PABLO

FUNDAMENTALS OF ELECTRICAL ENGINEERING.

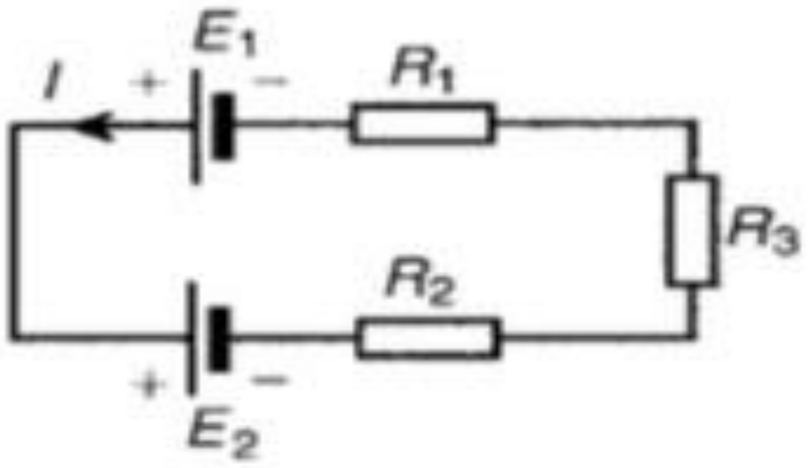
LECTURE 2 NOTES

N.T DUAH

- Kirchoff's Laws

o Kirchoff's Voltage Law

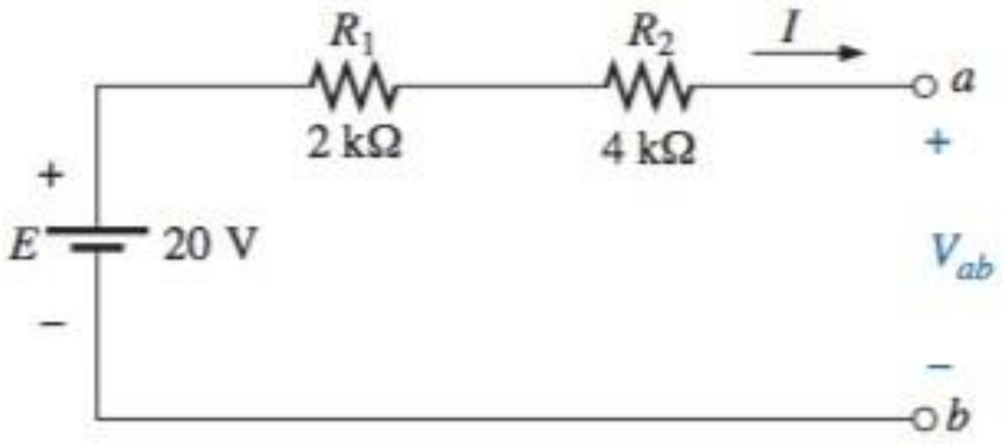
Kirchhoff's voltage law (KVL) states that the algebraic sum of the potential rises and drops around a closed loop (or path) is zero. i.e. the applied voltage of a series circuit equals the sum of the voltage drops across the series elements.

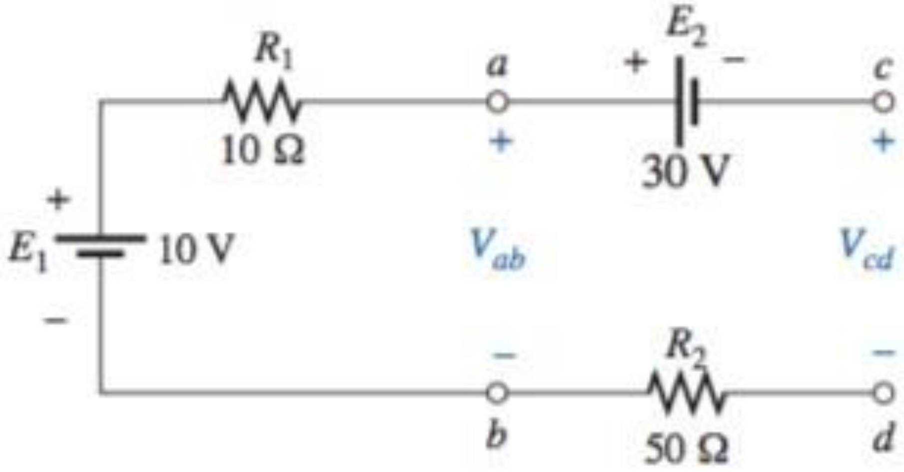


$$E_1 - E_2 = IR_1 + IR_2 + IR_3$$

- E.g

  Determine the voltage V_{ab} for the network. Determine the voltages V_{ab} and V_{cd} for the network.





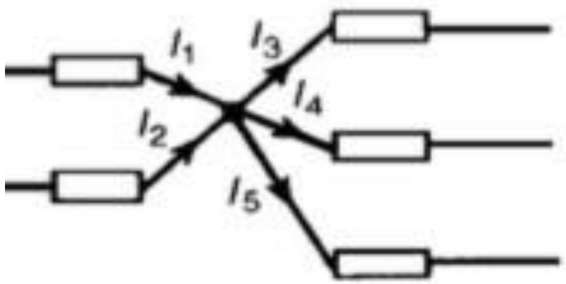
- Kirchoff's Current Law

Kirchhoff's current law (KCL) states that the algebraic sum of the currents entering and leaving an area, system, or junction is zero. i.e. the sum of the currents entering an area, system, or junction must equal the sum of the currents leaving the area, system, or junction.

$$I_1 + I_2 = I_3 + I_4 + I_5$$

or

$$I_1 + I_2 - I_3 - I_4 - I_5 = 0$$

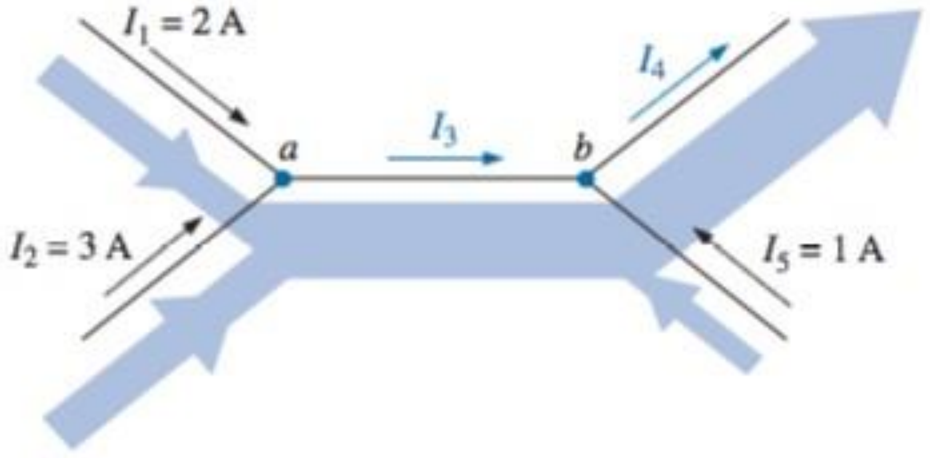


$$\sum_N I_n = 0$$

$n=1$

- E.g

Determine the currents I_3 and I_4



- Exercise

Find the currents and voltages in the circuit shown in Fig. 2.28.

Answer: $v_1 = 3$ V, $v_2 = 2$ V, $v_3 = 5$ V, $i_1 = 1.5$ A, $i_2 = 0.25$ A, $i_3 = 1.25$ A.

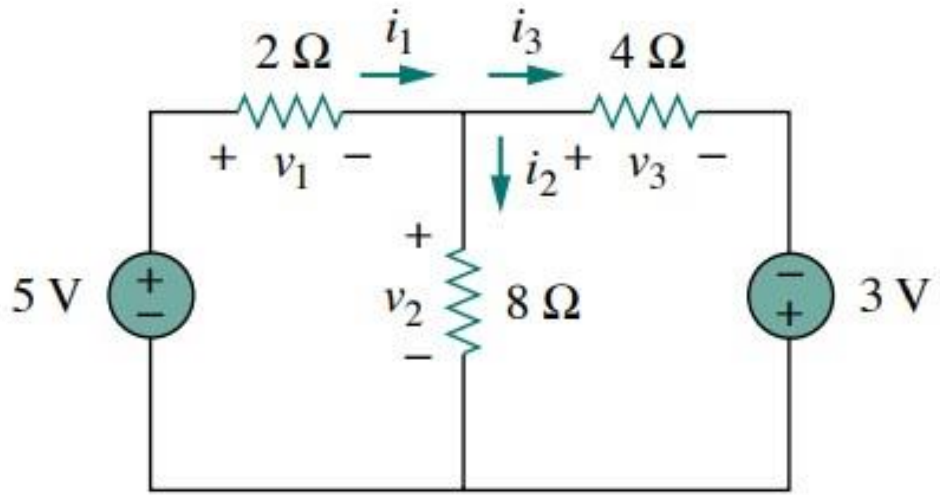


Figure 2.28 For Practice Prob. 2.8.

- Exercise

Find the currents and voltages in the circuit shown in Fig. 2.27(a).

