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DEEE 101:Basic Electricity

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Course content

- **The course is designed to equip students with fundamental of network theorems in DC circuits**

- **Introduction to Basic circuit elements Energy storage and dissipation. Capacitive, inductive and resistive circuits and it application. Self-inductance, mutual inductance.**
- ***Network Theorems*:**

  Kirchoff's Laws, superposition, Thevenin's, Norton's, Delta-star and star-delta transformations.

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- ***Alternating Voltage and Current*: Average and r.m.s values, harmonics, phasor representation of sinusoidal quantities, computation of sinusoidal quantities.**
- ***A.C. Circuits*: Active, reactive and apparent power, power factor, reactive and active loads and sources, solving single phase circuits using j operator and the concept of apparent power, solving 3-phase balanced and unbalanced loads.**

Reference material

- **Lecture note**

✞P. Y. Okyere, E. A. Frimpong, Fundamentals of Electric and Magnetic Circuits

✞J. W. Nilsson and S. A. Riedel, Electric circuits, Prentice hall, 7th ed., 2005

✞R. Boylestad, Introductory circuit analysis, Prentice Hall, 11th ed., 2007

- **Hughes E. , Electrical and Electronics Technology(10ed.,Pearson Education), ISBN 8131733661, 9788131733660**
- **Theraja B.L, Theraja R.K, A Text Book of Electrical Technology, ISBN 8121924413, 9788121924412**
- **Area of application: Basics of any electrical engineering career, ie Power, control etc**

Mode of delivery and assessment

➢ Mode of delivery

✦ Regular lectures/tutorial

➢ Assessment

- ✦ Assignments – 15%
- ✦ Announced/unannounced quizzes- 10%
- ✦ Mid-semester exam- 15%
- ✦ End of semester exam-60%

Basic electrical terms

- AC and DC: Abbreviations for alternating current and direct current respectively.
- *Current* - A movement of electricity analogous to the flow of a stream of water.
- ***Direct Current*** - An electric current flowing in one direction only (i.e. current produced using a battery).

- ***Alternating Current*** - a periodic electric current that reverses its direction at regular interval
- : The unit of intensity of electrical current (the measure of electrical flow), ***Amp or Ampere*** is abbreviated a or A.
- ***Circuit***: A complete path from the energy source through conducting bodies and back to the energy source. Or An interconnection of elements forming a closed path along which current can flow.

Basic circuit elements

- Basic elements of a circuit include:
  - Voltage sources
  - Current sourcesActive sources

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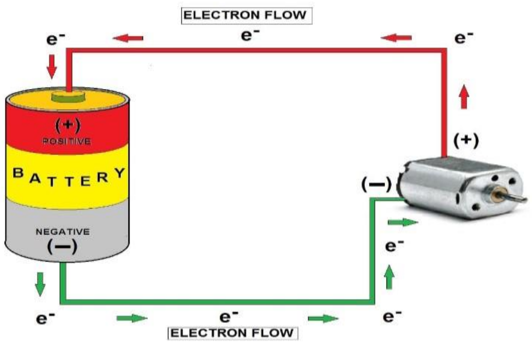
- Resisters
- Capacitors

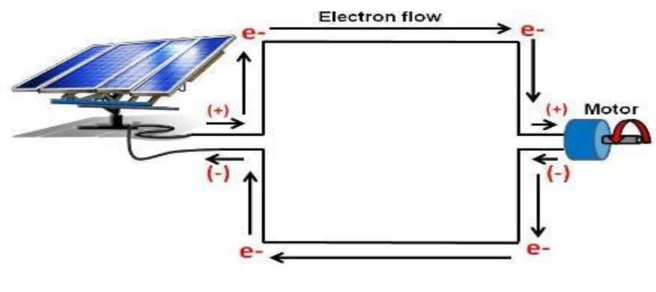
  passive sources
- inductors

Voltage, Resistance, current

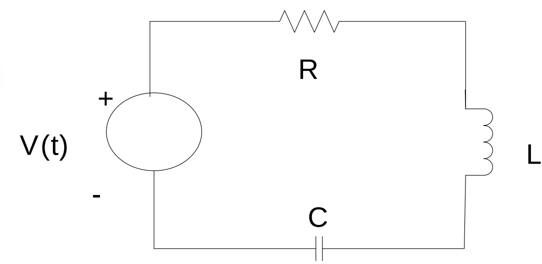
Voltage is the amount of energy a charge moves electrons from one point to another in a circuit. It is measured in volts (v)

Resistance is the material's ability to impede the flow of current. it is measured in ohms.





Solar Cell Circuit



Current is the rate of charge flow and it is measured in amperes (A).

Electric Circuit

- Elements of an electric circuit

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Active elements: Energy producing elements eg. Batteries, Generators, Solar cells, Transistor models

Passive elements: Energy using elements eg.
Resistors, inductors, capacitors

Circuit Basics

All electrical circuits require three elements.

(1) A source voltage, that is, an electron pump usually a battery or power supply. [ENERGY IN]

(2) A conductor to carry electrons from and to the voltage source (pump). The conductor is often a wire. [ENERGY TRANSFER]

(3) A load or resistance. A point where energy is extracted form the circuit in the form of heat, light, motion, etc. [ENERGY OUT]

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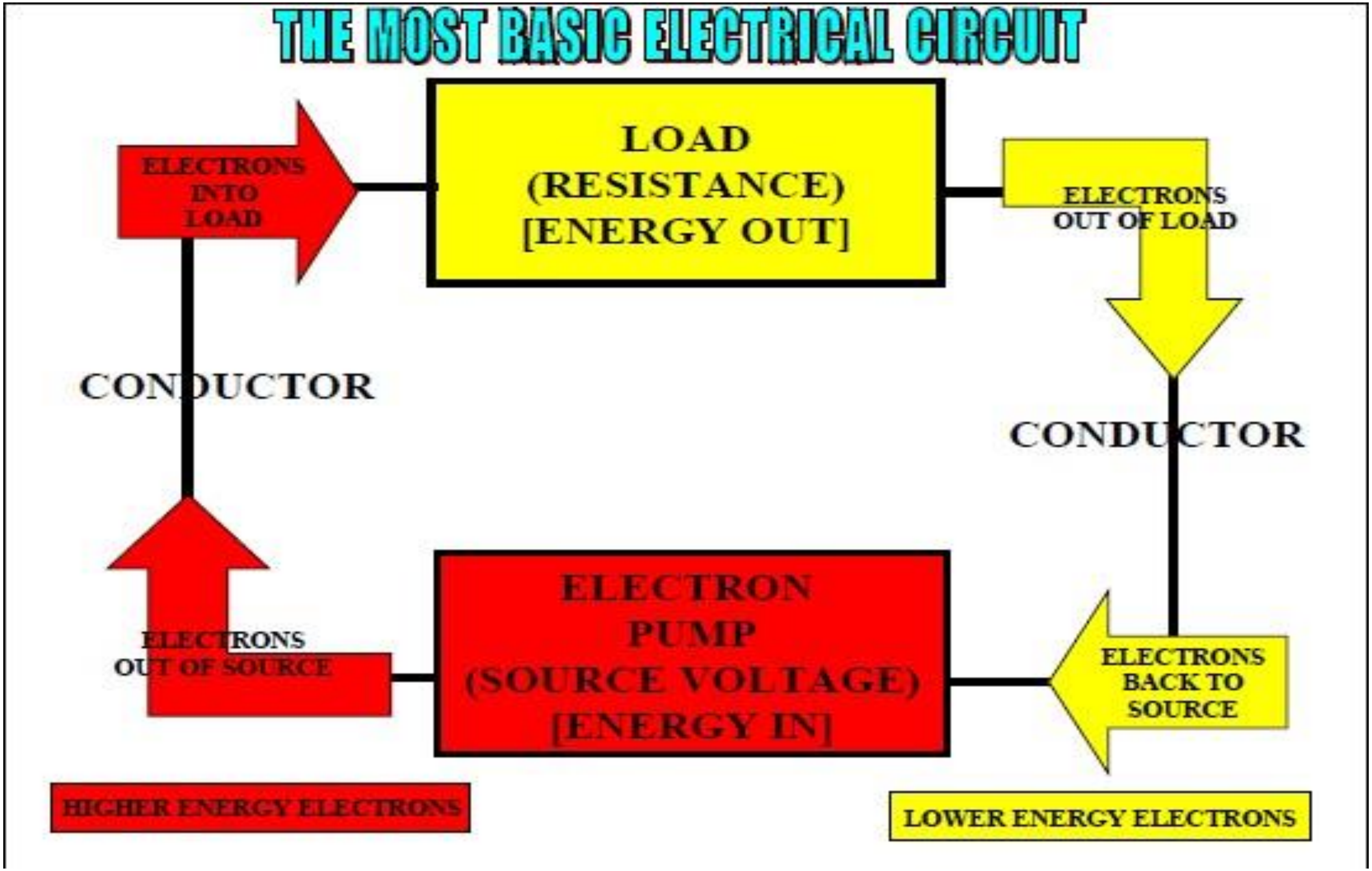
THE MOST BASIC ELECTRICAL CIRCUIT
ELECTRONS INTO LOAD
LOAD (RESISTANCE) [ENERGY OUT]
ELECTRONS OUT OF LOAD
CONDUCTOR
CONDUCTOR
ELECTRONS OUT OF SOURCE
ELECTRON PUMP (SOURCE VOLTAGE) [ENERGY IN]
ELECTRONS BACK TO SOURCE
HIGHER ENERGY ELECTRONS
LOWER ENERGY ELECTRONS

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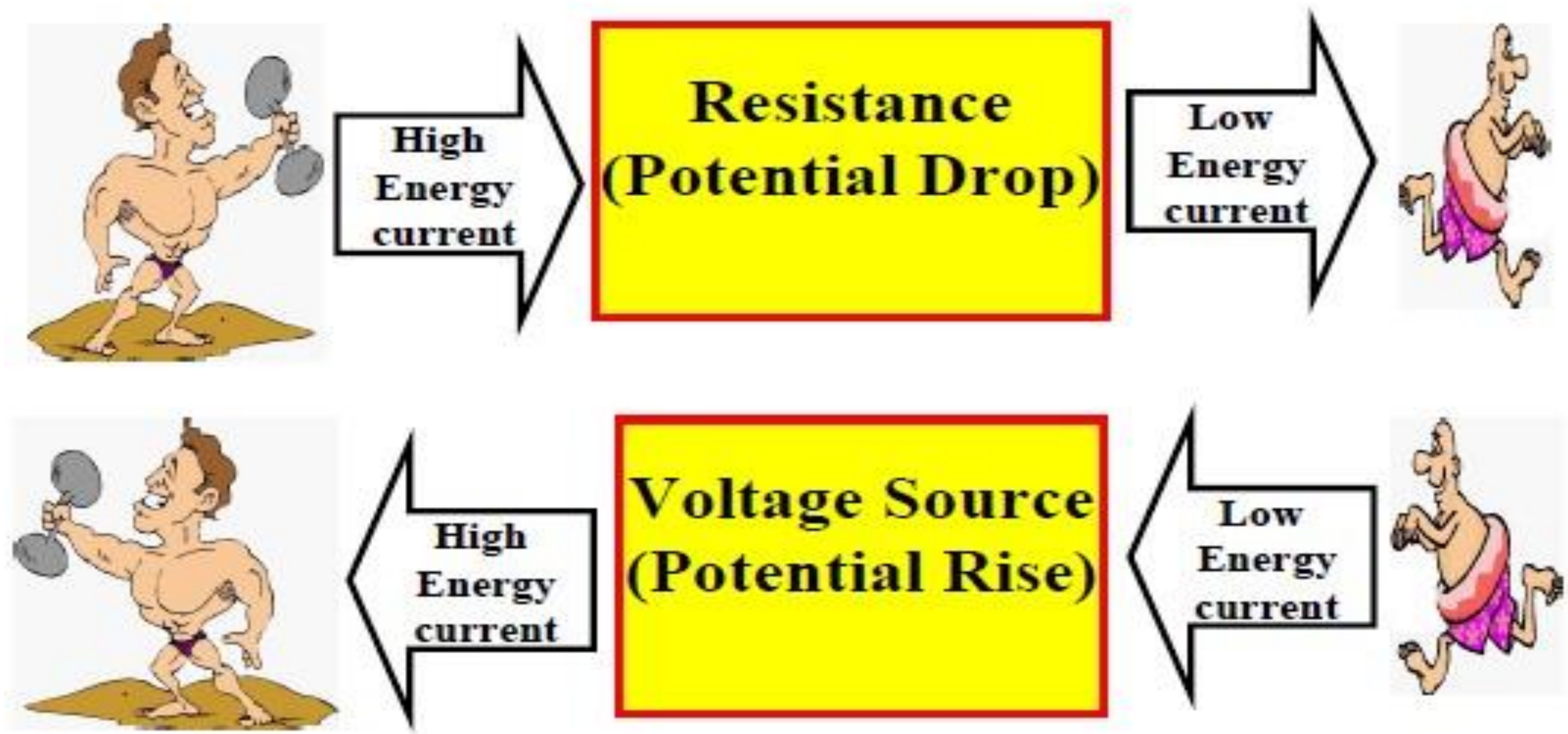
Potential Changes of Current in a Circuit



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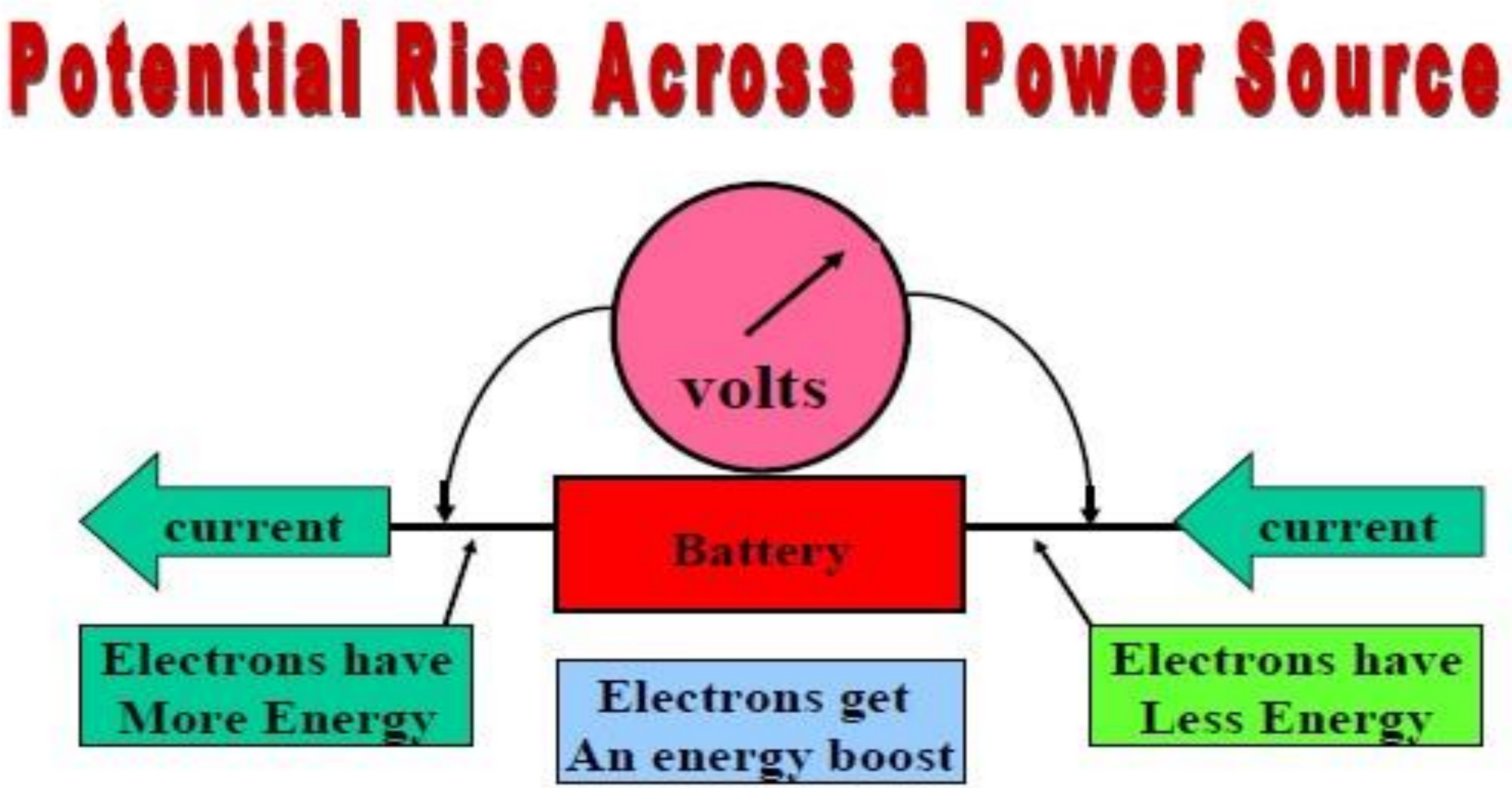
Potential Rise Across a Power Source
volts
current
Battery
current
Electrons have
More Energy
Electrons get
An energy boost
Electrons have
Less Energy

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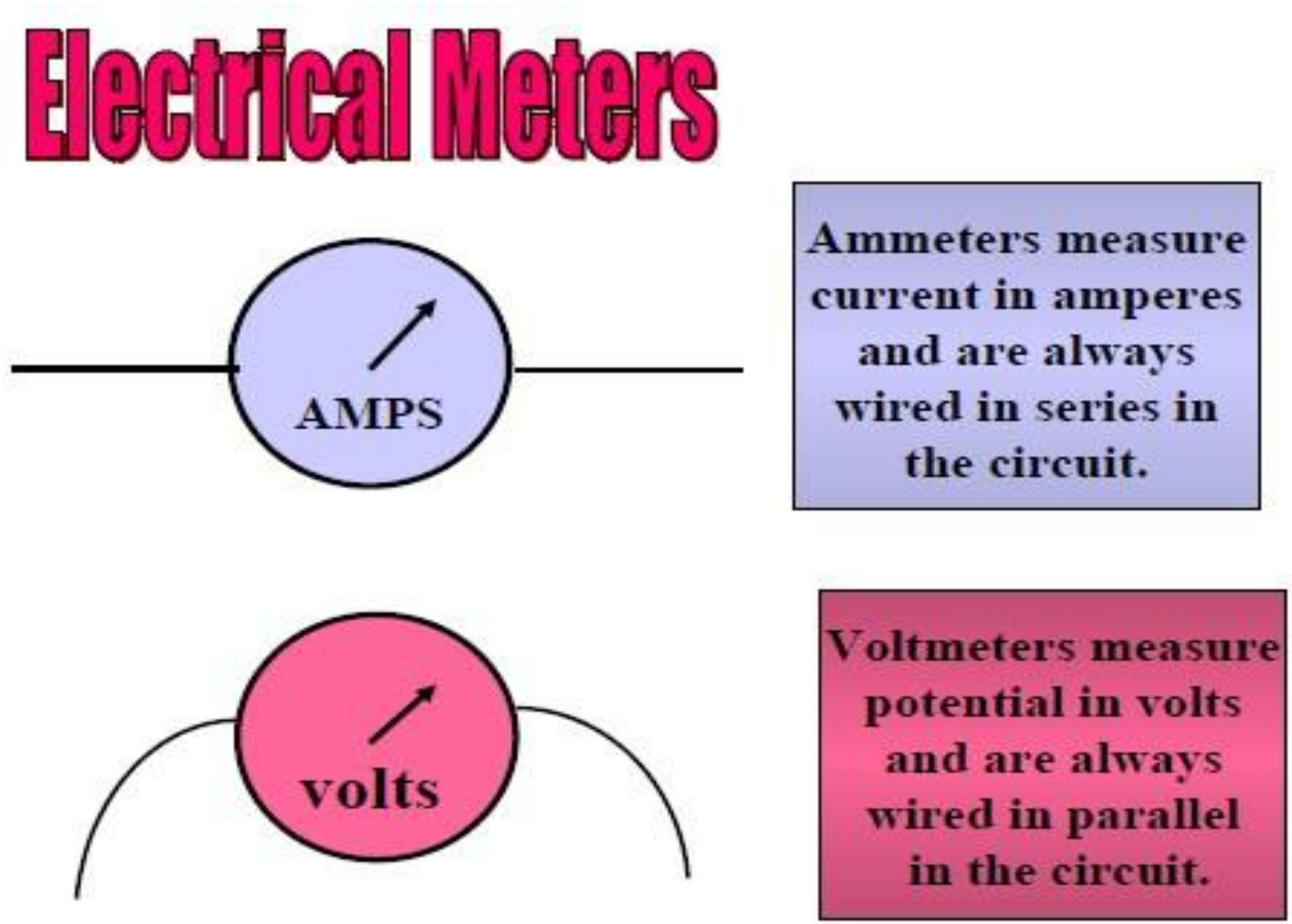
MEASUREABLE QUANTITIES THAT CAN BE OBTAINED FROM AN ELECTRICAL CIRCUIT

(1) VOLTAGE RISE – MEASURES THE ENERGY GIVEN TO ELECTRONS AS THEY LEAVE A VOLTAGE SOURCE. IT IS MEASURED IN VOLTS (+)

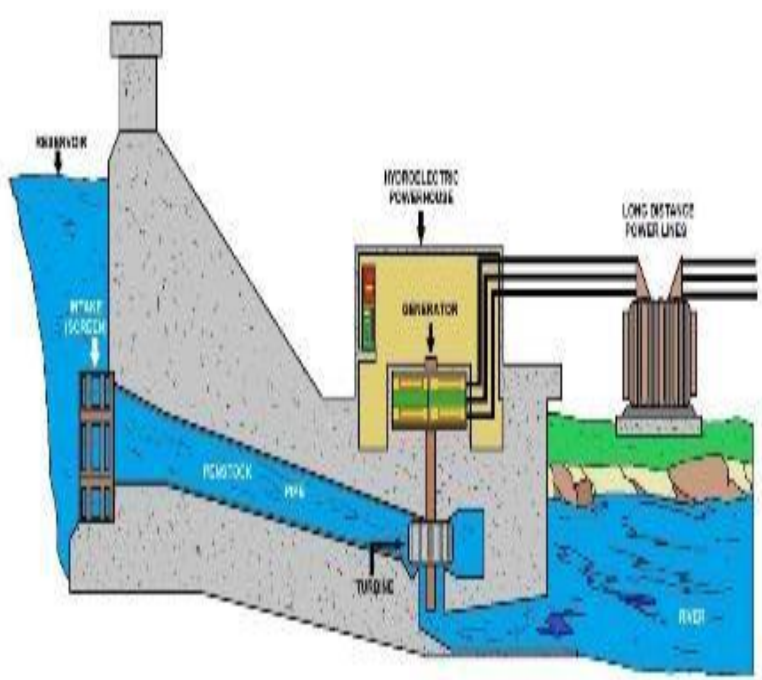
(2) VOLTAGE DROP – MEASURES THE ENERGY LOST BY TO ELECTRONS WHEN THEY LEAVE A RESISTANCE. IT IS MEASURED IN VOLTS (-)

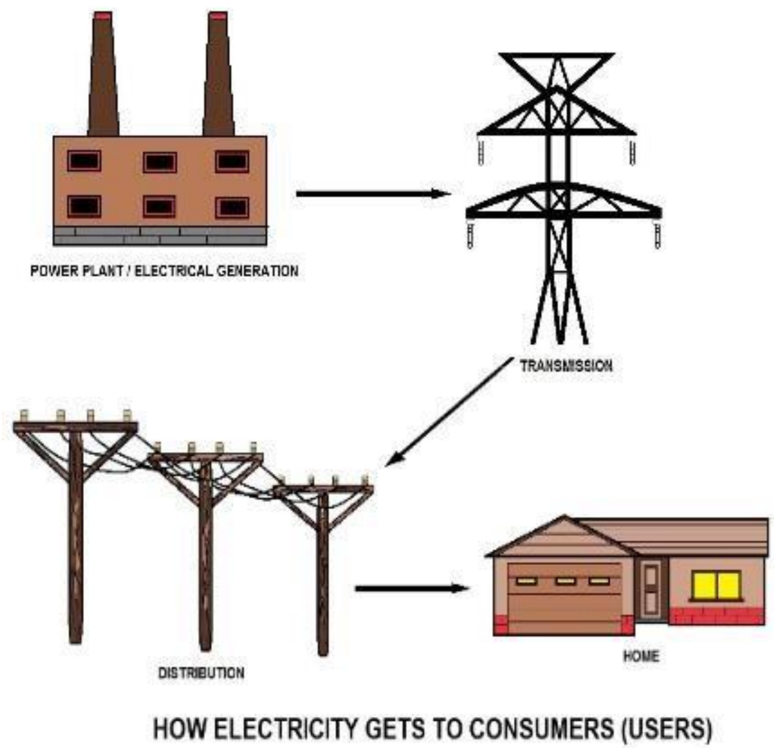
(3) CURRENT – MEASURES THE FLOW RATE THROUGH A CONDUCTOR. IT IS MEASURED IN AMPERES (AMPS)

(4)RESISTANCE – MEASURES THE OPPOSITION TO CURRENT FLOW THROUGH A CONDUCTOR OR RESISTOR
IT IS MEASURED IN OHMS (ITS SYMBOL IS OMEGA)

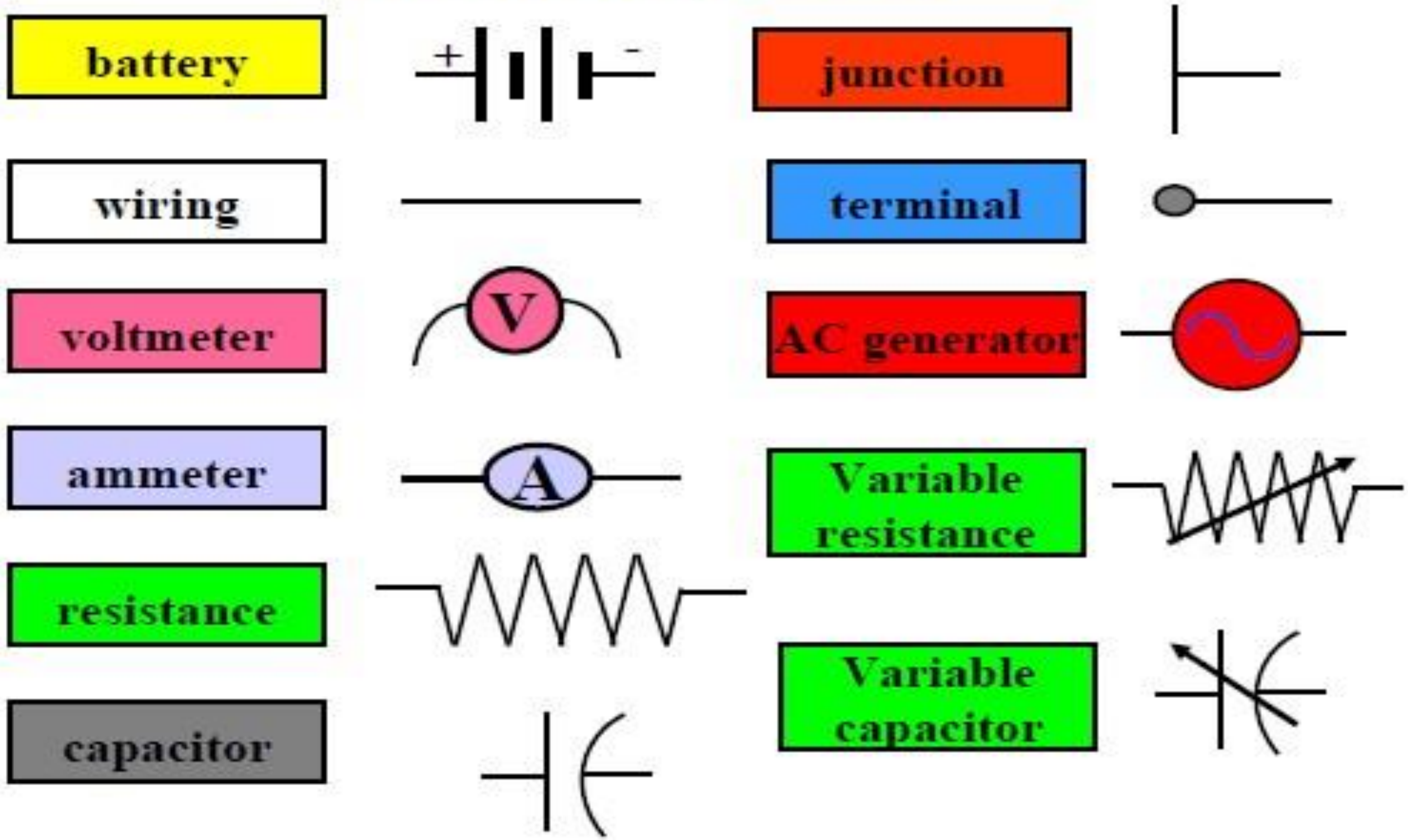

Electrical Meters
AMPS
Ammeters measure current in amperes and are always wired in series in the circuit.
volts
Voltmeters measure potential in volts and are always wired in parallel in the circuit.

Circuit for Electricity delivery to homes



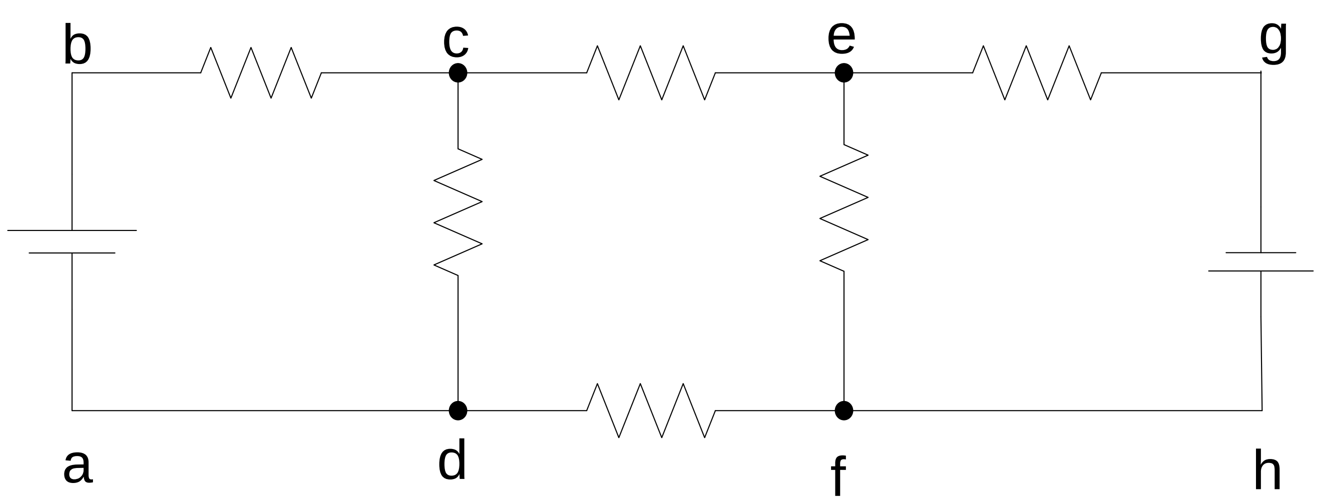


Electrical Symbols

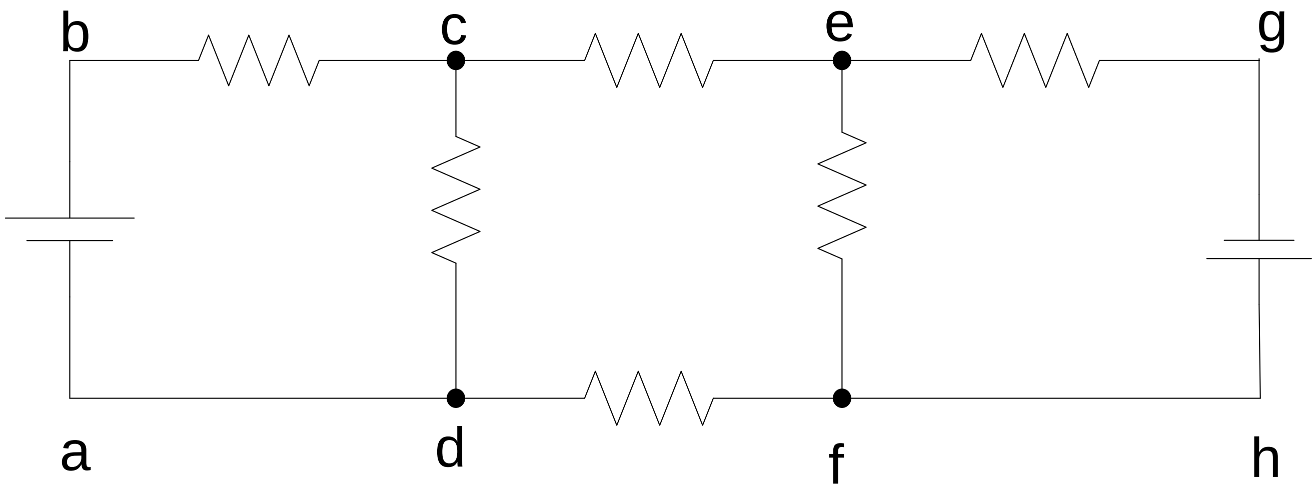


CIRCUIT TERMINOLOGIES

- **Node (Junction)** –A point where currents split or come together [points c, d, e and f]
- **Path** – Any connection where current flows [eg bc, be, fa]

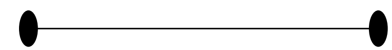


- **Branch** – A connection (path) between two nodes [eg. cd, cbad, df]
- **Loop/Mesh** – a closed path of a circuit [eg. cghdc]



CIRCUIT TERMINOLOGIES

- **Short-circuit**– A branch of theoretically zero resistance. It diverts to itself all currents that would have flown in adjacent branches (branches hooked to the same node) except branches with sources.



a b R_1 c R_3 d R_1 a R_2 R_4

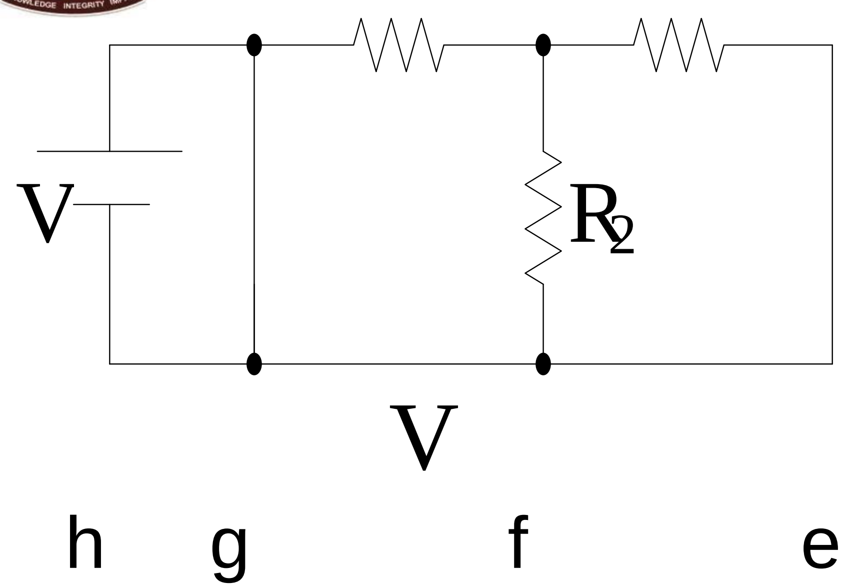


Fig. 1

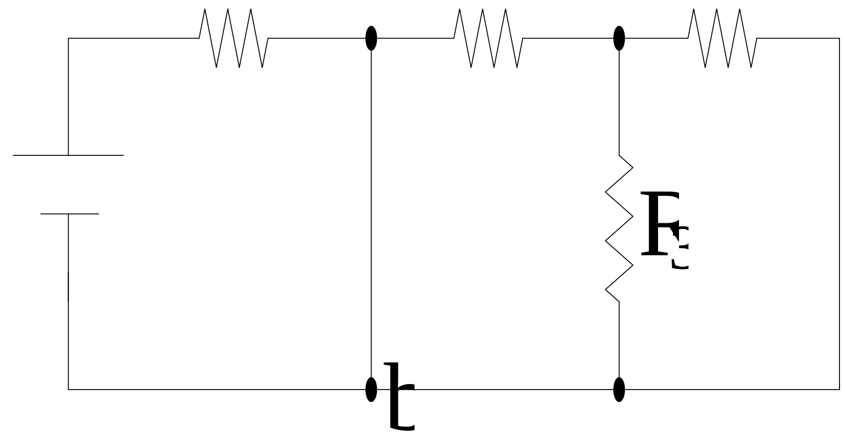


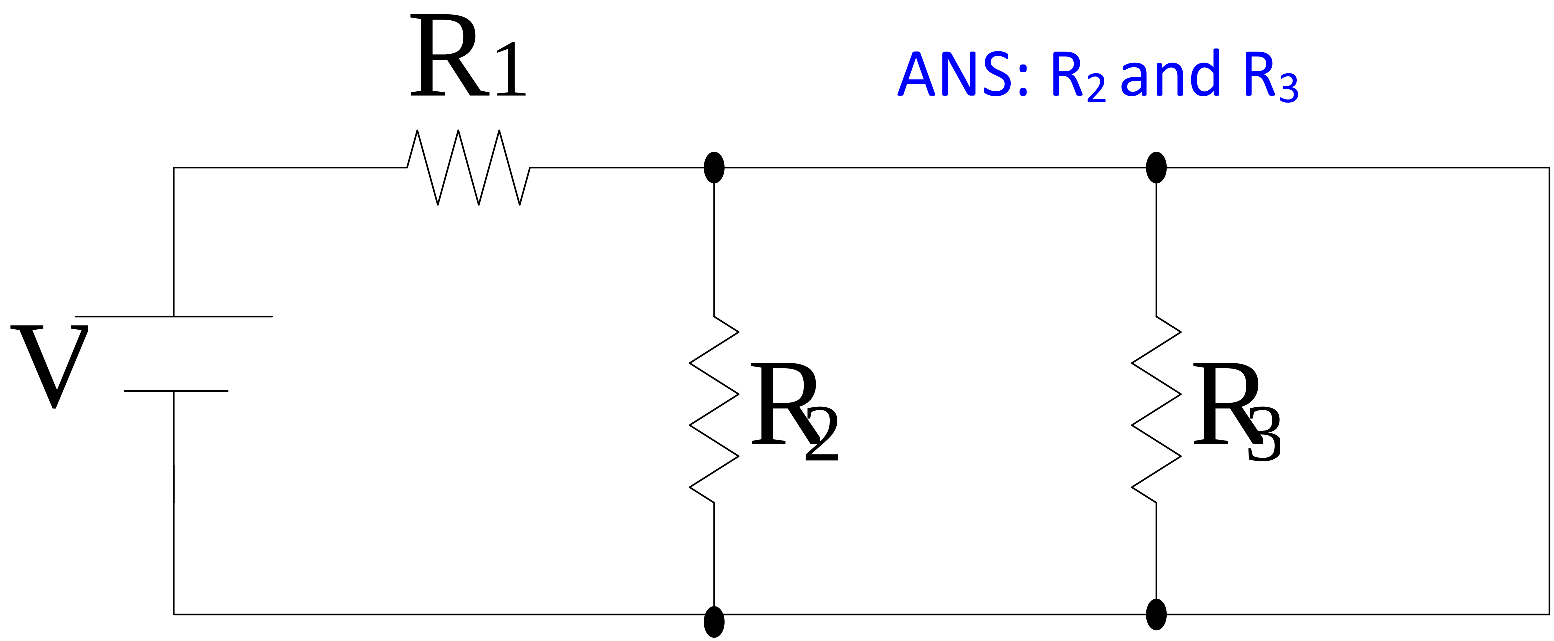
Fig. 2

Short-circuit *cont.*

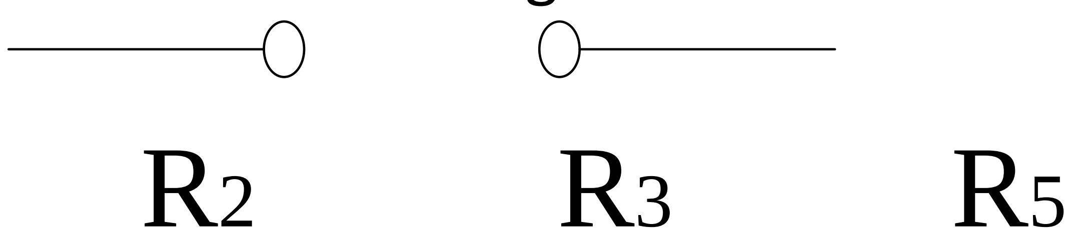
Self assessment

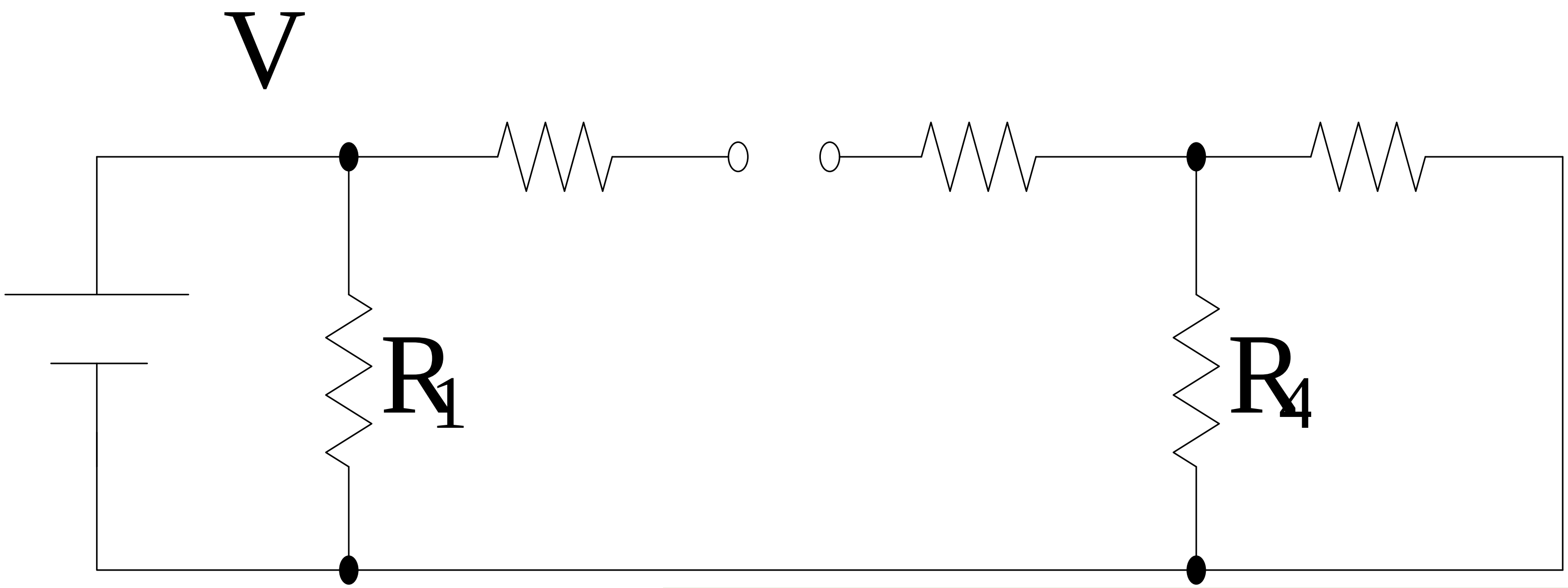
Which of the resistors in the circuit below have been shortcircuited?

