CENG 251: ELECTRIC CIRCUITS ANALYSIS AND DESIGN

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Content

- Basic Concepts
- Basic Laws
- Methods of Analysis
- Circuit Theorems
- Capacitors and Inductors
- First – Order Circuits
- Second – Order Circuits
- Laplace Transforms
- Operational Amplifiers
- Passive and Active Filters

Assessment

- Continuous assessment – 40 marks
  - Weekly quizzes and assignments - 20 marks
  - Mid-semester examination – 20 marks

- End of semester examination – 60mks
  - Multiple choice questions – 20 marks
  - Written examination – 40 marks

Further study

- Fundamentals of Electric Circuits (5^{th} Edition): Charles K. Alexander, Matthew N. O. Sadiku, 2013.

Click here for lecture materials

Basic Concepts

Basic Electrical Quantities

Quantity	Unit	Basic equation
Electric current (I): the time rate of change of charge.	amperes (A)	$i \triangleq \frac{dq}{dt}$ q – charge (C) t – time (s)
Voltage / potential difference (V): the energy required to move a unit charge through an element.	volts (V)	$v_{ab} \triangleq \frac{dw}{dq}$ w – energy (J) q – charge (C)
Power (P): the time rate of expending or absorbing energy.	watts (W)	$P = V.I$
Energy (E) the capacity to do work.	joules (J)	$w = \int_{t_0}^{t} p\,dt = \int_{t_0}^{t} vi\,dt$
Resistance (R): the physical property of an element or device that impedes the flow of current.	ohms (Ω)	$R = \frac{V}{I}$

Circuit Elements

- An element is the basic building block of a circuit
- **Circuit analysis** is the process of determining voltages across (or the currents through) the elements of the circuit.
- There are two types of elements, namely (1) passive element and (2) active element
- An active element generates energy, while a passive element does not generate energy.
- The most important active elements are voltage or current sources that generally deliver power to the circuit connected to them.
- There are two kinds of sources: independent and dependent sources.
- An **ideal independent source** provides a specified voltage or current that is completely independent of other circuit elements.
- An ideal dependent (or controlled) source is an active element in which the source quantity is controlled by another voltage or current.
- Dependent sources are usually designated by diamond-shaped symbols as shown in Fig. 1.

- There are four possible types of dependent sources, namely:
  1. A voltage-controlled voltage source (VCVS)
  2. A current-controlled voltage source (CCVS)
  3. A voltage-controlled current source (VCCS)
  4. A current-controlled current source (CCCS)

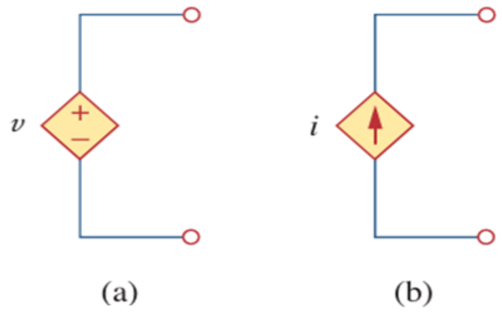
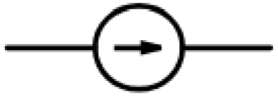


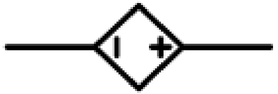
Figure 1
Symbols for: (a) dependent voltage source, (b) dependent current source.

Circuit Elements

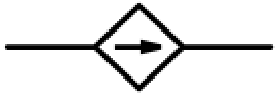
Independent voltage source



Independent current source



Dependent voltage source



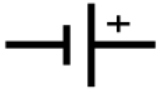
Dependent current source

Diode

Capacitor

Inductor

Resistor



DC voltage source

AC voltage source

Variable resistor

Basic Laws

Ohm's Law

- **Ohm's law** states that the voltage v across a resistor is directly proportional to the current i flowing through the resistor.

$$v \propto i$$
$$v = iR$$

- The resistance of any material with a uniform cross-sectional area A, depends on A and its length l, as shown in Fig. 2.

$$R = \rho \frac{\ell}{A}$$

- Where ρ is the resistivity of the material in ohm-meters.
- Conductance G is a measure of how well an element will conduct electricity, measured in mho (ohm spelled backwards).
- It's the reciprocal of R. $G = \frac{1}{R}$, unit ℧, SI unit is siemens (S).

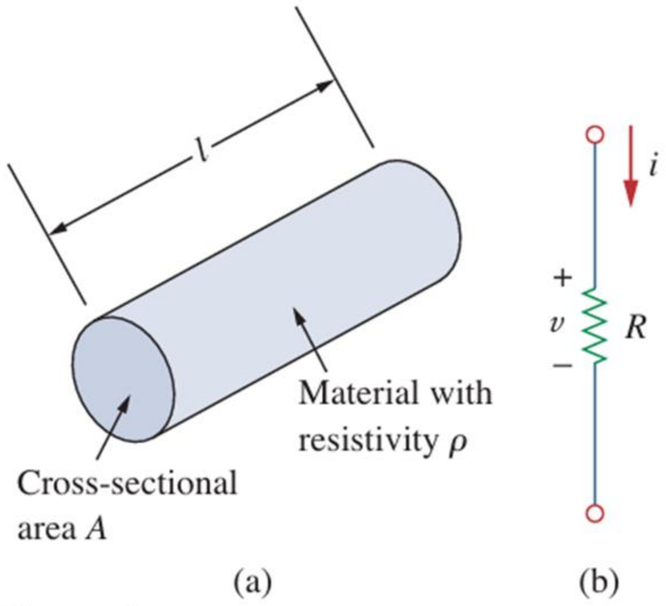
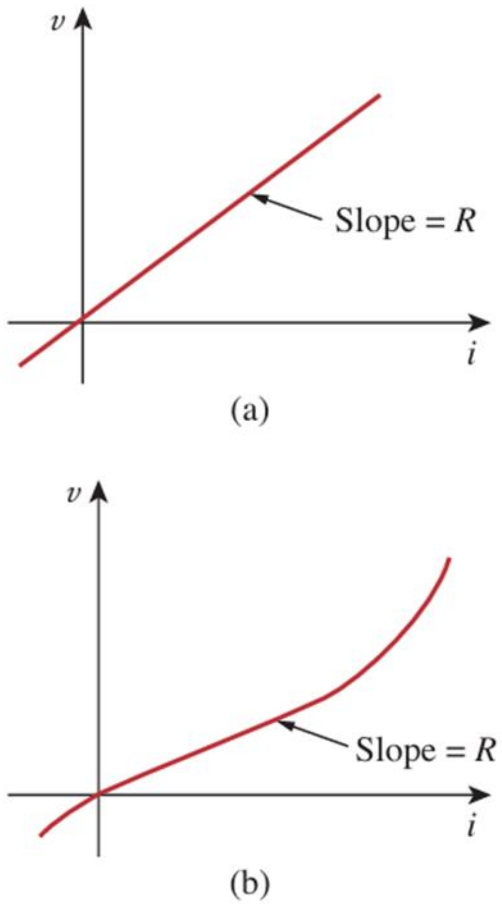


Figure 2

(a) Resistor, (b) Circuit symbol for resistance.

Ohm's Law

- It should be noted that not all resistors obey Ohm's law.
- A resistor that obeys Ohm's law is known as a linear resistor and has i-v characteristics as shown in Figure (a).
- A nonlinear resistor does not obey Ohm's law. Its resistance varies with current and its i-v characteristic is typically shown in Figure (b).



The i-v characteristic of: (a) a linear resistor, (b) a nonlinear resistor.

Branches, Nodes, and Loops

- A **branch** represents a single element.
- A **node** is the point of connection between two or more branches.
- A **loop** is any closed path in a circuit.
- Fig. 3 can be redrawn as Fig. 4 by combining the nodes connected by conducting wires.
- Two or more elements are in **series** if they exclusively share a single node and consequently carry the same current.
- Two or more elements are in **parallel** if they are connected to the same two nodes and consequently have the same voltage across them.