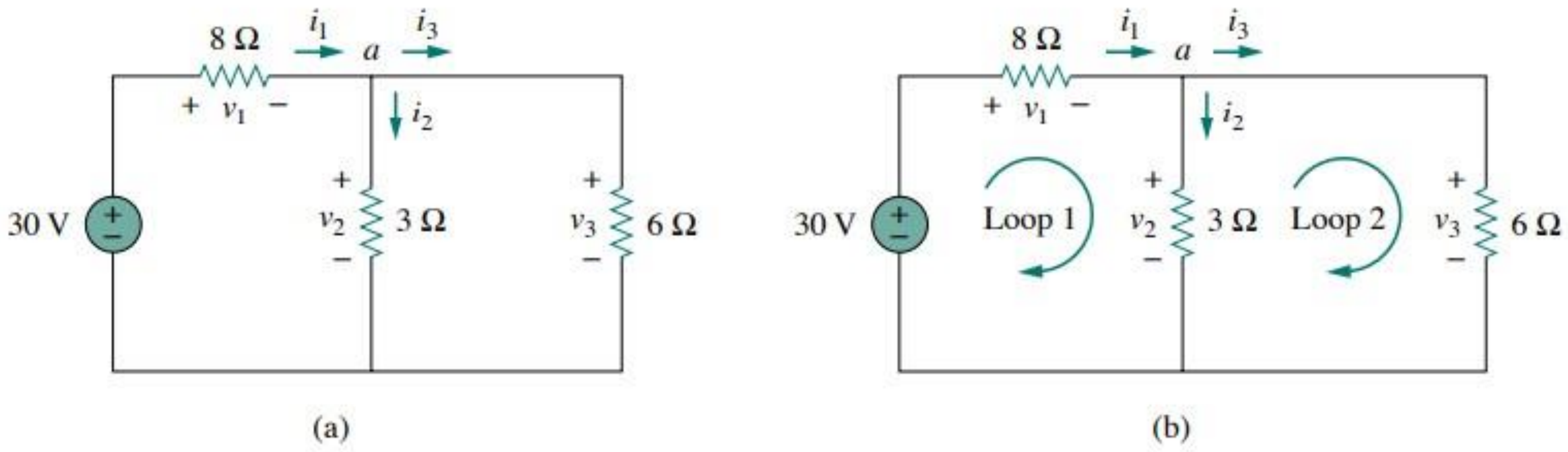


Figure 2.27 For Example 2.8.

- Series Circuits

Two elements are in series if

1. They have only one terminal in common (i.e., one lead of one is connected to only one lead of the other).
2. 2. The common point between the two elements is not connected to another current-carrying element.

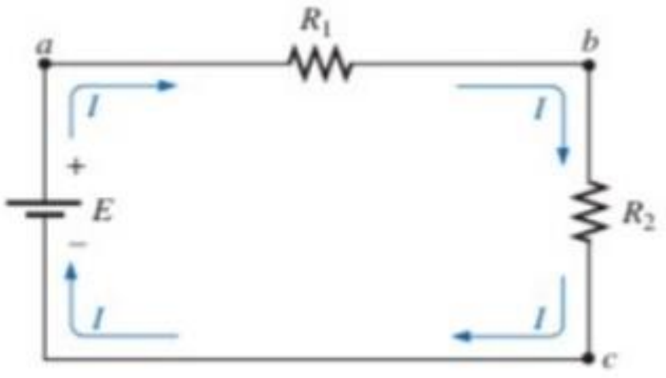


Figure 2.7: Series circuit

The circuit of Figure 2.7 has three elements joined at three terminal points (a, b, and c) to provide a closed path for the current I.

- Resistors In Series

In Fig. 2.7, the resistors R1 and R2 are in series because they have only point b in common. The other ends of the resistors are connected elsewhere in the circuit. For the same reason, the battery E and resistor R1 are in series (terminal a in common), and the resistor R2 and the battery E are in series (terminal c in common).

Since all the elements are in series, the network is called a series circuit. For series circuit, the current is the same through series elements.

The two resistors are in series, since the R1 and R2 are connected together without any branch between.

✦Applying ohm's law $v_1 = iR_1, v_2 = iR_2 ....(1)$

$$v = iR_{eq} \ldots\ldots. (3) \quad v = v_1 + v_2$$

$$= i(R_1 + R_2) \ldots\ldots.. (2)$$

Where $R\ R\ R_{eq} = \ +_1 \qquad _2 \ldots\ldots.(4)$

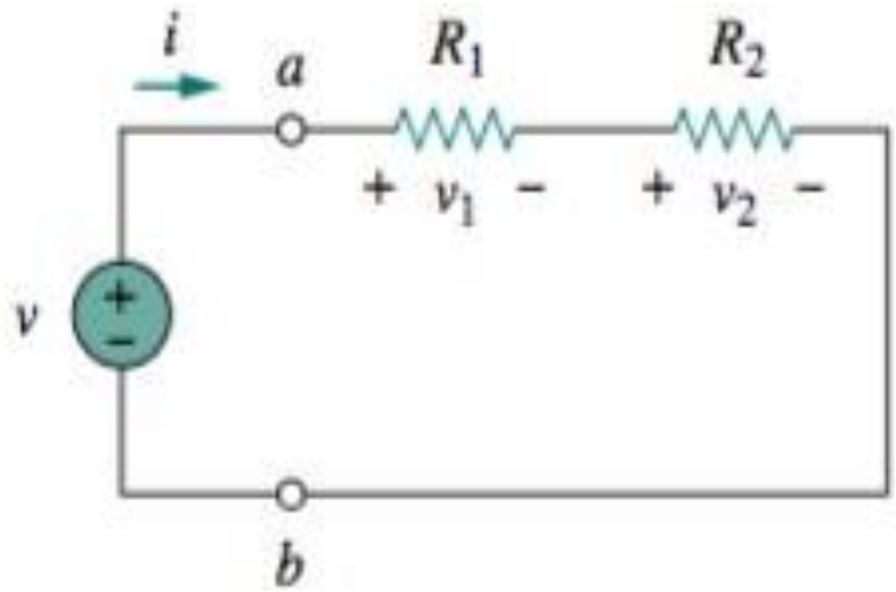
The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.

For N resistors in series then,

$$R_{eq} = R_1 + R_2 + \ldots\ldots + R_N = \sum_{n=1}^{N} R_n$$

- Voltage Divider Rule

In a series circuit, the voltage across the resistive elements will divide as the magnitude of the resistance levels. Voltage divider rule permits the voltage levels across resistors in series without first finding the current. $v_1 = iR_1, v_2 = iR_2 \ldots\ldots\ldots(1)$



$$v = v_1 + v_2 = i(R_1 + R_2) \ldots\ldots\ldots (2)$$

Substituting (2) into (1) $v_1 = \dfrac{R_1}{R_1 + R_2} v,$

$$v_2 = \frac{R_2}{R_1 + R_2} v$$

- In general, if a voltage divider has N resistors (R_1, R_2, . . . , R_N) in series with the source voltage *v*, the *nth* resistor (R_n) will have a voltage drop of

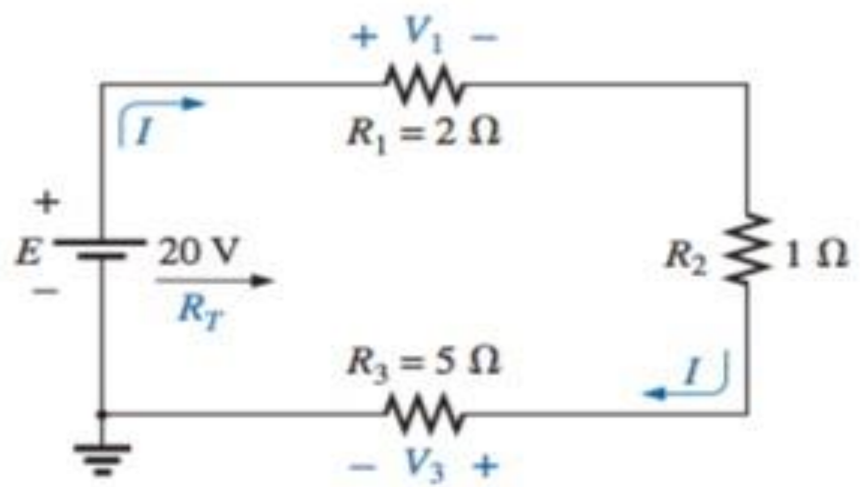
$$v_n = \frac{R_n v}{R_1 + R_2 + \dots + R_N}$$

- E.g

Find the total resistance for the series circuit a. Calculate the source current *I*.

b. Determine the voltages V_1, V_2, and V_3.

c. Calculate the power dissipated by R_1, R_2, and R_3.



d. Determine the power delivered by the source, and compare it to the sum of the power levels of part (d)

- Parallel Circuits

Two elements, branches, or networks are in parallel if they have two points in common.

In Fig. 2.10, for example, elements 1 and 2 have terminals a and b in common; they are therefore in parallel.

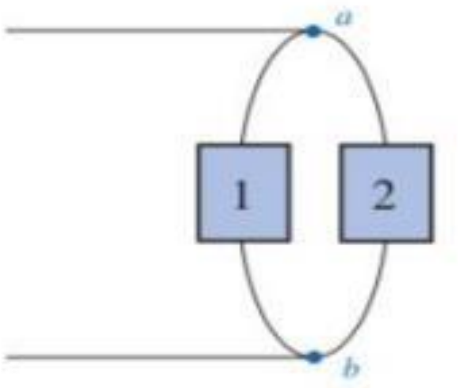


Figure 2.10:

❍Resistors In Parallel

Consider the circuit in Fig. 2.11, where two resistors are connected in parallel and therefore have the same voltage across them.

