PABLO

FUNDAMENTALS OF ELECTRICAL ENGINEERING.

LECTURE 4 NOTES

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Examples

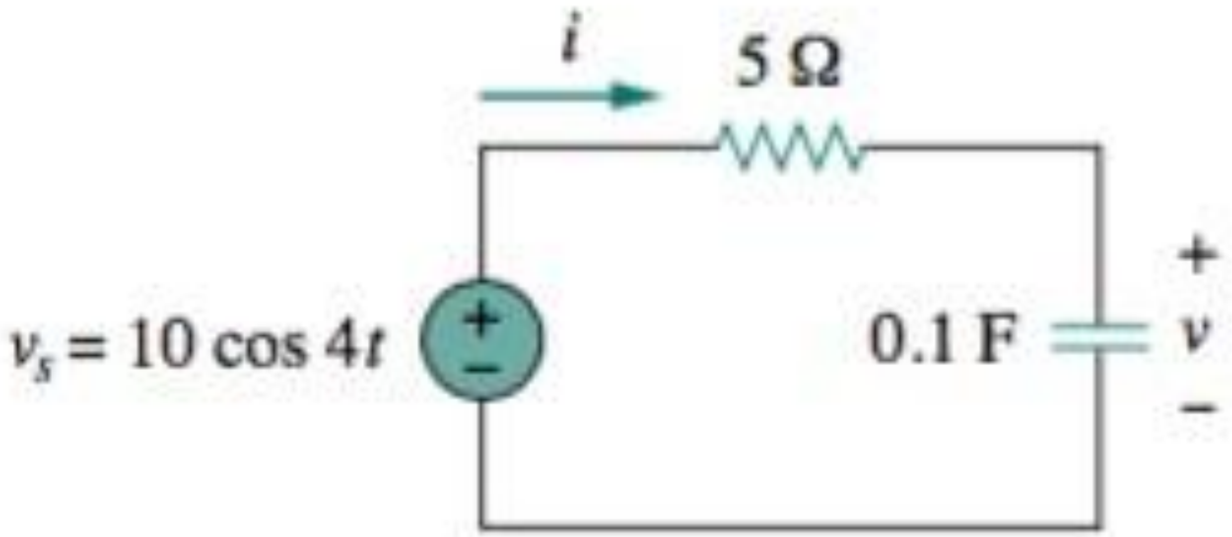
- A resistance of 7.0 Ω is connected in series with a pure inductance of 31.8 mH and the circuit is connected to a 100 V, 50 Hz, sinusoidal supply. Calculate:
- (a) the circuit current; (b) the phase angle.

Examples

- A capacitor of 8.0 μF takes a current of 1.0 A when the alternating voltage applied across it is 230 V. Calculate:
- (a) the frequency of the applied voltage;
- (b) the resistance to be connected in series with the capacitor to reduce the current in the circuit to 0.5 A at the same frequency;
- (c) the phase angle of the resulting circuit.

e.G

- Find v(t) and i(t) in the circuit



e.g.

- Calculate V_o in the circuit

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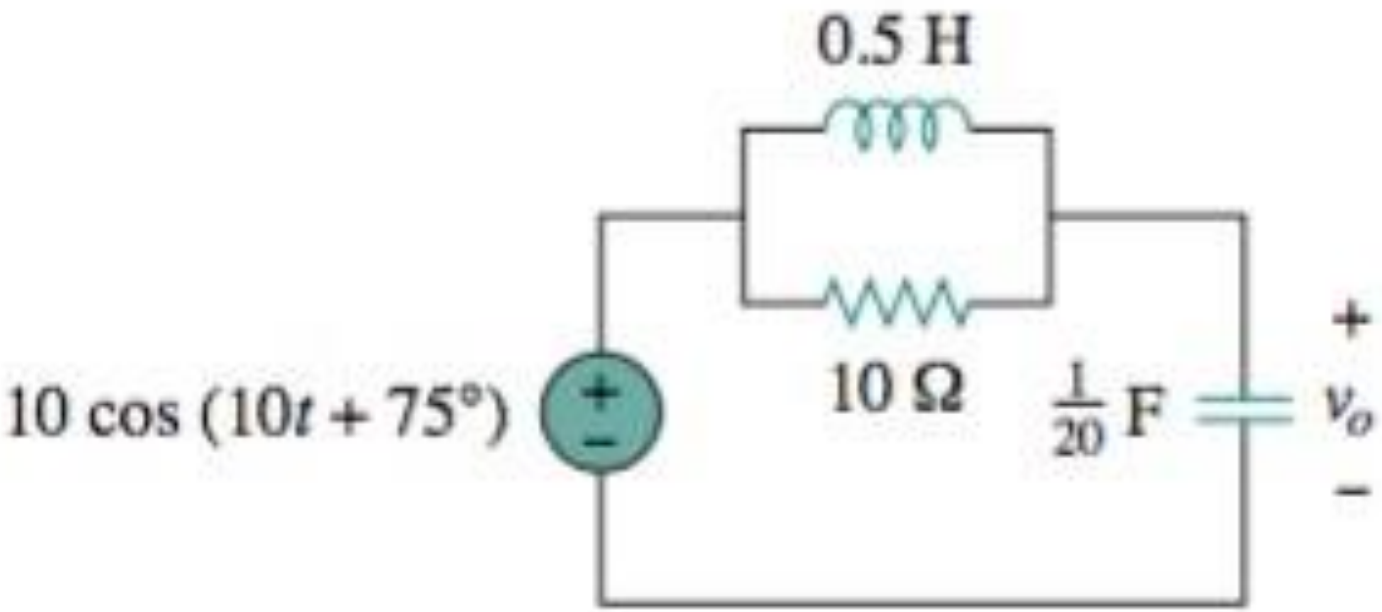


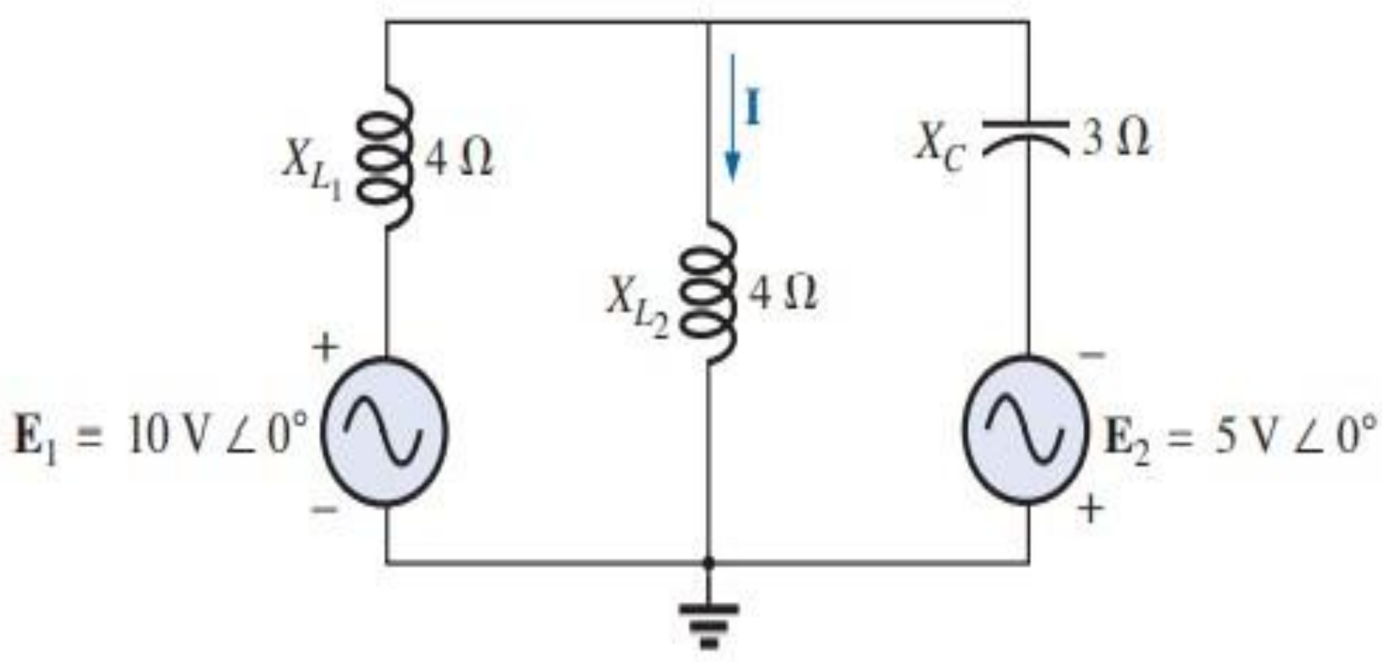
- Methods of Analysis for AC Circuit

- Superposition Theorem

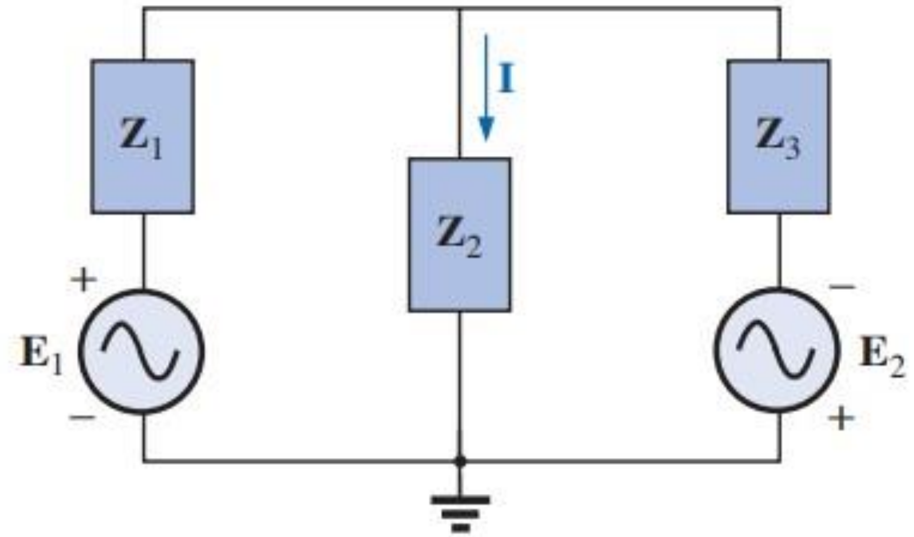
The only variation in applying this method to ac networks with independent sources is that we will now be working with impedances and phasors instead of just resistors and real numbers.

