ELNG 251: DC Machines and Transformers

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- Electromagnetism
- DC Machines
- DC Generators
- DC Motors
- Magnetic circuits
- Single-phase transformers
  - Ideal transformer
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- Three-phase transformers

Assessment

- Continuous assessment – 40 marks
  - Weekly quizzes and assignments - 20 marks
  - Mid-semester examination – 20 marks

- End of semester examination – 60mks
  - Multiple choice examination – 20 marks
  - Written examination – 40 marks

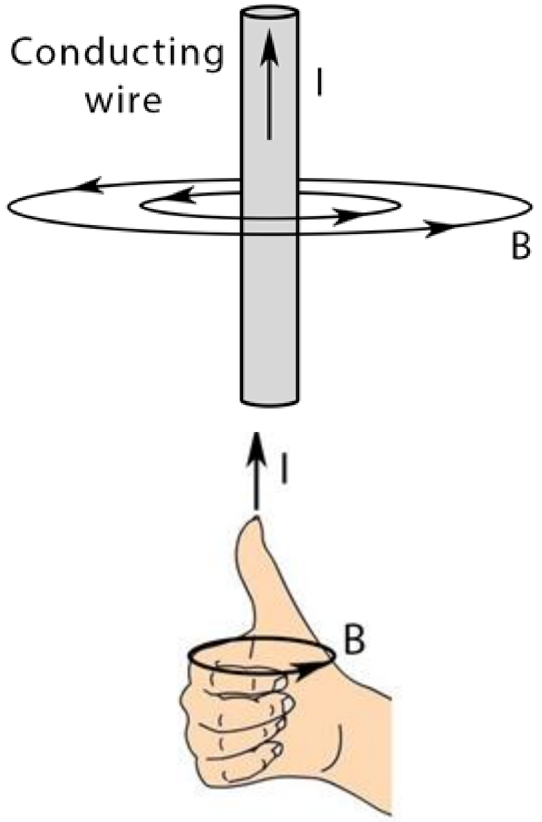
Further study

- Sahdev, S. K. (2017). Electrical machines. Cambridge University Press.

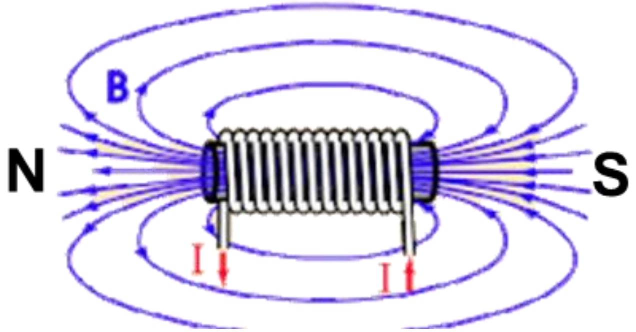
Click here for lecture materials

Electromagnetism

- **Electromagnetism** is the interaction or interrelation between electric charges and magnetic fields.
- In 1820 Danish physicist Hans Christian Ørsted discovered that an electric current creates a magnetic field around it.



- The magnetic field lines encircle the current-carrying wire.
- The magnetic field lines lie in a plane perpendicular to the wire.
- If the direction of the current is reversed, the direction of the magnetic field reverses.
- The strength of the field is directly proportional to the magnitude of the current.
- The strength of the field at any point is inversely proportional to the distance of the point from the wire.



Right hand thumb rule

- Thumb points in the direction of electric current and fingers curl around the current indicating the direction of the magnetic field.

