# ELNG 251: DC Machines and Transformers

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# Content

# Assessment

- Continuous assessment – 40 marks
  - Weekly quizzes and assignments - 20 marks
  - Mid-semester examination – 20 marks

- End of semester examination – 60mks
  - Multiple choice examination – 20 marks
  - Written examination – 40 marks

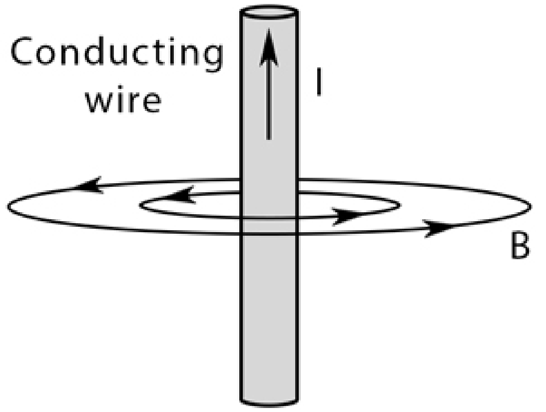
# Further study

- Sahdev, S. K. (2017). Electrical machines. Cambridge University Press.

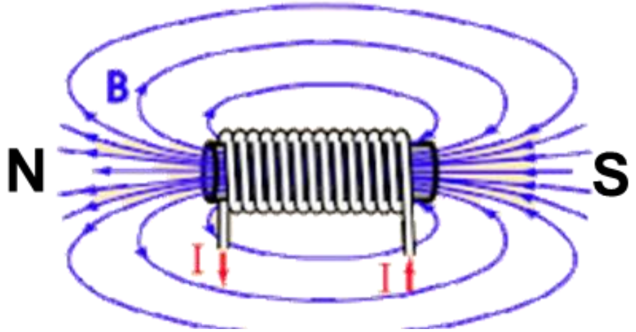
[Click here for lecture materials]

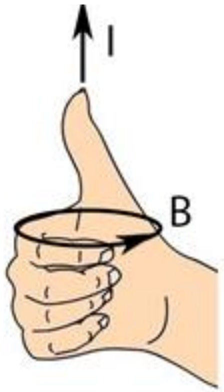
# Electromagnetism

- **Electromagnetism** is the interaction or interrelation between electric charges and magnetic fields.
- In 1820 Danish physicist Hans Christian Ørsted discovered that an electric current creates a magnetic field around it.




- The magnetic field lines encircle the current-carrying wire.
- The magnetic field lines lie in a plane perpendicular to the wire.
- If the direction of the current is reversed, the direction of the magnetic field reverses.
- The strength of the field is directly proportional to the magnitude of the current.
- The strength of the field at any point is inversely proportional to the distance of the point from the wire.




**Right hand thumb rule**

- Thumb points in the direction of electric current and fingers curl around the current indicating the direction of the magnetic field.

# Electromagnetism

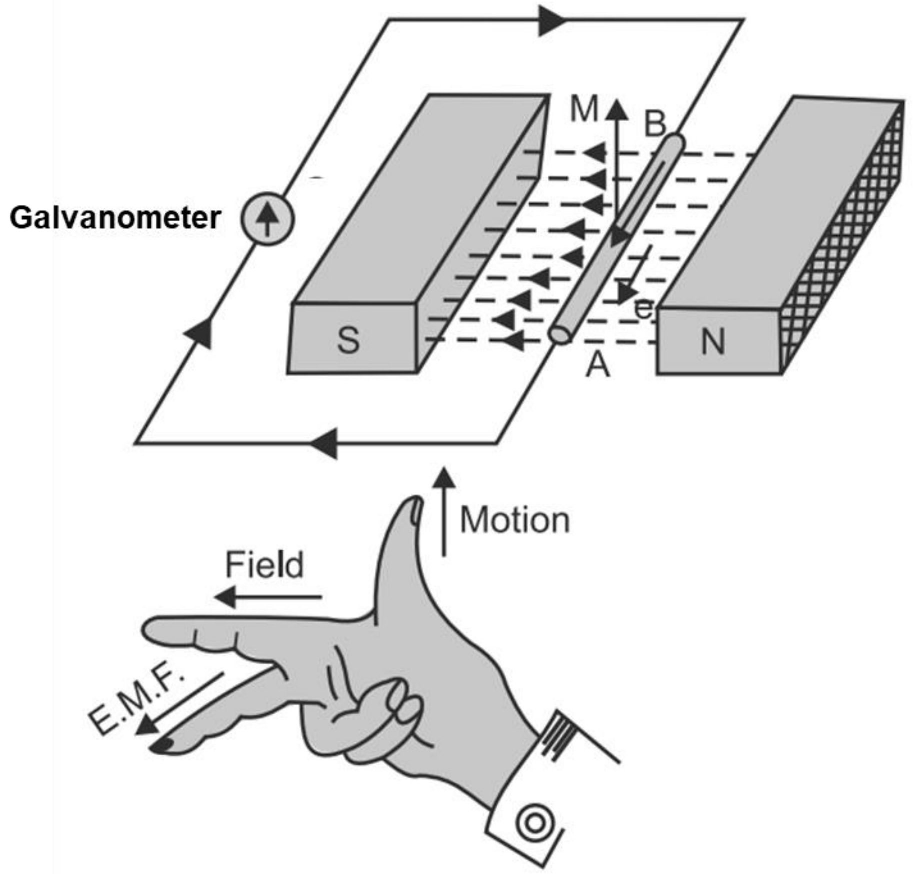
## Faraday's laws of electromagnetic induction

- **1st law:** Whenever a conductor cuts across a magnetic field, an emf is induced in the conductor
- ***OR*** Whenever the magnetic flux linking with any circuit (or coil) changes, an emf is induced in the circuit.
- **2nd law:** The magnitude of induced emf in a coil is directly proportional to the rate of change of flux linkages.
- $\boldsymbol{e = N \frac{d\emptyset}{dt}}$

$\boldsymbol{N}$ – Number of turns
$\emptyset$ - change in flux
$\boldsymbol{dt}$ – change in time

## Fleming's Right Hand Rule

- Stretch the 1st finger, middle finger and thumb of the right hand in mutually perpendicular manner such that the 1st finger points in the direction of the magnetic field and the thumb in the direction of the motion, the resulting induced emf is in the direction of the middle finger.




Direction of induced emf using the **RIGHT-HAND RULE**

**Worked Example**

A coil of 500 turns is linked with a flux of 2 mWb. If this flux is reversed in 4 ms, calculate the average emf induced in the coil.

**Solution**

Induced emf, $\boldsymbol{e = N \frac{d\emptyset}{dt}}$

$\boldsymbol{N}$ = 500 turns

$\boldsymbol{d\emptyset}$ = 2-(-2) = 4 mWb = 4 x $10^{-3}$ Wb

$\boldsymbol{dt}$ = 4 x $10^{-3}$ s

$$e = 500 \times \frac{4 \times 10^{-3}}{4 \times 10^{-3}} = \mathbf{500\ V}$$

- A coil of 250 turns is wound on a magnetic circuit of reluctance 100,000 At/Wb. If a current of 2 A flowing in the coil is reversed in 5 ms, find the average emf induced in the coil.

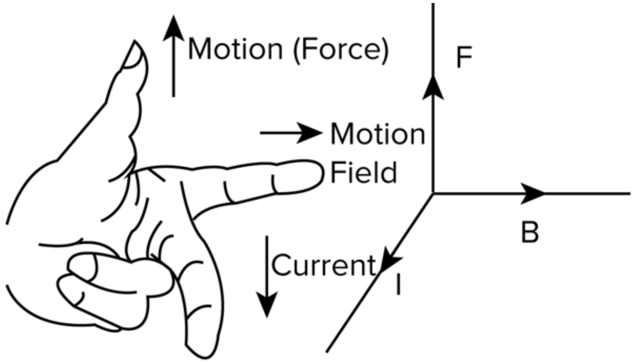
# Electromagnetism

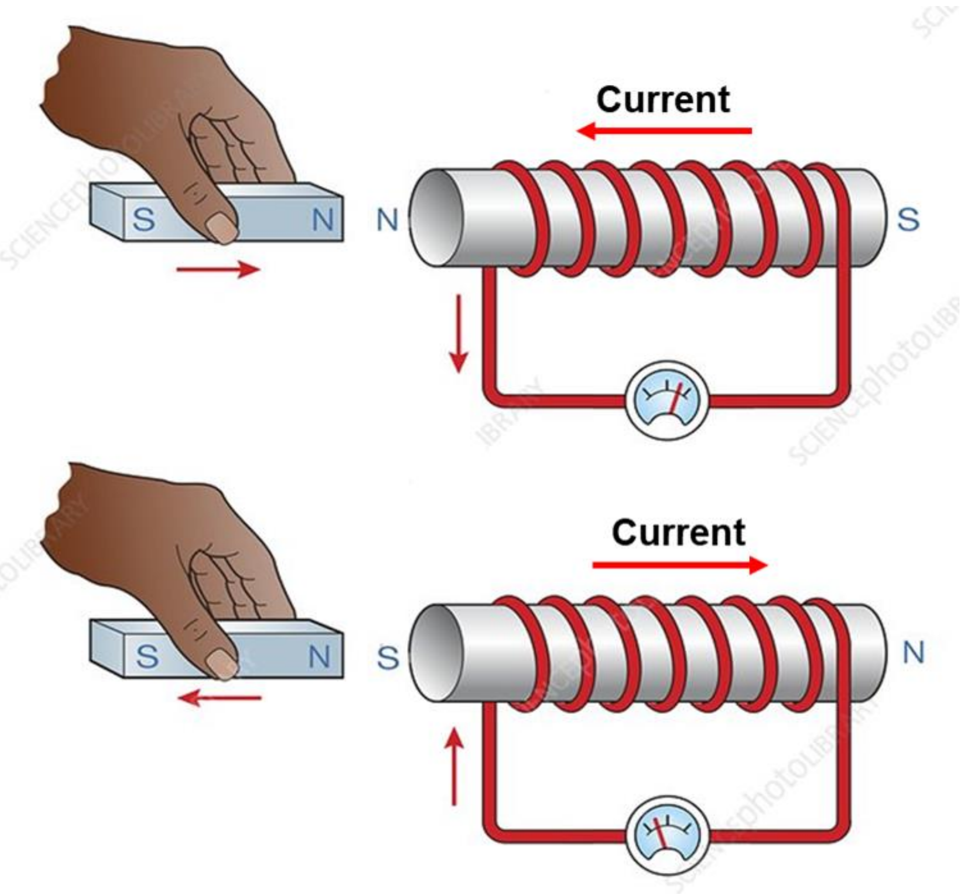
## Lenz's law

- states that the direction of the electric current induced in a conductor by a changing magnetic field is such that the magnetic field created by the induced current opposes changes in the initial magnetic field. It is named after physicist Heinrich Lenz, who formulated it in 1834.

## Fleming's Left Hand Rule

- Stretch the 1st finger, middle finger and thumb of the right hand in mutually perpendicular manner such that the 1st finger points in the direction of the magnetic field and the middle finger in the direction of the current, the thumb points in the direction of the force exerted on the conductor.



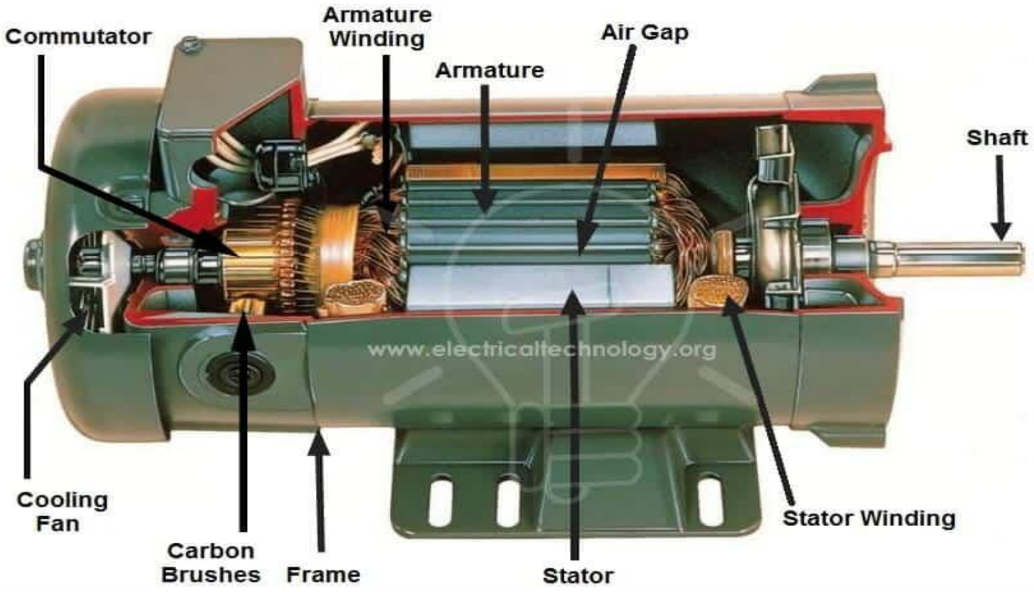



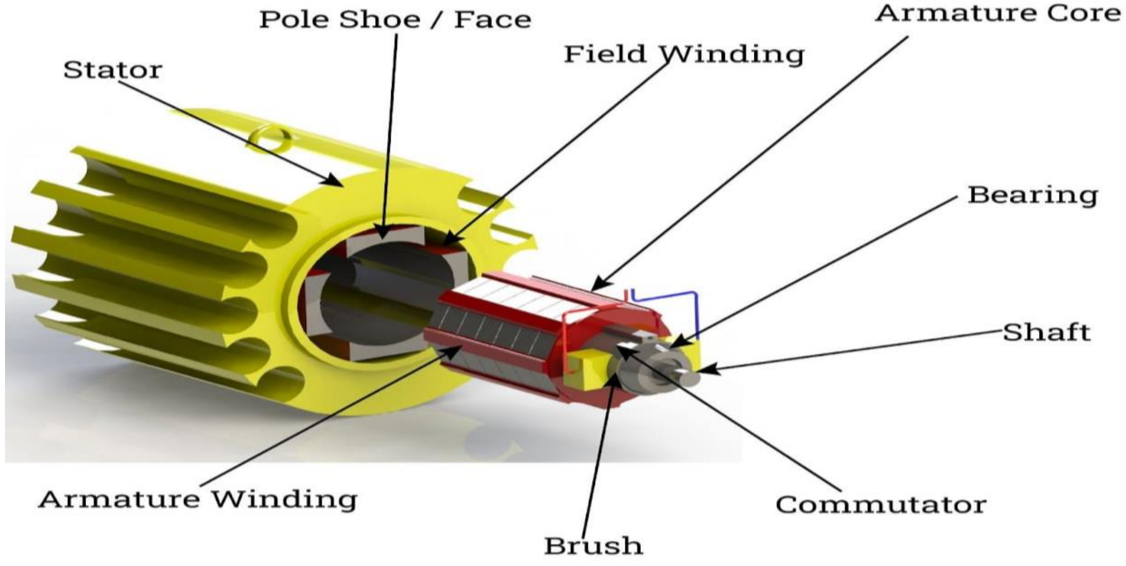

Induced current opposing magnetic field – **LENZ'S LAW**

[Click here for a video on Lenz's law]