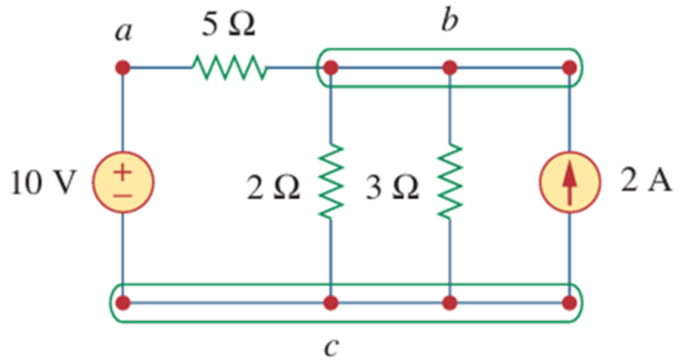


Figure 3
Nodes, branches, and loops.

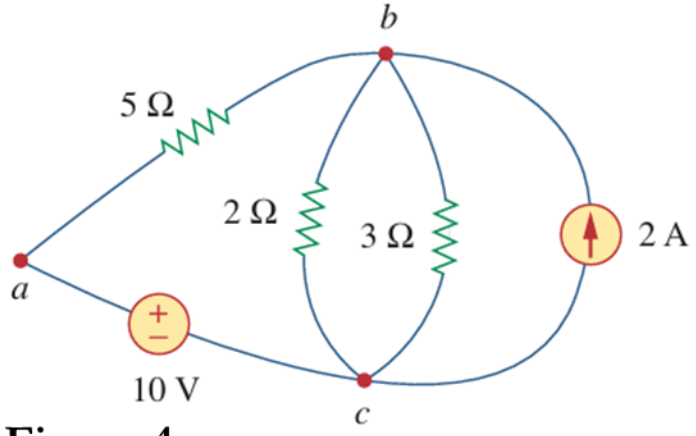


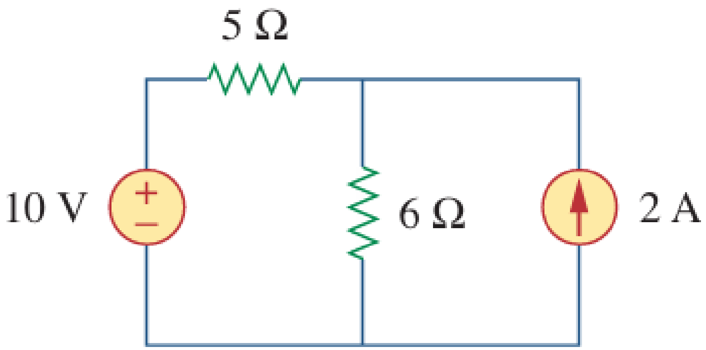
Figure 4
The three-node circuit of Fig. 2.10 is redrawn.

Branches, Nodes, and Loops

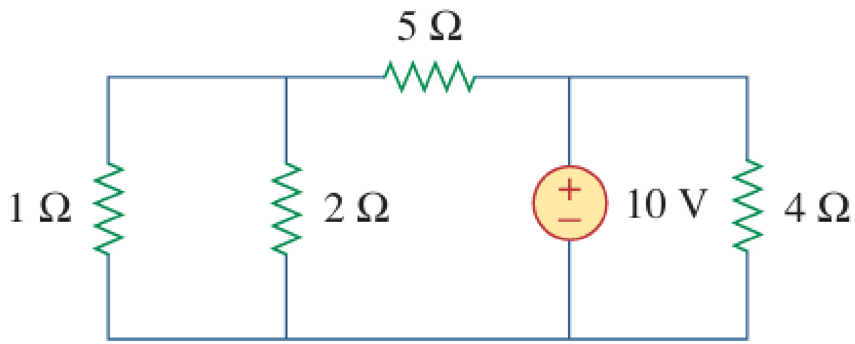
Problem

- Determine the number of branches and nodes in the circuits shown below. Identify which elements are in series and which are in parallel.

(a)



(b)



Kirchhoff's Laws

- Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.
- The sum of the currents entering a node is equal to the sum of the currents leaving the node.

$$i_1 + i_3 + i_4 = i_2 + i_5$$

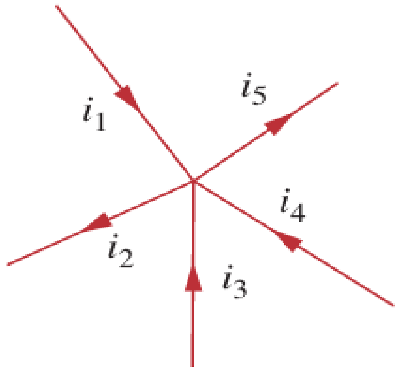


Figure 2.16
Currents at a node illustrating KCL.

- In Fig. 2.17, the total current entering the closed surface is equal to the total current leaving the surface.

- KCL can be applied to current sources in parallel such that

$$I_T = I_1 - I_2 + I_3$$

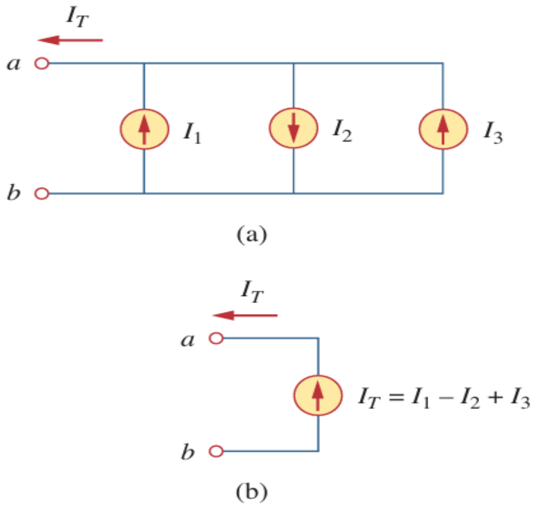


Figure 2.18
Current sources in parallel: (a) original circuit, (b) equivalent circuit.

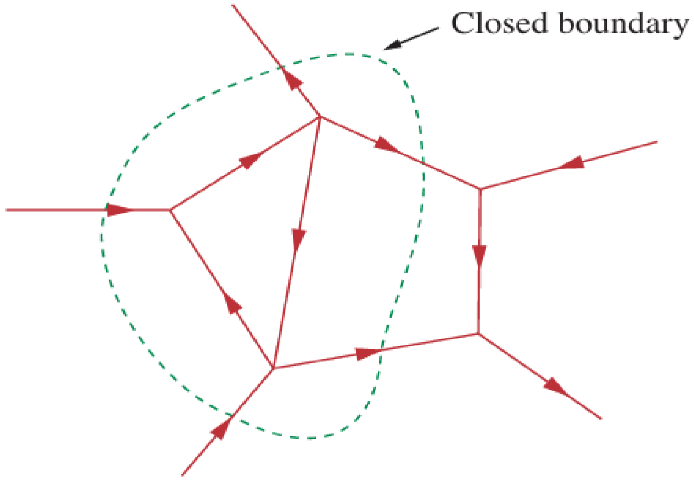


Figure 2.17
Applying KCL to a closed boundary.

Kirchhoff's Laws

- Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.
- Consider the circuit in Fig. 2.19. The sign on each voltage is the polarity of the terminal encountered first as we travel around the loop.

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$
$$v_1 + v_4 = v_2 + v_3 + v_5$$

- In other words,

$$Sum\ of\ voltage\ drops\ =\ Sum\ of\ voltage\ rises$$

- In Fig. 2.20, KVL can be applied to voltage sources in series such

$$V_{ab} = V_1 + V_2 - V_3$$

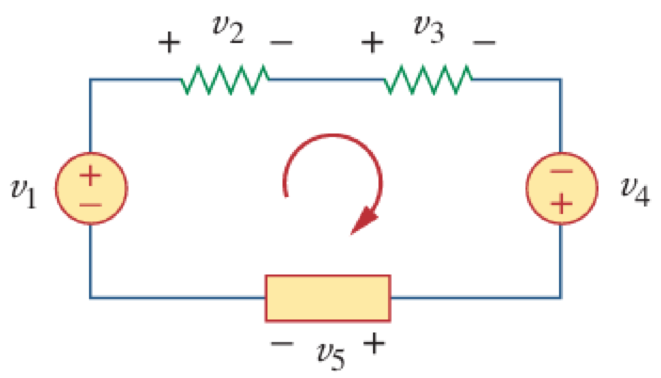


Figure 2.19
A single-loop circuit illustrating KVL.