- Example

Using the superposition theorem, find the current I through the 4Ω reactance (X_{L2}).



- Solution



For the redrawn circuit

$$\mathbf{Z}_1 = +j X_{L_1} = j\,4\ \Omega$$

$$\mathbf{Z}_2 = +j X_{L_2} = j\,4\ \Omega$$

$$\mathbf{Z}_3 = -j X_C = -j\,3\ \Omega$$

Considering the effects of the voltage source E_1

$$\mathbf{Z}_{2\|3} = \frac{\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_2 + \mathbf{Z}_3} = \frac{(j\,4\,\Omega)(-j\,3\,\Omega)}{j\,4\,\Omega - j\,3\,\Omega} = \frac{12\,\Omega}{j} = -j\,12\,\Omega$$

$$= 12\,\Omega\,\angle -90^\circ$$

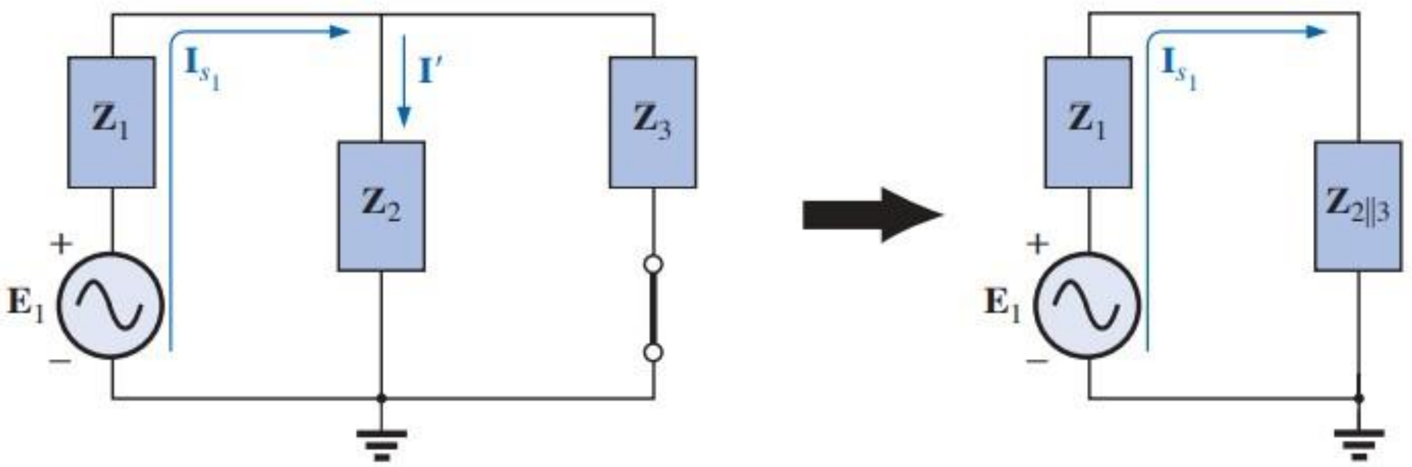
$$I_{s_1} = \frac{\mathbf{E}_1}{\mathbf{Z}_{2\|3} + \mathbf{Z}_1} = \frac{10\text{ V}\,\angle 0^\circ}{-j\,12\,\Omega + j\,4\,\Omega} = \frac{10\text{ V}\,\angle 0^\circ}{8\,\Omega\,\angle -90^\circ}$$

$$= 1.25\text{ A}\,\angle 90^\circ$$

$$\mathbf{I}' = \frac{\mathbf{Z}_3\mathbf{I}_{s_1}}{\mathbf{Z}_2 + \mathbf{Z}_3} \qquad \text{(current divider rule)}$$

$$= \frac{(-j\,3\,\Omega\,)(\,j\,1.25\text{ A})}{j\,4\,\Omega - j\,3\,\Omega} = \frac{3.75\text{ A}}{j\,1} = 3.75\text{ A}\,\angle -90^\circ$$

SINUSOIDAL ALTERNATING WAVEFORMS

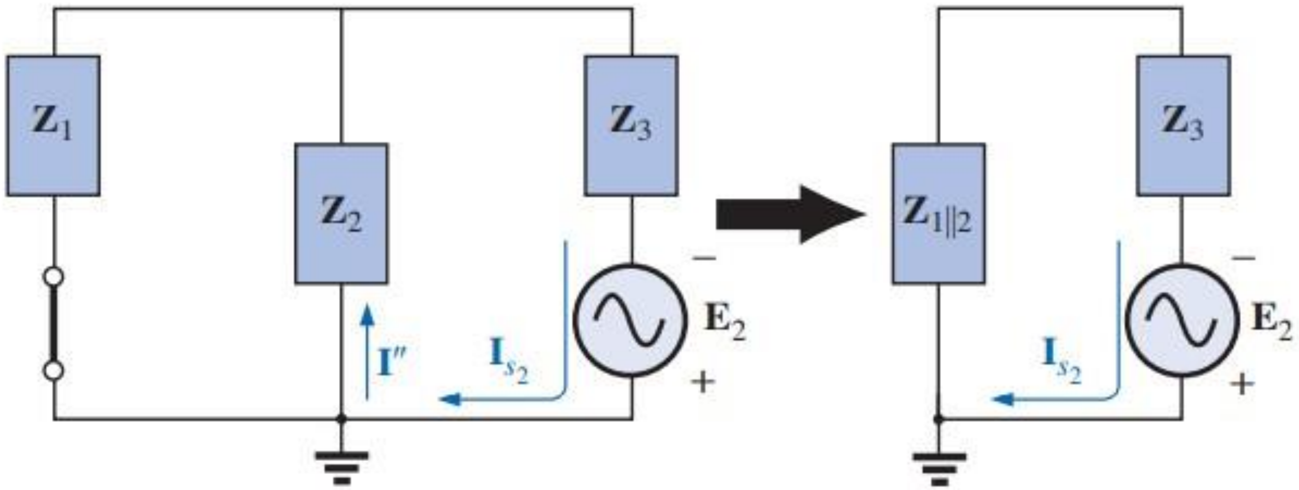


Considering the effects of the voltage source E2

$$\mathbf{Z}_{1\|2} = \frac{\mathbf{Z}_1}{N} = \frac{j\,4\,\Omega}{2} = j\,2\,\Omega$$

$$\mathbf{I}_{s_2} = \frac{\mathbf{E}_2}{\mathbf{Z}_{1\|2} + \mathbf{Z}_3} = \frac{5\text{ V}\angle 0^\circ}{j\,2\,\Omega - j\,3\,\Omega} = \frac{5\text{ V}\angle 0^\circ}{1\,\Omega\angle -90^\circ} = 5\text{ A}\angle 90^\circ$$