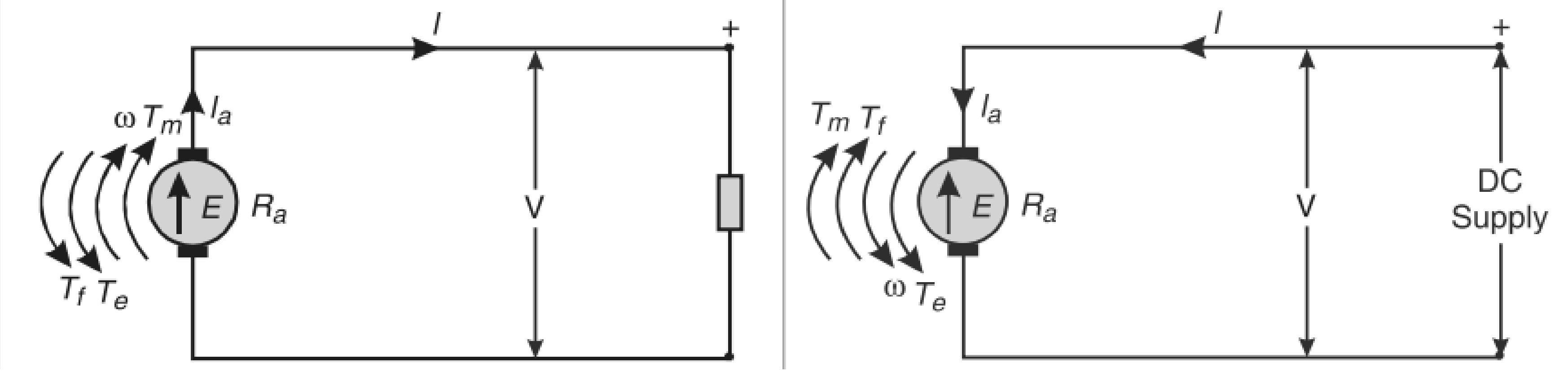
# DC machines

- A DC machine is an electro-mechanical energy conversion device.
- When it converts mechanical power into DC electrical power, it is known as a DC generator.
- On the other hand, when it converts DC electrical power into mechanical power it is known as a DC motor.
- Applications where large quantity of DC power is required are in chemical and metal extraction plants, for electroplating and electrolysis processes etc. At such places DC generators are used to deliver power.
- DC motors have several applications in industry and every day use; toys, windshield wipers, small appliances, power tools, electric vehicles, robotics, industrial automation, etc.



## Comparing DC generator and motor operation




| *Generator action* | *Motor action* |
|---|---|
| 1. In generating action, the rotation is due to mechanical torque, therefore, $T_m$ and $\omega$ are in the same direction. | 1. In motoring action, the rotation is due to electromagnetic torque, therefore, $T_e$ and $\omega$ are in the same direction. |
| 2. The frictional torque $T_f$ acts in opposite direction to rotation $\omega$. | 2. The frictional torque $T_f$ acts in opposite direction to rotation $\omega$. |
| 3. Electromagnetic torque $T_e$ acts in opposite direction to mechanical torque $T_m$ so that $\omega T_m = \omega T_e + \omega T_f$. | 3. Mechanical torque $T_m$ acts in opposite direction to electromagnetic torque $T_e$ so that $\omega T_e = \omega T_m + \omega T_f$. |
| 4. In generating action, an emf is induced in the armature conductors which circulates current in the armature when load is connected to it. Hence, e and i both are in the same direction. | 4. In motoring action, current is impressed to the armature against the induced emf (e), therefore current flows in opposite direction to that of induced emf. |
| 5. In generator action, $E > V$ | 5. In motor action, $E < V$ |
| 6. In generating action, the torque angle $\theta$ is leading. | 6. In motoring action, the torque angle $\theta$ is lagging. |
| 7. In generating action, mechanical energy is converted into electrical energy. | 7. In motoring action, electrical energy is converted into mechanical energy. |

# Armature winding of a dc machine

**Armature winding**

- The armature winding are the insulated conductors housed in the armature slot.

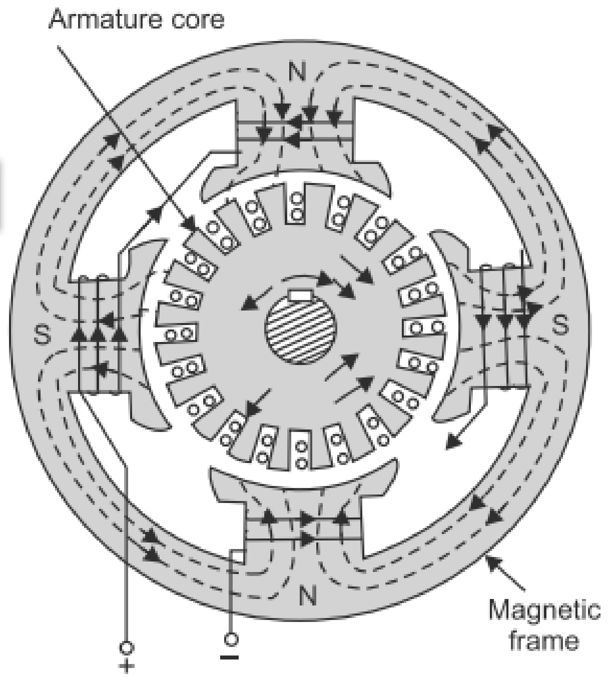
NB: DC motors and DC generators are collectively referred to as DC machine

- It is the place where one form of power is converted to the other form i.e., in case of generator, mechanical power is converted into electrical power and in case of motor, electrical power is converted into mechanical power.
- There are two types of armature windings:

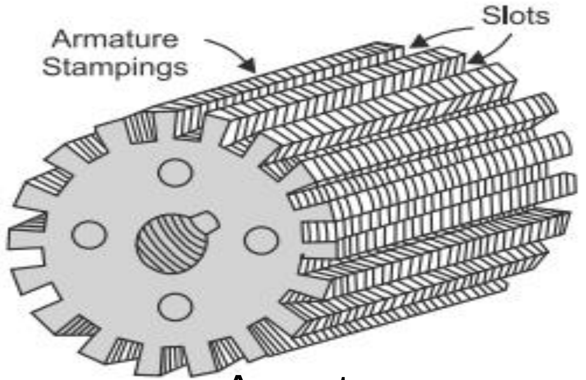
**(i) Lap winding:** In this winding, that the number of parallel paths (A) is equal to number of poles. In this case, the number of brushes is equal to the number parallel paths. Out of which half the brushes are positive and the remaining (half) are negative.

**(ii) Wave winding:** In this winding, the parallel paths are only two irrespective of the number of poles. In this case, is equal to two i.e., number of parallel paths.

Click here for further study on armature windings



Magnetic circuit of a DC machine



Armature core

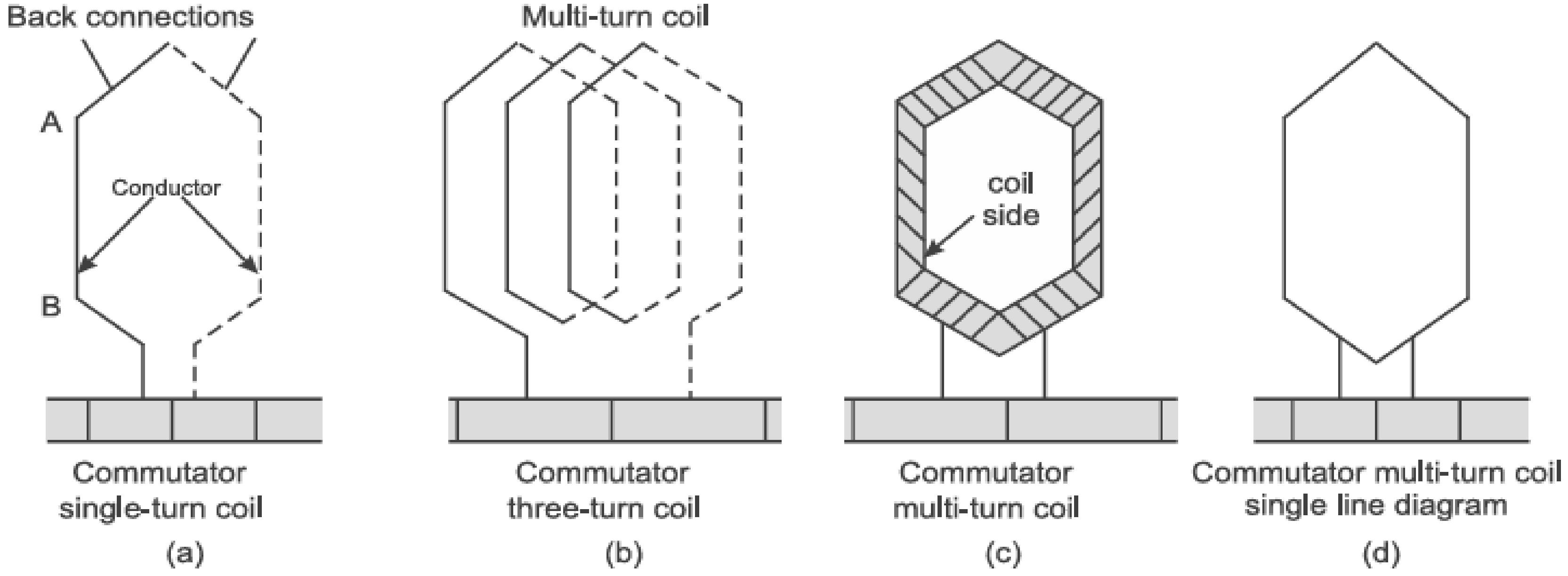
# Armature winding of a dc machine

**Conductor:** The length of wire embedded in armature core and lying within the magnetic field. Number of conductor is represented by $\boldsymbol{Z}$.

**Turn:** Two conductors lying in a magnetic field connected in series at the back.

$$Z = number\ of\ turns\ \times 2$$

**Coil:** A coil may be a single-turn coil having only two conductors, as shown in Fig (a), or it may be a multi-turn coil having more than two conductors as shown in Fig (b). In Fig. 4.17(b), a three-turn coil is shown. The bunch of three conductors may be wrapped by the cotton tape, as shown in Fig (c), before placing it in the armature slot. A multi-turn coil can be represented by single line diagram as shown in Fig (d).



# Emf and torque equation of a DC machine

## Emf equation

P = number of poles

$\emptyset$ = Flux per pole

Z = Number of conductors

A = Number of parallel paths

N = speed in rpm

- Total flux cut by one conductor

$$= P\emptyset$$

- Time in seconds taken in one revolution

$$= \frac{60}{N}$$

- Per Faraday's law, induced emf per conductor

$$e = \frac{\emptyset}{t} = \frac{P\emptyset}{60/N} = \frac{P\emptyset N}{60}$$

- Total emf for all conductors

$$\boldsymbol{E = \frac{P\emptyset N}{60} \times \frac{Z}{A}}$$

Where $\frac{Z}{A}$ is the number of conductors per parallel path

## Torque equation

$I_a$ = Armature current

$\omega$ = speed in rad/s = $\frac{2\pi}{60}N$

- The torque generated in a dc machine is due to electromagnet effect, hence it's called electromagnetic torque, $T_e$.
- The electrical power in the armature is equal to the electromagnetic torque, hence

$$EI_a = T\omega$$

$$\frac{P\emptyset NZ}{60A} \times I_a = T \times \frac{2\pi}{60}N$$

$$\boldsymbol{T = \frac{P\emptyset Z I_a}{2\pi A}}$$

# DC generator

- A DC generator is an **electro-mechanical** energy conversion device (electrical machine) that converts mechanical energy or power ($\omega T$) into DC electrical energy or power ($EI$).
- There are some applications where large quantity of DC power is required (such as in chemical and metal extraction plants, for electroplating and electrolysis processes etc.), at such places DC generators are used to deliver power.
- DC generators share the same internal composition and design with DC motors. The difference lies in the peripheral systems that support them.
- Motors often require control circuits for speed regulation, while generators require protection circuits for overvoltage and overcurrent.

**Working principle**

- The basic principle of a DC generator is electro-magnetic induction i.e., "*When a conductor cuts across the magnetic field, an emf is induced in it.*"
- The direction of the induced emf is determined by the **Fleming's Right Hand Rule**.