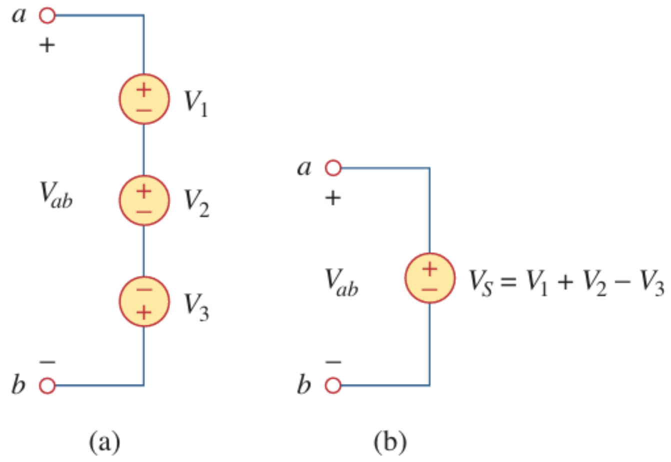
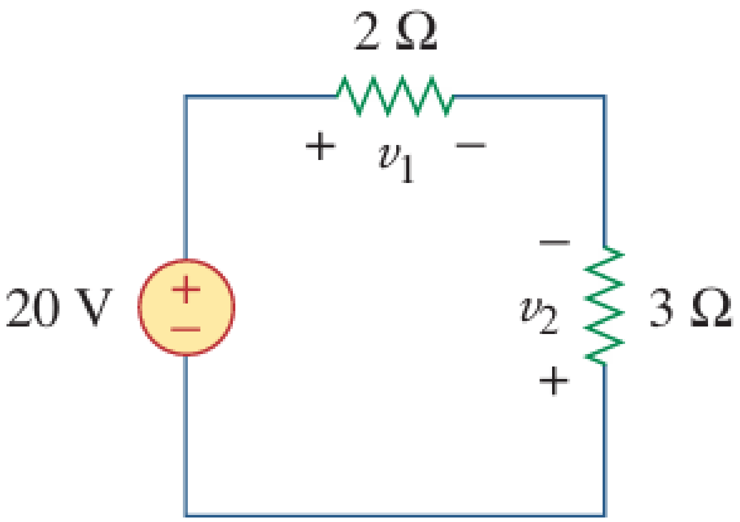


Figure 2.20
Voltage sources in series: (a) original circuit, (b) equivalent circuit.

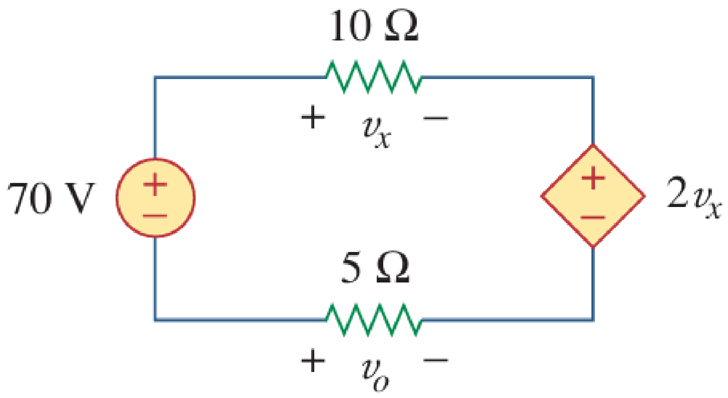
Kirchhoff's Laws

Problems

For the circuits below, find voltage v_1 and v_2.

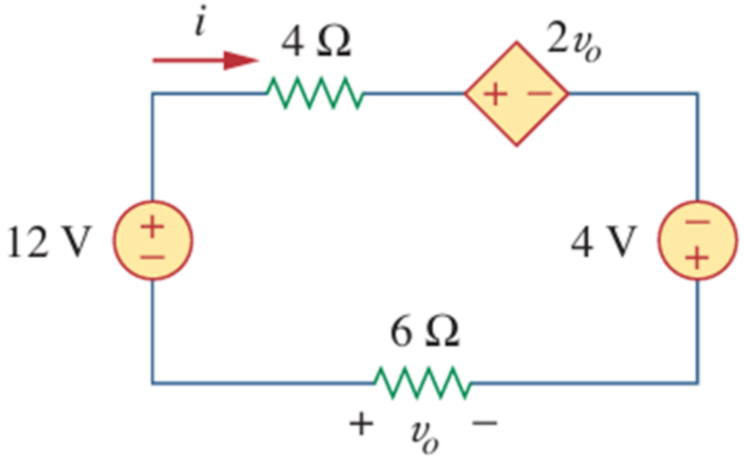


(a)



For the circuits below, find voltage v_x and v_o.

Determine v_o and i in the circuit below



(b)

Series Resistors and Voltage Division

- For two resistors in series as shown in Fig. 2.29

$$R_{eq} = R_1 + R_2$$

- The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.
- Per the voltage divider rule, the source voltage v is divided among the resistors in direct proportion to their resistances.

$$v_1 = \frac{R_1}{R_1 + R_2} v$$

$$v_2 = \frac{R_2}{R_1 + R_2} v$$

- In general, if a voltage divider has N resistors (R_1, R_2, R_3 . . . , R_N) in series with the source voltage v, the nth resistor (R_n) will have a voltage drop of

$$v_n = \frac{R_n}{R_1 + R_2 + R_3 \ldots + R_N} v$$

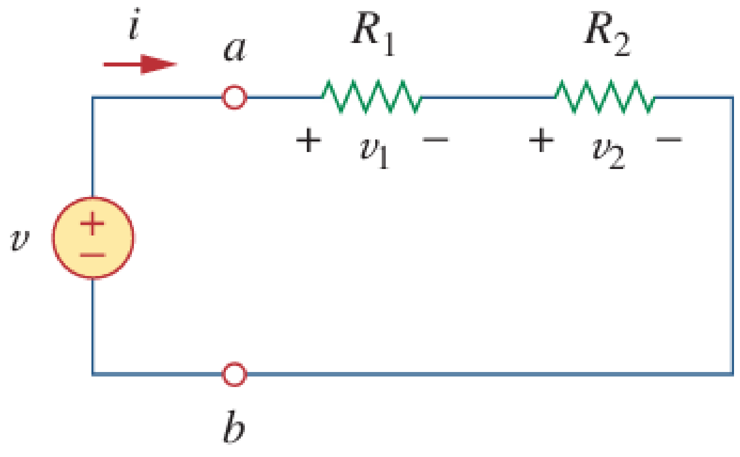


Figure 2.29
A single-loop circuit with two resistors in series.

Parallel Resistors and Current Division

- For two resistors in parallel as shown in Fig. 2.31.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

- In the general case of a circuit with N resistors in parallel. The equivalent resistance is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