and
$$\mathbf{I}'' = \frac{\mathbf{I}_{s_2}}{2} = 2.5\text{ A}\angle 90^\circ$$

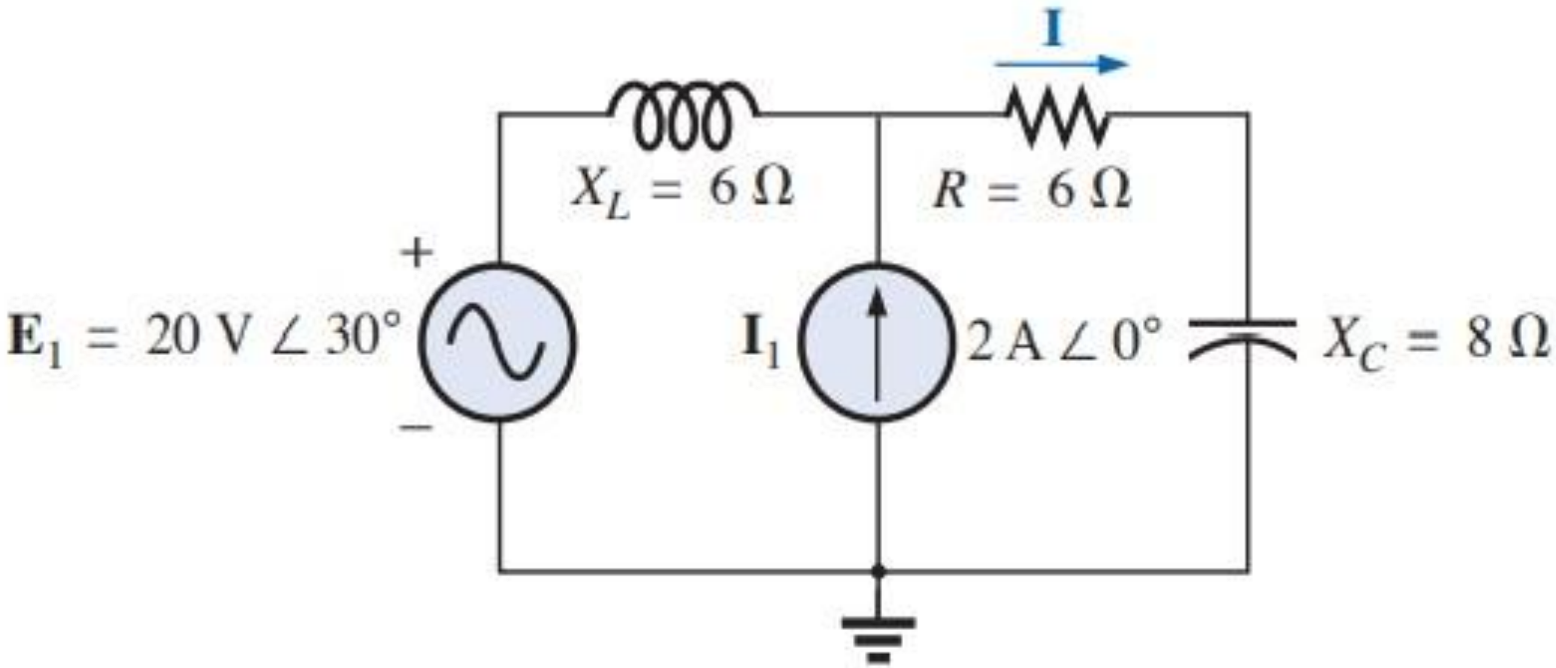


The resultant current through the 4Ω reactance X_{L2}

$$\mathbf{I} = \mathbf{I}' - \mathbf{I}''$$
$$= 3.75\text{ A}\angle -90° - 2.50\text{ A}\angle 90° = -j\,3.75\text{ A} - j\,2.50\text{ A}$$
$$= -j\,6.25\text{ A}$$
$$\mathbf{I} = \mathbf{6.25\text{ A}\angle -90°}$$

- Exercise

Using superposition, find the current I through the 6-Ω resistor.


I
$X_L = 6\ \Omega$
$R = 6\ \Omega$
+
$\mathbf{E}_1 = 20\ \text{V} \angle 30°$
−
$\mathbf{I}_1$
$2\ \text{A} \angle 0°$
$X_C = 8\ \Omega$

- Power In AC

We will now examine the total power equation in a slightly different form and will introduce two additional types of power: **apparent** and **reactive**.

For any system, the power delivered to a load at any instant is defined by the product of the applied voltage and the resulting current.

$$p = vi$$

In this case, since v and i are sinusoidal quantities, let us establish a general case where

$$v = V_m \sin(\omega t + \theta)$$
$$i = I_m \sin \omega t$$

Substituting the above equations for v and i into the power equation will result in

$$p = V_m I_m \sin \omega t \sin(\omega t + \theta)$$

If we now apply a number of trigonometric identities, the following form for the power equation will result:

$$\boxed{p = VI\cos\theta(1 - \cos 2\omega t) + VI\sin\theta(\sin 2\omega t)}$$

where V and I are the rms values.

The conversion from peak values V_m and I_m to rms values resulted from the operations performed using the trigonometric identities.

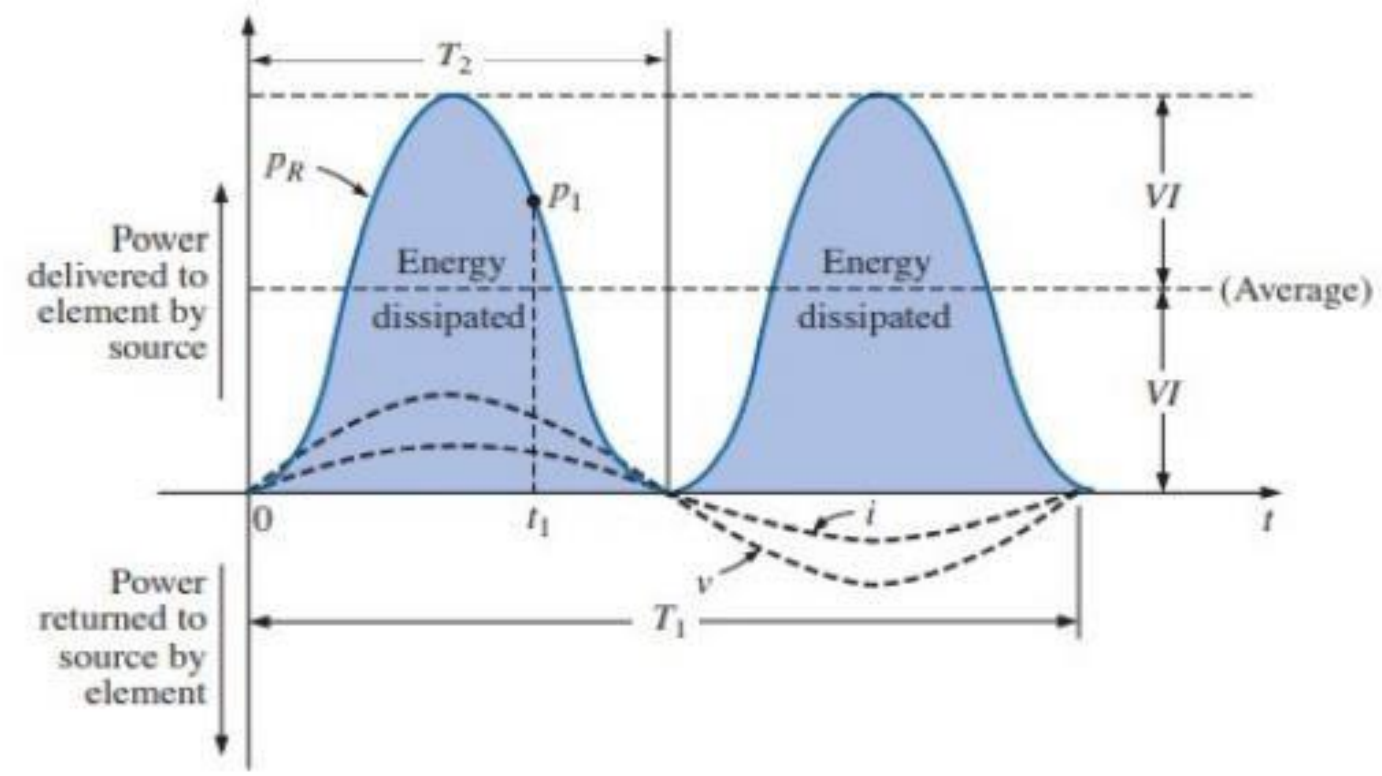
If Equation is expanded to the form

$$p = \underbrace{VI\cos\theta}_{\text{Average}} - \underbrace{VI\cos\theta}_{\text{Peak}}\underbrace{\cos 2\omega t}_{2x} + \underbrace{VI\sin\theta}_{\text{Peak}}\underbrace{\sin 2\omega t}_{2x}$$

the average power still appears as an isolated term that is time independent. Second, both terms that follow vary at a frequency twice that of the applied voltage or current, with peak values having a very similar format.

OResistive Circuit

For a purely resistive circuit, v and i are in phase, and θ = 0°.



Substituting θ = 0° into the power equation, we obtain

$$p_R = VI\cos(0°)(1 - \cos 2\omega t) + VI\sin(0°)\sin 2\omega t$$
$$= VI(1 - \cos 2\omega t) + 0$$

Or $$\boxed{p_R = VI - VI\cos 2\omega t}$$

where VI is the average or dc term and -VI cos 2ωt is a negative cosine wave with twice the frequency of either input quantity (v or i) and a peak value of VI.

Plotting the waveform for p_R, we see that

T1 = period of input quantities T2

= period of power curve p_R

✦Note that the power curve passes through two cycles about its average value of VI for each cycle of either v or i ($T_1 = 2T_2$ or $f_2 = 2f_1$). Consider also that since the peak and average values of the power curve are the same, the curve is always above the horizontal axis. This indicates that *the total power delivered to a resistor will be dissipated in the form of heat.*

The average (real) power is VI; or, as a summary

$$\boxed{P = VI = \frac{V_m I_m}{2} = I^2 R = \frac{V^2}{R}}$$

✦Apparent Power

From our analysis of dc networks (and resistive elements above), it would seem apparent that the power delivered to the load is simply determined by the product of the applied voltage and current, with no concern for the components of the load; that is, $P = VI$. However, we found that the power factor ($\cos\theta$) of the load will have a pronounced effect on the power dissipated, less pronounced for more reactive loads.

✦Although the product of the voltage and current is not always the power delivered, it is a power rating of significant usefulness in the description and analysis of sinusoidal ac networks and in the maximum rating of a number of electrical components and systems.

It is called the *apparent power* and is represented symbolically by S*.

