PABLO

FUNDAMENTALS OF ELECTRICAL ENGINEERING.

LECTURE 6 NOTES

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PABLO

Unit 8: Magnetism, Ohm’s law for magnetic circuit; Analogy between electric circuits and Magnetic Circuits.

- Review the basic principles of magnetism
- Use the concepts of reluctance and magnetic circuit equivalents to compute magnetic.
- flux and currents in simple magnetic structures.
- Generate the equivalent magnetic.
- circuit diagram, and calculate the total

- equivalent reluctance.
- Calculate the flux, flux density, and magnetic field intensity.

MAGNETIC CIRCUITS

Electromagnetics is the branch of electrical engineering (or physics) that deals with the analysis and application of electric and magnetic fields.

In electromagnetics, electric circuit analysis is applied at low frequency.

- **Magnetic fields**

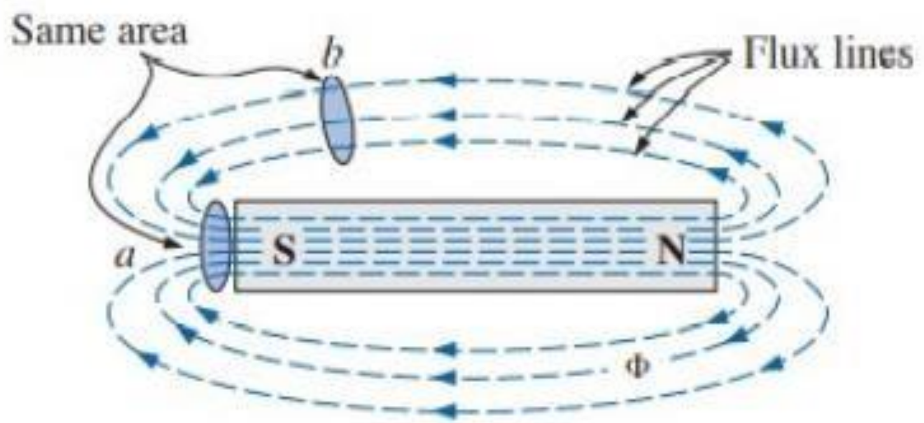
In the region surrounding a permanent magnet there exists a magnetic field, which can be represented by magnetic flux lines similar to electric flux lines.

✦ Magnetic flux lines, however, do not have origins or terminating points as do electric flux lines but exist in continuous loops.

Magnetic flux Φ is the amount of magnetic field produced by a magnetic source. The unit of magnetic flux is the Weber, Wb.

The magnetic flux lines radiate from the north pole to the south pole, returning to the north pole through the metallic bar.

✦Note the equal spacing between the flux lines within the core and the symmetric distribution outside the magnetic



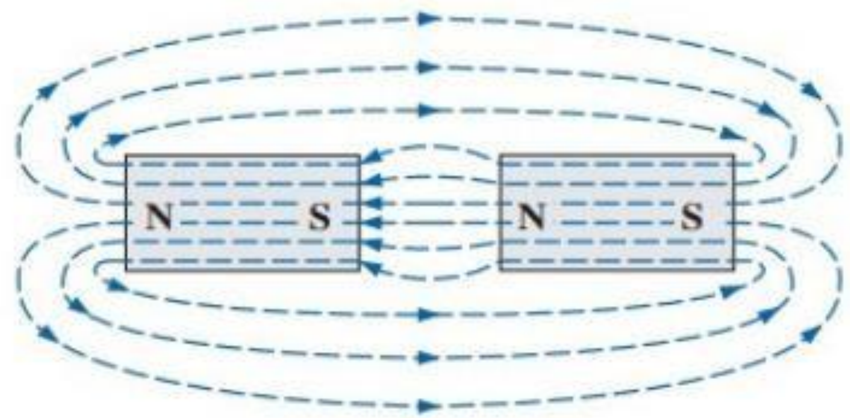
material.

Flux distribution for a permanent magnet.

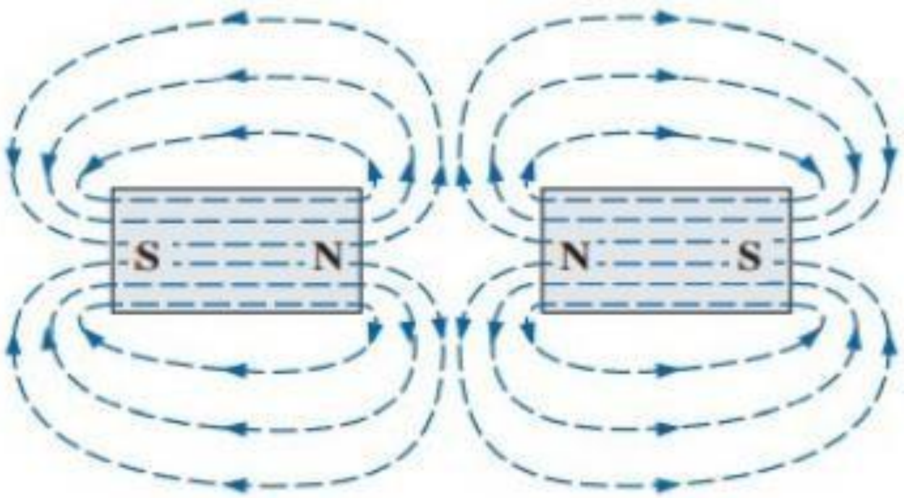
The magnetic field strength at a is twice that at b since twice as many magnetic flux lines are associated with the perpendicular plane at a than at b.

- ✦The strength of a magnetic field in a particular region is directly related to the density of flux lines in that region.

If unlike poles of two permanent magnets are brought together, the magnets will attract, and the flux distribution will be as shown below.

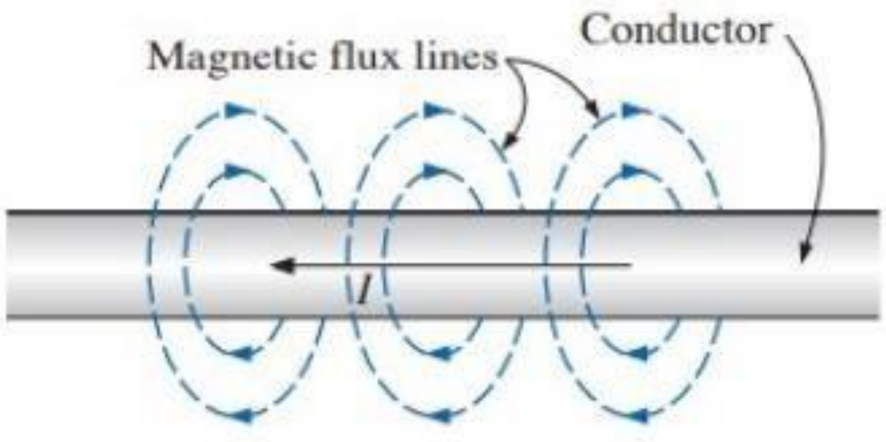


If like poles are brought together, the magnets will repel, and the flux distribution will be as shown below.