- Per the current divider rule,

$$i_1 = \frac{R_2}{R_1 + R_2} i$$

$$i_2 = \frac{R_1}{R_1 + R_2} i$$

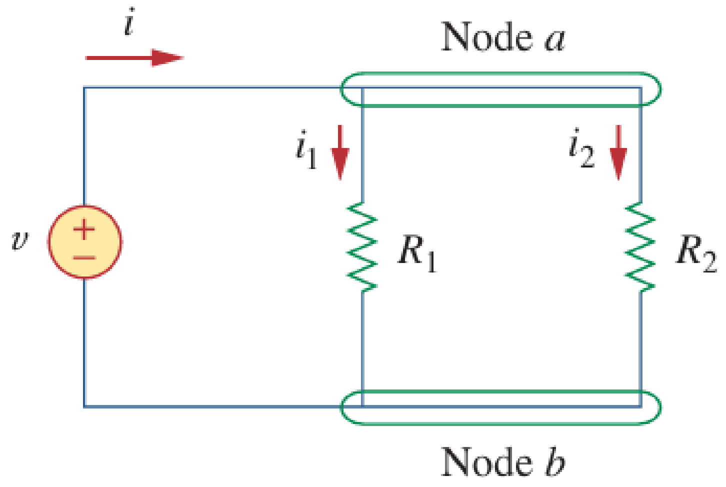
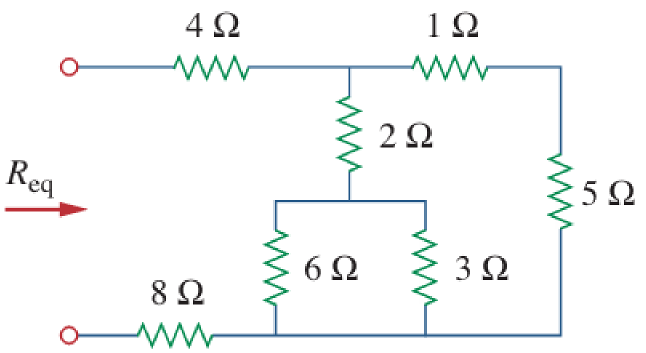
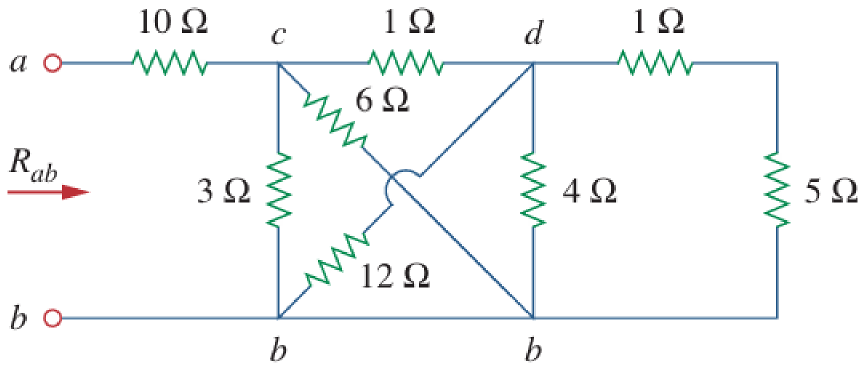


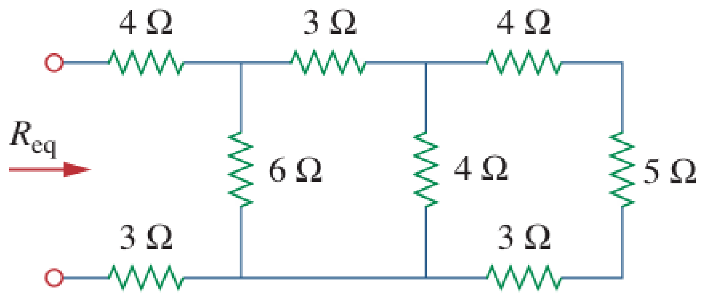
Figure 2.31
Two resistors in parallel.

Problems

- Find the equivalent resistances of the circuits shown below







Methods of Analysis

- **<u>Nodal Analysis with only current sources</u>**

Steps to Determine Node Voltages:

1. Select a node as the reference node. Assign voltages v_1, $v_2, \ldots, v_{n-1}$ to the remaining $n-1$ nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the $n-1$ nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

- Current flows from a higher potential to a lower potential in a resistor.

$$i = \frac{v_{\text{higher}} - v_{\text{lower}}}{R}$$

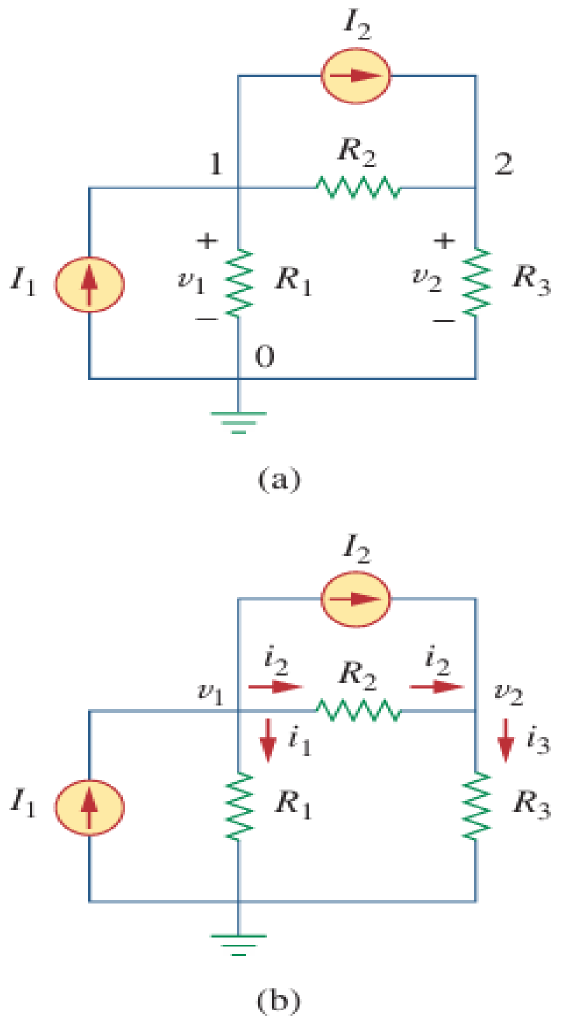


Figure 3.2
Typical circuit for nodal analysis.

At node 1,

$$I_1 = I_2 + i_1 + I_2$$

$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$$

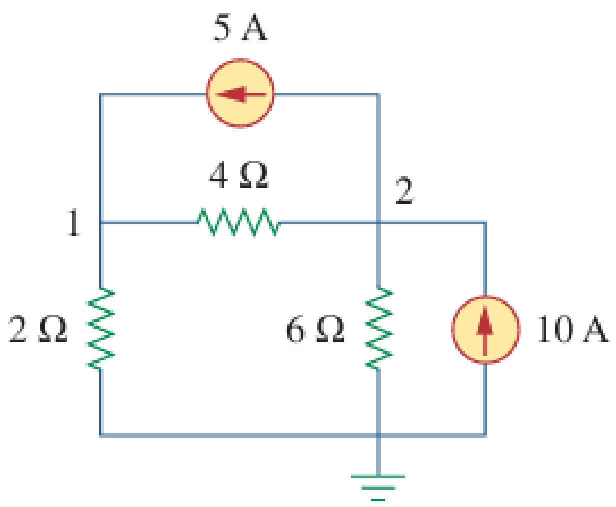
At node 2,

$$I_2 = i_3 + i_3$$

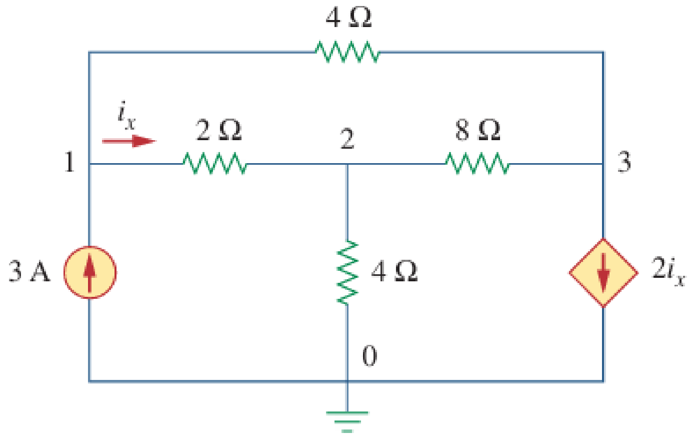
$$I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3}$$

- **Problems**

Calculate the node voltages in the circuits (a) and (b)



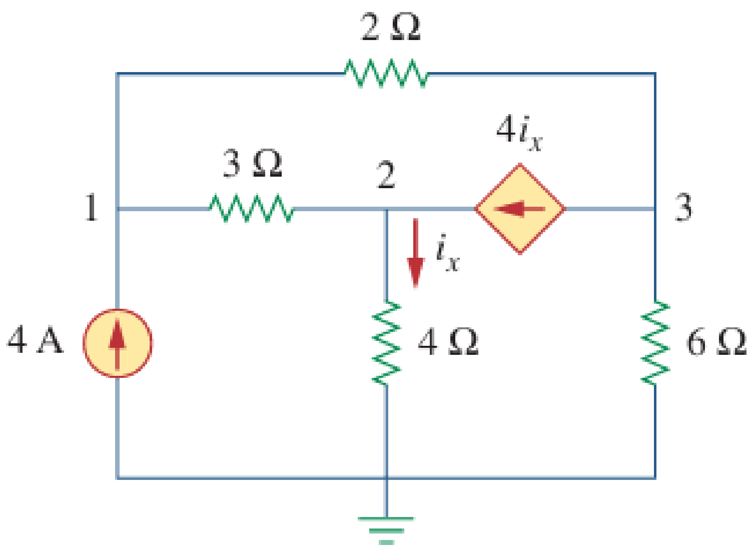
(a)



(b)

- **Assignment 1** – (Deadline: Thursday Jan 16, 2025, 23:59)

Determine the voltage at the three non-reference nodes



Nodal Analysis with Voltage Source

- **<u>Nodal Analysis with voltage sources</u>**
- CASE1: If a voltage source is connected between the reference node and a nonreference node, simply set the voltage at the non reference node equal to the voltage of the voltage source.
- In Fig. 3.7, for example, $v_1 = 10\,V$.
- CASE 2: If the voltage source (dependent or independent) is connected between two non-reference nodes, the two non-reference nodes form a **supernode**, apply both KCL and KVL to determine the node voltages.

