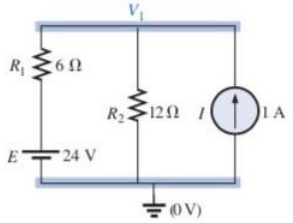


Figure 2.38:

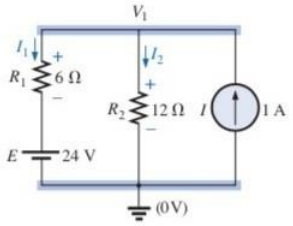


Figure 2.39:

Step 1 and 2: The network has two nodes, as shown in Fig. 2.38. The lower node is defined as the reference node at ground potential (zero volts), and the other node as V_1, the voltage from node 1 to ground.
Step 3: I_1 and I_2 are defined as leaving the node in Fig. 2.39, and Kirchoff's current law is applied as follows:

$$I = I_1 + I_2$$

The current I_2 is related to the nodal voltage V_1 by Ohm's law:

$$I_1 = \frac{V_{R1}}{R_2} = \frac{V_1}{R_2}$$

The current I_1 is also determined by Ohm's law as follows:

$$I_1 = \frac{V_{R1}}{R_1}$$

with

$$V_{R1} = V_1 - E$$

Substituting into the Kirchoff's current law equation:

$$I = \frac{V_1 - E}{R_1} + \frac{V_1}{R_2}$$

and rearranging, we have

$$I = \frac{V_1 - E}{R_1} + \frac{V_1}{R_2} = V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{E}{R_1}$$

or

$$V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{E}{R_1} + 1$$

Substituting numerical values, we obtain

$$V_1\left(\frac{1}{6\Omega} + \frac{1}{12\Omega}\right) = \frac{24V}{6\Omega} + 1A = 4A + 1A$$
$$V_1\left(\frac{1}{4\Omega}\right) = 5A$$
$$V_1 = 20V$$

The currents I_1 and I_2 can then be determined using the preceding equations:

$$I_1 = \frac{V_1}{R_1} - \frac{E}{R_1} = \frac{20V - 24V}{6\Omega} = \frac{-4V}{6\Omega} = -0.667A$$

The minus sign indicates simply that the current I_1 has a direction opposite to that appearing in Fig. 2.39

$$I_2 = \frac{V_1}{R_2} = 1.667A$$

2.17.4 Superposition Theorem

The superposition theorem, can be used to find the solution to networks with two or more sources that are not in series or parallel. The advantage of this method is that it does not require the use of a mathematical technique such as determinants to find the required voltages or currents. Instead, each source is treated independently, and the algebraic sum is found to determine a particular unknown quantity of the network.
The superposition theorem states the following:
The current through, or voltage across, an element in a linear bilateral network is equal to the algebraic sum of the currents or voltages produced independently by each source.
To consider the effects of each source independently requires that sources be removed and replaced without affecting the final result. To remove a voltage source when applying this theorem, the difference in potential between the terminals of the voltage source must be set to zero (short circuit); removing a current source requires that its terminals be opened (open circuit). Any internal resistance or conductance associated with the displaced sources is not eliminated but must still be considered. Figure 2.40 reviews the various substitutions required when removing an ideal source, and Figure 2.41 reviews the substitutions with practical sources that have an internal resistance.

Example 3.4

Using superposition theorem, Determine I_1 for the network of Fig. 2.42.

Solution:

As shown in Fig. 2.43(a), the source current will choose the short-circuit path, and I'1=0 A. If we applied the current divider rule,

$$I_1' = \frac{R_{SC} I}{R_{SC} + R_1} = \frac{(0\Omega) I}{0\Omega + 6\Omega} = 0A$$

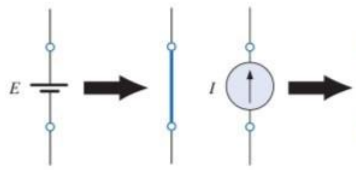


Figure 2.40: Removing the effects of ideal sources

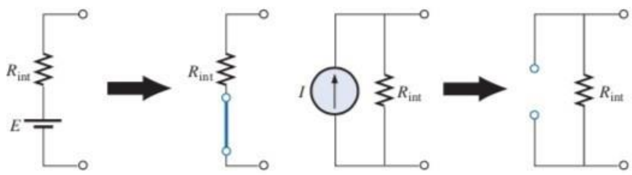


Figure 2.41: Removing the effects of practical sources

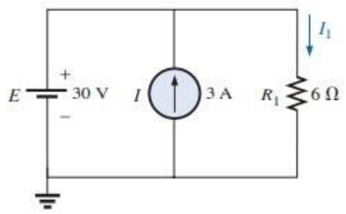


Figure 2.42:

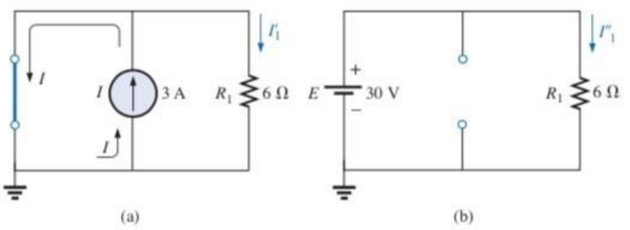


Figure 2.43:

Setting I to zero amperes will result in the network of Fig. 2.43(b), with the current source replaced by an open circuit. Applying Ohm's law

$$I''_1 = \frac{E}{R_1} = \frac{30V}{6\Omega} = 5A$$

Since I'_1 and I''_1 have the same defined direction in Fig. 2.43(a) and (b), the current I_1 is the sum of the two, and

$$I_1 = I'_1 + I''_1 = 0A + 5A = 5A$$

Note in this case that the current source has no effect on the current through the 6-Ω resistor. The voltage across the resistor must be fixed at 30 V because they are parallel elements.

Example 3.5
Using superposition, determine the current through the 4-Ω resistor of Fig. 2.44.

Solution

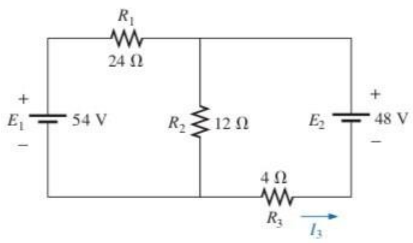


Figure 2.44:

Considering the effects of a 54-V source(Fig. 2.44):

$$R_T = R_1 + R_2 \| R_3 = 24\Omega + 12\Omega \| 4\Omega = 24\Omega + 3\Omega = 7\Omega$$
$$I = \frac{E_1}{R_T} = \frac{54V}{27\Omega} = 2A$$

Using the current divider rule.

$$I'_3 = \frac{R_2 I}{R_2 + R_3} = \frac{(12\Omega)(2A)}{16} = 15A$$

Considering the effects of the 48-V source (Fig. 2.45):

$$R_T = R_3 + R_1 \| R_2 = 4\Omega + 24\Omega \| 12\Omega = 4\Omega + 8\Omega = 12\Omega$$
$$I'_3 = \frac{E_2}{R_T} = \frac{48V}{12\Omega} = 4A$$

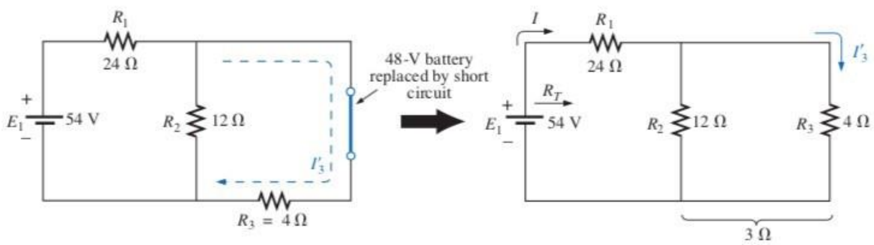


Figure 2.45: The effect of E1 on the current I3

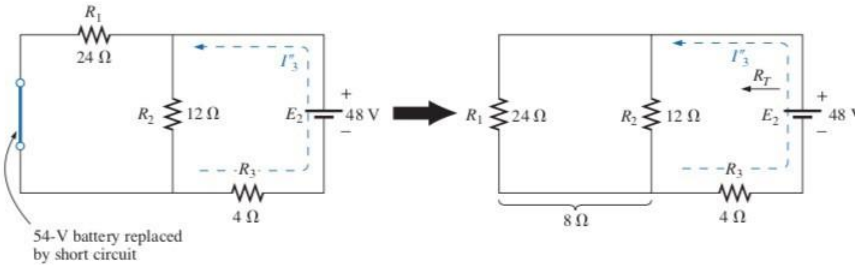


Figure 2.46:

The total current through the 4-Ω resistor (Fig. 2.47) is

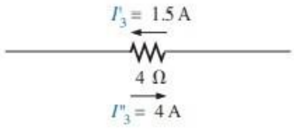


Figure 2.47: The resultant current for I_3

$$I_3 = I''_3 + I'_3 = 4A - 1.5A = 2.5A$$

(direction of I''_3)

2.17.5 Reciprocity Theorem

The reciprocity theorem is applicable only to single-source networks. The theorem states the following: The current I in any branch of a network, due to a single voltage source E anywhere else in the network, will equal the current through the branch in which the source was originally located if the source is placed in the branch in which the current I was originally measured. In other words, the location of the voltage source and the resulting current may be interchanged without a change in current. The theorem requires that the polarity of the voltage source have the same correspondence with the direction of the branch current in each position.

In the representative network of Fig. 2.48(a), the current I due to the voltage source E was determined. If the position of each is interchanged as shown in Fig. 2.48(b), the current I will be the same value as indicated

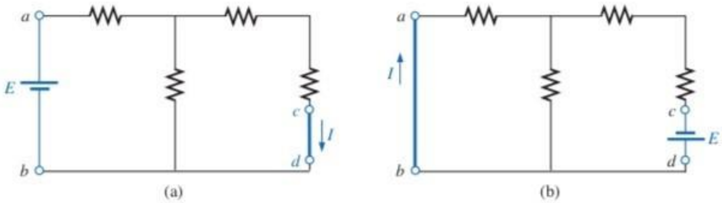


Figure 2.48:

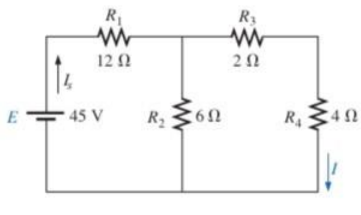


Figure 2.49: Finding the current I due to a source E

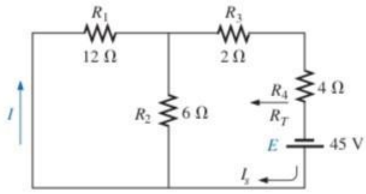


Figure 2.50: Interchanging the location of E and I of the Fig. 2.49 to demonstrate the validity of the reciprocity theorem.

For the network of Fig 2.49, which corresponds to that of Fig. 2.48(b), we find

$$R_T = R_4 + R_3 + R_1 \mathrm{k} R_2$$
$$= 4\Omega + 2\Omega + 12\Omega\mathrm{k}6\Omega = 10\Omega$$

and

$$I_s = \frac{E}{R_T} = \frac{45V}{10\Omega} = 4.5A$$

so that

$$I = \frac{(6\Omega)(4.5A)}{12\Omega + 6\Omega} = \frac{4.5A}{3} = 1.5A$$

$$R_T = R_1 + R_2 \,\|(R_3 + R_4) = 12\Omega + 6\Omega \,\|(2\Omega + 4\Omega)$$
$$= 12\Omega + 6\Omega \,\|6\Omega = 12\Omega + 3\Omega = 15\Omega$$

and

$$I_s = \frac{E}{R_T} = \frac{45V}{15\Omega} = 3A$$

with

$$I = \frac{3A}{2} = 1.5A$$

PABLO

Chapter 3

Sinusoidal Alternating Waveforms

At the end of the chapter students should be able to:
1. Define period and frequency of waveform.
2. Define instantaneous, peak, mean and rms values of a sine wave.
3. Understand and perform calculations on the general sinusoidal equation.
4. Calculate mean and rms values for given waveforms.
5. Understand the response of basic L, R and C elements to a sinusoidal voltage and current.
6. Define average power and power factor and perform calculations involving average power and power factor.
7. Perform calculations in ac networks using the superposition theorem.
8. Define Apparent and Reactive Power.
9. Perform calculations involving the resistive, inductive and capacitive circuits.

3.1 Introduction

The analysis thus far has been limited to dc networks. We will now turn our attention to the analysis of networks in which the magnitude of the source varies in a set manner. Of particular interest is the timevarying voltage that is commercially available in large quantities and is commonly called the ac voltage. (The letters ac are an abbreviation for alternating current.) Each waveform of Fig. 3.1 is an alternating waveform available from commercial supplies. The term alternating indicates only that the waveform alternates between two prescribed levels in a set time sequence (Fig. 3.1). The pattern of particular interest is the sinusoidal ac waveform for voltage of Fig. 3.1. Since this type of signal is encountered in the vast majority of instances.

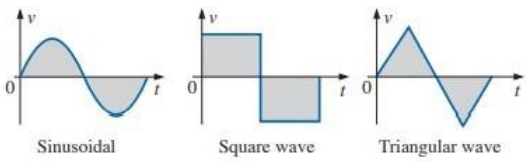


Figure 3.1: Alternating waveforms

3.2 Sinusoidal AC Voltage Characteristics And Definitions

Sinusoidal ac voltages are available from a variety of sources. The most common source is the typical home outlet, which provides an ac voltage that originates at a power plant; such a power plant is most commonly fueled by water power, oil, gas, or nuclear fusion. In each case an ac generator, as shown in Fig. 3.2(a), is the primary component in the energy-conversion process.

Definitions
The sinusoidal waveform of Fig. 3.3 with its additional notation will now be used as a model in defining a few basic terms. It is important to remember that the vertical scaling is in volts or amperes and the horizontal scaling is always in units of time.

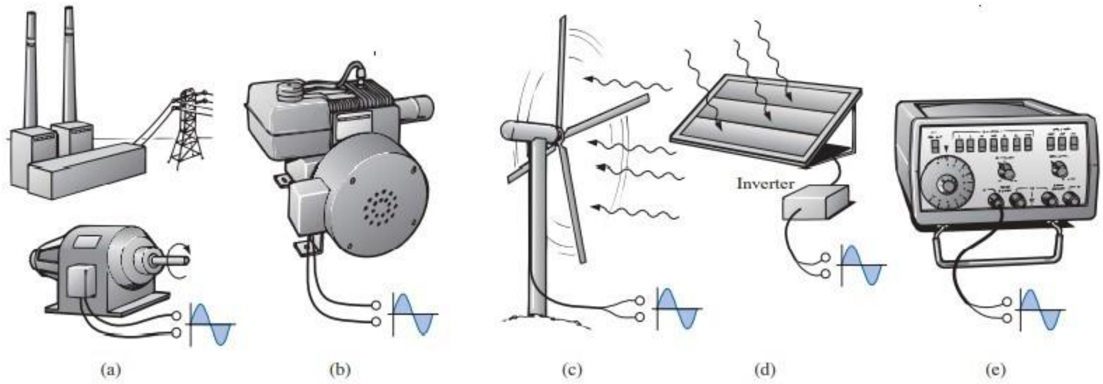


Figure 3.2: Various sources of ac power: (a) generating plant; (b) portable ac generator; (c) wind-power station; (d) solar panel; (e) function generator.

Waveform: The path traced by a quantity plotted as a function of some variable such as time, position, degrees, radians, temperature, and so on.
Instantaneous value: The magnitude of a waveform at any instant of time; denoted by lowercase letters (e_1, e_2).
Peak amplitude: The maximum value of a waveform as measured from its average, or mean, value, denoted by uppercase letters (such as E_m for sources of voltage and V_m for the voltage drop across a load). For the waveform of the average value is zero volts, and E_m is as defined by the figure.
Peak value: The maximum instantaneous value of a function as measured from the zero-volt level. For the waveform of the peak amplitude and peak value are the same, since the average value of the function is zero volts.
Peak-to-peak value: Denoted by E_{p-p} or V_{p-p}, the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks.
Periodic waveform: A waveform that continually repeats itself after the same time interval.
Period (T): the periodic function is the time of one complete cycle or the number of seconds per cycle. The time interval between successive repetitions of a periodic waveform as long as successive similar points of the periodic waveform are used in determining T.
Frequency (f): The number of cycles that occur in 1s. The reciprocal of period is frequency.

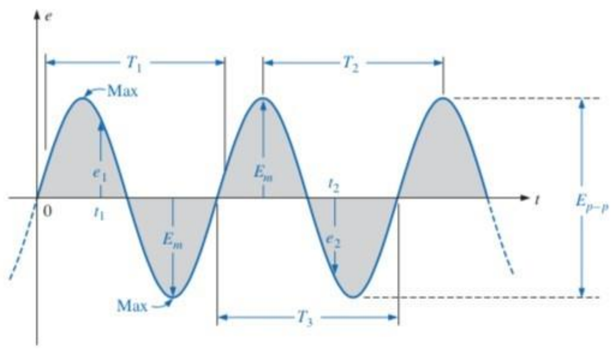


Figure 3.3: Important parameters for a sinusoidal voltage.