Its magnitude is determined by

$$\boxed{S = VI} \qquad \text{(volt-amperes, VA)}$$

Or, since $V = IZ \quad \text{and} \quad I = \dfrac{V}{Z}$

then and $\boxed{S = I^2 Z}$ (VA)

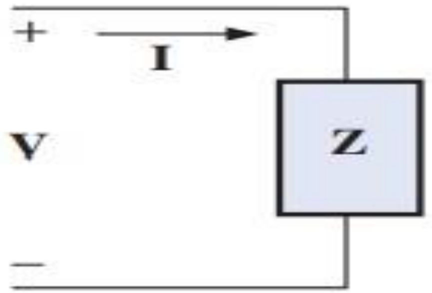


FIG. 19.4

$$\boxed{S = \frac{V^2}{Z}} \quad \text{(VA)}$$

The average power to the load of Fig. 19.4 is

However, $P = VI\cos\theta$

Therefore, $S = VI$

✦Reactive $\boxed{P = S\cos\theta}$ (W) Power

OInductive Circuit

For a purely inductive circuit (such as that in Fig. 19.6), v leads i by 90°, as shown in Fig. 19.7.

Substituting v = 90° into the power equation yields

$$
\begin{aligned}
p_L &= VI\cos(90^\circ)(1 - \cos 2\omega t) + VI\sin(90^\circ)(\sin 2\omega t) \\
&= 0 + VI\sin 2\omega t
\end{aligned}
$$

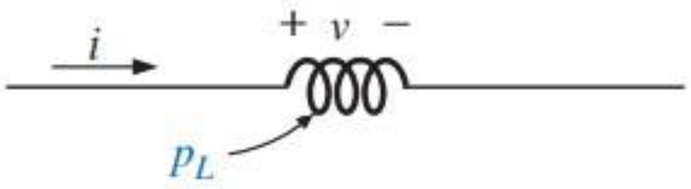


FIG. 19.6

Or

$$\boxed{p_L = VI\sin 2\omega t}$$

The power curve for a purely inductive load.

SINUSOIDAL ALTERNATING WAVEFORMS

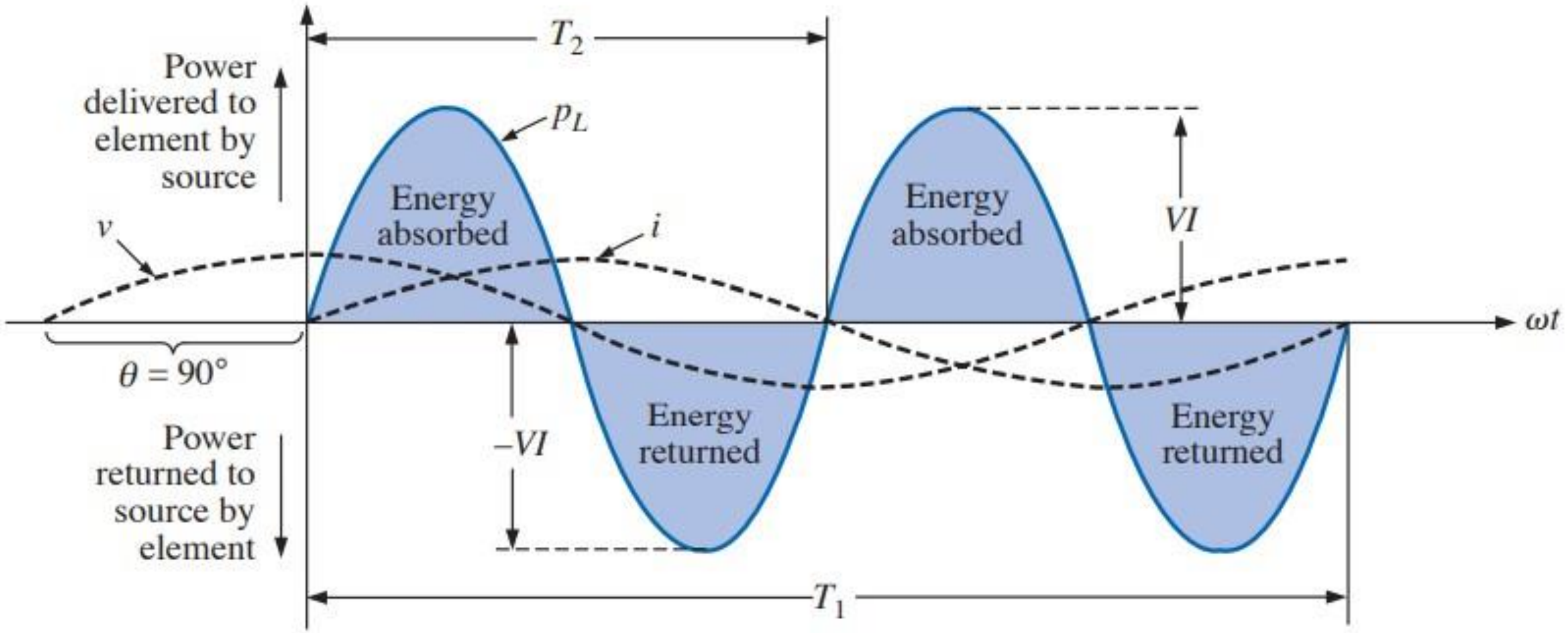


FIG. 19.7

where $VI\sin 2\omega t$ is a sine wave with twice the frequency of either input quantity (v or i) and a peak value of VI.

✦Note the absence of an average or constant term in the equation.

Plotting the waveform for p_L, we obtain

T_1 = period of either input quantity

T_2 = period of p_L curve

- In general, the reactive power associated with any circuit is defined to be VIsinθ.

The symbol for reactive power is Q, and its unit of measure is the volt-ampere reactive (VAR)*. Therefore,

$$\boxed{Q = VI \sin \theta} \qquad \text{(volt-ampere reactive, VAR)}$$

where θ is the phase angle between V and I.

For the inductor,

$$\boxed{Q_L = VI} \qquad \text{(VAR)}$$

or, since $V = IX_L$ or $I = V/X_L$,

$$\boxed{Q_L = I^2X_L} \qquad \text{(VAR)}$$

Or

$$\boxed{Q_L = \frac{V^2}{X_L}} \qquad \text{(VAR)}$$

The apparent power associated with an inductor is S = VI, and the average power is P = 0.

Capacitive Circuit

For a purely capacitive circuit (such as that in Fig. 19.8), i leads v by 90°, as shown in Fig. 19.9.

Substituting v = -90° into the power equation , we obtain

$$p_C = VI\cos(-90^\circ)(1 - \cos 2\omega t) + VI\sin(-90^\circ)(\sin 2\omega t)$$
$$= 0 - VI\sin 2\omega t$$

Or

$$\boxed{p_C = -VI\sin 2\omega t}$$

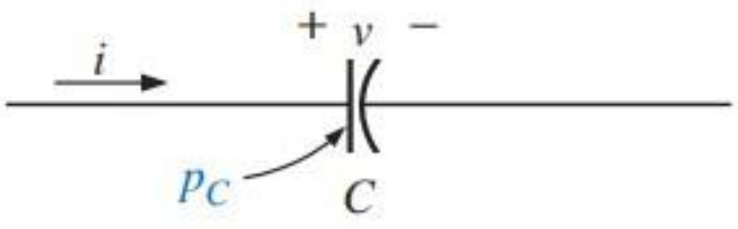
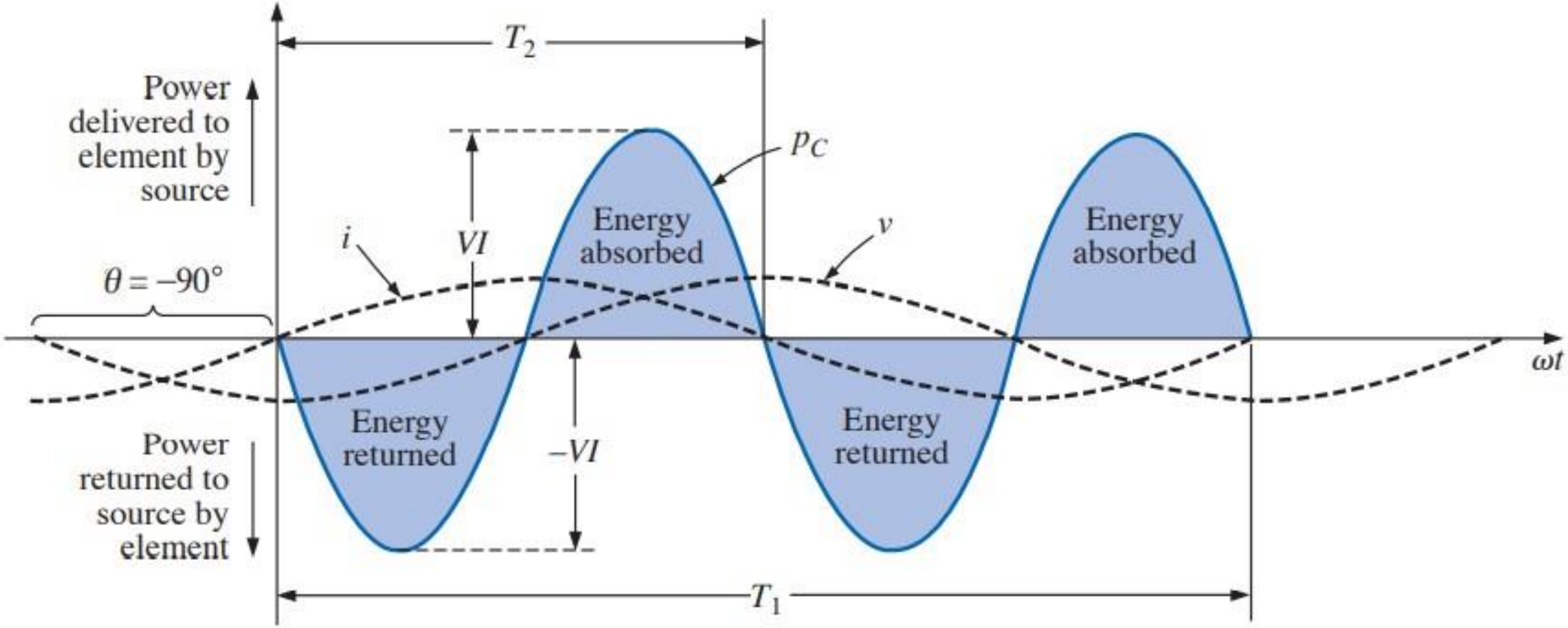


FIG. 19.8

The power curve for a purely capacitive load.