# DC generator

- In a DC generator, maximum emf is induced when the conductor cuts the magnetic field lines at $90^o$, any deviation from this decreases the emf.
- Since the conductor rotates, it only cuts the magnetic field at $90^o$ twice in its revolution, and $0^o$ twice when its perfectly parallel with the field.
- At $0^o$, no emf is induced.
- Hence, the emf is said to be dynamically induced as seen in Figure (a), where $e = Blv \sin\theta$

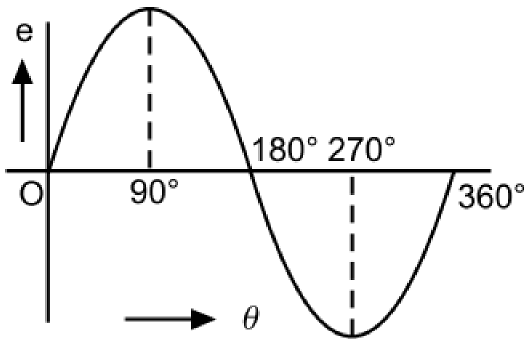
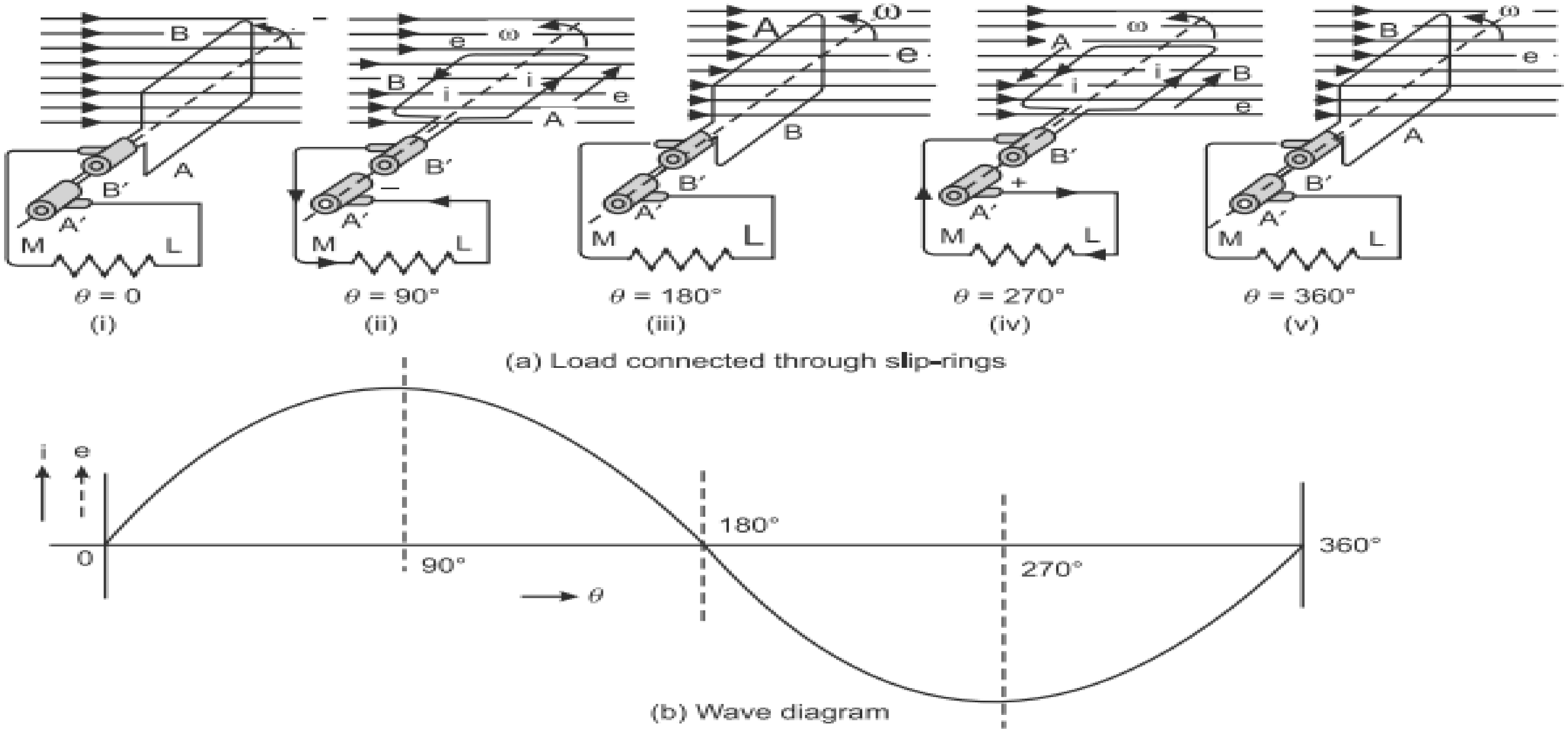



Figure (a): Wave shape of induced emf

How a DC generator works

# DC generator

- For simplicity, consider only one coil AB placed in the strong magnetic field. The two ends of the coil are joined to slip rings A’ and B’ respectively. Two brushes rest on these slip rings as shown in Figure (a).



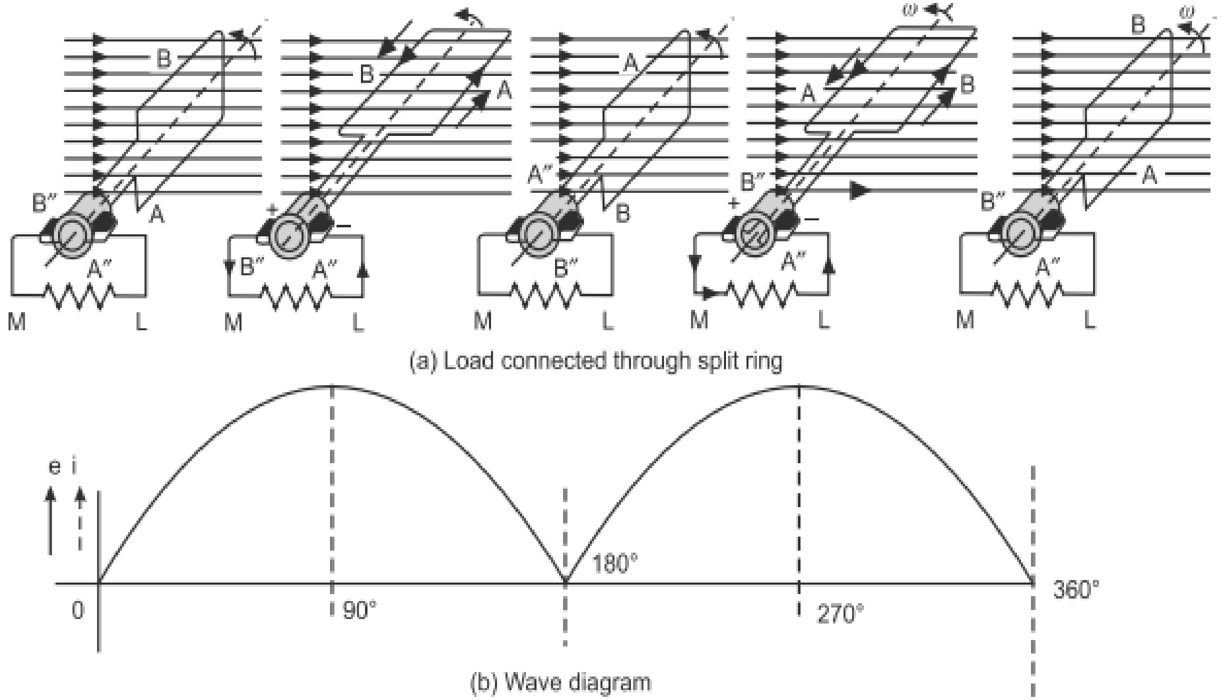

**Generated emf for external circuit connected through slip rings**

## Function of commutator

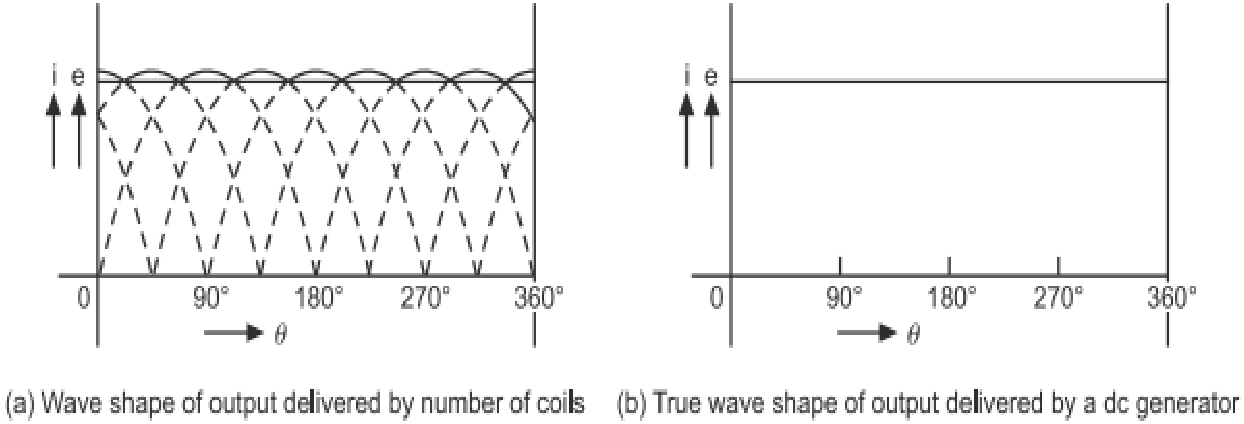
- The commutator is to converts the alternating current produced in the armature into direct current in the external circuit.

## Commutator action

- Now, consider that the two ends of the coil are connected to only one slip ring split into two parts (segment) i.e., A'' and B''. Each part is insulated from the other by a mica layer. Two brushes rest on these parts of the ring as shown in the figure below.
- The magnitude of emf induced in the coil at various instants will remain the same as shown in the previous slide, however, the flow of current in the external load will become unidirectional as been below




**Generated emf for external circuit connected through split ring**




**Wave shape of output delivered by a DC generator**

# DC generator

**Equivalent circuit of a DC generator**

$$\boldsymbol{E_g = V + I_a R_a}$$

$$I_a = I_L$$

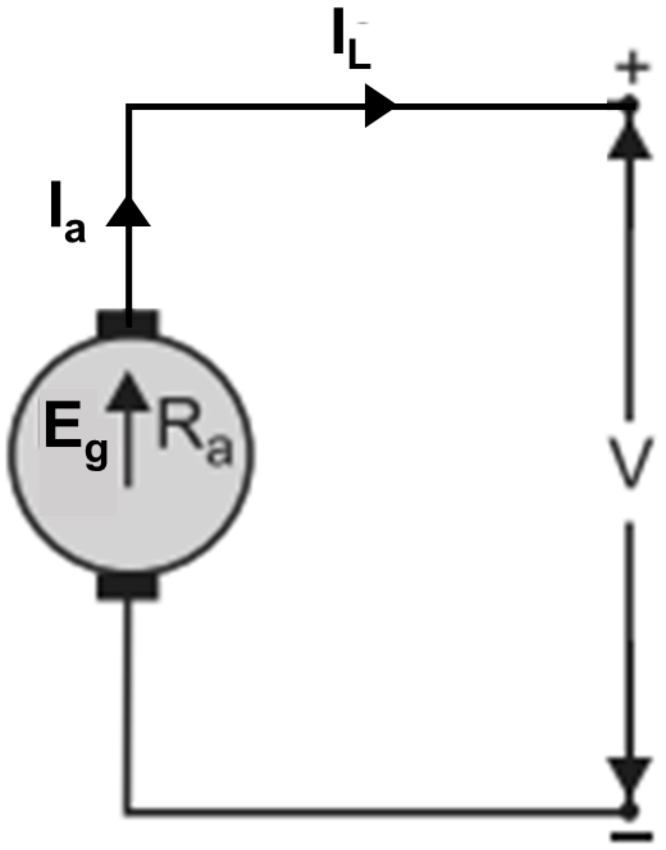
- Where $E_g$ is the generated voltage

$I_a$ is the armature current

$I_L$ is the load current

$V$ is the terminal voltage

$R_a$ is the armature resistance



# DC generator

**Problems**

1. A 12 kW, six-pole DC generator develops an emf of 240 at 1500 rpm. The armature has a lap connected winding. The average flux density over the pole pitch is 1.0 T. The length and diameter of the armature is 30 cm and 25 cm respectively. Calculate flux per pole, total number of active armature conductors. Power generated in the armature and torque developed when the machine is delivering 50 A current to the load.
2. The useful flux of a four-pole, lap wound DC generator is 0.07 Wb. Its armature carries 220 turns and each turn has 0.004 Ω resistance, if its armature current is 50 A, running at a speed of 900 rpm, determine its terminal voltage.
3. A DC generator is connected to a 220 V DC mains. The current delivered by the generator to the mains is 100A. The armature resistance is 0.1 Ω. The generator is driven at a speed of 500 rpm Calculate (i) the induced emf (ii) the electromagnetic torque (iii) the mechanical power input to the armature neglecting iron, winding and friction losses, (iv) Electrical power output from the armature, (v) armature copper loss.