Example 3.1
Determine the frequency of the waveform of Fig. 3.4.

Solution
From the figure, T=(25ms - 5ms)=20ms, and

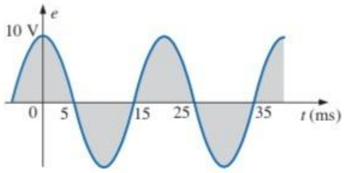


Figure 3.4:

$$f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}\,\text{s}} = \mathbf{50\ Hz}$$

3.3 The Sinusoidal Wave

A sinusoid is a signal that has the form of the sine or cosine function. The sinusoidal waveform is the only alternating waveform whose shape is unaffected by the response characteristics of R, L, and C elements. In other words, if the voltage across (or current through) a resistor, coil, or capacitor is sinusoidal in nature, the resulting current (or voltage, respectively) for each will also have sinusoidal characteristics. If a square wave or a triangular wave were applied, such would not be the case. The unit of measurement for the horizontal axis is the degree. A second unit of measurement frequently used is the radian (rad). The conversion equations between the two are the following:

$$\boxed{\text{Radians} = \left(\frac{\pi}{180^\circ}\right) \times (\text{degrees})} \quad \boxed{\text{Degrees} = \left(\frac{180^\circ}{\pi}\right) \times (\text{radians})}$$

The velocity with which the radius vector rotates about the center, called the angular velocity, can be determined from the following equation:

$$\boxed{\text{Angular velocity} = \frac{\text{distance (degrees or radians)}}{\text{time (seconds)}}} \quad \boxed{\omega = \frac{\alpha}{t}}$$

Since ω is typically provided in radians per second, the angle a obtained using is usually in radians. The time required to complete one revolution is equal to the period (T) of the sinusoidal waveform. The radians subtended in this time interval are 2Π. Substituting, we have

$$\boxed{\omega = \frac{2\pi}{T}}$$

If f = 1/T. Thus,

$$\boxed{\omega = 2\pi f} \quad \text{(rad/s)}$$

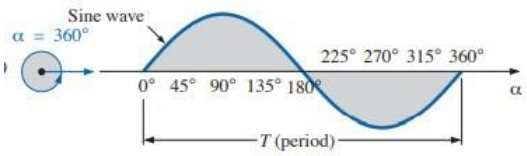


Figure 3.5: Generating a sinusoidal waveform through the vertical projection of a rotating vector.

3.4 General Format For The Sinusoidal Voltage Or Current

The basic mathematical format for the sinusoidal waveform is

$A_m sin\alpha$

where A_m is the peak value of the waveform and a is the unit of measure for the horizontal axis, as shown in Fig. 3.6.

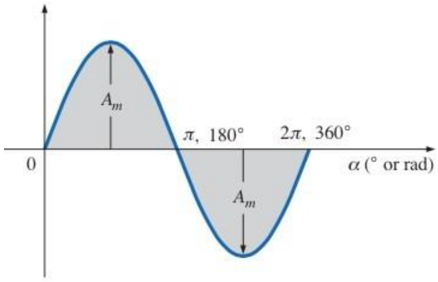


Figure 3.6:

The equation a α=ωt states that the angle a through which the rotating vector of Fig. 3.5 will pass is determined by the angular velocity of the rotating vector and the length of time the vector rotates. The general format of a sine wave can also be written

$A_m sin\omega t$

For electrical quantities such as current and voltage, the general format is

i=$I_m sin\omega t$=$I_m sin\alpha$ e=$E_m sin\omega t$=E_m=$E_m sin\alpha$

where the capital letters with the subscript m represent the amplitude, and the lowercase letters i and e represent the instantaneous value of current or voltage, respectively, at any time t. This format is particularly important since it presents the sinusoidal voltage or current as a function of time,

3.4.1 Phase Relations

Thus far, we have considered only sine waves that have maxima at $\pi/2$ and 3 π /2, with a zero value at 0, π, and 2 π. If the waveform is shifted to the right or left of 0 , the expression becomes

$$\boxed{A_m \sin(\omega t \pm \theta)}$$

where θ is the angle in degrees or radians that the waveform has been shifted. The phase relationship between two waveforms indicates which one leads or lags, and by how many degrees or radians.
If the waveform passes through the horizontal axis with a positive- going (increasing with time) slope before 0 , as shown in Fig. 3.7, the expression is

$$\boxed{A_m \sin(\omega t + \theta)}$$

At ωt=α =0 , the magnitude is determined by Am sin θ . If the waveform passes through the horizontal axis with a positive-going slope after 0 , as shown in Fig. 3.8, the expression is

$$\boxed{A_m \sin(\omega t - \theta)}$$

3.4.2 Average Value

The average or mean value of a symmetrical alter- nating quantity, (such as a sine wave), is the average value measured over a half cycle, (since over a complete cycle the average value is zero).

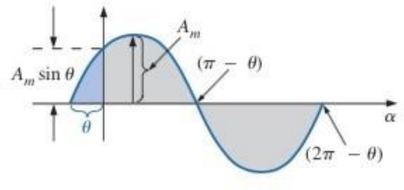


Figure 3.7:

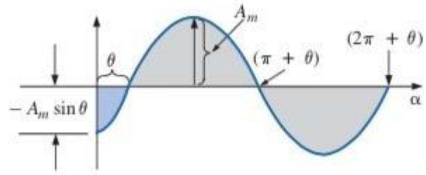


Figure 3.8:

Average or mean value = (area under the curve)/(length of base)
The procedure of calculus that gives the exact solution is known as integration. Finding the area under the positive pulse of a sine wave using integration,

$$\text{Area} = \int_0^{\pi} A_m \sin\alpha \, d\alpha$$

Average value of sinusoidal signal
The average of a sinusoidal signal is usually take for a half-cycle, thus T = π
Thus the average of x (t) = $X_m \sin\theta d\theta$

$$\begin{aligned}
X_0 &= \frac{1}{\pi}\int_0^{\pi} X_m \sin\theta d\theta \\
&= \frac{1}{\pi} X_m (-\cos\theta)_0^{\pi} \\
&= \frac{1}{\pi} X_m (\cos\theta)_{\pi}^{0} \\
&= \frac{1}{\pi} X_m (\cos 0 - \cos\pi) \\
&= \frac{2}{\pi} X_m \\
&= 0.637 X_m
\end{aligned}$$

3.4.3 Effective(rms) Values

How is it possible for a sinusoidal ac quantity to deliver a net power if, over a full cycle, the net current in any one direction is zero (average value = 0)? It would almost appear that the power delivered during the positive portion of the sinusoidal waveform is withdrawn during the negative portion. and since the two are equal in magnitude, the net power delivered is zero. However, understand that irrespective of direction, current of any magnitude through a resistor will deliver power to that resistor. The effective value of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current. The effective value of a periodic signal is its root mean square (rms) value
. Thus the rms of x (t) = $X_m \sin\theta d\theta$

Example 3.2

Determine the rms value of the current waveform in Fig. 3.9. If the current is passed through a 2 ohms resistor, find the average power absorbed by the resistor.

Solution

$$
\begin{aligned}
X_{rms} = \; &= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} X_m^2 \sin^2\theta d\theta} \\
&= \sqrt{\frac{X_m^2}{2\pi}\int_0^{2\pi}\left(\frac{1}{2}(1-\cos 2\theta d\theta\right)} \\
&= \sqrt{\frac{X_m^2}{4\pi}\left(\theta - \frac{\sin 2\theta}{2}\right)_0^{2\pi}} \\
&= \sqrt{\frac{X_m^2}{4\pi}\left(2\pi - 0 - \frac{(\sin 4\pi - \sin 0)}{2}\right)} \\
&= \sqrt{\frac{X_m^2}{4\pi}(2\pi)} = \sqrt{\frac{X_m^2}{2}} \\
&= \frac{X_m}{\sqrt{2}} = 0.707X_m
\end{aligned}
$$

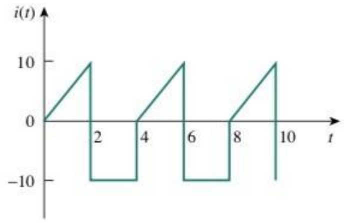


Figure 3.9:

$$
i(t) = \begin{cases} 5t, & 0 < t < 2 \\ -10, & 2 < t < 4 \end{cases}
$$

The rms value is

$$
I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} = \sqrt{\frac{1}{4}\left[\int_0^2 (5t)^2\,dt + \int_2^4 (-10)^2\,dt\right]}
$$

$$
= \sqrt{\frac{1}{4}\left[25\frac{t^3}{3}\Big|_0^2 + 100t\Big|_2^4\right]} = \sqrt{\frac{1}{4}\left(\frac{200}{3} + 200\right)} = 8.165\text{ A}
$$

The power absorbed by a 2-Ω resistor is

$$
P = I_{\text{rms}}^2 R = (8.165)^2(2) = 133.3\text{ W}
$$

3.5 Response Of Basic R, L And C Elements To A Sinusoidal Voltage Or Current

3.5.1 Resistor

For power-line frequencies and frequencies up to a few hundred kilo-hertz, resistance is, for all practical purposes, unaffected by the frequency of the applied sinusoidal voltage or current. For this frequency region, the resistor R of Fig. 3.10 can be treated as a constant, and Ohm's law can be applied as follows For v = Vm sin ωt,

A plot of v and i in Fig. 3.11 reveals that for a purely resistive element, the voltage across and the current through the element are in phase, with their peak values related by Ohm's law

3.5.2 Inductor

The voltage across an inductor is directly related to the rate of change of current through the coil. Consequently, the higher the frequency, the greater will be the rate of change of current through the coil, and the greater the magnitude of the voltage. the inductance of a coil will determine the rate of change of the flux linking a coil for a particular change in current through the coil. The higher the inductance, the greater the

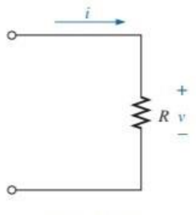


Figure 3.10: Determining the sinusoidal response for a resistive element

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

$$\boxed{I_m = \frac{V_m}{R}}$$

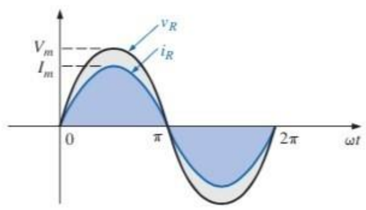


Figure 3.11: The voltage and current of a resistive element are in phase

rate of change of the flux linkages, and the greater the resulting voltage across the coil. The inductive voltage, therefore, is directly related to the frequency (or, more specifically, the angular velocity of the sinusoidal ac current through the coil) and the inductance of the coil. For the inductor of Fig. 3.12.We recall that:

$$v_L = L \frac{di_L}{dt}$$

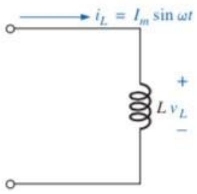


Figure 3.12:

and, applying differentiation,

$$\frac{di_L}{dt} = \frac{d}{dt}(I_m \sin \omega t) = \omega I_m \cos \omega t$$

therefore,

$$v_L = L \frac{di_L}{dt} = L(\omega I_m \cos \omega t) = \omega L I_m \cos \omega t$$

or

$$v_L = V_m \sin(\omega t + 90°)$$

where

$V_m = \omega L I_m$

Note that the peak value of v_L is directly related to ω (= $2\pi f$) and L

The quantity ωL, called the reactance (from the word reaction) of an inductor, is symbolically represented by X_L and is measured in ohms; that is,

$$X_L = \omega L (Ohms, \Omega)$$

In an Ohm's law format, its magnitude can be determined from

$$X_L = \frac{V_m}{I_m} \, (Ohms, \Omega)$$

A plot of v_L and i_L in Fig. 3.13 reveals that for an inductor, v_L leads i_L by 90 , or i_L lags v_L by 90 .

If a phase angle is included in the sinusoidal expression for i_L, such

$$i_L = I_m \sin(\omega t \pm \theta)$$

$$v_L = \omega L I_m \sin(\omega t \pm \theta + 90^\circ)$$

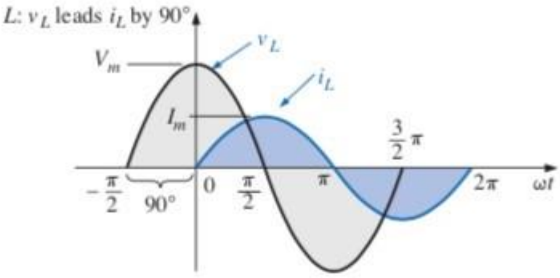


Figure 3.13:

3.5.3 Capacitor

For the capacitor, we will determine i for a particular voltage across the element. For capacitive networks, the voltage across the capacitor is limited by the rate at which charge can be deposited on, or released by, the plates of the capacitor during the charging and discharging phases, respectively. In other words, an instantaneous change in voltage across a capacitor is opposed by the fact that there is an element of time required to deposit charge on (or release charge from) the plates of a capacitor, and V = Q/C. Since capacitance is a measure of the rate at which a capacitor will store charge on its plates,

for a particular change in voltage across the capacitor, the greater the value of capacitance, the greater will be the resulting capacitive current.

Also, for a particular capacitance, the greater the rate of change of voltage across the capacitor, the greater the capacitive current. The current of a capacitor is therefore directly related to the frequency (or, again more specifically, the angular velocity) and the capacitance of the capacitor.

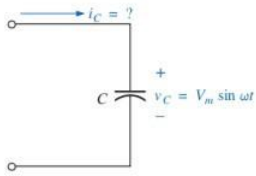


Figure 3.14:

For a capacitor

$$i_C = C\frac{dv_C}{dt}$$

and, applying differentiation, $\frac{dv_C}{dt} = \frac{d}{dt}(V_m \sin\omega t) = \omega V_m \cos\omega t$

Therefore

$$i_C = C\frac{dv_C}{dt} = C(\omega V_m \cos\omega t) = \omega C V_m \cos\omega t$$

where

$i_c = I_m \sin(\omega t + 90^\circ)$ $I_m =$
$\omega C V_m$

Note that the peak value of i_C is directly related to ω (= 2πf) and C, as predicted in the discussion above. A plot of v_C and i_C in Fig. 3.15 reveals that for a capacitor, i_C leads v_C by 90 , or v_C lags i_C by 90 . If a phase angle is included in the sinusoidal expression for v_C, such as

$v_c = V_m \sin(\omega t \pm \theta)$ $i_c = \omega C V_m$
$\sin(\omega t \pm \theta + 90^\circ)$

In an Ohm's law format, its magnitude can be determined from

$$= \frac{V_m}{I_m} = \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$$

The quantity 1/ ωC, called the reactance of a capacitor, is symbolically represented by X_C and is measured in ohms;

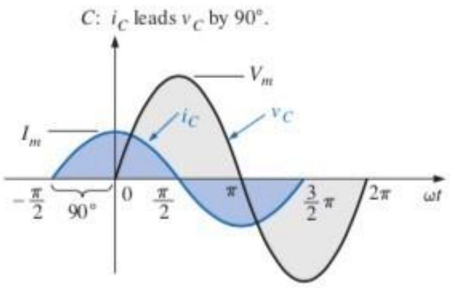


Figure 3.15:

It is possible to determine whether a network with one or more ele- ments is predominantly capacitive or inductive by noting the phase relationship between the input voltage and current. If the source current leads the applied voltage, the network is predominantly capacitive, and if the applied voltage leads the source current, it is predominantly inductive.

Example 3.3

The voltage across a resistor is indicated. Find the sinusoidal expression for the current if the resistor is 10 Ω. Sketch the curves for v and i.

a. v = 100 sin 377t
b. v = 25 sin(377t + 60)

Solution a.

$$I_m = \frac{V_m}{R} = \frac{100V}{10\Omega} = 10A$$

(v and i are in phase), resulting in

$i = 10\sin 377t$ b.

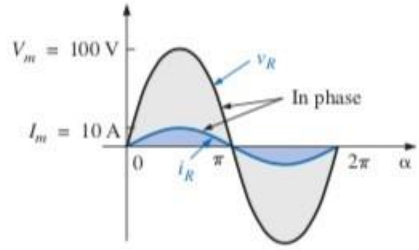
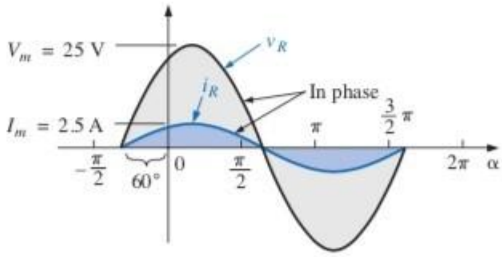


Figure 3.16: (a)

$$I_m = \frac{V_m}{R} = \frac{25V}{10\Omega} = 2.5A$$

(v and i are in phase), resulting in

$$i = 2.5\sin(377t + 60^\circ)$$



(b)

3.5.4 DC, High, and Low-Frequency Effects on L and C

For dc circuits, the frequency is zero, and the reactance of a coil is
$X_L = 2\pi fL = 2\pi(0)L = 0\ \omega$
In dc circuit, short-circuit is the equivalence for the inductor. At very high frequencies, X_L is very large, and for some practical applications the inductor can be replaced by an open circuit.