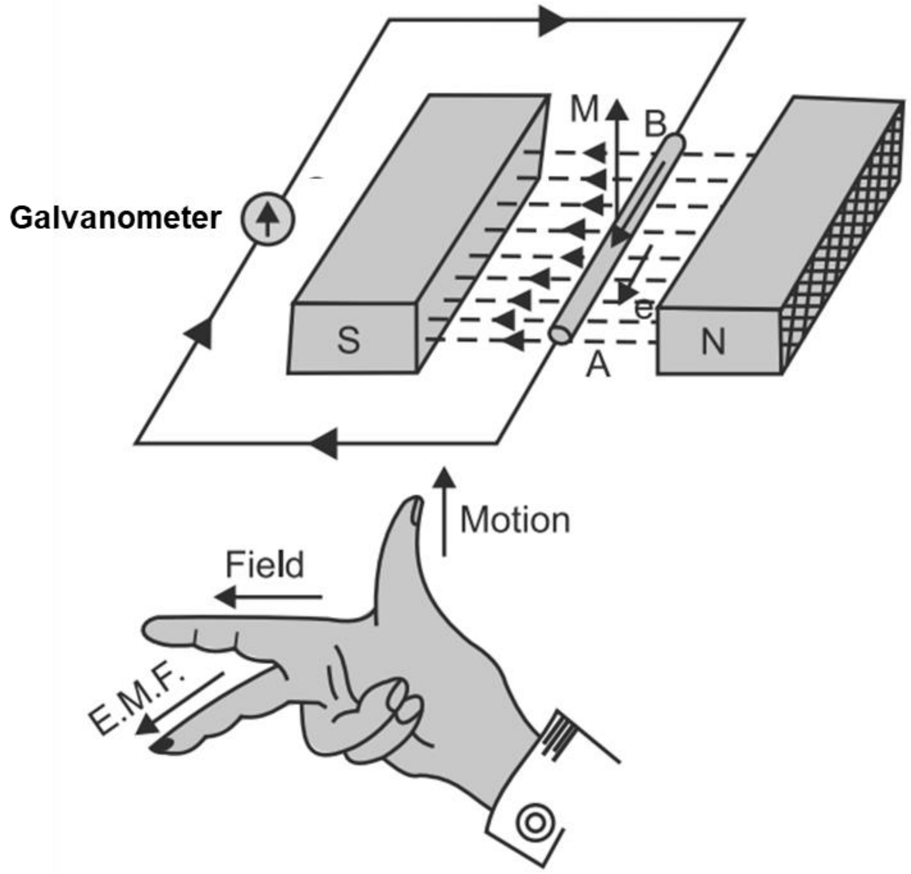
Faraday's laws of electromagnetic induction

- **1st law:** Whenever a conductor cuts across a magnetic field, an emf is induced in the conductor
- ***OR*** Whenever the magnetic flux linking with any circuit (or coil) changes, an emf is induced in the circuit.
- **2nd law:** The magnitude of induced emf in a coil is directly proportional to the rate of change of flux linkages.
- $e = N\frac{d\emptyset}{dt}$

N – Number of turns
$\emptyset$ - change in flux
dt – change in time

Fleming's Right Hand Rule

- Stretch the 1st finger, middle finger and thumb of the right hand in mutually perpendicular manner such that the 1st finger points in the direction of the magnetic field and the thumb in the direction of the motion, the resulting induced emf is in the direction of the middle finger.



Direction of induced emf using the **RIGHT-HAND RULE**

Worked Example

A coil of 500 turns is linked with a flux of 2 mWb. If this flux is reversed in 4 ms, calculate the average emf induced in the coil.