ANS: R_2 and R_3

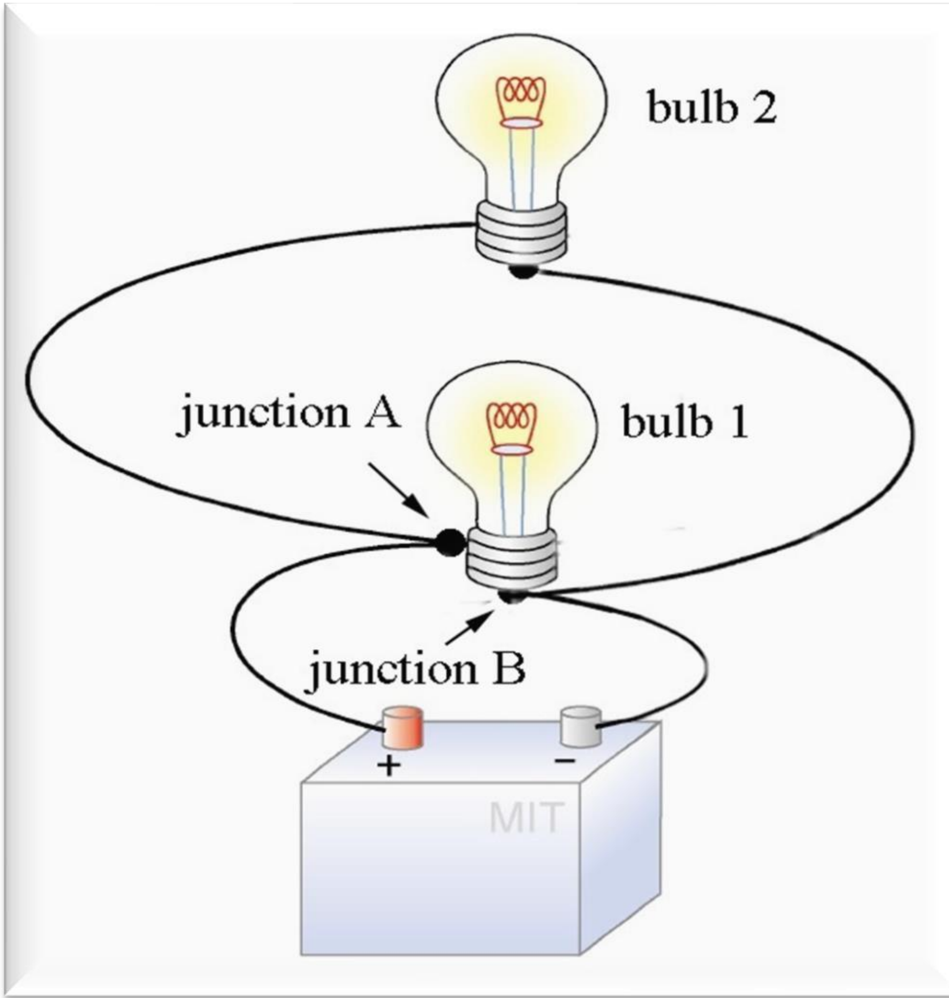


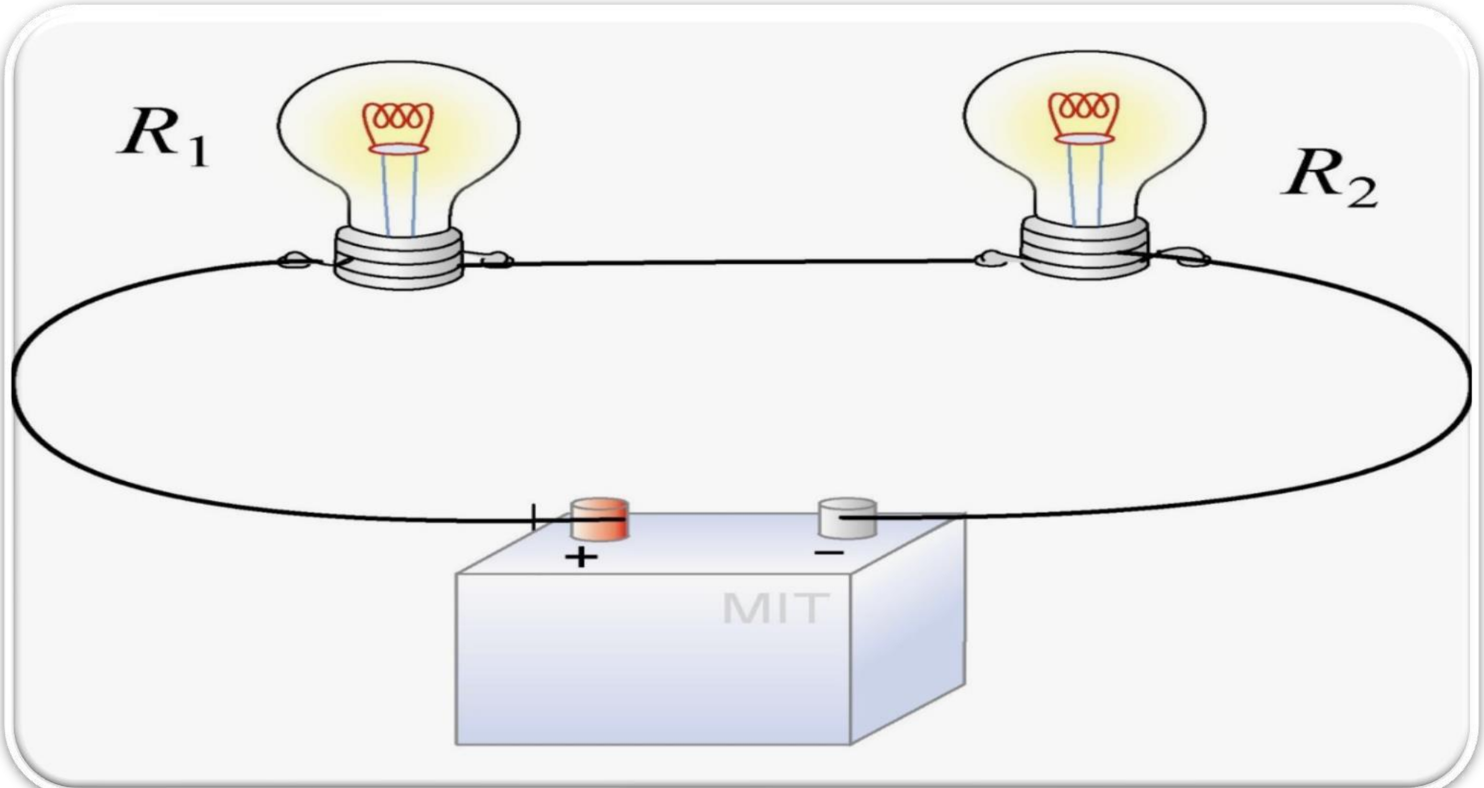
- Open circuit – A branch of theoretically infinite resistance. It prevents current from flowing in its branch.





LOAD/RESISTORS IN PARALLEL


bulb 2
junction A
bulb 1
junction B
+
−
MIT


R_1
R_2
+
−
MIT

Resistors are in series when the same current flows through them. There is NO JUNCTION between them.

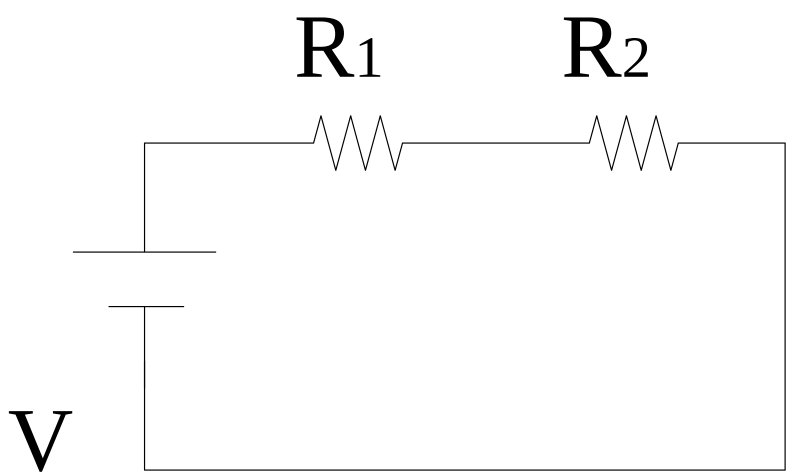


Fig. 1

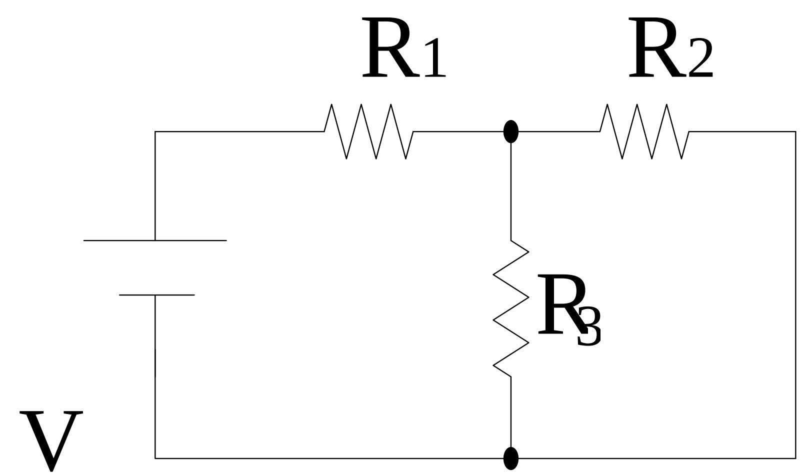


Fig. 2

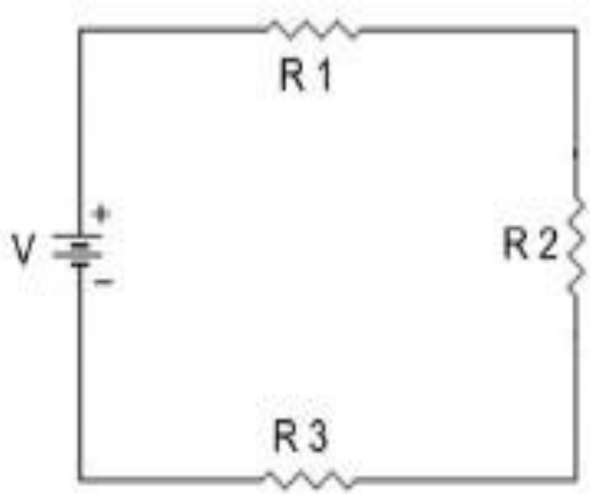
In Fig. 1 : R_1 and R_2 are in series

In Fig. 2: None of the resistors are in series

Resistors in series- Summary

In a series circuit ,

- The current through each of the components is the same, and the voltage across the components is the sum of the voltages across each component



- A series circuit is a circuit in which the current can only flow through one path.
- Current is the same at all points in a series circuit

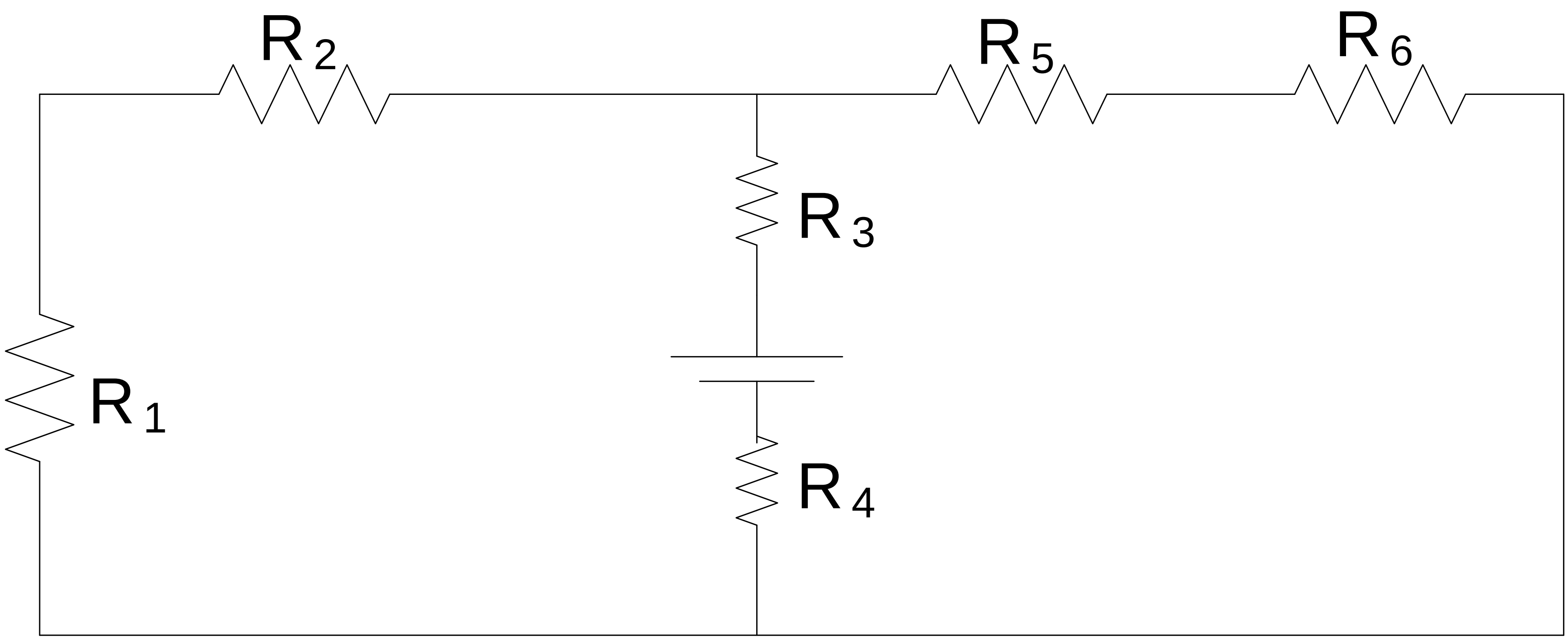
$$R_{total} = R_1 + R_2 + R_3$$

$$V_{total} = V_1 + V_2 + V_3$$

RESISTORS IN SERIES

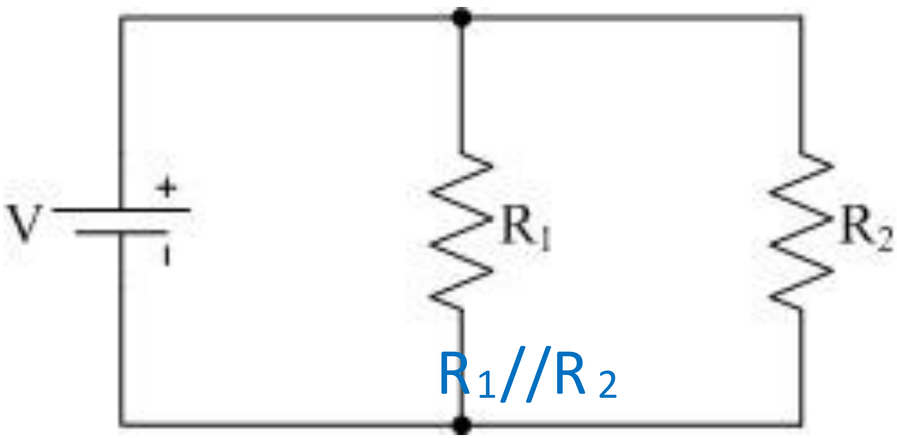
Self assessment 1

Which of the following resistors are in series?

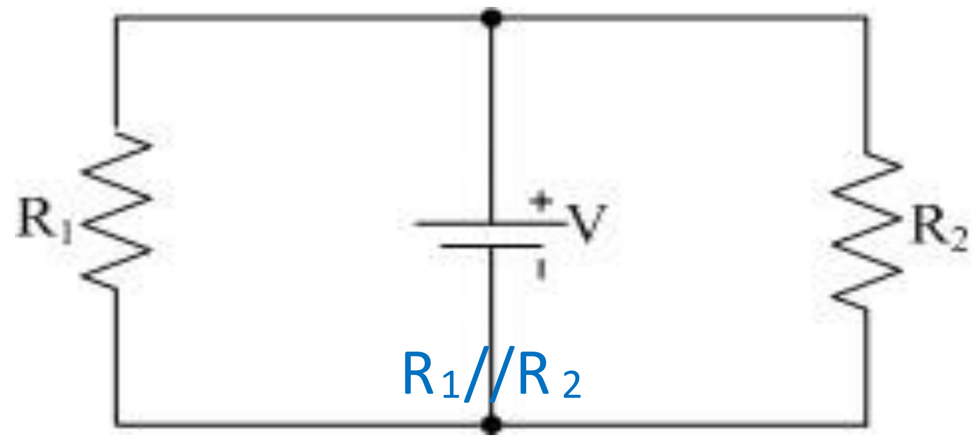


ANS: R_1&R_2, R_3 & R_4 and R_5 & R_6

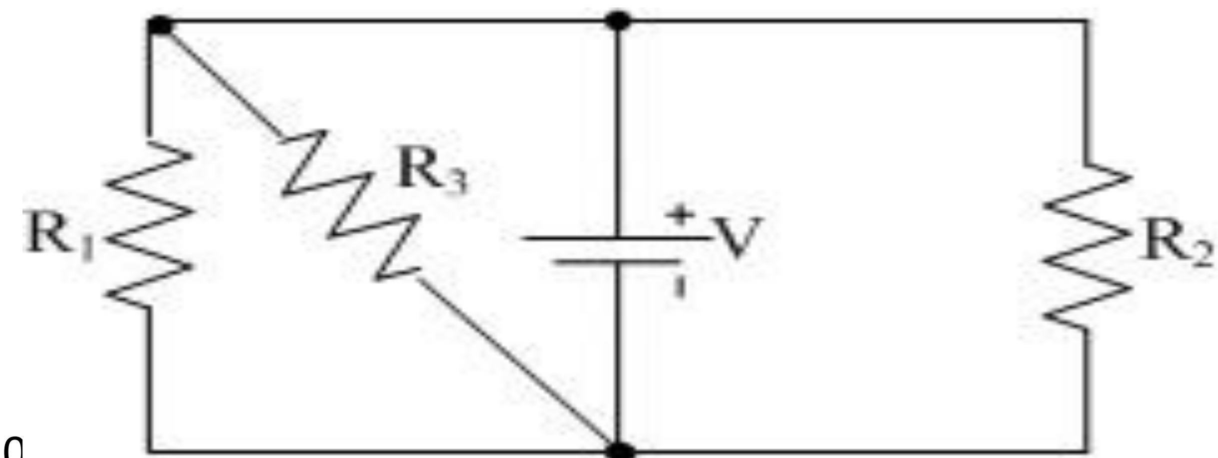
RESISTORS IN PARALLEL



$R_1//R_2$



$R_1//R_2$



$R_1//R_2$

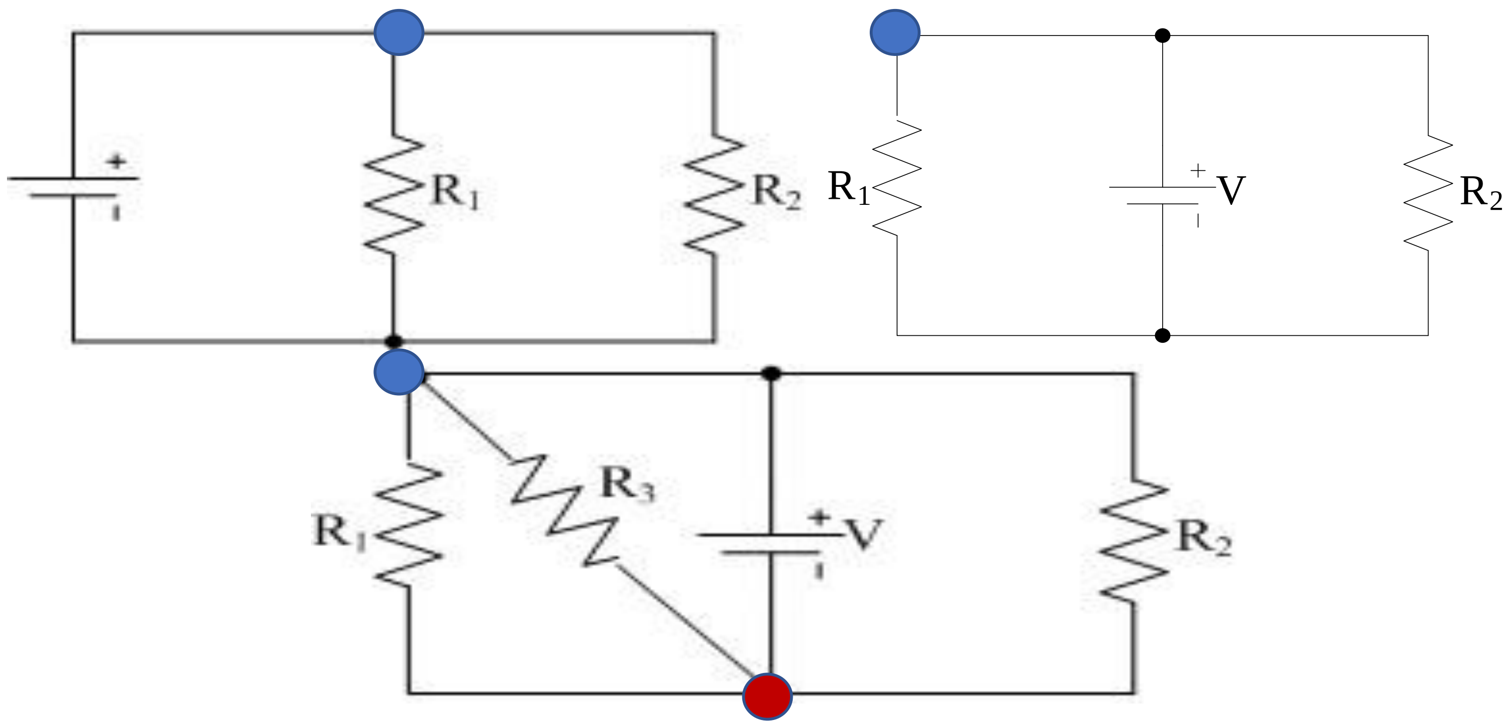
$R_1//R_3$

$R_3//R_2$

Resistors are said to be in parallel when the voltage across them is the same.

RESISTORS IN PARALLEL

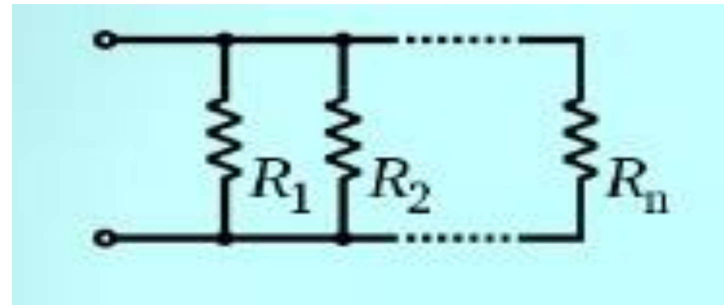
TWO resistors are in parallel if it is possible to traverse them without passing through another element.



PARARELL CIRCUITS- summary

➝In a parallel circuit,

- **The voltage across each of the components is the same, and the total current is the sum of the currents through each component.**
- In contrast, in a parallel circuit, there are multiple paths for current flow.
- Different paths may contain different current flow.
- This is also based on Ohms Law

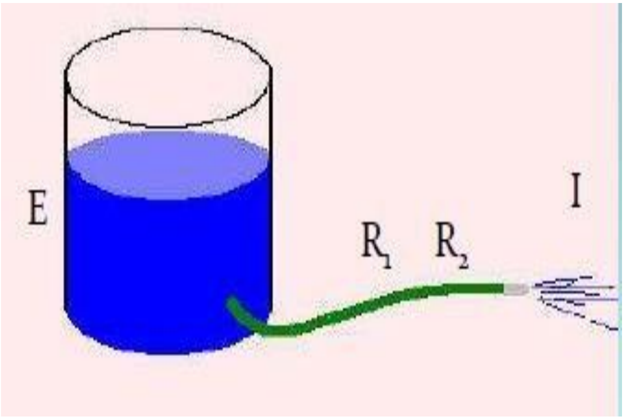


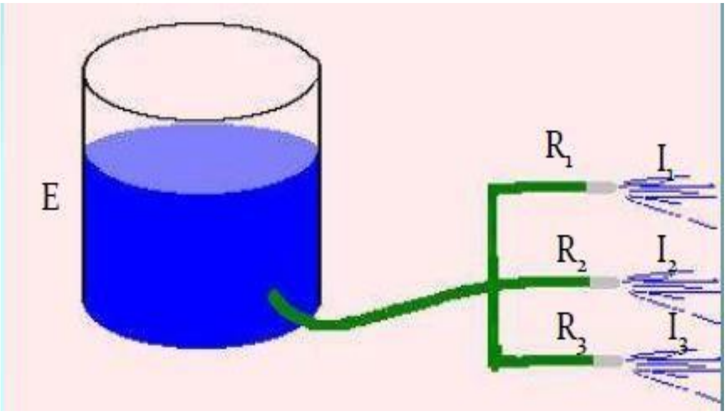
$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_n}$$

$$I_{total} = I_1 + I_2 + I_3$$

- Total resistance will be less than the smallest resistor

SIMPLE ANALOGY OF SERIES/PARARELL CONNECTION





Same current flowing through but different E

Same E available to both R1,R2 and R3 but R1 and R2 different I

Parallel circuits have two distinct advantages over series circuits:

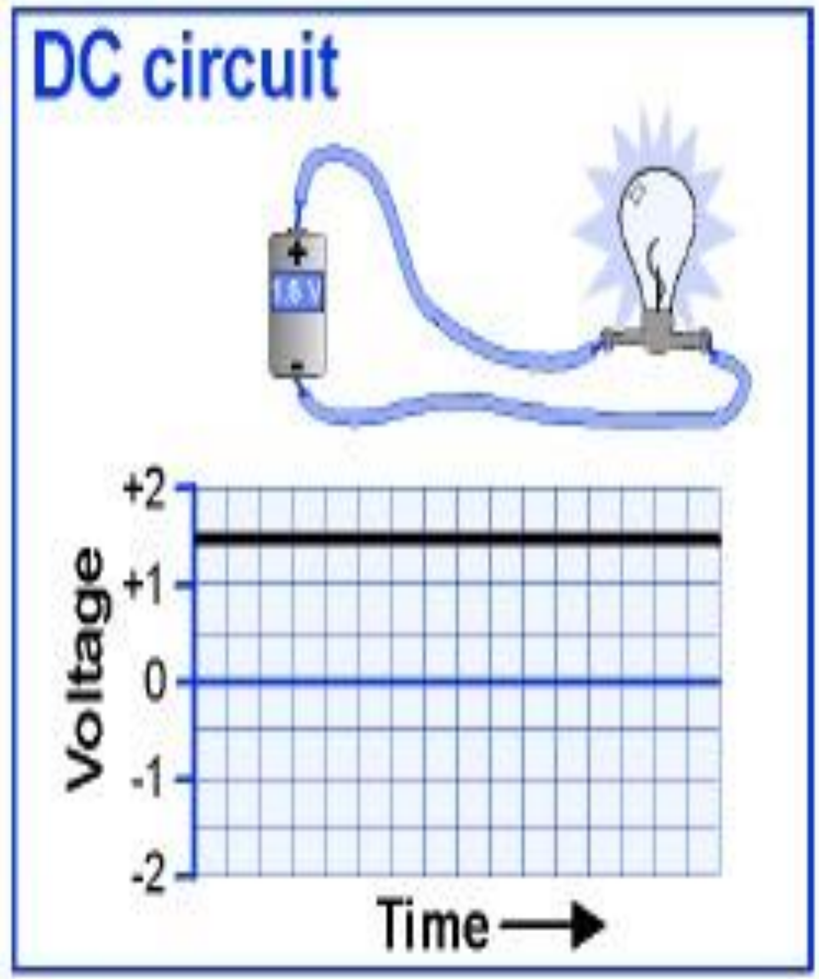
- Each device in the circuit sees the full battery voltage.

- Each device in the circuit may be turned off independently without stopping the current flowing to other devices in the circuit.

Parallel resistors

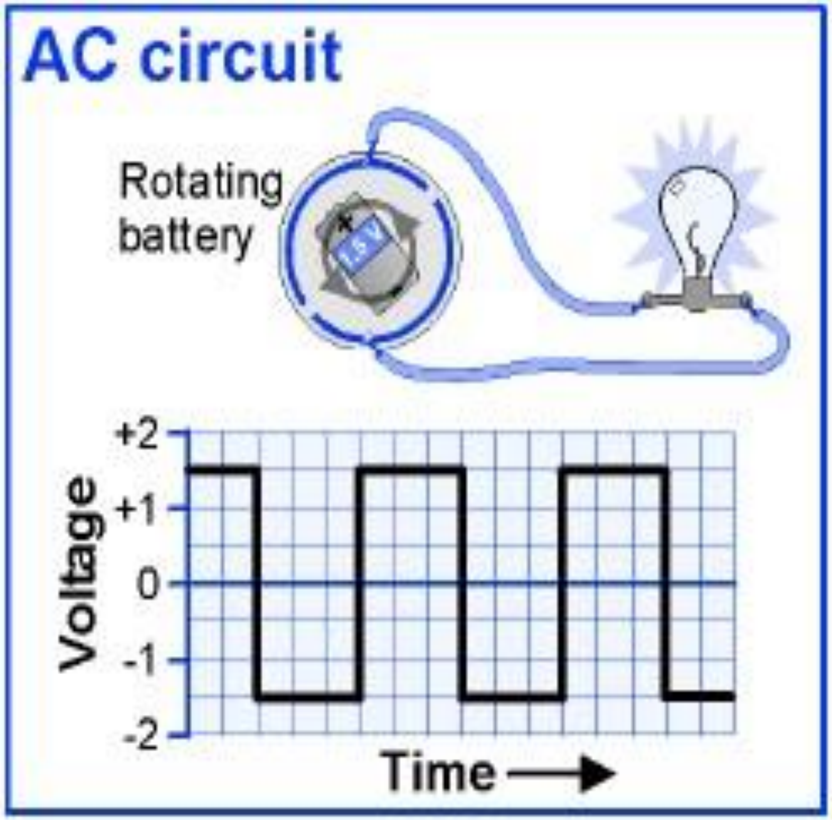
DC vs. AC

- The current from a battery is always in the same direction
- One end of the battery is positive and the other end is negative.
- The direction of current flows from positive to negative
- This is called direct current, or DC



DC vrs AC

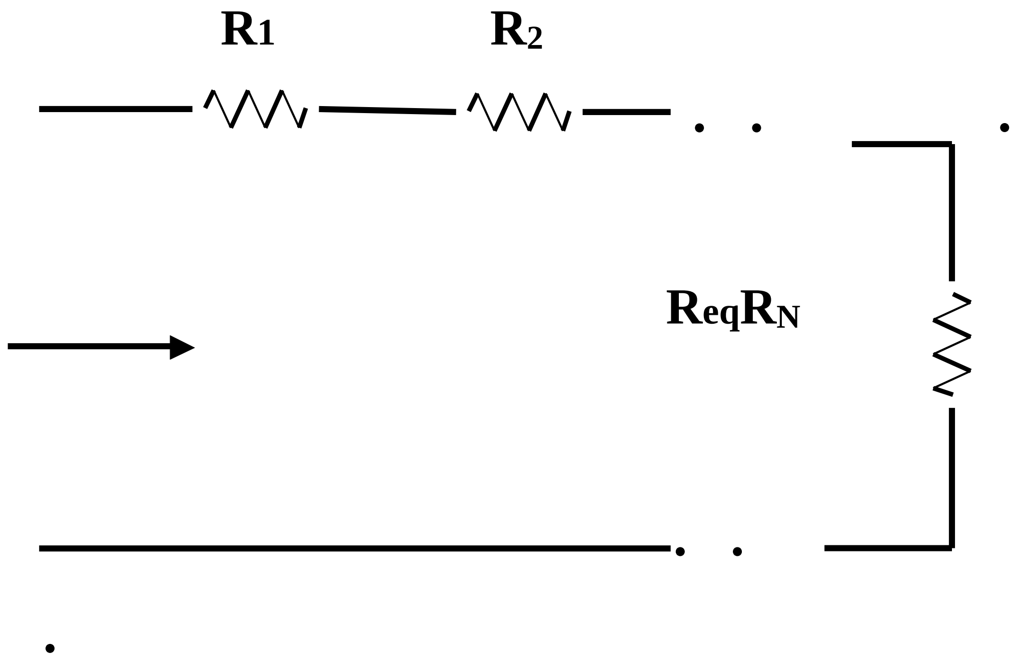
- If voltage alternates, so does current.
- When the voltage is positive, the current in the circuit is clockwise.
- When the voltage is negative the current is the opposite direction.
- This type of current is called alternating current, or AC.



Effective resistance of a circuit

Equivalent Resistance

We know the following for series resistors:

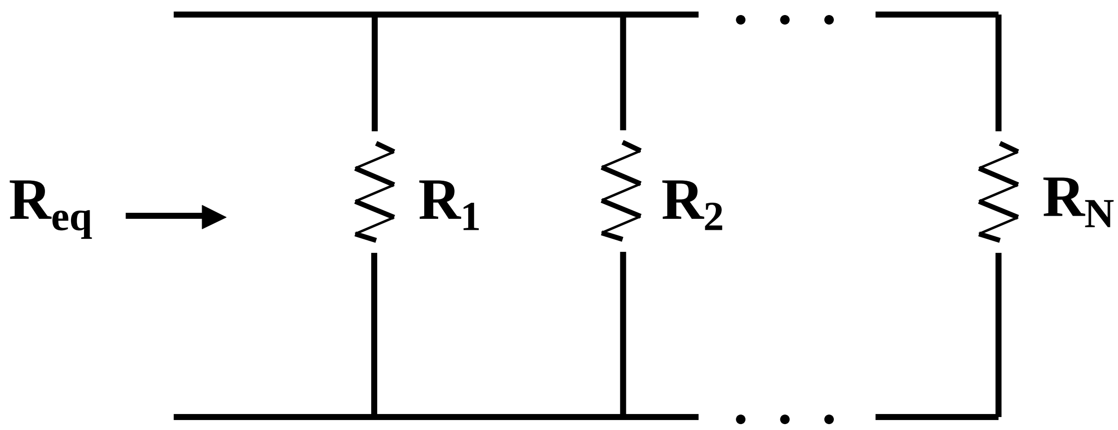


Resistors in series.

$$R_{eq} = R_1 + R_2 + \ldots + R_N$$

Equivalent Resistance:

We know the following for parallel resistors:



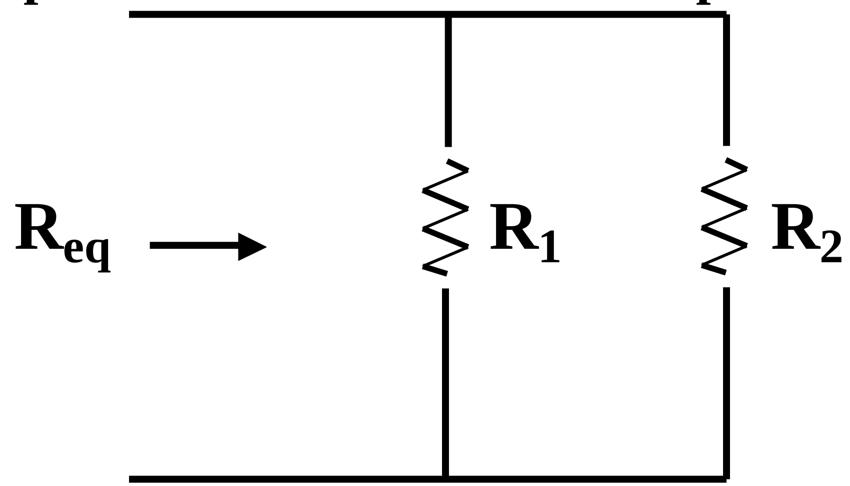
1 1 1 + 1

$$\overline{\phantom{R_{eq}}} = \underline{} + \underline{} \ldots + \underline{}$$

R_{eq} R_1 R_2 R_N

Equivalent Resistance

For the special case of two resistors in parallel:

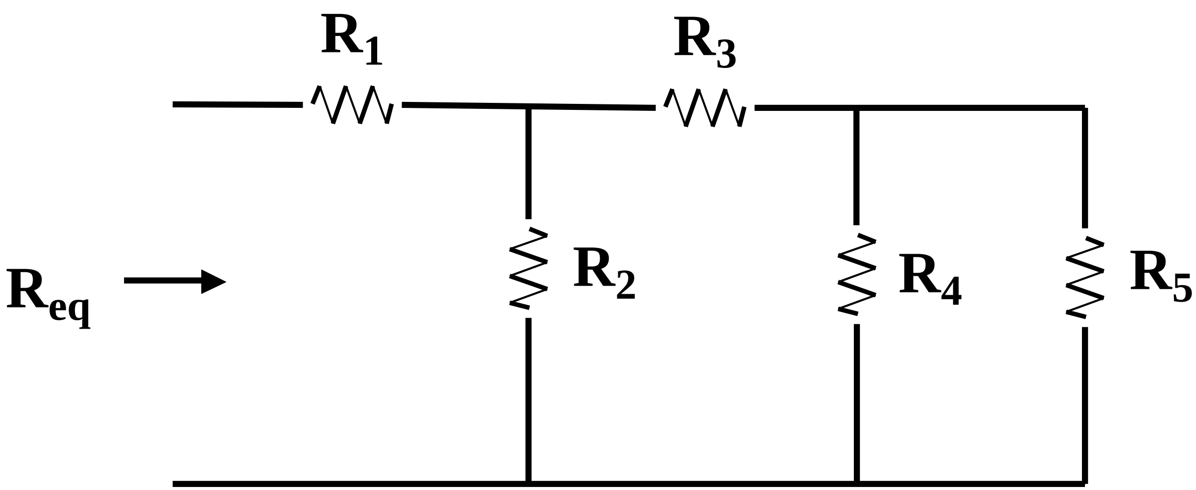


When resistors R_1 and R_2 are in parallel, the total resistance R_{eq} is given by:

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Equivalent Resistance: **Resistors in combination.**

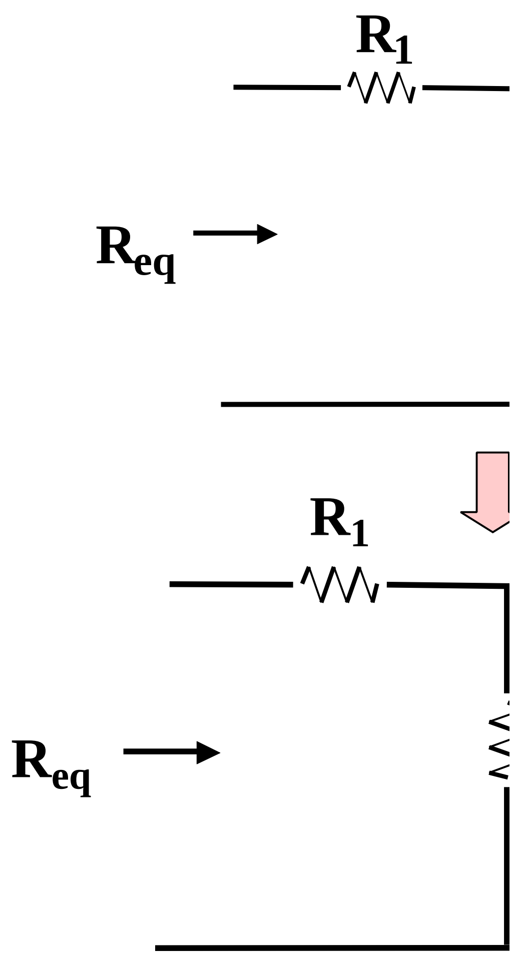
- **By combination we mean we have a mix of series and Parallel. This is illustrated below.**



To find the equivalent resistance we usually start at the output of the circuit and work back to the input.