$\boxed{X_L = 0\ \Omega}$ dc, $f = 0$ Hz
$\boxed{X_L \Rightarrow \infty\ \Omega \quad \text{as } f \Rightarrow \infty \text{ Hz}}$

The capacitor can be replaced by an open-circuit equivalence in dc circuits since f = 0, and

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(0)C} \Rightarrow \infty\ \Omega$$

At very high frequencies, for finite capacitances,
$\boxed{X_C \cong 0\ \Omega}$ f = very high frequencies

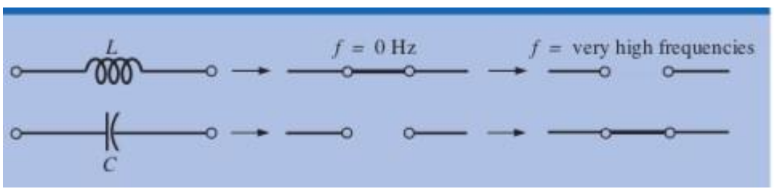


Figure 3.17: Effect of high and low frequencies on the circuit model of an inductor and a capacitor

3.6 Average Power and Power Factor

For any load in a sinusoidal ac network, the voltage across the load and the current through the load will vary in a sinusoidal nature. The questions then arise, How does the power to the load determined by the product v • i vary, and what fixed value can be assigned to the power since it will vary with time?

If

$$v = V_m \sin(\omega t + \theta_v) \quad i = I_m \sin(\omega t + \theta_i)$$

then the power is defined by

$$p = vi = V_m \sin(\omega t + \theta_v) I_m \sin(\omega t + \theta_i)$$
$$= V_m I_m \sin(\omega t + \theta_v)\sin(\omega t + \theta_i)$$

Using the trigonometric identity

$$\sin A \sin B = \frac{\cos(A - B) - \cos(A + B)}{2}$$

the function $\sin(\omega t + \theta_v)\sin(\omega t + \theta_i)$ becomes

$$\sin(\omega t + \theta_v)\sin(\omega t + \theta_i)$$
$$= \frac{\cos[(\omega t + \theta_v) - (\omega t + \theta_i)] - \cos[(\omega t + \theta_v) + (\omega t + \theta_i)]}{2}$$
$$= \frac{\cos(\theta_v - \theta_i) - \cos(2\omega t + \theta_v + \theta_i)}{2}$$

so that

$$p = \overbrace{\left[\frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\right]}^{\text{Fixed value}} - \overbrace{\left[\frac{V_m I_m}{2}\cos(2\omega t + \theta_v + \theta_i)\right]}^{\text{Time-varying (function of } t)}$$

A plot of v, i, and p on the same set of axes is shown in fig. 3.18.

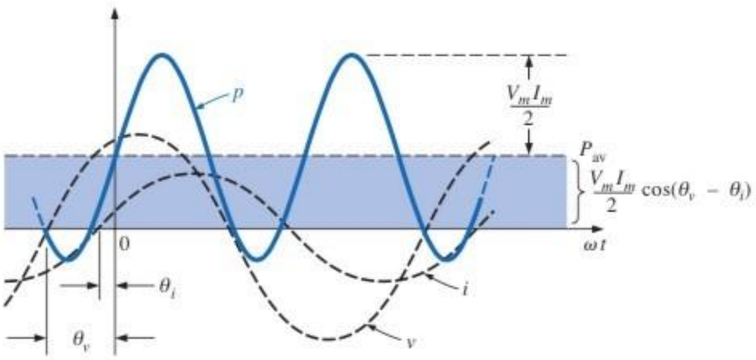


Figure 3.18:

Note that the second factor in the preceding equation is a cosine wave with an amplitude of $V_m I_m/2$ and with a frequency twice that of The average value of this term is zero over one cycle, producing no net transfer of energy in any one direction. The first term in the preceding equation, however, has a constant magnitude (no time dependence) and therefore provides some net transfer of energy. This term is referred to as the average power or real power and is the power delivered to and dissipated by the load. It corresponds to the power calculations performed for dc networks. The angle (θv - θi) is the phase angle between v and i. Since $\cos(-\alpha) = \cos \alpha$, the magnitude of average power delivered is independent of whether v leads i or i leads v .

Defining v as equal to kθv - θi k, Where

$$P = \frac{V_m I_m}{2}\cos\theta(watts, W)$$

where P is the average power in watts. This equation can also be written, and

$$P = \left(\frac{V_m}{\sqrt{2}}\right)\left(\frac{I_m}{\sqrt{2}}\right)\cos\theta$$

$$V_{eff} = \frac{V_m}{\sqrt{2}}$$

$$I_{eff} = \frac{I_m}{\sqrt{2}}$$

therefore

$$P = V_{eff} I_{eff} \cos\theta$$

3.6.1 Resistor

In a purely resistive circuit, since v and i are in phase, —θv - θi—= θ = 0 , and cos θ = cos 0 = 1, so that

$$P = \frac{V_m I_m}{2} = V_{eff} I_{eff}(W)$$

$$I_{eff} = \frac{V_{eff}}{R}$$

$$P = \frac{V^2{}_{eff}}{R} = I_{eff}^2 R$$

3.6.2 Inductor

In a purely inductive circuit, since v leads i by 90 , —θ v - θi —= θ —-90 —= 90 . Therefore,

$$P = \frac{V_m I_m}{2}\cos 90^\circ = \frac{V_m I_m}{2}(0) = 0W$$

The average power or power dissipated by the ideal inductor (no associated resistance) is zero watts.

3.6.3 Capacitor

In a purely capacitive circuit, since i leads v by 90 , —θv - θi—= θ —-90 — = 90 . Therefore,

$$P = \frac{V_m I_m}{2}\cos 90^\circ = \frac{V_m I_m}{2}(0) = 0W$$

The average power or power dissipated by the ideal capacitor (no associated resistance) is zero watts.

Example 3.4

Find the average power dissipated in a network whose input current and voltage are the following:

$i = 5\sin(\omega t + 40^\circ)$ v

$= 10\sin(\omega t + 40^\circ)$

Solution

Since v and i are in phase, the circuit appears to be purely resistive at the input terminals. Therefore,

$$P = \frac{V_m I_m}{2} = \frac{(10V)(5A)}{2} = 25W$$

$$R = \frac{V_m}{I_m} = \frac{10V}{5A} = 2\Omega$$

$$P = \frac{V^2{}_{eff}}{R} = \frac{[(0.707)(10V)]^2}{2} = 25W$$

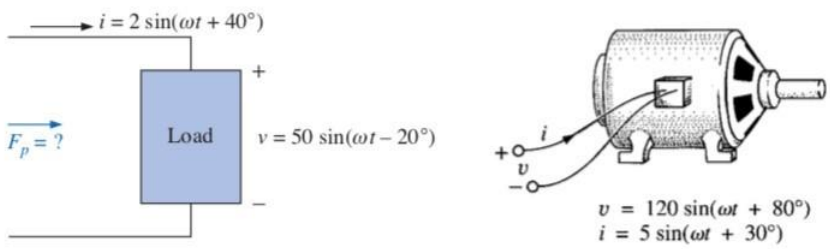
$$P = I_{eff}^2 R = [(0.707)(5A)]^2(2) = 25W$$

3.6.4 Power Factor

In the equation P = $(V_m I_m/2)\cos\theta$, the factor that has significant control over the delivered power level is the cos θ . No matter how large the voltage or current, if cos θ= 0, the power is zero; if cos θ = 1, the power delivered is a maximum. Since it has such control, the expression was given the name power factor and is defined by
Power factor = F_p = cos θ

For a purely resistive load, the phase angle between v and I is 0 and F_p = cos θ = cos 0 =1. The power delivered is a maximum of $(V_m I_m/2)$ cos θ. For a purely reactive load (inductive or capacitive), the phase angle between v and i is 90 and F_p = cos θ = cos 90 = 0. The power delivered is then the minimum value of zero watts, even though the current has the same peak value. For situations where the load is a combination of resistive and reactive elements, the power factor will vary between 0 and 1. The more resistive the total impedance, the closer the power factor is to 1; the more reactive the total impedance, the closer the power factor is to 0. The terms leading and lagging are often written in conjunction with the power factor. They are defined by the current through the load. If the current leads the voltage across a load, the load has a leading power factor. If the current lags the voltage across the load, the load has a lagging power factor. In other words, capacitive networks have leading power factors, and inductive networks have lagging power factors.

Example 3.5
Determine the power factors of the following loads, and indicate whether they are leading or lagging:



Solution
a.

$F_p = \cos\theta = \cos|40^\circ - (-20^\circ)| = 0.5 lagging$ b.

$F_p = \cos\theta = \cos|80^\circ - 30^\circ| = \cos 50^\circ = 0.6428 lagging$

3.7 Phasors

Sinusoids are easily expressed in terms of phasors, which are more convenient to work with than sine and cosine functions. A phasor is a complex number that represents the amplitude and phase of a sinusoid. Phasor algebra for sinusoidal quantities is applicable only for waveforms having the same frequency.

Convert the following from the time to the phasor domain:

Time Domain	Phasor Domain
a. $\sqrt{2}(50)\sin\omega t$	$\mathbf{50 \angle 0°}$
b. $69.6\sin(\omega t + 72°)$	$(0.707)(69.6)\angle 72° = \mathbf{49.21 \angle 72°}$
c. $45\cos\omega t$	$(0.707)(45)\angle 90° = \mathbf{31.82 \angle 90°}$

Write the sinusoidal expression for the following phasors if the frequency is 60 Hz:

Phasor Domain	Time Domain
a. $\mathbf{I} = 10\ \angle 30^\circ$	$i = \sqrt{2}(10)\sin(2\pi 60t + 30^\circ)$
	and $i = \mathbf{14.14\ sin(377\mathit{t} + 30^\circ)}$
b. $\mathbf{V} = 115\ \angle -70^\circ$	$v = \sqrt{2}(115)\sin(377t - 70^\circ)$
	and $v = \mathbf{162.6\ sin(377\mathit{t} - 70^\circ)}$

3.8 Network Theorem AC

3.8.1 Superposition

The only variation in applying this method to ac networks with independent sources is that we will now be working with impedances and phasors instead of just resistors and real numbers.

Example 3.6

Using the superposition theorem, find the current I through the 4Ω reactance (X_{L2})

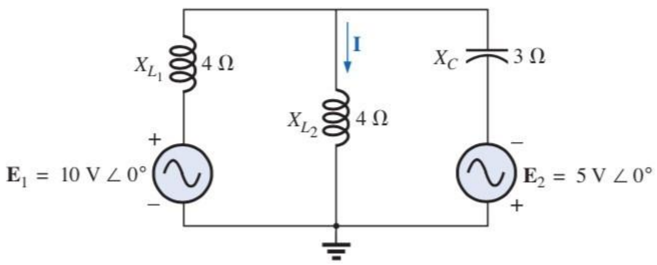
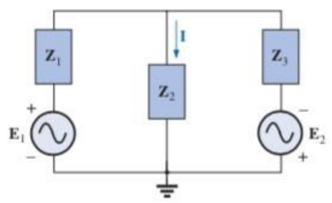


Figure 3.19:

Solution

For the redrawn circuit



$$Z_1 = +jX_{L1} = j4\Omega$$
$$Z_2 = +jX_{L2} = j4\Omega$$
$$Z_3 = -JX_C = -j3\Omega$$

Considering the effects of the voltage source E_1

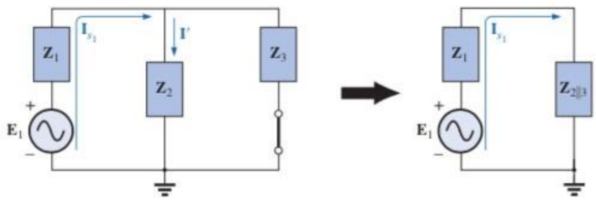
$$Z_{2\|3} = \frac{Z_2 Z_3}{Z_2+Z_3} = \frac{(j4\Omega)(-j3\Omega)}{j4\Omega - j3\Omega} = \frac{12\Omega}{j} = -j12\Omega$$
$$= 12\Omega\angle-90^\circ$$
$$I_{s1} = \frac{E_1}{Z_{2\|3}+Z_1} = \frac{10V\angle 0^\circ}{-j12\Omega + j4\Omega} = \frac{10V\angle 0^\circ}{8\Omega\angle -90^\circ}$$
$$= 1.25A\angle 90^\circ$$

and

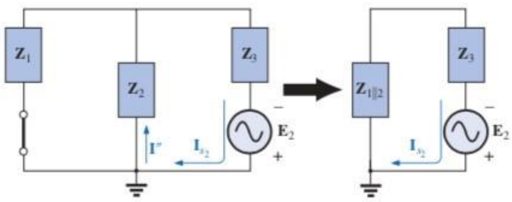
$$I' = \frac{Z_3 I_{s1}}{Z_2 + Z_3}$$

(current divider rule)

$$= \frac{(-j3\Omega)(j1.25A)}{j4\Omega - j3\Omega} = \frac{3.75A}{j1} = 3.75A\angle -90^\circ$$



Considering the effects of the voltage source E_2



$$Z_{1\|2} = \frac{Z_1}{N} = \frac{j4\Omega}{2} = j2\Omega$$

$$I_{s2} = \frac{E_2}{Z_{1\|2}+Z_3} = \frac{5V\angle 0^\circ}{j2\Omega - j3\Omega} = \frac{5V\angle 0^\circ}{1\Omega\angle -90^\circ} = 5A\angle 90^\circ$$

and

$$I'' = \frac{I_{s2}}{2} = 2.5A\angle 90^\circ$$

The resultant current through the 4Ω resistance X_{L2}(Fig. 3.19) is

$$I = I' - I''$$
$$= 3.75A\angle -90^\circ - 2.50A\angle 90^\circ = -j3.75A - j2.50A$$
$$= -j6.25A$$
$$I = 6.25A\angle -90^\circ$$

3.9 Power In AC

The discussion of power included only the average power delivered to an ac network. We will now examine the total power equation in a slightly different form and will introduce two additional types of power: apparent and reactive. For any system, the power delivered to a load at any instant is defined by the product of the applied voltage and the resulting current. The chosen v and i include all possibilities because, if the load is purely resistive, θ= 0 . If the load is purely inductive or capacitive, θ=90 or θ= -90 , respectively.

In this case, since v and i are sinusoidal quantities, let us establish a general case where

$$v = V_m \sin(\omega t + \theta) \quad i = I_m \sin\omega t$$

Substituting the above equations for v and i into the power equation will result in

$$p = V_m I_m \sin\omega t \sin(\omega t + \theta)$$

If we now apply a number of trigonometric identities, the following form for the power equation will result:

$$p = V I \cos\theta(1 - \cos 2\omega t) + V I \sin\theta(\sin 2\omega t) \qquad (3.1)$$

where V and I are the rms values. The conversion from peak values V_m and I_m to rms values resulted from the operations performed using the trigonometric identities.

If Equation is expanded to the form

$$p = \underbrace{VI\cos\theta}_{\text{Average}} - \underbrace{VI\cos\theta}_{\text{Peak}}\cos\underbrace{2\omega t}_{2x} + \underbrace{VI\sin\theta}_{\text{Peak}}\sin\underbrace{2\omega t}_{2x}$$

the average power still appears as an isolated term that is time independent. Second, both terms that follow vary at a frequency twice that of the applied voltage or current, with peak values having a very similar format.

Resistive Circuit

For a purely resistive circuit, v and i are in phase, and $\theta = 0$, as appearing in Fig. 3.20. Substituting $\theta = 0$ into Eq.(3.1), we obtain

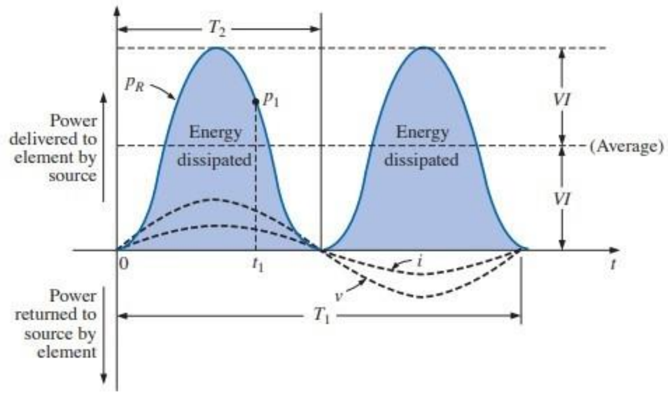


Figure 3.20:

$$\begin{aligned} p_R &= V\,I\cos(0^\circ)(1 - \cos 2\omega t) + V\,I\sin\theta(\sin 2\omega t) \\ &= V\,I(1 - \cos 2\omega t) + 0 \end{aligned} \tag{3.2}$$

or

$$p_R = V\,I - V\,I\cos 2\omega t \tag{3.3}$$

where VI is the average or dc term and VI cos 2ωt is a negative cosine wave with twice the frequency of either input quantity (v or i) and a peak value of VI.