From Ohm's law, $v = i_1 R_1 = i_2 R_2$.......(5)

Rearranging $i_1 = \dfrac{v}{R_1}, i_2 = \dfrac{v}{R_2}$ ......(6)

Applying KCL at node *a* $i = i_1 + i_2$.....(7)

Substituting (6) into (7)

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}} \quad .....(8)$$

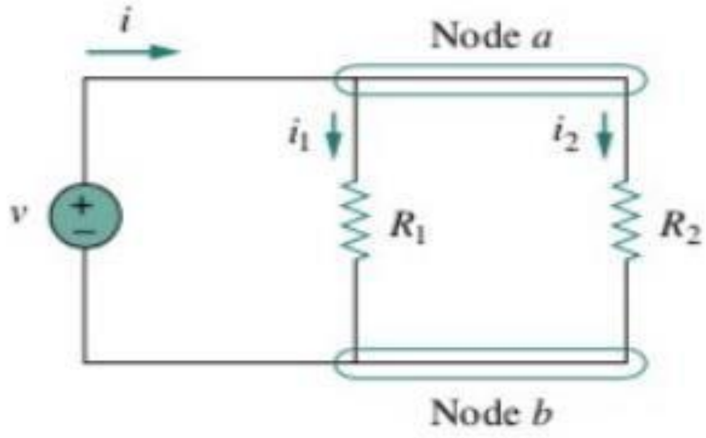


Figure 2.11:

Therefore,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \text{......(9)}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \text{........(10)}$$

The equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum. For N number of resistors in parallel,

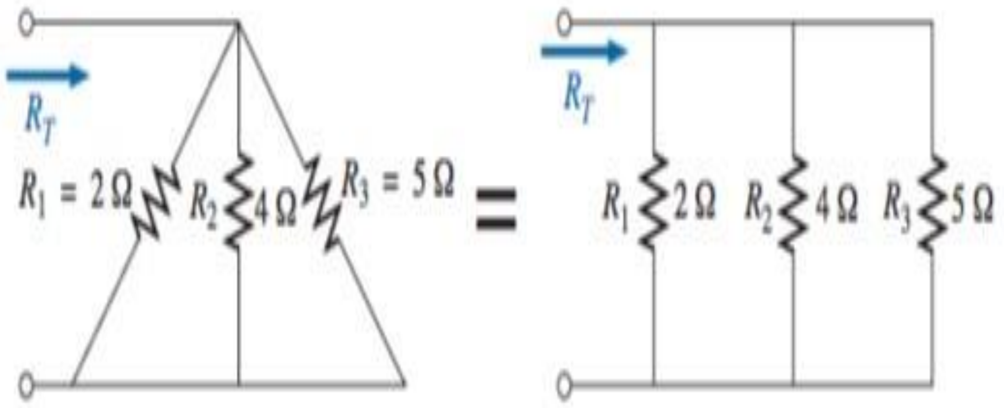
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}$$

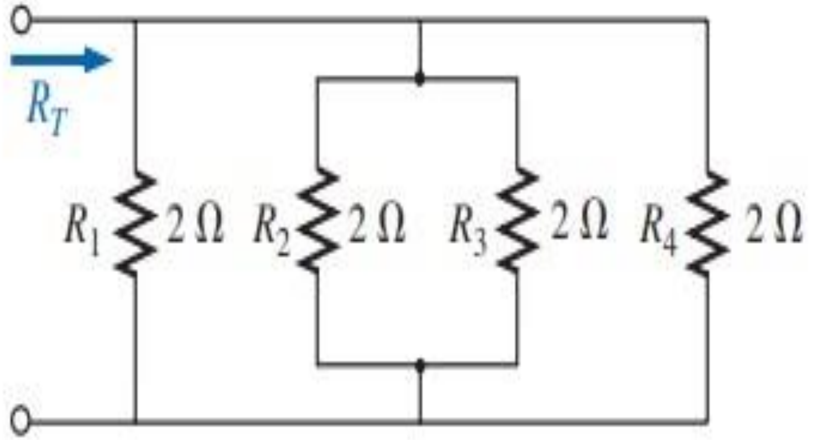
- E.g

Determine the total resistance for the networks below.

a.

b.

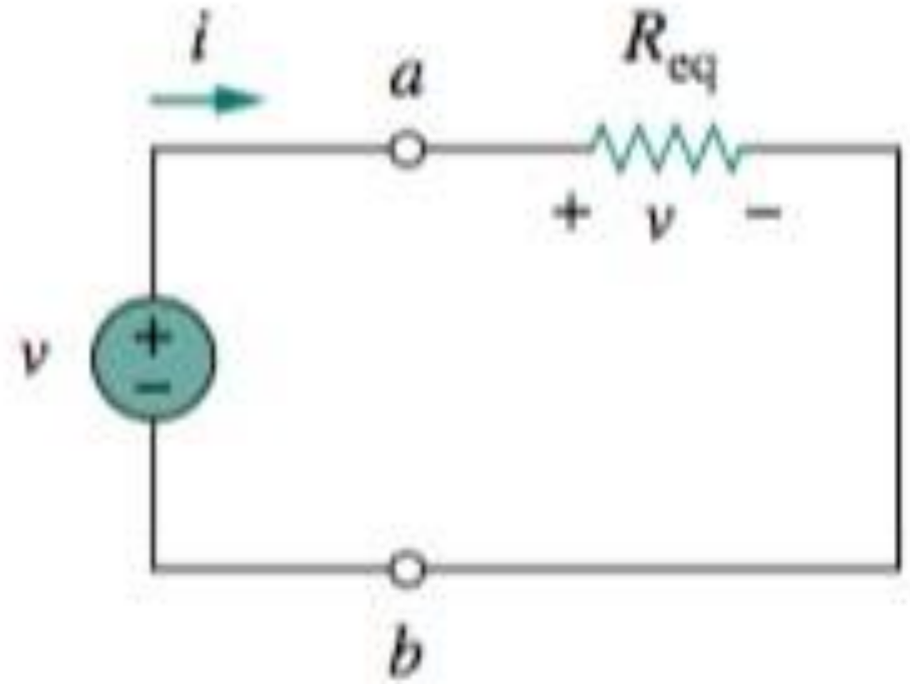




- Current Divider Rule

The current divider rule (CDR) will determine how the current entering a set of parallel branches will split between the elements. For two parallel elements of equal value, the current will divide equally. For parallel elements with different values, the smaller the resistance, the greater the share of input current. For parallel elements of different values, the current will split with a ratio equal to the inverse of their resistor values.

$$v = iR_{eq} = \frac{iR_1R_2}{R_1+R_2} \text{......(11)}$$



Combining (6) and (11)

$$i_1 = \frac{R_2 i}{R_1+R_2}, \quad i_2 = \frac{R_1 i}{R_1+R_2}$$

- Exercise

Using Kirchhoff's voltage law, determine the unknown voltages for the network of Fig. 5.16.

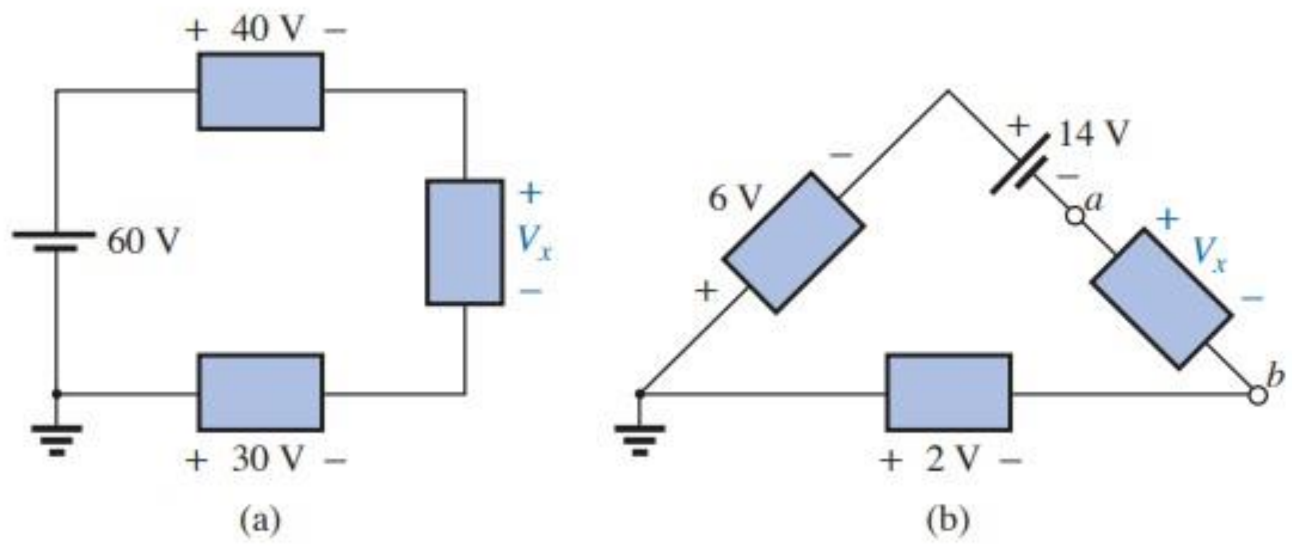


FIG. 5.16

Ans;

(a)50V (b)-18V

- Exercise

Determine the currents I_3 and I_5 of Fig. 6.29 through applications of Kirchhoff's current law.