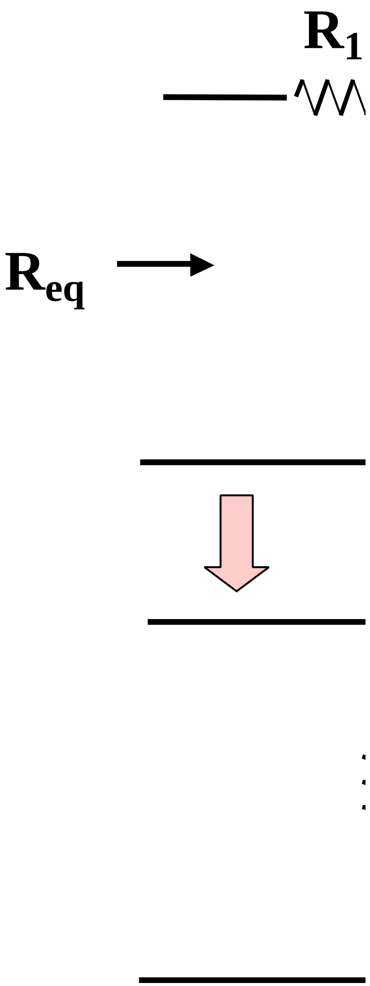
Resistors in combination.

$$R_x = \frac{R_4 R_5}{R_4 + R_5}$$



$$R_y = R_x + R_3$$

Equivalent Resistance: Resistors in combination.



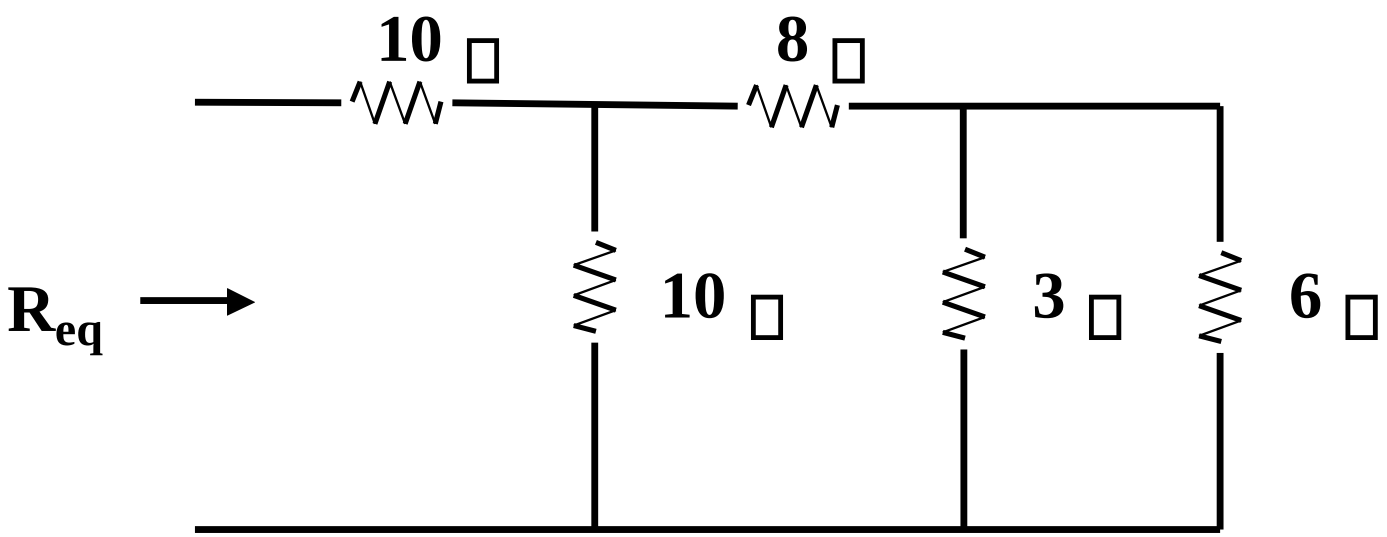
RR

$$R_Z = R_{22} + {}_Y R_Y$$

$$R_{eq} = R_Z + R_1$$

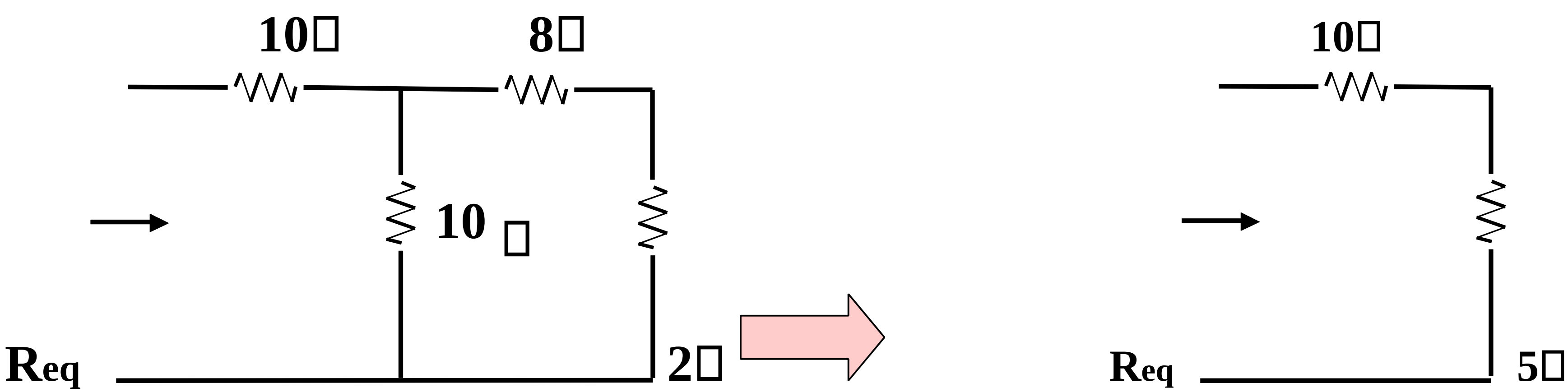
Resistors in combination.

Example Given the circuit below. Find R_{eq}.



Resistors in combination.

We start at the right hand side of the circuit and work to the . left



Ans: $R_{eq} = 15\,\Omega$

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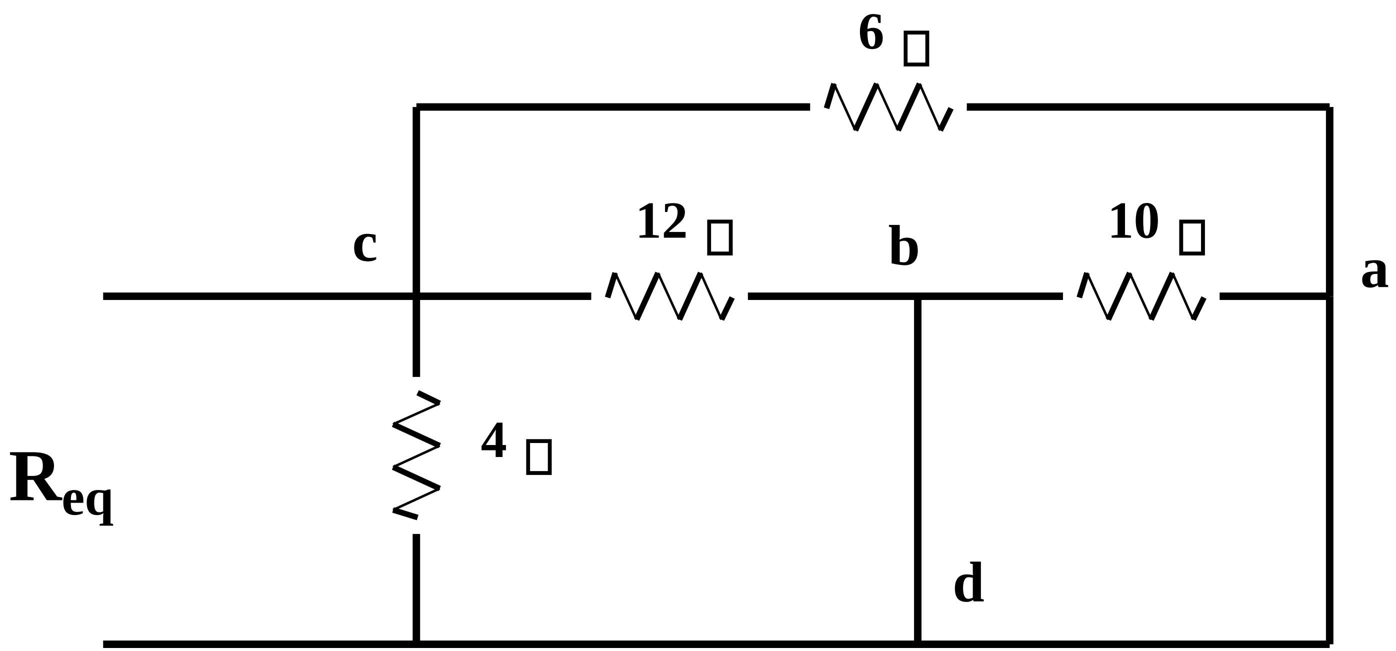
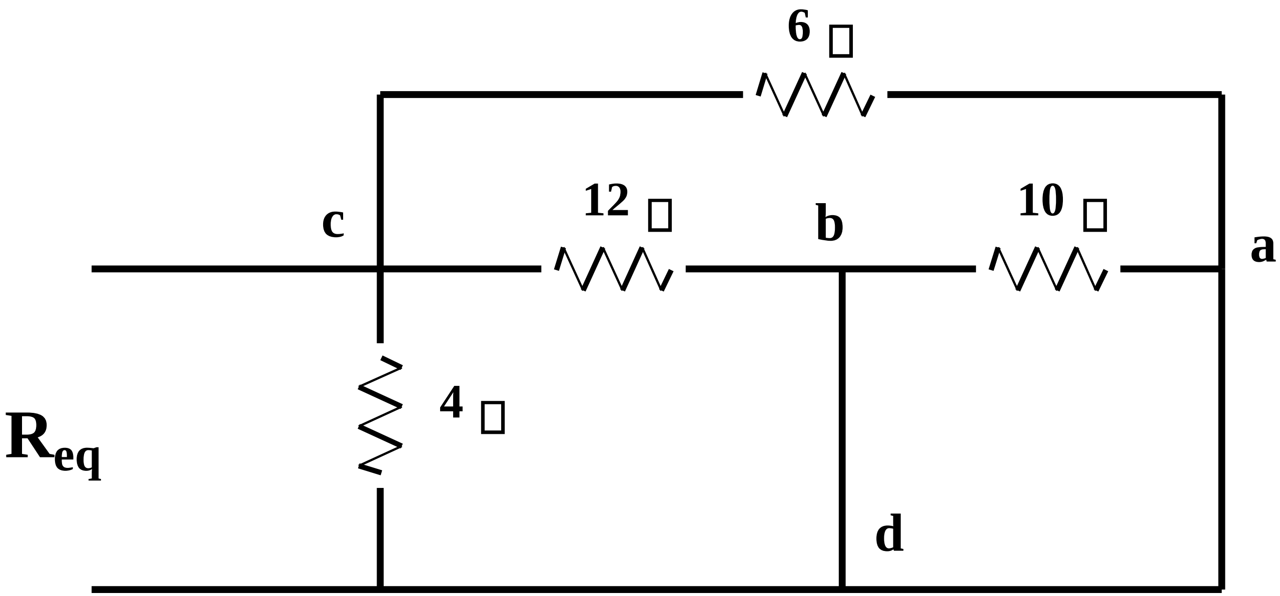
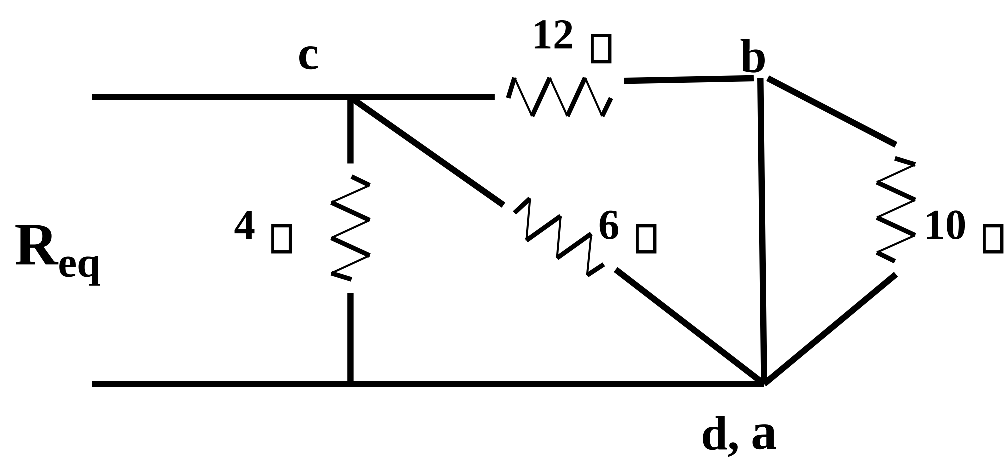
Resistors in combination. Given the circuit shown below. Find R_{eq}.



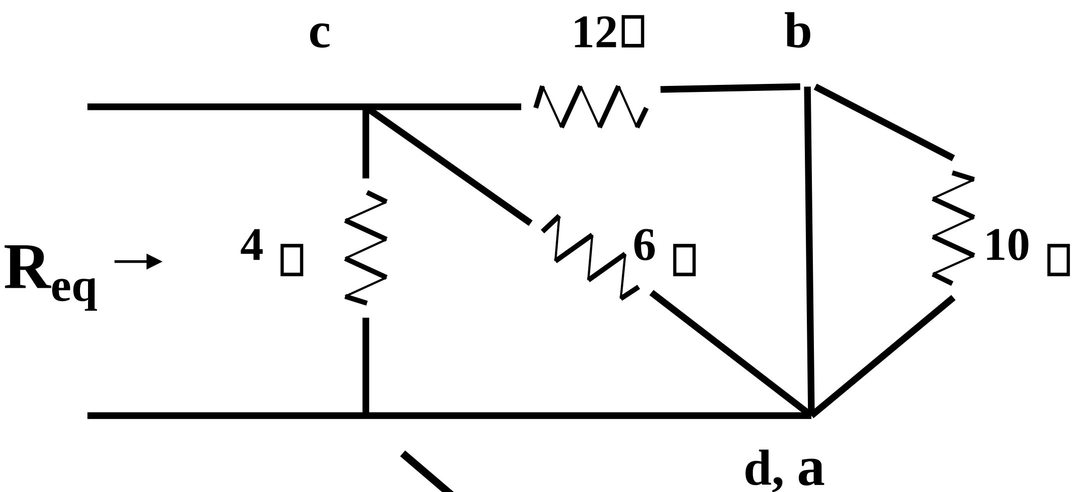
Equivalent Resistance: **Resistors in combination.**


6
12
10
c
b
a
4
R_{eq}
d

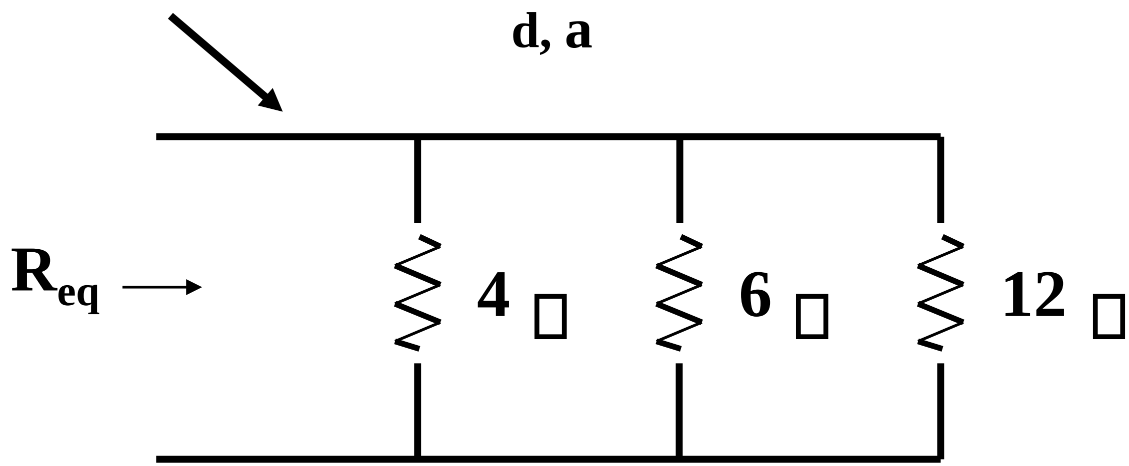

c
12
b
4
6
10
R_{eq}
d, a

<u>Equivalent Resistance:</u> Resistors in combination.

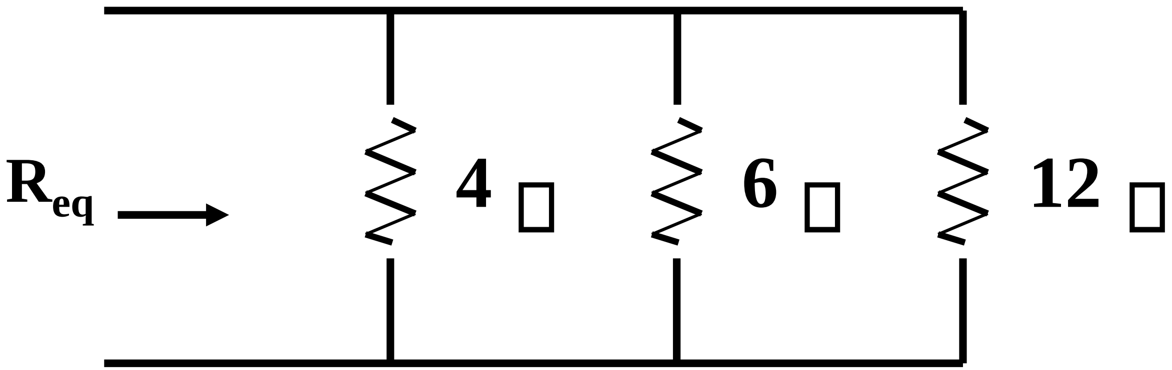
Continued.

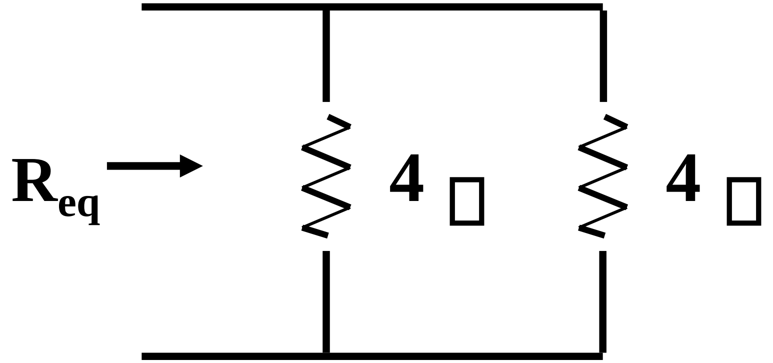


10 Ω resistor shorted out



Equivalent Resistance: Resistors in combination.





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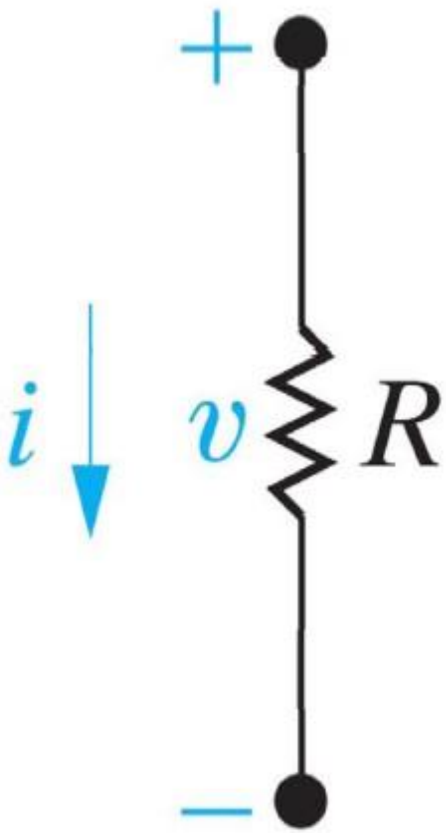
Ohm's Law



Resistance is the material's ability to impede the flow of electrons

Resistance

$$v = R \cdot i$$

Conductance

$$i = \frac{v}{R} = G \cdot v$$

$$p = v \cdot i = R \cdot i^2 = \frac{v^2}{R}$$

A resistor absorbs power

Ohm's Law

- States that for a linear circuit the current flowing through it is proportional to the potential difference across it so the greater the potential difference across any two points the bigger will be the current flowing through it.
- It is discovered by the German Physicist George Simon Ohm.

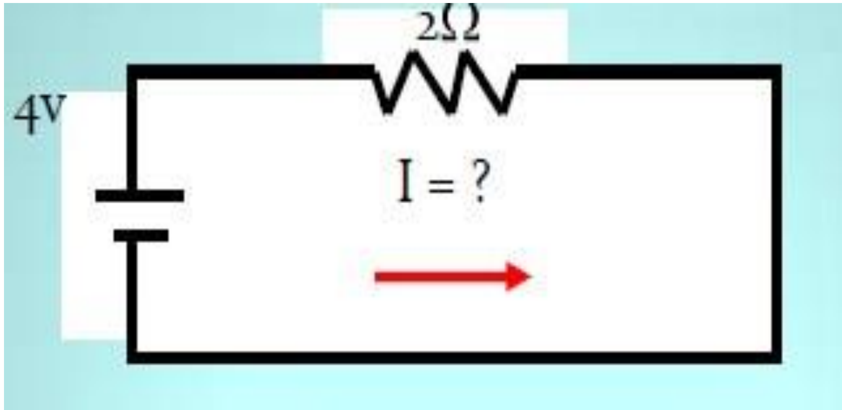
$$V = IR,$$

$$I = V/R,$$

$$R = V/I$$

Series Circuit Analysis

- Example #1
- A 4v battery is placed in a series circuit with a 2Ohm resistor. What is the total current that will flow through the circuit?



Solution

Given:

$V = 4v$

$R = 2\ \Omega$

$I = ?$

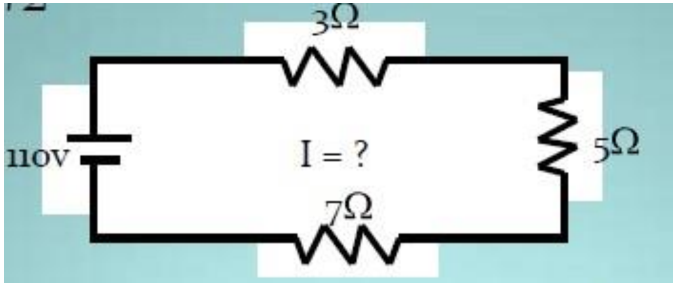
$I = V/R$

$= 4v/2\ \Omega$

$I = 2A$

Example # 2

A 110v supplies a load with a resistance of 3Ω ,5Ω , and 7Ω respectively, find the current in the circuit?



Solution

Given:

$V = 110\ V$

$R_1 = 3\Omega$

$R_2 = 5\Omega$

$R_3 = 7\Omega$

$I = ?$

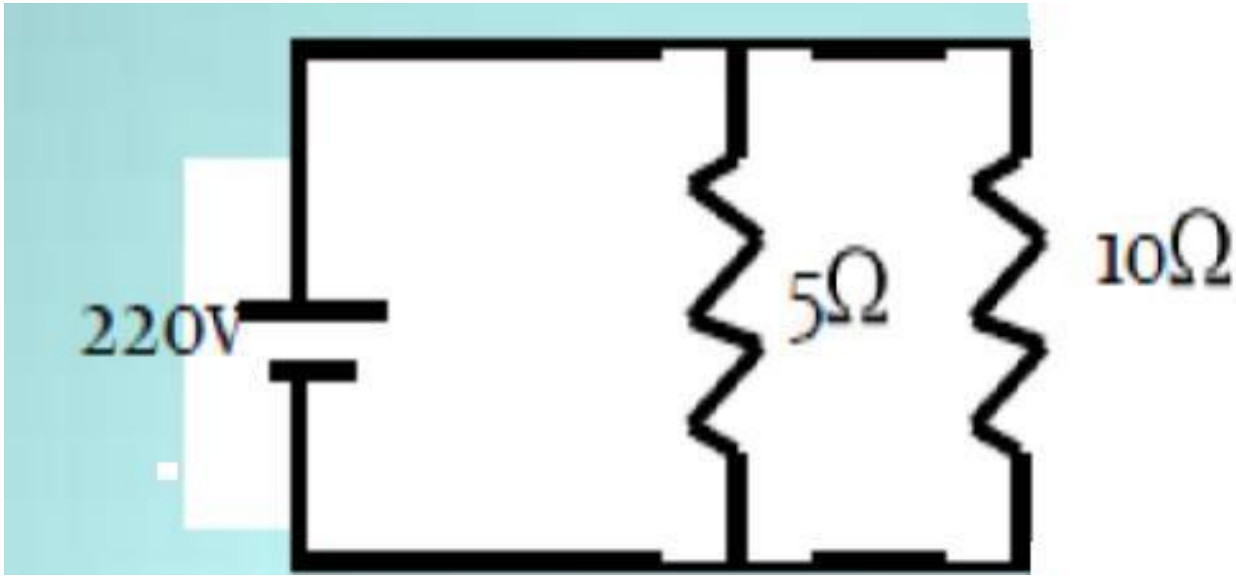
$$R_{total} = R_1 + R_2 + R_3$$
$$= 3 + 5 + 7$$
$$= 12\ \Omega$$

$$I = V/R$$
$$= 110/12$$
$$= 9.17\ A$$

Parallel circuit analysis

Example # 3

- A 220v is connected in parallel with the load. It has a resistance of 5ohms and 10ohms. Find the Total current and the I_1 and the I_2



Solution

Given:

- $V = 220v$
- $R_1 = 5\Omega$
- $R_2 = 10\Omega$
- $I_1 = V / R_1$

  $= 220 / 5$

  $= 44\ A$

  $I_2 = V / R_2$

  $= 220 / 10$

  $= 22\ A$
- $R_T = R_1 R_2 / R_1 + R_2$
  - $= (5 \times 10) / 15$
  - $= 3.33\ \Omega$

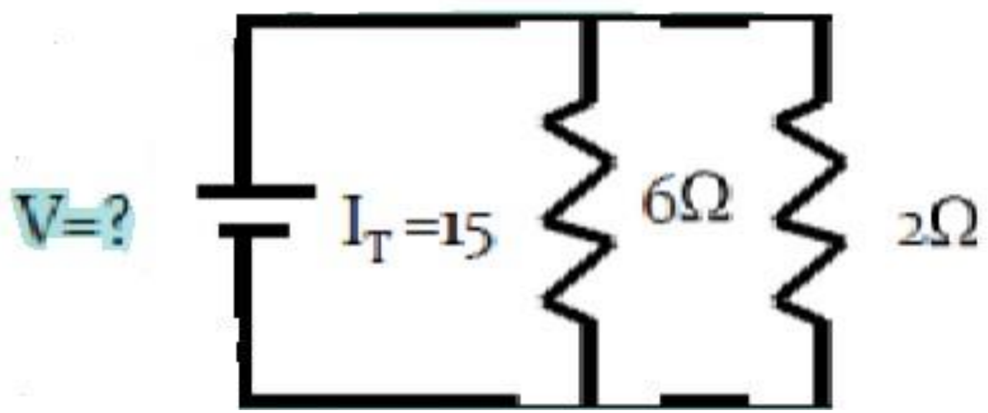
  $I_T = V / R_T$

  $= 220/3.33$

  $= 66.06A$

Example 4

- Find the total voltage and the total resistance of the load if the total current is 15A and it has a $R_1 = 6$ ohms and $R_2 = 2$ ohms.



Solution

Given:

- $R_1 = 6\Omega$
- $R_2 = 2\Omega$
- $I_T = 15\ A$
- $R_T = R_1 R_2 / R_1 + R_2$
    - $= (6 \times 2) / 8$
    - $= 1.5\ \Omega$

$V_T = I_T / R_T$

$= 15\ A \times 1.5\ \Omega$

$= 22.5\ v$

- $I_1 = V / R_1$

$= 22.5 / 6$

$= 3.75\ A$

$I_2 = V / R_2$

$= 22.5 / 2$

$= 11.25\ A$

Electric Power, AC, and DC Electricity

- The watt (W) is a unit of power.
- Power is the rate at which energy moves or is used.
- Since energy is measured in joules, power is measured in joules per second.
- One joule per second is equal to one watt.

Power in electric circuits

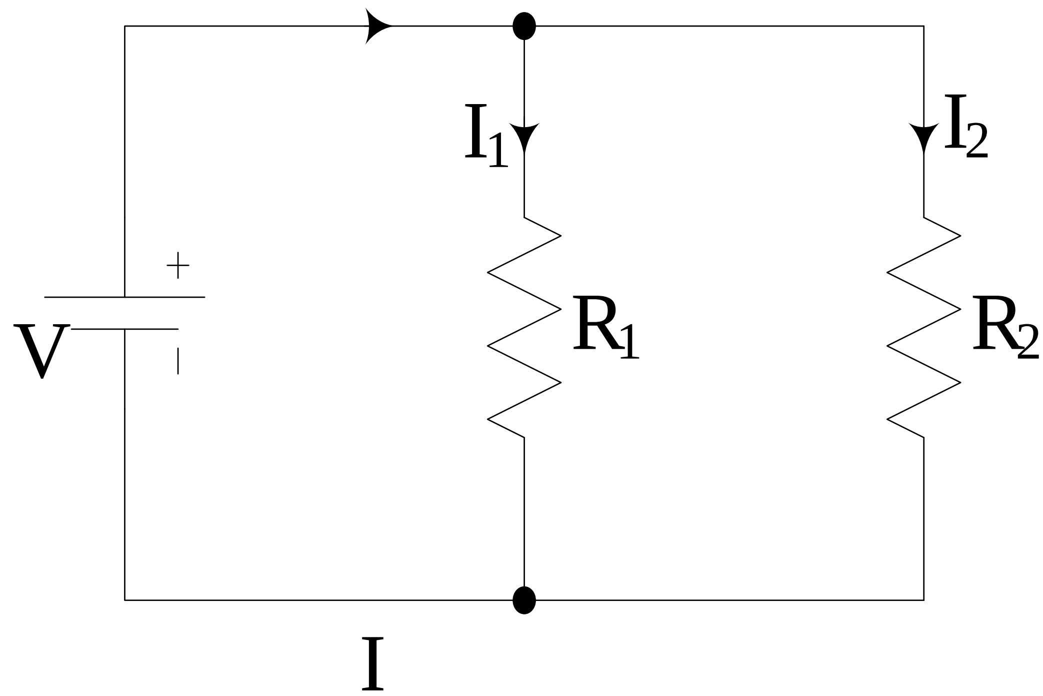
- One watt is a pretty small amount of power. In everyday use, larger units are more convenient to use.
- A kilowatt (kW) is often used and is equal to 1,000 watts. The other common unit of power often seen on electric motors is the horsepower.
- One horsepower is 746 watts.

Current division rule

The current division rule is applied to share current between parallel branches. Consider the circuits below

$$\overline{}$$
$$R_1 + R_2$$

$$V \qquad IR \qquad I \frac{R_1 R_2}{+ R_2}$$



$$RR$$

$$R_T = \; 1\;2$$

$= \quad {}_T =$

R_1

$R_1 R_2 \qquad R_1 \qquad R_1$

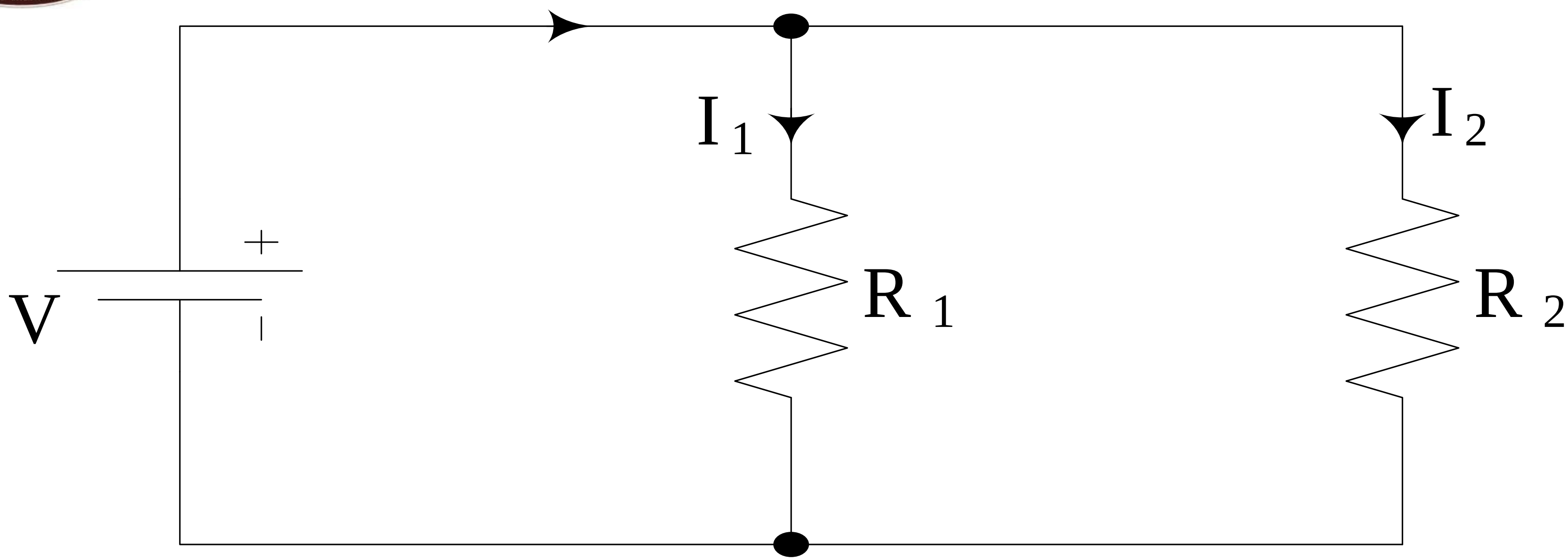
$I \qquad \underline{R_1 R_2} \qquad \underline{1} \qquad I \qquad R_2$

$V \qquad \underline{R_1 + R_2} \qquad = I \qquad$ □ $=$

$I_1 = \; = \qquad R_1 + R_2 \qquad R_1 \qquad R_1 + {}_{59} R_2$

CURRENT DIVISION RULE

Similarly, I

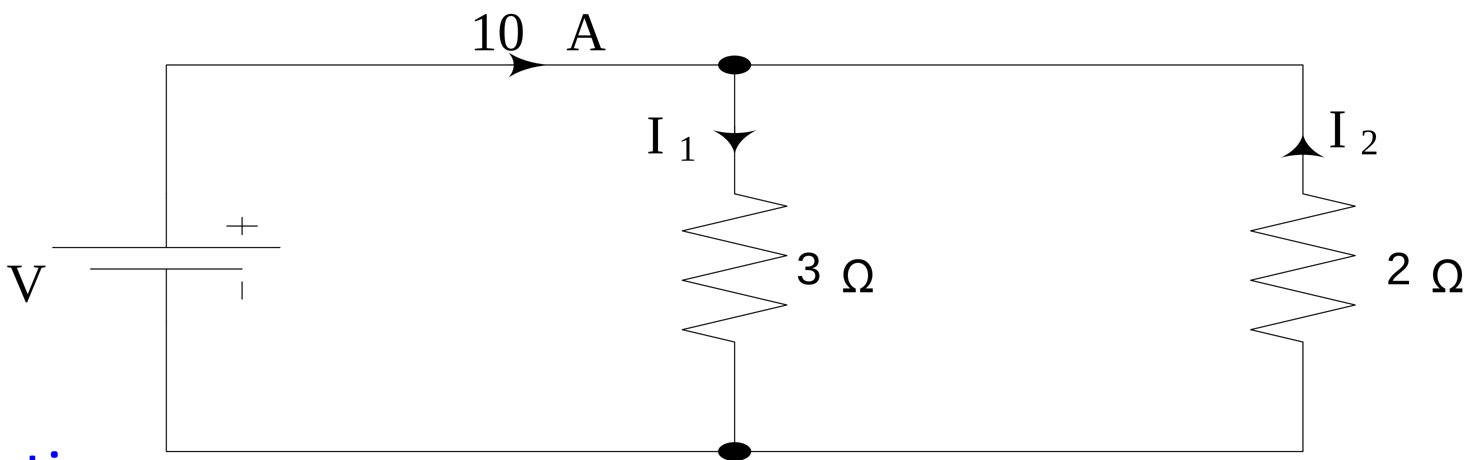


$$R_1 \; R_1 + R_2$$

$$I_2 = I \;\underline{\qquad\qquad\qquad\qquad}$$

Example 1

Find the values of I_1 and I_2 in the circuit below.