SINUSOIDAL ALTERNATING WAVEFORMS



where $-VI \sin 2\omega t$ is a negative sine wave with twice the frequency of either input (v or i) and a peak value of VI. Again, note the absence of an average or constant term.

Plotting the waveform for p_C gives us

T_1 = period of either input quantity

T_2 = period of p_C curve

The reactive power associated with the capacitor is equal to the peak value of the p_C curve, as follows:

$$\boxed{Q_C = VI} \qquad \text{(VAR)}$$

But, since $V=IX_c$ and $I=V/X_c$ the reactive power to the capacitor can also be written

$$\boxed{Q_C = I^2 X_C} \qquad \text{(VAR)}$$

And

$$\boxed{Q_C = \frac{V^2}{X_C}} \qquad \text{(VAR)}$$

The apparent power associated with the capacitor is

$$\boxed{S = VI} \qquad \text{(VA)}$$

and the average power is P = 0.

- Power Factor

In the equation $P = (V_m I_m/2)\cos\theta$, the factor that has significant control over the delivered power level is the $\cos\theta$.

✦ No matter how large the voltage or current, if $\cos\theta = 0$, the power is zero; if $\cos\theta = 1$, the power delivered is a maximum. Since it has such control, the expression was given the name power factor and is defined by

$$\text{Power Factor} = F_p = \cos\theta = \frac{P}{S}$$

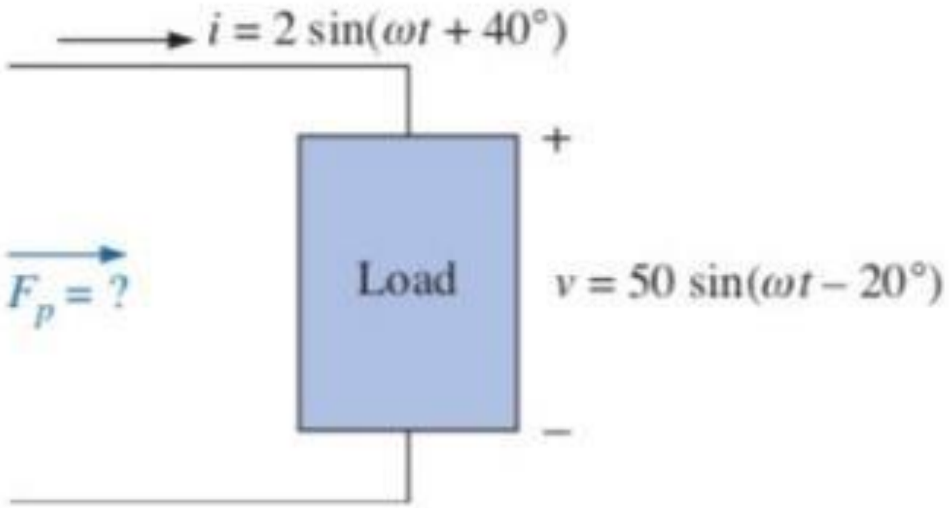
- For a **purely resistive load**, the phase angle between v and I is 0°and Fp = cos θ = cos 0°=1. The power delivered is a maximum of $(V_m I_m/2)$ cos θ.
- For a **purely reactive load** (inductive or capacitive), the phase angle between v and i is 90° and F_p = cos θ = cos 90° = 0. The power delivered is then the minimum value of zero watts, even though the current has the same peak value.

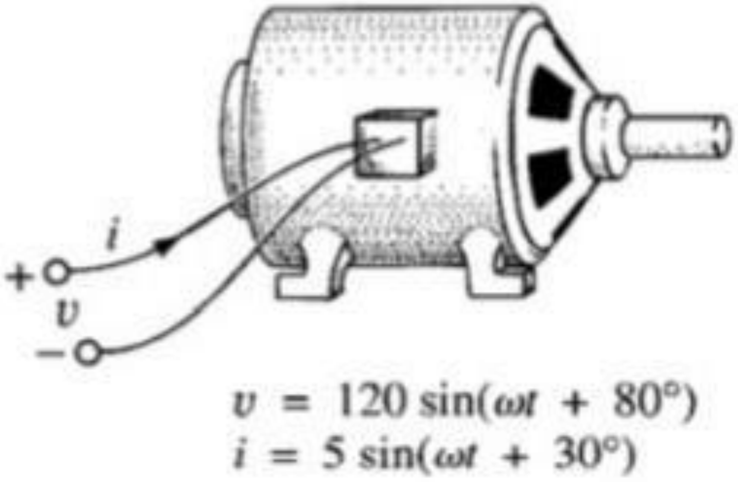
- For situations where the load is a combination of resistive and reactive elements, the power factor will vary between 0 and 1. The more resistive the total impedance, the closer the power factor is to 1; the more reactive the total impedance, the closer the power factor is to 0.
- The terms leading and lagging are often written in conjunction with the power factor. They are defined by the current through the load. If the current leads the voltage across a load, the load has a leading power factor. If the

current lags the voltage across the load, the load has a lagging power factor.

- In other words, capacitive networks have leading power factors, and inductive networks have lagging power factors.
  - Example

Determine the power factors of the following loads, and indicate whether they are leading or lagging:


$i = 2 \sin(\omega t + 40°)$
+
Load
$v = 50 \sin(\omega t - 20°)$
$F_p = ?$
−


i
+
v
−
$v = 120 \sin(\omega t + 80°)$
$i = 5 \sin(\omega t + 30°)$

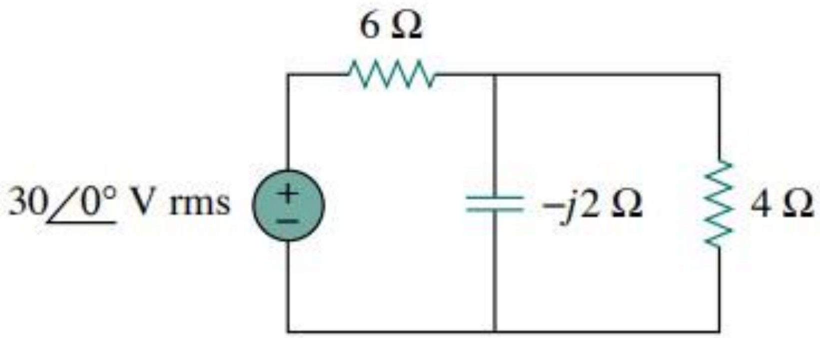
✦Solution

a. $F_p = \cos\theta = \cos|40° - (-20°)| = \cos 60° = \mathbf{0.5\ leading}$

b. $F_p = \cos\theta\ |80° - 30°| = \cos 50° = \mathbf{0.6428\ lagging}$

- Exercise

Determine the power factor of the entire circuit of Fig. 11.18 as seen by the source. Calculate the average power delivered by the source.



- Power Triangle

The three quantities average power, apparent power, and reactive power can be related in the vector domain by

$$\boxed{\mathbf{S} = \mathbf{P} + \mathbf{Q}}$$

With

$$\mathbf{P} = P \angle 0^\circ \qquad \mathbf{Q}_L = Q_L \angle 90^\circ \qquad \mathbf{Q}_C = Q_C \angle -90^\circ$$

For an inductive load, the phasor power **S**, as it is often called, is defined by

$$\mathbf{S} = P + jQ_L$$

SINUSOIDAL ALTERNATING WAVEFORMS

For a capacitive load, the phasor power **S** is defined by

$$\mathbf{S} = P - j\,Q_C$$

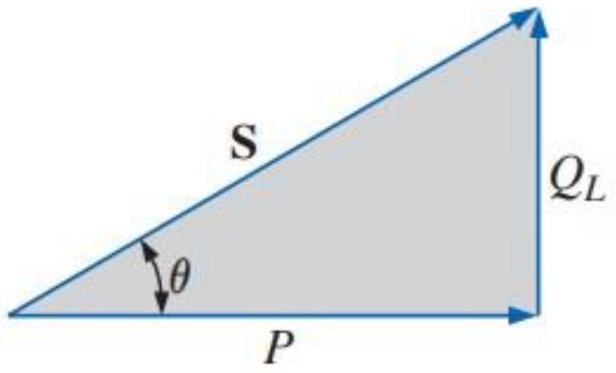


FIG. 19.10
Power diagram for inductive loads.