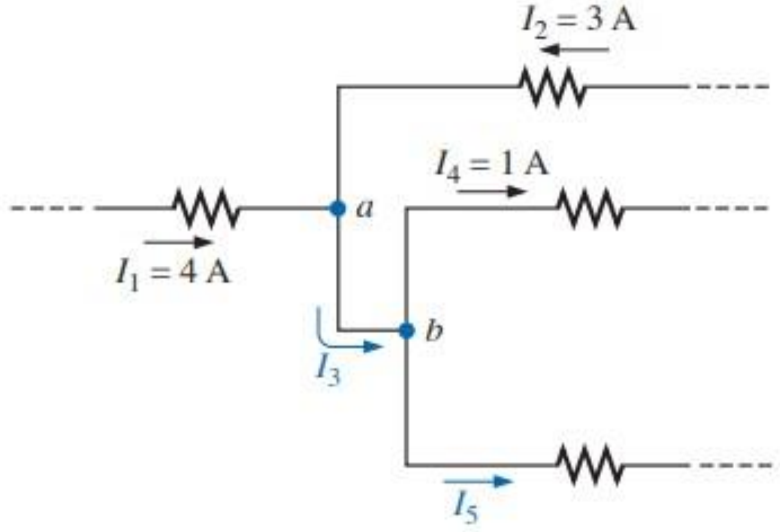


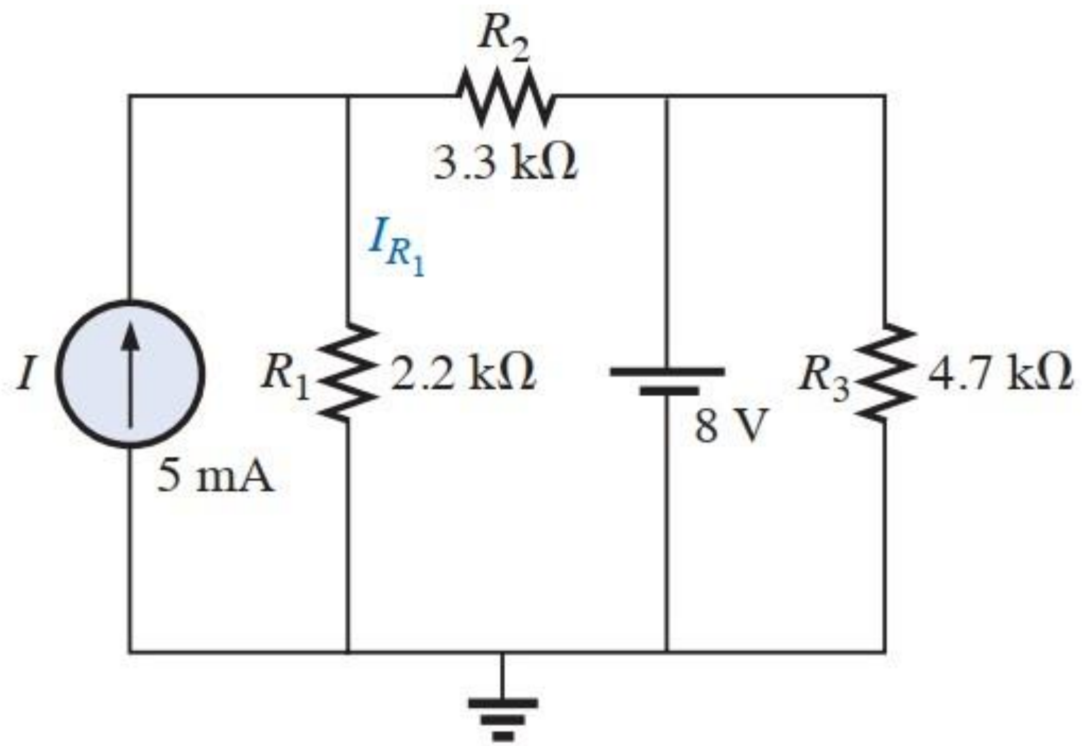
FIG. 6.29

Ans;

I_3=7A

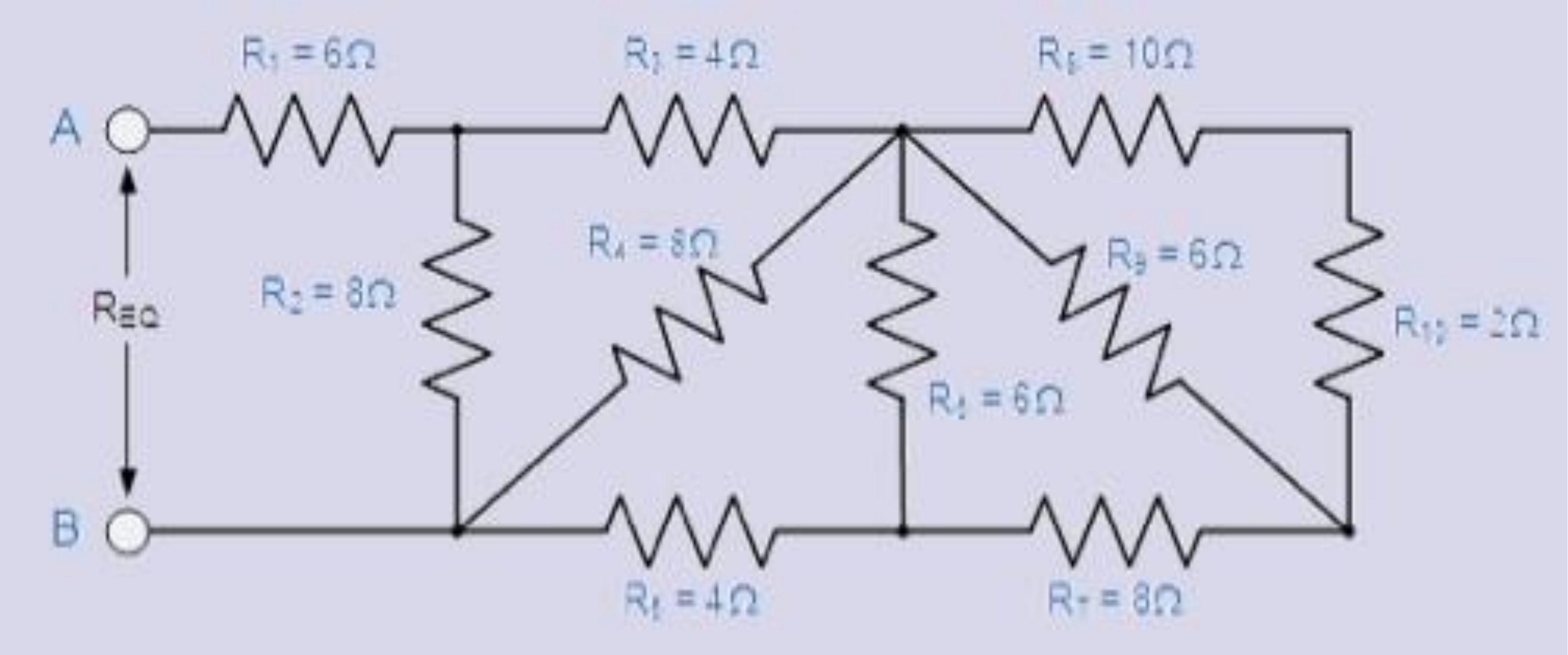
I_5=6A

Using Kirchhoff's, find the current through R1 for each network



❍Exercise

Find the total resistance as seen from terminal AB.



Exercise

Using the voltage divider rule, determine the voltages V_1 and V_3 for the series circuit of Fig. 5.28.

Ans;

V_1=6V

V_3=24V

- Exercise

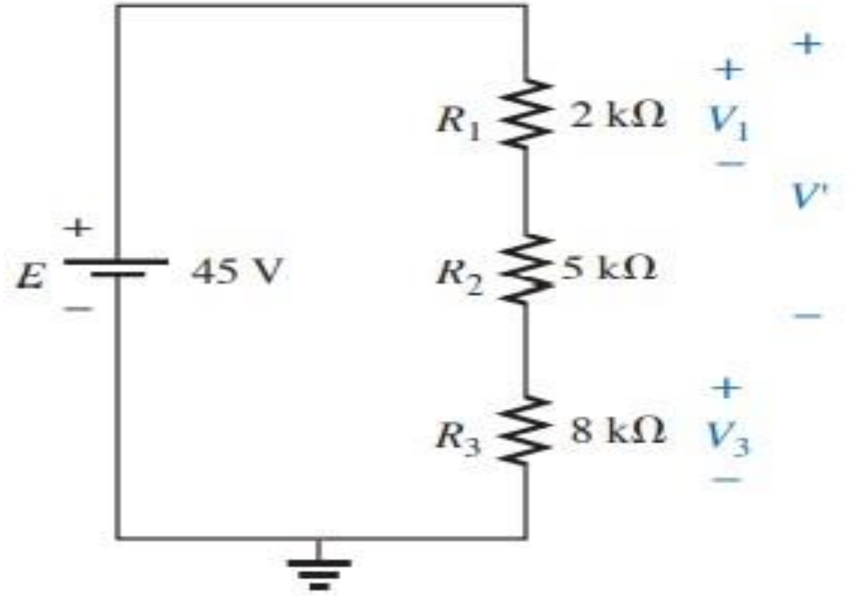


FIG. 5.28

Determine the current I_2 for the network of Fig. 6.35 using the current divider rule.