Plotting the waveform for p_R (Fig. 3.20), we see that

T_1 = period of input quantities

T_2 = period of power curve p_R

Note that in Fig. 3.20 the power curve passes through two cycles about its average value of VI for each cycle of either v or i ($T_1 = 2T_2$ or $f_2 = 2f_1$). Consider also that since the peak and average values of the power curve are the same, the curve is always above the horizontal axis. This indicates that the total power delivered to a resistor will be dissipated in the form of heat.

The power returned to the source is represented by the portion of the curve below the axis, which is zero in this case. The power dissipated by the resistor at any instant of time t_1 can be found by simply substituting the time t_1 into Eq. (3.3) to find p_1, as indicated in Fig. 3.20. The average (real) power from Eq. (3.3), or Fig. 3.20, is VI; or, as a summary,

$$P = VI = \frac{V_m \mathrm{I}_m}{2} = I^2 R = \frac{V^2}{R}(watts, W)$$

3.9.1 Apparent Power

From our analysis of dc networks (and resistive elements above), it would seem apparent that the power delivered to the load is simply determined by the product of the applied voltage and current, with no concern for the components of the load; that is, P = VI. However, we found that the power factor ($\cos\theta$) of the load will have a pronounced effect on the power dissipated, less pronounced for more reactive loads. Although the product of the voltage and current is not always the power delivered, it is a power rating of significant usefulness in the description and analysis of sinusoidal ac networks and in the maximum rating of a number of electrical components and systems. It is called the apparent power and is represented symbolically by S.* Since it is simply the product of voltage and current, its units are volt-amperes (VA). Its magnitude is determined by

$$S = V\,I(volt - amperes, V\,A) \tag{3.4}$$

or since

$$V = IZ$$

and

$$I = \frac{V}{Z}$$

then

$$S = I^2 Z(V\,A) \tag{3.5}$$

and

$$s = \frac{V^2}{Z}(V\,A) \tag{3.6}$$

The average power to the load of

$$P = V\,I\cos\theta$$

However,

$$S = IV$$

Therefore,

$$P = S\cos\theta(W) \tag{3.7}$$

The power factor of a circuit, therefore, is the ratio of the average power to the apparent power ot the percentage of apparent power that is converted to real power.

3.9.2 Reactive Power

Inductive Circuit

For a purely inductive circuit (such as that in Fig. 3.21), v leads i by 90 , as shown in Fig. 3.22. Therefore, in Eq. (3.1), θ = 90 . Substituting θ = 90 into Eq. (3.1) yields

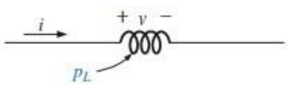


Figure 3.21: Defining the power level for a purely inductive load

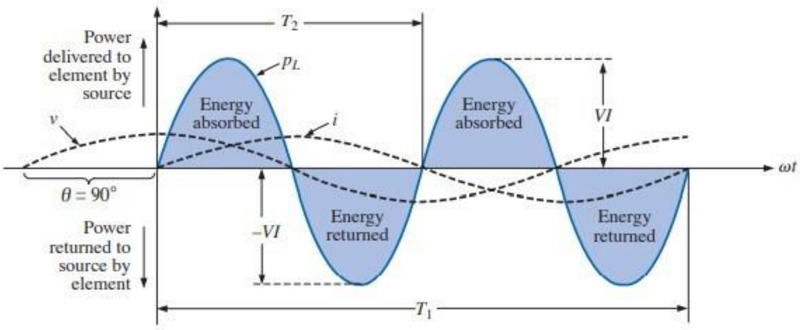


Figure 3.22: The power curve for a purely inductive load.

$$P_L = V\,I\cos(90^\circ)(1 - \cos 2\omega t) + V\,I\sin(90^\circ)(\sin 2\omega t)$$

$$= 0 + VI\sin 2\omega t \quad (3.8)$$

$$p_L = VI\sin 2\omega t \quad (3.9)$$

where VI sin 2ωt is a sine wave with twice the frequency of either input quantity (v or i) and a peak value of VI. Note the absence of an average or constant term in the equation. Plotting the waveform for p_L (Fig. 3.22), we obtain

T_1 = period of either input quantity

T_2 = period of pL curve

Note that over one full cycle of p_L (T_2), the area above the horizontal axis in Fig. 3.22 is exactly equal to that below the axis. This indicates that over a full cycle of p_L, the power delivered by the source to the inductor is exactly equal to that returned to the source by the inductor.

The net flow of power to the pure (ideal) inductor is zero over a full cycle, and no energy is lost in the transaction.

The power absorbed or returned by the inductor at any instant of time t_1 can be found simply by substituting t_1 into Eq. (3.9). The peak value of the curve VI is defined as the reactive power associated with a pure inductor.

In general, the reactive power associated with any circuit is defined to be VIsinθ , a factor appearing in the second term of Eq. (3.1). Note that it is the peak value of that term of the total power equation that produces no net transfer of energy. The symbol for reactive power is Q, and its unit of measure is the voltampere reactive (VAR).* Therefore,

$$Q = VI\sin\theta (VAR) \quad (3.10)$$

where v is the phase angle between V and I.

For the inductor,

$$Q_L = VI \quad (3.11)$$

or, since V=IXL or I=V/XL.

$$Q_L = I^2 X_L \quad (3.12)$$

or

$$Q_L = \frac{V^2}{X_L} \quad (3.13)$$

If the average power is zero, and the energy supplied is returned within one cycle, why is reactive power of any significance? At every instant of time along the power curve that the curve is above the axis (positive), energy must be supplied to the inductor, even though it will be returned during the negative portion of the cycle. This power requirement during the positive portion of the cycle requires that the generating plant provide this energy during that interval. Therefore, the effect of reactive elements such as the inductor can be to raise the power requirement of the generating plant, even though the reactive power is not dissipated.

Capacitive Circuit

For a purely capacitive circuit (such as that in Fig. 3.23), i leads v by 90 , as shown in Fig. 3.24. Therefore, in Eq. (3.1), θ= -90 . Substituting θ = -90 into Eq. (3.1), we obtain

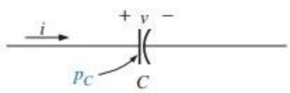


Figure 3.23: Defining the power level for a purely capacitive load.

$$p_c = VI\cos(-90^\circ)(1 - \cos 2\omega t) + VI\sin(-90^\circ)(\sin 2\omega t)$$
$$= 0 - VI\sin 2\omega t$$

$$p_c = -VI\sin 2\omega t$$

where -VI sin 2ωt is a negative sine wave with twice the frequency of either input (v or i) and a peak value

of VI. Again, note the absence of an average or constant term.
Plotting the waveform for p_C (Fig. 3.24) gives us

T_1 = period of either input quantity

T_2 = period of p_C curve

The power delivered by the source to the capacitor is exactly equal to that returned to the source by the capacitor over one full cycle. The net flow of power to the pure (ideal) capacitor is zero over a full cycle, and no energy is lost in the transaction.

The reactive power associated with the capacitor is equal to the peak value of the p_C curve, as follows:

$$Q_c = V\,I(VAR)$$

But, since $V=IX_c$ and $I=V/X_c$ the reactive power to the capacitor can also be written

$$Q_c = IX_C(VAR)$$

And

$$Q_c = \frac{V^2}{X_c}(VAR)$$

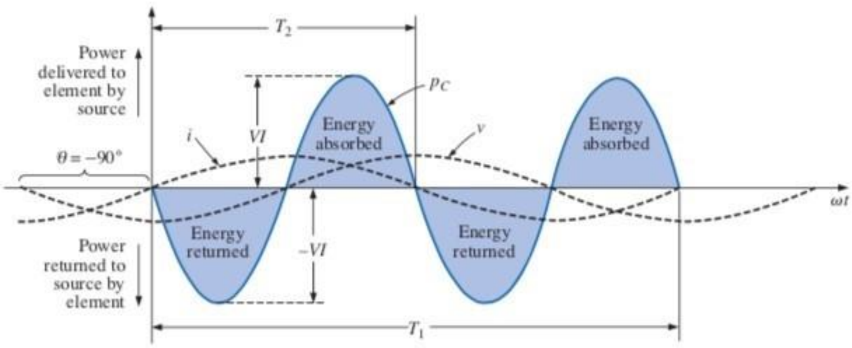


Figure 3.24: The power curve for a purely capacitive load

3.10 Power Triangle

The three quantities average power, apparent power, and reactive power can be related in the vector domain by

$$S = P + Q$$

with

$$P = P\angle 0°$$
$$Q_L = Q_L\angle 90°$$
$$Q_C = Q_C\angle -90°$$

For an inductive load, the phasor power S, as it is often called, is defined by

$$S = P + jQ_L$$

For a capacitive load, the phasor power S is defined by

$$S = P - jQ_C$$

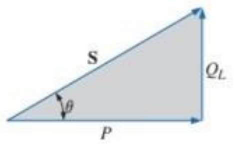


Figure 3.25: Power diagram for inductive loads

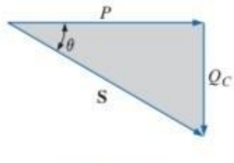


Figure 3.26: Power diagram for capacitive loads

Chapter 4

Three-phase Systems

At the end of the chapter students should be able to:

1. Describe a three-phase supply.
2. Appreciate the advantages of three-phase power systems.
3. Understand a star connection, and draw a complete phasor diagram for a balanced, star connected load.
4. Understand a delta connection and draw a phasor diagram for balanced, delta connected load.
5. Understand the phase sequence.
6. Calculate power in three-phase systems using $P=\sqrt{3}V_LI_LCos\varphi$.

4.1 Introduction

An ac generator designed to develop a single sinusoidal voltage for each rotation of the shaft (rotor) is referred to as a single-phase ac generator. If the number of coils on the rotor is increased in a specified manner, the result is a polyphase ac generator, which develops more than one ac phase voltage per rotation of the rotor. Circuits or systems in which the ac sources operate at the same frequency but different phases are known as polyphase.
A three-phase system is produced by a generator consisting of three sources having the same amplitude and frequency but out of phase with each other by 120°.

The figure below depicts a three-phase four-wire system, where V_p is the magnitude of the source voltage and ϕ is the phase.

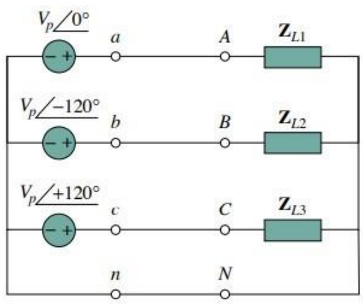


Figure 4.1: Three-phase four-wire system

In general, three-phase systems are preferred over single-phase systems for the transmission of power for many reasons, including the following:

1. Three-phase equipment and motors have preferred running and starting characteristics compared to singlephase systems because of a more even flow of power to the transducer than can be delivered with a single-phase supply.
2. The lighter lines are easier to install, and the supporting structures can be less massive and farther apart.3. Thinner conductors can be used to transmit the same kVA at the same voltage, which reduces the amount of copper required (typically about 25percent less) and in turn reduces construction and maintenance costs.
4. In general, most larger motors are three phase because they are essentially self-starting and do not require a special design or additional starting circuitry.

The frequency generated is determined by the number of poles on the rotor (the rotating part of the generator) and the speed with which the shaft is turned. Throughout the United States the line frequency is 60Hz, whereas in Europe the chosen standard is 50Hz.

4.2 The Three-Phase Generator

The three-phase generator of Fig. 4.2 has three induction coils placed 120 apart on the stator, as shown symbolically by Fig. 4.3. Since the three coils have an equal number of turns, and each coil rotates with the same angular velocity, the voltage induced across each coil will have the same peak value, shape, and frequency.

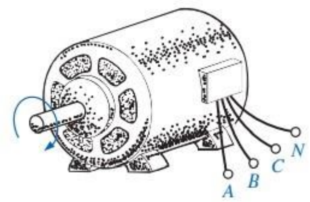


Figure 4.2: Three-phase generator

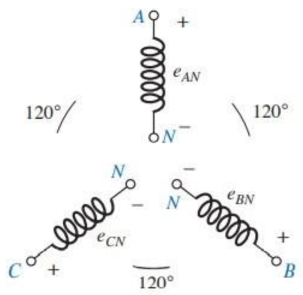


Figure 4.3: Induced voltages of a three-phase generator

In particular, note that at any instant of time, the algebraic sum of the three phase voltages of a three-phase generator is zero.

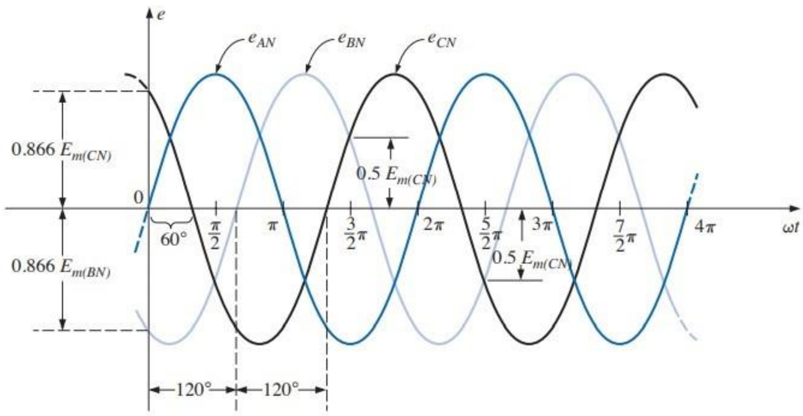
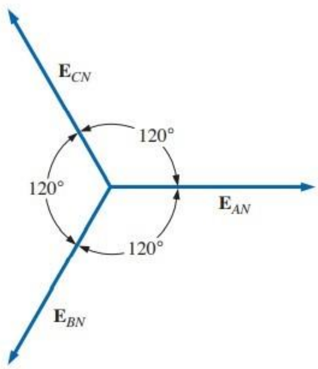


Figure 4.4: Phase voltage of a three-phase generator

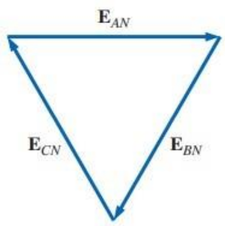
The sinusoidal expression for each of the induced voltages;

$e_{AN} = E_{m(AN)} \sin\omega t$ $e_{BN} = E_{m(BN)} \sin(\omega t - 120^\circ)$ $e_{CN} = E_{m(CN)}$
$\sin(\omega t - 240^\circ) = E_{m(CN)} \sin(\omega t + 120^\circ)$



$$E_{AN} = E_{AN}\angle 0^\circ$$
$$E_{BN} = E_{BN}\angle -120^\circ$$
$$E_{CN} = E_{CN}\angle +120^\circ$$

By rearranging the phasors as shown in Fig. and applying a law of vectors which states that the vector sum of any number of vectors drawn such that the “head” of one is connected to the “tail” of the next, and that the head of the last vector is connected to the tail of the first is zero, we can conclude that the phasor sum of the phase voltages in a three-phase system is zero. That is,



$$E_{AN} + E_{BN} + E_{CN} = 0$$

4.3 The Y-Connected Generator

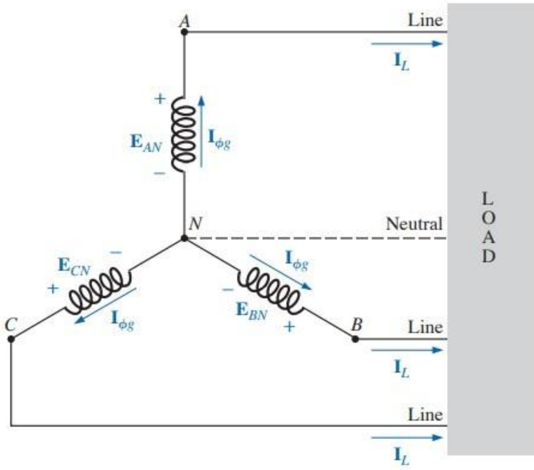


Figure 4.5: Y-connected generator

The point at which all the terminals are connected is called the neutral point. If a conductor is not attached from this point to the load, the system is called a Y-connected, three-phase, three-wire generator.
If the neutral is connected, the system is a Y-connected, three-phase, four-wire generator.
The three conductors connected from A, B, and C to the load are called lines.
For the Y-connected system, the line current equals the phase current for each phase; that is

$$I_L = I_{\varphi g}$$

where φ is used to denote a phase quantity and g is a generator parameter.
The voltage from one line to another is called a line voltage. On the phasor diagram (Fig. 4.6) it is the

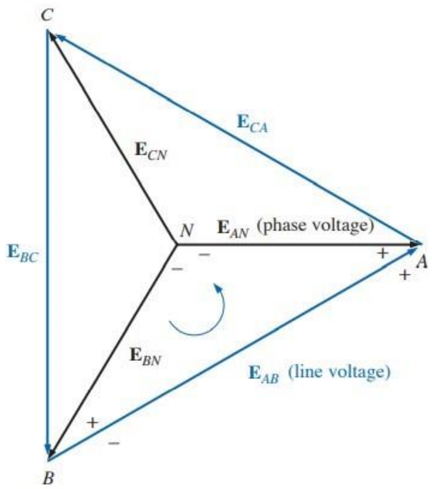


Figure 4.6: Line and phase voltages of the Y-connected three-phase generator

phasor drawn from the end of one phase to another in the counter-clockwise direction. Applying Kirchoff's voltage law around the indicated loop of Fig. 4.6, we obtain

$$E_{AB} - E_{AN} + E_{BN} = 0$$

or

$$E_{AB} = E_{AN} - E_{BN} = E_{AN} + E_{NB}$$

Determining a line voltage for a three-phase generator.