As indicated in the introduction, a magnetic field is present around every wire that carries an electric current.

The direction of the magnetic flux lines can be found simply by placing the thumb of the right hand in the direction of conventional current flow and noting the direction of the fingers. (This method is commonly called the right-hand rule).

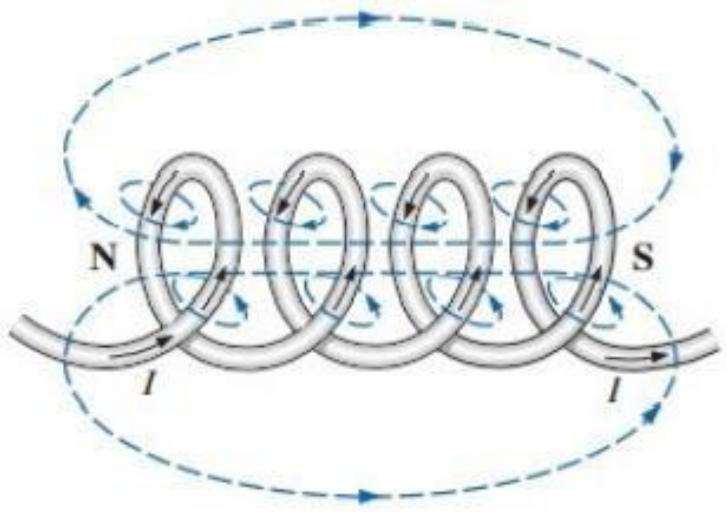


Magnetic flux lines around a current-carrying conductor.

If the conductor is wound in a single-turn coil, the resulting flux will flow in a common direction through the center of the coil.

Flux distribution of a single-turn coil

A coil of more than one turn would produce a magnetic field that would exist in a continuous path through and around the coil.



Flux distribution of a current-carrying coil.

- **Flux Density**

The number of flux lines per unit area is called the flux density, is denoted by the capital letter B, and is measured in teslas.

Its magnitude is determined by the following equation:

$$B = \frac{\Phi}{A}$$

B=teslas(T)

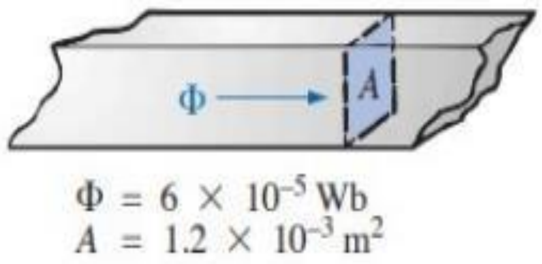
Φ=webers(Wb)

A=square meters(m^2)

✦ 1 T = 1 Wb/m2

○Example

For the core in the figure below, determine the flux density B



in teslas.

✦Solution

$$B = \frac{\Phi}{A} = \frac{6 \times 10^{-5} Wb}{1.2 \times 10^{-3} m^2} = 5 \times 10^{-2} T$$

- **Permeability**

If cores of different materials with the same physical dimensions are used in an electromagnet, the strength of the magnet will vary in accordance with the core used.

✦This variation in strength is due to the greater or lesser number of flux lines passing through the core.

The **permeability(μ)** of a material, therefore, is a measure of the ease with which magnetic flux lines can be established in the material.

✦It is similar in many respects to conductivity in electric circuits.

The permeability of free space μ_o (vacuum) is

$$\mu_o = 4\pi \times 10^{-7} \frac{Wb}{A.m}$$

As indicated above, μ has the units of Wb/A · m.

- Materials that have permeabilities slightly less than that of free space are said to be diamagnetic, and those with permeabilities slightly greater than that of free space are said to be paramagnetic.

Magnetic materials, such as iron, nickel, steel, cobalt, and alloys of these metals, have permeabilities hundreds and even thousands of times that of free space.

Materials with these very high permeabilities are referred to as ferromagnetic.

The ratio of the permeability of a material to that of free space is called its relative permeability; that is,

$$\mu_r = \frac{\mu}{\mu_o}$$

- **Reluctance**

The resistance of a material to the flow of charge (current) is determined for electric circuits by the equation

$$R = \rho \frac{l}{A} (ohms, \Omega)$$

The reluctance of a material to the setting up of magnetic flux lines in the material is determined by the following equation:

$$\boxed{\mathcal{R} = \frac{l}{\mu A}} \qquad \text{(rels, or At/Wb)}$$

- where R is the reluctance, l is the length of the magnetic path, and A is the cross-sectional area. The t in the units At/Wb is the number of turns of the applied winding.

- **Ohm's Law For Magnetic Circuits**

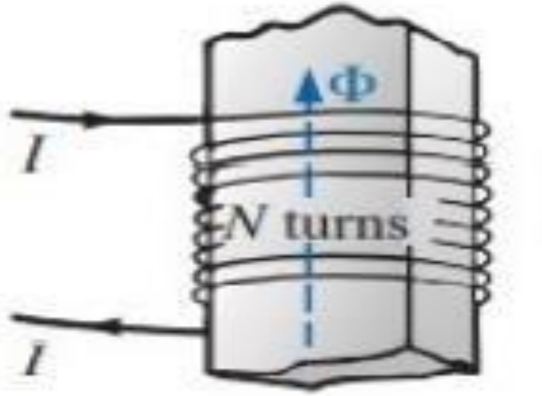
$$\text{Effect} = \frac{\text{cause}}{\text{opposition}}$$

For magnetic circuits, the effect desired is the flux. The cause is the magnetomotive force (mmf), which is the external force (or "pressure") required to set up the magnetic flux lines within the magnetic material. The opposition to the setting up of the flux is the reluctance.

Substituting, we have

$$\boxed{\Phi = \frac{\mathscr{F}}{\mathscr{R}}}$$

The magnetomotive force is proportional to the product of the number of turns around the core (in which the flux is to be established) and the current through the turns of wire (in



the figure below).

Defining the components of a magnetomotive force

- **Magnetizing Force**

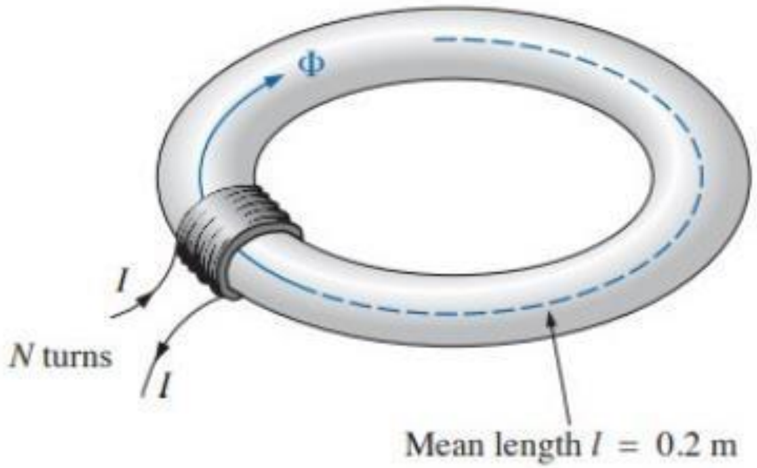
The magnetomotive force per unit length is called the magnetizing force (H). In equation form,