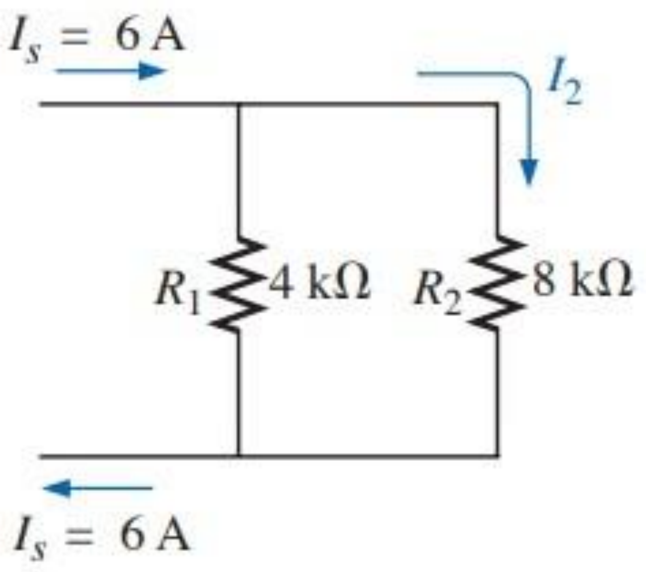


FIG. 6.35
Example 6.17.

Ans; 2A

- Voltage Sources In Parallel

Voltage sources are placed in parallel as shown in Fig. 6.40 only if they have the same voltage rating.

The primary reason for placing two or more batteries in parallel of the same terminal voltage would be to increase the current rating (and, therefore, the power rating) of the source.

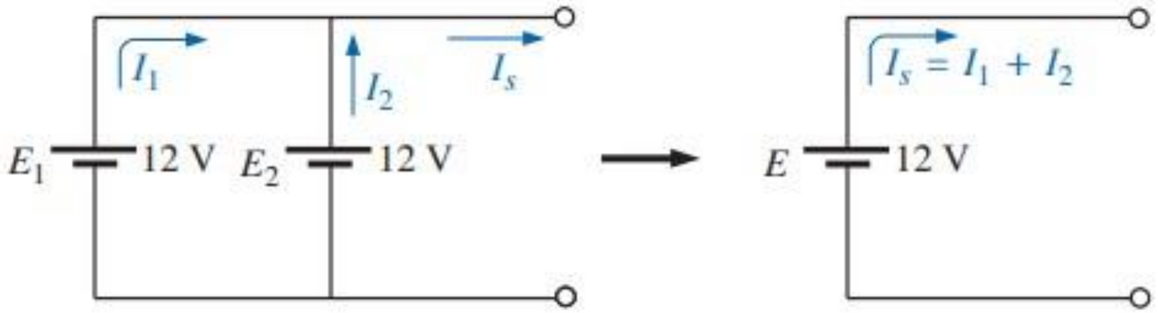
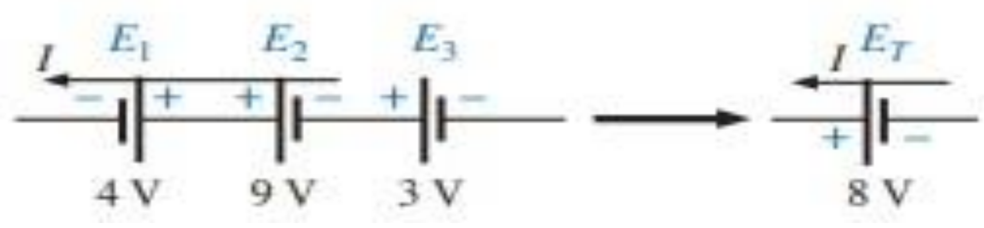
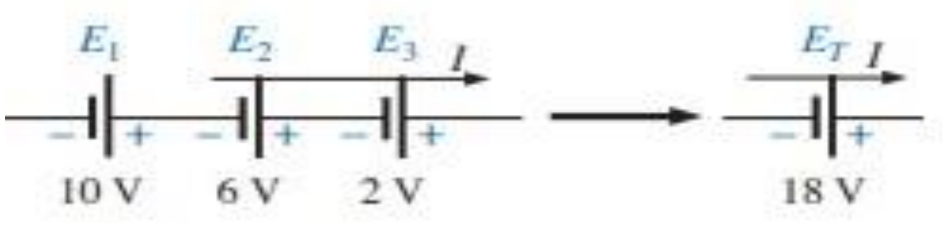


FIG. 6.40

- Voltage Sources In Series

Voltage sources can be connected in series, as shown in the figure below, to increase or decrease the total voltage applied to a system.



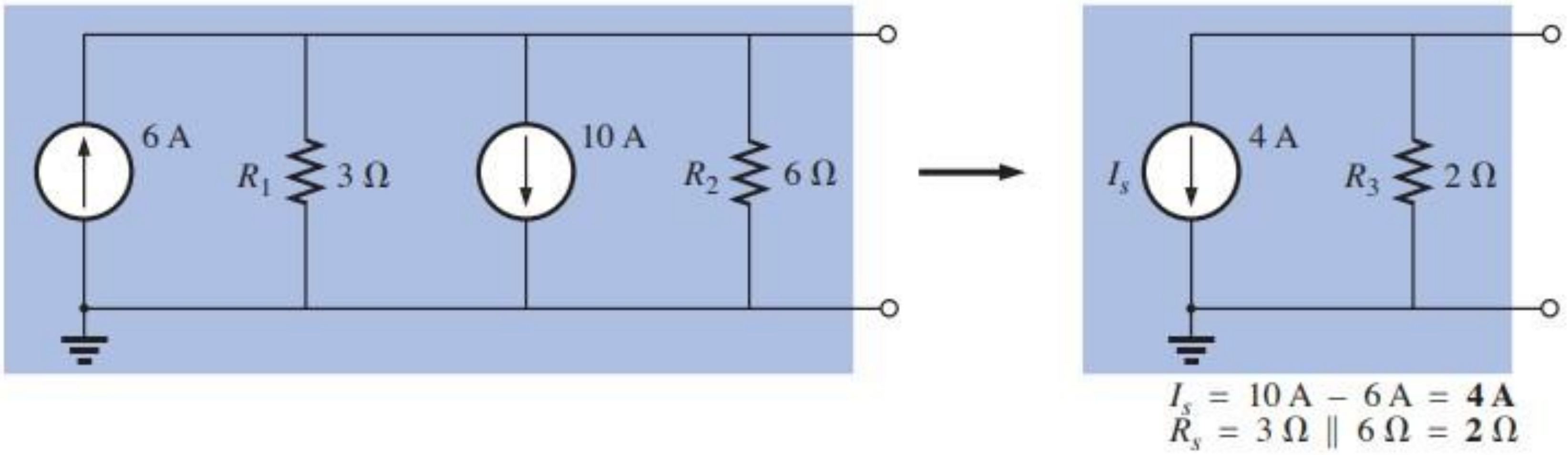


$$E_T = E_2 + E_3 - E_1 = 9\,\text{V} + 3\,\text{V} - 4\,\text{V} = 8\,\text{V}$$

$$E_T = E_1 + E_2 + E_3 = 10\,\text{V} + 6\,\text{V} + 2\,\text{V} = 18\,\text{V}$$

- **Current Sources In Parallel**

If two or more current sources are in parallel, they may all be replaced by one current source having the magnitude and direction of the resultant, which can be found by summing the currents in one direction and subtracting the sum of the currents in the opposite direction.



- Current Sources In Series

The current through any branch of a network can be only single-valued. For the situation indicated at point a in Fig. 8.18, we find by application of Kirchhoff's current law that the current leaving that point is greater than that entering—an impossible situation. Therefore, *current sources of different current ratings are not connected in series*, just as voltage sources of different voltage ratings are not connected in parallel.