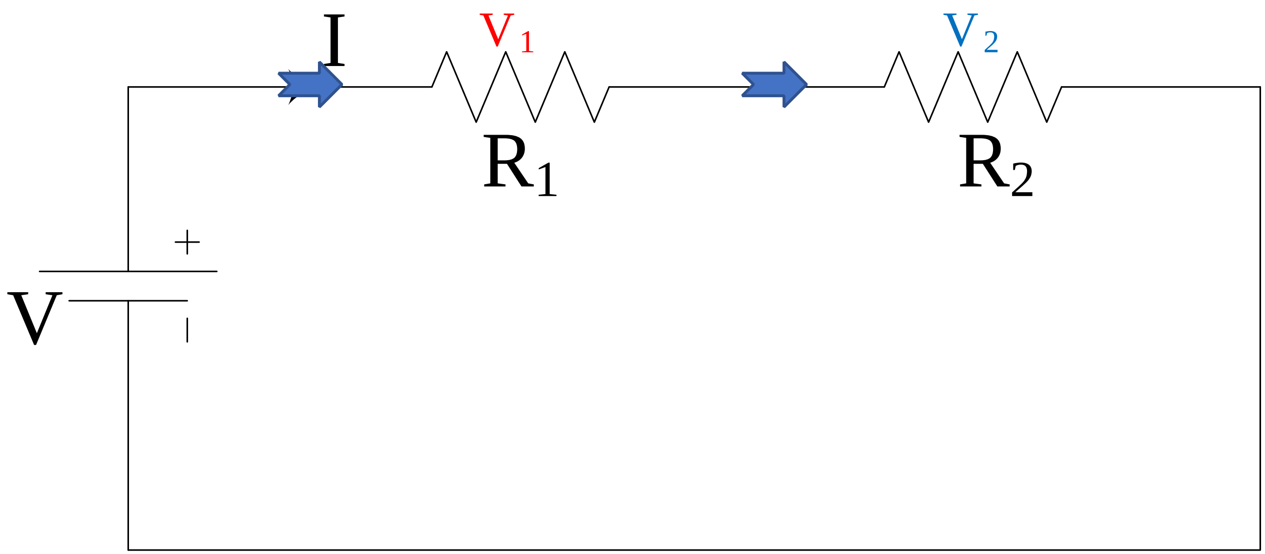
Solution

$$I_1 = \frac{R_2}{R_1 + R_2} I = \frac{2}{2+3} \times 10 = 4\,A$$

$$I_2 = -\frac{R_1}{R_1 + R_2} I = -\frac{3}{2+3} \times 10 = -6\,A$$

VOLTAGE DROP

- Any time a voltage drives current through a resistor, some of the voltage drops across the resistor.
- The magnitude of the drop is the product of the resistance and current

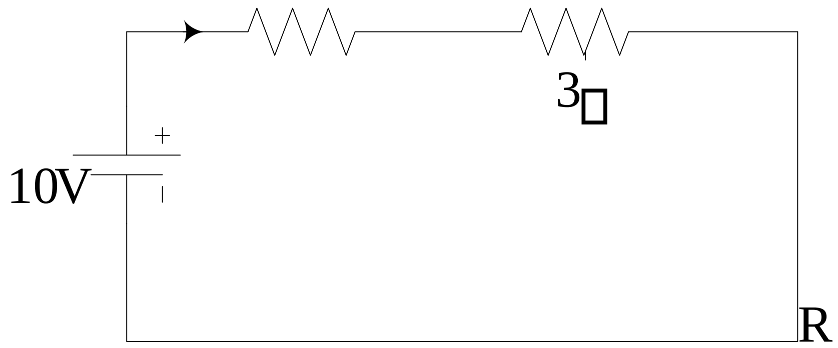


$$V_2 = V - V_1$$

Example

Find the values of I and R in the circuit below.

I 4V



Solution

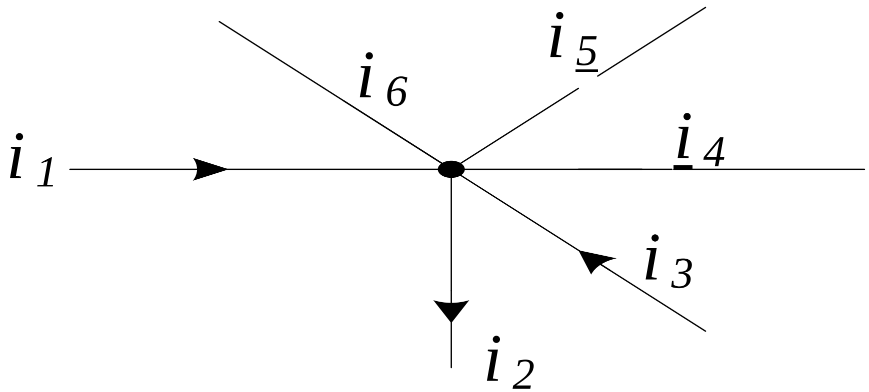
Voltage across 3Ω resistor = 10 – 4 = 6V

Current in 3Ω resistor = I = 6/3 = 2A

Resistance R = 4V/I = 4/2 = 2Ω

The KCL

The sum of currents entering a node equals the sum of currents leaving the node.



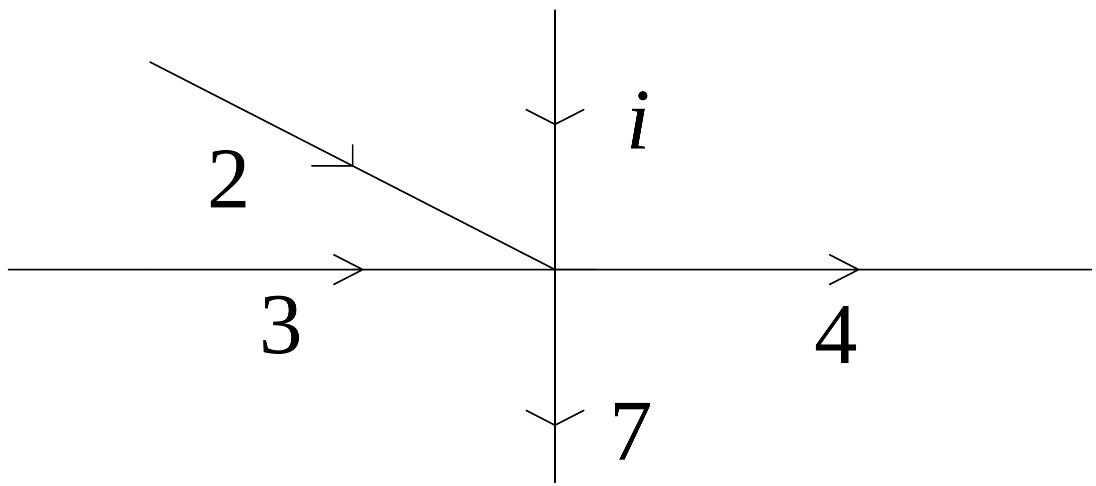
$$i_1 + i_3 + i_5$$

$$i_2 + i_4 + i_6$$

$$\Rightarrow i_1 + i_3 + i_5 = i_2 + i_4 + i_6$$

- **Example**

Find the value of i in the figure below.

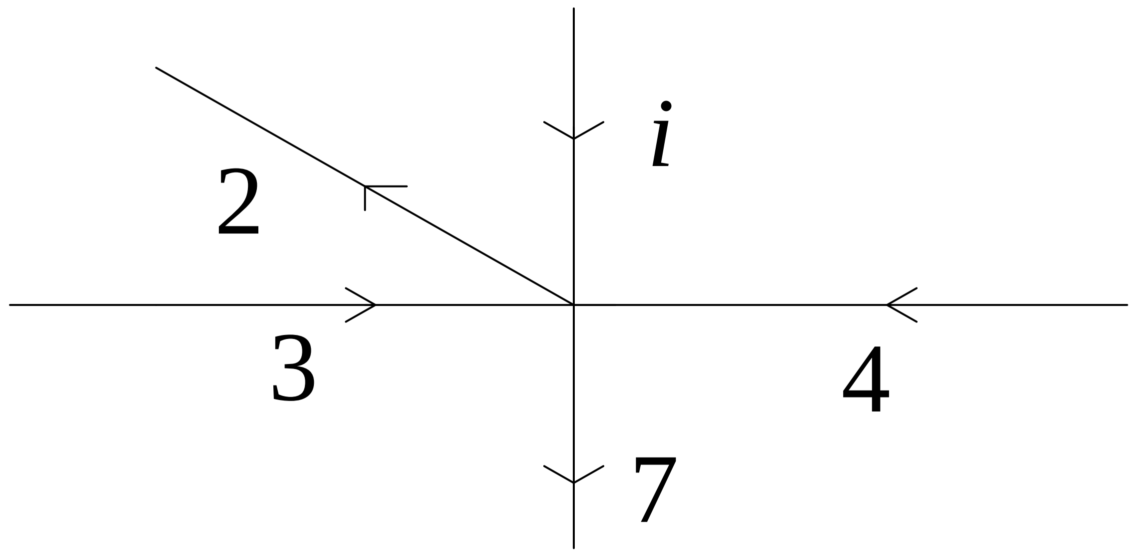


Solution

$$i + 2 + 3 = 4 + 7 \quad i + 5 = 11 \quad i$$
$$= 6$$

Self assessment

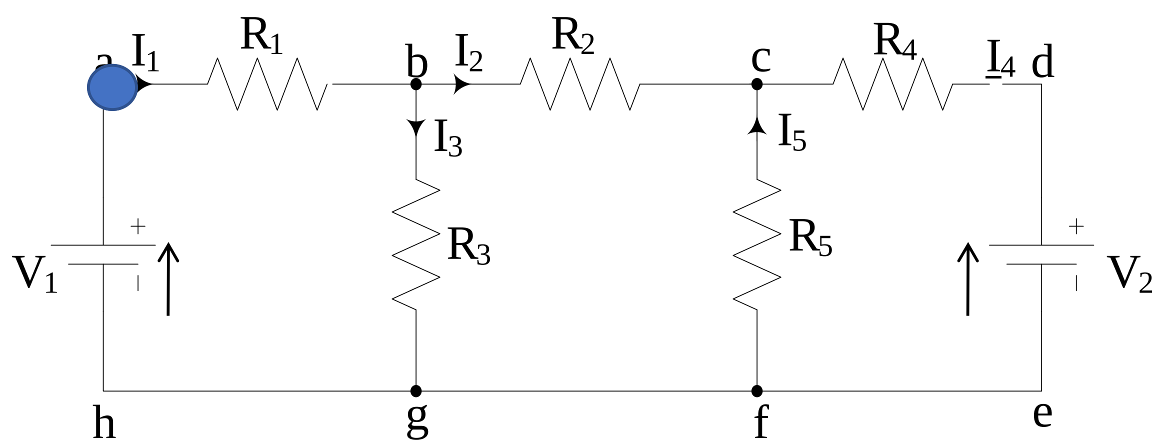
Find the value of i in the figure below.



ANS $i = 2$

The law

The algebraic sum of the voltages in a loop (closed path) equals zero. Alternatively, in a loop, the algebraic sum of voltage sources equals the algebraic sum of voltage drops.

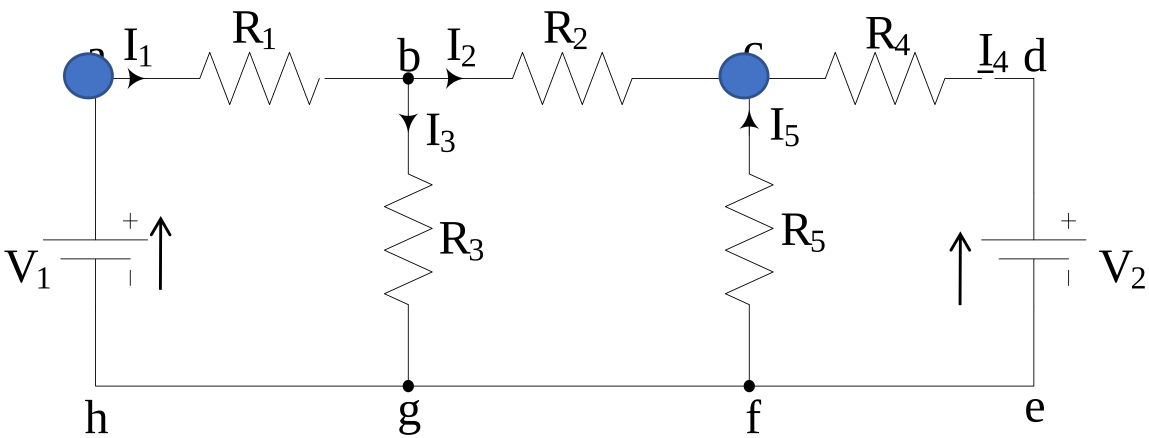


Loop abgha $V_1 = I_1R_1 + I_3R_3$

Loop adeha

$$V_1 - V_2 = I_1R_1 + I_2R_2 - I_4R_4$$

KIRCHHOFF'S VOLTAGE LAW(KVL)



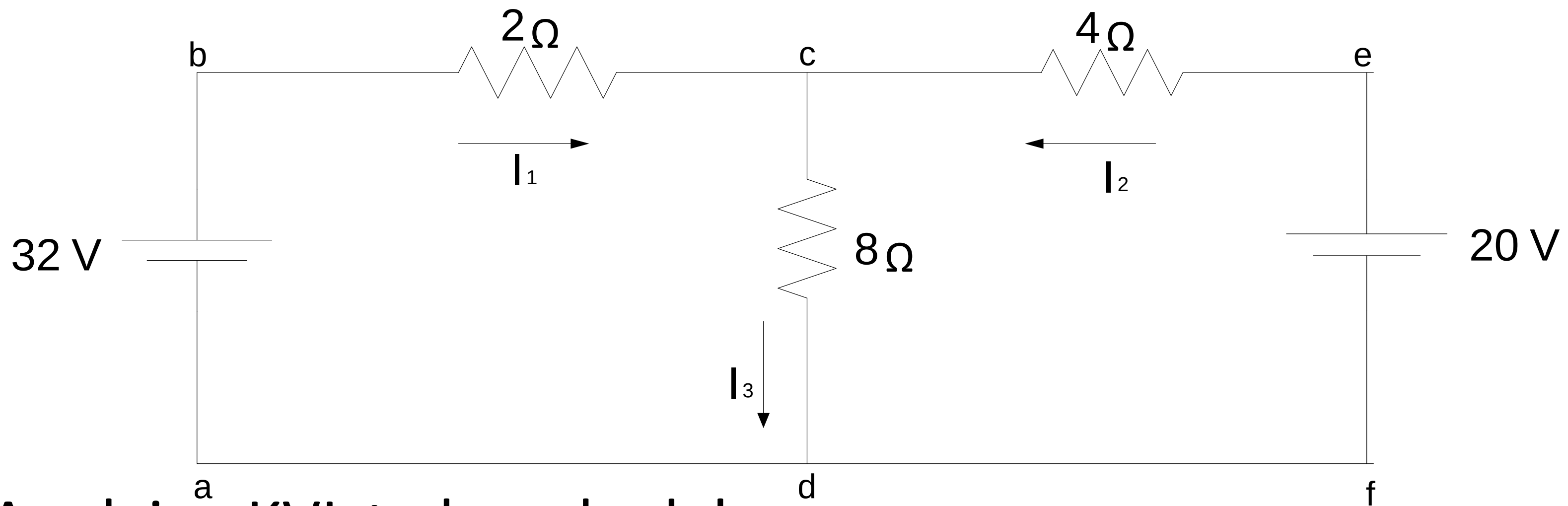
Loop cbgfc $0 = -I_2R_2 + I_3R_3 + I_5R_5$

Loop acfha $V_1 = I_1R_1 + I_2R_2 - I_5R_5$

KIRCHHOFF'S VOLTAGE LAW(KVL)

Example 1

Find the current in all parts of the circuit below.



- Applying KVL to loop bcdab $32 - 2I_1 - 8I_3 = 0$

$$\Rightarrow 32 = 2I_1 + 8I_3 \quad (1)$$

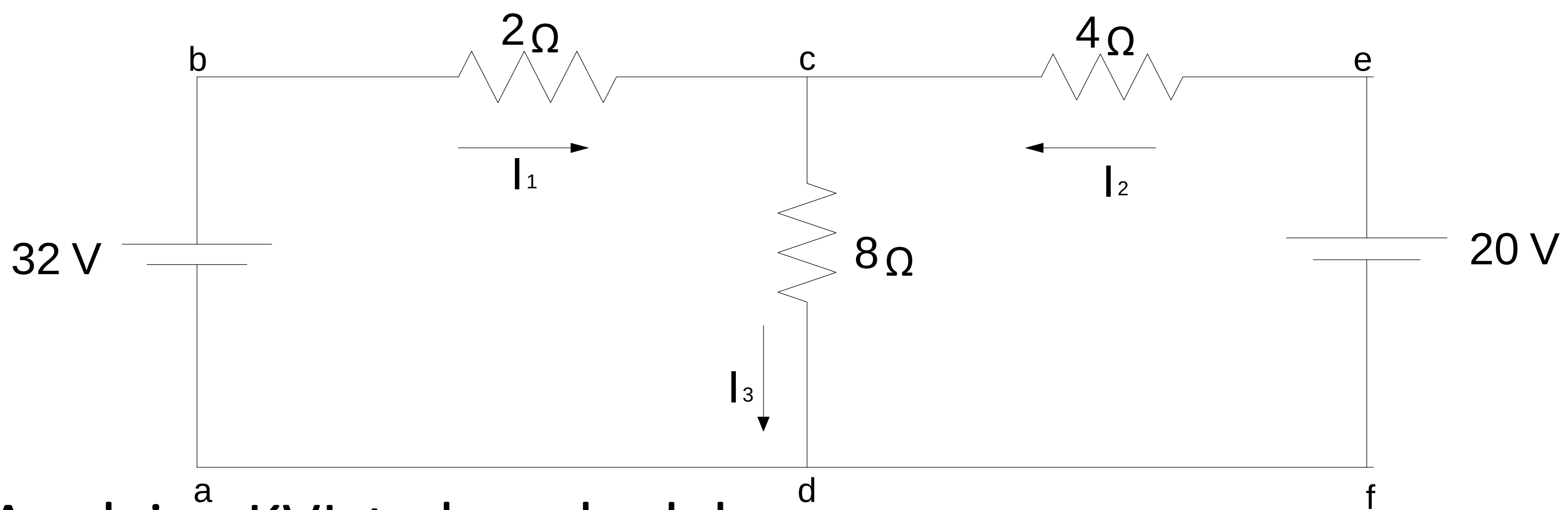
- Applying KVL to loop ecdfe $20 - 4I_2 - 8I_3 = 0$

$$\Rightarrow 20 = 4I_2 + 8I_3 \quad (2)$$

KIRCHHOFF'S VOLTAGE LAW(KVL)

Example 1

Find the current in all parts of the circuit below.



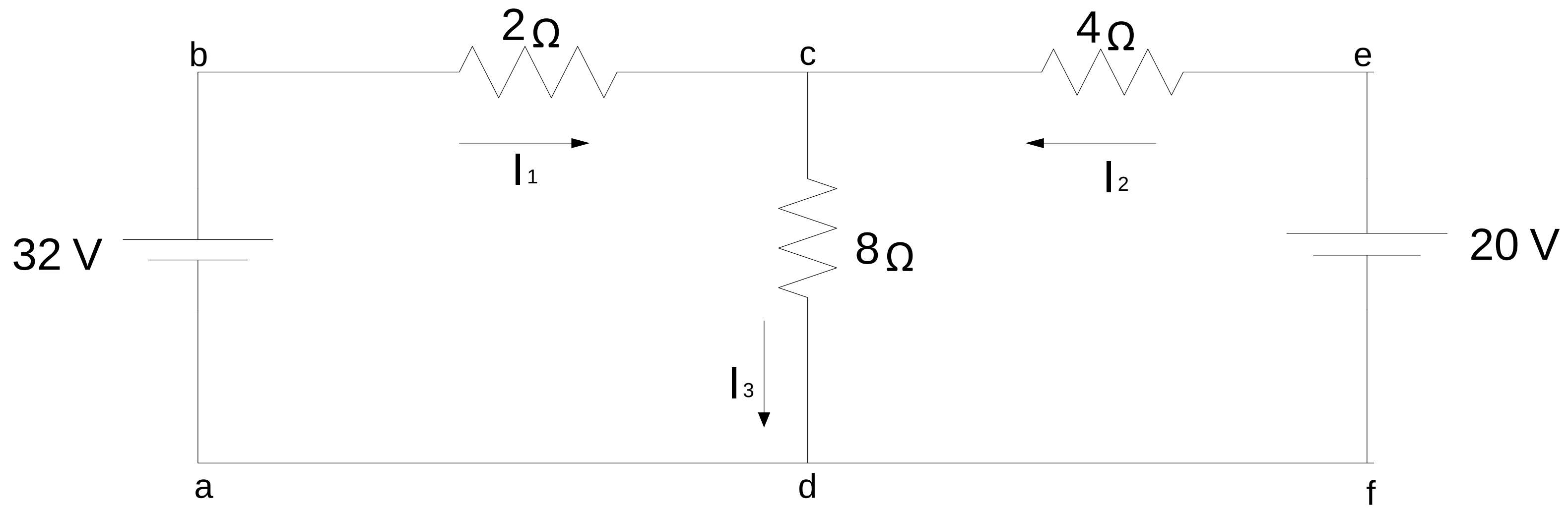
- Applying KVL to loop bcdab $32 - 2I_1 - 8I_3 = 0$

$\Rightarrow 32 = 2I_1 + 8I_3 \quad (1)$

- Applying KVL to loop ecdfe $\quad 20 - 4I_2 - 8I_3 = 0$

$\Rightarrow 20 = 4I_2 + 8I_3$

(2)



KIRCHHOFF'S VOLTAGE LAW(KVL)

- Applying KCL to node c: $I_3 = I_1 + I_2$ (3)

Solving the equations simultaneously yields

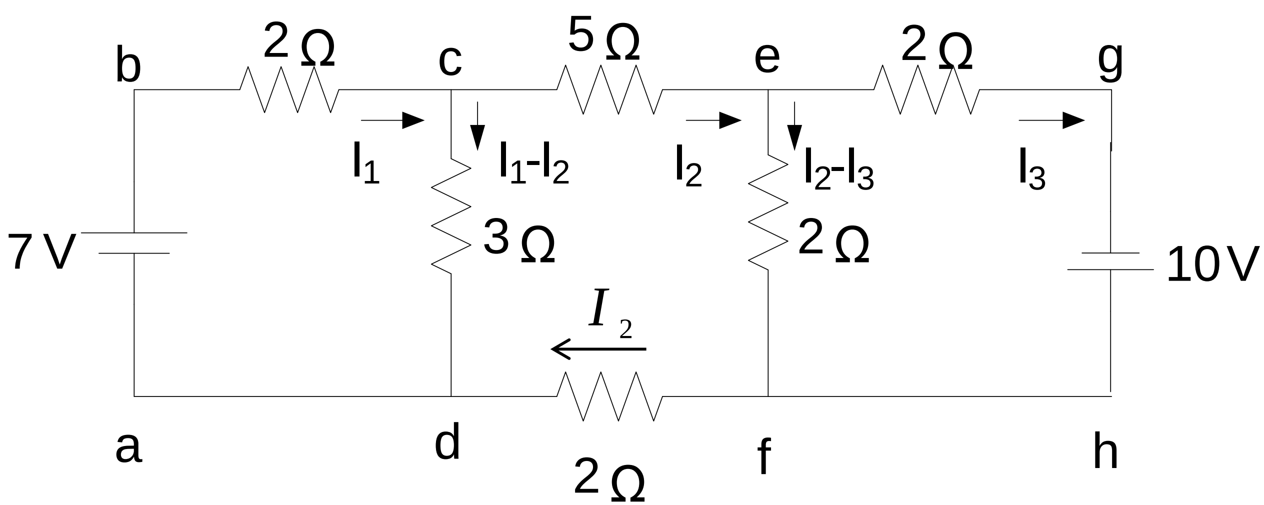
$$I_1 = 4A, \quad I_2 = -1A$$

$$\text{and} \quad I_3 = 3A$$

Example 2

KIRCHHOFF'S VOLTAGE LAW(KVL)

Find the currents in all parts of the circuit below.



Solution

Apply KVL to loop cefdc

$$5I_2 + 2(I_2 - I_3) + 2I_2 - 3(I_1 - I_2) = 0$$

Theorem:

Any linear circuit connected between two terminals can be replaced by a Thevenin’s voltage(V_{TH}) in series with a Thevenin's resistance (R_{TH}).

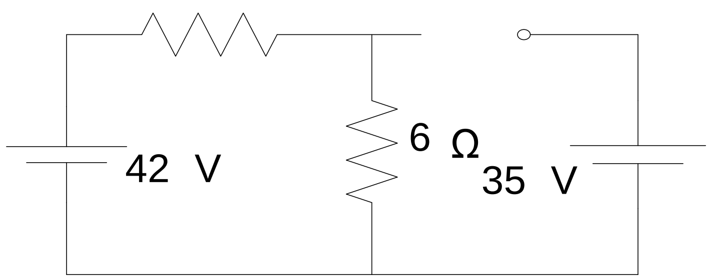
V_{TH} is the open-circuit voltage across the two terminals

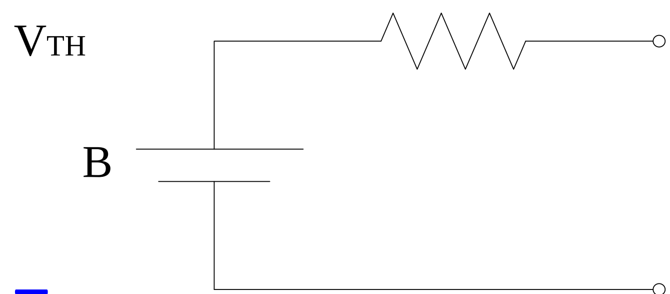
R_{TH} is the resistance seen from the two terminals when all sources have been deactivated

12 Ω A___B R_{TH}

A

Thevenin's Theorem

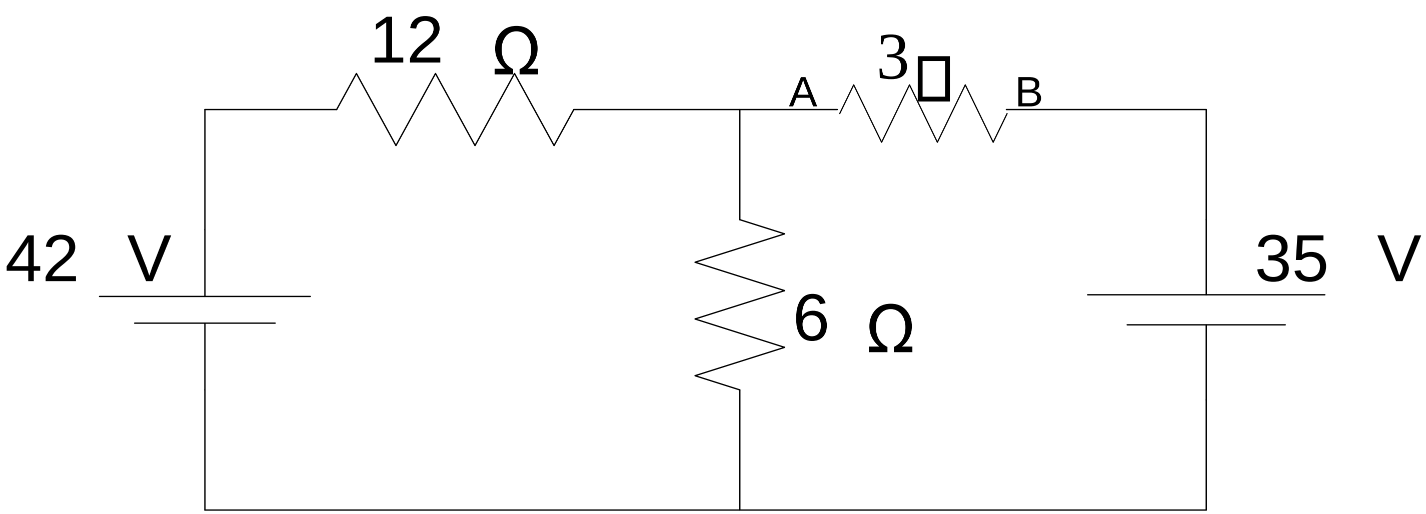




To find the current through a resistor in a circuit, the following steps are taken:

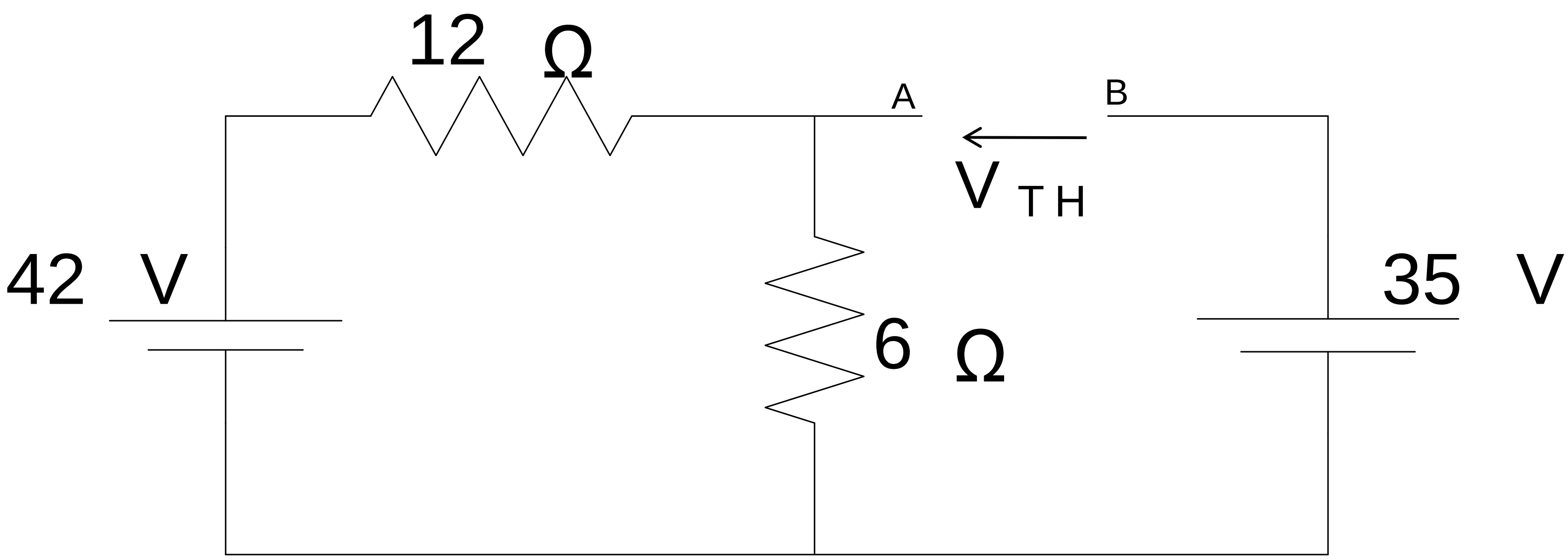
1. **Remove the resistor from the circuit and mark the two terminals.**

Thevenin's Theorem



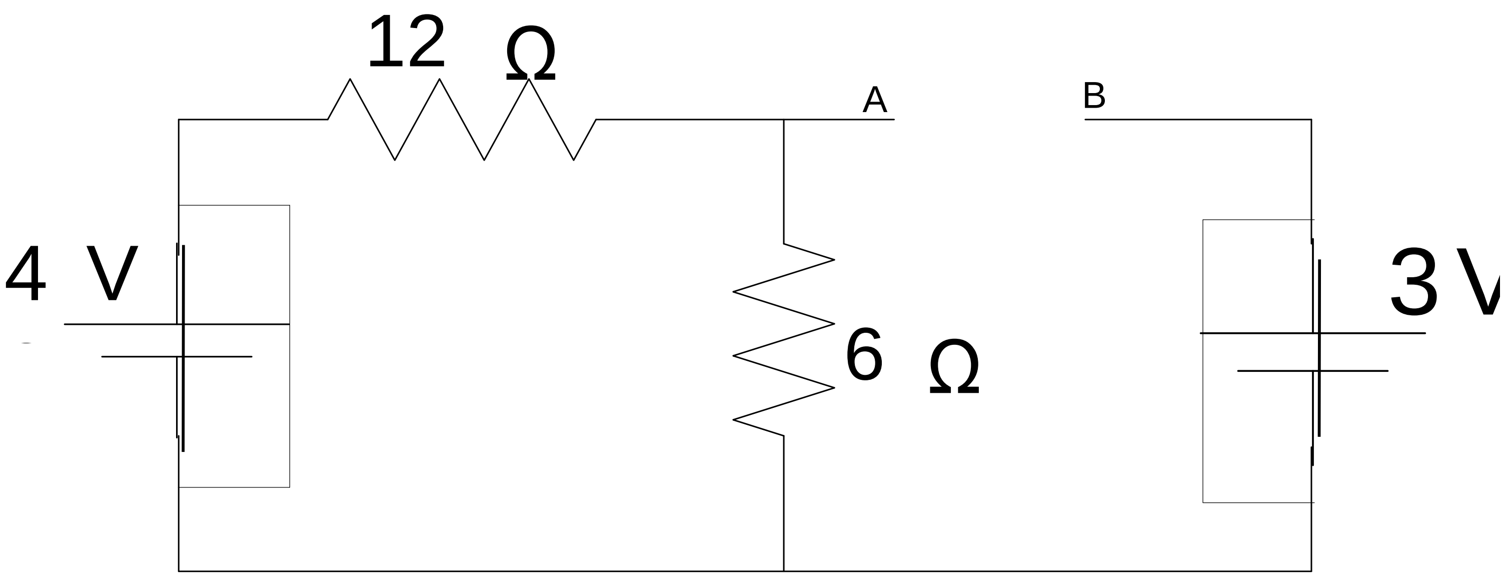
2. Find the open-circuit voltage (V_{TH}) across the two terminals by applying KVL. Treat V_{TH} as a source.