$$\boxed{H = \frac{\mathscr{F}}{l}} \qquad \text{(At/m)}$$

Substituting for the magnetomotive force will result in

$$\boxed{H = \frac{NI}{l}} \qquad \text{(At/m)}$$

✦ For the magnetic circuit of the figure below, if NI=40At and



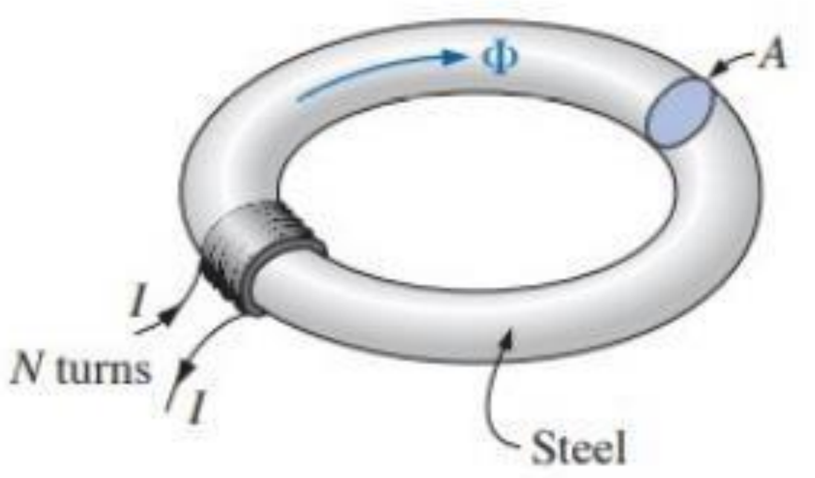
l=0.2m, then

$$H = \frac{NI}{l} = \frac{40At}{0.2m} = 200At/m$$

The flux density and the magnetizing force are related by the following equation: $B=\mu H$

- **Hysteresis**

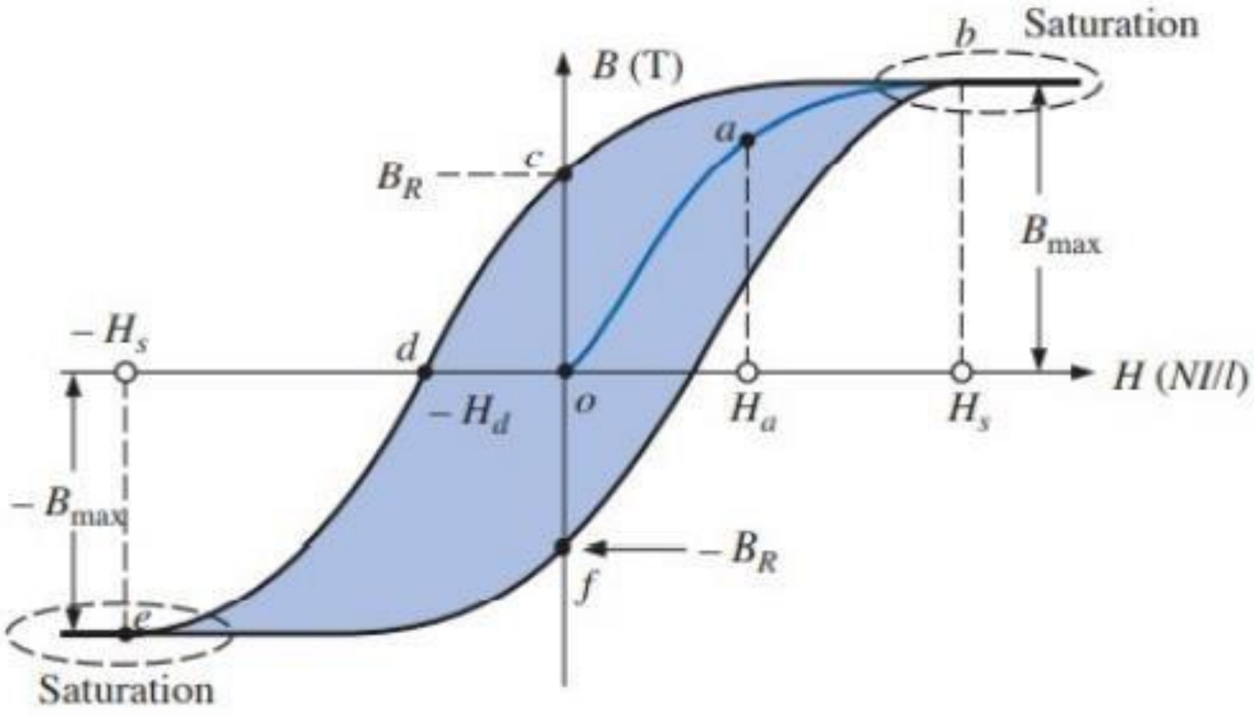
A typical B-H curve for a ferromagnetic material such as steel can be derived using the setup below.



Series magnetic circuit used to define the hysteresis curve.

The core is initially unmagnetized and the current I = 0. If the current I is increased to some value above zero, the magnetizing force H will increase to a value determined by

$$H \uparrow = \frac{NI \uparrow}{l}$$



- The flux Φ and the flux density B ($B = \Phi/A$) will also increase with the current I (or H). If the material has no residual magnetism, and the magnetizing force H is increased from zero to some value H_a, the B-H curve will follow the path shown above between o and a.
- If the magnetizing force is reduced to zero by letting I decrease to zero, the curve will follow the path of the curve between *b* and *c*. The flux density B_R, which remains when the magnetizing force is zero, is called the residual flux

density.It is this residual flux density that makes it possible to create permanent magnets.