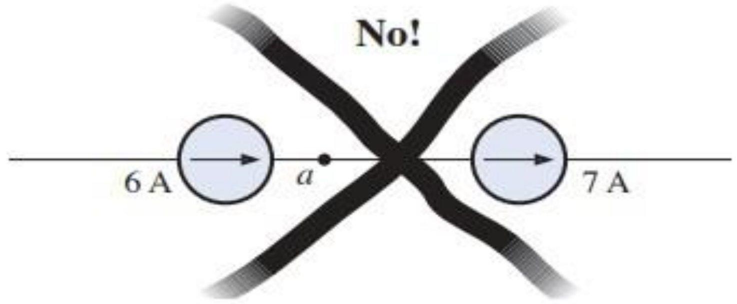
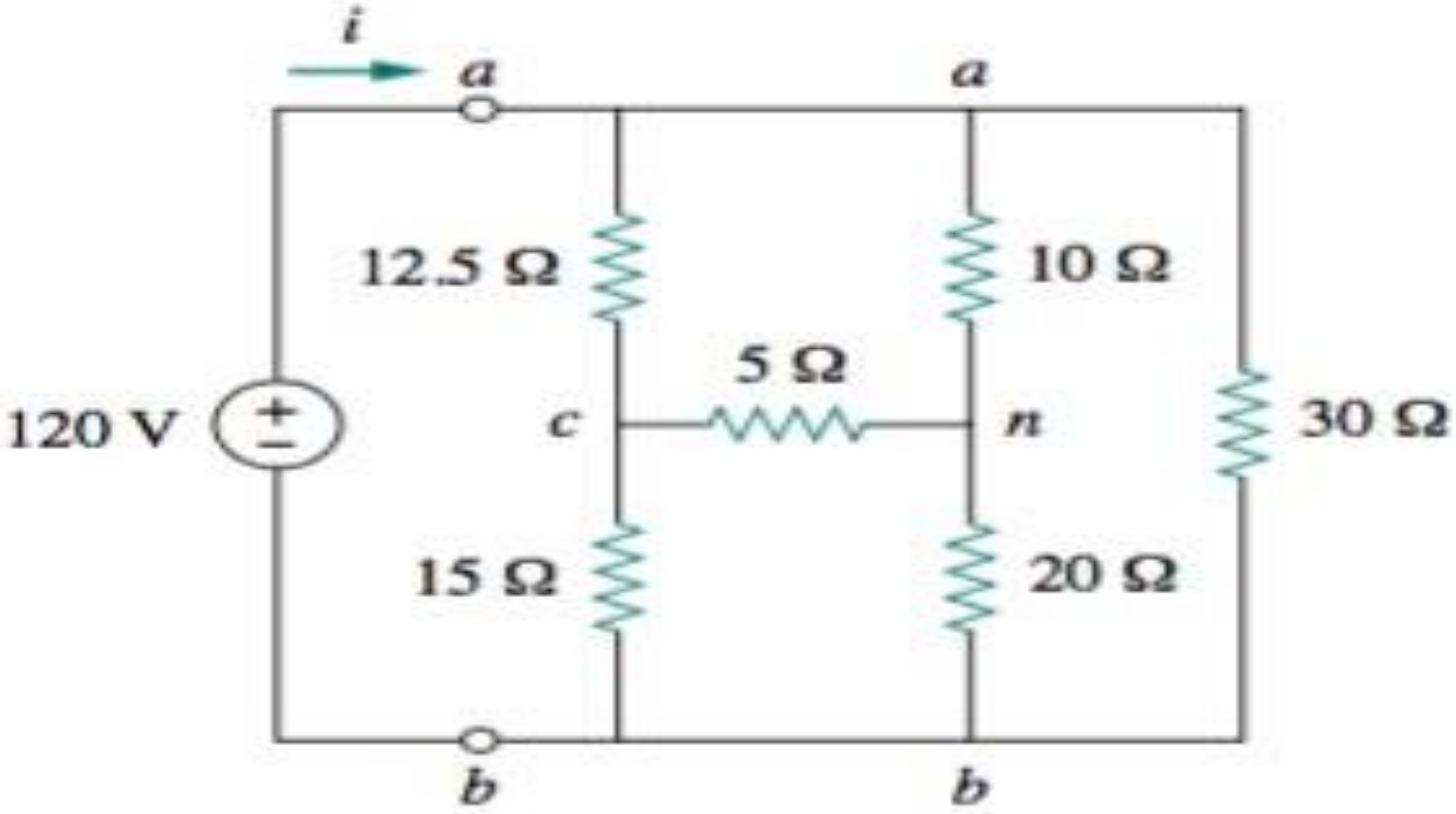
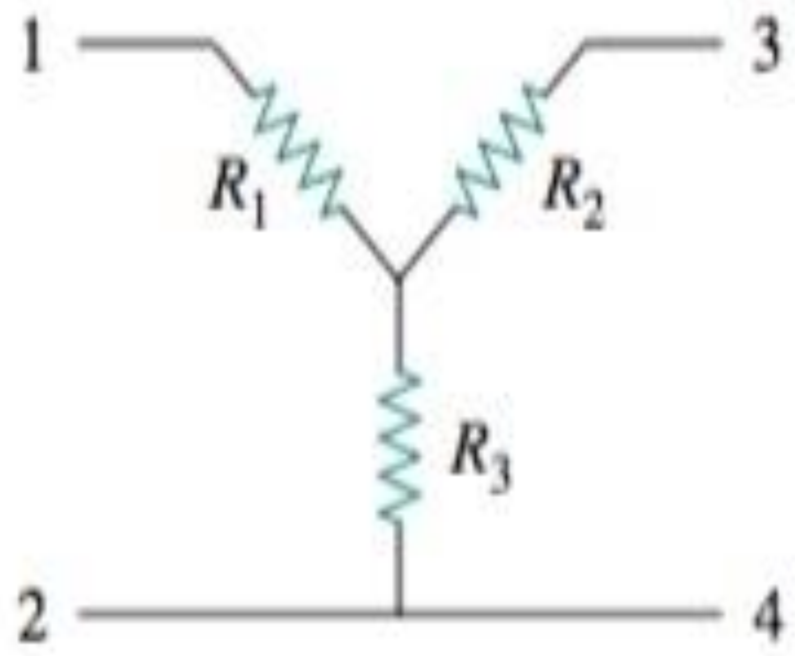


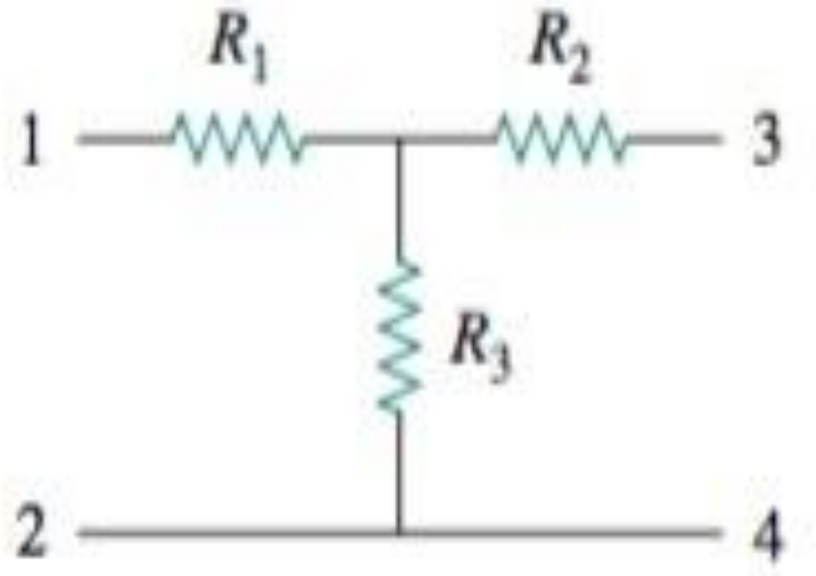
FIG. 8.18
Invalid situation.