> **A supernode is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.**

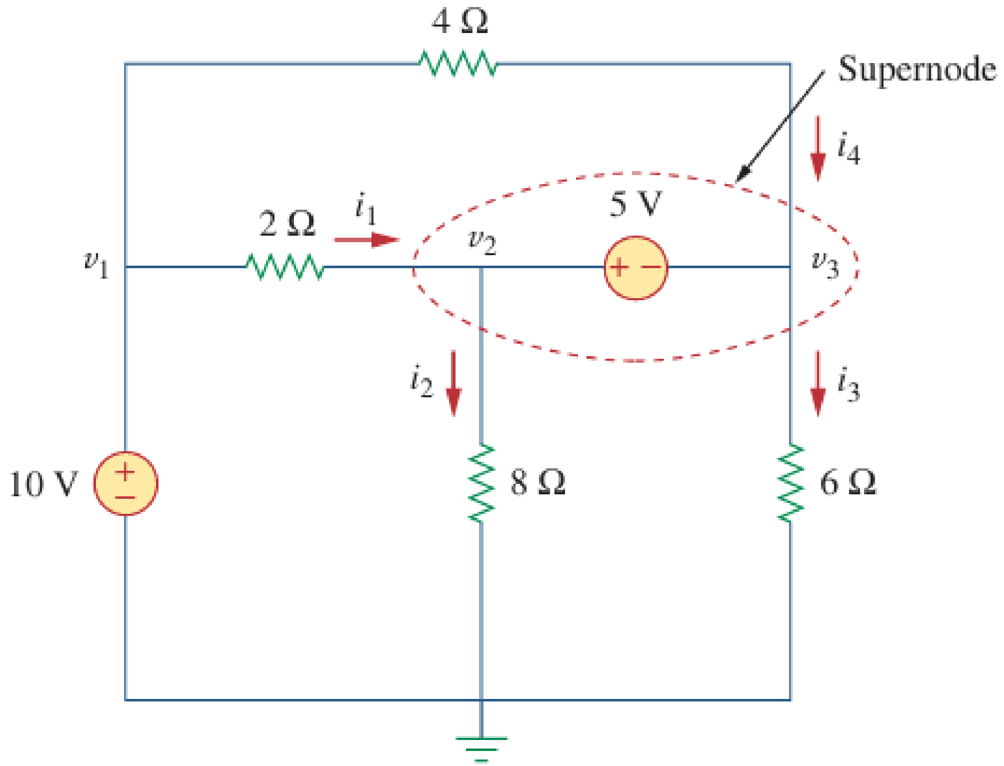


Figure 3.7
A circuit with a supernode.

Nodal Analysis with Voltage Source

- At the supernode,

$$i_1 + i_4 = i_2 + i_3$$

$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2 - 0}{8} + \frac{v_3 - 0}{6}$$

- Applying KVL to the supernode,

$$-v_2 + 5 + v_3 = 0 \quad \Rightarrow \quad v_2 - v_3 = 5$$

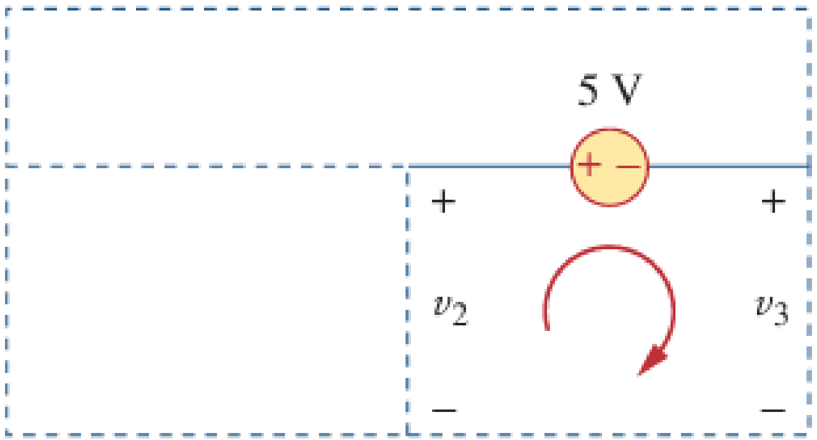
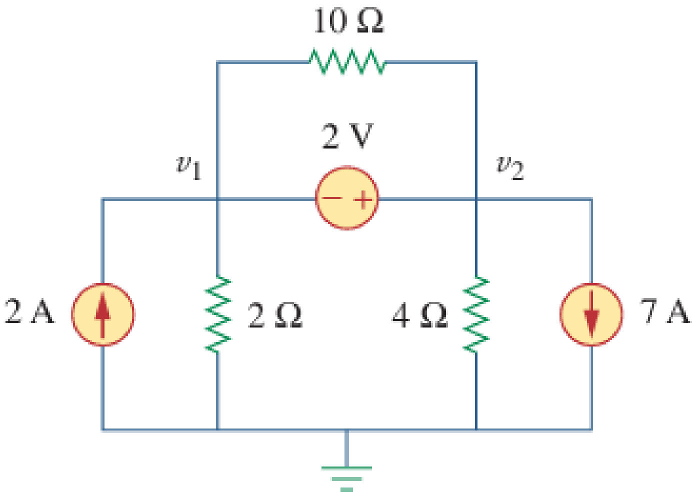
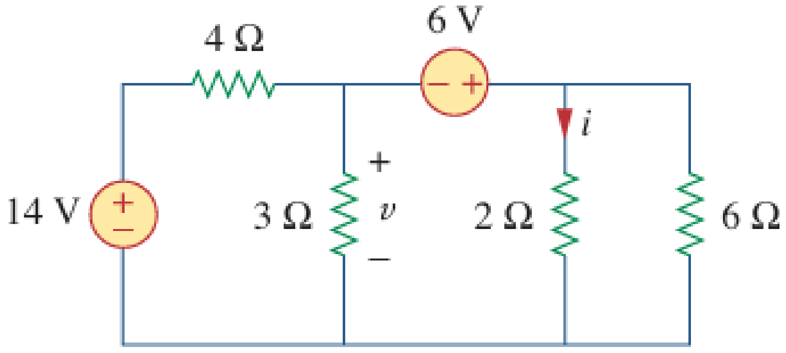


Figure 3.8
Applying KVL to a supernode.

Problems

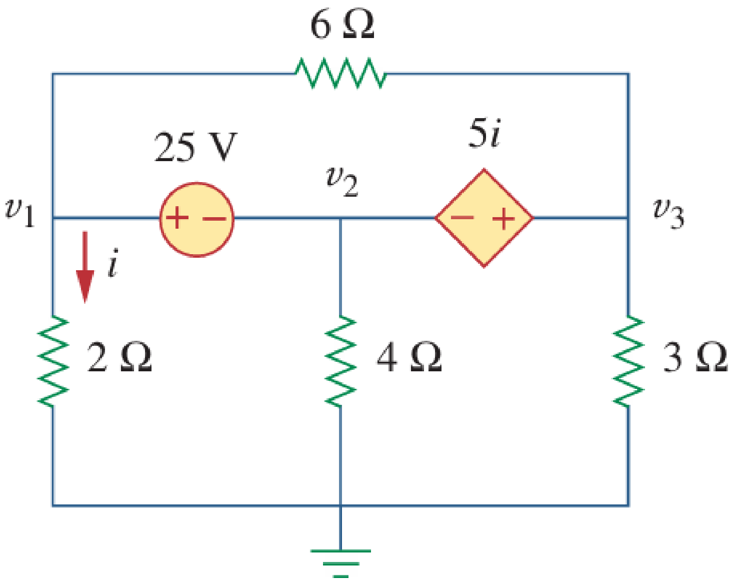
Find the node voltages in the circuit below





Find v and i.