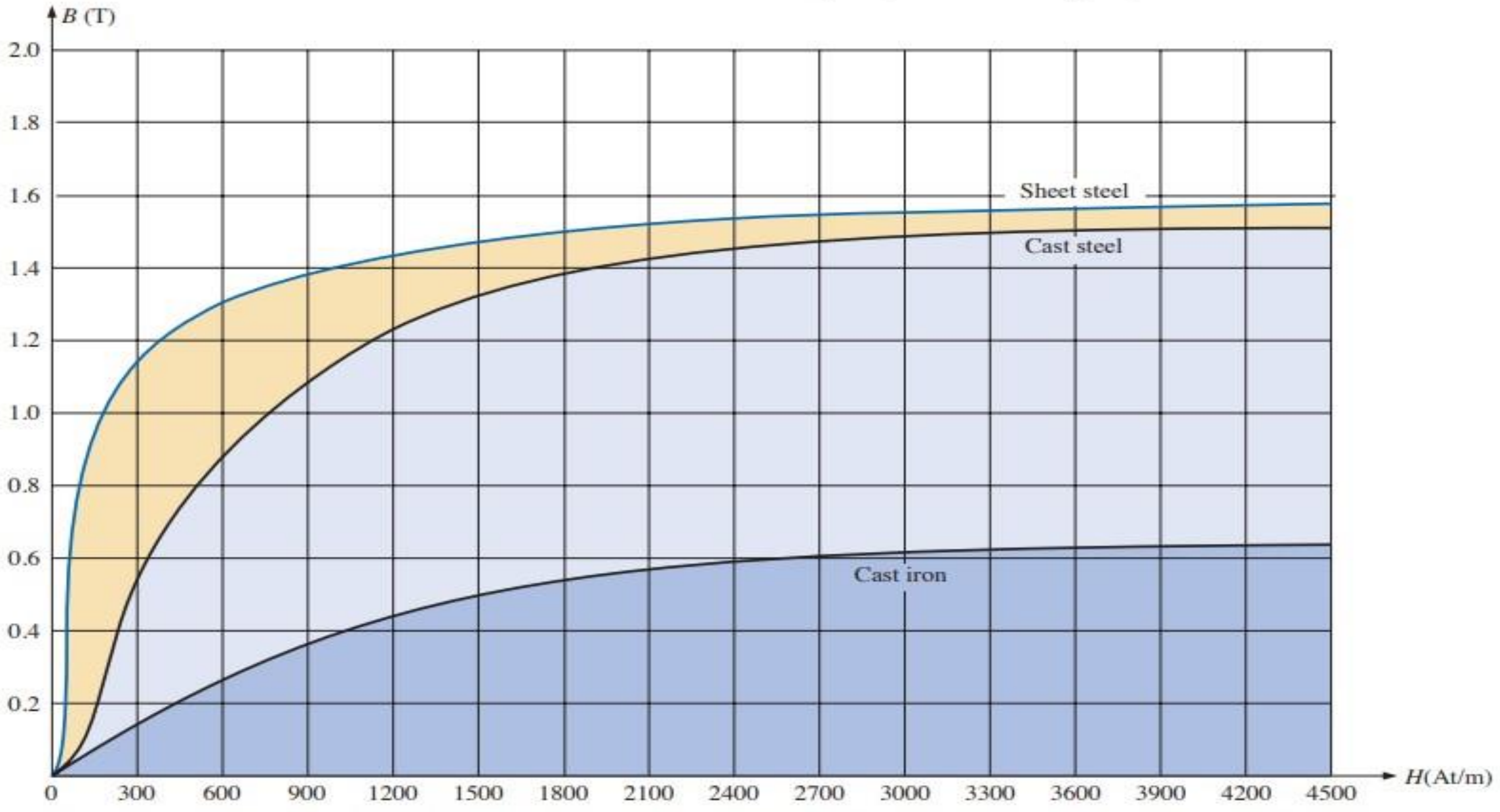
✦If the coil is now removed from the core, the core will still have the magnetic properties determined by the residual flux density, a measure of its “retentivity.”

✦If the current I is reversed, developing a magnetizing force, H, the flux density B will decrease with an increase in I. Eventually, the flux density will be zero when $-H_d$ (the portion of curve from *c* to *d*) is reached.

- ✦ As the force -*H* is increased until saturation again occurs and is then reversed and brought back to zero, the path *def* will result.
- ✦ If the magnetizing force is increased in the positive direction (+*H*), the curve will trace the path shown from *f* to *b*. The entire curve represented by *bcdefb* is called the hysteresis curve for the ferromagnetic material.

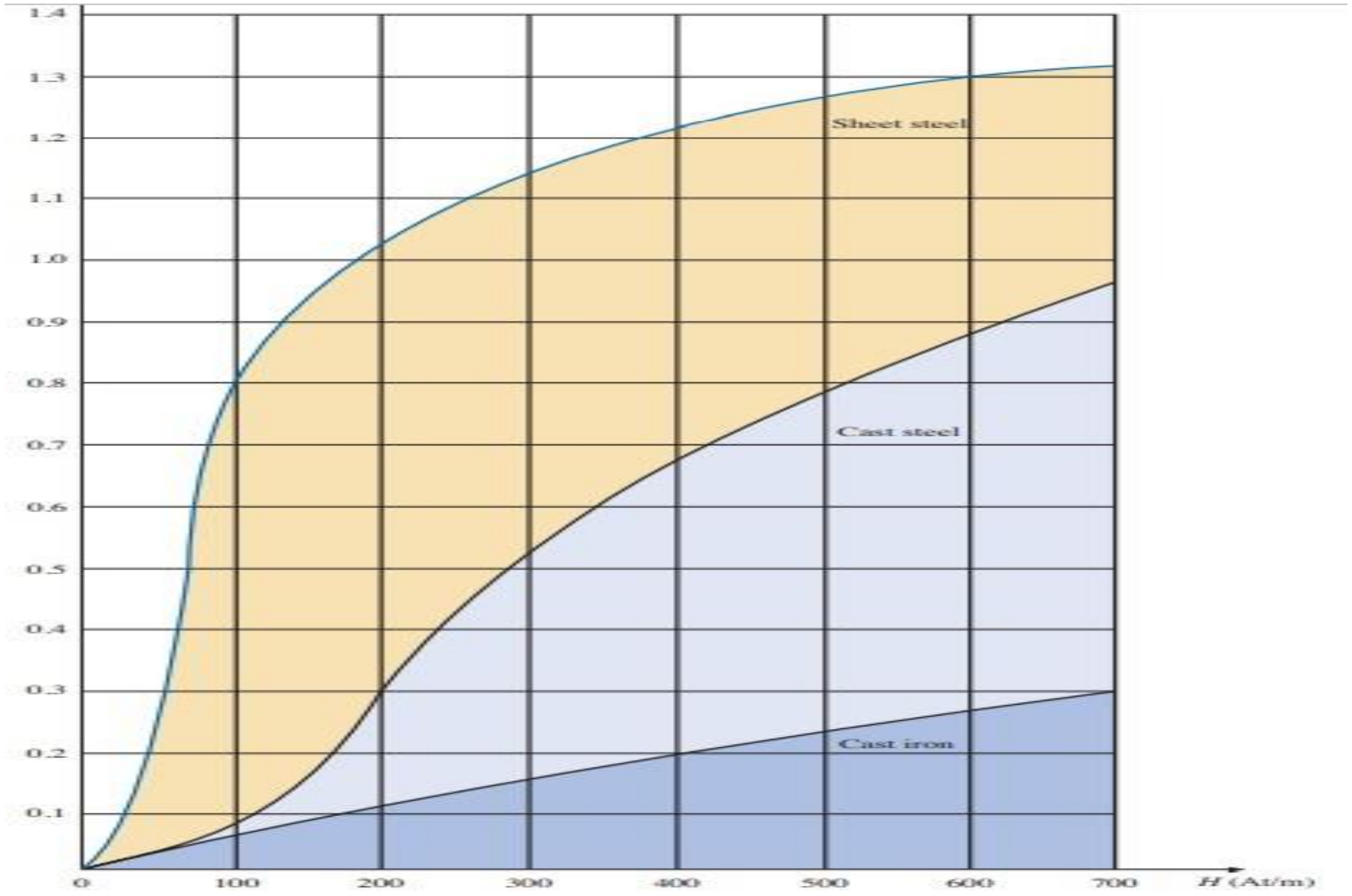
Normal magnetization curve for three ferromagnetic materials.