- Wye-Delta And Delta-WyeTransformations

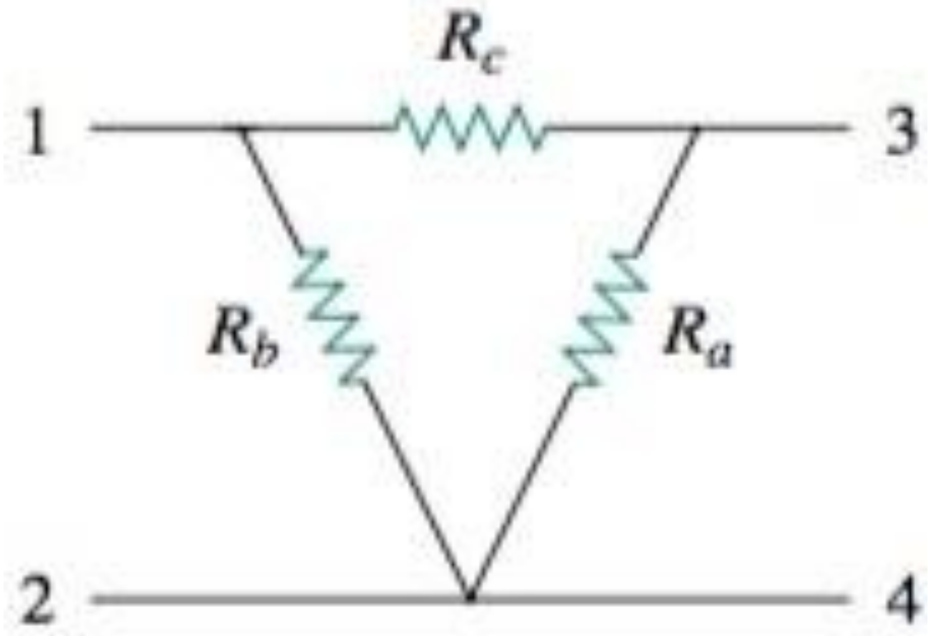


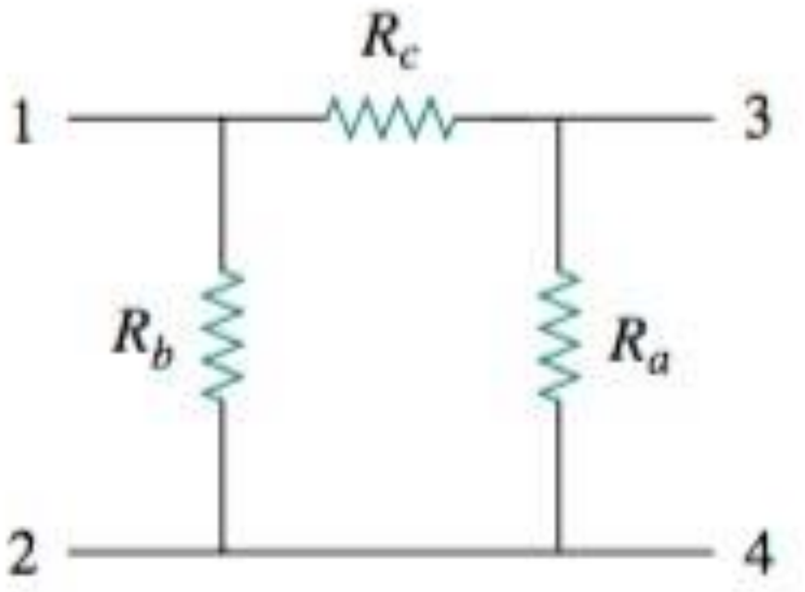
Wye connection



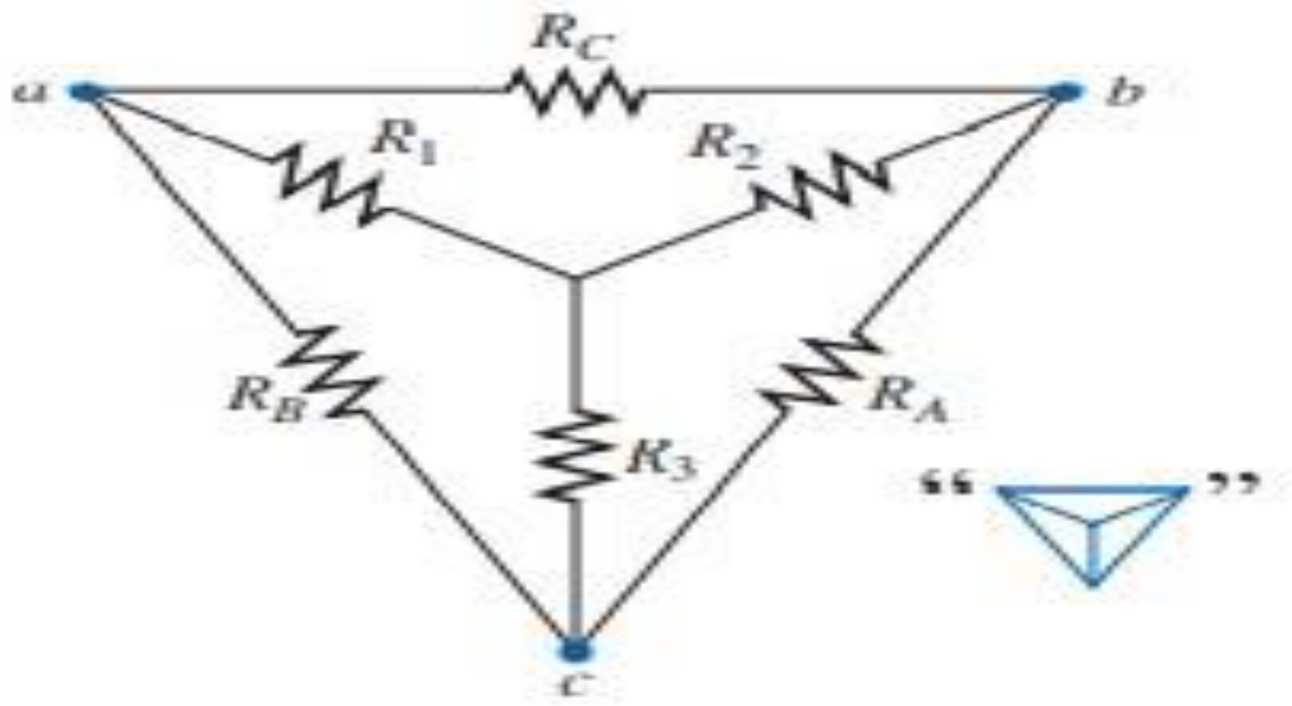


Delta Connection





Superposition of wye and delta networks as an aid in transforming one to other.



- Delta to Wye transformation

$$\begin{aligned} R_{12}(Y) &= R_1 + R_3 \\ R_{12}(\Delta) &= R_b \,||\, (R_a + R_c) \end{aligned} \quad (2.21)$$

Setting $$R_{12}(Y) = R_{12}(\Delta)$$

gives $$R_{12} = R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} \quad (2.22)$$

Similarly, $$R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} \quad (2.23)$$

$$R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \quad (2.24)$$

Subtracting Eq. (2.24) from Eq. (2.22),we get

$$R_1 - R_2 = \frac{R_c(R_b - R_a)}{R_a + R_b + R_c} \qquad (2.25)$$

Adding Eqs.(2.23) and (2.25), we gives

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \qquad (2.26)$$

and subtracting Eq.(2.25) from Eq.(2.23) yields

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} \tag{2.27}$$

Subtracting Eq. (2.26) from Eq. (2.22), we obtain

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Each resistor in the Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors.

- Wye to Delta transformation

To obtain the conversion formulas for transforming a wye network to an equivalent delta network, we note from Eqs. (2.26) to (2.28) that

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_aR_bR_c(R_a+R_b+R_c)}{(R_a+R_b+R_c)^2} = \frac{R_aR_bR_c}{R_a+R_b+R_c} \qquad (2.29)$$

Dividing Eq. (2.30) by each of Eqs. (2.26) to (2.28) leads to the following equations

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \tag{2.30}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \tag{2.31}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \tag{2.32}$$

Each resistor in the Δ network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor.

From Eqs. (2.30) to (2.32) and Fig. 2.30, the conversion rule for Y to Δ is as follows:

Each resistor in the Δnetwork is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor. The Y and Δ networks are said to be balanced when

$$R_1 = R_2 = R_3 = R_Y, R_a = R_b = R_c = R_\Delta$$

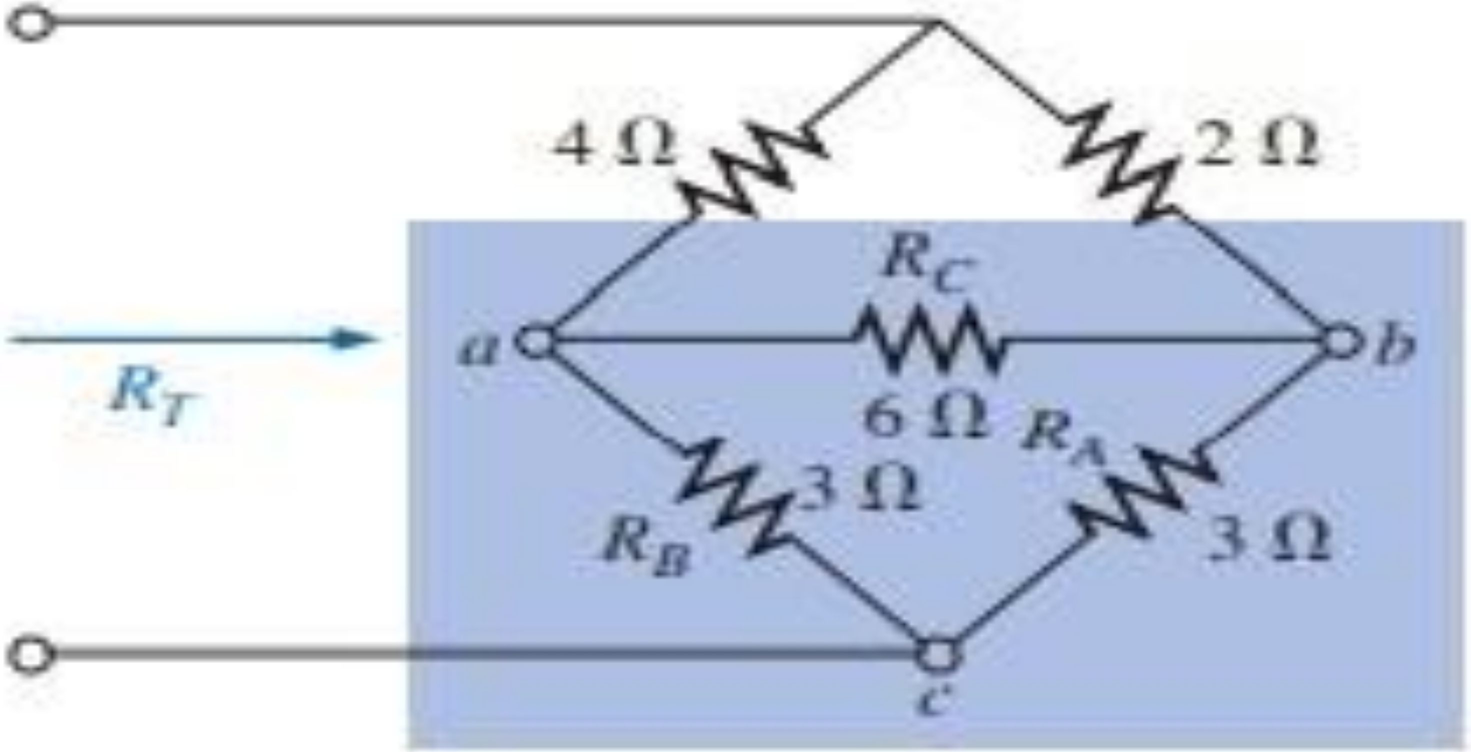
Under these conditions, conversion formulas become

$$R_Y = \frac{R_\Delta}{3}$$

or $$R_\Delta = 3R_Y$$

- Example

Find the total resistance of the network.