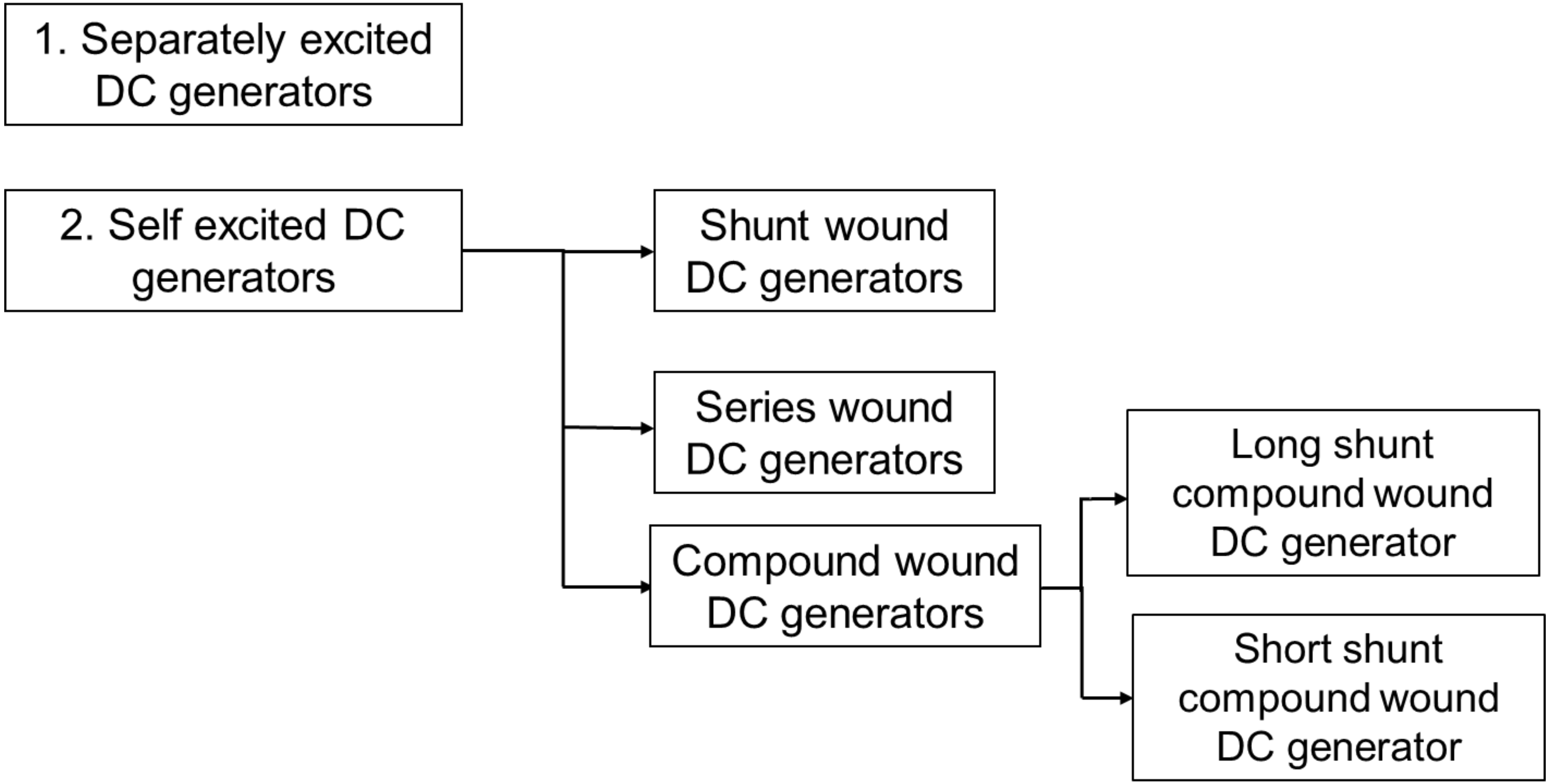
# DC generator

**Assignment 1 – (Deadline Mon, Jan 20, 2025, 23:59)**

1. A four-pole, lap wound DC machine is having 500 conductors on its armature and running at 1000 rpm. The flux per pole is 30 m Wb Calculate the voltage induced in the armature winding. What will be induced emf if the armature is rewound for wave winding?
2. A 4-pole machine has an armature with 90 slots and 8 conductors per slot, the flux per pole is 0.05 Wb and runs at 1200 rpm. Determine induced emf if winding is (i) lap connected (ii) wave connected.
3. A wave wound armature of a six-pole generator has 51 slots. Each slot contains 20 conductors. The voltage required to be generated is 250 V. What would be the speed of coupled prime mover if flux per pole is 0.07 Wb. If the armature is rewound as lap wound machine and run by same prime mover, what will be the generated voltage.
4. The armature of a four-pole 200 V, lap wound generator has 400 conductors and rms of 300 rpm. Determine the useful flux per pole. If the number of turns in each field coil is 900, what is the average induced emf in each field coil on breaking its connection if the magnetic flux set-up by it dies away completely in 0.1 second?

# Types of DC generators

• DC generators are classified based on the method of exciting their field winding.



# Applications of the different types of DC generators

- The various types of DC generators have different applications due to their various characteristics.

**Separately excited DC generators:**

- These generators are more costly than self excited generators as they require a separate source for their field excitation.
- However, their response to changes in field resistance is quicker and more precise.
- Hence, they are employed where quick and definite response to control is important such as the Ward–Leonard System of speed control.

**Shunt-wound DC generators:**

- As they provide constant terminal voltage, they are best suited for battery charging.
- Along with field regulators, they are also used for light and power supply purposes.

# Applications of the different types of DC generators

**Series-wound DC generators:**

- These generators have very few applications. Their best application is in the DC locomotives, where they supply field current for regenerative braking.
- They are also employed in series arc lighting.
- Another application of these generators is as series boosters for increasing DC voltage across the feeders.

**Compound-wound DC generators**

*(i)* *Over-compounded type:* These are more suited for lighting and power services, as they compensate for the voltage drop in the lines and voltage at the terminals of the load remains constant.

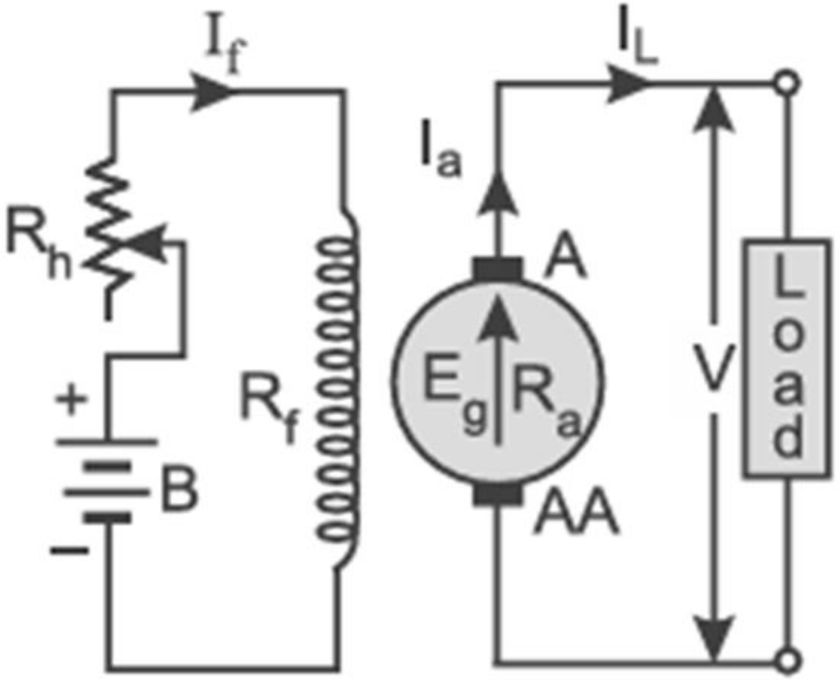
*(ii)* *Differential-compounded type:* They are usefully employed as are welding sets. In such cases, the generator is practically short-circuited every time the electrode touches the metal plates to be welded.

# Separately excited DC generators

- In a separately excited DC generator, the field winding is excited with current from an **external DC source**.
- $I_a = I_L$ where $I_a$ is the armature current and $I_L$ is the line current.
- Terminal voltage, $\boldsymbol{V = E_g - I_a R_a}$
- If contact brush drop per brush ($v_b$) is known,

$$\boldsymbol{V = E_g - I_a R_a - 2v_b}$$

- Power developed = $E_g I_a$;
- Power output = $V I_L = V I_a$



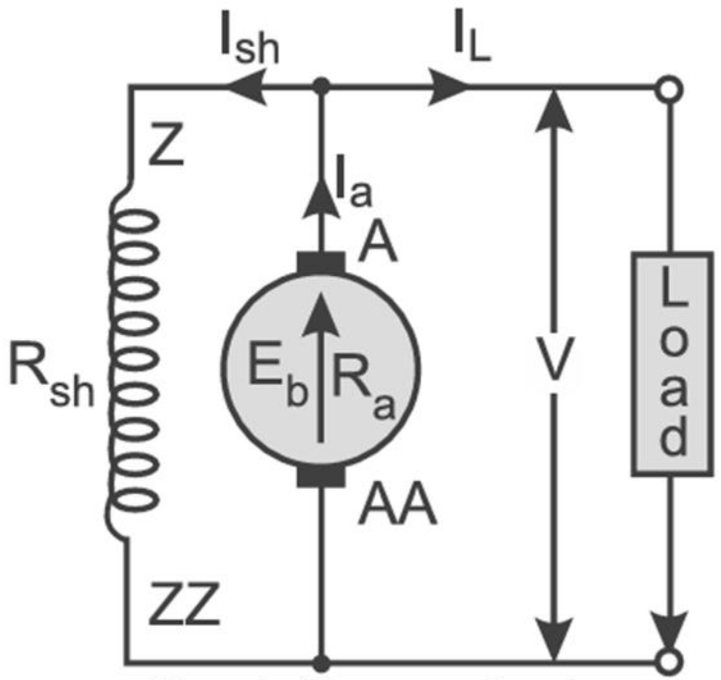

Circuit diagram for separately excited DC generators

# Self excited DC generators

- In a self excited DC generator, the field winding is excited with current from the generator itself.
- The field coils may be connected in parallel with the armature, in series with the armature or partly in series and partly in parallel with the armature winding.
- They may be classified as
  (i) Shunt wound DC generators
  (ii) Series wound DC generators
  (iii) Compound wound DC generators.

# Shunt wound DC generators

- In a shunt wound DC generator, the field winding is connected across the armature winding forming, a parallel or shunt circuit.
- Therefore, full terminal voltage is applied across the field winding.
- A very small current $I_{sh}$ flows through it because this winding has many turns of fine wire having very high resistance $R_{sh}$ (of the order of 100 Ω.
- Shunt field current, $I_{sh} = \frac{V}{R_{sh}}$
- Where $R_{sh}$ is the shunt field winding resistance. The field current $I_{sh}$ is practically constant at all loads, therefore, the DC shunt machine is considered a **constant flux machine**.
- Armature current, $I_a = I_{sh} + I_L$
- Terminal voltage, $\boldsymbol{V = E_g - I_a R_a}$
- Including brush contact drop, $\boldsymbol{V = E_g - I_a R_a - 2v_b}$
- Power developed = $E_g I_a$ ; Power output = $V I_L$



Circuit diagram for shunt wound DC generator

# Shunt wound DC generators

**Problems**

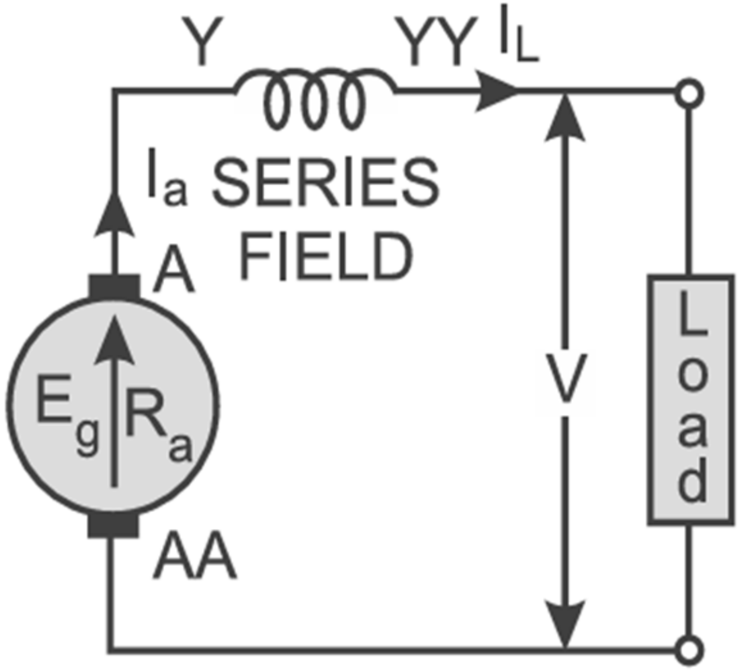
1. A 12-pole DC shunt generator has 50 slots on its armature with 12 conductors per slot with wave winding. The armature and field winding resistance is 0.5 Ω and 60 Ω respectively. The generator is supplying a resistive load of 15 Ω at terminal voltage of 300 V when running at a speed of 625 rpm. Find the armature current, the generated emf and the flux per pole.
2. A four-pole shunt generator with lap connected armature has field and armature resistance of 50 and 0.1 respectively. The generator is supplying a load of 2.4 kW at 100 V. Calculate the armature current, current in each conductor and generated emf.
3. A four-pole DC shunt generator with a wave wound armature having 390 conductors has to supply a load of 500 lamps each of 100 W at 250 V. Allowing 10 V for the voltage drop in the connecting leads between the generator and the load and brush drop of 2 V. Calculate the speed at which the generator should be driven. The flux per pole is 30 m Wb and the value of $R_a$ = 0·05 Ω and $R_{sh}$ = 65 Ω.

# Series wound DC generators

- In a DC series wound generator, the field winding is connected in series with the armature winding forming a series circuit.
- Therefore, full line current $I_L$ or armature current $I_a$ flows through it. Since the series field winding carries full load current, it has a few turns of thick wire having low resistance (usually of the order of less than one ohm).
- Series current, $I_{se} = I_a = I_L$
- Terminal voltage, $\boldsymbol{V = E_g - I_a R_a - I_{se} R_{se}}$

$$\boldsymbol{V = E_g - I_a R_a - I_a R_{se} = E_g - I_a (R_a + R_{se})}$$

- Including brush contact drop, $\boldsymbol{V = E_g - I_a (R_a + R_{se}) - 2v_b}$
- Power developed = $E_g I_a$
- Power output = $V I_L = V I_a$



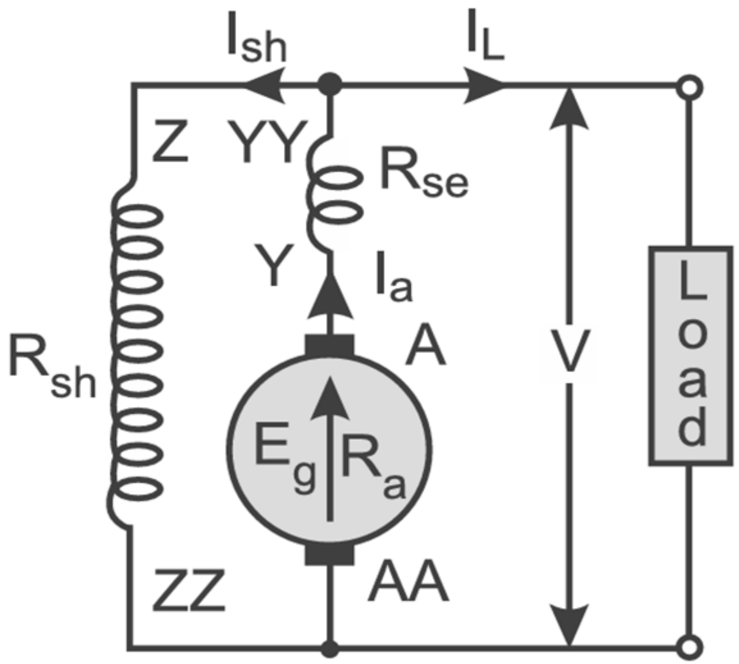

Circuit diagram for DC series wound generator

# Compound wound DC generators

- In a compound wound DC generator, there are two sets of field windings on each pole. One of them is connected in series (having few turns of thick wire) and the other is connected in parallel (having many turns of fine wire) with armature. A compound wound generator may be;

***(a) Long shunt:*** In this generator, the shunt field winding is connected in parallel with the combination of both armature and series field winding.