MAGNETIC CIRCUITS



Expanded view of the Normal Magnetization Curve for the low magnetizing force region.

MAGNETIC CIRCUITS



• Ampere's Circuital Law

There is a broad similarity between the analyses of electric and magnetic circuits. This has already been demonstrated to some extent for the quantities.

	Electric Circuits	Magnetic Circuits
Cause	E	F_m
Effect	I	Φ
Opposition	R	$\Re$

If we apply the "cause" analogy to Kirchhoff's voltage law ($\Sigma V=0$), we obtain the following:

$$\Sigma F_m = 0 \quad \text{(for magnetic circuits)}$$

which, in words, states that the algebraic sum of the rises and drops of the mmf around a closed loop of a magnetic circuit is equal to zero; that is, the sum of the rises in mmf equals the sum of the drops in mmf around a closed loop.

- The equation above is referred to as Ampere's circuital law. When it is applied to magnetic circuits, sources of mmf are expressed by the equation

$$F_m = NI \ (\text{At})$$

- The equation for the mmf drop across a portion of a magnetic circuit can be found by applying the relationships listed in the table ; that is, for electric circuits

$$V = IR$$

resulting in the following for magnetic circuits:

$$F_m = \Phi\Re \ (\mathrm{At})$$

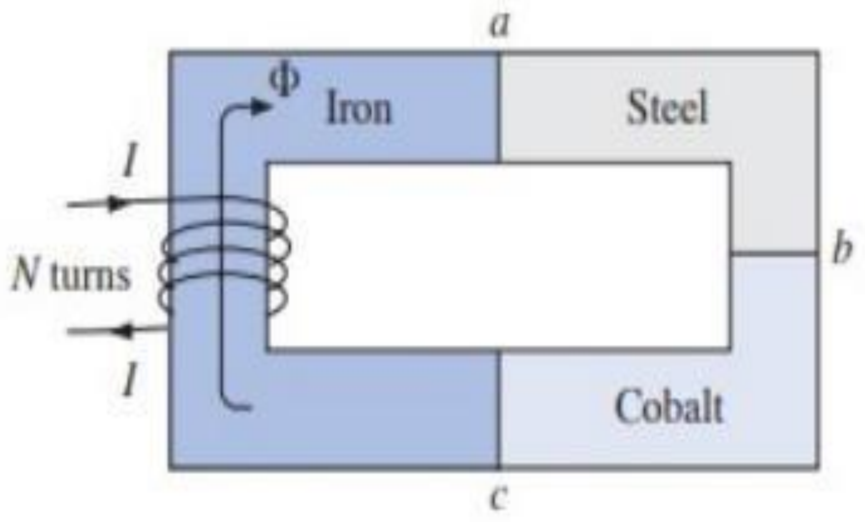
where Φ is the flux passing through a section of the magnetic circuit and R is the reluctance of that section.

- Consider the magnetic circuit appearing in the figure below constructed of three different ferromagnetic materials. Applying Ampere's circuital law, we have

$$\sum F_m = 0$$

$$\underbrace{+HI}_{Rise} - \underbrace{H_{ab}l_{ab}}_{Drop} - \underbrace{H_{bc}l_{bc}}_{Drop} - \underbrace{H_{ca}l_{ca}}_{Drop} = 0$$

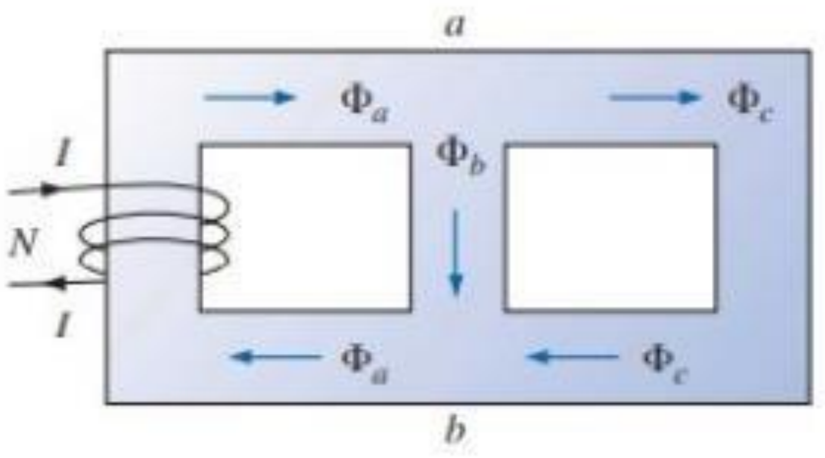
$$\underbrace{NI}_{impressed\_mmf} = \underbrace{H_{ab}l_{ab} + H_{bc}l_{bc} + H_{ca}l_{ca}}_{mmf\_drops}$$



All the terms of the equation are known except the magnetizing force for each portion of the magnetic circuit, which can be found by using the B-H curve if the flux density B is known.

- **The Flux Φ**

If we continue to apply the relationships described in the previous section to Kirchhoff's current law, we will find that the sum of the fluxes entering a junction is equal to the sum of the fluxes leaving a junction; that is, for the circuit of the figure below,



Flux distribution of a series-parallel magnetic network.

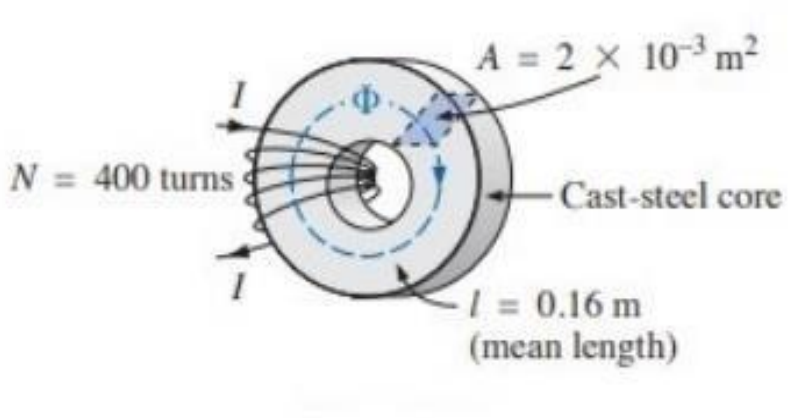
$\Phi_a = \Phi_b + \Phi_c$ (at junction a)
or
$\Phi_b + \Phi_c = \Phi_a$ (at junction b)
both of which are equivalent.