The length x is

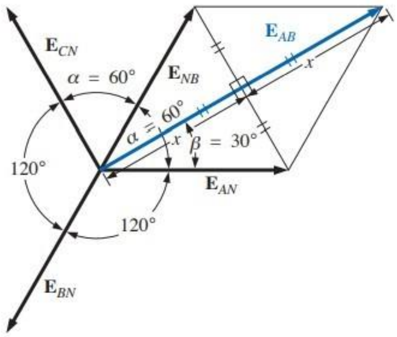


Figure 4.7: Determining a line voltage for a three-phase generator.

$$x = E_{AN} \cos 30° = \frac{\sqrt{3}}{2} E_{AN}$$

and

$$E_{AB} = 2x = (2)\frac{\sqrt{3}}{2} E_{AN} = \sqrt{3} E_{AN}$$

Noting from the phasor diagram that θ of

$$E_{AB} = \beta = 30^{\circ}$$

, the result is

$$E_{AB} = E_{AB} \angle 30° = \sqrt{3} E_{AN} \angle 30°$$

and

$$E_{CA} = \sqrt{3} E_{CN} \angle 150°$$
$$E_{BC} = \sqrt{3} E_{BN} \angle 270°$$

In words, the magnitude of the line voltage of a Y-connected generator is $\sqrt{3}$ times the phase voltage:

$$E_L = \sqrt{3} E_{\varphi}$$

with the phase angle between any line voltage and the nearest phase voltage at 30°.

4.3.1 Phase Sequence (Y-Connected)

The phase sequence can be determined by the order in which the phasors representing the phase voltages pass through a fixed point on the phasor diagram if the phasors are rotated in a counter-clockwise direction.

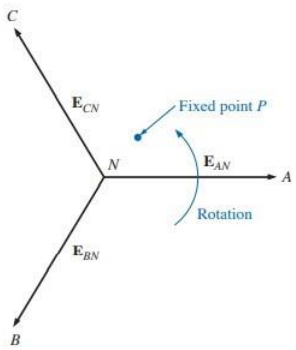


Figure 4.8: Determining the phase sequence from the phase voltages of a three-phase generator

For example, in Fig. 4.8 the phase sequence is ABC. However, since the fixed point can be chosen anywhere on the phasor diagram, the sequence can also be written as BCA or CAB.

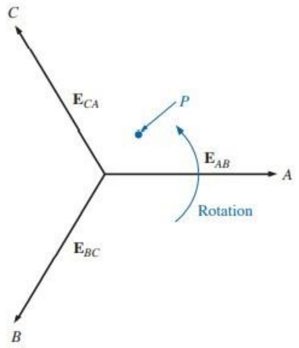


Figure 4.9: Determining the phase sequence from the line voltages of a three-phase generator

The phase sequence can also be described in terms of the line voltages. Drawing the line voltages on a phasor diagram in Fig. 4.9, we are able to determine the phase sequence by again rotating the phasors in the counter-clockwise direction. In this case, however, the sequence can be determined by noting the order of the passing first or second subscripts. In the system of Fig. 4.9, for example, the phase sequence of the first subscripts passing point P is ABC, and the phase sequence of the second subscripts is BCA. But we know that BCA is equivalent to ABC, so the sequence is the same for each.

If the sequence is given, the phasor diagram can be drawn by simply picking a reference voltage, placing it on the reference axis, and then drawing the other voltages at the proper angular position.
In phasor notation,
Line voltages

$$E_{AB} = E_{AB}\angle 0°(reference)$$
$$E_{CA} = E_{CA}\angle -120°$$
$$E_{BC} = E_{BC}\angle +120°$$

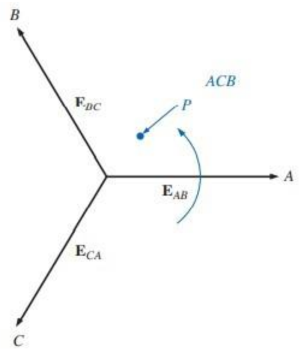


Figure 4.10: Drawing the phasor diagram from the phase sequence.

4.3.2 The Y-Connected Generator With A Y-Generator Load

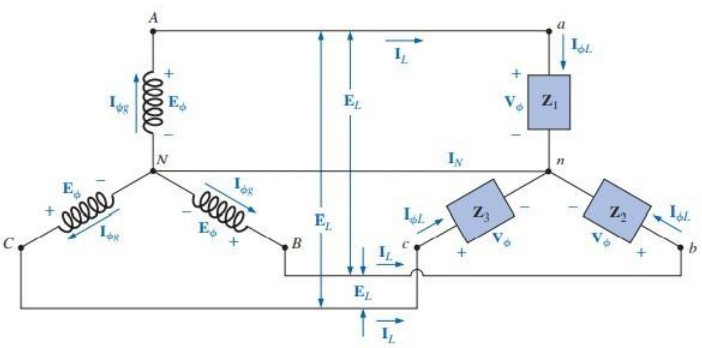


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Figure 4.11: Y-connected generator with a Y-connected load.

If the load is balanced, the neutral connection can be removed without affecting the circuit in any manner; that is, if

$$Z_1 = Z_2 = Z_3$$

then IN will be zero.

$$I_{\varphi g} = I_L = I_{\varphi L}$$

For a balanced or an unbalanced load, since the generator and load have a common neutral point, then

$$V_\phi = E_\phi$$
$$E_L = \sqrt{3}V_\phi$$

Example 4.1

The phase sequence of the Y-connected generator in Fig. 4.12 is ABC. a. Find the phase angles θ_2 and θ_3.

b. Find the magnitude of the line voltages.

c. Find the line currents.

d. Verify that, since the load is balanced, I_N=0.

Solutions:

a. For an ABC phase sequence, $\theta_2 = -120^\circ$ and $\theta_3 = +120^\circ$

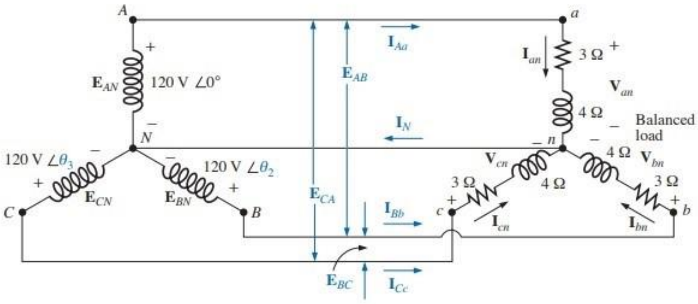

Figure 4.12:

b. $E_L = \sqrt{3}\, E_\varphi = (1.73)(120V) = 208V$. Therefore,

$$E_{AB} = E_{BC} = E_{CA} = 208V$$

c. $V_\varphi = E_\varphi$. Therefore,

$V_{an}=E_{AN}$ $V_bn=E_{BN}$ $V_{cn}=E_{CN}$

$$I_{\phi L} = I_{an} = \frac{V_{an}}{Z_{an}} = \frac{120V\angle 0^\circ}{3\Omega + j4\Omega} = \frac{120V\angle 0^\circ}{5\Omega\angle 53.13^\circ} = 24A\angle -53.13^\circ$$

$$I_{bn} = \frac{V_{bn}}{Z_{bn}} = \frac{120V\angle -120^\circ}{5\Omega\angle 53.13^\circ} = 24A\angle -173.13^\circ$$

$$I_{cn} = \frac{V_{cn}}{Z_{cn}} = \frac{120V\angle +120^\circ}{5\Omega\angle 53.13^\circ} = 24A\angle 66.87^\circ$$

and, since $I_L = I_\varphi$,

$$I_{Aa} = I_{an} = 24A\angle -53.13^\circ$$
$$I_{Bb} = I_{bn} = 24A\angle -173.13^\circ$$
$$I_{Cc} = I_{cn} = 24A\angle 66.87^\circ$$

d. Applying Kirchhoff's current law, we have

$$I_N = I_{Aa} + I_{Bb} + I_{Cc}$$

In rectangular form,

$$I_{Aa} = 24A\angle -53.13° = 14.40A - j19.20A$$
$$I_{Bb} = 24A\angle -173.13° = -22.83A - j2.87A$$
$$I_{Cc} = 24A\angle 66.87° = 9.43A + j22.07A$$
$$\Sigma(I_{Aa} + I_{Bb} + I_{Cc}) = 0 + j0$$

and I_N is in fact equal to zero, as required for a balanced load.

4.4 The Y-Δ System

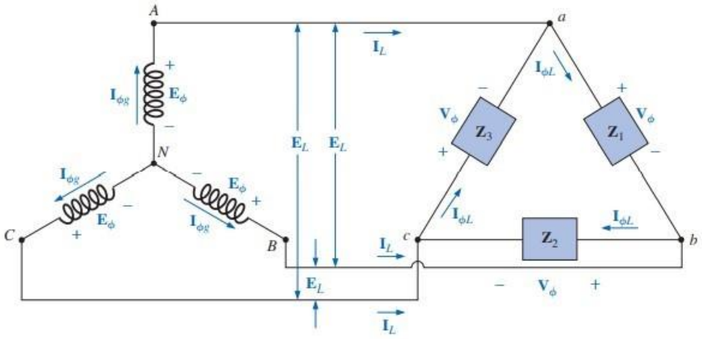

Figure 4.13: Y-connected generator with a Δ-connected load

For a balanced load

$$Z_1 = Z_2 = Z_3$$

The voltage across each phase of the load is equal to the line voltage of the generator for a balanced or an unbalanced load:

$$V_\phi = E_L$$
$$I_L = \sqrt{3}I_\phi$$

Example 4.2

For the three-phase system of Fig. 4.14: a.
Find the phase angles θ_2 and θ_2.
b. Find the current in each phase of the load.
c. Find the magnitude of the line currents.**Solutions:**

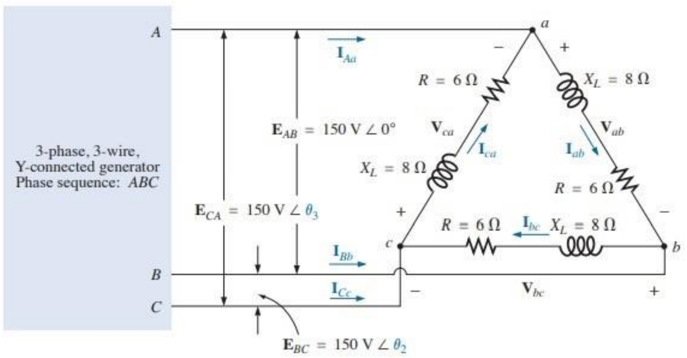

Figure 4.14:

a. For an ABC sequence,
θ_2=-120° and θ_3=+120° b. $V_\varphi = E_L$. Therefore,
$V_{ab}=E_{AB}$ $V_{ab}=E_{CA}$ $V_{ab}=E_{BC}$
The phase currents are

$$I_{ab} = \frac{V_{ab}}{Z_{ab}} = \frac{150V\angle 0^\circ}{6\Omega+j8\Omega} = \frac{150V\angle 0^\circ}{10\Omega\angle 53.13^\circ} = 15A\angle -53.13^\circ$$
$$I_{bc} = \frac{V_{bc}}{Z_{bc}} == \frac{150V\angle -120^\circ}{10\Omega\angle 53.13^\circ} = 15A\angle -173.13^\circ$$
$$I_{ca} = \frac{V_{ca}}{Z_{ca}} == \frac{150V\angle +120^\circ}{10\Omega\angle 53.13^\circ} = 15A\angle 66.87^\circ$$

c. $I_L = \sqrt{3} I_{\varphi}$=(1.73A)(15A)=25.95A. Therefore,

$$I_{Aa} = I_{Bb} = I_{Cc} = 25.95A$$

4.5 The Δ-Connected Generator

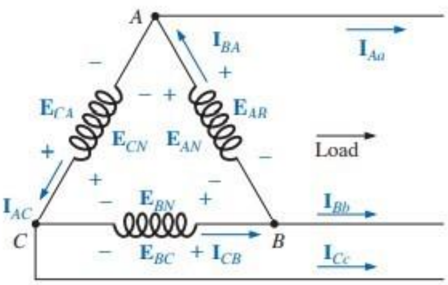

Figure 4.15: Δ-connected generator

Δ-connected ac generator. In this system, the phase and line voltages are equivalent and equal to the voltage induced across each coil of the generator; that is,

Phase sequence ABC;

$E_{AB}=E_{AN}$ and $e_{AN}=\sqrt{2}E_{AN}\sin\Omega t$

$E_{BC}=E_{BN}$ and $e_{BN}=\sqrt{2}E_{BN}\sin(\Omega t-120^\circ)$

$E_{CA}=E_{CN}$ and $e_{CN}= 2E_{CN}\sin(\Omega t+120^\circ)$ or

$$E_L = E_{\varphi g}$$

Unlike the line current for the Y-connected generator, the line current for the D-connected system is not equal to the phase current. The relationship between the two can be found by applying Kirchoff's current law at one of the nodes and solving for the line current in terms of the phase currents; that is, at node A.

$I_{BA}=I_{Aa}+I_{AC}$ or $I_{Aa}=I_{BA}-I_{AC}=I_{BA}+I_{CA}$

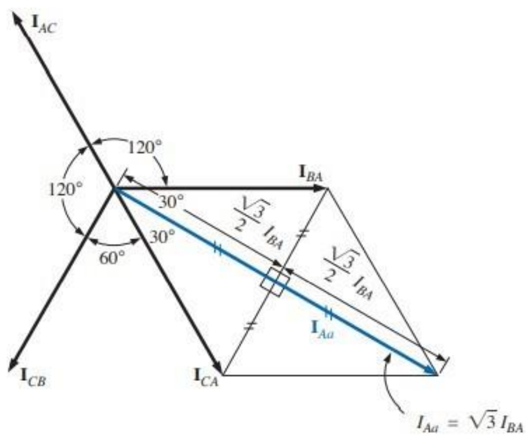

Figure 4.16: Determining a line current from the phase currents of a Δ-connected, three-phase generator.

$$I_{Aa} = \sqrt{3} I_{BA} \angle -30^\circ$$
$$I_{Bb} = \sqrt{3} I_{CB} \angle -150^\circ$$
$$I_{Cc} = \sqrt{3} I_{AC} \angle 90^\circ$$
$$I_L = \sqrt{3} I_{\phi g}$$

with the phase angle between a line current and the nearest phase current at 30°.