- Shunt field current, $I_{sh} = \frac{V}{R_{sh}}$
- Series field current, $I_{se} = I_a = I_L + I_{sh}$
- Terminal voltage, $V = E_g - I_a R_a - I_{se} R_{se} = \boldsymbol{E_g - I_a(R_a + R_{se})}$
- Including brush contact drop, $\boldsymbol{V = E_g - I_a(R_a + R_{se}) - 2v_b}$
- Power developed = $E_g I_a$ ; Power output = $V I_L$



Circuit diagram for long shunt DC compound generator

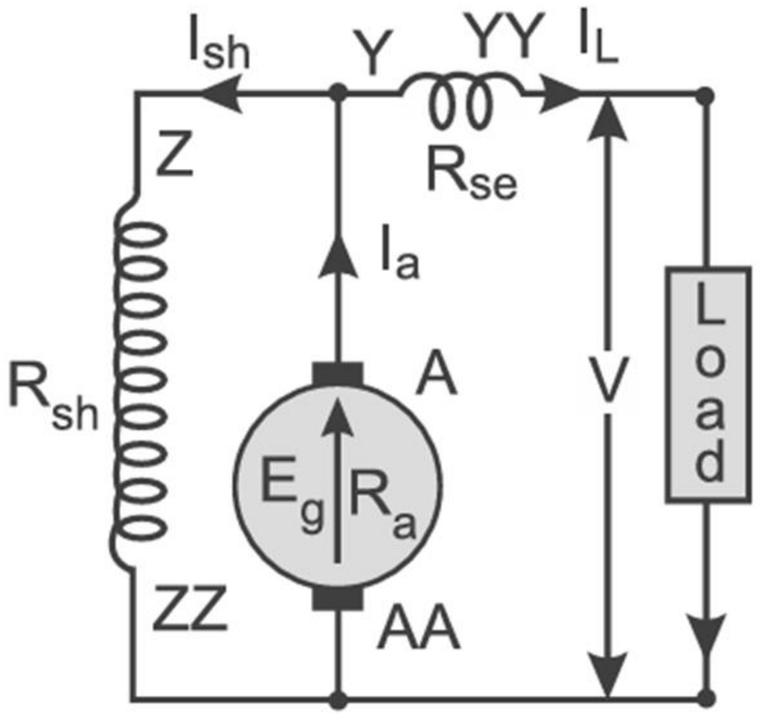
# Compound wound DC generators

***(b) Short shunt:*** In this generator, the shunt field winding is connected in parallel with only the armature winding and the series field winding in series with the load.

- Series field current, $I_{se} = I_L = I_a - I_{sh}$
- Shunt field current, $I_{sh} = \dfrac{V + I_L\, R_{se}}{R_{sh}}$
- Terminal voltage, $\boldsymbol{V = E_g - I_a R_a - I_L R_{se}}$

Including brush contact drop, $\boldsymbol{V = E_g - I_a R_a - I_L R_{se} - 2v_b}$

- Power developed = $E_g I_a$ ;        Power output = $V I_L$




Circuit diagram of short shunt compound DC generator

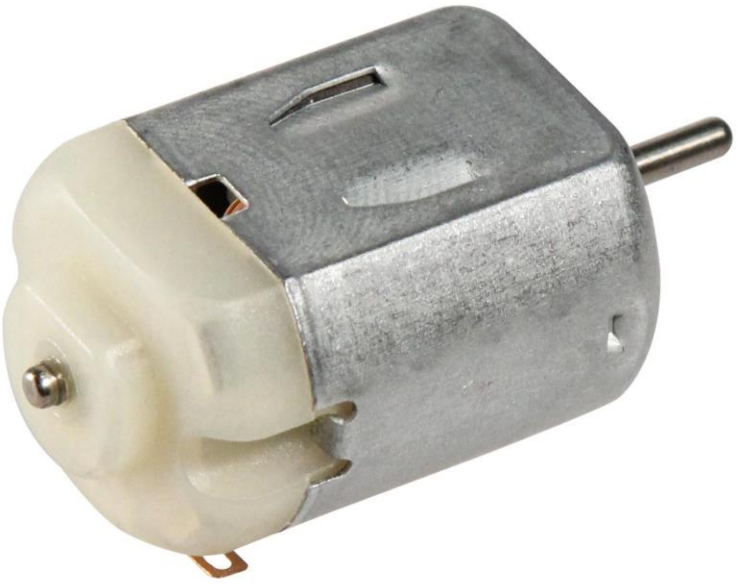
# Compound wound DC generators

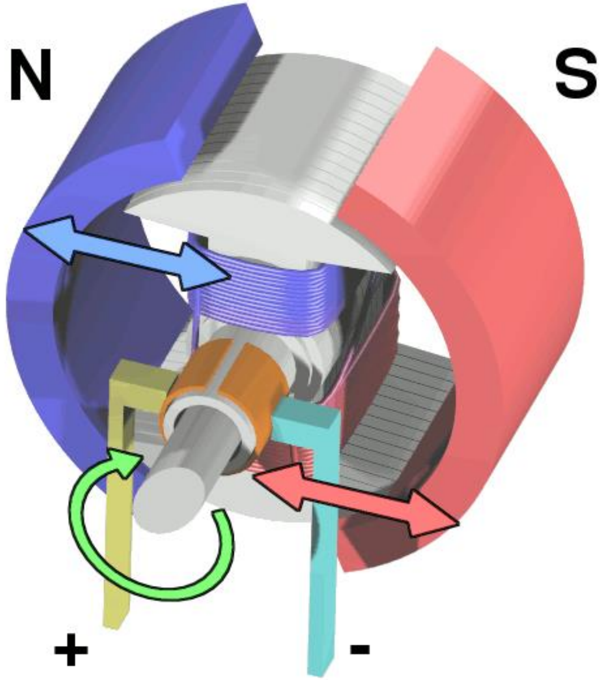
## Problems

- A load of 7.5 kW at 230V is supplied by a short-shunt cumulatively compound DC generator. If the armature, series and shunt field resistances are 0·4, 0·3 and 100 Ω respectively. Calculate the induced emf and the load resistance. *Calculate the emf if this was a long shunt generator.*
- A load of 20 kW at 230 V is supplied by a compound DC generator. If the series, shunt field and armature resistances are 0·05, 115 and 0.1 Ω respectively. Calculate the generated emf when the generator is connected as *(i) long shunt (ii) short shunt*.

# DC motor

- A DC motor is an **electro-mechanical** energy conversion device (electrical machine) that converts DC electrical energy or power (EI) into mechanical energy or power (ωT).
- DC motors have extensive applications in industrial automation, transportation, robotics, aerospace, home appliances, medical devices, etc.

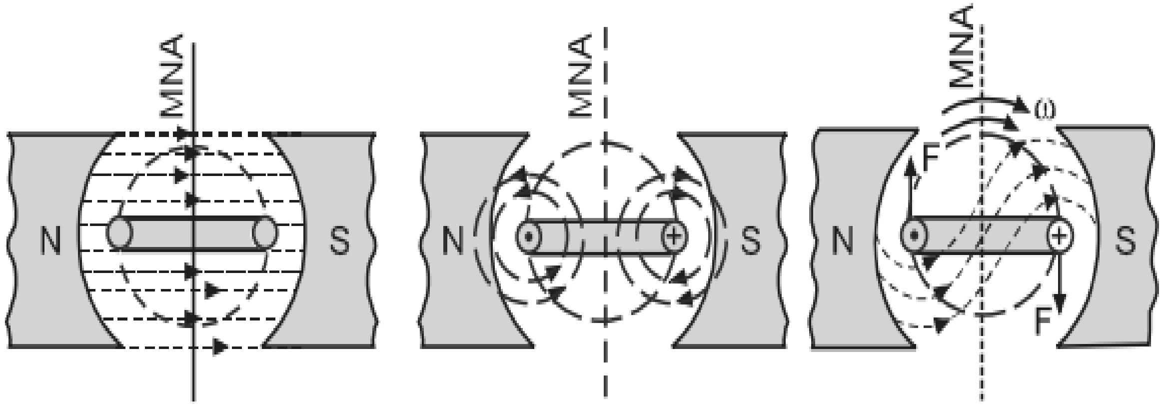




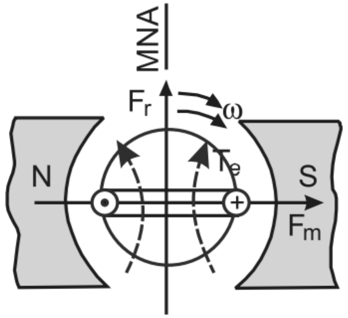
## Working principle

- The operation of a DC motor is based on the principle that when a current carrying conductor is placed in a magnetic field, a mechanical force is experienced by it.
- The direction of this force is determined by **Fleming's Left Hand Rule** and its magnitude is given by the relation: $F = B\ i\ l$ (newton)

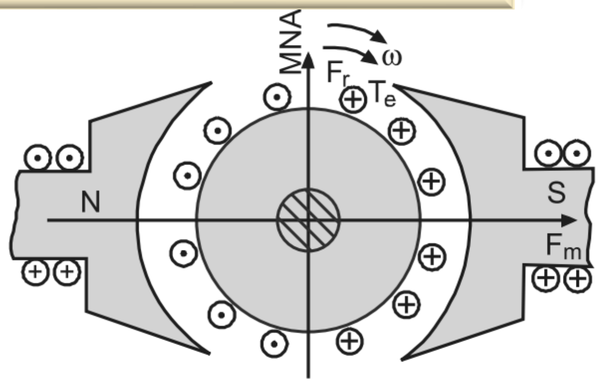
Where, $B$ is the magnetic flux density in Tesla (T), $i$ is the current in the conductor and $l$ is the length of the conductor




(a) Main field (b) Field due to current carrying coil (c) Resultant field




Position of main field ($F_m$) and rotor field




Motor action

the direction of flow of current in each conductor or coil side must be reversed when it passes through the magnetic neutral axis (MNA). A commutator is used to achieve this.

[How a DC motor works]

# DC motor

**Function of a Commutator**

- The function of a commutator in DC motors is to reverse the direction of flow of current in each armature conductor when it passes through the M.N.A. to obtain continuous torque.

**Back EMF**

- Due to the armature conductors (coil) cutting across the magnetic field of the main poles, a back emf is induced. “Back” because **the direction of this emf is opposite that of the applied voltage** in the coils.
- The back EMF plays a crucial role in DC motor operation.
  - **Speed regulation:** The back EMF helps regulate the motor's speed by opposing the applied voltage.
  - **Current limitation:** The back EMF reduces the current flowing through the coil, preventing excessive currents and potential damage.
  - **Efficiency:** The back EMF contributes to the motor's efficiency, as it reduces energy losses due to excessive currents.

# DC motor

- **Shaft torque:** This is the actual torque seen at the output of the motor after windage and frictional losses. Hence $T_{sh}\,\omega = Pe - P_{losses}$ , where $P_e$ is the electrical power, and $P_{losses}$ is the power losses.
- **Brake horse power (BHP):** In the case of motors, the mechanical power (H.P.) available at the shaft is known as brake horsepower.

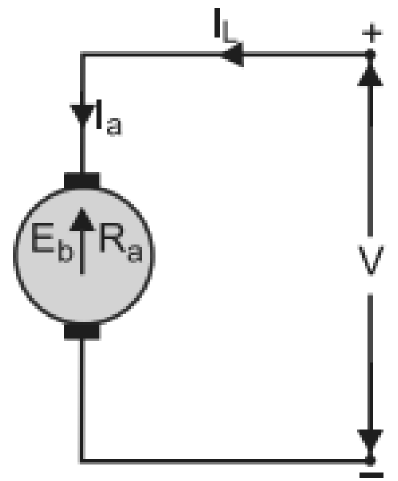
$$BHP = \frac{T_{sh}\,\omega}{745.7}$$

- Hence, BHP is shaft power ($P_{sh}$) converted to units of horse power.

# DC motor

$$E_b = \frac{P\Phi NZ}{60\ A}$$

$E_b \propto \Phi N$ since $\frac{PZ}{60\ A}$ are constants




The supply voltage (V) is always greater than the induced or back emf ($E_b$) (i.e., V > $E_b$ ). Therefore, current is always supplied to the motor from the mains and the relation among the various quantities will be; $\mathbf{E_b = V - I_a R_a}$.

Hence the armature current is: $\boldsymbol{I_a = \frac{V - E_b}{R_a}}$

- When mechanical load applied on the motor increases, its speed decreases which reduces the value of $E_b$. As a result the value ($V - E_b$ ) increases which consequently increases $I_a$. Hence, motor draws extra current from the mains. Thus, the back emf regulates the input power as per the extra load.

# DC motor

## Work example

A 50 HP, 400 V, 4 pole, 1000 rpm, DC motor has flux per pole equal to 0.027 Wb. The armature having 1600 conductors is wave connected. Calculate the gross torque when the motor takes 70 ampere.

## Solution

Torque developed, $T = \frac{P \emptyset Z I_a}{2\pi A}$

Where, P = 4 poles; ∅ = 0.027 Wb; $I_a$ = 70A; A = 2 (wave connected)

$$T = \frac{4 \times 0.027 \times 1600 \times 70}{2\pi \times 2} = 963\ Nm$$

## Assignment

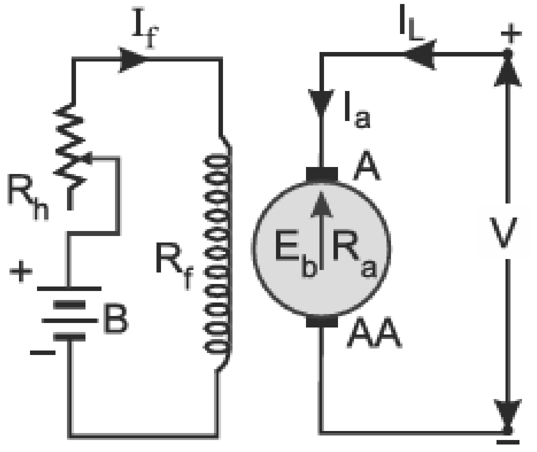
- The induced emf in a DC machine is 200 V at a speed of 1200 rpm. Calculate the electromagnetic torque developed at an armature current of 15 A.
- A four-pole DC motor has a wave-wound armature with 594 conductors. The armature current is 40 A and flux per pole is 7.5 m Wb. Calculate H.P. of the motor when running at 1440 rpm.
- A DC motor has 6-poles with lap wound armature. What will be its brake horse power when it draws a current of 340 A and rotates at 400 rpm. The flux per pole is 0.05 Wb and the armature carries 864 turns, Neglect mechanical losses.