Example

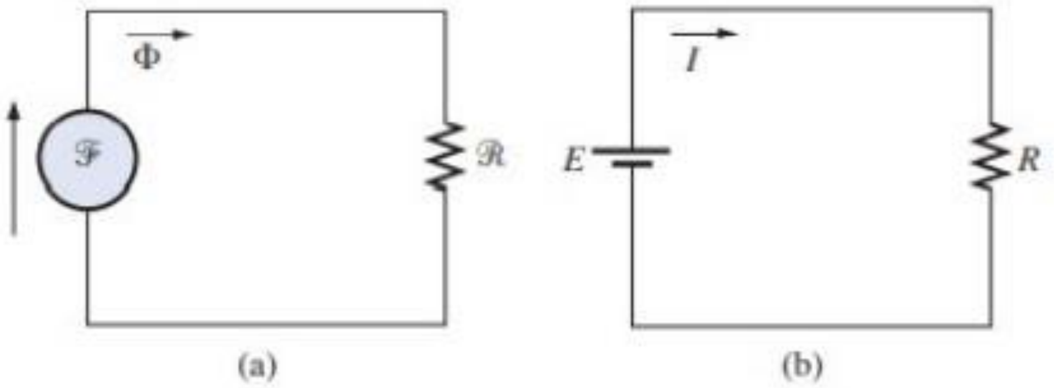
For the series magnetic circuit of the figure below:

a. Find the value of I required to develop a magnetic flux of $\Phi=4 \times 10{-}4\,Wb$.

b. b. Determine μ and μ_{o} for the material under these conditions.



✦Solution



(a)Magnetic circuit equivalent and (b)electric circuit analogy.

Section	Φ (Wb)	A (m^2)	B (T)	H (At/m)	l (m)	Hl (At)
One continuous section	4×10^{-4}	2×10^{-3}			0.16	

a. The flux density B is

$$B = \frac{\Phi}{A} = \frac{4 \times 10^{-4} Wb}{2 \times 10^{-3} m^2} = 2 \times 10^{-1} T = 0.2T$$

Using the B-H curves, we can determine the magnetizing force H:

H (cast steel) = 170 At/m

Applying Ampere's circuital law yields

NI= Hl

And $$I = \frac{Hl}{N} = \frac{(170 At/m)(0.16m)}{400t} = 68mA$$

b. The permeability of the material can be found using

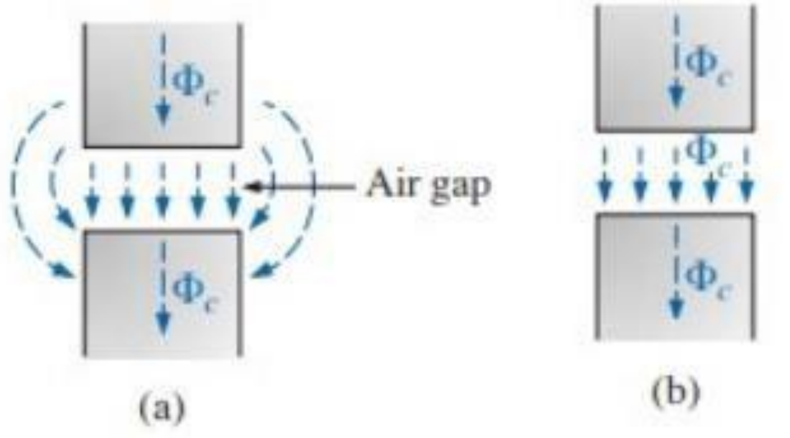
$$\mu = \frac{B}{H} = \frac{0.2T}{170At/m} = 1.176 \times 10^{-3} Wb/A.m$$

and the relative permeability is

$$\mu_r = \frac{\mu}{\mu_o} = \frac{1.176 \times 10^{-3}}{4\pi \times 10^{-7}} = 935.83$$

- **Air Gaps**

The spreading of the flux lines outside the common area of the core for the air gap in figure(a) is known as fringing. For our purposes, we shall neglect this effect and assume the flux distribution to be as in figure(b).



Air gaps (a)with fringing; (b)ideal.

 The flux density of the air gap in Fig. (b) above is given by

$$B_g = \frac{\Phi_g}{A_g}$$

where, for our purposes,

$$\Phi_g = \Phi_{core}$$
$$A_g = A_{core}$$

For most practical applications, the permeability of air is taken to be equal to that of free space. The magnetizing force of the air gap is then determined by

$$H_g = \frac{B_g}{\mu_o}$$

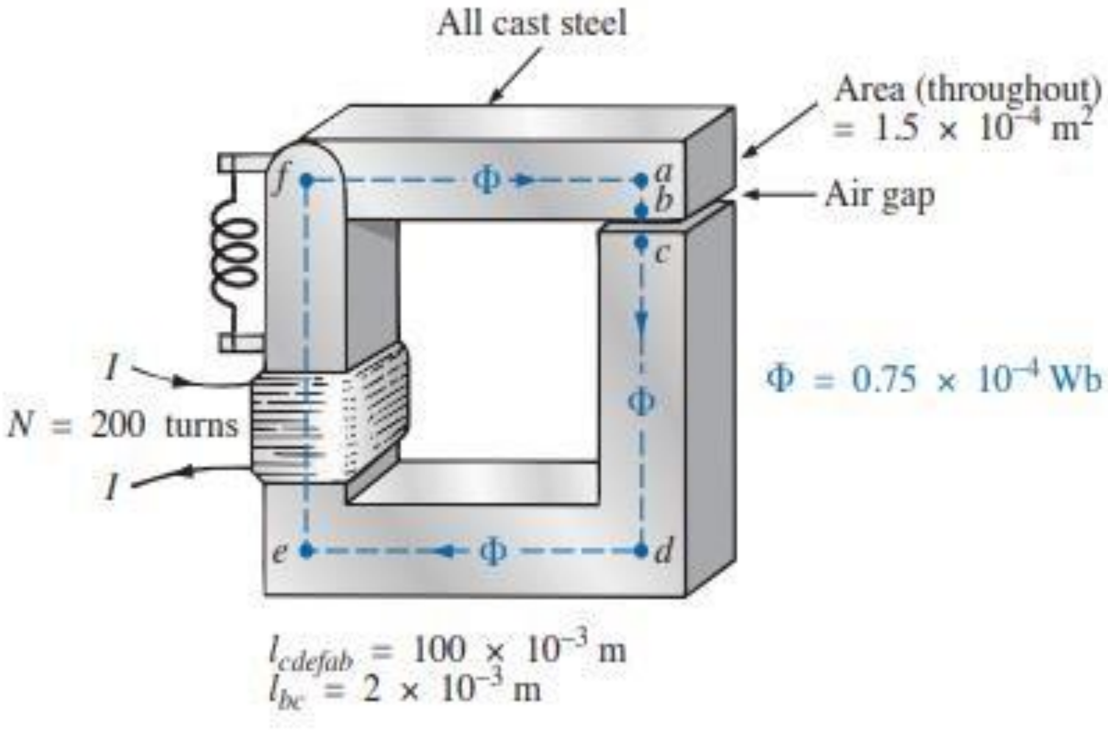
and the mmf drop across the air gap is equal to $H_g l_g$. An equation for H_g is as follows:

$$H_g = \frac{B_g}{\mu_o} = \frac{B_g}{4\pi \times 10^{-7}}$$

And $$\boxed{H_g = (7.96 \times 10^5)B_g}$$

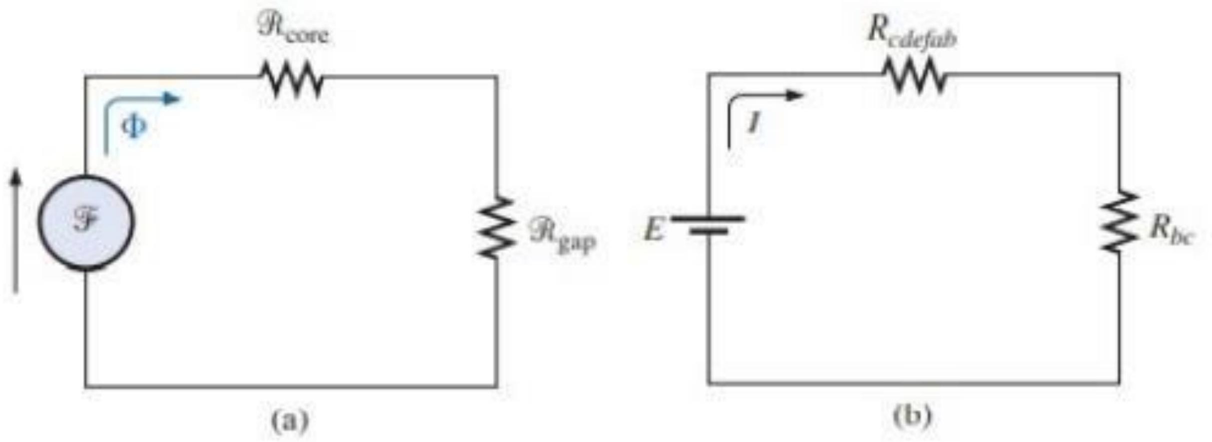
- **Example**

Find the value of I required to establish a magnetic flux of Φ=0.75×10^{-4}Wb in the series magnetic circuit of the figure



below.

✦Solution



(a)Magnetic circuit equivalent and (b)electric circuit analogy for the relay

$$B = \frac{\Phi}{A} = \frac{0.75 \times 10^{-4} Wb}{1.5 \times 10^{-4} m^2} = 0.5T$$

From the B-H curves

H(cast steel) $\sim$= 280At/m

$$H_g = (7.96 \times 10^5)B_g = (7.96 \times 10^5)(0.5\ \text{T}) = 3.98 \times 10^5\ \text{At/m}$$

The mmf drops are

$$H_{\text{core}}l_{\text{core}} = (280\ \text{At/m})(100 \times 10^{-3}\ \text{m}) = 28\ \text{At}$$
$$H_g l_g = (3.98 \times 10^5\ \text{At/m})(2 \times 10^{-3}\ \text{m}) = 796\ \text{At}$$

Applying Ampère's circuital law,

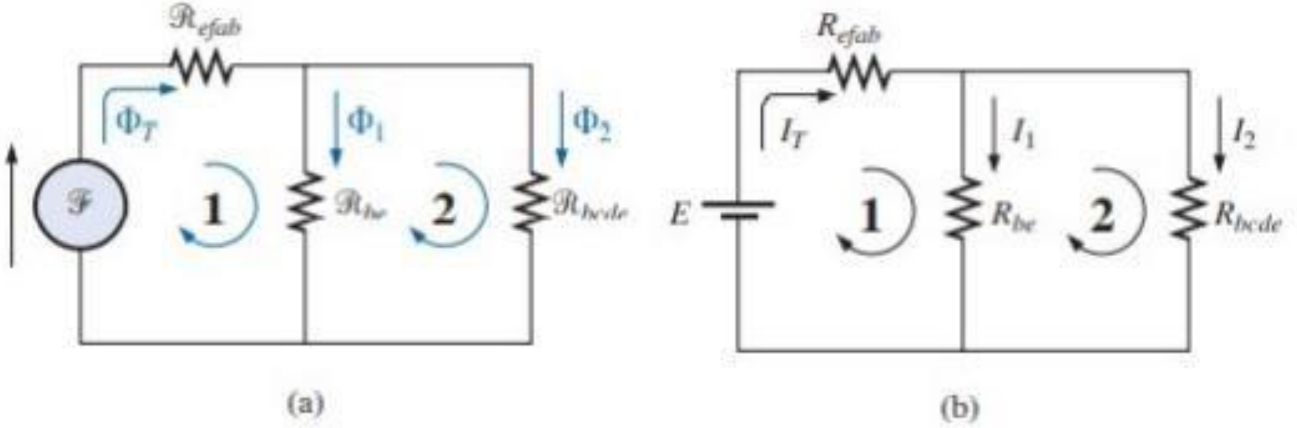
$$\begin{aligned} NI &= H_{\text{core}}l_{\text{core}} + H_g l_g \\ &= 28\ \text{At} + 796\ \text{At} \\ (200\ \text{t})I &= 824\ \text{At} \\ I &= \mathbf{4.12\ A} \end{aligned}$$

- **Series-Parallel Magnetic Circuits**

❍Example

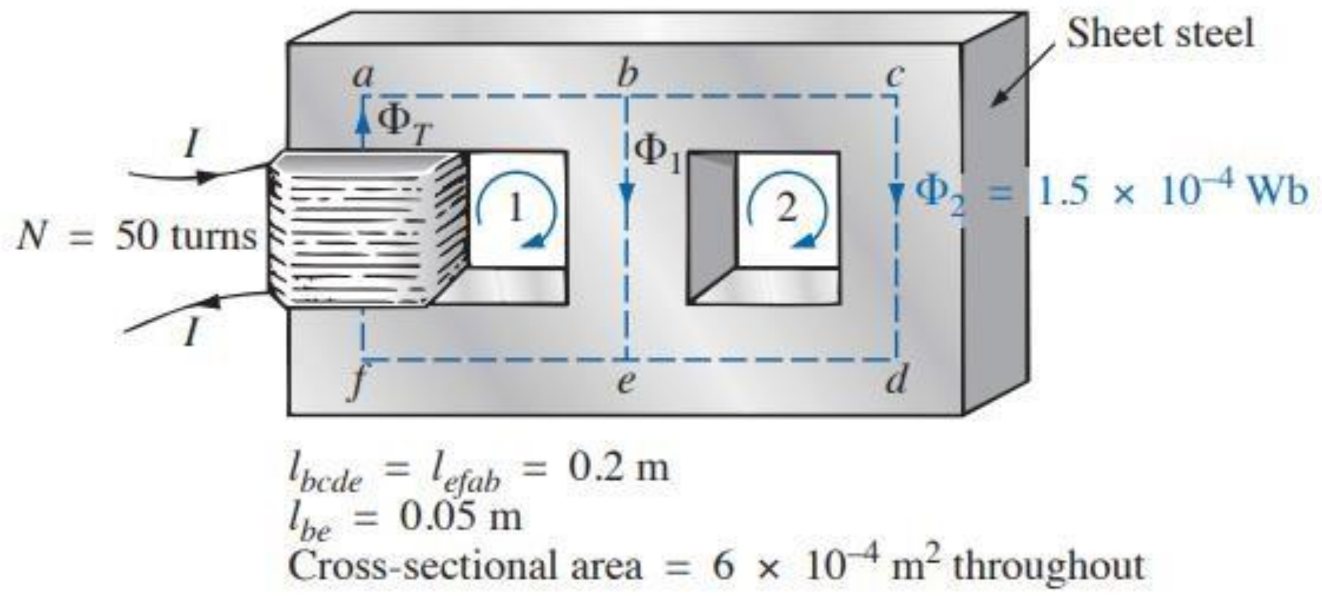
Determine the current I required to establish a flux of

1.5×10^{-4} Wb in the section of the core indicated in the figure below.