4.5.1 Phase Sequence (Δ-connected Generator)

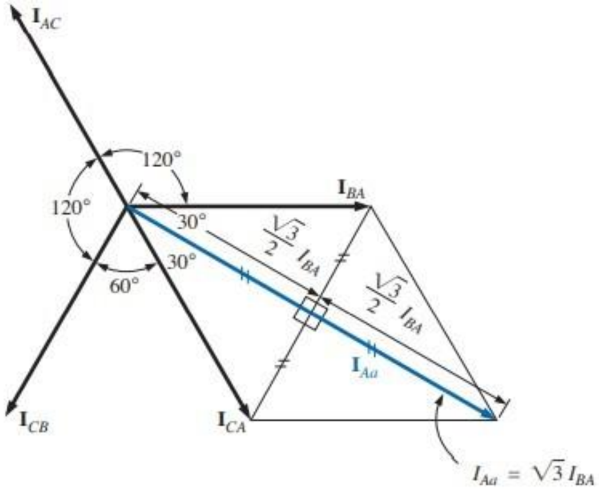

Figure 4.17: Determining the phase sequence for a Δ-connected, three-phase generator.

$$E_{AB} = E_{AB} \angle 0^\circ$$
$$E_{BC} = E_{BC} \angle -120^\circ$$
$$E_{CA} = E_{CA} \angle 120^\circ$$

4.6 The Δ-Δ,Δ-Y Three-phase Systems

Example 4.3

For the Δ-Δ system shown in Fig. (4.18):

a. Find the phase angles θ_2 and θ_3 for the specified phase sequence.
b. Find the current in each phase of the load.
c. Find the magnitude of the line currents.**Solutions:**

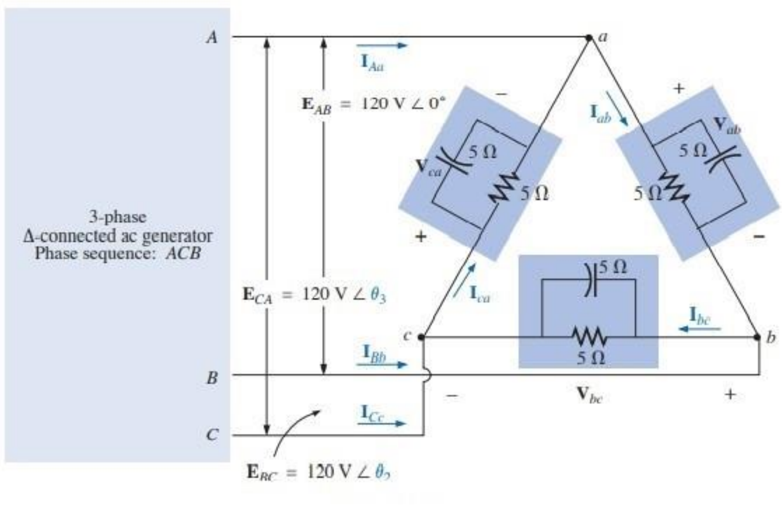

Figure 4.18:

a. For an ABC phase sequence,θ_2=120° and θ_3=-120°

b. $V_\varphi=E_L$. Therefore,

$V_{ab}=E_{AB}$ $V_{ca}=E_{CA}$ $V_{bc}=E_{BC}$

The phase currents are

$$I_{ab}=\frac{V_{ab}}{Z_{ab}}=\frac{120V\angle 0^\circ}{\frac{(5\Omega\angle 0^\circ)(5\Omega\angle -90^\circ)}{5\Omega-j5\Omega}}=\frac{120V\angle 0^\circ}{\frac{25\Omega\angle -90^\circ}{7.071\angle -45^\circ}}$$
$$=\frac{120V\angle 0^\circ}{3.45\Omega\angle -45^\circ}=33.9A\angle 45^\circ$$
$$I_{bc}=\frac{V_{bc}}{Z_{bc}}=\frac{120V\angle 120^\circ}{3.45\Omega\angle -45^\circ}=33.9A\angle 165^\circ$$
$$I_{ca}=\frac{V_{ca}}{Z_{ca}}=\frac{120V\angle -120^\circ}{3.45\Omega\angle -45^\circ}=33.9A\angle -75^\circ$$

c. $I_L=\sqrt{3}I_\varphi$=(1.73)(34A)=58.82A. Therefore, $I_{Aa}=I_{Bb}=I_{Cc}$=58.82A

Example 4.4

For the Δ-Y system shown in Fig. (4.19):

a. Find the voltage across each phase of the load.

b. Find the magnitude of the line voltages.**Solutions:**

a. $I_{\varphi L}=I_L$. Therefore,

$$I_{an}=I_{Aa}=2A\angle 0^\circ$$
$$I_{bn}=I_{Bb}=2A\angle -120^\circ$$
$$I_{cn}=I_{Cc}=2A\angle 120^\circ$$

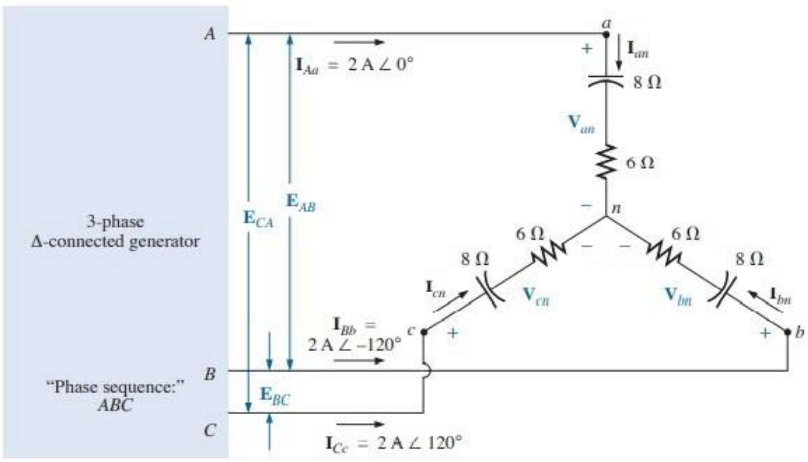

Figure 4.19:

The phase voltages are

$$V_{an}=I_{an}Z_{an}=(2A\angle 0^\circ)(10\Omega\angle -53.13^\circ)=20V\angle -53.13^\circ$$
$$V_{bn}=I_{bn}Z_{bn}=(2A\angle -120^\circ)(10\Omega\angle -53.13^\circ)=20V\angle -173.13^\circ$$
$$V_{cn}=I_{cn}Z_{cn}=(2A\angle 120^\circ)(10\Omega\angle -53.13^\circ)=20V\angle 66.87^\circ$$

b. $E_L=\sqrt{3}V_\varphi$=(1.73)(20V)=34.6V. Therefore, $E_{BA}=E_{CB}=E_{AC}$=34.6V

4.7 Power In Three-phase Systems

4.7.1 Y-Connected Balanced Load

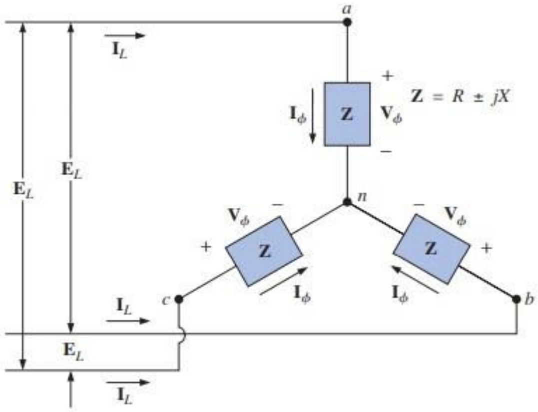

Figure 4.20:

Average Power

$$P_\phi = V_\phi I_\phi \cos\theta_{I_\theta}^{V_\phi} = I_\phi^2 R_\phi = \frac{V_R^2}{R_\phi}(watts, W)$$

where $\theta_{I_\theta}^{V_\phi}$ indicates that θ is the phase angle between V_φ and I_φ. The total power to the balanced load is

$$P_T = 3P_\varphi(W)$$

$V_\phi = \frac{E_L}{\sqrt{3}}$ and $I_\varphi = I_L$ or,
since then
$P_T = 3\frac{E_L}{\sqrt{3}} I_L \cos\theta_{I_\phi}^{V_\phi}$ But

$$\left(\frac{3}{\sqrt{3}}\right)(1) = \left(\frac{3}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

Therefore,

$$P_T = \sqrt{3} E_L I_L \cos\theta_{I_\phi}^{V_\phi} = 3I_L^2 R_\phi (W)$$

Reactive Power

$$P_T = 3\frac{E_L}{\sqrt{3}} I_L \cos\theta_{I_\phi}^{V_\phi}$$

But

$$\left(\frac{3}{\sqrt{3}}\right)(1) = \left(\frac{3}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

Therefore,

$$P_T = \sqrt{3} E_L I_L \cos\theta_{I_\phi}^{V_\phi} = 3I_L^2 R_\phi (W)$$

The total reactive power of the load is
$Q_T = 3Q_\varphi$ (VAR) or, proceeding in the same manner as above, we have
$Q_T = \sqrt{3} E_L I_L \sin\theta_{I_\phi}^{V_\phi} = 3I_L^2 X_\phi$ (VAR)

Apparent Power

The apparent power of each phase is
$S_\varphi = V_\varphi I_\varphi$ (VA)
The total apparent power of the load is

$S_T=3S_\varphi$ (VA) or, as before,

$S_T=\sqrt{3}E_LI_L$ (VA)

Power Factor

$F_p=\frac{P_T}{S_T}=\cos\theta^{V_\phi}_{I_\phi}$ (leading or lagging)

Example 4.5

For the Y-connected load of Fig. (4.21):

a. Find the average power to each phase and the total load.

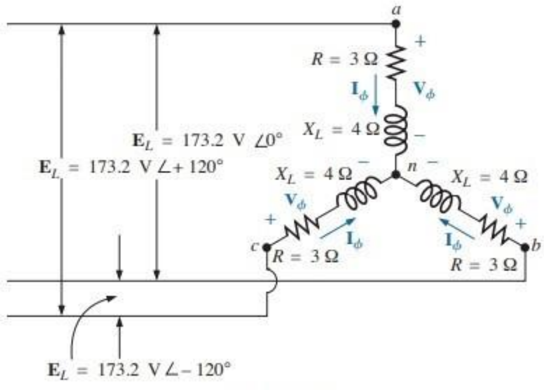

Figure 4.21:

b. Determine the reactive power to each phase and the total reactive power.

c. Find the apparent power to each phase and the total apparent power.

d. Find the power factor of the load.

Solutions:

a. The average power is

$$P_\phi = V_\phi I_\phi \cos\theta^{V_\phi}_{I_\theta} = (100V)(20A)\cos 53.13^\circ = (2000)(0.6) = 1200W$$
$$P_\phi = I_\phi^2 R_\phi = (20A)^2(3\Omega) = (400)(3) = 1200W$$
$$P_\phi = \frac{V_R^2}{R_\phi} = \frac{(60V)^2}{3\Omega} = \frac{3600}{3} = 1200W$$
$$P_T = 3P_\phi = (3)(1200W) = 3600W$$

or

$$P_T = \sqrt{3}E_L I_L \cos\theta^{V_\phi}_{I_\phi} = (1.732)(173.2V)(20A)(0.6) = 3600W$$

b. The reactive power is

$$Q_\phi = V_\phi I_\phi \sin\theta^{V_\phi}_{I_\phi} = (100V)(20A)\sin 53.13^\circ = (2000)(0.8) = 1600VAR$$

or

$$Q_\phi = I_\phi^2 X_\phi = (20A)^2(4\Omega) = (400)(4) = 160VAR$$
$$Q_T = 3Q_\phi = (3)(1600VAR) = 4800VAR$$

or

$$Q_T = \sqrt{3}E_L I_L \sin\theta^{V_\phi}_{I_\phi} = (1.732)(173.2V)(20A)(0.8) = 4800VAR$$

c. The apparent power is

$S_\varphi = V_\varphi I_\varphi = (100V)(20A) = 2000VA$ $S_T = 3S_\varphi = (3)(2000VA) = 6000VA$

or

$S_T = \sqrt{3}E_L I_L = (1.732)(173.2V)(20A) = 6000VA$

d. The power factor is

$$F_p = \frac{P_T}{S_T} = \frac{3600W}{6000VA} = 0.6lagging$$

4.7.2 Δ-Balanced Load

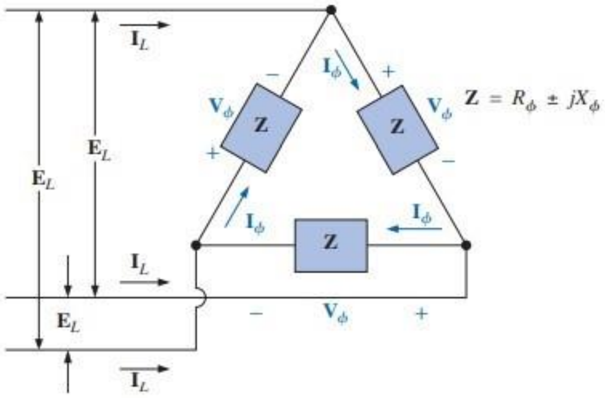

Figure 4.22: Δ-connected balanced load.

Average Power

$$P_\phi = V_\phi I_\phi \cos\theta_{I_\phi}^{V_\phi} = I_\phi^2 R_\phi = \frac{V_R^{\phi}}{R_\phi}(W)$$
$$P_T = 3P_\phi(W)$$

Reactive Power

$$Q_\phi = V_\phi I_\phi \sin\theta_{I_\phi}^{V_\phi} = I_\phi^2 X_\phi = \frac{V_x^2}{X_\phi}(VAR)$$
$$Q_T = 3Q_\phi(VAR)$$

Apparent Power

$$S_\phi = V_\phi I_\phi(VA)$$
$$S_T = 3S_\phi = \sqrt{3}E_L I_L(VA)$$

Power Factor

$$F_p = \frac{P_T}{S_T}$$

Example 4.6

For the Δ-Y connected load of Fig. (4.23), find the total average, reactive, and apparent power. In addition, find the power factor of the load.

Solution: Consider the Δ and Y separately.

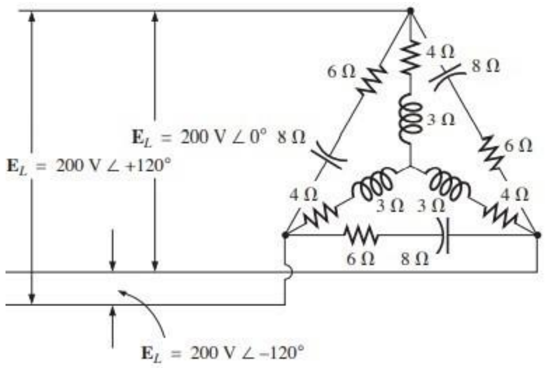

Figure 4.23:

For the Δ:

$$I_\phi = \frac{E_L}{Z_\Delta} = \frac{200V}{10\Omega} = 20A$$
$$P_{T_\Delta} = 3I_\phi^2 R_\phi = (3)(20A)^2\,(6\Omega) = 7200W$$
$$Q_{T_\Delta} = 3I_\phi^2 X_\phi = (3)(20A)^2(8\Omega)9600VAR(C)$$
$$S_{T_\Delta} = 3V_\phi I_\phi = (3)(200V)(20A) = 12,000VA$$

For the Y:

$$Z_Y = 4\Omega + j3\Omega = 5\Omega\angle 36.87°$$
$$I_\phi = \frac{E_L/\sqrt{3}}{Z_Y} = \frac{200V/\sqrt{3}}{5\Omega} = \frac{116V}{5\Omega} = 23.12A$$
$$P_{T_Y} = 3I_\phi^2 R_\phi = (3)(23.12)^2(4\Omega) = 6414.41W$$
$$Q_{T_Y} = 3I_\phi^2 X_\phi = (3)(23.12)^2(3\Omega) = 4810.81VAR(L)$$
$$S_{T_Y} = 3V_\phi I_\phi = (3)(116V)(23.12A) = 8045.76VA$$

For the total load:

$$P_T = P_{T_\Delta} + P_{T_Y} = 7200W + 6414.41W = 13,614.41W$$
$$Q_T = Q_{T_\Delta} - Q_{T_Y} = 9600VAR(C) - 4810.81VAR(I) = 4789.19VAR(C)$$
$$S_T = \sqrt{P_T^2 + Q_T^2} = \sqrt{(13,614.41W)^2 + (4789.19VAR)^2} = 14,432.2VA$$
$$F_p = \frac{P_T}{S_T} = \frac{13,614.41W}{14,432.20VA} = 0.943 leading$$

4.8 Unbalanced, Three-phase, Four-wire, Y-Connected Load

Since the neutral is a common point between the load and source, no matter what the impedance of each phase of the load and source, the voltage across each phase is the phase voltage of the generator:

$$V_\varphi = E_\varphi$$

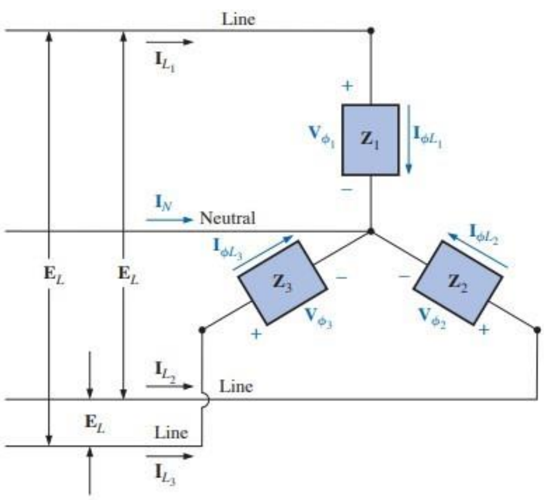

Figure 4.24: Unbalanced Y-connected load.

The phase currents can therefore be determined by Ohm's law:

$$I_{\phi_1} = \frac{V_{\phi_1}}{Z_1} = \frac{E_{\phi_1}}{Z_1}$$

and so

$$I_N = I_{\varphi 1} + I_{\varphi 2} + I_{\varphi 3} = I_{L1} + I_{L2} + I_{L3}$$

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Chapter 5

Magnetic Circuits

At the end of the chapter students should be able to:
1. Describe the magnetic field around a permanent magnet.
2. State the laws of magnetic attraction and repulsion for two magnets in close proximity.
3. Define magnetic flux, Φ, and magnetic flux density, B, and state their units.
4. Perform simple calculations involving B.
5. Define permeability and reluctance and state its units.
6. Define magnetomotive force and magnetizing force, H, and state their units.
7. Understand the B-H curve for different magnetic materials.
8. Appreciate how a hysteresis loop is obtained.
9. Compare electrical and magnetic quantities.
10. Perform calculations on series-parallel magnetic circuits.