# Types of DC motors

- The construction of the various types of DC motors is the same as their corresponding types in DC generators. The only difference is that, in DC motors, the flow of current is from the terminal to the armature.

**Separately excited DC motor**

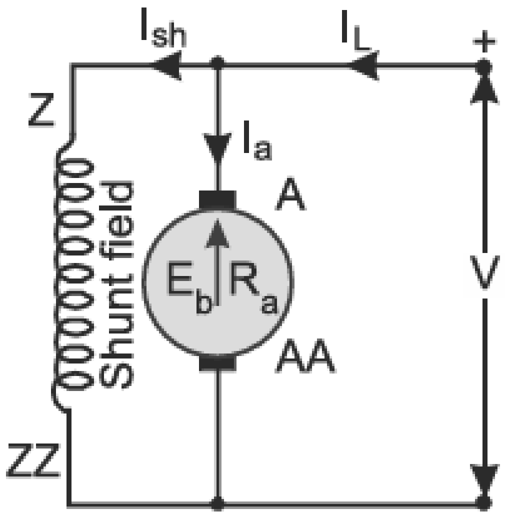
- $\boldsymbol{E_b = V - I_a R_a}$
- $\boldsymbol{E_b = V - I_a R_a - 2v_b}$ - when voltage drops at brushes is considered. $\boldsymbol{v_b}$ is the voltage drop per brush



Separately excited DC motor

**Self excited DC motors:** These motors can be further classified as;

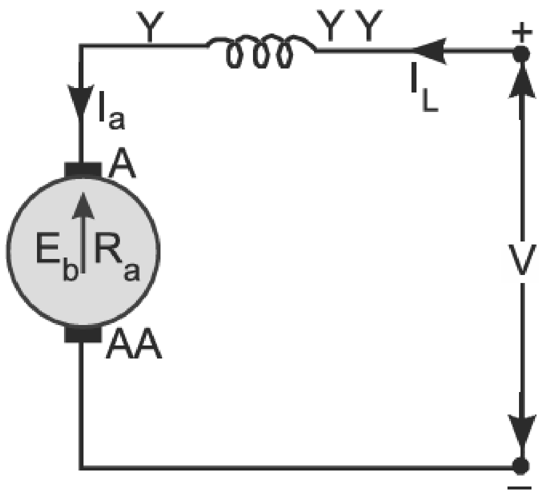
- ***(i) Shunt motors:***
- $I_{sh} = \frac{V}{R_{sh}}$
- $I_a = I_L - I_{sh}$
- $\boldsymbol{E_b = V - I_a R_a}$
- $\boldsymbol{E_b = V - I_a R_a - 2v_b}$



Shunt wound DC motor

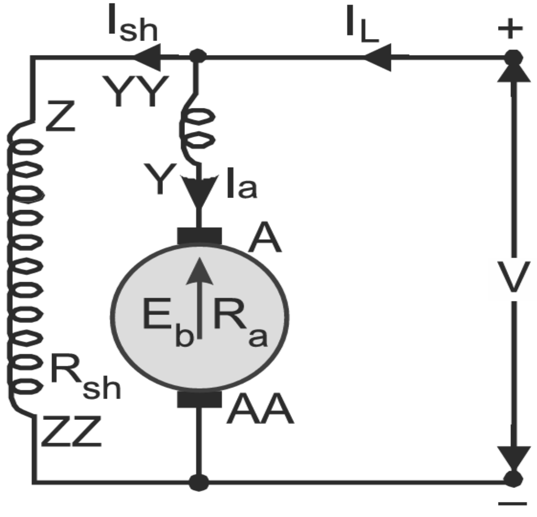
# Types of DC motors

- ***(ii) Series motors:***
- $I_a = I_L = I_{se}$
- $\boldsymbol{E_b = V - I_a R_a - I_L R_{se}}$
- $\boldsymbol{E_b = V - I_a (R_a + R_{se})}$
- $\boldsymbol{E_b = V - I_a (R_a + R_{se}) - 2v_b}$




Series wound DC motor

- ***(ii) Compound motors:***
- $I_{sh} = \frac{V}{R_{sh}}$
- $I_{se} = I_a$
- $I_a = I_L - I_{sh}$
- $\boldsymbol{E_b = V - I_a R_a - I_L R_{se}}$
- $\boldsymbol{E_b = V - I_a (R_a + R_{se})}$
- $\boldsymbol{E_b = V - I_a (R_a + R_{se}) - 2v_b}$




Compound DC motor

# Types of DC motors

**Examples**

1) The armature resistance of a DC shunt motor is 0.5 Ω, it draws 20 A from 220 V mains and is running at a speed of 80 radians per second. Determine (i) Induced emf (ii) Electromagnetic torque (iii) Speed in rpm.

2) A 400 V DC motor takes an armature current of 100 A when its speed is 1000 rpm If the armature resistance is 0·25 Ω, calculate the torque produced in Nm.

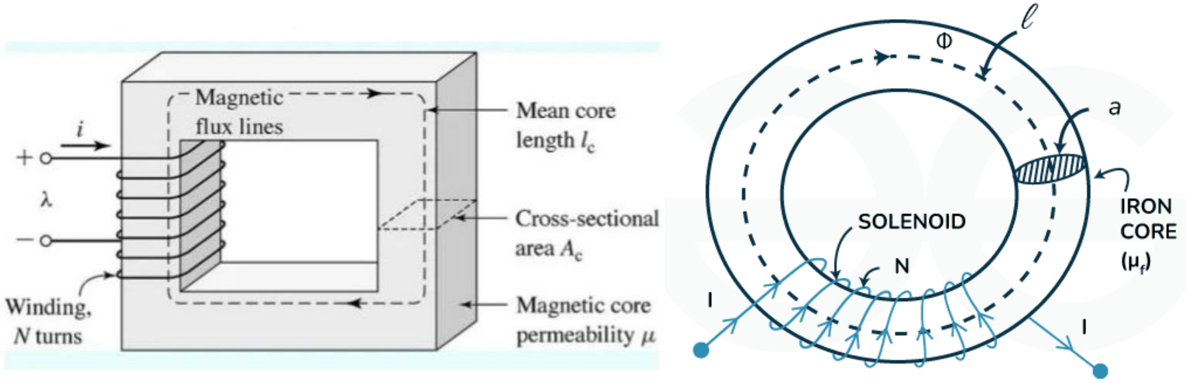
3) The armature and series field winding resistance of a 220 V, four-pole DC series motor is 0.75 Ω. It has 782 wave wound armature conductors. If it draws 40 A from the supply mains and has a flux of 25 mWb, determine its speed and gross torque developed.

4) The armature and shunt field resistance of a four-pole, lap wound DC shunt motor is 0.05 Ω and 25 Ω respectively. If its armature contains 500 conductors, find the speed of the motor when it takes 120 A from a DC mains of 100 V supply. Flux per pole is 2 × 10–2 Wb.

5) A 6-pole, 440 V DC motor has 936 wave wound armature conductors. The useful flux per pole is 25 m Wb. The torque developed is 45·5 kg-m. Calculate the following, if armature resistance is 0·5 Ω; (i) Armature current (ii) Speed

# Magnetic circuits

- A **magnetic circuit** is a closed path followed by magnetic flux.

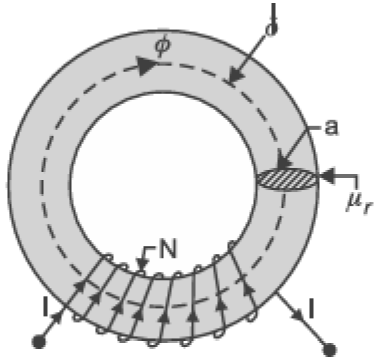
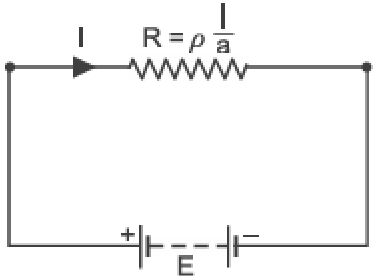



Magnetic circuits

- **Magnetic flux (∅):** The amount of magnetic lines of force set up in a magnetic circuit. Its unit is weber (Wb). It is analogous to electric current in an electric circuit.
- **Magnetic flux density (B)** at a point is the flux per unit area at right angles to the flux at that point. Its unit is Wb/m$^2$ or Tesla (T).
- **Magnetic field:** The region around a magnet where its poles exhibit a force of attraction or repulsion.
- **Permeability ($\mu$):** The ability of a material to conduct magnetic lines of force through it. Air/vacuum has the lowest permeability represent as $\mu_o$ which is $4\pi \times 10^{-7}$ H/m.
- **Relative permeability ($\mu_r$):** the ratio of the permeability of a material to the permeability of air or vacuum.
- **Magnetic field intensity (H):** The force acting on a unit north pole (1 Wb) when placed at a point in the magnetic field.
- **Magnetomotive force (mmf):** The magnetic pressure which sets-up or tends to set-up magnetic flux in a magnetic circuit.
- **Reluctance (S):** The opposition offered to the magnetic flux by a magnetic circuit.

# Magnetic circuits

## Comparison between Magnetic and Electric Circuits

| Magnetic Circuits | Electrical Circuits |
|---|---|
|   **Fig. 1.3** Magnetic circuit |   **Fig. 1.4** Electric circuit |

### Similarities

| Magnetic Circuits | Electrical Circuits |
|---|---|
| 1. The closed path for magnetic flux is called magnetic circuit. | 1. The closed path for electric current is called electric circuit. |
| 2. Flux = mmf/reluctance | 2. Current = emf/resistance |
| 3. Flux, $\phi$ in Wb | 3. Current, $I$ in ampere |
| 4. mmf in AT | 4. emf in V |
| 5. Reluctance, $S = \frac{l}{a\mu} = \frac{l}{a\mu_0\,\mu_r}$ AT/Wb | 5. Resistance, $R = \rho\frac{l}{a}\,\Omega$ or $R = \frac{1}{\sigma}\frac{l}{a}\,\Omega$ |
| 6. Permeance = 1/reluctance | 6. Conductance = 1/resistance |
| 7. Permeability, μ | 7. Conductivity, $\sigma = 1/\rho$ |
| 8. Reluctivity | 8. Resistivity |
| 9. Flux density, $B = \frac{\phi}{a}$ Wb/m$^2$ | 9. Current density, $J = \frac{I}{a}$ A/m$^2$ |
| 10. Magnetic intensity, $H = NI/l$ | 10. Electric intensity, $E = V/d$ |

### Dissimilarities

| Magnetic Circuits | Electrical Circuits |
|---|---|
| 1. In fact, the magnetic flux does not flow but it sets-up in the magnetic circuit (basically molecular poles are aligned). | 1. The electric current (electrons) actually flows in an electric circuit. |
| 2. For magnetic flux, there is no perfect insulator. It can be set-up even in the non-magnetic materials like air, rubber, glass etc. with reasonable mmf | 2. For electric current, there are large number of perfect insulators like glass, air, rubber, etc., which do not allow it to follow through them under normal conditions. |
| 3. The reluctance ($S$) of a magnetic circuit is not constant rather it varies with the value of $B$. It is because the value of $\mu_r$ changes considerably with the change in $B$. | 3. The resistance ($R$) of an electric circuit is almost constant as its value depends upon the value of $\rho$ which is almost constant. However, the value of $\rho$ and $R$ may vary slightly if temperature changes. |
| 4. Once the magnetic flux is set-up in a magnetic circuit, no energy is expanded. However, a small amount of energy is required at the start to create flux in the circuit. | 4. Energy is expanded continuously, so long as the current flows through an electric circuit. This energy is dissipated in the form of heat. |

# Magnetic circuits

- The figure shows a solenoid having N turns wound on an iron core (ring).
- When current ($I$) ampere is passed through the solenoid, magnetic flux Ø is set-up in the core.
- Flux density in the core material, $\mathrm{B} = \frac{\emptyset}{a}$ Wb/m$^2$

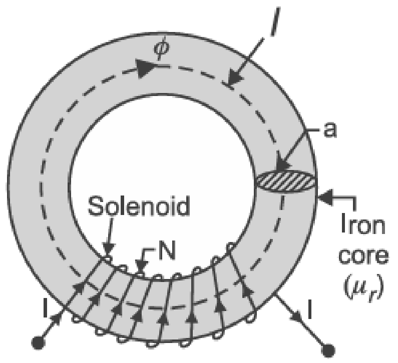



**Fig. 1.2** Magnetic circuit

- Magnetising force in the core material, $\mathrm{H} = \frac{B}{\mu_o \mu_r} = \frac{\emptyset}{a\ \mu_o \mu_r}$ AT/m

- According to work law, the work done in moving a unit pole once round the magnetic circuit (or path) is equal to the ampere-turns enclosed by the magnetic circuit.
- $Hl = NI$ or $\frac{\emptyset}{a\ \mu_o \mu_r} \times l = NI$ or $\emptyset = \frac{NI}{l/a\ \mu_o \mu_r}$ or $\emptyset = \frac{NI}{l/a\ \mu_o \mu_r}$ or
- $\emptyset = \frac{mmf}{S} \rightarrow \frac{magnetomotive\ force}{reluctance}$

where;
$\boldsymbol{l}$ – mean length of magnetic circuit
$\boldsymbol{a}$ – cross-sectional area of core
$\boldsymbol{\mu_o}$– permeability of air
$\boldsymbol{\mu_r}$– relative permeability of core

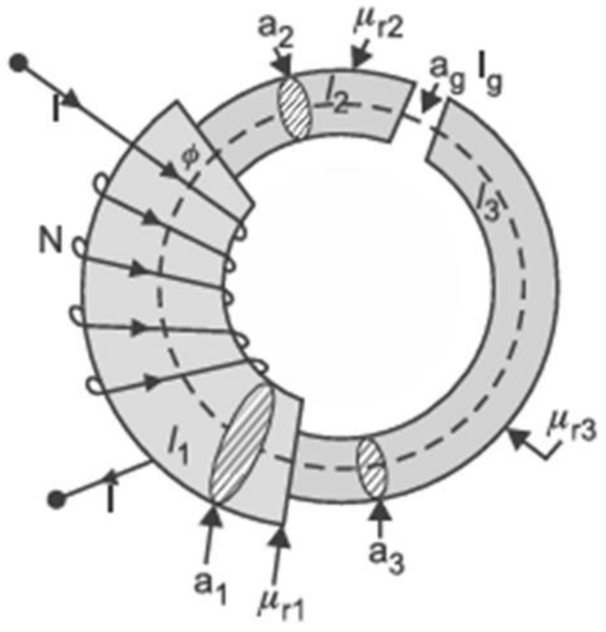
# Magnetic circuits

**Problems**

1. An iron ring of 400 cm mean circumference is made from round iron of cross-section 20 $cm^2$. Its relative permeability is 500. If it is wound with 400 turns, what current would be required to produce a flux of 0·001 Wb?
2. A coil of insulated wire of 500 turns and of resistance 4 is closely wound on iron ring. The ring has a mean diameter of 0·25 m and a uniform cross-sectional area of 700 mm2. Calculate the total flux in the ring when a DC supply of 6V is applied to the ends of the winding. Assume a relative permeability of 550.