Assignment 2 – (Deadline: Sunday, Jan 19, 2025, 20:00)



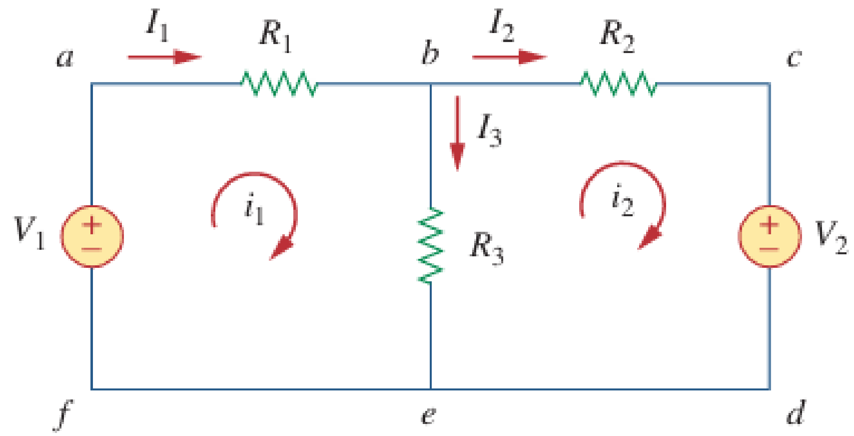
Using nodal analysis, find v_1, v_2, and v_3 in the circuit above

Mesh Analysis

- A mesh is a loop which does not contain any other loops within it.

Steps to Determine Mesh Currents:

1. Assign mesh currents $i_1, i_2, \ldots, i_n$ to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.



Applying KVL to mesh 1

$$-V_1 + R_1 i_1 + R_3(i_1 - i_2) = 0$$

$$(R_1 + R_3)i_1 - R_3 i_2 = V_1$$

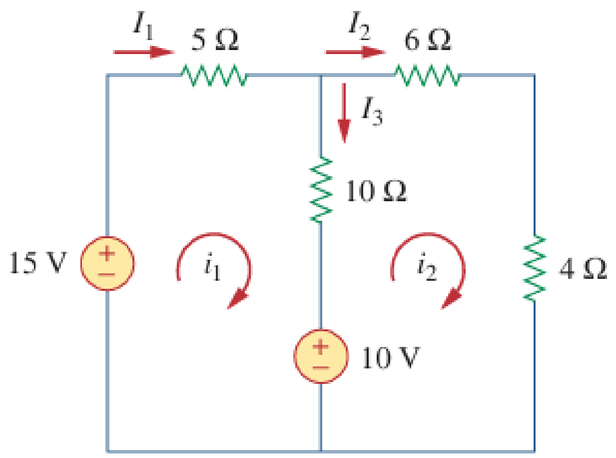
Applying KVL to mesh 2

$$R_2 i_2 + V_2 + R_3(i_2 - i_1) = 0$$

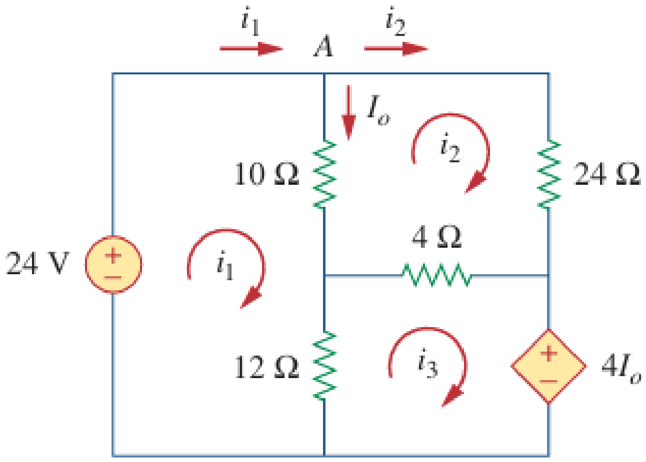
$$-R_3 i_1 + (R_2 + R_3)i_2 = -V_2$$

Mesh Analysis

Problems



Find the branch currents using mesh analysis.



Use mesh analysis to find the mesh currents in the circuit

Mesh Analysis with Current Sources

- CASE 1: When a current source exists only in one mesh: Consider the circuit in Fig. 3.22 for example, set $i_2 = -5\,A$ and write a mesh equation for the other mesh in the usual way.

$$-10 + 4i_1 + 6(i_1 - i_2) = 0 \quad \Rightarrow \quad i_1 = -2\text{ A}$$

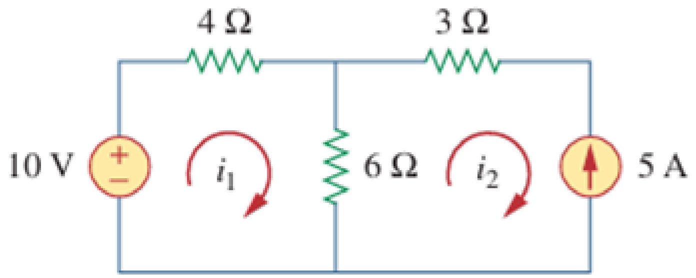


Figure 3.22
A circuit with a current source.

- CASE 2: When a current source exists between two meshes: Consider the circuit in Fig. 3.23(a), for example. Create a supermesh by excluding the current source and any elements connected in series with it, as shown in Fig. 3.23(b).

A supermesh results when two meshes share a current source (dependent or independent).

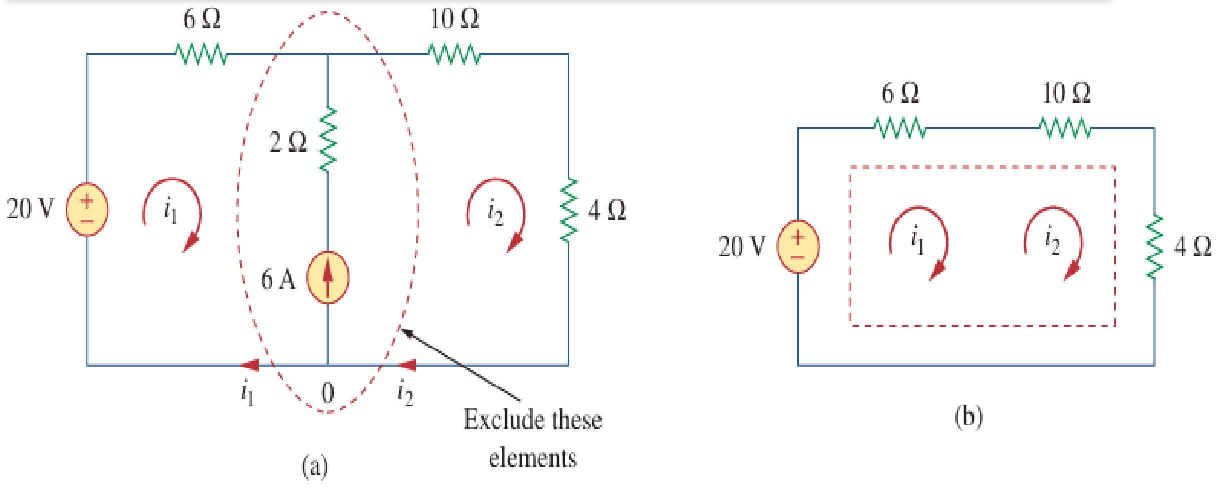


Figure 3.23
(a) Two meshes having a current source in common, (b) a supermesh, created by excluding the current source.