Thevenin's Theorem



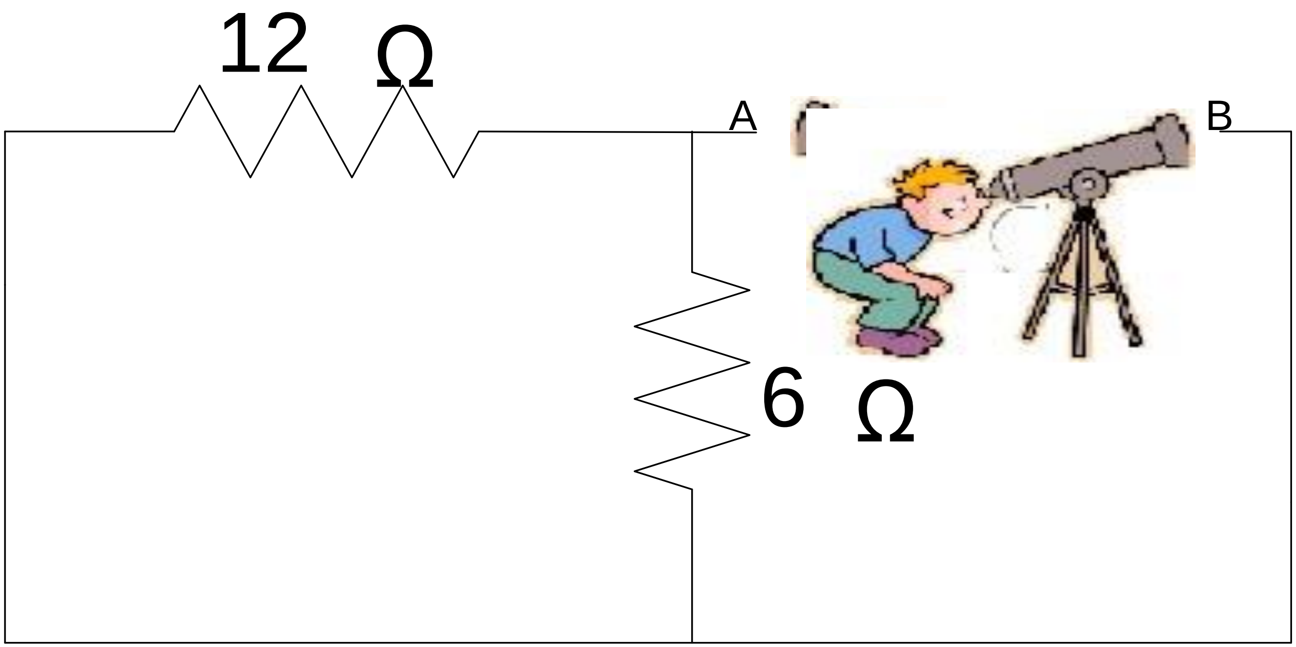
3. **Recall the circuit created before step 2 and deactivate all sources. Short-circuit voltage sources and Open-circuit current sources.**

Thevenin's Theorem



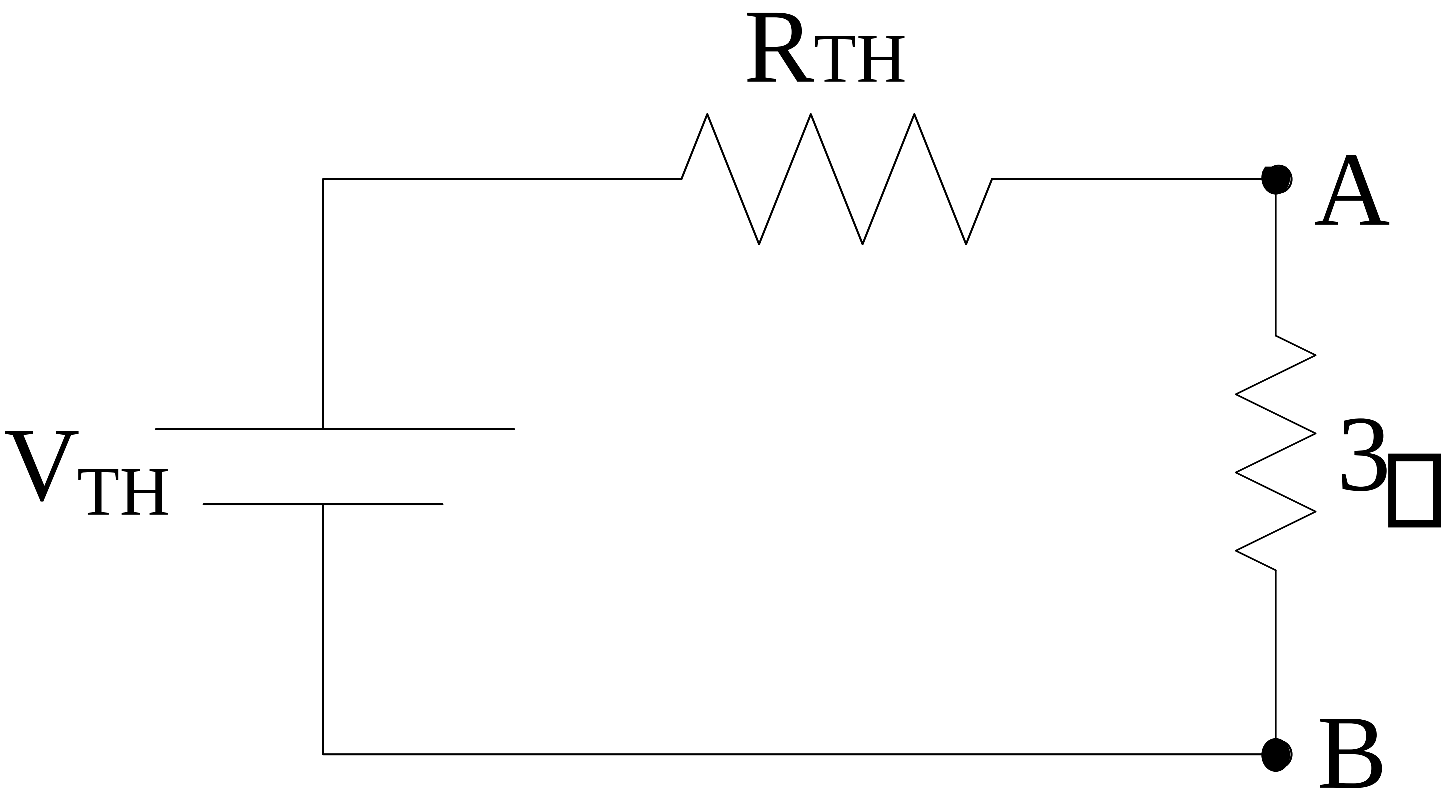
THEVENIN'S THEOREM

4. **Find the total resistance of the circuit resulting from step 3 as seen from the two terminals**



THEVENIN’S THEOREM

5. Reproduce the Thevenin's equivalent circuit and connect the resistor whose current is to be found.



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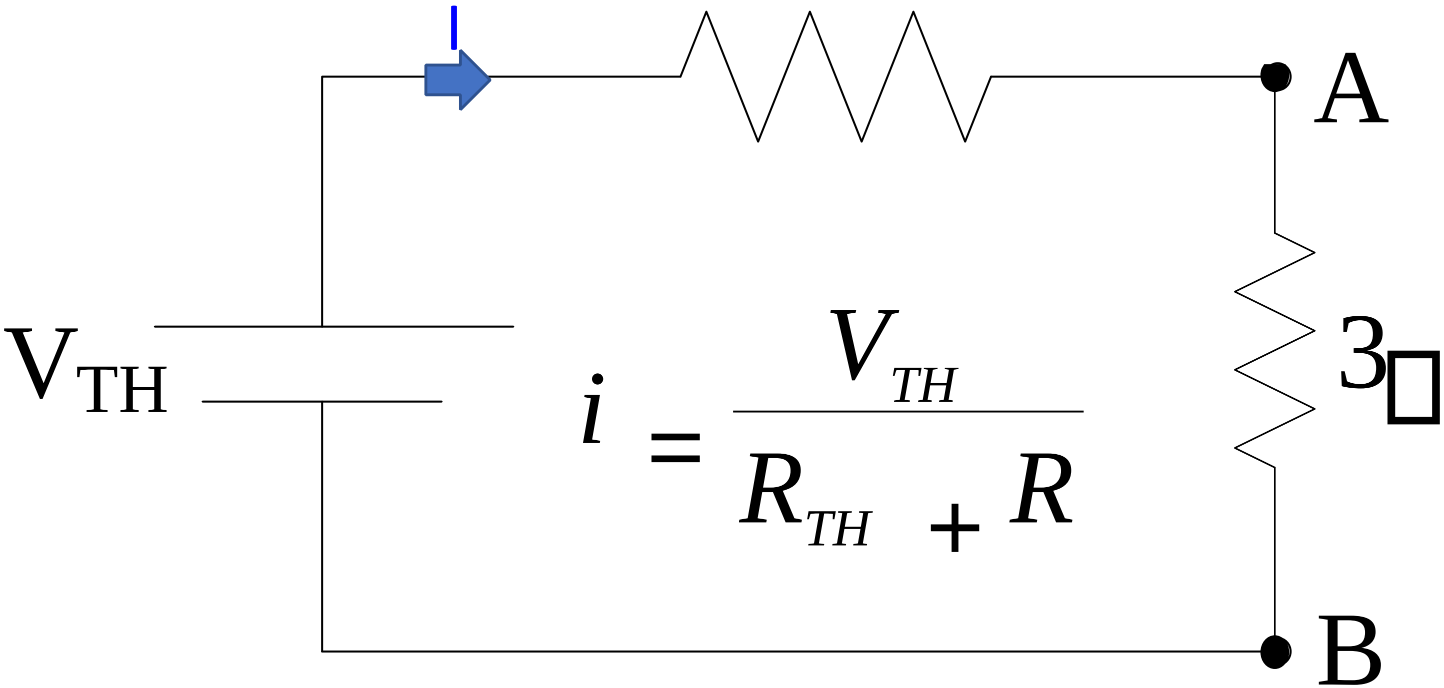
THEVENIN'S THEOREM

6. **Calculate the current in the circuit in step 5. This is the current being sought.**

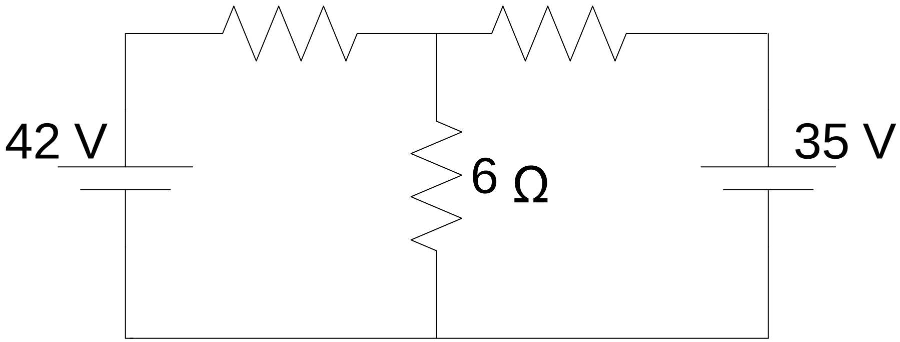
R_{TH}



$$i = \frac{V_{TH}}{R_{TH} + R}$$

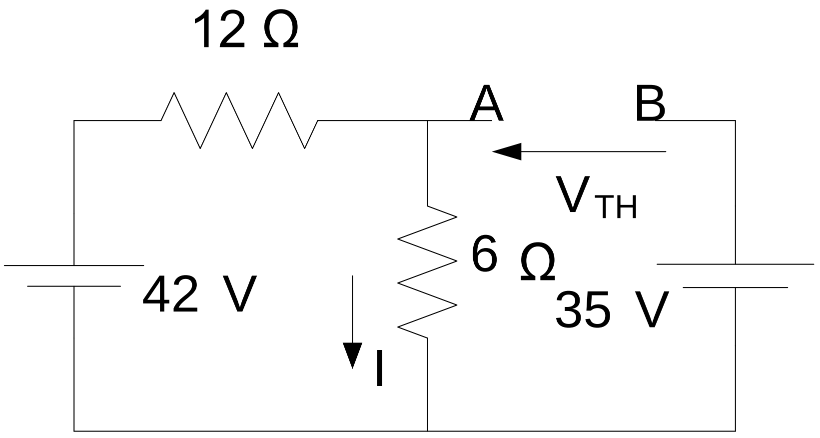
THEVENIN'S THEOREM

Example 1

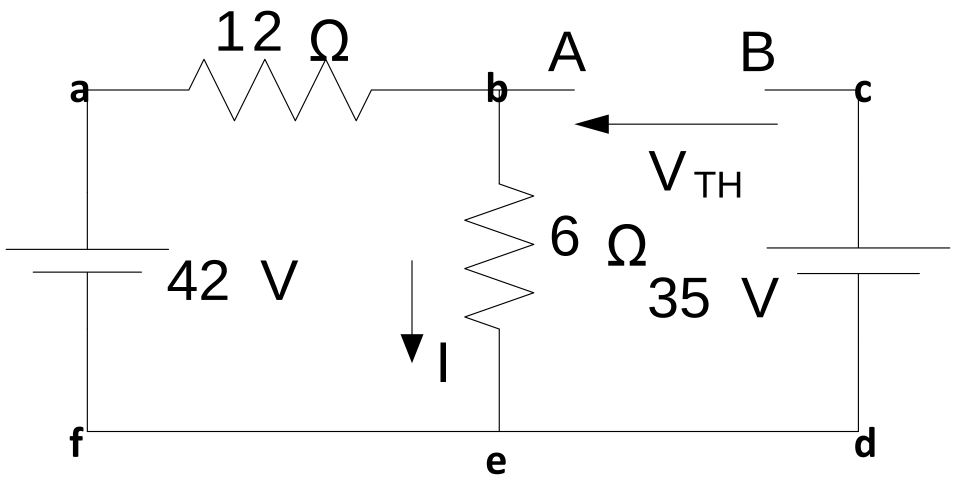


Using Thevenin's theorem, determine the current in the 3-Ω resistor of the circuit below. 12 Ω 3 Ω

Solution



Steps 1 & 2



THEVENIN'S THEOREM

Applying KVL to loop dcbed: $35 + V_{TH} = 6I$ **(1)**

Applying KVL to loop fabef: $42 = (12+6)I$

$\Rightarrow I \quad \frac{7}{3} \quad = A$

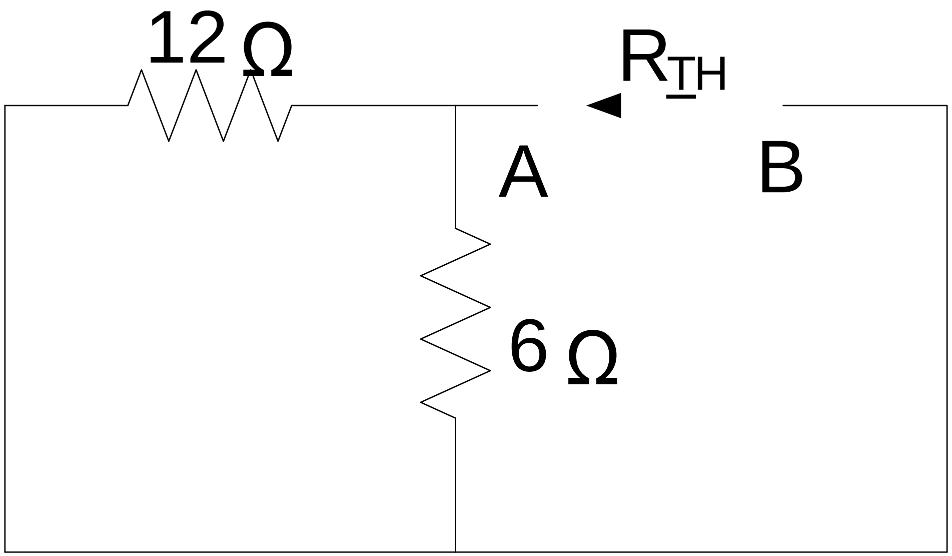
$$\frac{7}{3}$$

THEVENIN'S THEOREM

Substituting for I in equation 1: $35 + V_{TH} = 6()$

$$V_{TH} = -21V$$

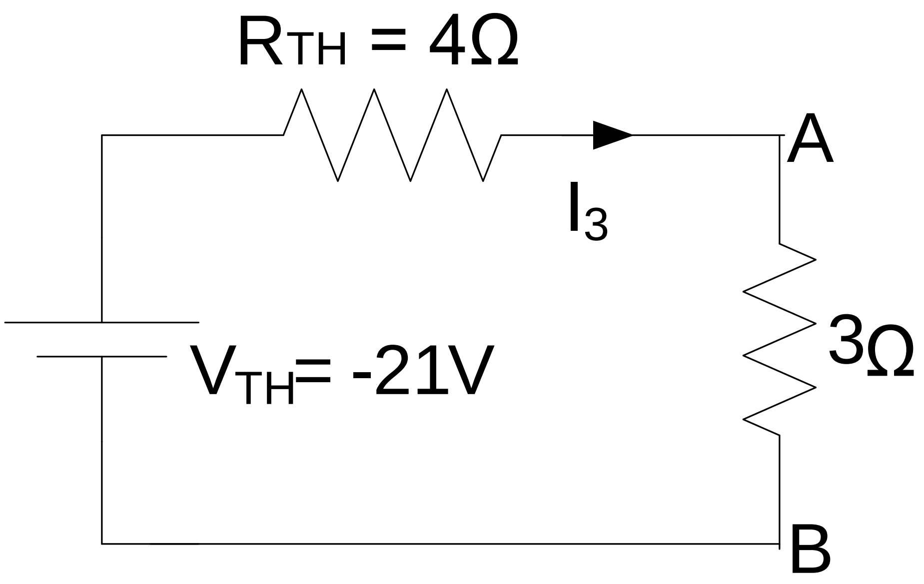
Steps 3 & 4



$$R_{TH} = 12//6 = \frac{12 \times 6}{12+6} = 4\Omega$$

THEVENIN'S THEOREM

Steps 5 & 6



V^{TH} 21 3A

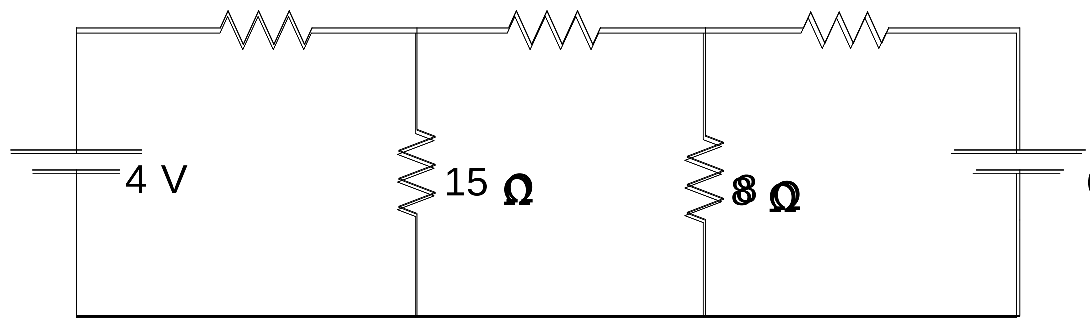
$$I_3 = \frac{\quad}{\quad} \quad \frac{\quad}{\quad} = -$$

$$= - \; R_{TH} + 3 \quad 4 + 3$$

Example 2

Find the current in the 10-Ω resistor of the circuit below using Thevenin's theorem. 5^5 Ω^Ω 1010 ΩΩ 1212 ΩΩ

THEVENIN'S THEOREM

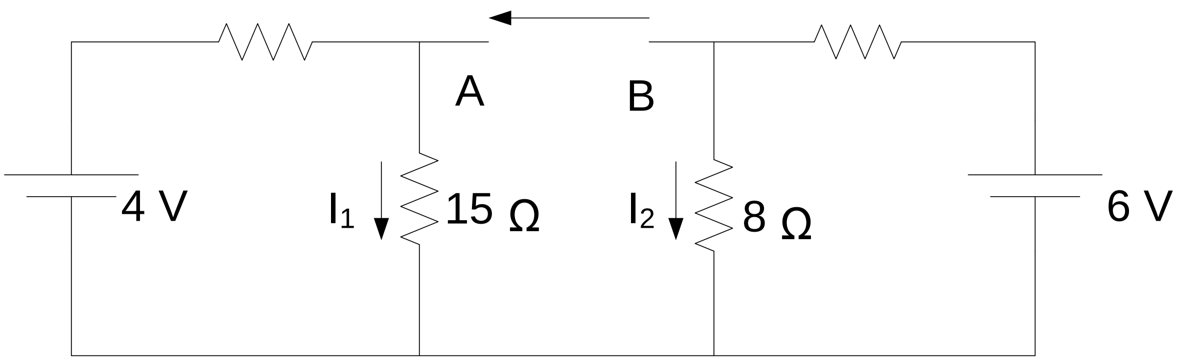


Solution

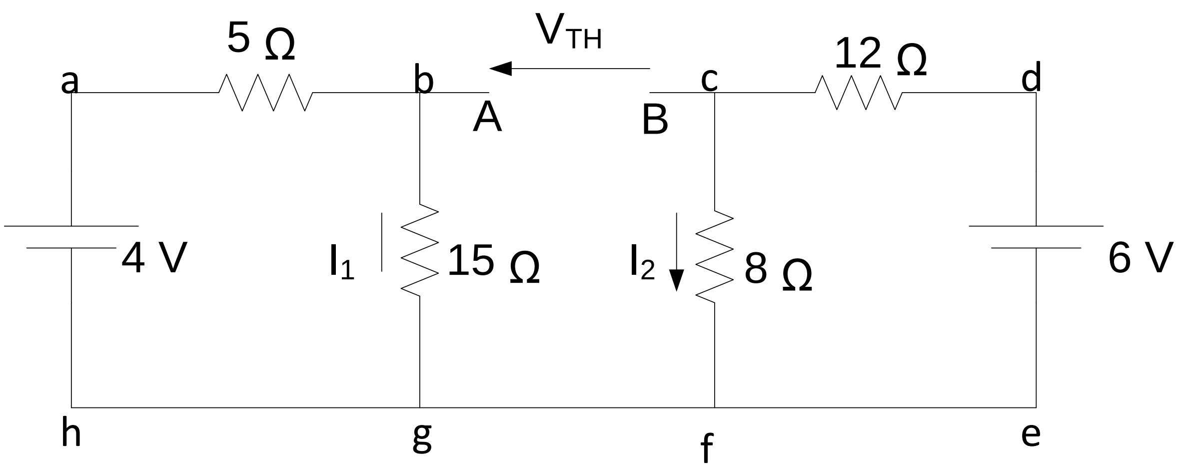
5 Ω V_{TH} 12 Ω

THEVENIN'S THEOREM

Steps 1 & 2



THEVENIN'S THEOREM



Applying KVL to loop cbgfc: $V_{TH} = 15I_1 - 8I_2$

(1)

$\frac{1}{5}$

Applying KVL to loop abgha: $4 = (5+15)I_1 \Rightarrow I_1 = A$

Applying KVL to loop dcfed: $6 = (12+8)I_2$

$$\frac{3}{10}$$

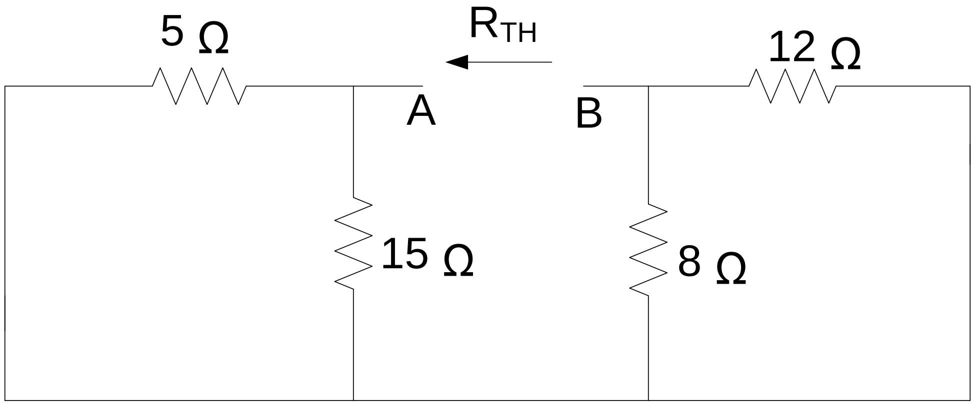
$\Rightarrow I_2 = A$

Substituting for I_1 and I_2 in equation 1 yields:

THEVENIN'S THEOREM

$$V_{TH} = 15\left(\frac{1}{5}\right) - 8\left(\frac{3}{10}\right) = \frac{3}{5}V$$

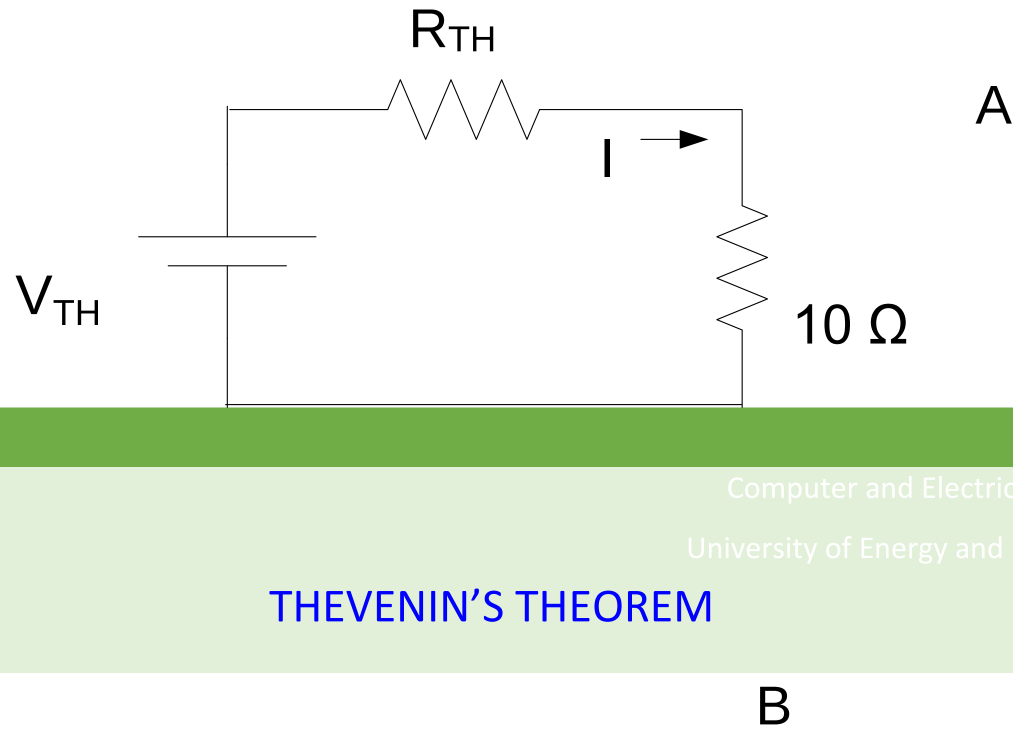
Steps 3 & 4



THEVENIN'S THEOREM

$$R_{TH} = (5//15)+(12//8) = \frac{171}{20}\,\Omega$$

Steps 5 & 6



Assignment 2

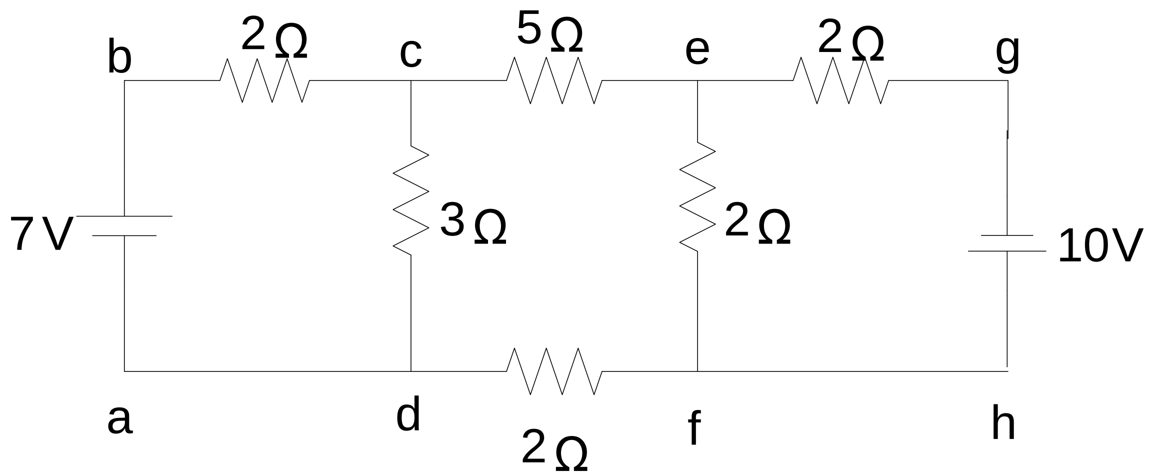
$$V^{TH} \quad = \quad 3 \quad = 3^{\square 20} = 0.032\ \text{A}$$

$$I \quad =$$

$$R_{TH} + 10\,5\square\square171\_\_\_ + 10\square\square\,5\square371$$

$$\square\ 20 \qquad \square$$

Use Thevenin’s theorem to find the current in the 5Ω resistor of the circuit below.



Submission date: God willing a week today

Submission time: Before lecture starts

Where to submit: Teaching Assistant’s office

Theorem:

NORTON'S Theorem

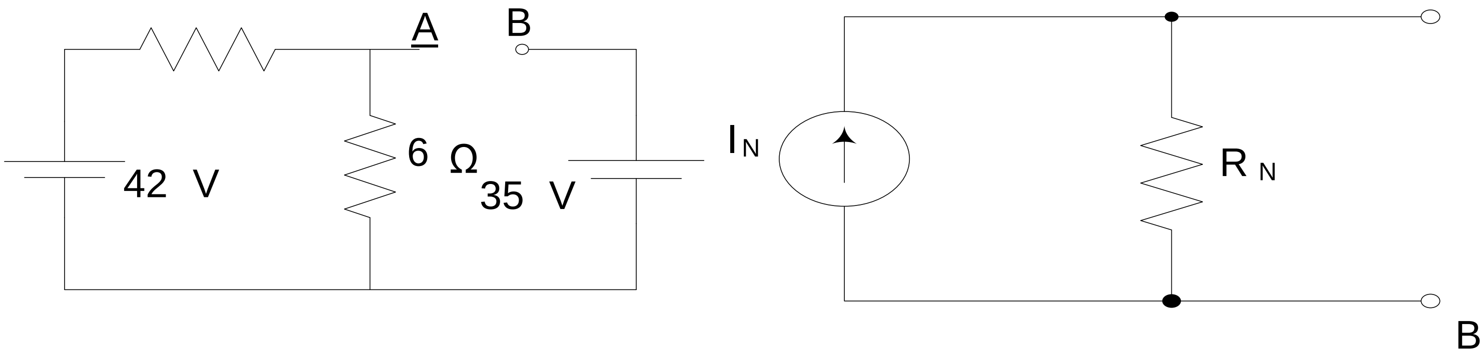
Any linear circuit connected between two terminals can be replaced by a Norton's current(I_N) in parallel with a Norton's resistance (R_N).

I_N is the short-circuit current between the two terminals

R_N is the resistance seen from the two terminals when all sources have been deactivated ($R_N = R_{TH}$)

12 Ω A

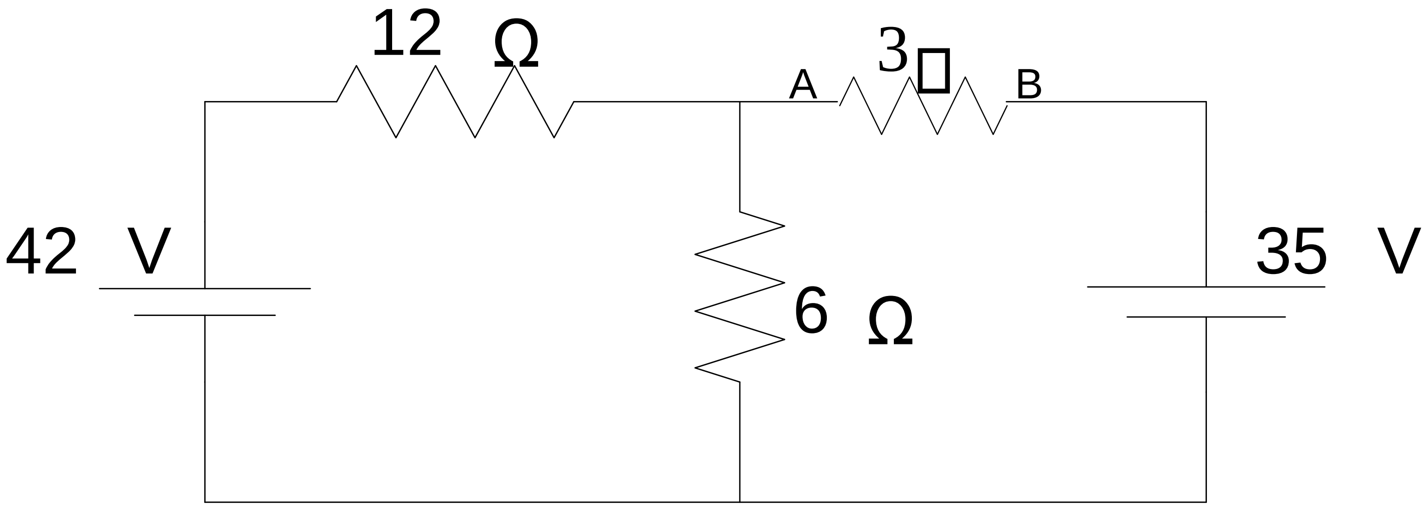
NORTON'S Theorem



To find the current through a resistor in a circuit, the following steps are taken:

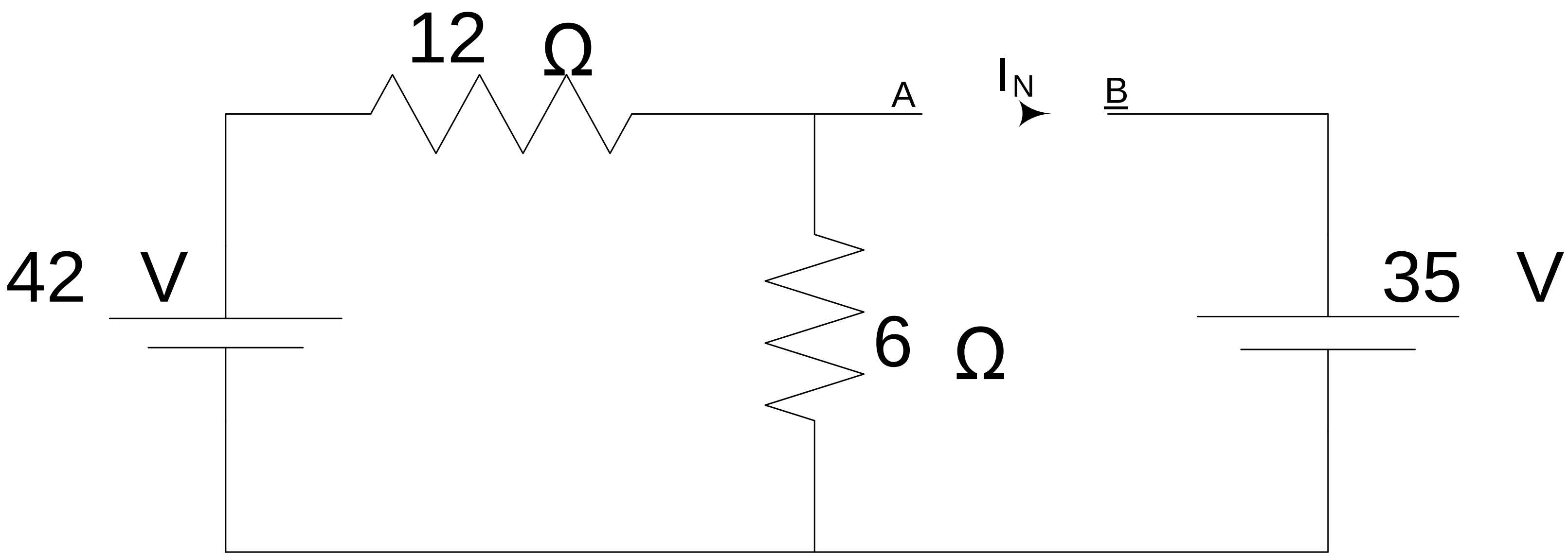
1. **Remove the resistor from the circuit and mark the two terminals.**

NORTON'S Theorem

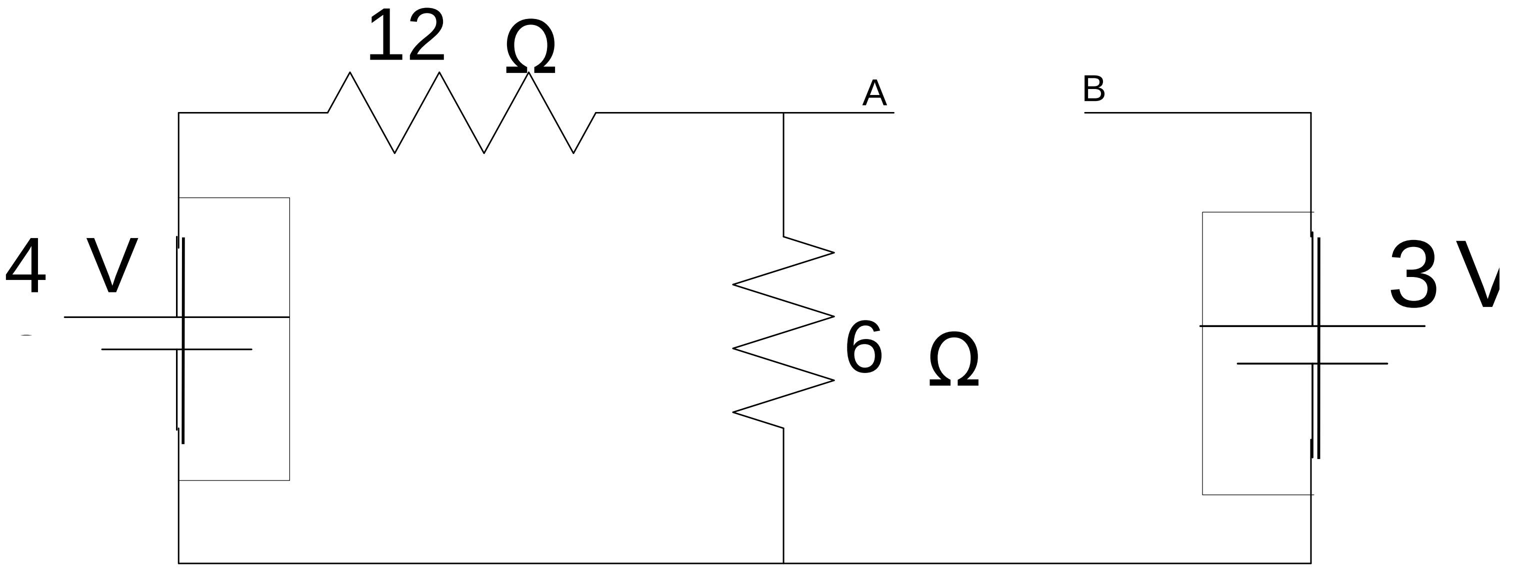


2. Find the short-circuit current (I_N) through the two terminals by applying KVL.

NORTON'S Theorem

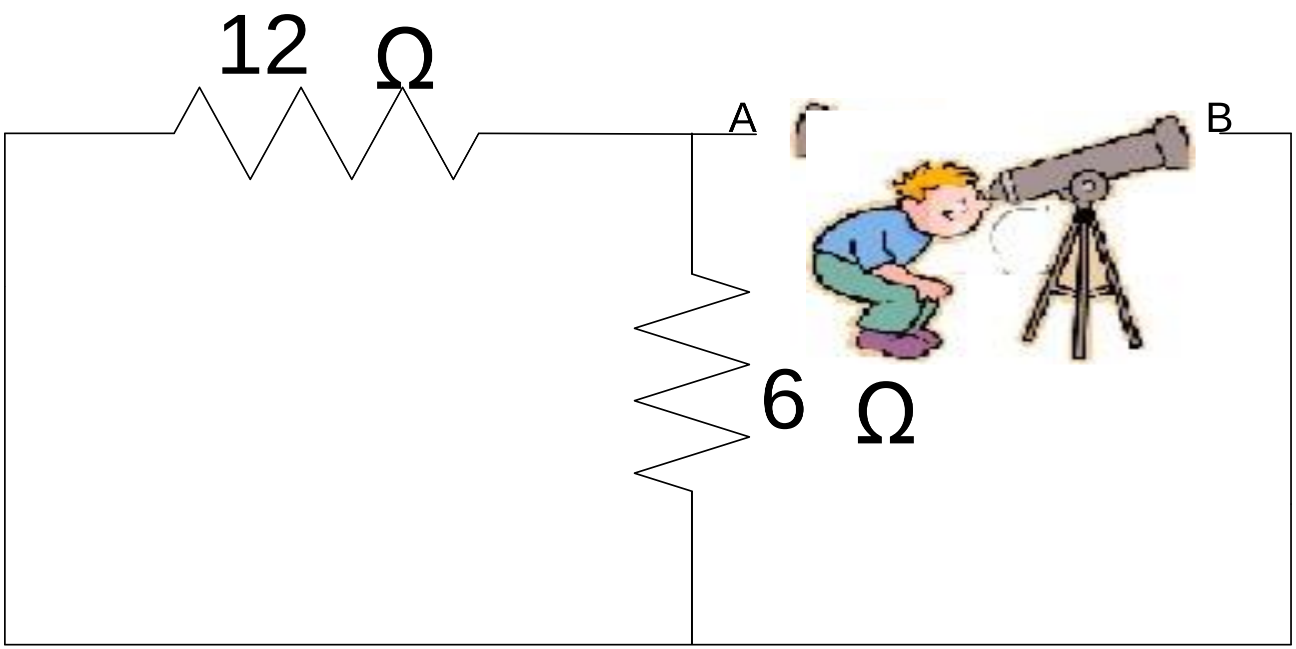


3. **Recall the circuit created before step 2 and deactivate all sources. Short-circuit voltage sources and Open-circuit current sources.**


12 Ω
A
B
4 V
6 Ω
3 V

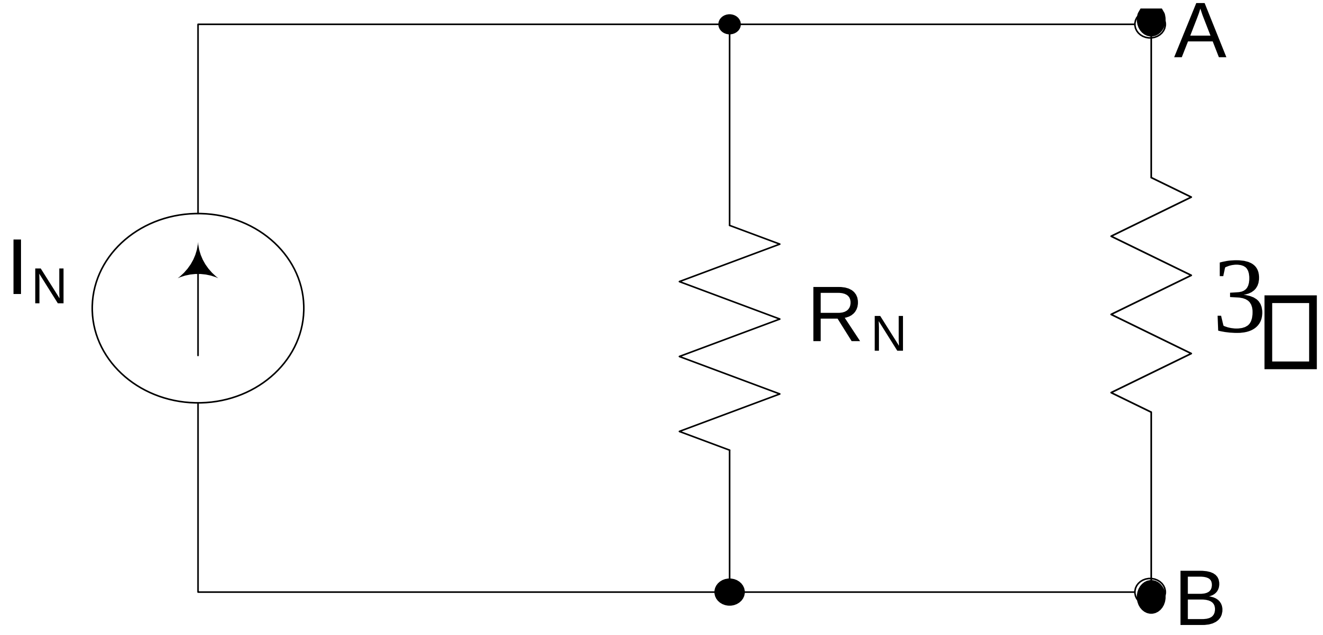
NORTON'S THEOREM

4. Find the total resistance of the circuit resulting from step 3 as seen from the two terminals



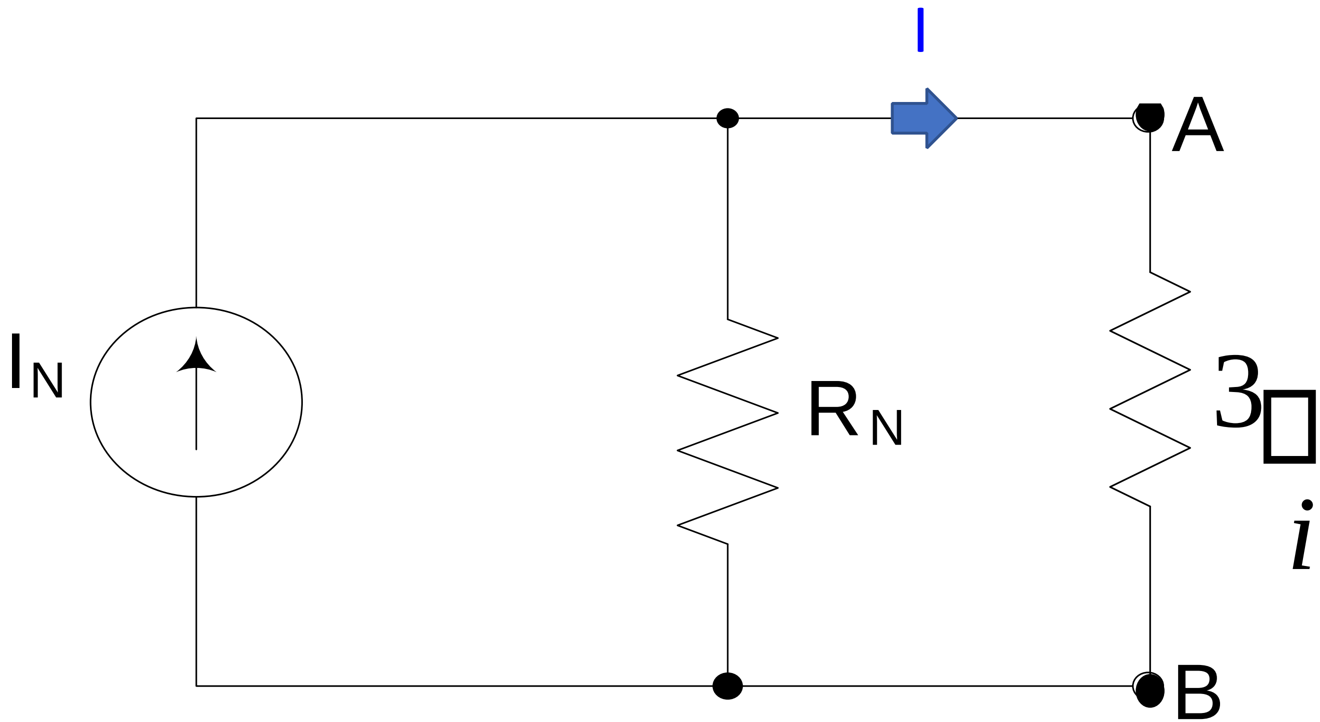
NORTON'S THEOREM

5. **Reproduce the Norton's equivalent circuit and connect the resistor whose current is to be found.**