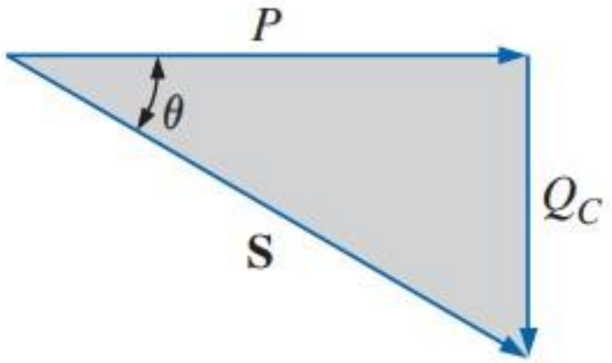


FIG. 19.11
Power diagram for capacitive loads.

Since the reactive power and average power are always angled 90° to each other, the three powers are related by the Pythagorean theorem; that is,

$$\boxed{S^2 = P^2 + Q^2}$$

Therefore, the third power can always be found if the other two are known.

It is particularly interesting that the equation $\boxed{\mathbf{S} = \mathbf{V}\mathbf{I}^*}$ will provide the vector form of the apparent power of a system. Here, V is the voltage across the system, and I* is the complex conjugate of the current.

Consider, for example, the simple R-L circuit of Fig. 19.15, where

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_T} = \frac{10\text{ V}\angle 0^\circ}{3\ \Omega + j\,4\ \Omega} = \frac{10\text{ V}\angle 0^\circ}{5\ \Omega\angle 53.13^\circ} = 2\text{ A}\angle -53.13^\circ$$

The real power (the term real being derived from the positive real axis of the

$$P = I^2R = (2\text{ A})^2(3\ \Omega) = 12\text{ W}$$

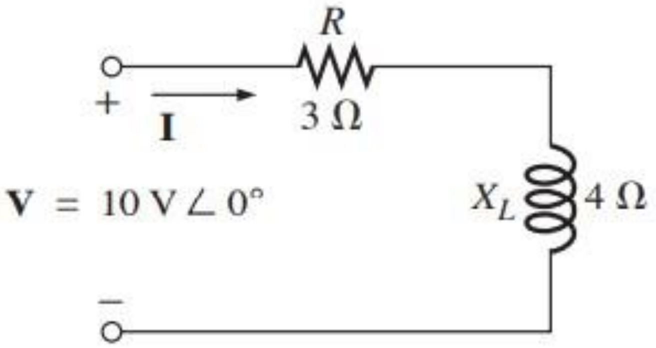


FIG. 19.15

$$Q_L = I^2X_L = (2\text{ A})^2(4\ \Omega) = 16\text{ VAR }(L)$$

complex plane) is and the reactive power is

$$\mathbf{S} = P + j\,Q_L = 12\text{ W} + j\,16\text{ VAR }(L) = 20\text{ VA}\ \angle 53.13^\circ$$

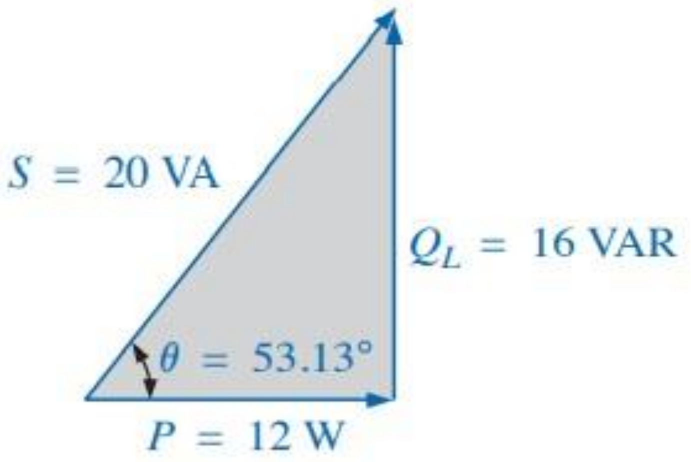


FIG. 19.16
The power triangle for the circuit of Fig. 19.15.

With

as shown in Fig. 19.16.

$$\mathbf{S} = \mathbf{VI}^* = (10\ \text{V} \angle 0°)(2\ \text{A} \angle +53.13°) = 20\ \text{VA} \angle 53.13°$$

The angle v associated with **S** is the power-factor angle of the network. Since

$$P = VI \cos \theta$$

Or $$P = S \cos \theta$$

Then $$\boxed{F_p = \cos \theta = \frac{P}{S}}$$

- Resonance

Resonance is a condition in an RLC circuit in which the capacitive and inductive reactances are equal in magnitude, thereby resulting in a purely resistive impedance.

✦There are two types of resonant circuits: series and parallel.

A resonant circuit (series or parallel) must have an inductive and a capacitive element. A resistive element will always be present due to the internal resistance of the source (R_s), the internal resistance of the inductor (R_l), and any added resistance to control the shape of the response curve (R_{design}).

○Series Resonance

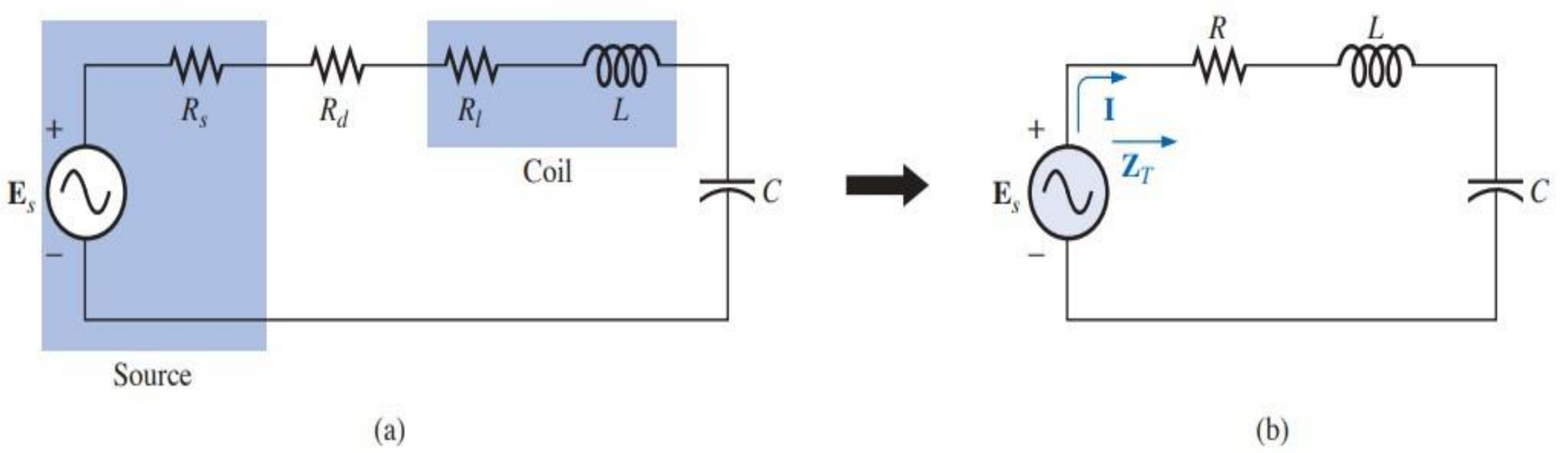


FIG. 20.2

Series resonant circuit.

The “cleaner” appearance of Fig. 20.2(b) is a result of combining the series resistive elements into one total value.

That is, $\boxed{R = R_s + R_l + R_d}$

The total impedance of this network at any frequency is determined by

$$\mathbf{Z}_T = R + j X_L - j X_C = R + j (X_L - X_C)$$

The resonant conditions described in the introduction will

occur when $$\boxed{X_L = X_C}$$

removing the reactive component from the total impedance equation. The total impedance at resonance is then simply

$$\boxed{\mathbf{Z}_{T_s} = R}$$

representing the minimum value of ZT at any frequency. The subscript s will be employed to indicate series resonant conditions.

The resonant frequency can be determined in terms of the inductance and capacitance by examining the defining equation for resonance. $X_L = X_C$

Substituting yields

$$\omega L = \frac{1}{\omega C} \quad \text{and} \quad \omega^2 = \frac{1}{LC}$$

And

$$\boxed{\omega_s = \frac{1}{\sqrt{LC}}}$$

Or

$$\boxed{f_s = \frac{1}{2\pi\sqrt{LC}}}$$

f = hertz (Hz)
L = henries (H)
C = farads (F)

The current the circuit at resonance is

$$\mathbf{I} = \frac{E \angle 0°}{R \angle 0°} = \frac{E}{R} \angle 0°$$

which you will note is the maximum current for the circuit of Fig. 20.2 for an applied voltage E since Z_T is a minimum value. Consider also that the input voltage and current are in phase at resonance.