# Series Magnetic Circuits

- A magnetic circuit that has a number of parts of different dimensions and materials carrying the same magnetic field is called a **series magnetic circuit.**




Series magnetic circuit

Total reluctance of the magnetic circuit,

$$S = S_1 + S_2 + S_3 + S_g$$

$$= \frac{l_1}{a_1 \mu_0 \mu_{r1}} + \frac{l_2}{a_2 \mu_0 \mu_{r2}} + \frac{l_3}{a_3 \mu_0 \mu_{r3}} + \frac{l_g}{a_g \mu_0}$$

$$\text{Total mmf} = \phi S$$

$$= \phi\left(\frac{l_1}{a_1 \mu_0 \mu_{r1}} + \frac{l_2}{a_2 \mu_0 \mu_{r2}} + \frac{l_3}{a_3 \mu_0 \mu_{r3}} + \frac{l_g}{a_g \mu_0}\right)$$

$$= \frac{B_1 l_1}{\mu_0 \mu_{r1}} + \frac{B_2 l_2}{\mu_0 \mu_{r2}} + \frac{B_3 l_3}{\mu_0 \mu_{r3}} + \frac{B_g l_g}{\mu_0}$$

$$= H_1\, l_1 + H_2\, l_2 + H_3\, l_3 + H_g\, l_g$$

# Series Magnetic Circuits

**Problems**

- An iron ring of mean length 50 cm and relative permeability 300 has an air gap of 1 mm. If the ring is provided with winding of 200 turns and a current of 1 A is allowed to flow through, find the flux density across the airgap.
- A coil of 1000 turns is wound on a laminated core of steel having a cross-section of 5 cm2. The core has an air gap of 2 mm cut at right angle. What value of current is required to have an air gap flux density of 0·5 T? Permeability of steel may be taken as infinity.

# Parallel Magnetic Circuits

- A magnetic circuit which has two or more paths for magnetic flux is called a **parallel magnetic circuit**.

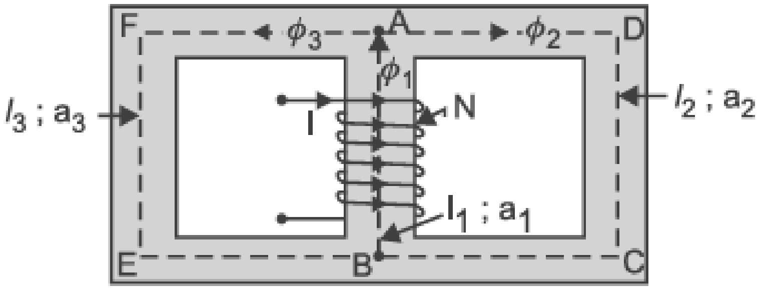



**Fig. 1.6** Parallel magnetic circuit

$$\phi_1 = \phi_2 + \phi_3$$

The two magnetic paths $ADCB$ and $AFEB$ are in parallel. The ATs required for this parallel circuit is equal to the ATs required for any one of the paths.

If $S_1$ = reluctance of path $BA$ i.e., $l_1/a_1\,\mu_0\,\mu_{r1}$

$S_2$ = reluctance of path $ADCB$ i.e., $l_2/a_2\,\mu_0\,\mu_{r2}$

$S_3$ = reluctance of path $AFEB$ i.e., $l_3/a_3\,\mu_0\,\mu_{r3}$

$\therefore$ Total mmf required = mmf required for path $BA$ + mmf required path $ADCB$ or path $AFEB$.

i.e., Total mmf or $AT = \phi_1 S_1 + \phi_2 S_2 = \phi_1 S_1 + \phi_3 S_3$

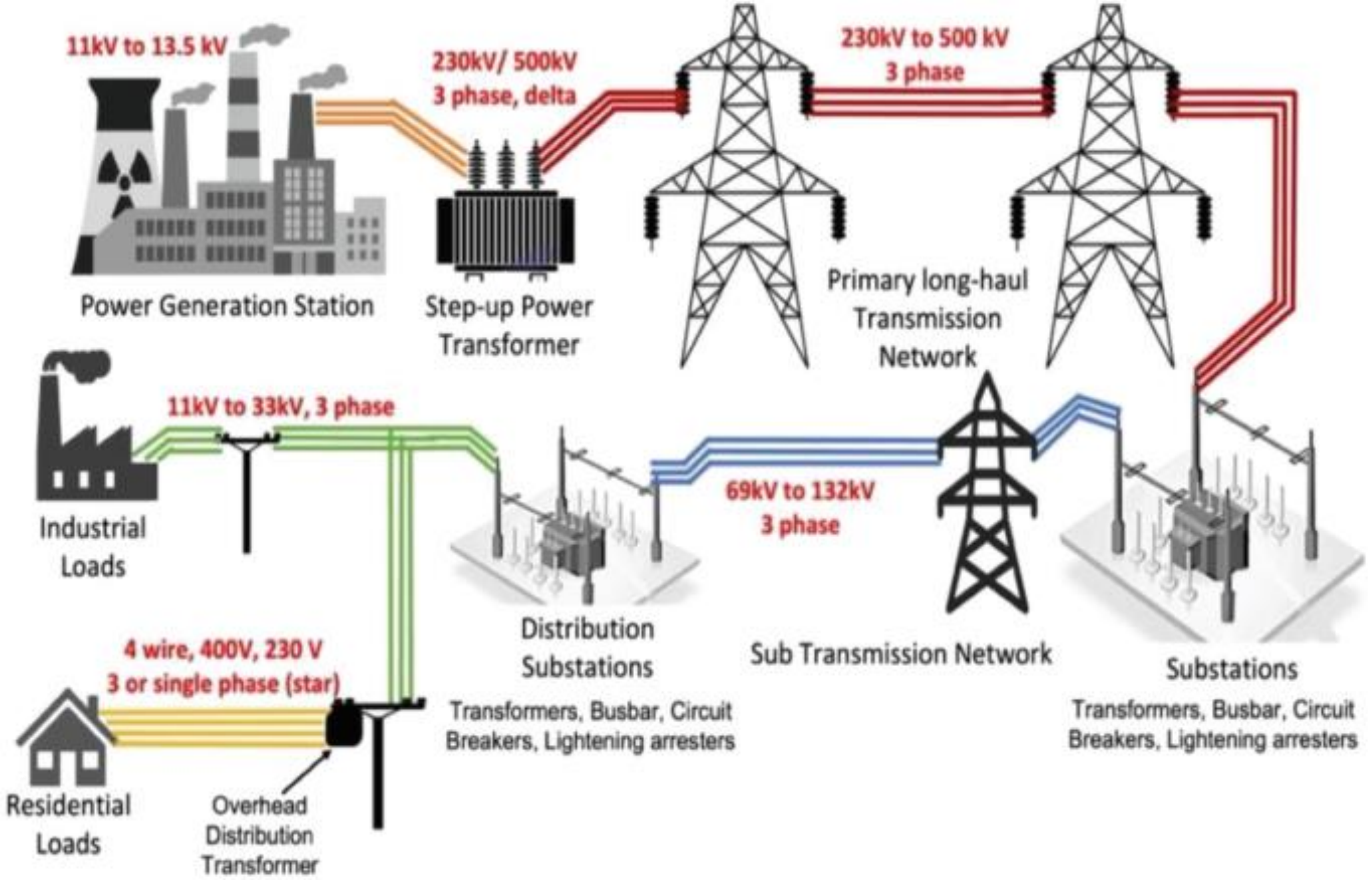
# Voltage Transformers

- Transformers are considered to be the backbone of power systems.
- A device that changes the voltage of an alternating current (AC) electrical signal by either increasing (step-up) or decreasing (step-down) the voltage, without changing the frequency or power.
- Step-down transformer – $V_1 > V_2$
- Step-up transformer – $V_2 > V_1$

# Applications of Transformers

- To change the level of voltage and current in electric power systems.
- As impedance-matching device for maximum power transfer in low-power electronic and control circuits.
- As a coupling device is electronic circuits.
- To isolate one circuit from another, since primary and secondary are not electrically connected.
- To measure voltage and currents; these are known as instrument transformers.

# Transformers on Power Systems

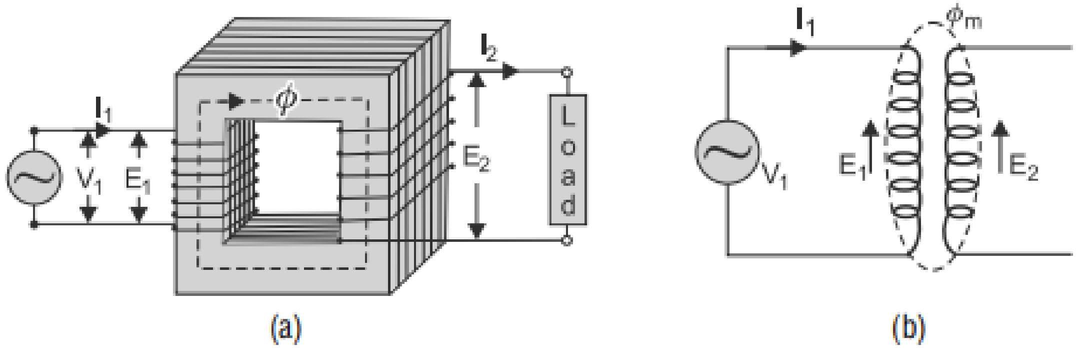


# Working Principle of a Transformer

## Working principle

- A single-phase transformer consists of two windings placed over a laminated silicon steel core. The winding having less number of turns is called low-voltage winding and the winding having more number of turns is called high voltage winding.

- Once AC supply of voltage V1 is given to primary winding, an alternating flux is set-up in the magnetic core which links with the primary and secondary winding.
- Consequently, self-induced emf E1 and mutually-induced emf E2 are induced in primary and secondary, respectively.
- These induced emf's are developed in phase opposition to V1 as per Lenz's law.
- The self-induced emf in the primary is also called back emf since it acts in opposite direction to the applied voltage




(a) Single-phase transformer (core and windings) (b) Flux linking with primary and secondary

**Turn ratio:** The ratio of primary to secondary turns is called turn ratio, i.e., turn ratio, a = $N_1/ N_2$.
**Transformation ratio:** The ratio of secondary voltage to primary voltage is called the voltage transformation ratio of the transformer. It is represented by K and it's equal to the turn ratio

$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{1}{a}$$

# An ideal transformer

- In an ideal transformer, the following assumptions are made

a) coefficient of coupling (k) is unity.
b) primary and secondary windings are pure inductors having infinitely large values.
c) leakage flux and leakage inductances are zero.
d) self and mutual inductances are zero having no reactance or resistance.
e) efficiency is 100%, having no loss due to resistance, hysteresis or eddy current.
f) transformation ratio (or turn ratio) is equal to the ratio of its secondary to primary terminal voltage and also as the ratio of its primary to secondary current.
g) Its core has a permeability of infinite value.