5.1 Introduction

Electromagnetics is the branch of electrical engineering (or physics) that deals with the analysis and application of electric and magnetic fields. In electromagnetics, electric circuit analysis is applied at low frequencies.

5.2 Magnetic Fields

In the region surrounding a permanent magnet there exists a magnetic field, which can be represented by magnetic flux lines similar to electric flux lines. Magnetic flux lines, however, do not have origins or terminating points as do electric flux lines but exist in continuous loops.
Magnetic flux Φ is the amount of magnetic field produced by a magnetic source. The unit of magnetic flux is the Weber, Wb. If the flux linking one turn in a circuit changes by one weber in one second, a voltage of one volt will be induced in that turn.

The magnetic flux lines radiate from the north pole to the south pole, returning to the north pole through the metallic bar. Note the equal spacing between the flux lines within the core and the symmetric distribution outside the magnetic material. These are additional properties of magnetic flux lines in homogeneous materials (that is, materials having uniform structure or composition throughout).

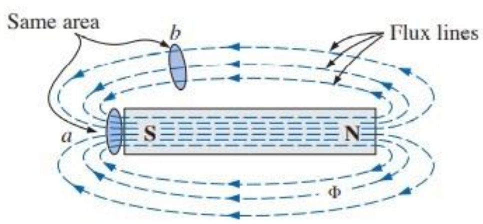

Figure 5.1: Flux distribution for a permanent magnet.

The magnetic field strength at a is twice that at b since twice as many magnetic flux lines are associated with the perpendicular plane at a than at b. The strength of a magnetic field in a particular region is directly related to the density of flux lines in that region.
If unlike poles of two permanent magnets are brought together, the magnets will attract, and the flux distribution will be as shown in Fig. 5.2

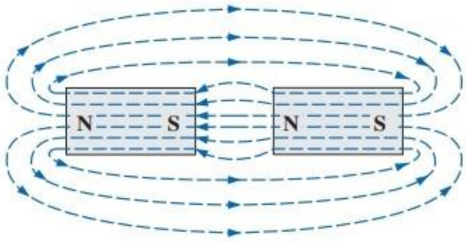

Figure 5.2: Flux distribution for two adjacent, opposite poles.

If like poles are brought together, the magnets will repel, and the flux distribution will be as shown in Fig. 5.3

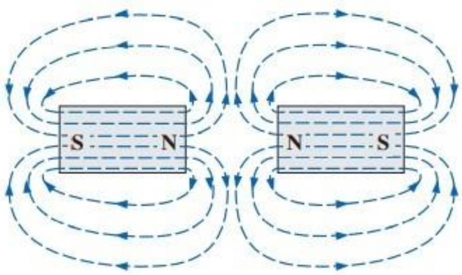

Figure 5.3: Flux density for two adjacent, like poles.

As indicated in the introduction, a magnetic field (represented by concentric magnetic flux lines, as in Fig. 5.4) is present around every wire that carries an electric current. The direction of the magnetic flux lines can be found simply by placing the thumb of the right hand in the direction of conventional current flow and noting the direction of the fingers. (This method is commonly called the right-hand rule).

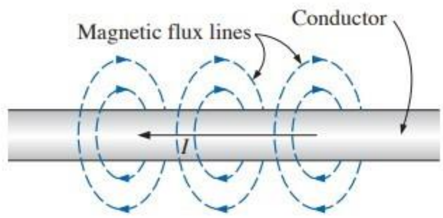

Figure 5.4: Magnetic flux lines around a current-carrying conductor.

If the conductor is wound in a single-turn coil (Fig. 5.5), the resulting flux will flow in a common direction through the center of the coil. A coil of more than one turn would produce a magnetic field that would exist in a continuous path through and around the coil (Fig. 5.6).

Other areas of application for electromagnetic effects are shown in Fig.5.7. The flux path for each is indicated in each figure.

5.3 Flux Density

The number of flux lines per unit area is called the flux density, is denoted by the capital letter B, and is measured in teslas. Its magnitude is determined by the following equation:

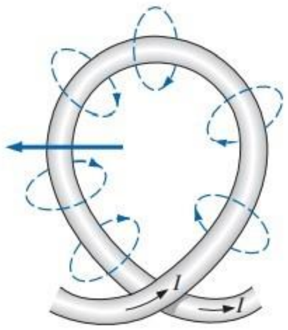

Figure 5.5: Flux distribution of a single-turn coil

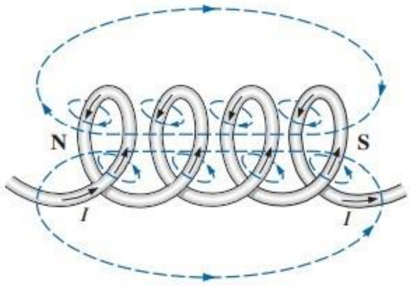

Figure 5.6: Flux distribution of a current-carrying coil.

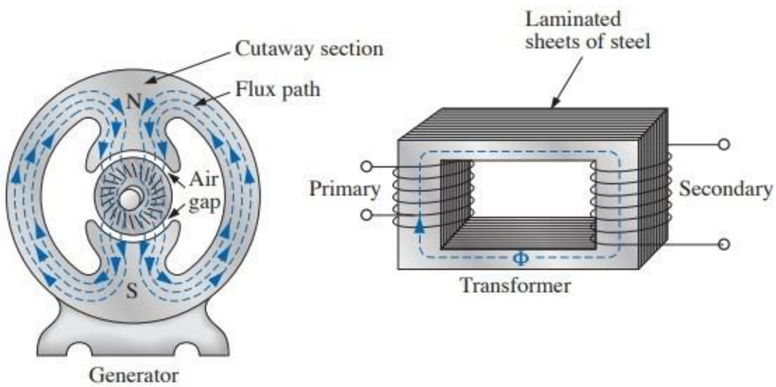

Figure 5.7: Some areas of application of magnetic effect.

$$B = \frac{\Phi}{A}$$

B=teslas(T)
Φ=webers(Wb)
A=square meters(m^2) where Φ is the number of flux lines passing through the area A (Fig. 5.8). 1

T = 1 Wb/m^2

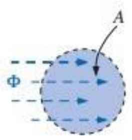

Figure 5.8: Defining the flux density B.

Example 5.1
For the core Fig. (5.9), determine the flux density B in teslas. **Solution:**

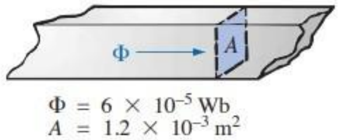

Figure 5.9:

$$B = \frac{\Phi}{A} = \frac{6 \times 10^{-5} Wb}{1.2 \times 10^{-3} m^2} = 5 \times 10^{-2} T$$

5.4 Permeability

If cores of different materials with the same physical dimensions are used in an electromagnet , the strength of the magnet will vary in accordance with the core used. This variation in strength is due to the greater or lesser number of flux lines passing through the core. Materials in which flux lines can readily be set up are said to be magnetic and to have high permeability. The permeability (μ) of a material, therefore, is a measure of the ease with which magnetic flux lines can be established in the material. It is similar in many respects to conductivity in electric circuits. The permeability of free space μ_o (vacuum) is

$$\mu_o = 4\pi \times 10^{-7} \frac{Wb}{A.m}$$

As indicated above, μ has the units of Wb/A$\bullet$ m.

Materials that have permeabilities slightly less than that of free space are said to be diamagnetic, and those with permeabilities slightly greater than that of free space are said to be paramagnetic. Magnetic materials, such as iron, nickel, steel, cobalt, and alloys of these metals, have permeabilities hundreds and even thousands of times that of free space. Materials with these very high permeabilities are referred to as ferromagnetic.

The ratio of the permeability of a material to that of free space is called its relative permeability; that is,

$$\mu_r = \frac{\mu}{\mu_o}$$

5.5 Reluctance

The resistance of a material to the flow of charge (current) is determined for electric circuits by the equation

$$R = \rho \frac{l}{A} (ohms, \Omega)$$

The reluctance of a material to the setting up of magnetic flux lines in the material is determined by the following equation:

$\Re = \frac{l}{\mu A}$ (rels, or At/Wb)

where $\Re$ is the reluctance, l is the length of the magnetic path, and A is the cross-sectional area. The t in the units At/Wb is the number of turns of the applied winding.

Ohm's Law For Magnetic Circuits

Effect $= \frac{cause}{opposition}$

For magnetic circuits, the effect desired is the flux. The cause is the magnetomotive force (mmf), which is the external force (or "pressure") required to set up the magnetic flux lines within the magnetic material. The opposition to the setting up of the flux is the reluctance.

Substituting, we have

$\Phi = \frac{F_m}{\Re}$

The magnetomotive force is proportional to the product of the number of turns around the core (in which the flux is to be established) and the current through the turns of wire (Fig. 5.10). In equation form,

F_m=NI (ampere-turns, At)

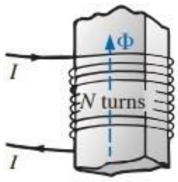

Figure 5.10: Defining the components of a magnetomotive force

5.6 Magnetizing Force

The magnetomotive force per unit length is called the magnetizing force (H). In equation form,
H = $\frac{F_m}{l}$ (At/m)

Substituting for the magnetomotive force will result in
H = $\frac{NI}{l}$ (At/m)
For the magnetic circuit of Fig. 5.11, if NI=40At and l=0.2m, then

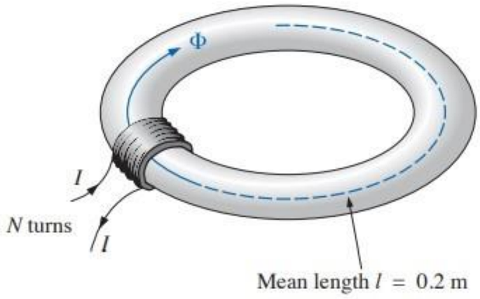

Figure 5.11:

$$H = \frac{NI}{l} = \frac{40At}{0.2m} = 200At/m$$

The applied magnetizing force has a pronounced effect on the resulting permeability of a magnetic material. As the magnetizing force increases, the permeability rises to a maximum and then drops to a minimum, as shown in Fig. for three commonly employed magnetic materials.

The flux density and the magnetizing force are related by the following equation:
B=μH
This equation indicates that for a particular magnetizing force, the greater the permeability, the greater will be the induced flux density.

5.7 Hysteresis

A typical B-H curve for a ferromagnetic material such as steel can be derived using the setup of

The core is initially unmagnetized and the current I = 0. If the current I is increased to some value above zero, the magnetizing force H will increase to a value determined by

$$H \uparrow = \frac{NI \uparrow}{l}$$

The flux Φ and the flux density B (B = Φ/A) will also increase with the current I (or H). If the material has no residual magnetism, and the magnetizing force H is increased from zero to some value H_a, the B-H curve will follow the path shown in Fig. between o and a.

If the magnetizing force H is increased until saturation (H_s) occurs, the curve will continue as shown in the figure to point b. When saturation occurs, the flux density has, for all practical purposes, reached its maximum value. Any further increase in current through the coil increasing H = NI/l will result in a very small increase in flux density B.

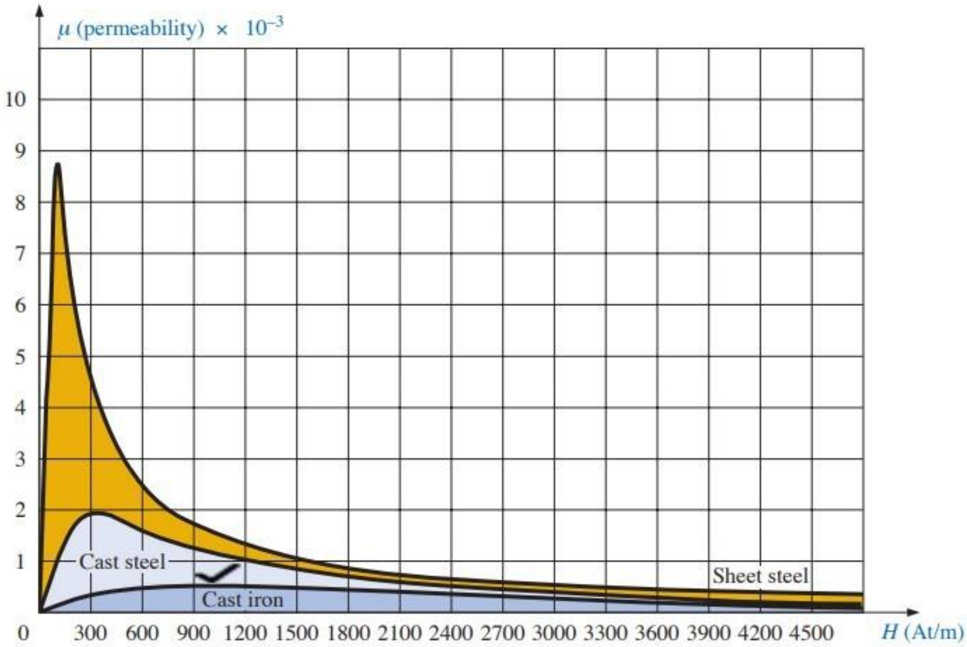

Figure 5.12: Variation of μ with the magnetizing force.

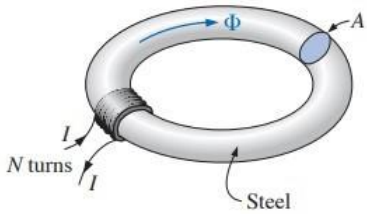

Figure 5.13: Series magnetic circuit used to define the hysteresis curve.

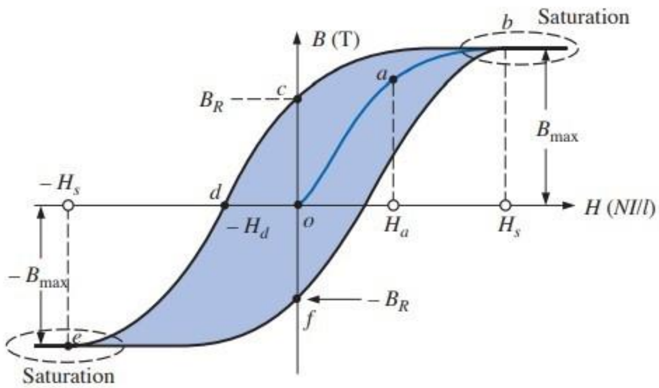

Figure 5.14: Hysteresis curve.

If the magnetizing force is reduced to zero by letting I decrease to zero, the curve will follow the path of the curve between b and c. The flux density B_R, which remains when the magnetizing force is zero, is called the residual flux density. It is this residual flux density that makes it possible to create permanent magnets. If the coil is now removed from the core of Fig. 5.13, the core will still have the magnetic properties determined by the residual flux density, a measure of its "retentivity." If the current I is reversed, developing a magnetizing force, -H, the flux density B will decrease with an increase in I. Eventually, the flux density will be zero when $-H_d$ (the portion of curve from c to d) is reached. The magnetizing force $-H_d$ required to "coerce" the flux density to reduce its level to zero is called the coercive force, a measure of the coercivity of the magnetic sample. As the force -H is increased until saturation again occurs and is then reversed and brought back to zero, the path def will result. If the magnetizing force is increased in the positive direction (+H), the curve will trace the path shown from f to b. The entire curve represented by bcdefb is called the hysteresis curve for the ferromagnetic material.

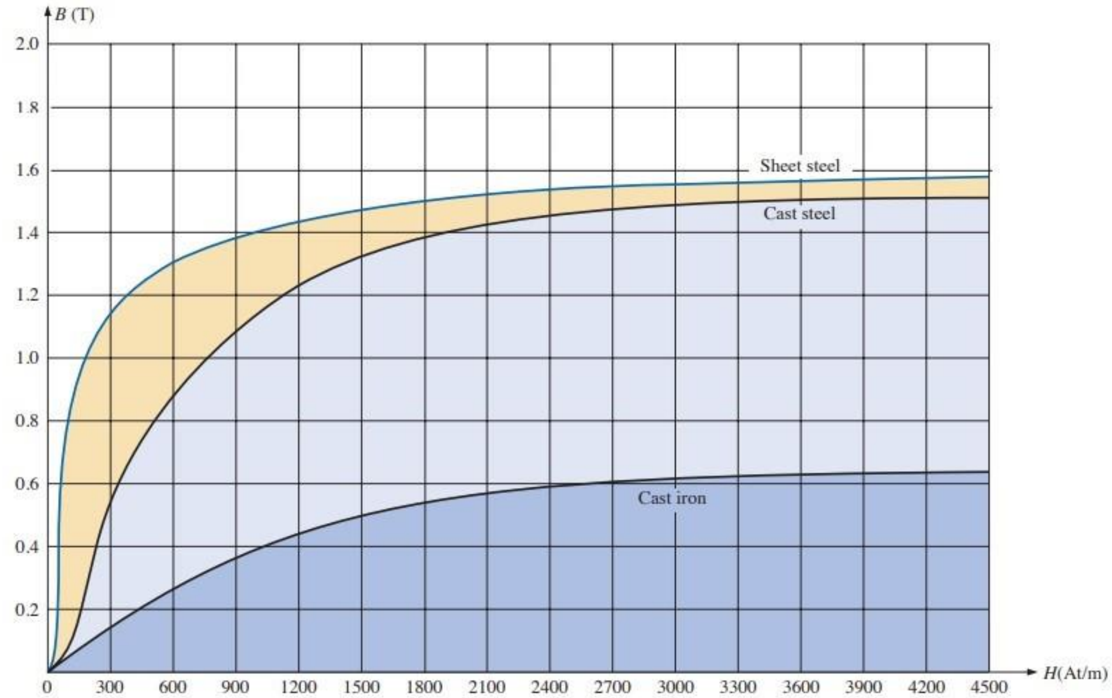

Figure 5.15: Normal magnetization curve for three ferromagnetic materials.