$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

$$6i_1 + 14i_2 = 20 \quad (1)$$

apply KCL to the node where the two meshes meet.

$$i_2 = i_1 + 6 \quad \textbf{OR} \quad \boldsymbol{i_2 - i_1 = 6} \quad (2)$$

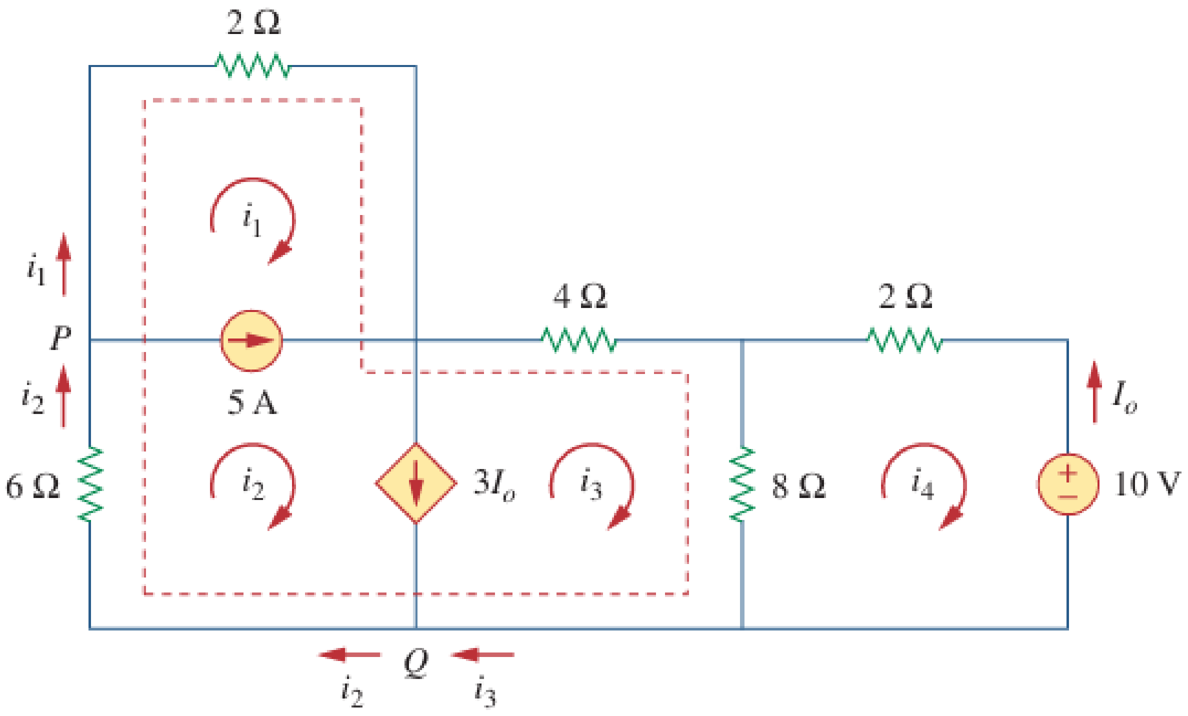
Solving equation (1) and (2) simultaneously,

$$\boldsymbol{i_1 = -3.2\,A} \qquad \boldsymbol{i_2 = 2.8\,A}$$

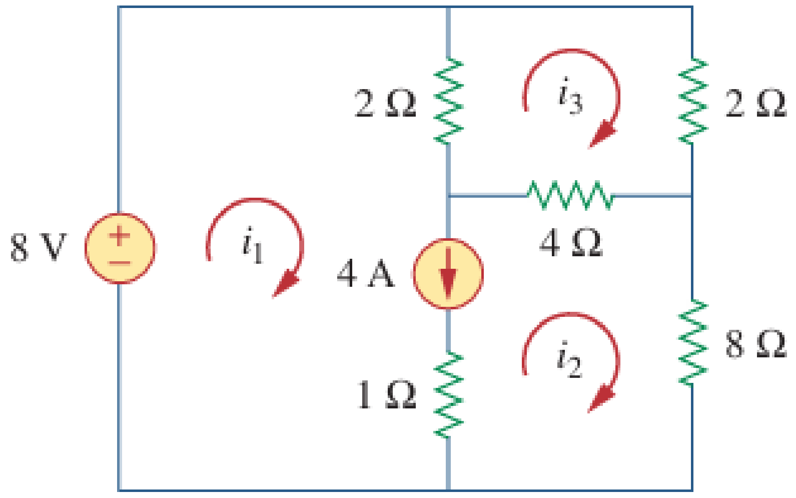
Mesh Analysis with Current Sources

Problems

Use mesh analysis to determine i_1, i_2, i_3 and i_4 in the circuit below.



Assignment 3 – (Deadline: Tuesday, Jan 28, 2025, 23:59)



Use mesh analysis to determine i_1, i_2 and i_3 in the circuit above.

Nodal Analysis vs. Mesh Analysis

Difference between modal analysis and mesh analysis	
Nodal analysis	Mesh analysis
Used to find node voltages	Used to find branch and mesh currents
Can solve non-planar circuits	Cannot solve non-planar circuit
Primarily depends on KCL	Primarily depends on KVL
A supernode is when a voltage source is found in between two non-reference nodes	A supermesh is when a current source is shared by two meshes
Supernode is secondarily solved using KVL	Supermesh is secondarily solved using KCL

Circuit Theorems

Linearity Property of Circuits

- The linearity property is a combination of both the **homogeneity (scaling) property** and the **additivity property**.
- The homogeneity property requires that if the input is multiplied by a constant, then the output is multiplied by the same constant.
- The additivity property requires that the response to a sum of inputs is the sum of the responses to each input applied separately.

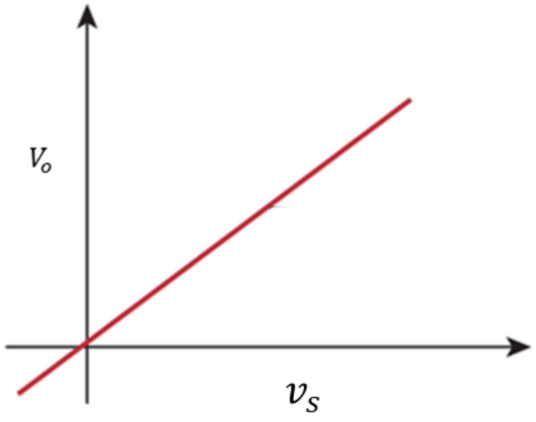


Fig: *An illustrative diagram showing how the output voltage (V_o) linear relates to the source voltage (V_s) in a linear circuit*

> **A linear circuit is one whose output is linearly related (or directly proportional) to its input.**

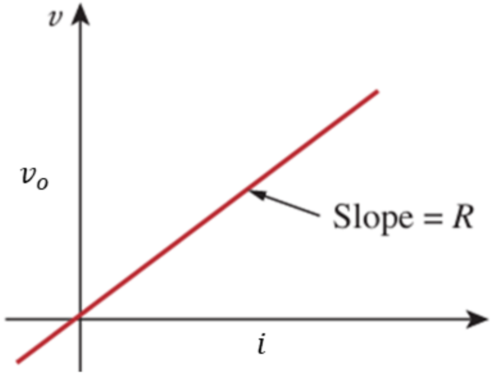


Fig: *An illustrative diagram showing how current linearly relates to voltage*

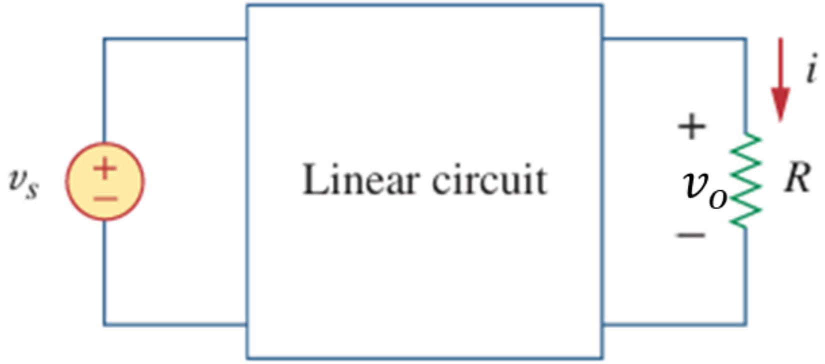


Fig: *A linear circuit with input v_s and output $v_o - i$.*

Problems

Assume $I_o = 1$ A and use linearity to find the actual value of I_o in the circuit of Fig. 4.4.