NORTON'S THEOREM

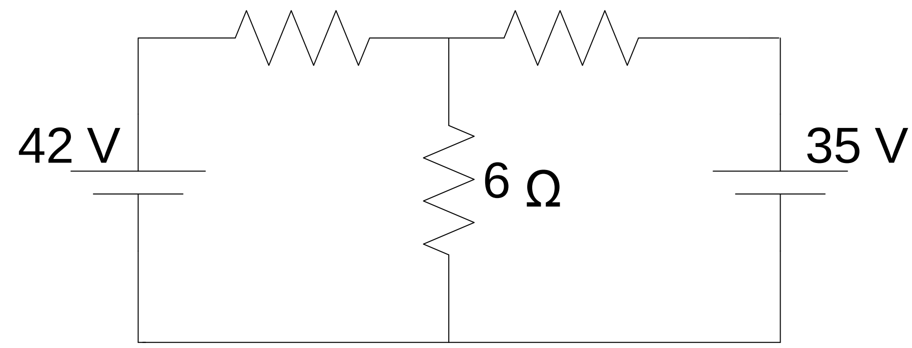
6. Calculate the current in the circuit in step 5. This is the current being sought for.



$$i = \frac{R_N}{R_N + 3} \square I_N$$

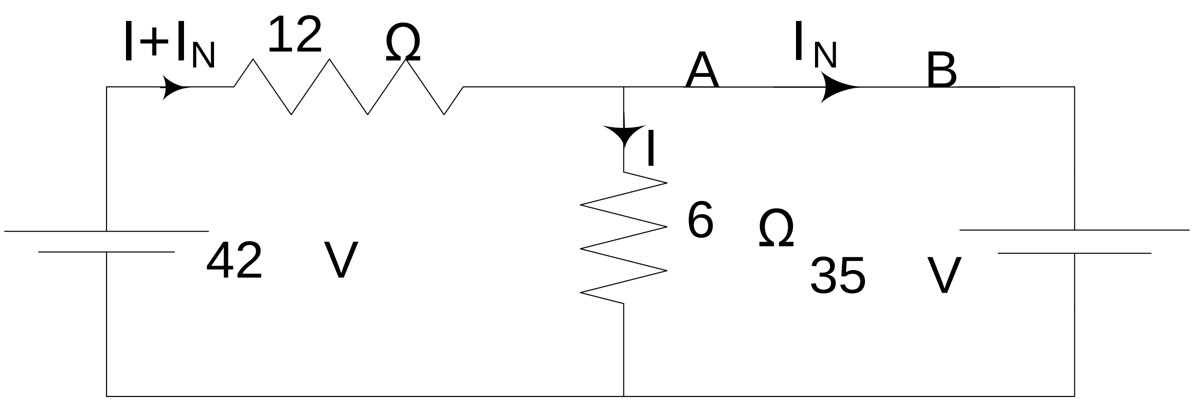
Example 1

Using Norton’s theorem, determine the current in the 3-Ω resistor of the circuit below. 12 Ω 3 Ω

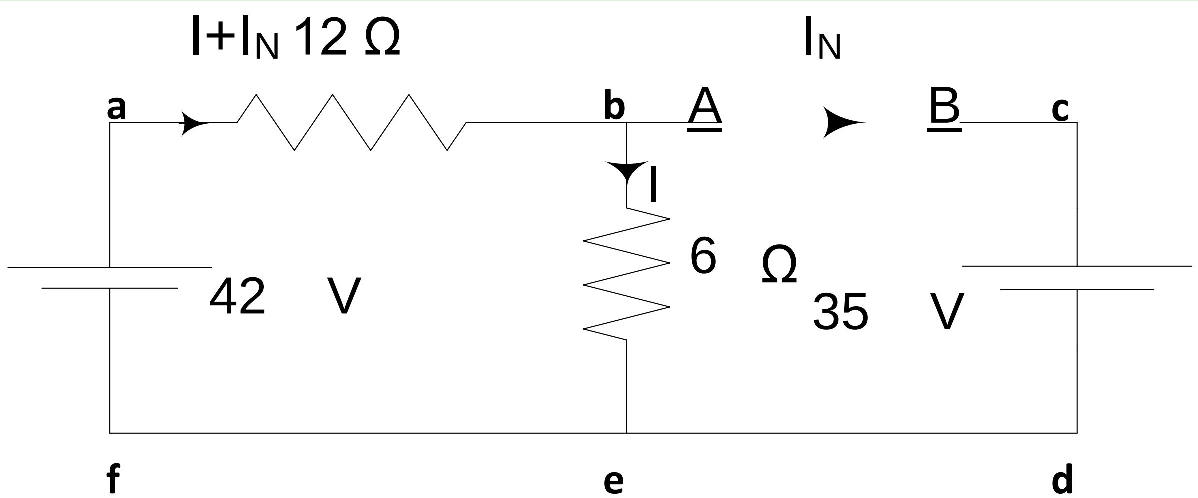


Solution

Steps 1 & 2



NORTON'S THEOREM



Applying KVL to loop abefa: $42 = 12(I+I_N) + 6I$

$$42 = 18I + 12I_N \qquad \textbf{(1)}$$

Applying KVL to loop cbedc: $35 = 6I$

$$\Rightarrow I = \frac{35}{6}\ A$$

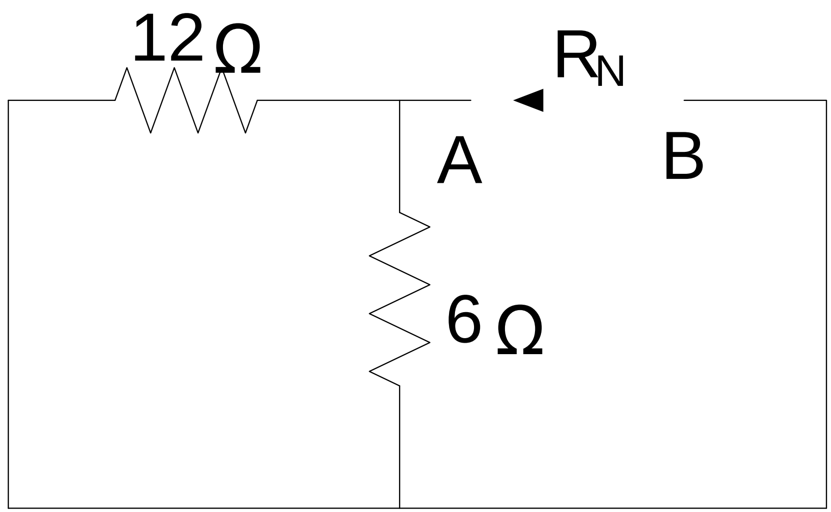
Substituting for I in equation 1: $42 = 18\left(\frac{35}{6}\right) + 12I_N$

$$-21$$

$I_N = A$

4

Steps 3 & 4

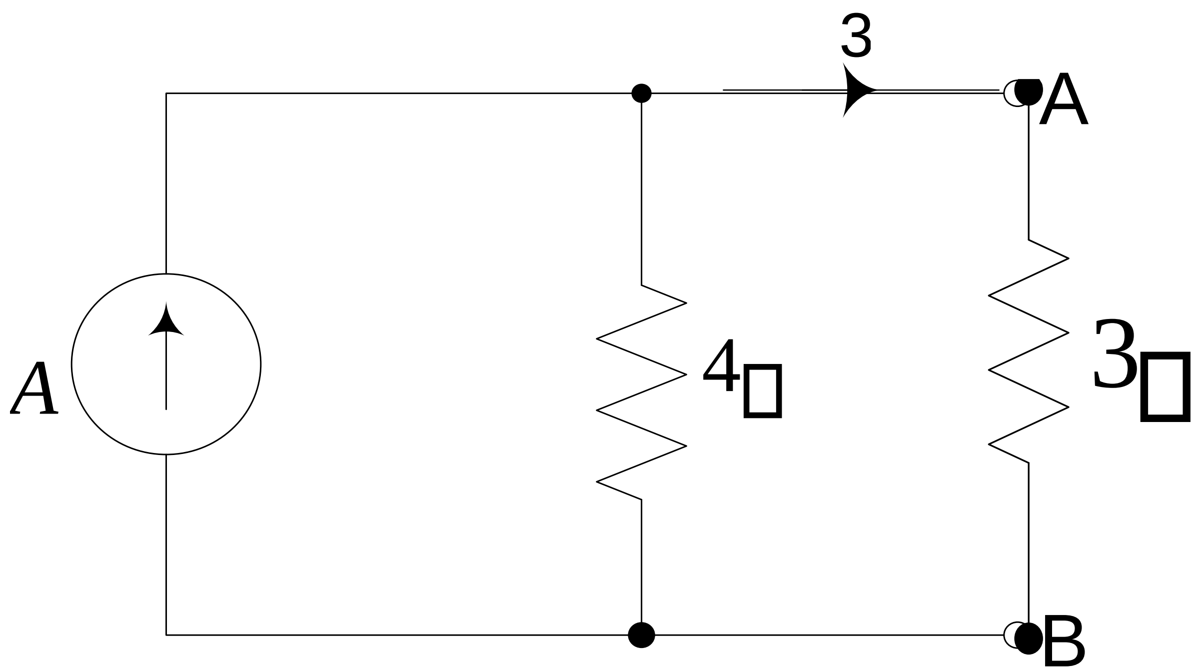


$$R_N = 12//6 = \frac{12 \times 6}{12+6}$$

$$= 4\Omega$$

$$\frac{-21}{4}$$

Steps 5 & 6

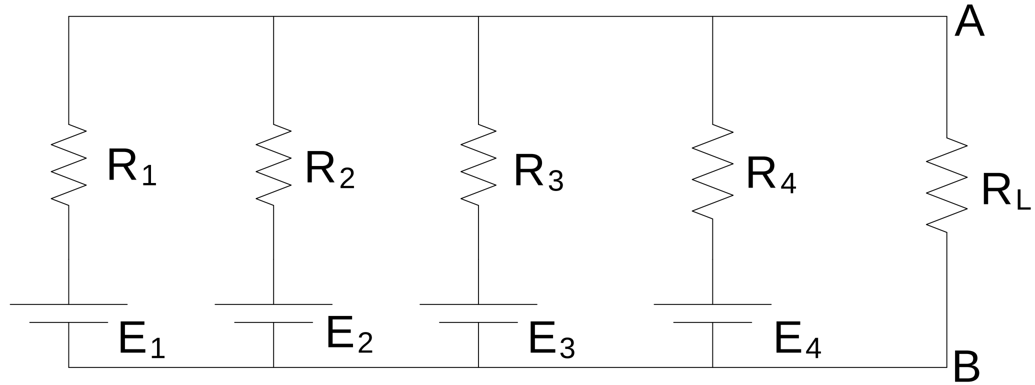


$$I_3 = \frac{4}{4+3} \square \frac{-21}{4} = -3A$$

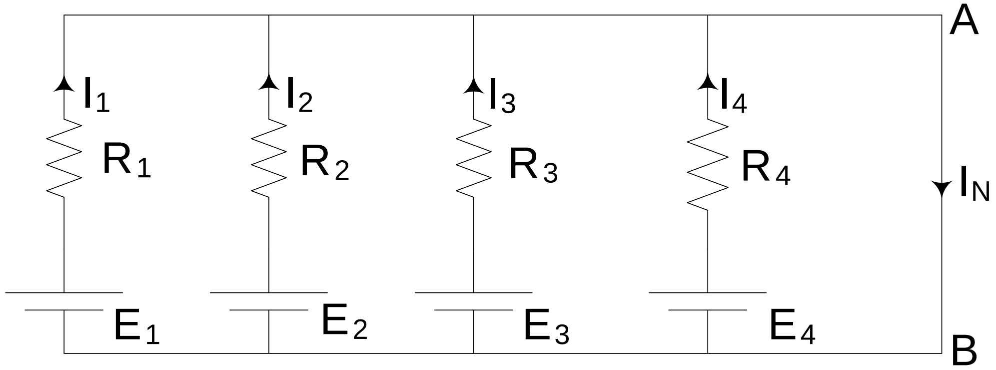
NORTON'S THEOREM

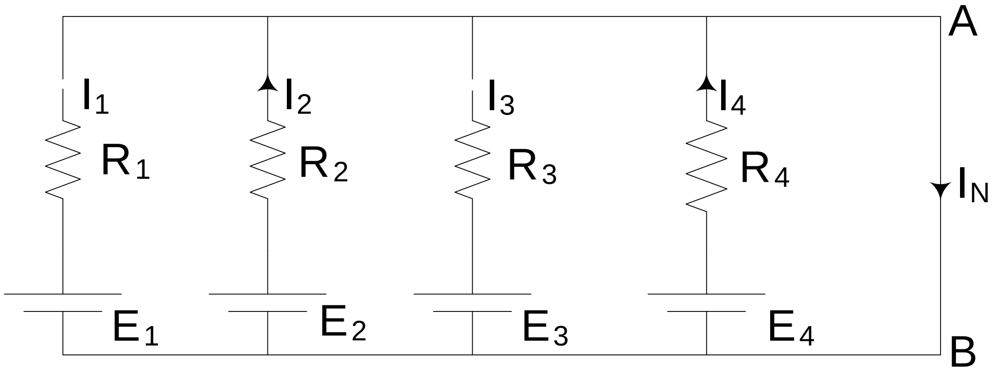
Example 2

Determine the current in the load resistor R_L



Solution





Solution

Applying KCL $$I_N = I_1 + I_2 + I_3 + I_4$$

NORTON'S THEOREM

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University of Energy and Natural Resources, Sun ani

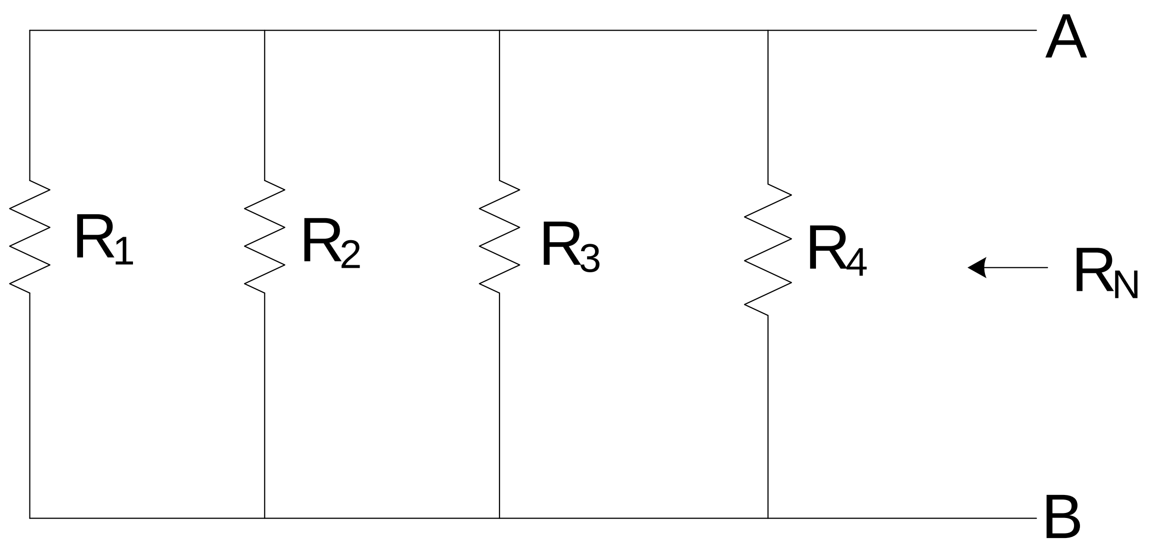
NORTON'S THEOREM

$$E_1 \; E_2 \; E_3 \; E_4$$

$$= \; + \; + \; +$$

$$R_1 \; R_2 \; R_3 \; R_4$$

Finding R_N



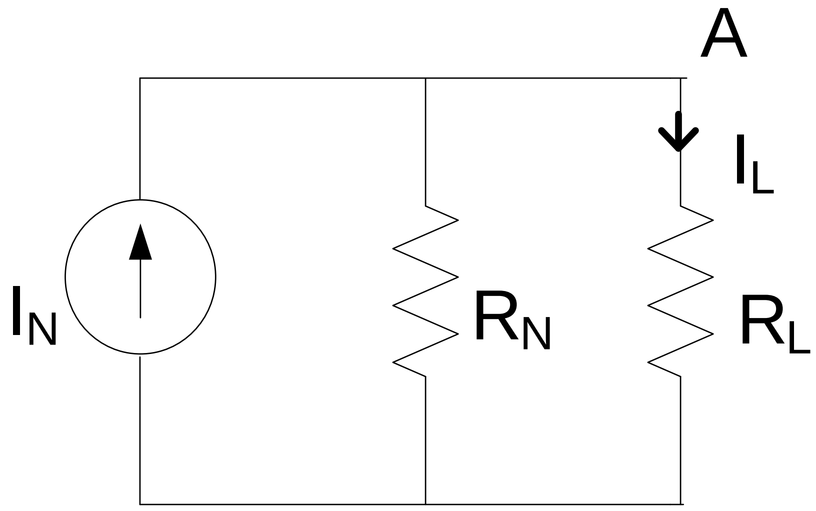
$$\frac{1}{\quad\quad} = \frac{1}{\quad\quad} + \frac{1}{\quad\quad} + \frac{1}{\quad\quad} + \frac{1}{\quad\quad}$$

NORTON'S THEOREM

$$R_N \; R_1 \; R_2 \; R_3 \; R_4$$

Finding I

L



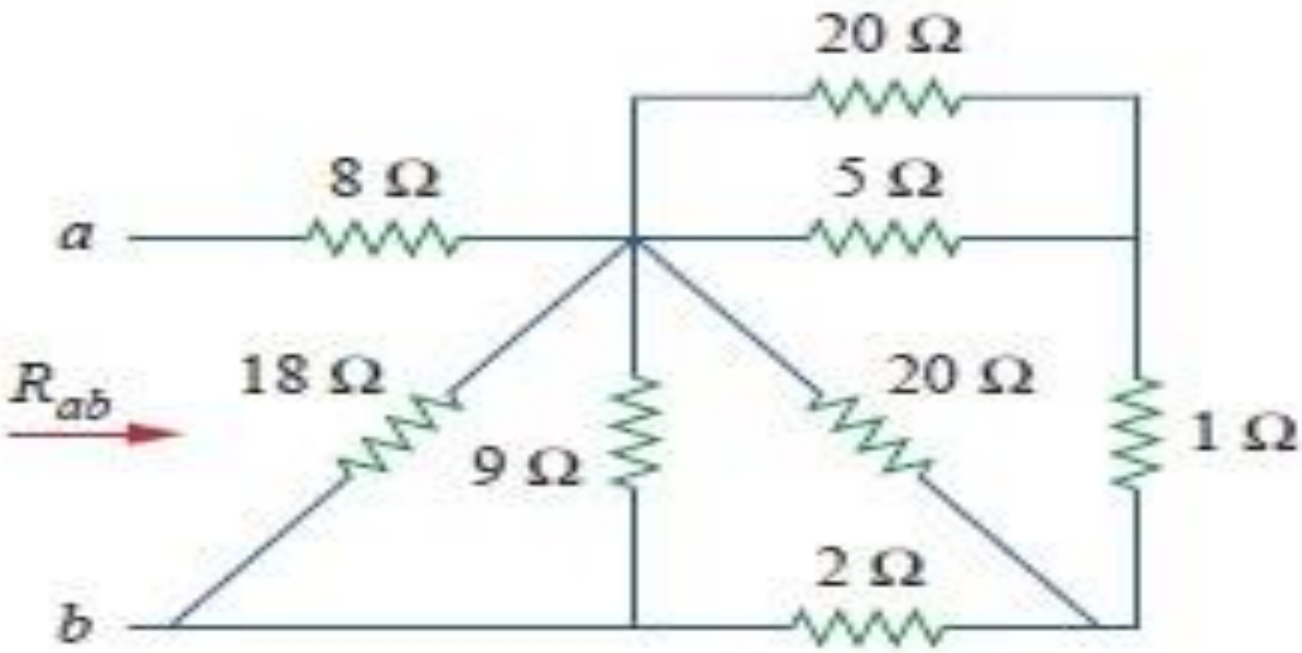
B

$$R_N$$

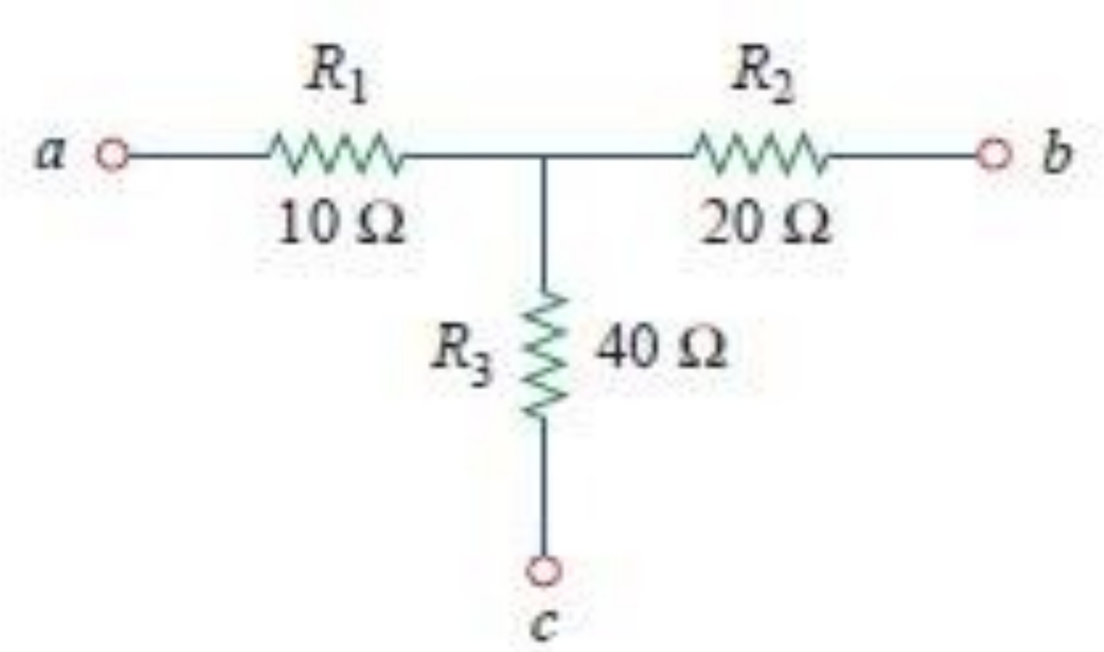
$$I_N$$

$$I_L = \square\ R_N + R_L$$

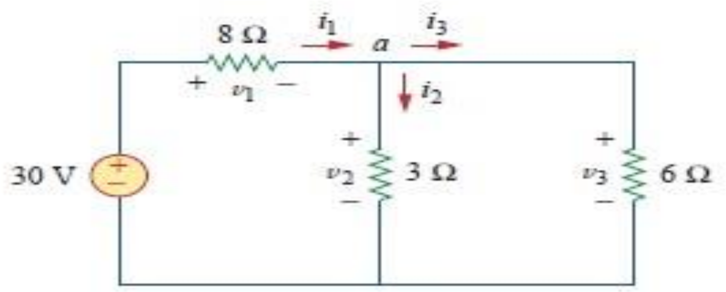
1. Find the equivalent resistance in the circuit.


20 Ω
8 Ω
5 Ω
a
R_{ab}
18 Ω
9 Ω
20 Ω
1 Ω
2 Ω
b

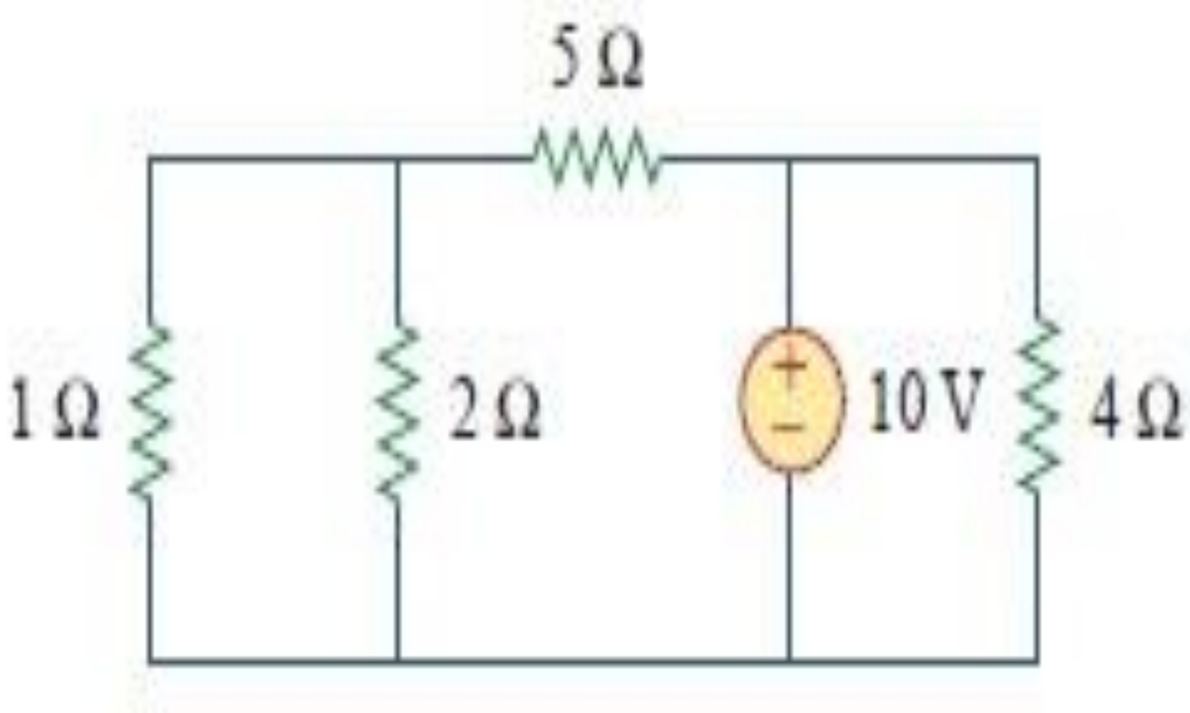
2. Transform the circuit to delta



3. Find the currents and voltages in all the branches



4. Find the current in the 1 ohm

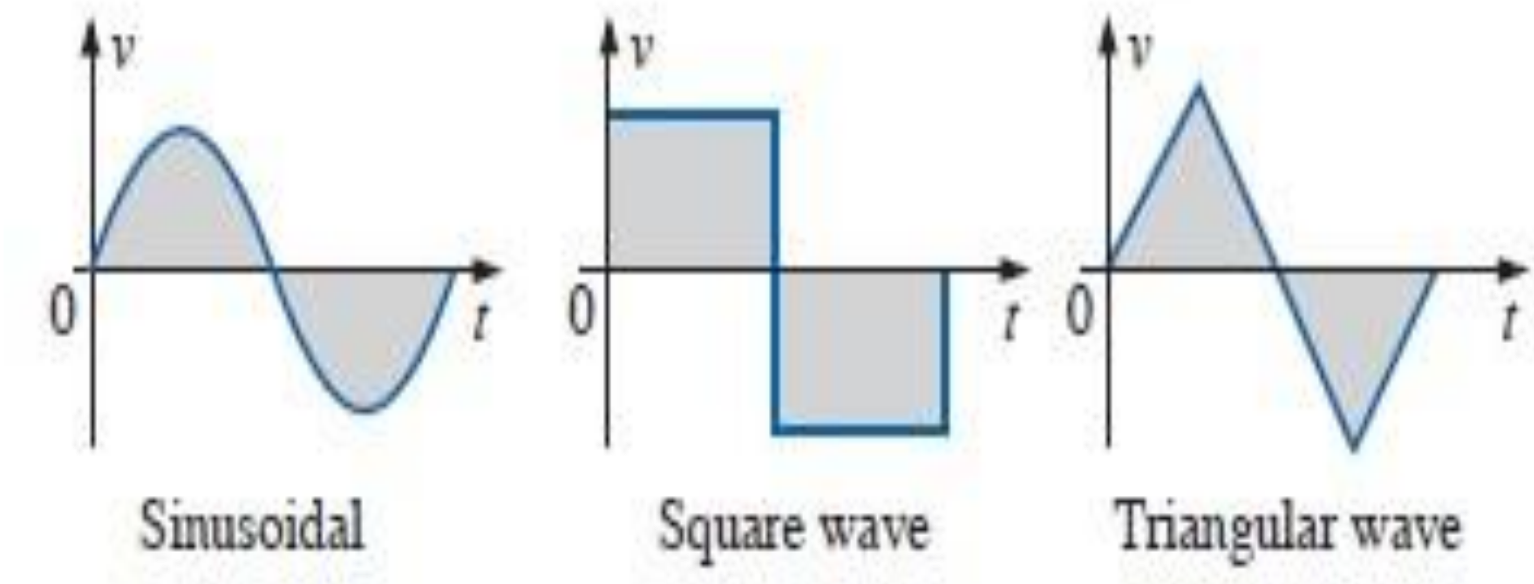


resistor

ALTERNATING CURRENT CIRCUITS

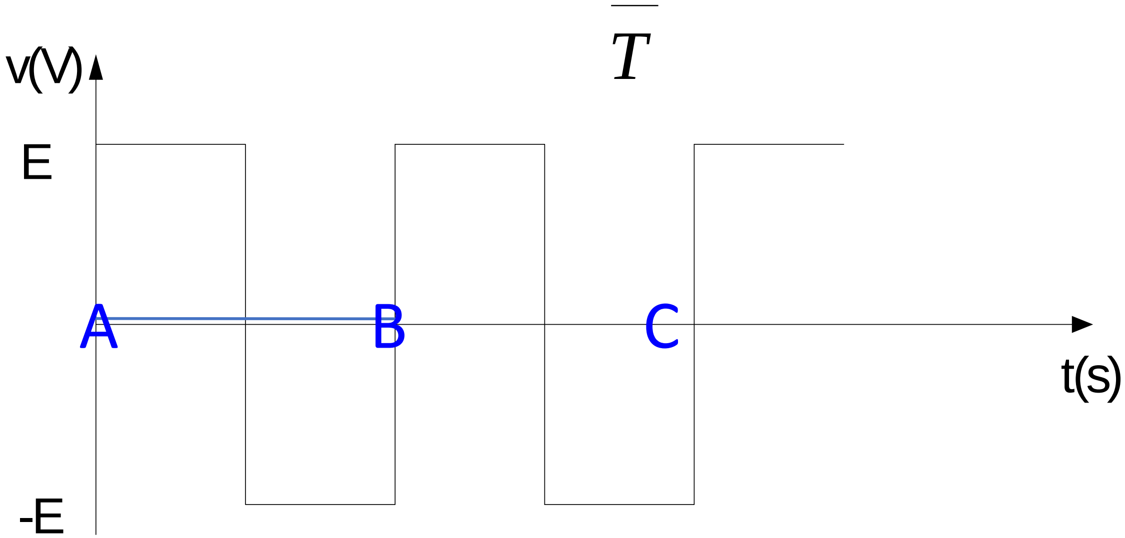
- Alternating current (AC) circuits are circuits with currents and voltages which are time-varying

- Examples of AC waveforms are



TERMINOLOGIES IN AC CIRCUITS

- **Amplitude (peak):** The maximum deviation of the function from its center position
- **Cycle:** A repeating portion of a function (wave).
- **Period (T):** The duration of a cycle



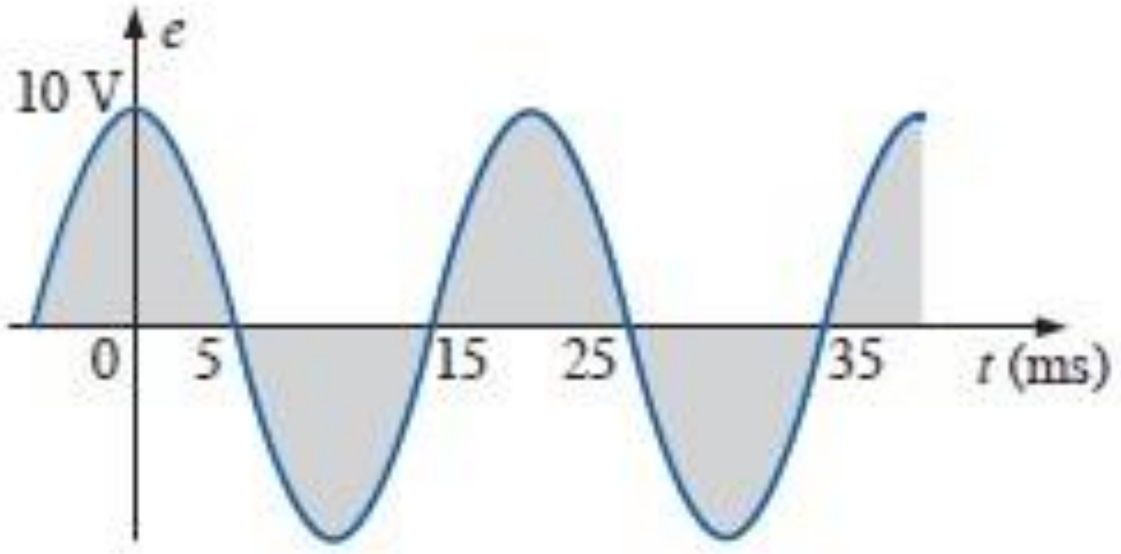
- **Frequency(f):** The number of cycles that occur in 1second

  The inverse of period. $f = 1$

Determine the frequency of the waveform

Solution: From the figure, $T = (25 \text{ ms} - 5 \text{ ms}) = 20 \text{ ms}$, and

$$f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}\,\text{s}} = 50 \text{ Hz}$$


e
10 V
0
5
15
25
35
t (ms)

Instantaneous value: The magnitude of a waveform at any instant of time; denoted by lowercase letters (e_1, e_2).

Peak amplitude: The maximum value of a waveform as measured from its *average*, or *mean*, value, denoted by uppercase letters (such as E_m for sources of voltage and V_m for the voltage drop across a load). For the waveform of Fig. 13.3, the average value is zero volts, and E_m is as defined by the figure.

Peak value: The maximum instantaneous value of a function as measured from the zero-volt level. For the waveform of Fig. 13.3, the peak amplitude and peak value are the same, since the average value of the function is zero volts.

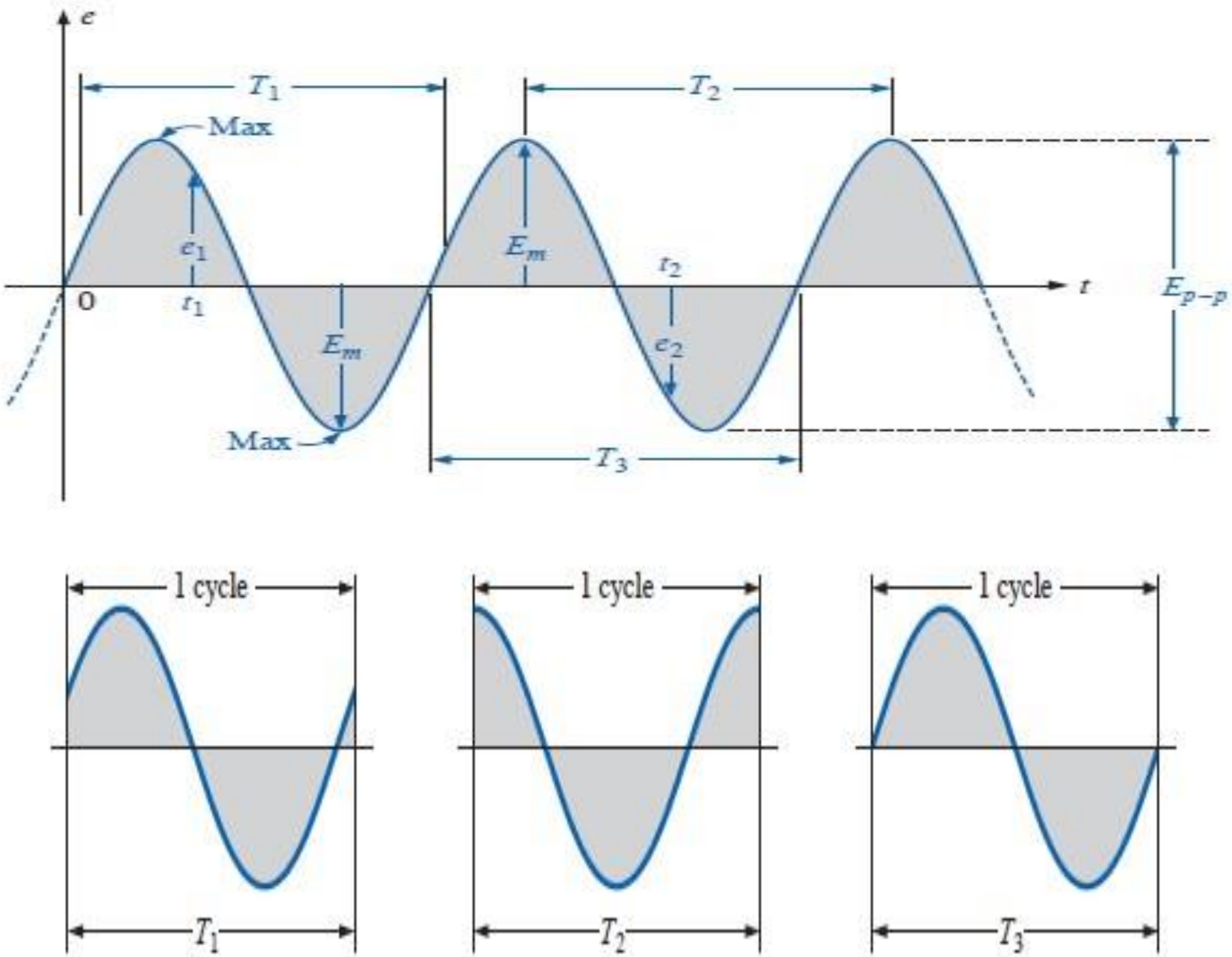
Peak-to-peak value: Denoted by E_{p-p} or V_{p-p}, the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks.