(a)Magnetic circuit equivalent and (b)electric circuit analogy for the series-parallel system

$$B_2 = \frac{\Phi_2}{A} = \frac{1.5 \times 10^{-4}\ \text{Wb}}{6 \times 10^{-4}\ \text{m}^2} = 0.25\ \text{T}$$



✦**Solution** From the magnetization curve

$$H_{bcde} \cong 40 \text{ At/m}$$

Applying Ampère's circuital law around loop 2 of figures (a) and (b)

$$\Sigma_{\circlearrowright} \mathcal{F} = 0$$

$$H_{be}l_{be} - H_{bcde}l_{bcde} = 0$$

$$H_{be}(0.05 \text{ m}) - (40 \text{ At/m})(0.2 \text{ m}) = 0$$

$$H_{be} = \frac{8 \text{ At}}{0.05 \text{ m}} = 160 \text{ At/m}$$

From the magnetization curve

$$B_l \cong 0.97\text{T}$$

And

$$\Phi_1 = B_1 A = (0.97T)(6 \times 10^{-4} m^2) = 5.82 \times 10^{-4} Wb$$

The results are entered in the table below.

The table reveals that we must turn our attention to section efab:

Section	Φ (Wb)	A (m^2)	B (T)	H (At/m)	l (m)	Hl (At)
bcde	1.5×10^{-4}	6×10^{-4}	0.25	40	0.2	8
be	5.82×10^{-4}	6×10^{-4}	0.97	160	0.05	8
efab		6×10^{-4}			0.2	

$$\Phi_T = \Phi_1 + \Phi_2 = 5.82 \times 10^{-4}\,\text{Wb} + 1.5 \times 10^{-4}\,\text{Wb}$$
$$= 7.32 \times 10^{-4}\,\text{Wb}$$
$$B = \frac{\Phi_T}{A} = \frac{7.32 \times 10^{-4}\,\text{Wb}}{6 \times 10^{-4}\,\text{m}^2}$$
$$= 1.22\,\text{T}$$

$H_{efab} \sim = 400\text{At}$

Applying Ampere's circuital law,

$$+NI - H_{efab}l_{efab} - H_{be}l_{be} = 0$$
$$NI = (400\,\text{At/m})(0.2\,\text{m}) + (160\,\text{At/m})(0.05\,\text{m})$$
$$(50\,\text{t})I = 80\,\text{At} + 8\,\text{At}$$
$$I = \frac{88\,\text{At}}{50\,\text{t}} = \mathbf{1.76\,A}$$

To demonstrate that μ is sensitive to the magnetizing force H, the permeability of each section is determined as follows.

For section bcde,

$$\mu = \frac{B}{H} = \frac{0.25\ \text{T}}{40\ \text{At/m}} = 6.25 \times 10^{-3}$$

$$\mu_r = \frac{\mu}{\mu_o} = \frac{6.25 \times 10^{-3}}{12.57 \times 10^{-7}} = \mathbf{4972.2}$$

For section be,

$$\mu = \frac{B}{H} = \frac{0.97\ \text{T}}{160\ \text{At/m}} = 6.06 \times 10^{-3}$$

$$\mu_r = \frac{\mu}{\mu_o} = \frac{6.06 \times 10^{-3}}{12.57 \times 10^{-7}} = \mathbf{4821}$$

For section efab,

$$\mu = \frac{B}{H} = \frac{1.22\ \text{T}}{400\ \text{At/m}} = 3.05 \times 10^{-3}$$

$$\mu_r = \frac{\mu}{\mu_o} = \frac{3.05 \times 10^{-3}}{12.57 \times 10^{-7}} = \mathbf{2426.41}$$

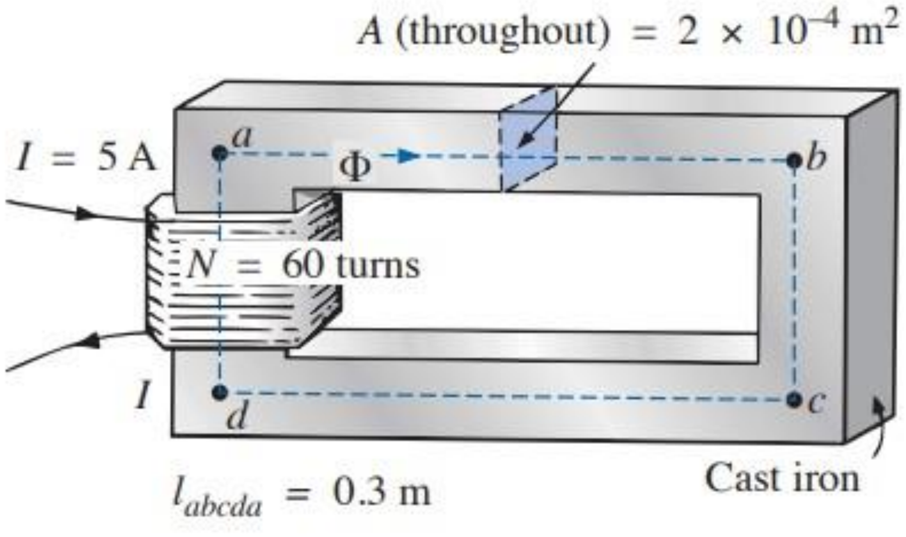
- **Determining Φ**

$$H = \frac{NI}{l} \qquad H \rightarrow B \qquad (B\text{-}H \text{ curve})$$

And $\Phi = BA$

Example

Calculate the magnetic flux Φ for the magnetic circuit of the figure below.



✦Solution

By Ampere's circuital law,

$$NI = H_{abcda} l_{abcda}$$

or
$$H_{abcda} = \frac{NI}{l_{abcda}} = \frac{(60 \text{ t})(5 \text{ A})}{0.3 \text{ m}}$$
$$= \frac{300 \text{ At}}{0.3 \text{ m}} = 1000 \text{ At/m}$$

and
$$B_{abcda} \text{ (from Fig. 11.23)} \cong 0.39 \text{ T}$$

Since $B = \Phi/A$, we have

$$\Phi = BA = (0.39 \text{ T})(2 \times 10^{-4} \text{ m}^2) = \mathbf{0.78 \times 10^{-4} \text{ Wb}}$$

- **Exercise**

Find the magnetic flux Φ for the series magnetic circuit of the figure below for the specified impressed mmf.