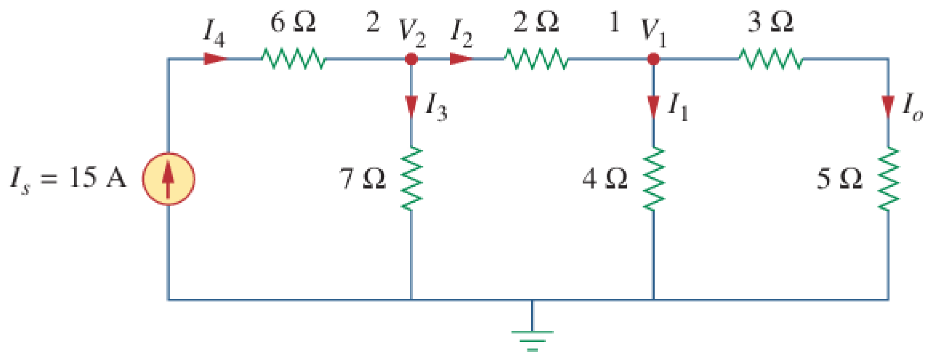


Figure 4.4
For Example 4.2.

Assume that $V_o = 1$ V and use linearity to calculate the actual value of V_o in the circuit of Fig. 4.5.

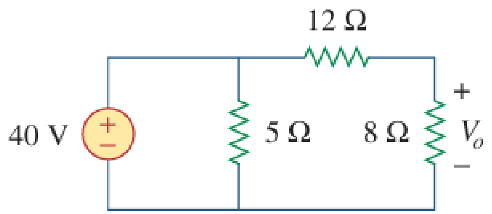
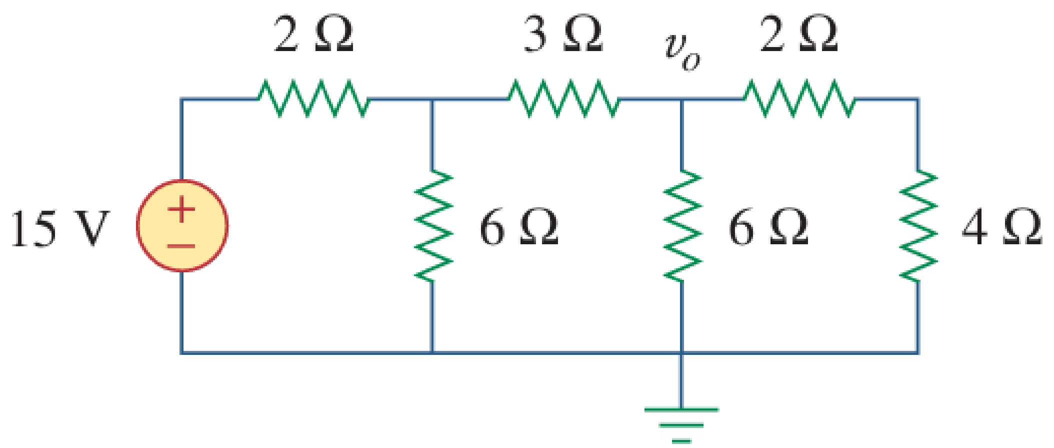


Figure 4.5
For Practice Prob. 4.2.

Assignment 4

In the circuit below, V_o is the 5V at a certain value of the source voltage, determine the actual value of V_o.

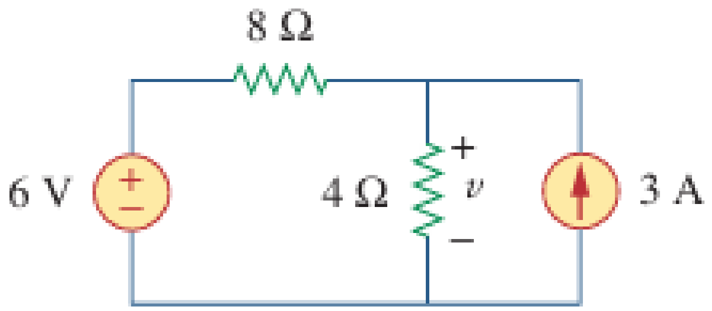


Superposition

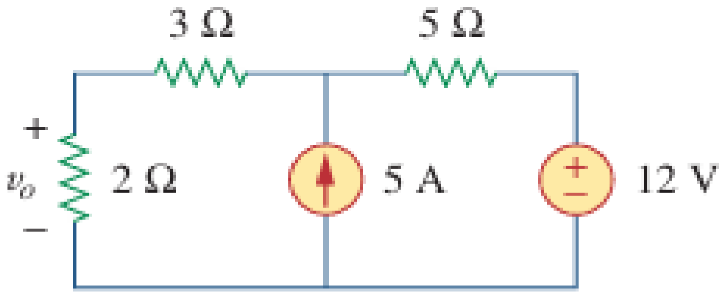
- The **superposition principle** states that the voltage across (or current through) an element in a ***linear circuit*** is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

Steps to Apply Superposition Principle:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using the techniques covered in Chapters 2 and 3.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.



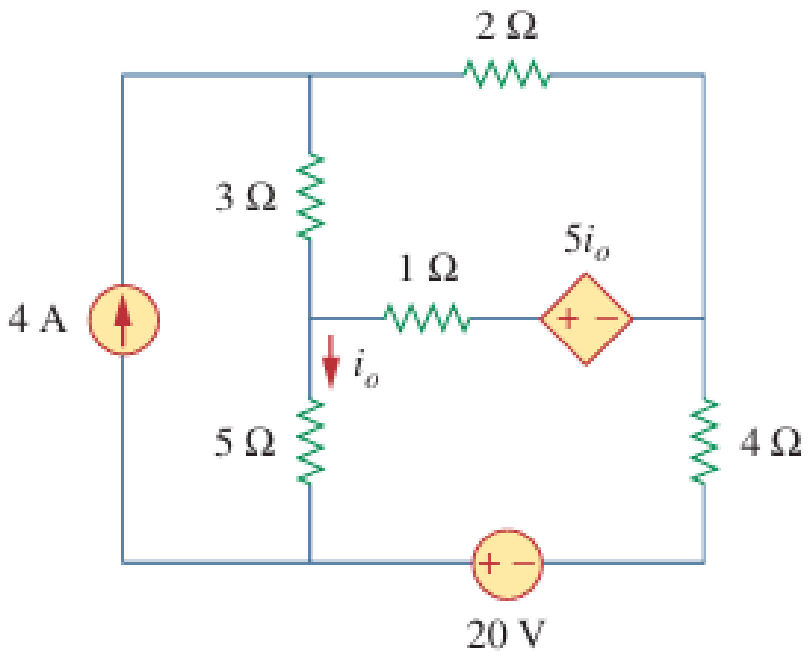
Use the superposition theorem to find v in the circuit above



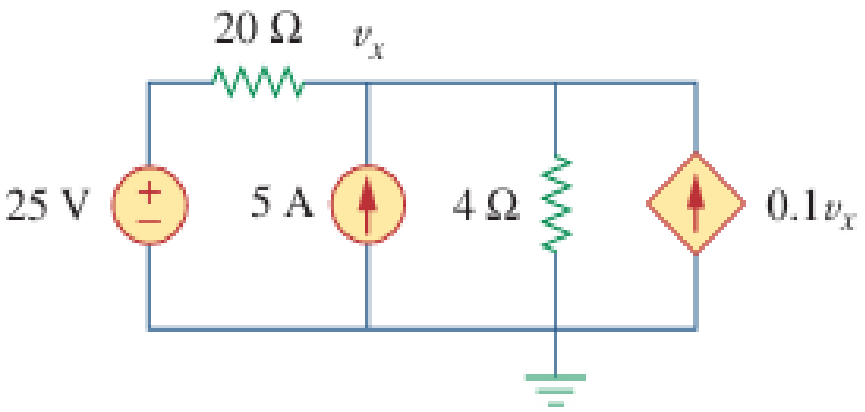
Using the superposition theorem, find v_o in the circuit

Problems

(1) Find i_o in the circuit below



(2) Find v_x using superposition



Assignment 5

Find i using the superposition theorem