Periodic waveform: A waveform that continually repeats itself after the same time interval. The waveform of Fig. 13.3 is a periodic waveform.

Period (T): The time interval between successive repetitions of a periodic waveform (the period $T_1 = T_2 = T_3$ in Fig. 13.3), as long as successive *similar points* of the periodic waveform are used in determining T.

Cycle: The portion of a waveform contained in *one period* of time. The cycles within T_1, T_2, and T_3 of Fig. 13.3 may appear different in Fig. 13.4, but they are all bounded by one period of time and therefore satisfy the definition of a cycle.

Period T



AVERAGE VALUE

✞Average value: The average value of a periodic function is its dc value.

If $$i = f(t)$$

Then $$I_{av} = \frac{1}{T}\int_0^T f(t)dt = \frac{area[f(t)]}{T}$$

AVERAGE VALUE

The following steps are followed when finding average values of waveforms:

1. **Identify a cycle of the wave**

2. **Note the period**

3. **Find the area of the cycle**

4. **Divide the area by the period**

ROOT MEAN SQUARE VALUE

The Root Mean Square (RMS) or Effective value of an alternating quantity is the value of a direct current which when flowing through a given resistance for a given time produces the same heat as produced by the alternating current when flowing through the same resistance.

The RMS value of an alternating current $i = f(t)$ is:

$$I_{rms} = \left[\frac{1}{T}\int_0^T \left[f(t)\right]^2 dt\right]^{\frac{1}{2}} = \sqrt{\frac{area\left[(f(t))^2\right]}{T}}$$

ROOT MEAN SQUARE VALUE

The following steps are taken when finding the RMS value of a waveform :

1. **Identify a cycle of the waveform**

2. **Note the period**

3. **Square the cycle**

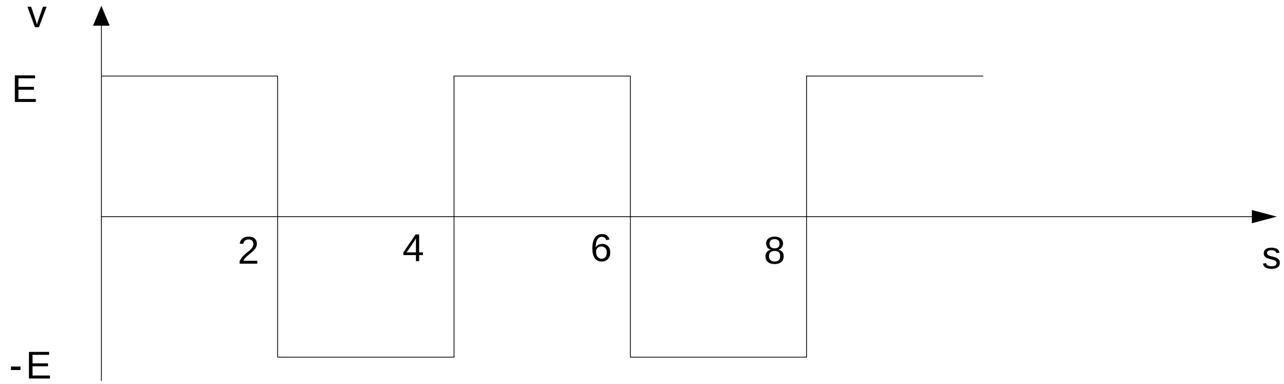
4. **Find the area under the squared cycle**

5. **Divide the area by the period**

6. **Take the square root of the result**

Example 1

Find the average and rms values of the waveform below.



Solution

Average Value

- Cycle spans from 0 to 4
- Period= 4s

- Area of cycle $=(2\square E)+(2\square -E)$

$$=0 \; Area \; 0$$

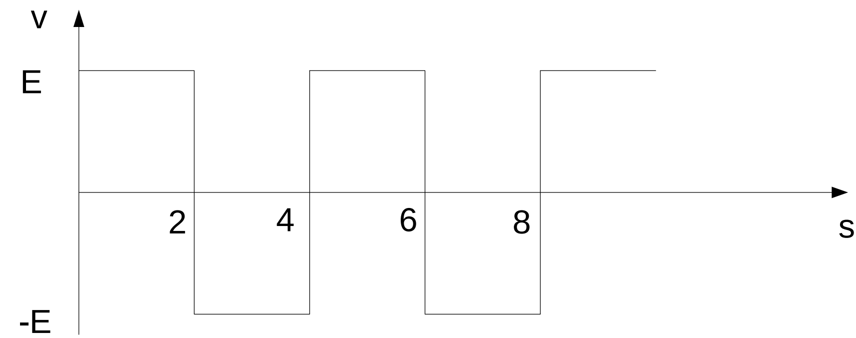
$$\square V_{avg} = \underline{\hspace{6em}} = \underline{\hspace{1em}} = 0V \; period \; 4$$

ROOT MEAN SQUARE VALUE

RMS value

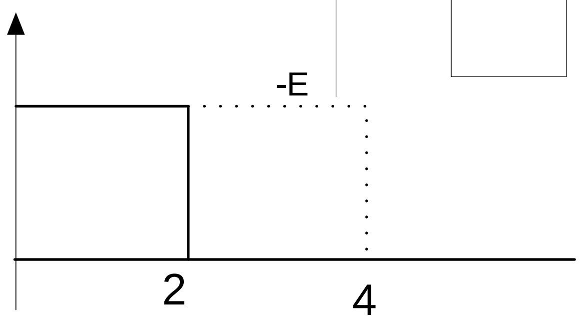
- Cycle spans 0 to 4
- Period = 4s

- Squared cycle E^2



- Area covered by squared cycle

$$=4\square E^2 = 4E^2$$



- Division of area by period $= \frac{4E^2}{4} = E^2$

ROOT MEAN SQUARE VALUE

- Taking square root

$$V = E_2 = E$$

rms

SINUSOIDAL VOLTAGES AND CURRENT

Voltages and currents of commercial ac generators have the following expressions:

$$v = V_m \sin \omega t \text{ or } v = V_m \sin 2\pi f t$$

V_m is the peak voltage f is the frequency in Hz

ω is the angular frequency in radian per second. It specifies how many oscillations occur in a unit time interval

$$i = I_m \sin \omega t \quad \text{or} \quad i = I_m \sin 2\pi f t$$

I_m is the peak current f is the frequency in Hz

ω is the angular frequency in radian per second. It specifies how many oscillations occur in a unit time interval

RMS VALUE OF SINUSOIDAL QUANTITIES

The RMS value of a sinusoidal voltage v 1 $V_m \sin 2\square ft$ is given by:

$$1 \; T \; 2 \; - \; \sin_2 2\square ftdt\square\square_{12}$$

$$V = \square\square\square T \; \square_0 V_m \qquad \square$$

$$1 \; \square\square \; 2 \; T\square\square$$

$$\left[\frac{1}{T}\int_0^T \frac{V_m^2}{2}(1-\cos 4\pi ft)\,dt\right]^{\frac{1}{2}}$$

$$=\left[\frac{1}{T}\left[\,\right]_0^T \frac{\;}{2}\right]$$

$$\left[\frac{V_m^2 T}{2T}\right]^{\frac{1}{2}} \quad \boxed{=\frac{V_m}{\sqrt{2}}}$$

RMS VALUE OF SINUSOIDAL QUANTITIES

Similarly,

The RMS value of a sinusoidal current $i = I_m \sin 2\pi ft$ is given by:

$$\boxed{I = \frac{I_m}{\sqrt{2}}}$$

Example 1 RMS VALUE OF SINUSOIDAL QUANTITIES

Find the rms values of the following quantities:

(a) $i = 10\sqrt{2}\sin 100\pi t$ **(b)** $v = 20\sin 100\pi t$

Solution

(a) $I = \dfrac{10\sqrt{2}}{\sqrt{2}} = 10$

(b) $$V = \frac{20}{\sqrt{2}} = 14.14$$

HARMONICS

Non-sinusoidal periodic voltages and currents can be expressed as the sum of sine waves in which the lowest frequency is *f* and all other frequencies are integral multiples of *f*.

For example, a square wave $v(t)$ of amplitude E can be expressed as:

$$v(t) = \frac{4E}{\pi}\left[\sin 2\pi ft + \frac{1}{3}\sin 6\pi ft + \frac{1}{5}\sin 10\pi ft + - - -\right]$$

HARMONICS

- **Any quantity which contains multiple frequencies is a harmonic quantity.**
- **The frequency of which others have been expressed as multiples of is the fundamental frequency.**
- **An odd multiple of the fundamental is an odd harmonic.**

RMS VALUE OF A HARMONIC QUANTITY

- **An even multiple of the fundamental is an even harmonic.**

The effective value of a harmonic quantity is obtained by:

- **First obtaining the square of the rms value of each term**
- **Adding the obtained squared rms values**
- **Taking the square root of the sum**

$$v(t) = a_o + a_1 \sin(\omega t + \theta_1) + a_2 \sin(2\omega t + \theta_2) + a_3 \sin(3\omega t + \theta_3) + \ldots\ldots$$

$$V = \sqrt{a_o^{2} + \left[\frac{a_1}{\sqrt{2}}\right]^2 + \left[\frac{a_2}{\sqrt{2}}\right]^2 + \left[\frac{a_3}{\sqrt{2}}\right]^2 + \cdots\cdots\cdots}$$

RMS VALUE OF A HARMONIC QUANTITY

Example

Find the RMS value of the current

$$i(t) = 2 + 5\sin wt + 3\sqrt{2}\sin\left(3wt + 30^0\right)$$

Solution

2 2

$$I = \sqrt{2^2 + \left(\frac{5}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}}\right)^2}$$

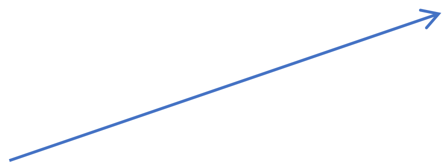
$$= \mathbf{5.05}$$

PHASORS

- **Phasors are used to represent sinusoidal quantities to avoid drawing the sine waves.**

- **A phasor is a straight line whose length is proportional to the rms voltage or current it represents.**

- **To show the phase angle or phase displacement between voltages and currents, the phasors bear an arrow.**



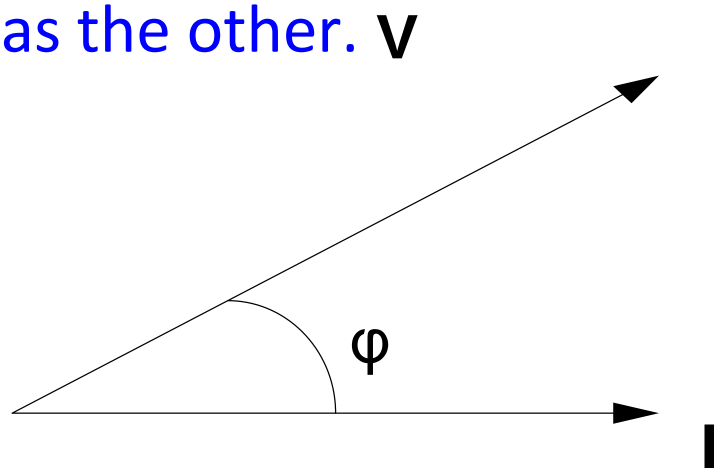
PHASORS

- **Two phasors are said to be in phase when they point in the same direction. The phase angle between them is then zero.**

PHASORS

- Two phasors are said to be out of phase when they point in different directions.

- The phase angle between them is the angle through which one of them has to be rotated to make it point in the same direction as the other.



EXAMPLE 13.12 What is the phase relationship between the sinusoidal waveforms of each of the following sets?

a. $v = 10 \sin(\omega t + 30°)$

$i = 5 \sin(\omega t + 70°)$

b. $i = 15 \sin(\omega t + 60°)$

$v = 10 \sin(\omega t - 20°)$

c. $i = 2 \cos(\omega t + 10°)$

$v = 3 \sin(\omega t - 10°)$

d. $i = -\sin(\omega t + 30°)$

$v = 2 \sin(\omega t + 10°)$

e. $i = -2 \cos(\omega t - 60°)$

$v = 3 \sin(\omega t - 150°)$

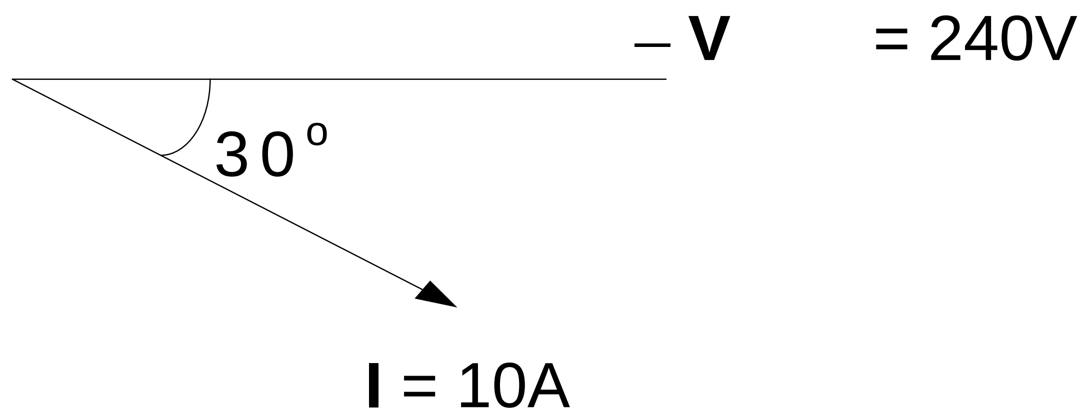
- **It is used to show at a glance the magnitude and phase relations among the various quantities within a network. This is often helpful in the analysis of the network.**

- **Example**

  A 50 Hz source having rms voltage of 240 V delivers a rms current of 10 A to a circuit. The current lags the voltage by 30°. (a) Draw the phasor diagram for the circuit. (b) Express the voltage and current as functions of time.

- **Solution**

(a) Take V as the reference



$$v(t) = 240\sqrt{2}\sin 100\pi t$$

(b)

$$i(t) = 10\sqrt{2}\sin(100\pi t - 30^{\circ})$$

ADDITION AND SUBTRACTION OF SINUSOIDAL QUANTITIES

- **The sum of sinusoidal quantities is obtained by taking the vector sum of their phasors.**
- **The difference of sinusoidal quantities is obtained by first reversing the subtracted quantity and adding it as a vector to the other phasors.**
- **A sinusoidal quantity is reversed by adding 180^0 to its angle**
- **Only sinusoidal quantities of the SAME FREQUENCY can be added or subtracted.**

ADDITION AND SUBTRACTION OF SINUSOIDAL QUANTITIES

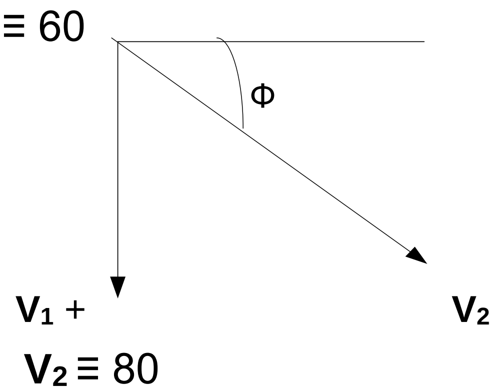
- **Example 1**

Let $v_1(t) = 60 \sin \omega t$ and $v_2(t) = 80 \sin (\omega t - 90°)$.

Determine (a) $v_1 + v_2$ and (b) $v_1 - v_2$

Solution

(a) Phasor diagram



$$\left|V_1 + V_2\right| = \sqrt{60^2 + 80^2} = 100$$

$$\phi = \tan^{-1}\left(\frac{80}{60}\right) = 53^0$$

ADDITION AND SUBTRACTION OF SINUSOIDAL QUANTITIES

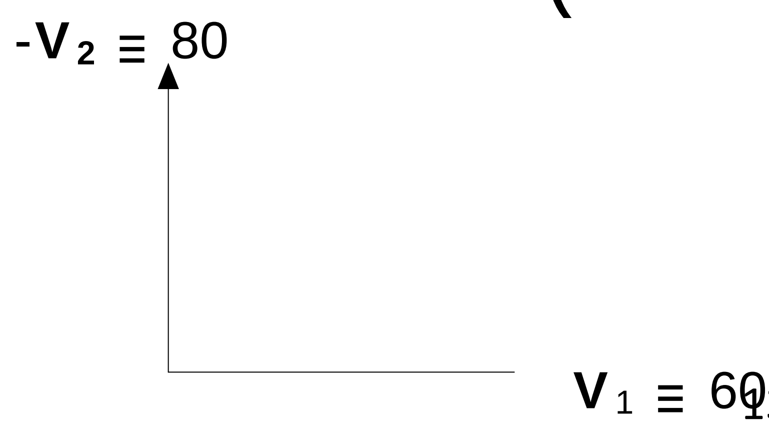
$$\therefore v_1 + v_2 = 100\left(\sin\omega t - 53^0\right)$$

(b) 1 2 1 2

$$= 60\sin\omega t + 80\sin\left(\omega t - 90^0 + 180^0\right)$$

$$= 60\sin\omega t + 80\sin\left(\omega t + 90^0\right)$$

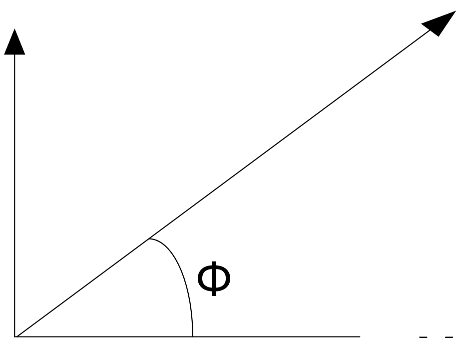
Phasor diagram



$$v - v = v + (-v)$$

ADDITION AND SUBTRACTION OF SINUSOIDAL QUANTITIES

-



V2 ≡ 80 **V1** – **V2**

V1 ≡ 60

$$\left|V_1 - V_2\right| = \sqrt{60^2 + 80^2} = 100$$

$$\square = \tan^{-1} \square\square \frac{80}{60} \square\square = 53^{0}$$

$$\square v_1 - v_2 = 100 sin\left(\square t + 53^{0}\right)$$

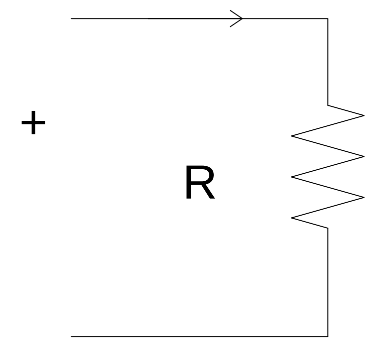
IMPEDANCE (Z)

- **The opposition to current flow in ac circuits owing to the presence of combinations of resistive, inductive and capacitive elements.**
- **Opposition due to inductance (L) is called inductive reactance(X_L).**
- **Opposition due to capacitance is called capacitive reactance(X_C).**

IMPEDANCE (Z)

Phase relationship between the current and voltage in a resistor v i

$$i = \frac{v}{R}$$



Let $v = V_m \sin \omega t$

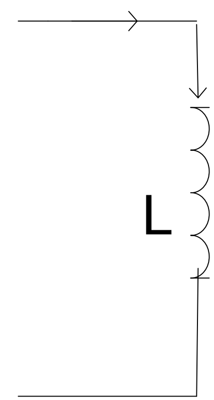
$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = I_m \sin \omega t$$

R R

It is noted that the voltage across and the current through a resistor are in phase.

IMPEDANCE (Z)

- **Phase relationship between the current and voltage in an inductor**



di $+$ dt v **Let** $i = I_m \sin\square t$

$v = L\square\square I_m \cos\square t$

$$= \omega L I_m \sin\left(\omega t + 90^0\right) = V_m$$

$$\sin\left(\omega t + 90^0\right)$$

$$v = L\,\underline{\qquad} \qquad V_L = I_L X_L$$

It is noted that the current through an inductor lags the voltage by 90^0.

$$X_L = \omega L$$

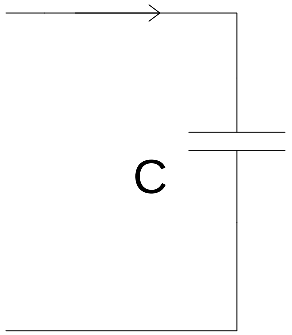
IMPEDANCE (Z)

- **Phase relationship between the current and voltage** i **in a capacitor**

$$dv$$

$$V_C = I_C X_C$$

+V



$i = c$ Let

$v = V_m$

$\sin□t \; i$

$=$

$C□□V_m$

$\cos□t$

$= □CV_m$

$$sin(\omega t$$

$$+90^0)$$

$$=$$

$$I_m(sin\omega$$

$$t+90^0)$$

$$dt$$ -

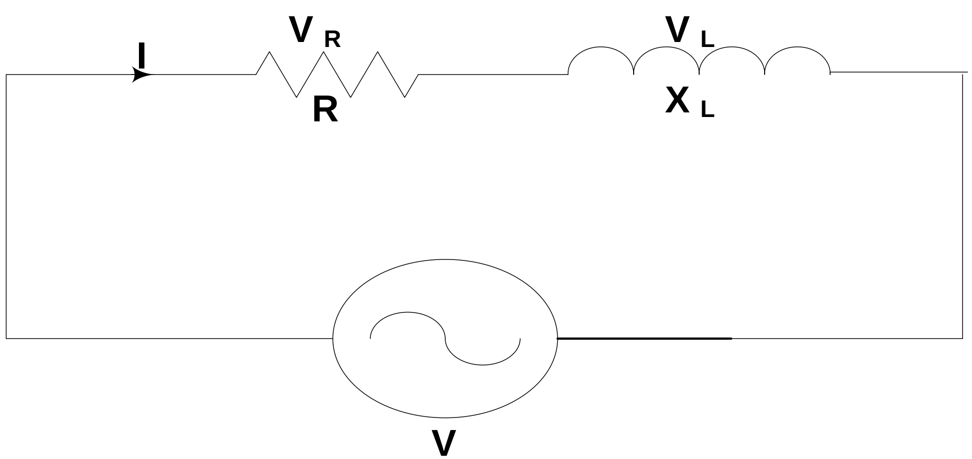
It is noted that0 the current through a capacitor leads the voltage by 90 .

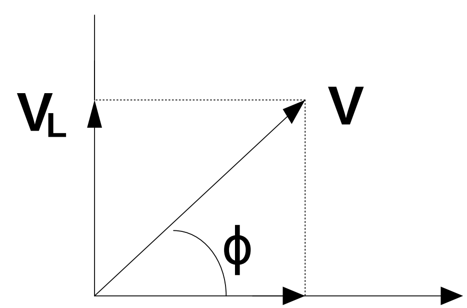
$$X_c = \frac{1}{\square c}$$

144

IMPEDANCE (Z)

- **Series circuit containing R and L**





VR

I

$$V = V_R + V_L$$

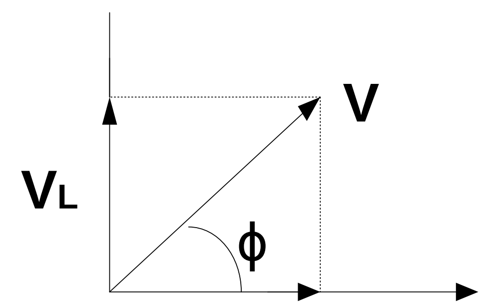
$$V^2 = V_R^2 + V_L^2 \quad Z = \sqrt{R^2 + X_L^2} \quad (IZ)^2 = (IR)^2$$

$$+ (IX_L)^2 \quad Z = \sqrt{R^2 + X_L^2}$$

IMPEDANCE (Z)

- **Phase angle between current and voltage in a series RL circuit**

$$\square = \tan^{-1} \square\square \; \frac{X_L}{} \; \square\square$$



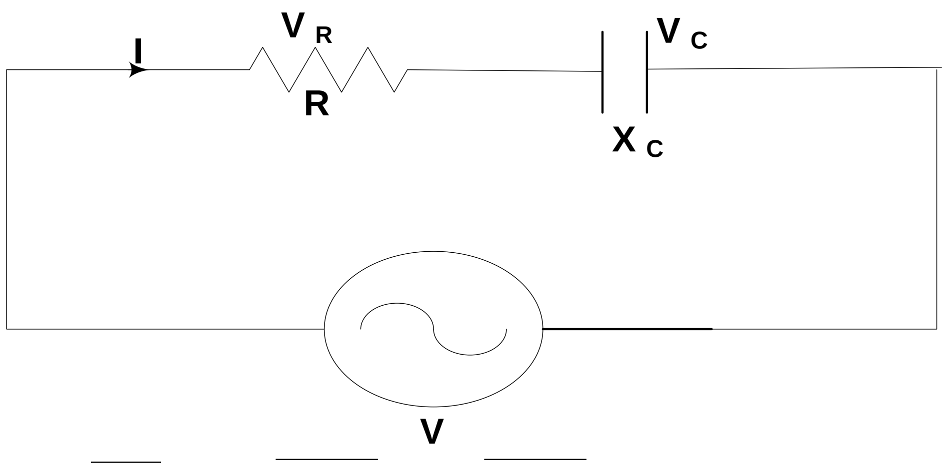
$\square R \square$

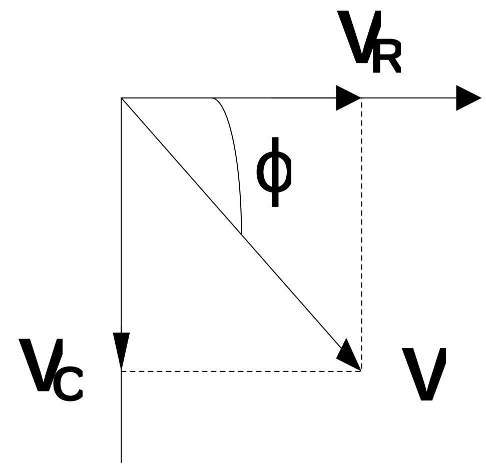
V_R I

- **The current in a series RL circuit lags the voltage but not by 90^0**

IMPEDANCE (Z)

- **Series circuit containing R and C**





$$V = V_{R\,2} + V_{C\,2}$$

$$Z_{\,2} = R_2 + X_{C\,2}$$

$$V_{\,2} = V_R + V_C$$

2 2

Z

$$\doteq \sqrt{R + X_C}$$

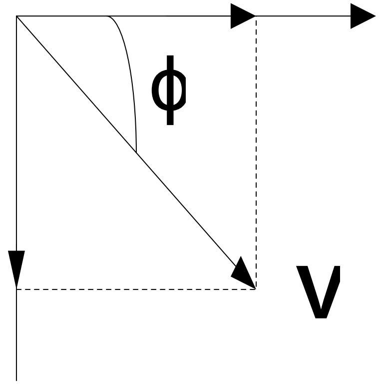
$$(IZ)^2 = (IR)^2 + (IX_C)^2$$

IMPEDANCE (Z)

- **Phase angle between current and voltage in a series RC circuit**

V_R I

$$\phi = \tan^{-1}\left(\frac{X_C}{\ }\right)$$

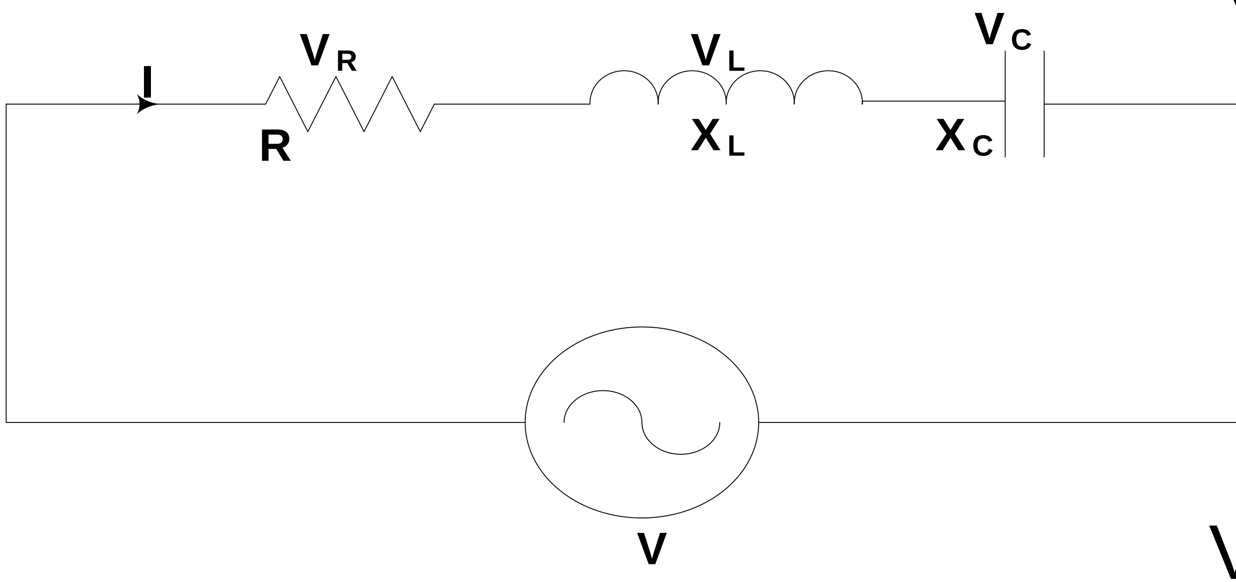


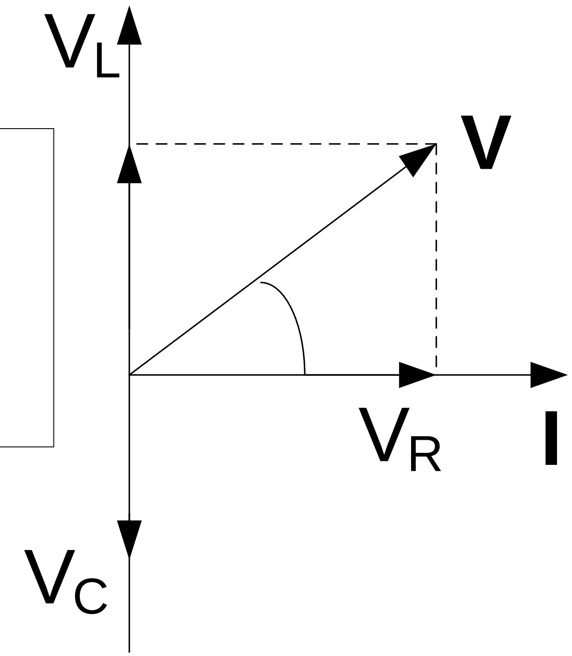
$\square\ R\ \square$ VC

- The current in a series RC circuit leads the voltage but not by 90^0

IMPEDANCE (Z)

- Series circuit containing R, L and C





$$\overline{V} = \overline{V_R} + \overline{V_L} + \overline{V_c}$$

$$V^2 = V_R^2 + \left(V_L - V_C\right)^2$$

IMPEDANCE (Z)

$$V^2 = V_R^2 + (V_L - V_C)^2$$

$$= (IR)^2 + (IX_L - IX_c)^2$$

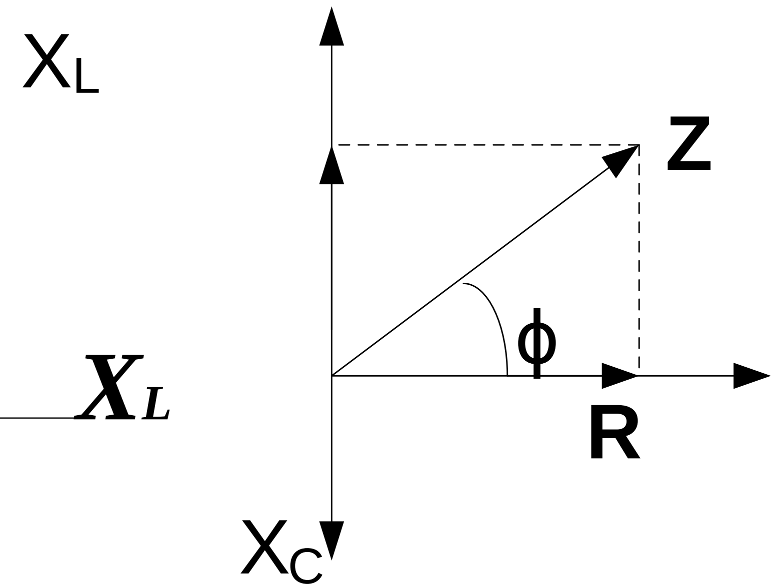
$$V = \sqrt{(IR)^2 + (IX_L - IX_c)^2}$$

$$IZ = I\sqrt{R^2 + (X_L - X_c)^2}$$

$$\square Z = \sqrt{R^2 + (X_L - X_c)^2}$$

IMPEDANCE (Z)

- **Phase angle between current and voltage in a series RLC circuit**



$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

- **Current in a series RLC circuit may lead or lag the voltage depending on the relative values of X_L and X_C**

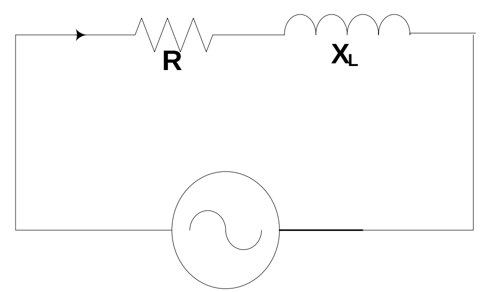
IMPEDANCE (Z)

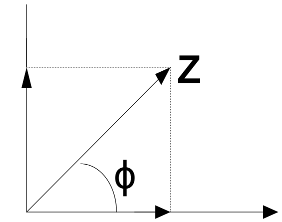
- **Example 1**

A coil has R=12Ω and L=0.1H. It is connected across a 100V, 50Hz supply. Calculate (a) the reactance and impedance of the coil (b) the current and (c) the phase difference or angle between the current and the applied

I

voltage. X_L





Solution R

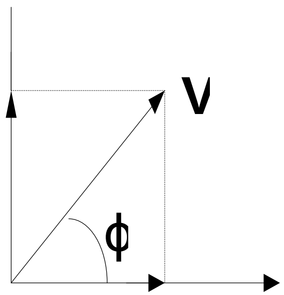
I

(a) $X_L = 2\pi fL = 2 \times \pi \times 50 \times 0.1 = 31.416\Omega$ $Z = \sqrt{R^2 + X_L^2} = \sqrt{12^2 + 31.416^2}$

$$= 33.630\Omega$$

IMPEDANCE (Z)

(b) $$I = \frac{V}{Z} = \frac{100}{33.630} = 2.974A$$ X_L



R **I**

(c) $\square = \tan^{-1}\square\square\square \; \frac{X_L}{R} \; \square\square\square = \tan^{-1}\square\square\square$

$\frac{31}{12.4}16\square\square\square$

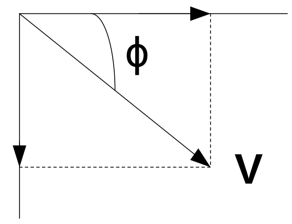
$= 69.09^0$

IMPEDANCE (Z)

✞Example 2

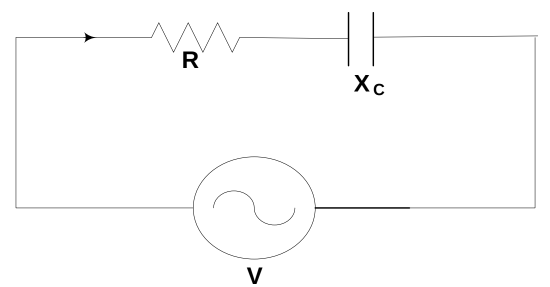
A metal filament lamp, rated at 750W, 100V is to be connected in series with a capacitor across a 230V, 50Hz

supply. Calculate (a) the capacitance required and (b) the phase angle between the current and supply voltage.



R I

$$X_C = \frac{V_C}{I_C}$$



Xc

Solution

(a) $V^2 = V_R^2 + V_C^2$

$$\sqrt{^2 \quad ^2}$$

$$V_C = \sqrt{V^2 - V_R^2} = 230 - 100$$

$$= 207.123V$$

IMPEDANCE (Z)

$$P_R\ 750\ \text{(a)}\ I_R = I_C =$$

$$I == \_\_\_\_ = 7.5A\ V_R\ 100$$

$$V_C\ \_\_\_\_\ 207.123$$

$$\square\ X_C = \quad = \_\_\_\_\_\_ = 27.616\square$$

$$I_C \quad 7.5$$

$$1 \quad 1 \; c = \underline{\qquad\qquad} =$$

Hence,

$$2□fX_C$$

$$2□□50□27.616 =$$

$$115□F$$

IMPEDANCE (Z)

(b) $\theta = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{V_C}{V_R}\right)$

$$= \tan^{-1}\left(\frac{207.123}{100}\right)$$

$$= 64.23^0$$

POWER IN AC CIRCUITS

- There are three kinds of power in ac circuits

1. Apparent Power (S) which is measured in Voltamperes (VA)
2. Active Power (P) which is measured in Watts (W).

   Active Power is also called Actual Power, Useful Power, True Power, Real Power or simply, Power

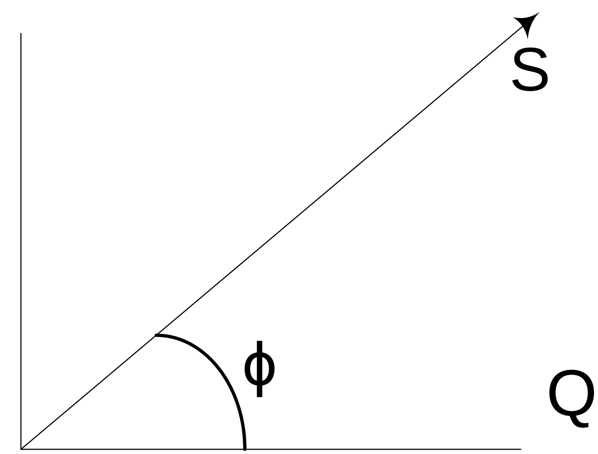
3. **Reactive Power (Q) which is measured in Voltamperes reactive (VAR)**

POWER IN AC CIRCUITS

- **The following relationships exist between S, P and Q**

$$S = VI$$

$$S^2 = P^2 + Q^2$$



$$P = S\cos\theta \quad Q = S\sin\theta \quad \cos\theta$$

is called power factor (*pf*)

$$pf = \frac{P}{S}$$

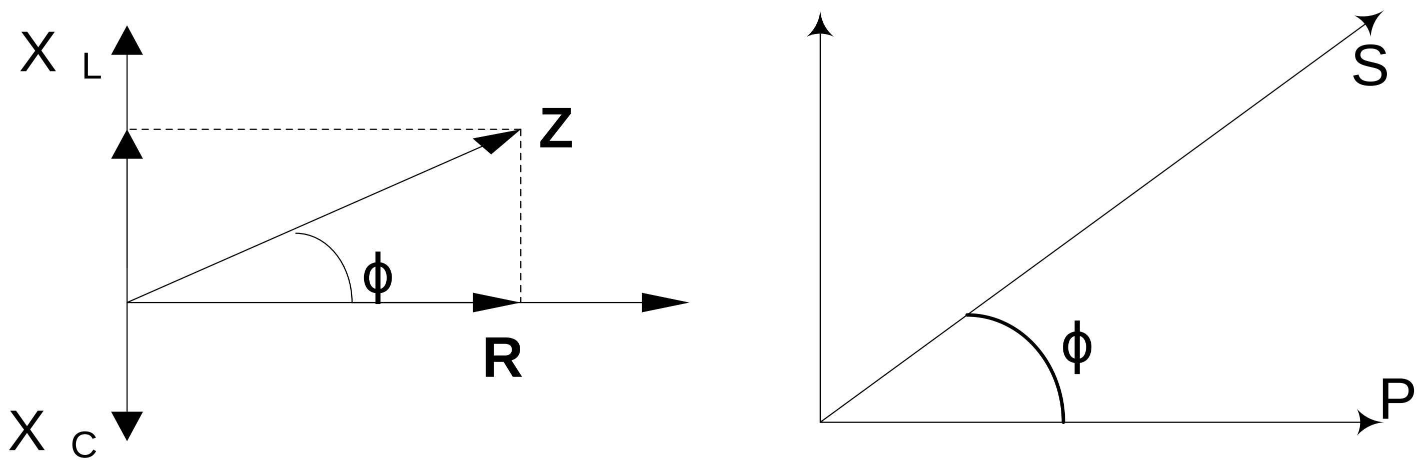
POWER IN AC CIRCUITS

- Power factor may be said to be lagging or leading.