# Emf equation of an ideal transformer

**Emf equation**

RMS value of emf induced in the primary winding

$$E_1 = 4.44\phi_m f N_1$$

$$\frac{E_1}{N_1} = 4.44\phi_m f$$

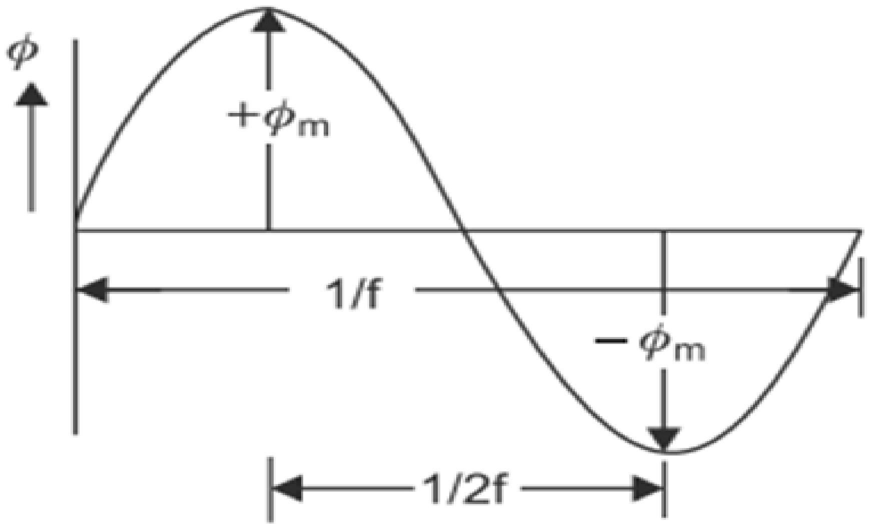
- RMS value of emf induced in the secondary winding

$$E_2 = 4.44\phi_m f N_2$$

$$\frac{E_2}{N_2} = 4.44\phi_m f$$

Where, $\phi_m$ is the maximum flux per turn,

$f$ is the frequency of the AC supply




Wave diagram of flux

$$\frac{E_1}{N_1} = \frac{E_2}{N_2}$$

Known as the voltage per turn constant

**Try**

Derive the emf equation of an ideal transformer

# Transformers

## Example

- What will be the number of primary and secondary turn of a single-phase 2310/220V, 50 Hz transformer which has an emf of 13V per turn approximately

## Solution

Here, $\frac{E_1}{N_1} = \frac{E_2}{N_2} = 13\text{ V (given)};\ E_1 = 2310\text{ V};\ E_2 = 220\text{ V}$

$\therefore$ Primary turns, $N_1 = \frac{E_1}{13} = \frac{2310}{13} = 177.69 \cong \mathbf{178}$ (*Ans.*)

Secondary turns, $N_2 = \frac{E_2}{13} = \frac{220}{13} = 16.92 \cong \mathbf{17}$ (*Ans.*)

## Assignment

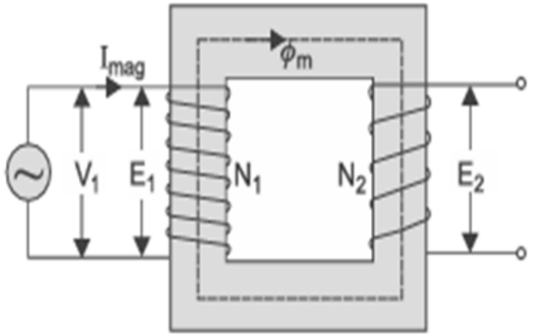
1) A power transformer has 1000 primary turns and 100 secondary turns. The cross-sectional area of the core is 6 sq. cm and the maximum flux density while in operation is 10 000 Gauss. Calculate turns per volt for the primary and secondary windings.
2) The primary and secondary of a 25 kVA transformer has 500 and 40 turns, respectively. If the primary is connected to 3000 V, 50 Hz mains, calculate (i) primary and secondary currents at full load; (ii) The secondary emf and (iii) The maximum flux in the core. Neglect magnetic leakage, resistance of the winding and the primary no-load current in relation to the full load current.
3) The primary and secondary turns of a single-phase transformer are 400 and 1100, respectively. The net cross-sectional area of the core is 60 cm2. When its primary is connected 500V, 50 Hz supply, calculate the value of maximum flux density in the core and the emf induced in secondary winding.
4) The emf per turn of an 11 kV /415 V, 50 Hz single-phase core type transformer is 15 V. The maximum flux density in the core is 2.5 T. Find number of primary and secondary turns and net cross sectional area of core.

# Transformer on no load

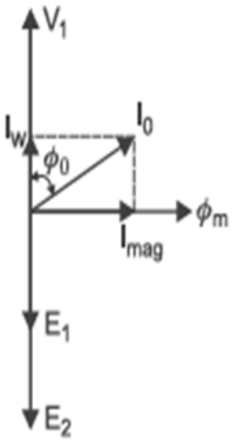
- A transformer is said to be on no-load when its secondary winding is kept open and no-load is connected across it.
- As such, $I_2 = 0$.
- Hence, the secondary winding does not cause any effect on the magnetic flux set-up in the core or on the current drawn by the primary. But the losses cannot be ignored.
- At no-load, a transformer draws a small current $I_0$ (usually 2 to 10% of the rated value).
- This current has to supply the iron losses (hysteresis and eddy current losses) in the core and a very small amount of copper loss in the primary (the primary copper losses are so small as compared to core losses that they are generally neglected moreover secondary copper losses are zero as $I_2$ is zero).

# Transformer on no load

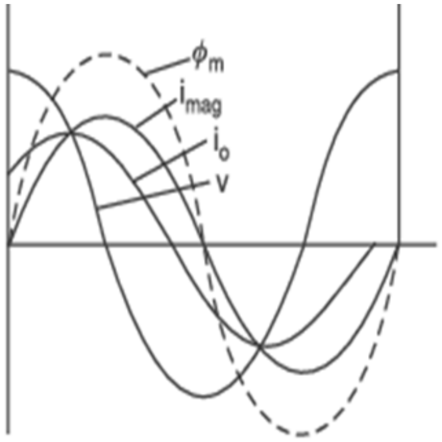
- The no-load current $I_0$ has two components, $I_w$ (working component) and $I_{mag}$ (magnetizing component)




(a) Circuit diagram

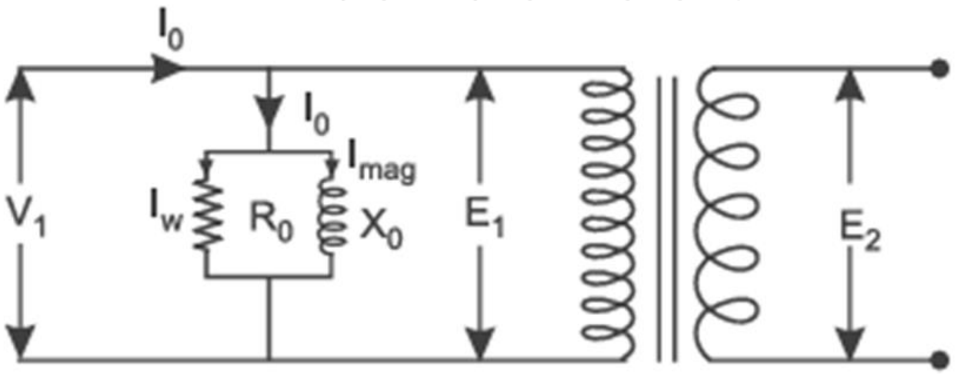



(b) Phasor diagram




(c) Wave diagram

**Transformer on no load**




**Equivalent circuit of a transformer on no load**

Working component, $I_w = I_0 \cos \phi_0$

Magnetising component, $I_{mag} = I_0 \sin \phi_0$

No-load current, $I_0 = \sqrt{I_w^2 + I_{mag}^2}$

Primary p.f. at no-load, $\cos \phi_0 = \dfrac{I_w}{I_0}$

No-load power input, $P_0 = V_1 I_0 \cos \phi_0$

Exciting resistance, $R_0 = \dfrac{V_1}{I_w}$

Exciting reactance, $X_0 = \dfrac{V_1}{I_{mag}}$

# Transformer on no load

## Examples

- A 230/110 V single-phase transformer has a core loss of 100 W. If the input under no-load condition is 400 VA, find core loss current, magnetising current and no-load power factor angle.
- A 230V, 50 Hz transformer has 200 primary turns. It draws 5 A at 0.25 p.f lagging at no-load. Determine: (i) Maximum value of flux in the core; (ii) Core loss; (iii) Magnetising current (iv) Exciting resistance and reactance of the transformer. Also draw its equivalent circuit.
- At open circuit, transformer of 10 kVA, 500/250 V, 50 Hz draws a power of 167 watt at 0.745 A, 500 V. Determine the magnetising current, wattful current, no-load power factor, hysteresis angle of advance, equivalent resistance and reactance of exciting circuit referred to primary side.

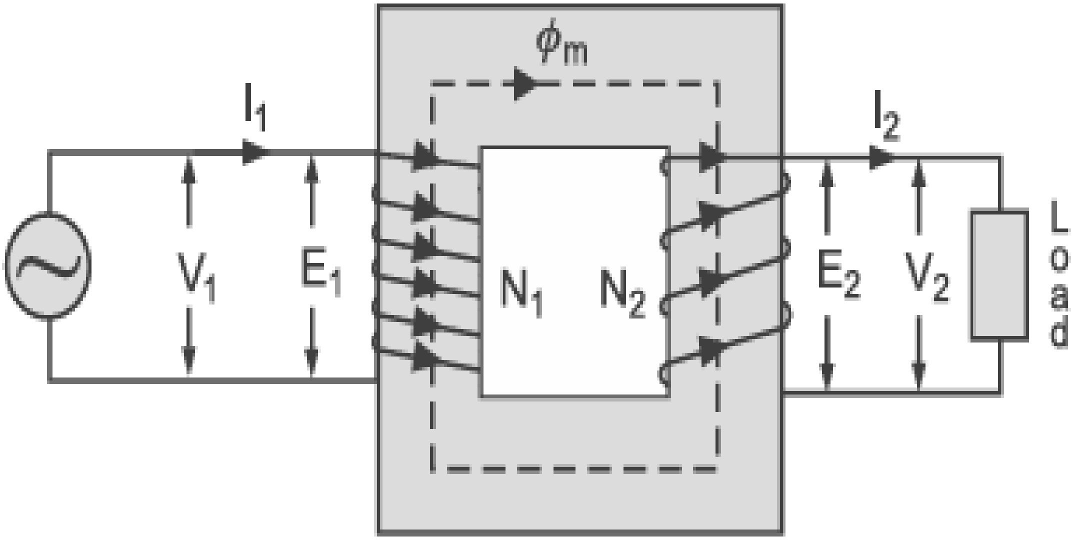
## Assignment 5

- The primary and secondary of a single-phase transformer carry 500 and 100 truns, respectively. The mean length of the flux in the iron core is 200 cm and the joints are equivalent to an air gap of 0.1 mm. If the maximum value of flux density is to be 1.1 Wb/m2 when a potential difference of 2200 volt at 50 cycles is applied to the primary, calculate (i) the cross-sectional area of the core (ii) the secondary voltage on no-load (iii) the primary current and power factor on no-load.

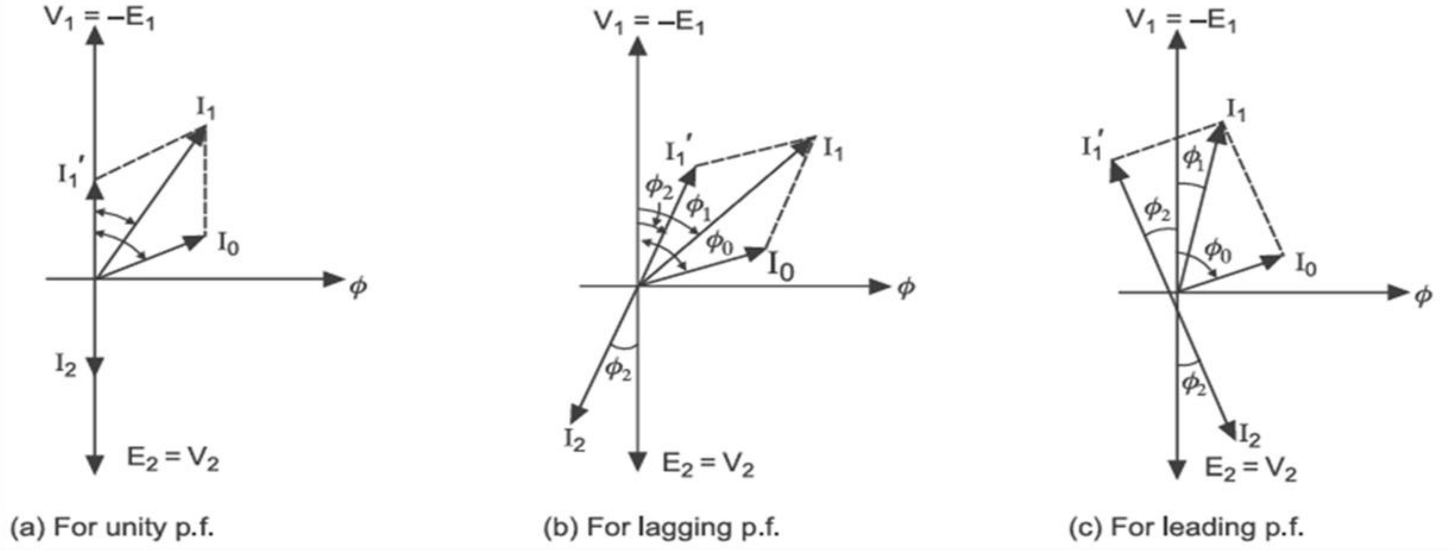
# Transformer on load

- When a certain load is connected across the secondary, a current $I_2$ flows through it.
- The magnitude of current $I_2$ depends upon terminal voltage $V_2$ and impedance of the load.
- The phase angle of secondary current $I_2$ with respect to $V_2$ depends upon the nature of the load, whether the load is resistive, inductive or capacitive.
- The connected load also causes a primary current $I'_1$ which combines with the no-load current $I_0$ to form the total current drawn at the primary $I_1$.
- The algebraic sum of $I'_1$ and $I_0$ gives $I_1$.
- Unlike in an ideal transformer, the transformation ratio,

$$\mathrm{K} = \frac{I_1}{I'_2} = \frac{E_2}{E_1}$$



# Phasor diagram of a transformer on load




**Resistive load** **Inductive load** **Capacitive load**

**Phasor diagram on-load (neglecting winding resistance and leakage reactance)**

$$\overline{I_1} = \overline{I_0} + \overline{I_1'} \text{ or } \overline{I_1} = \sqrt{(I_0)^2 + (I_1')^2 + 2I_0 I_1' \cos\theta}$$

$$\cos\phi_1 = \frac{I_0 \cos\phi_0 + I_1' \cos\phi_2}{I_1}$$

*Alternately*

The primary current $I_1$ can also be determined by resolving the vectors, i.e.,

$$I_v = I_0 \cos\phi_0 + I'_1 \cos\phi_2 \qquad [\text{where } \sin\phi_0 = \sin\cos^{-1}(\cos\phi_0)$$

$$I_H = I_0 \sin\phi_0 + I'_1 \sin\phi_2 \qquad \text{and } \sin\phi_2 = \sin\cos^{-1}(\cos\phi_2)]$$

$$I_1 = \sqrt{(I_v)^2 + (I_H)^2}$$

# Transformer on load

## Examples

- A 440/110 V, single-phase transformer draws a no-load current of 5 A at 0.2 p.f. lagging. If a current of 120 A at 0.8 p.f lagging is supplied by the secondary, calculate the primary current and p.f.

- A single-phase transformer with a ratio of 6600/400 V (primary to secondary voltage) takes a no-load current of 0·7 A at 0·24 power factor lagging. If a current of 120 A at a power factor of 0·8 lagging is supplied by its secondary. Estimate the current drawn by the primary winding.