- Solution

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{12\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega} \longleftarrow$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(3\ \Omega)(3\ \Omega)}{12\ \Omega} = \frac{9\ \Omega}{12} = \mathbf{0.75\ \Omega}$$

Replacing the Δ by the Y, as shown in Fig. 8.81, yields

$$R_T = 0.75\ \Omega + \frac{(4\ \Omega + 1.5\ \Omega)(2\ \Omega + 1.5\ \Omega)}{(4\ \Omega + 1.5\ \Omega) + (2\ \Omega + 1.5\ \Omega)}$$

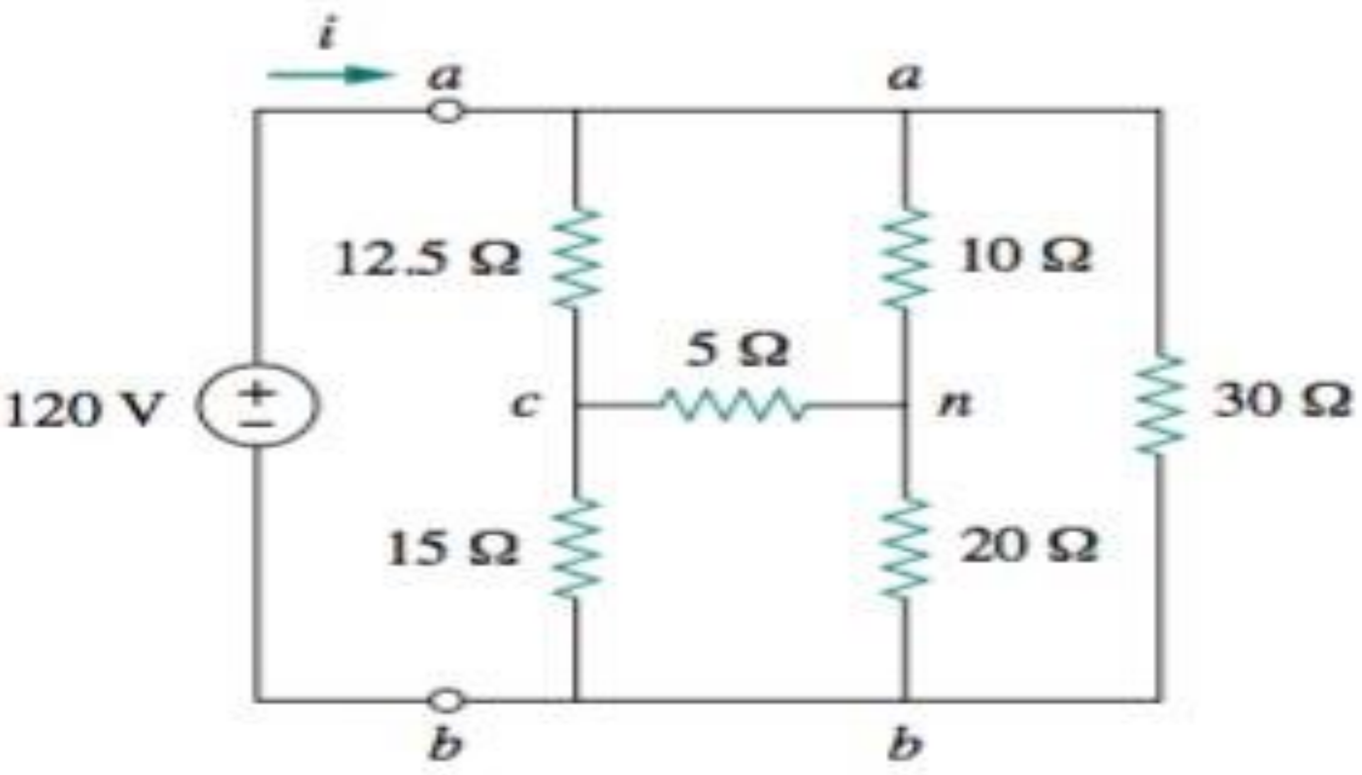
$$= 0.75\ \Omega + \frac{(5.5\ \Omega)(3.5\ \Omega)}{5.5\ \Omega + 3.5\ \Omega}$$

$$= 0.75\ \Omega + 2.139\ \Omega$$

$$R_T = \mathbf{2.889\ \Omega}$$

- Exercise

Obtain the equivalent resistance R_{ab} for the circuit and use it to find current i.



- Superposition Theorem

The current through, or voltage across, an element in a linear bilateral network is equal to the algebraic sum of the currents or voltages produced independently by each source.

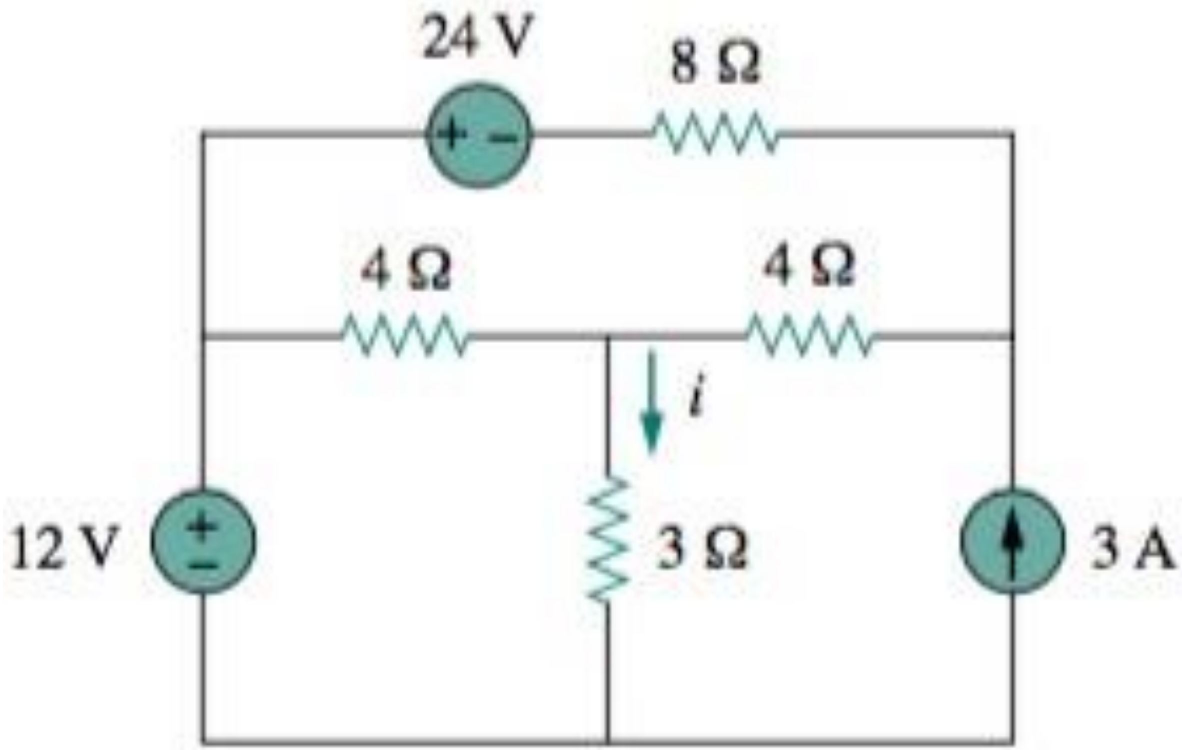
✦**Steps to Apply Superposition Principle:**

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source.
2. Repeat step 1 for each of the other independent sources.

3. Find the total contribution by adding algebraically all the contribution due to the independent sources.

- Exercise

Use superposition theorem to find the current *i*.



- Reciprocity Theorem

The reciprocity theorem is applicable only to single-source networks. The theorem states the following:
The current I in any branch of a network, due to a single voltage source *E* anywhere else in the network, will equal the current through the branch in which the source was originally located if the source is placed in the branch in which the current *I* was originally measured.

In other words, the location of the voltage source and the resulting current may be interchanged without a change in current.

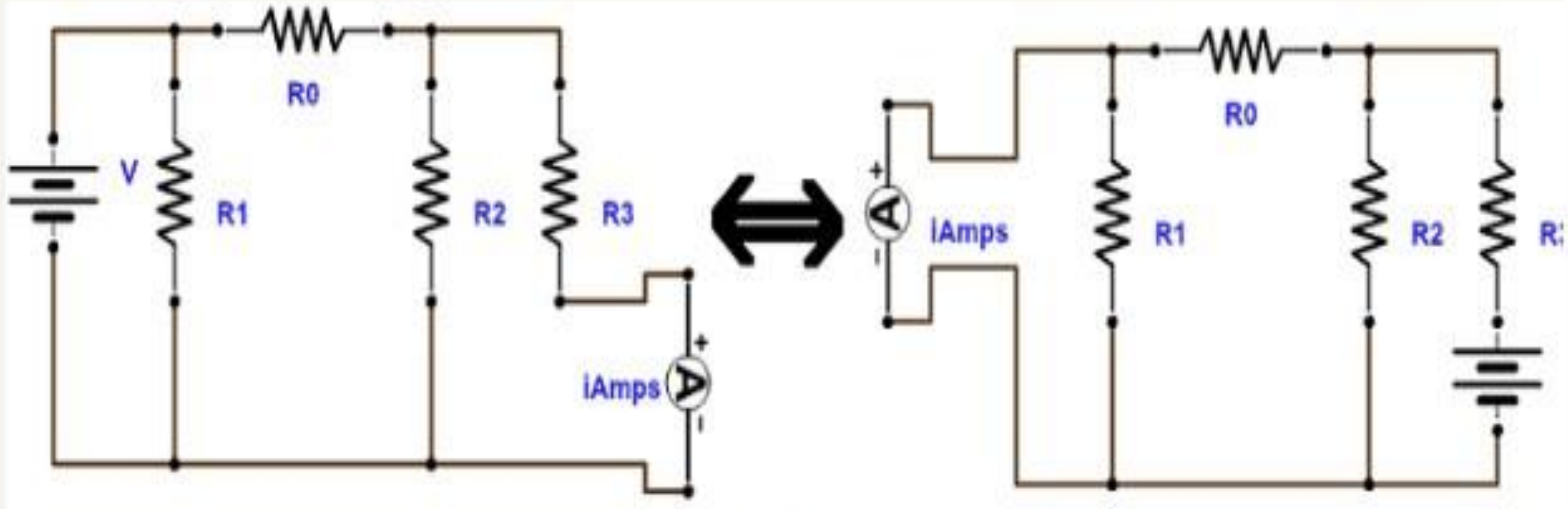


Fig: Reciprocity Theorem

- Exercise

Using reciprocity theorem find the ammeter current.

DC CIRCUIT THEORY