Since the current is the same through the capacitor and inductor, the voltage across each is equal in magnitude but 180° out of phase at resonance:

$$\left.\begin{aligned}\mathbf{V}_L &= (I\angle 0^\circ)(X_L\angle 90^\circ) = IX_L\angle 90^\circ \\ \mathbf{V}_C &= (I\angle 0^\circ)(X_C\angle -90^\circ) = IX_C\angle -90^\circ\end{aligned}\right\}\begin{array}{l}180^\circ\\ \text{out of}\\ \text{phase}\end{array}$$

and, since $X_L = X_C$, the magnitude of V_L equals V_C at resonance;

that is, $\boxed{V_{L_s} = V_{C_s}}$

The average power to the resistor at resonance is equal to I^2R,

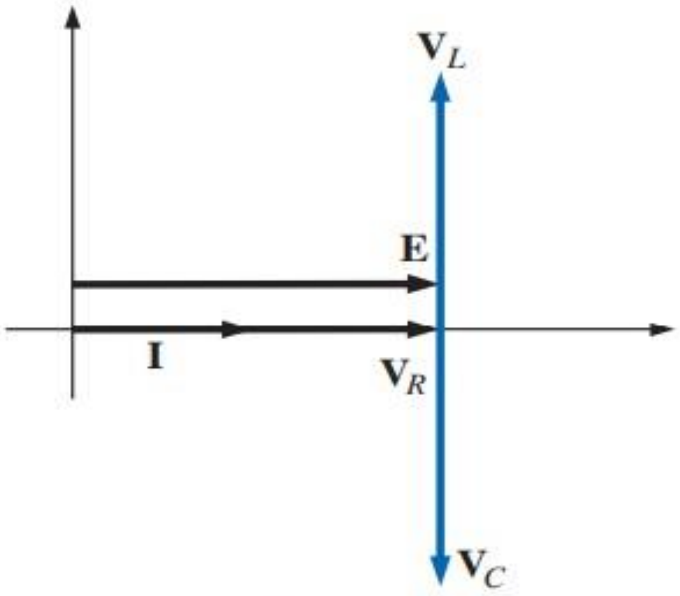


FIG. 20.3
Phasor diagram for the series resonant circuit at resonance.

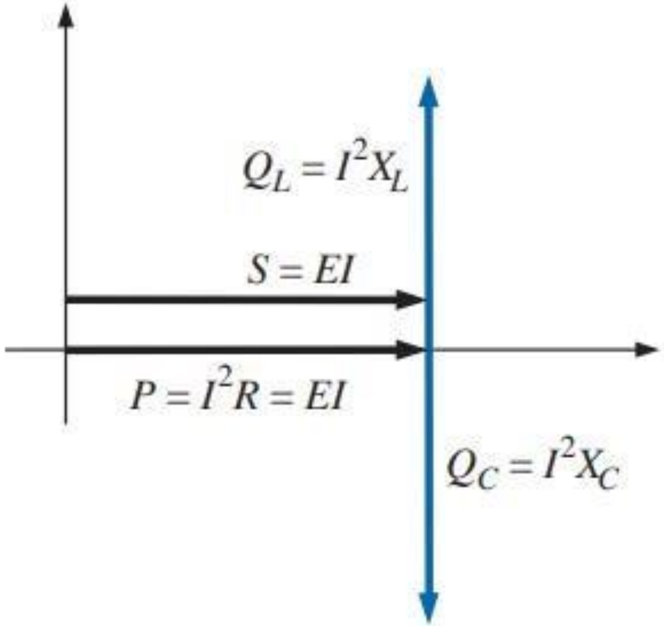


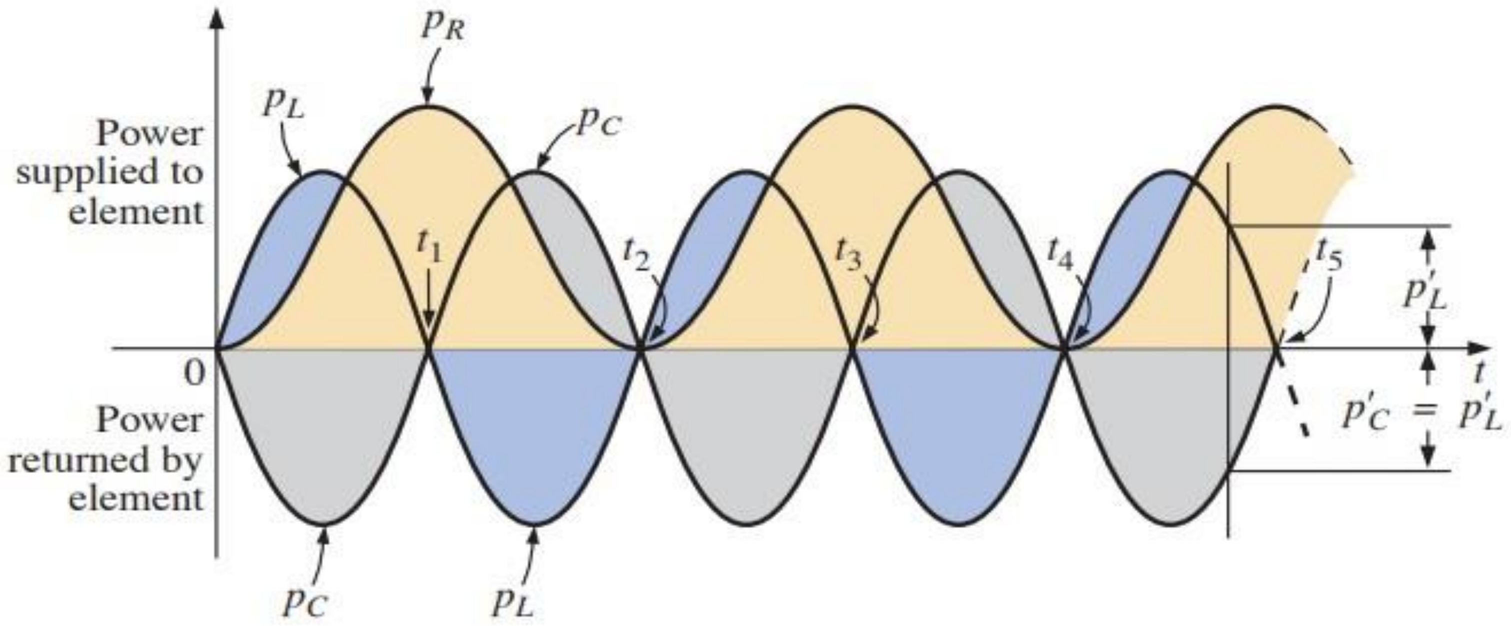
FIG. 20.4
Power triangle for the series resonant circuit at resonance.

and the reactive power to the capacitor and inductor are I^2X_C and I^2X_L, respectively.

The power factor of the circuit at resonance is $F_p = \cos\theta = \frac{P}{S}$

And $\boxed{F_{p_s} = 1}$

Power curves at resonance for the series resonant circuit.



Power
supplied to
element
Power
returned by
element
0
p_R
p_L
p_C
t_1
t_2
t_3
t_4
t_5
p'_L
$p'_C = p'_L$
t
p_C
p_L

THREE-PHASE SYSTEMS

An ac generator designed to develop a single sinusoidal voltage for each rotation of the shaft (rotor) is referred to as a single-phase ac generator.

✦ If the number of coils on the rotor is increased in a specified manner, the result is a polyphase ac generator, which develops more than one ac phase voltage per rotation of the rotor.

✦Circuits or systems in which the ac sources operate at the same frequency but different phases are known as polyphase.

THREE-PHASE SYSTEMS

A three-phase system is produced by a generator consisting of three sources having the same amplitude and frequency but out of phase with each other by 120◦.

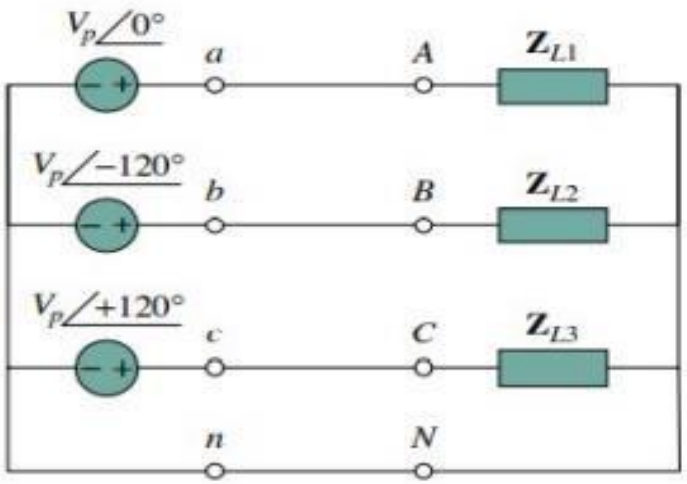


Figure 4.1: Three-phase four-wire system

The figure below depicts a three-phase four-wire system, where V_p is the magnitude of the source voltage and ϕ is the phase.

- The Three-Phase Generator

The three-phase generator of Fig. 4.2 has three induction coils placed 120° apart on the stator, as shown symbolically by Fig. 4.3.