- **Power factor is lagging when current lags voltage**
- **Power factor is leading when current leads voltage**

POWER IN AC CIRCUITS

✞ **Relationships between the three passive elements , P and Q.**Q



1. **Resistors consume only P**
2. **Inductors consume only Q**

3. **Capacitors do not consume P and Q. They rather supply Q or reduce the consumption of Q.**

✞Example 1

A single-phase motor connected to a 400-V, 50-Hz supply is developing 10 kW with efficiency of 84 per cent and a power factor of 0.7 lagging. Calculate (a) the input kVA (b) the active and reactive components of the current and (c) the reactive kVA.

✞**Solution** $P_{in} = \underline{\qquad} \quad \overline{\qquad\quad} \; P_{out} = 10 = 11.905kW$

□ 0.84

(a)

POWER IN AC CIRCUITS

$$P^{in} \qquad 11.905$$

$$S = \underline{\qquad} = \underline{\qquad\qquad} = 17.007kVA\ pf$$

$$0.7$$

(b) $S = VI \Rightarrow I = \dfrac{S}{V} = \dfrac{17.007 \times 10^3}{400}$

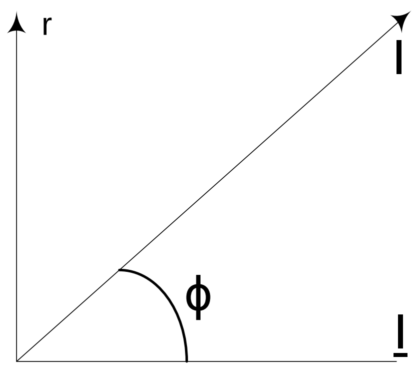
$= 42.518A$

$I_a = I\cos\phi = 42.518 \times 0.7$

$= 29.766A$

$I_r = \sqrt{I^2 - I_a^2} =$

$30.361A$



POWER IN AC CIRCUITS

(b) $Q = VI \sin\Box = VI_r = 400\Box 30.361$

$= 12.144 kVAR$

POWER IN AC CIRCUITS

✞Example 2

An emf whose instantaneous value is given by 283sin(314t + π/4)V is applied to an inductive circuit and the current in the circuit is 5.66sin(314t – π/6)A. Determine (a) the frequency of the emf (b) the R and L (c) the power absorbed.

✞Solution $314\ 2\square f = 314\ \square\ f = \_\_\_\_$

$= 50Hz$

(a)

$$2\square$$

$$Z = \frac{V}{I} = \frac{283}{\sqrt{2}} \quad \frac{5.66}{\sqrt{2}} \qquad \square = 50\square$$

(b)

POWER IN AC CIRCUITS 283 5.66

(c) P

$\overline{\sqrt{2}}$ $\overline{\sqrt{2}}$

$$= VI\cos\square = \square\cos 75^0$$

$$= 207.286W$$

USING COMPLEX NUMBERS TO SOLVE AC CIRCUIT PROBLEMS

- **The ability to make a vector quantity appear as a scalar quantity using complex numbers is utilized in the analysis of ac circuits.**
- **All the mathematical manipulations in complex algebra hold when employing complex numbers in analyzing ac circuits.**
- **The operator ‘i’ is replaced with ‘j’ in other to avoid confusing it with current.**

TO SOLVE AC CIRCUIT PROBLEMS

- **The three passive elements are represented as follows:**

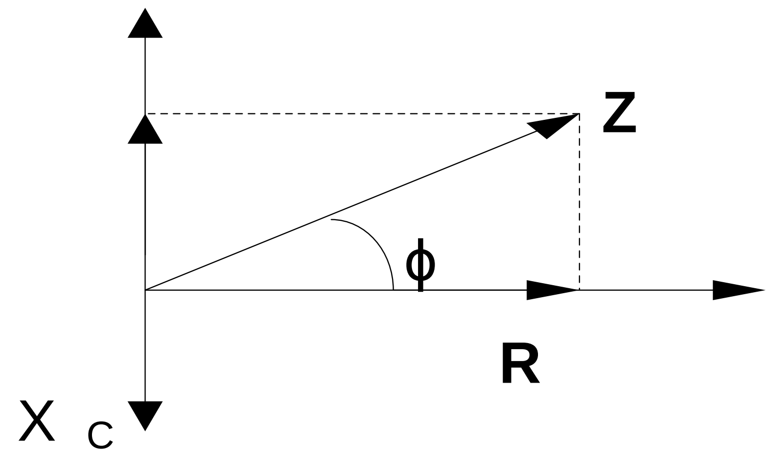
1. R as a complex number is R
2. X_L as a complex number is jX_L

$$X \qquad\qquad\qquad - jX$$

3. $_C$ **as a complex number is**

C X_L

4. Series impedance



$$Z = R + j\left(X_L - X_C\right)$$

USING COMPLEX NUMBERS
TO SOLVE AC CIRCUIT PROBLEMS

✞Example 1

Express in rectangular and polar notations, the impedance of each of the following circuits at a frequency of 50 Hz: (a) a resistance of 20 Ω (b) a resistance of 20 Ω in series with an inductance of 0.1 H (c) a resistance of 50 Ω in series with a capacitance of 40 μF.

✞Solution

(a) $Z = 20 + j0 = 20\angle 0^0$ (b) $X_L = 2\pi f L = 2\pi \times 50 \times 0.1 = 31.416\Omega$

$$\square Z = 20 + j31.416$$

$$= 37.242\square 57.52^{0}$$

USING COMPLEX NUMBERS TO SOLVE AC CIRCUIT PROBLEMS

(c) 1 1

$$X_C = \underline{\qquad\qquad} = \underline{\qquad\qquad\qquad\qquad\qquad} 2\pi fC$$

$$2\times\pi\times 50\times 40\times 10^{-6} \quad Z = 50 - j79.577 = 79.577\,\Omega$$

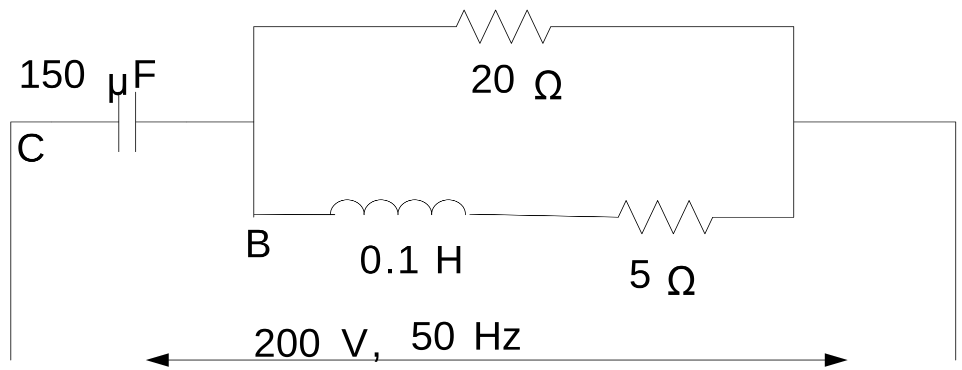
$$= 93.981\angle -57.86^{0}$$

USING COMPLEX NUMBERS
TO SOLVE AC CIRCUIT PROBLEMS

✞**Example 2**

A circuit is arranged as indicated in the figure below, the values being as shown. Calculate the value of the current in each branch and its phase relative to the supply voltage.

A

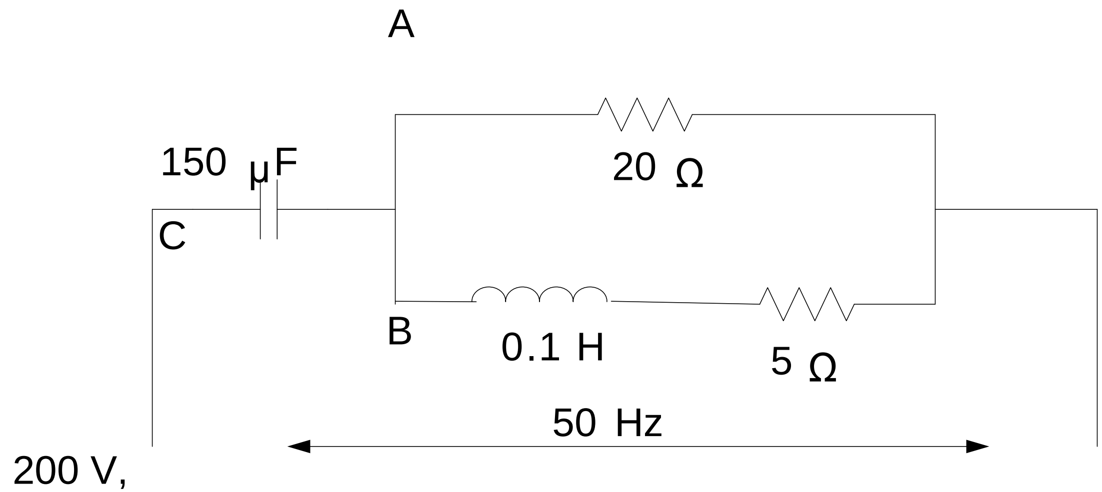


Solution

$$Z_A = 20 + j0 \quad Z_B = R + jX_L = 5 + j31.4 \quad Z_C = -jX_C = -j21.2\Omega$$

$$Z_{AB} = Z_A // Z_B = 15.84\angle 29.48^0$$

TO SOLVE AC CIRCUIT PROBLEMS



$$= 13.78 + j7.8$$

$$Z_T = Z_C + Z_{AB}$$

$$= -j21.2 + (13.78 + j7.8)$$

$$= 19.22\angle -44.2^0$$

Choosing the voltage as the reference phasor,0

$$V \; 200\square 0 \; I_C = I_T = \underline{\qquad} Z =$$

$$19.22\square - 44.2_0$$

$$T$$

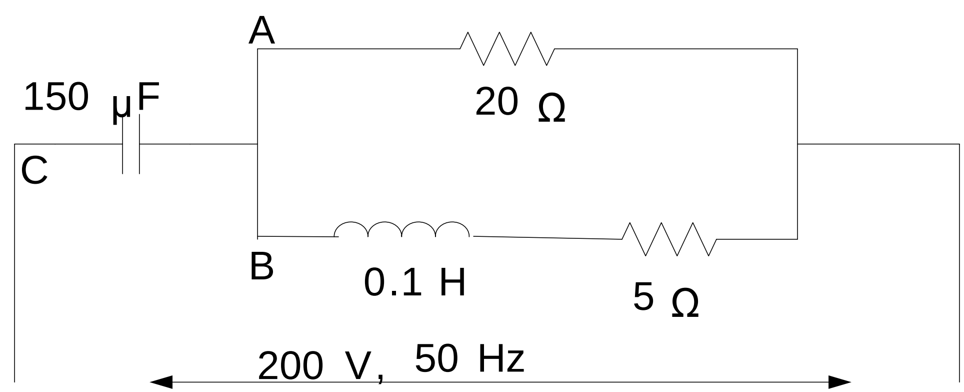
$$= 10.4\square 44.2^0$$

Current leads supply voltage by 44.2^0

$$V_{AB} = IZ_{AB} = (10.4\angle 44.2^0)\times(15.84\angle 29.48^0)$$

$$= 164.8\angle 73.68^0$$

$$I_A = \frac{V_{AB}}{Z_A} = \frac{164.8\angle 73.68^0}{20} = 8.24\angle 73.68^0$$



172

Current leads supply voltage by 73.68^0

USING COMPLEX NUMBERS
TO SOLVE AC CIRCUIT PROBLEMS

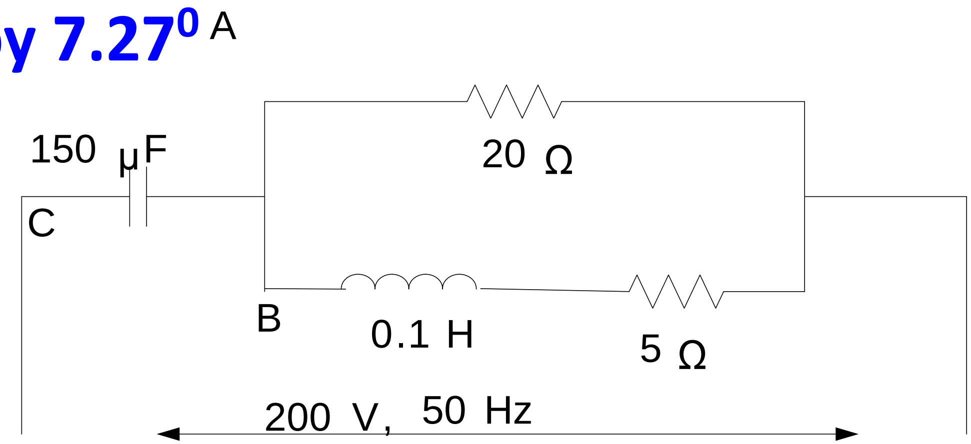
$$V_{AB} = \frac{\qquad}{} 164.8\angle 73.68^0{}_0 =$$

$$5.18\angle -7.27^0$$

$I_B =$ $31.79□80.95$

Z_B

Current lags supply voltage by 7.27^0



USING COMPLEX NUMBERS
TO SOLVE AC CIRCUIT PROBLEMS

$$S = VI^*$$

$$= P + jQ$$

- Q is positive when the current lags the voltage
- Q is negative when the current leads the voltage

CALCULATION OF COMPLEX POWER

- **Example 1**

The potential difference across and the current in a circuit are represented by 100 + j200 v and 10 + j5 a respectively. Calculate the power and reactive voltamperes (or vars).

- **Solution**

*

$$S = VI^* = (100 + j200)(10 + j5) = (100 + j200)(10 - j5)$$

$$= 2000 + j1500$$

$$P = 2000W \qquad Q = 1500VAR$$ 175

CALCULATION OF COMPLEX POWER

✞Example 2

A small installation consists of the following loads connected in parallel across a single-phase 240V, 50Hz supply:

(a) a fan motor taking an input of 1.5kVA at 0.75pf lag,

(b) a 1000W radiator operating at unity power factor

(c) a number of fluorescent lamps taking a total load of 1.2kVA at 0.95pf lagging

Find the total current, kW, kVA and power factor of the load.

THREE-PHASE CIRCUITS

- A single-phase generator produces a single sinusoidal voltage.
- A 3-phase generator on the other hand produces three equal voltages which are out of phase with one another by 120^o.

THREE-PHASE CIRCUITS

- **The three voltages are generated in three separate windings arranged in a special way in the machine.**
- **A 3-phase system is a power supply system consisting of three voltages which are 120^o out of phase with one another.**

- **Three-phase systems have the following advantages over single phase systems**
  - **Three-phase motors, generators and transformers are simpler, cheaper and more efficient**
  - **Three-phase transmission lines can deliver more power for a given weight and cost**

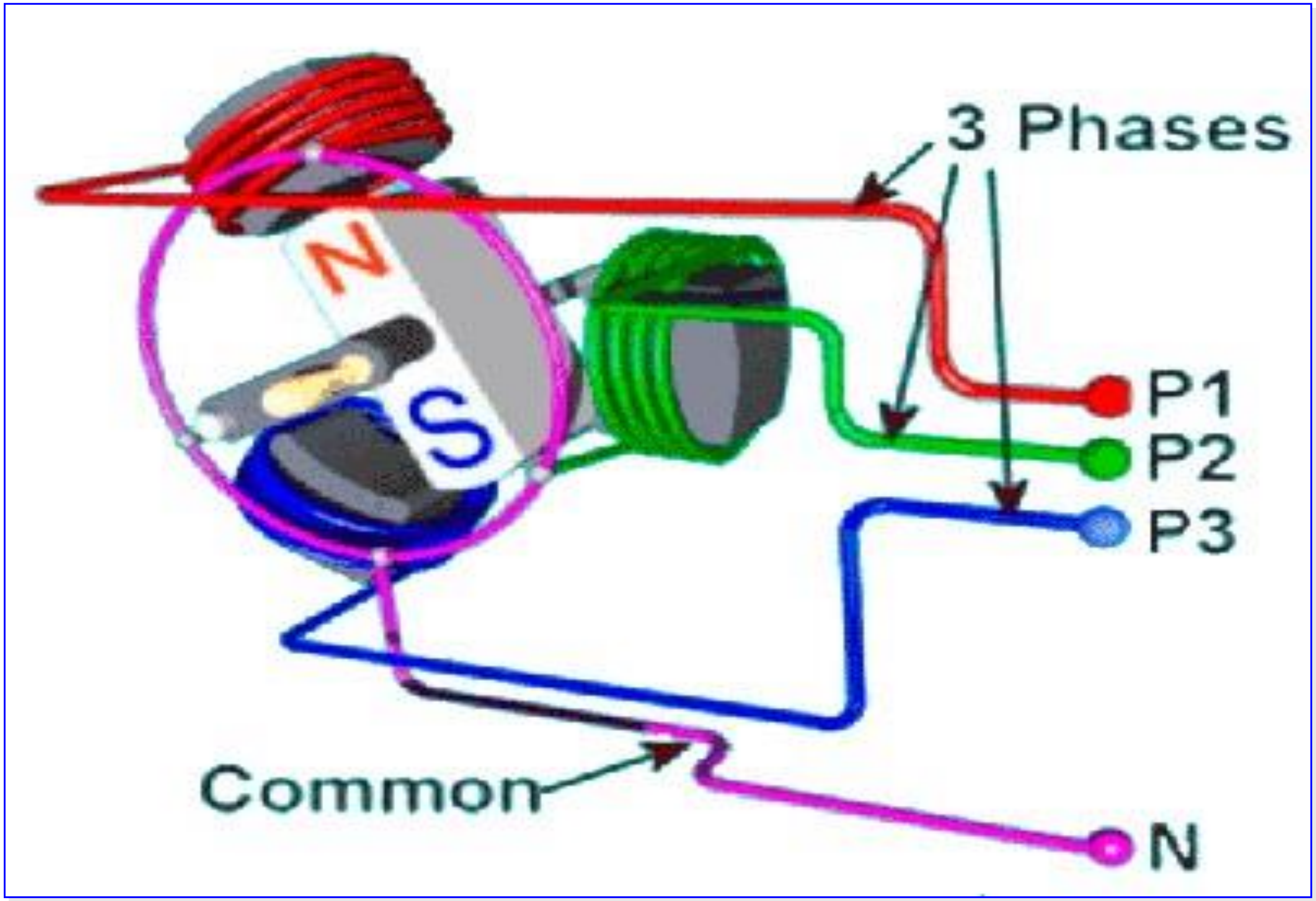
THREE-PHASE CIRCUITS

- **The voltage regulation of three-phase transmission lines is inherently better**

- **A 1-phase supply can be obtained from a 3-phase one**

Winding arrangement of a three-phase

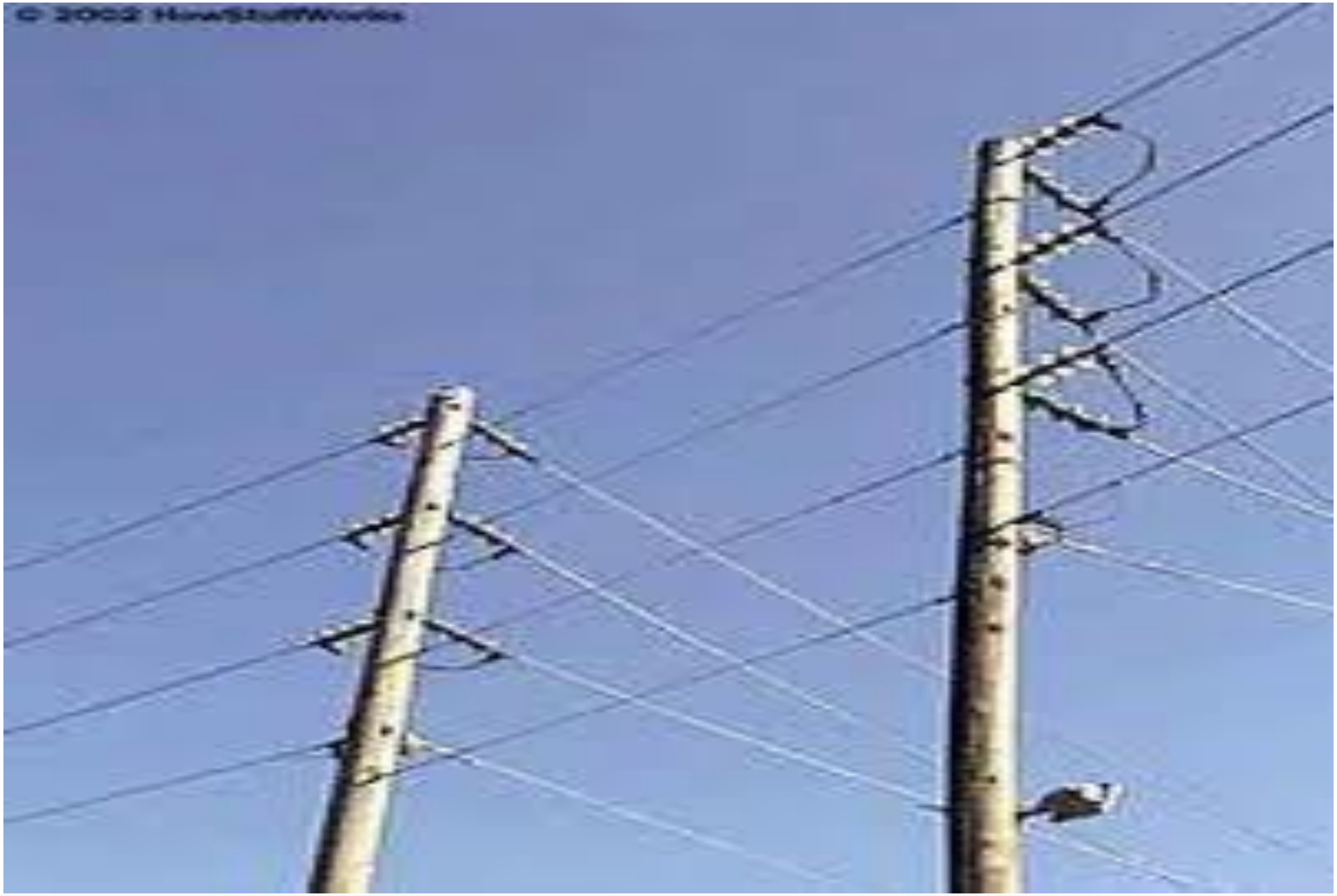
generator



A three-phase transmission line

11 kV Distribution Feeder

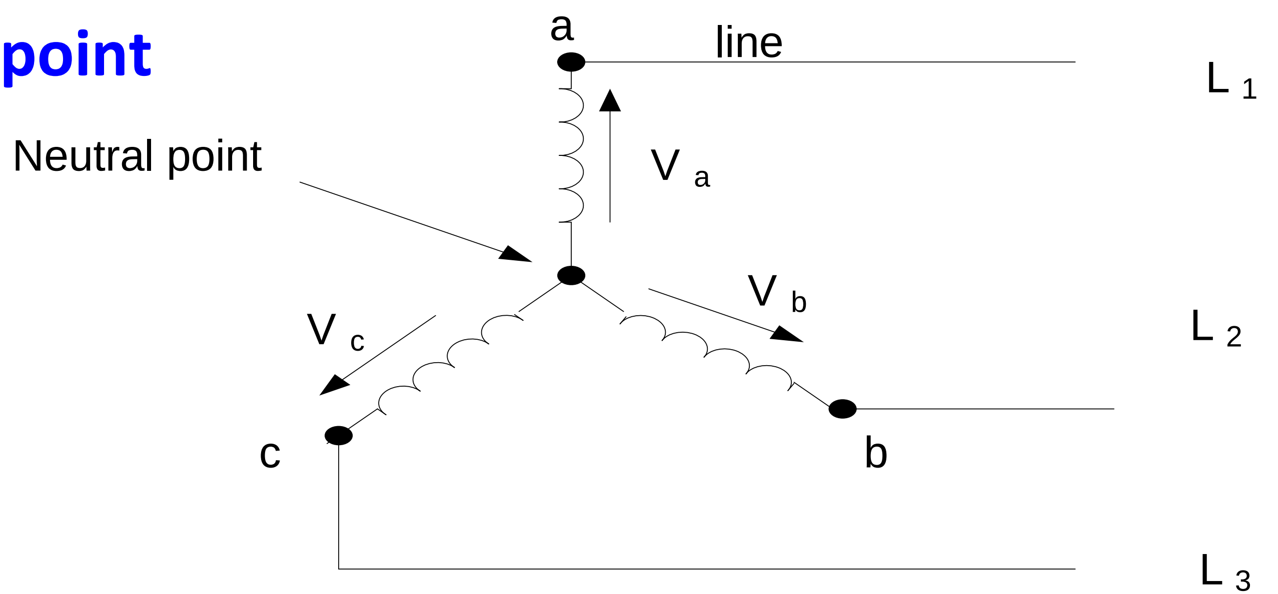
A Distribution Line



THREE-PHASE CIRCUITS

♱The two main connections of three-phase windings

1. **A star arrangement where all winding have a common point**

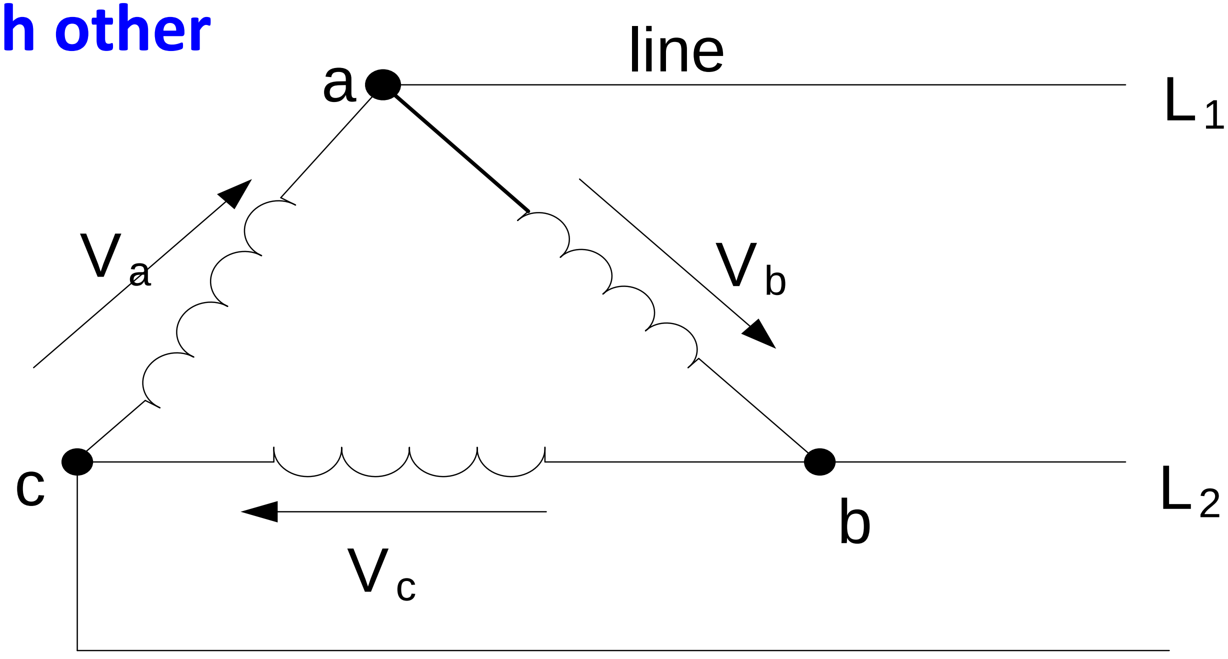


✈Letters a, b and c , colours red (R), yellow (Y) and blue (B) or numbers 1, 2 and 3 are used to name the

windings

THREE-PHASE CIRCUITS

2. A delta arrangement where all winding are connected to each other

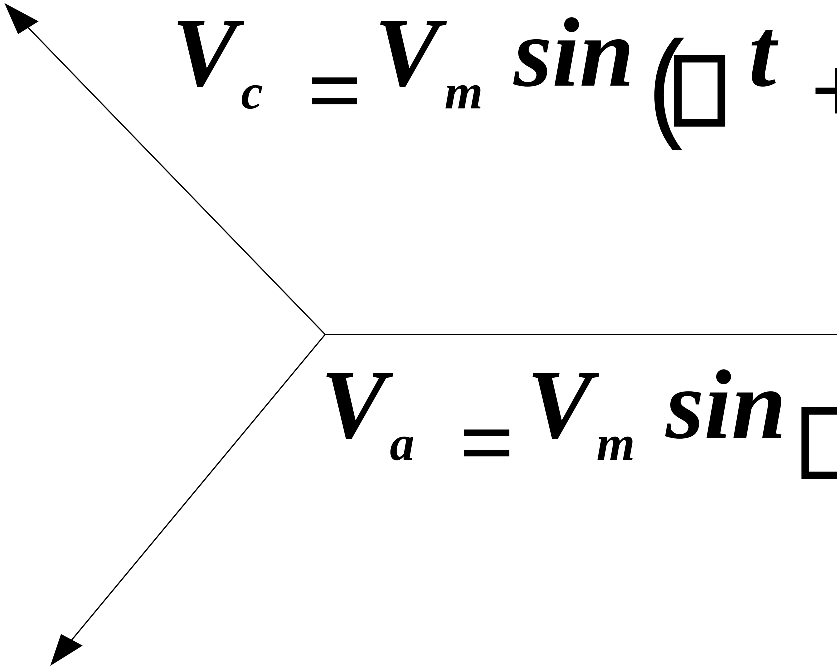


- **Letters a, b and c , colours red (R), yellow(Y) and blue (B) or numbers 1, 2 and 3 are used to name the windings**

- **The phasor diagram for the three-voltages (in star or delta) is indicated below.** V_3,V_c or V_B

V_2, V_b

$$V_c = V_m \sin(\omega t + 120^o)$$ or



V_Y $$V_b = V_m \sin(\omega t - 120^o)$$

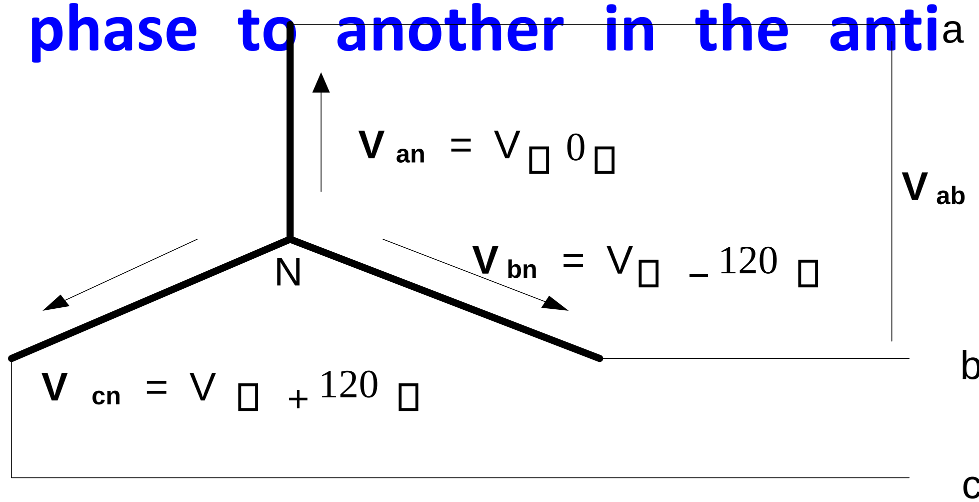
V_1, V_a or V_R

$$V_a = V_m \sin \omega t$$

THREE-PHASE CIRCUITS

Line and phase voltages

- The voltage from one line to another is called a line-toline voltage or simple a line voltage
- The voltage across each winding is a phase voltage
- On a phasor diagram, a line voltage is drawn from the end of one phase to another in the anti clockwise direction



✞Relationship between line and phase voltages for a star connection

$$V_{ab} = V_{an} - V_{bn} = V\square 0^o$$

$$- V\square - 120^0 = V(1 -$$

$$\cos(- \ 120) - j\sin(-$$

$$120)) \ = V\square\square \ \ 3 + j$$

$$3\square\square \ = \ 3V\square 30_0$$

$$= V\left(1 - 1\angle -120^{0}\right)$$

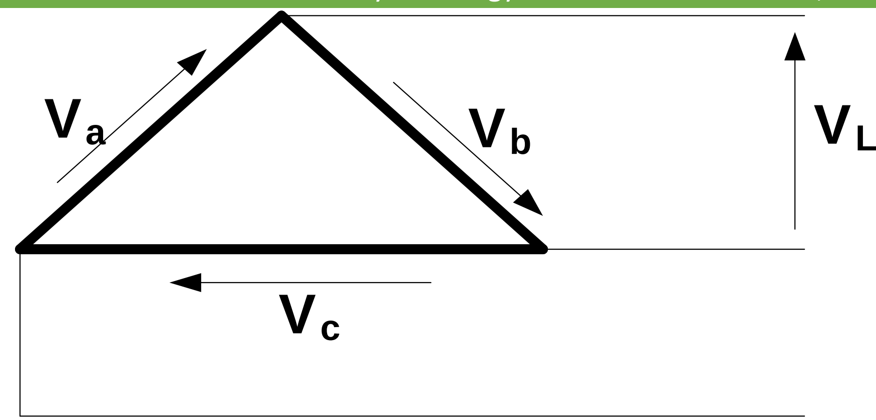
$$2 \qquad\qquad 2$$

$\sqrt{3}$

Hence, for a star connection, the line voltage is

$$V_L = 3V_p$$ **times the phase voltage.**

- **Relationship between Line and phase voltages for a delta connection**

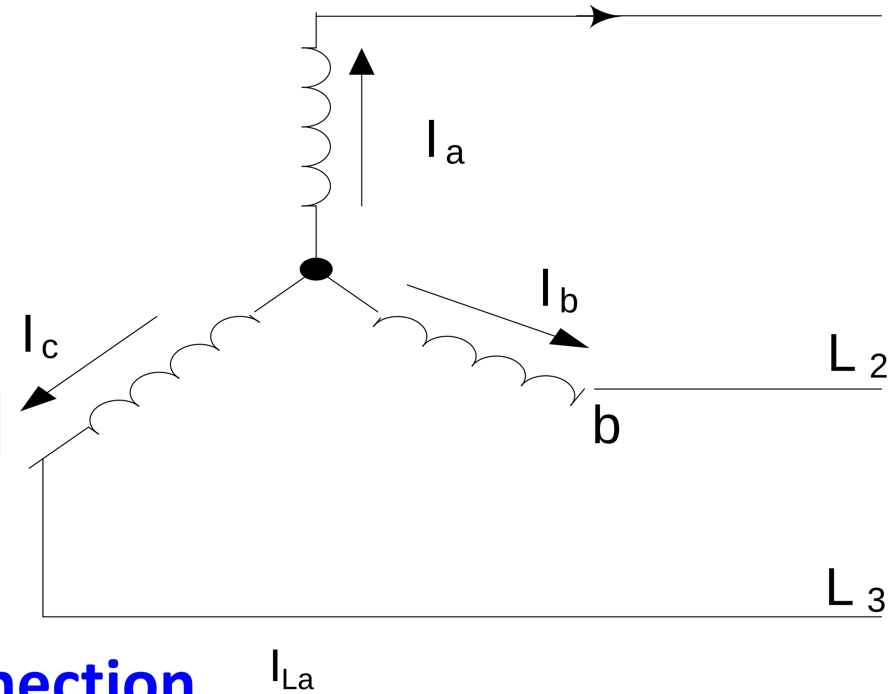


$$V_L = V_p$$

THREE-PHASE CIRCUITS

Relationship between Line and phase current for a star connection a I_L L_1

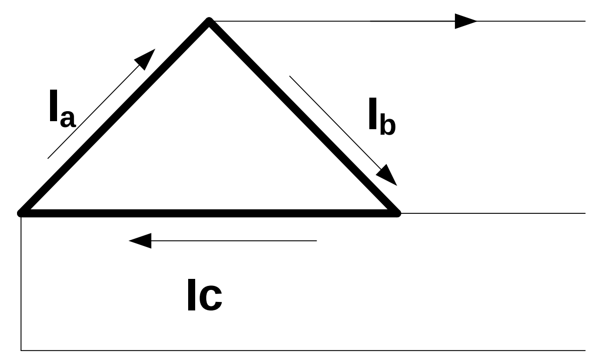
$$I_L = I_p$$



c

THREE-PHASE CIRCUITS

✞Relationship between Line and phase currents for a delta connection



$$I_{La} = I_a - I_b = I\angle 0^o -$$

$$I\angle -120^0 = I(1 - \cos(-$$

$$120) - j\sin(-120)) = I \quad 3 + j$$

$$3 \quad = 3I \, 30_0$$

$$= I\left(1 - 1 \, -120^0\right)$$

$$2 \qquad 2$$

$$\sqrt{3}$$

Hence, for a delta connection, the line current is $\sqrt{\ }$

$$I_L = 3I_p$$ **times the phase current**

Analysis of three-phase balanced circuits

- A balanced three-phase circuit is that in which identical loads are connected in each phase.
- The currents that flow in a balanced three-phase system are equal in magnitude and also 120° out of phase.
- A balanced three-phase circuit is analysed by considering just one phase
- When finding total power, the per phase power is multiplied by three
- 1-phase power factor is the same as 3-phase

THREE-PHASE CIRCUITS

✞Example 1

Three identical resistors are connected in star across a 3phase, 415-V supply. If each resistor has a resistance of 50 ohms, calculate (a) the voltage across each resistor (b) the current in each resistor (c) the total power supplied to the load

Solution(a) $V_p = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 240$

$I = \frac{V_p}{} \quad 240 \quad 4.8A$

(b) $_p = R = 50 =$

(c) $P_p = V_p I_p = 240 \times 4.8 = 1152W$

$\therefore P_T = 3 \times P_p = 3 \times 1152 = 3456W$

♰Example 2

Three identical impedances are connected in delta across a 3-phase, 415-V supply. If the line current is 10 A, calculate (a) the current in each impedance (b) the value of each impedance.

♰♰**Solution** (a) $I_p = I_L = 10 = 5.78A$

$$3 \quad \sqrt{3}$$

$$V_p 415 \quad 71.80\Omega$$

$$Z_p = = =$$

(b) $I_p \quad 5.78$

Example 3

A 3-phase, 450-V system supplies a balanced deltaconnected load of 12 kW at 0.8 power factor lagging.

Calculate (a) the phase currents (b) the line currents and (c) the effective impedance per phase.

- **Solution**

- (a) $P_p = V_p I_p \cos\square\ 12$ 3

P

$\square 10$

3

$\square\ I_p = \frac{\quad}{\quad}{}_p = 3 \quad = 11.1A$

$$V_p \cos\square\ 450\square 0.8$$

(b) $I_L = \sqrt{3} I_p = \sqrt{3} \square 11.1 = 19.2\,A$

(c) $Z_p = \dfrac{V_p}{I_p}$ o

$$= \frac{450\angle 0^{0}}{11.1\angle -\cos^{-1}(0.8)} = 40.5\angle 36.9^{0}$$

- **Power in three-phase circuits**
  - **The total apparent power of a balanced three-phase circuit (star or delta) is given by:**

$$S = \sqrt{3}V_L I_L$$

- The total active power of a balanced three-phase circuit (star or delta) is given by:

$$P = \sqrt{3}V_L I_L \cos\phi$$

- The total reactive power of a balanced three-phase circuit (star or delta) is given by:

$$Q = \sqrt{3}V_L I_L \sin\phi$$