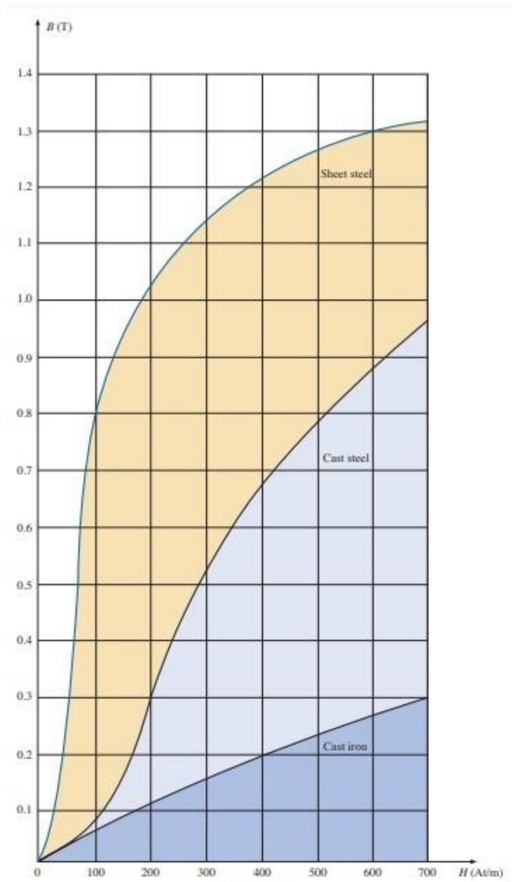

Figure 5.16: Expanded view of Fig. 5.15 for the low magnetizing force region.

5.8 Ampere's Circuital Law

There is a broad similarity between the analyses of electric and magnetic circuits. This has already been demonstrated to some extent for the quantities.

	Electric Circuits		Magnetic Circuits
	E	F_m	
	I	Φ	
	R	$\Re$	

If we apply the "cause" analogy to Kirchhoff's voltage law ($\Sigma V = 0$), we obtain the following:

$$\Sigma F_m = 0 \tag{5.1}$$

(for magnetic circuits)

which, in words, states that the algebraic sum of the rises and drops of the mmf around a closed loop of a magnetic circuit is equal to zero; that is, the sum of the rises in mmf equals the sum of the drops in mmf around a closed loop.
Equation(5.1) is referred to as Amp`ere's circuital law. When it is applied to magnetic circuits, sources of mmf are expressed by the equation F_m=NI (At)

The equation for the mmf drop across a portion of a magnetic circuit can be found by applying the relationships listed in Table ; that is, for electric circuits,

$$V = IR$$

resulting in the following for magnetic circuits:

$F_m = \Phi <$ (At)

where Φ is the flux passing through a section of the magnetic circuit and < is the reluctance of that section. The reluctance, however, is seldom calculated in the analysis of magnetic circuits. A more practical equation for the mmf drop is
F_m=Hl (At) where H is the magnetizing force on a section of a magnetic circuit and l is the length of the

section.

Consider the magnetic circuit appearing in Fig.17 constructed of three different ferromagnetic materials. Applying Amp`ere's circuital law, we have

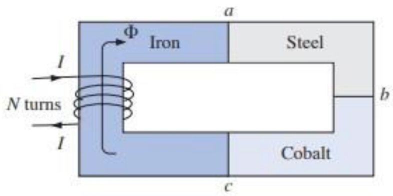

Figure 5.17: Series magnetic circuit of three different materials.

$$\sum F_m = 0$$
$$\underbrace{+HI}_{Rise} - \underbrace{H_{ab}l_{ab}}_{Drop} - \underbrace{H_{bc}l_{bc}}_{Drop} - \underbrace{H_{ca}l_{ca}}_{Drop} = 0$$
$$\underbrace{NI}_{impressed-mmf} = \underbrace{H_{ab}l_{ab} + H_{bc}l_{bc} + H_{ca}l_{ca}}_{mmf-drops}$$

All the terms of the equation are known except the magnetizing force for each portion of the magnetic circuit, which can be found by using the B-H curve if the flux density B is known.

5.9 The Flux Φ

If we continue to apply the relationships described in the previous section to Kirchhoff's current law, we will find that the sum of the fluxes entering a junction is equal to the sum of the fluxes leaving a junction; that is, for the circuit of Fig.5.18,

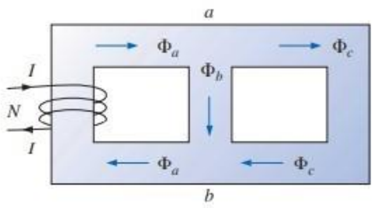

Figure 5.18: Flux distribution of a series-parallel magnetic network.

$\Phi_a = \Phi_b + \Phi_c$ (at junction a) or
$\Phi_b + \Phi_c = \Phi_a$ (at junction b) both of
which are equivalent.

Example 5.2

For the series magnetic circuit of Fig. (5.19):

a. Find the value of I required to develop a magnetic flux of $\Phi = 4 \times 10^{-4}$Wb.

b. Determine μ and μ_o for the material under these conditions.

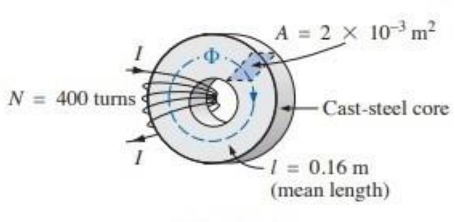

Figure 5.19:

Solutions:

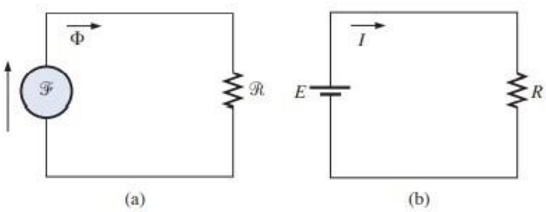

Figure 5.20: (a)Magnetic circuit equivalent and (b)electric circuit analogy.

Section	Φ (Wb)	A (m²)	B (T)	H (At/m)	l (m)	Hl (At)
One continuous section	4×10^{-4}	2×10^{-3}			0.16	

a. The flux density B is

$$B = \frac{\Phi}{A} = \frac{4 \times 10^{-4} Wb}{2 \times 10^{-3} m^2} = 2 \times 10^{-1} T = 0.2T$$

Using the B-H curves of Fig. 5.16, we can determine the magnetizing force H:
H (cast steel) = 170 At/m
Applying Amp`ere's circuital law yields

$$NI = Hl$$

and

$$I = \frac{Hl}{N} = \frac{(170At/m)(0.16m)}{400t} = 68mA$$

b. The permeability of the material can be found using

$$\mu = \frac{B}{H} = \frac{0.2T}{170At/m} = 1.176 \times 10^{-3}Wb/A.m$$

and the relative permeability is

$$\mu_r = \frac{\mu}{\mu_o} = \frac{1.176 \times 10^{-3}}{4\pi \times 10^{-7}} = 935.83$$

5.10 Air Gaps

The spreading of the flux lines outside the common area of the core for the air gap in Fig. 5.21(a) is known as fringing. For our purposes, we shall neglect this effect and assume the flux distribution to be as in Fig. 5.21(b).

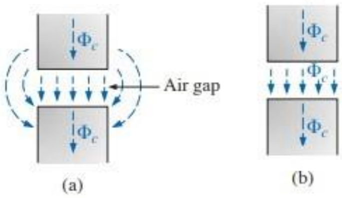

Figure 5.21: Air gaps (a)with fringing; (b)ideal.

The flux density of the air gap in Fig. 5.21(b) is given by

$$B_g = \frac{\Phi_g}{A_g}$$

where, for our purposes,

$$\Phi_g = \Phi_{core}$$

$$A_g = A_{core}$$

For most practical applications, the permeability of air is taken to be equal to that of free space. The magnetizing force of the air gap is then determined by

$$H_g = \frac{B_g}{\mu_o}$$

and the mmf drop across the air gap is equal to $H_g l_g$. An equation for H_g is as follows:

$$H_g = \frac{B_g}{\mu_o} = \frac{B_g}{4\pi \times 10^{-7}}$$
$$H_g = (7.96 \times 10^5)B_g(At/m)$$

Example 5.3

Find the value of I required to establish a magnetic flux of $\Phi = 0.75 \times 10^{-4}$Wb in the series magnetic circuit of Fig. (5.22). **Solution:**

$$B = \frac{\Phi}{A} = \frac{0.75 \times 10^{-4}Wb}{1.5 \times 10^{-4}m^2} = 0.5T$$

PABLO

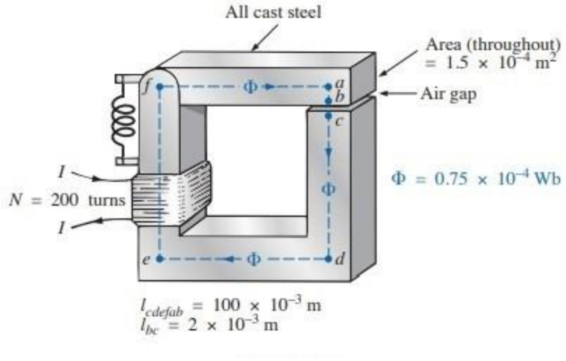

Figure 5.22:

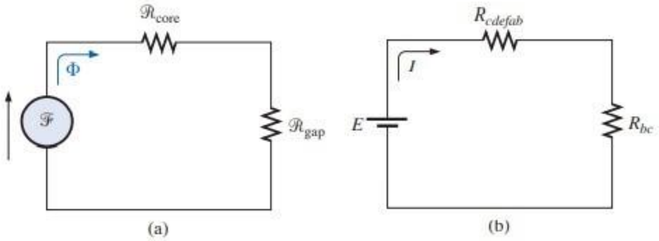

Figure 5.23: (a)Magnetic circuit equivalent and (b)electric circuit analogy for the relay of Fig. 5.22

From the B-H curves of Fig. 5.16,

$$H(cast\_steel) \cong 280At/m$$

$$H_g = (7.96 \times 10^5)B_g = (7.96 \times 10^5)(0.5T) = 3.98 \times 10^5 At/m$$

The mmf drops are

$$H_{core}l_{core} = (280At/m)(100 \times 10^{-3}m) = 28At$$
$$H_g l_g = (3.98 \times 10^5 At/m)(2 \times 10^{-3}m) = 796At$$

Applying Ampere's circuital law,

$$NI = H_{core}l_{core} + H_g l_g = 28At + 796At\ (200t)I$$
$$= 824At\ I = 4.12A$$

5.11 Series-Parallel Magnetic Circuits

Example 5.4 Determine the current I required to establish a flux of 1.5×10^{-4}Wb in the section of the core indicated in Fig. 5.24. **Solution:**

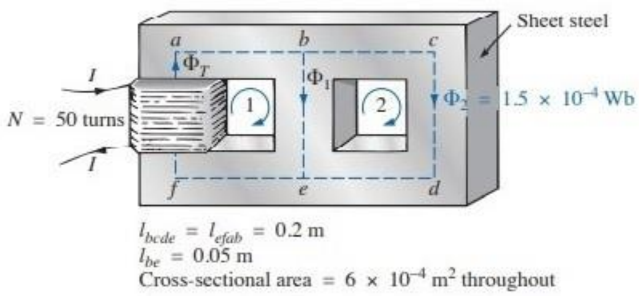

Figure 5.24:

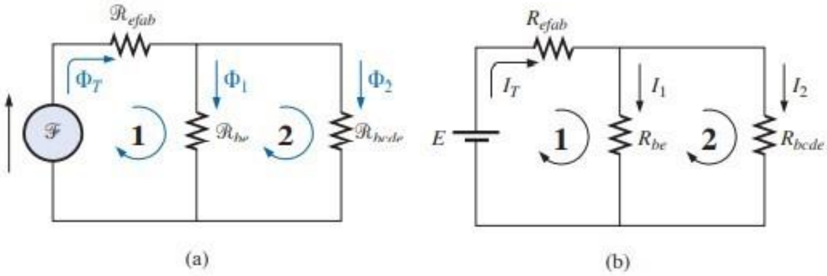

Figure 5.25: (a)Magnetic circuit equivalent and (b)electric circuit analogy for the series-parallel system of Fig. 5.24

$$B_2 = \frac{\Phi_2}{A} = \frac{1.5 \times 10^{-4}Wb}{6 \times 10^{-4}m^2} = 0.25T$$

From Fig. 5.16,

$$\begin{aligned}&H_{bcde} \cong 40At/m\\&\sum F_m = 0\\&H_{be}l_{be} - H_{bcde}l_{bcde} = 0\\&H_{be}(0.05m) - (40At/m)(0.2m) = 0\\&H_{be} = \tfrac{8At}{0.05m} = 160At/m\end{aligned}$$

From Fig. 5.16,

$$B_1 \tilde{=} 0.97T$$

and

$$\Phi_1 = B_1A = (0.97T)(6 \times 10^{-4}m^2) = 5.82 \times 10^{-4}Wb$$

The results are entered in the table below
The table reveals that we must turn our attention to section efab:

Section	Φ (Wb)	A (m²)	B (T)	H (At/m)	l (m)	Hl (At)
bcde	1.5×10^{-4}	6×10^{-4}	0.25	40	0.2	8
be	5.82×10^{-4}	6×10^{-4}	0.97	160	0.05	8
efab		6×10^{-4}			0.2	

$$\begin{aligned}&\Phi_T = \Phi_1 + \Phi_2 = 5.82 \times 10^{-4}Wb + 1.5 \times 10^{-4}Wb = 7.32 \times 10^{-4}Wb\\&B = \tfrac{\Phi_T}{A} = \tfrac{7.32\times10^{-4}Wb}{6\times10^{-4}m^2} = 1.22T\end{aligned}$$

From Fig. 5.15,

$$H_{efab} \sim= 400At$$

Applying Ampere's circuital law,

$$\begin{aligned}&+NI - H_{efab}l_{efab} - H_{be}l_{be} = 0\\&NI = (400At/m)(0.2m) + (160At/m)(0.05m)\\&(50t)I = 80At + 8At\\&I = \tfrac{88At}{50t} = 1.76A\end{aligned}$$

To demonstrate that μ is sensitive to the magnetizing force H, the permeability of each section is determined as follows. For section bcde,

$$\mu = \frac{B}{H} = \frac{0.25T}{40At/m} = 6.25 \times 10^{-3}$$

and For section be,

$$\mu_r = \frac{\mu}{\mu_o} = \frac{6.25 \times 10^{-3}}{12.57 \times 10^{-7}} = 4972.2$$

$$\mu = \frac{B}{H} = \frac{0.97T}{160At/m} = 6.606 \times 10^{-3}$$

and For section efab,

$$\mu_r = \frac{\mu}{\mu_o} = \frac{6.06 \times 10^{-3}}{12.57 \times 10^{-7}} = 4821$$

$$\mu = \frac{B}{H} = \frac{1.22T}{400At/m} = 3.05 \times 10^{-3}$$

and

$$\mu_r = \frac{\mu}{\mu_o} = \frac{3.05 \times 10^{-3}}{12.57 \times 10^{-7}} = 2426.41$$

5.12 Determining Φ

H = $\frac{NI}{l}$ H $\rightarrow$ B (B-H curve)

$$\Phi = BA$$

Example 5.5

Calculate the magnetic flux Φ for the magnetic circuit of Fig. 5.26.

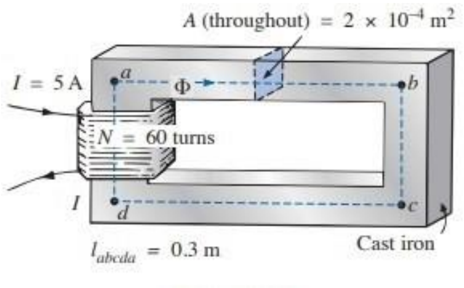

Figure 5.26:

Solution

By Ampere's circuital law,

$$NI = H_{abcda} l_{abcda}$$

or

$$H_{abcda} = \frac{NI}{l_{abcda}} = \frac{(60t)(5A)}{0.3m} = \frac{300At}{0.3m} = 1000At/m$$

and B_{abcda} (from Fig.) ~=0.39T

Since B=Φ/A, we have

$$\Phi = BA = (0.39T)(2 \times 10^{-4} m^2) = 0.78 \times 10^{-4} Wb$$