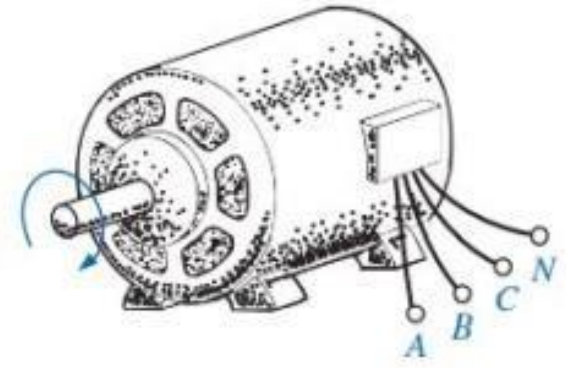


Figure 4.2: Three-phase generator

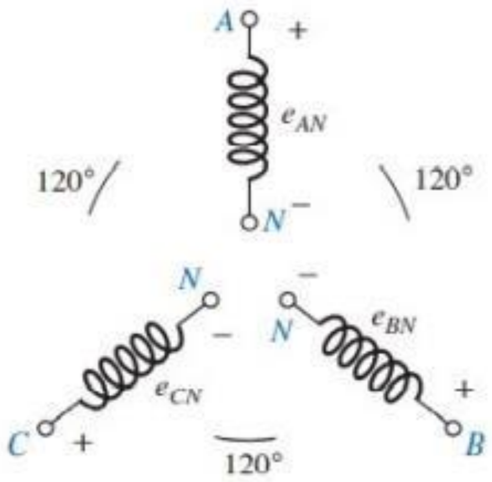


Figure 4.3: Induced voltages of a three-phase generator

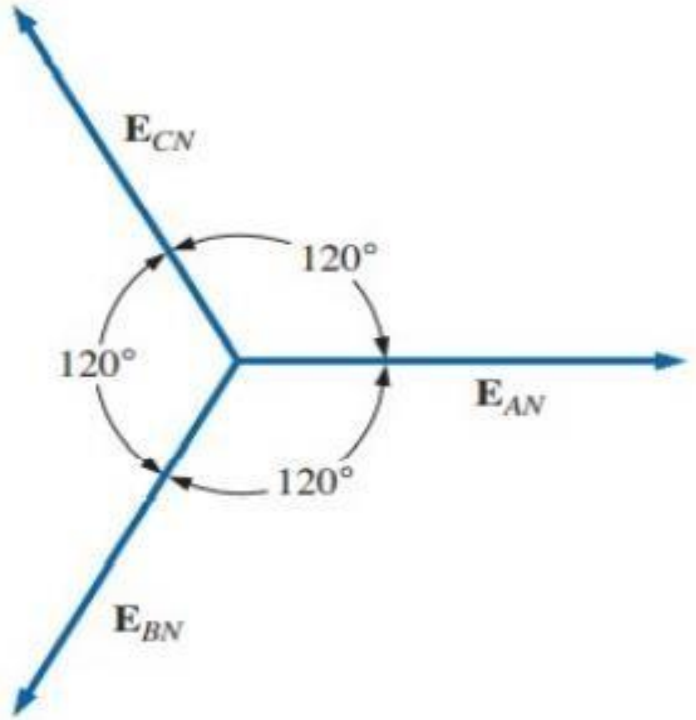
Since the three coils have an equal number of turns, and each coil rotates with the same angular velocity, the voltage induced across each coil will have the same peak value, shape, and frequency.

✦In particular, note that at any instant of time, the algebraic sum of the three phase voltages of a three-phase generator is zero.

The sinusoidal expression for each of the induced voltages;

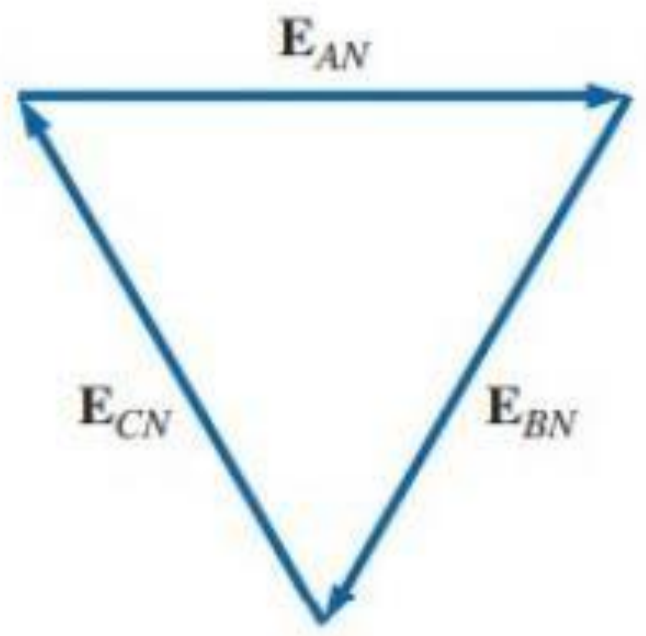
THREE-PHASE SYSTEMS

$$e_{AN} = E_{m(AN)} \sin \omega t$$
$$e_{BN} = E_{m(BN)} \sin(\omega t - 120^\circ)$$
$$e_{CN} = E_{m(CN)} \sin(\omega t - 240^\circ) = E_{m(CN)} \sin(\omega t + 120^\circ)$$



$$\mathbf{E}_{AN} = E_{AN}\angle 0^\circ$$
$$\mathbf{E}_{BN} = E_{BN}\angle -120^\circ$$
$$\mathbf{E}_{CN} = E_{CN}\angle +120^\circ$$

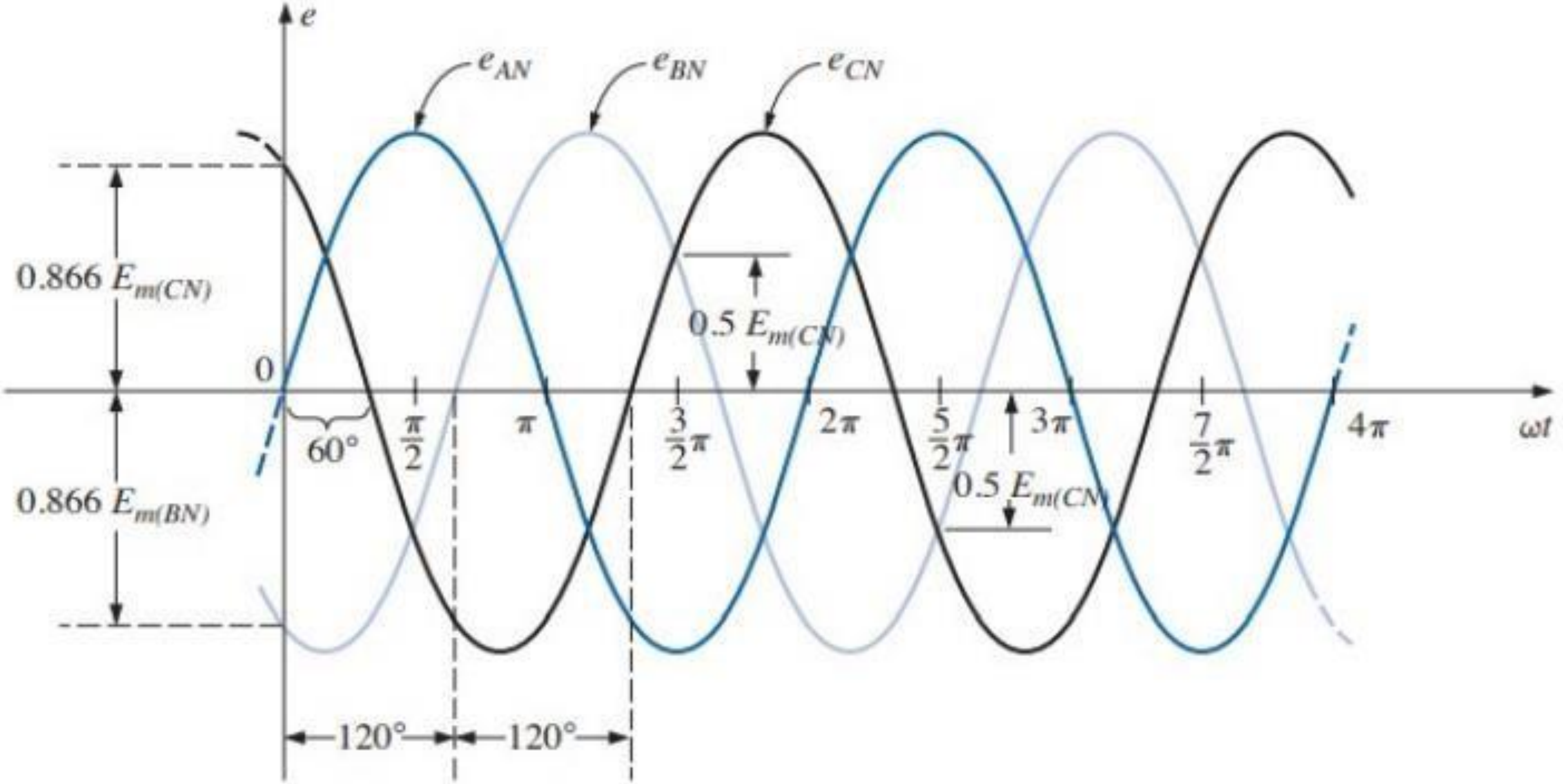
We can conclude that the phasor sum of the phase voltages in a three-phase system is zero. That is,



$$E_{AN} + E_{BN} + E_{CN} = 0$$

THREE-PHASE SYSTEMS

Phase voltage of a three-phase generator.


e
e_{AN}
e_{BN}
e_{CN}
$0.866\ E_{m(CN)}$
$0.866\ E_{m(BN)}$
0
60°
$\frac{\pi}{2}$
π
$\frac{3}{2}\pi$
2π
$\frac{5}{2}\pi$
3π
$\frac{7}{2}\pi$
4π
ωt
$0.5\ E_{m(CN)}$
$0.5\ E_{m(CN)}$
120°
120°