Solution

Induced emf, $\boldsymbol{e = N \frac{d\emptyset}{dt}}$

$\boldsymbol{N}$ = 500 turns

$\boldsymbol{d\emptyset}$ = 2-(-2) = 4 mWb = 4 x 10^{-3} Wb

$\boldsymbol{dt}$ = 4 x 10^{-3} s

$$e = 500 \times \frac{4 \times 10^{-3}}{4 \times 10^{-3}} = \mathbf{500\ V}$$

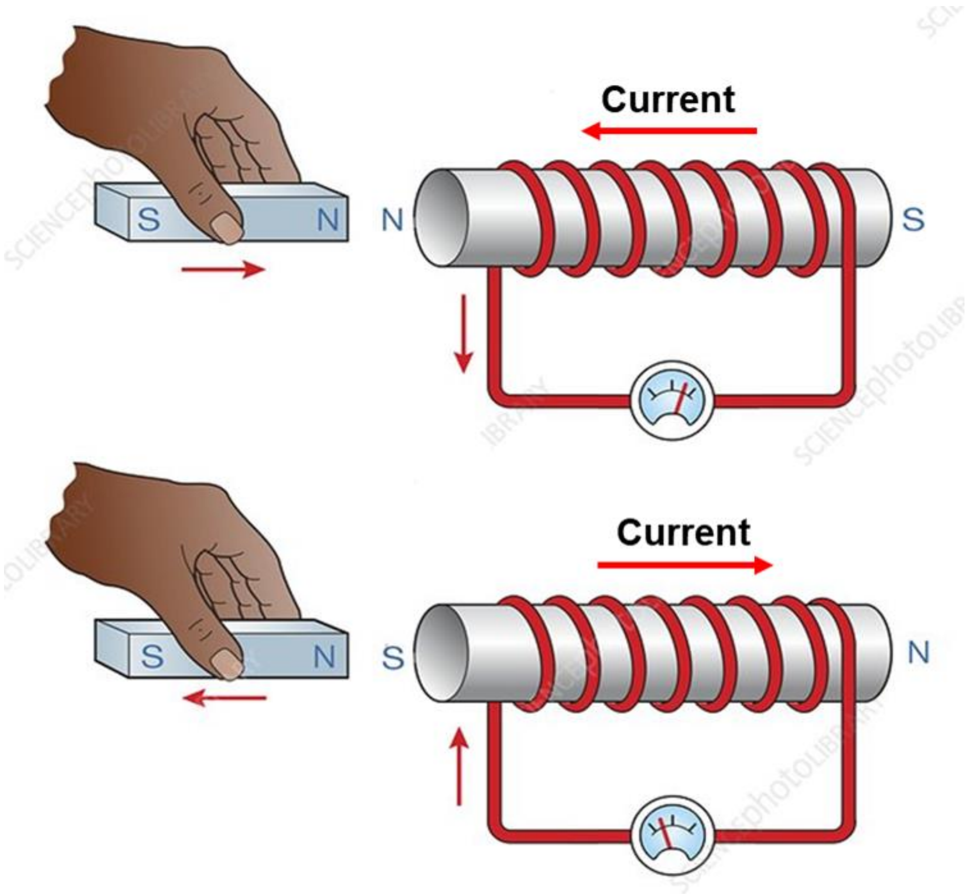
- A coil of 250 turns is wound on a magnetic circuit of reluctance 100,000 At/Wb. If a current of 2 A flowing in the coil is reversed in 5 ms, find the average emf induced in the coil.

Lenz's law

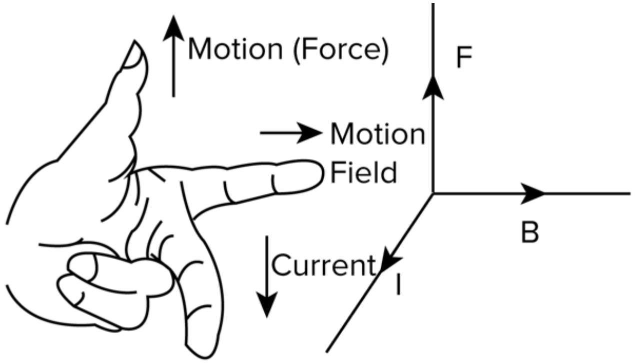
- states that the direction of the electric current induced in a conductor by a changing magnetic field is such that the magnetic field created by the induced current opposes changes in the initial magnetic field. It is named after physicist Heinrich Lenz, who formulated it in 1834.

Fleming's Left Hand Rule

- Stretch the 1st finger, middle finger and thumb of the right hand in mutually perpendicular manner such that the 1st finger points in the direction of the magnetic field and the middle finger in the direction of the current, the thumb points in the direction of the force exerted on the conductor.



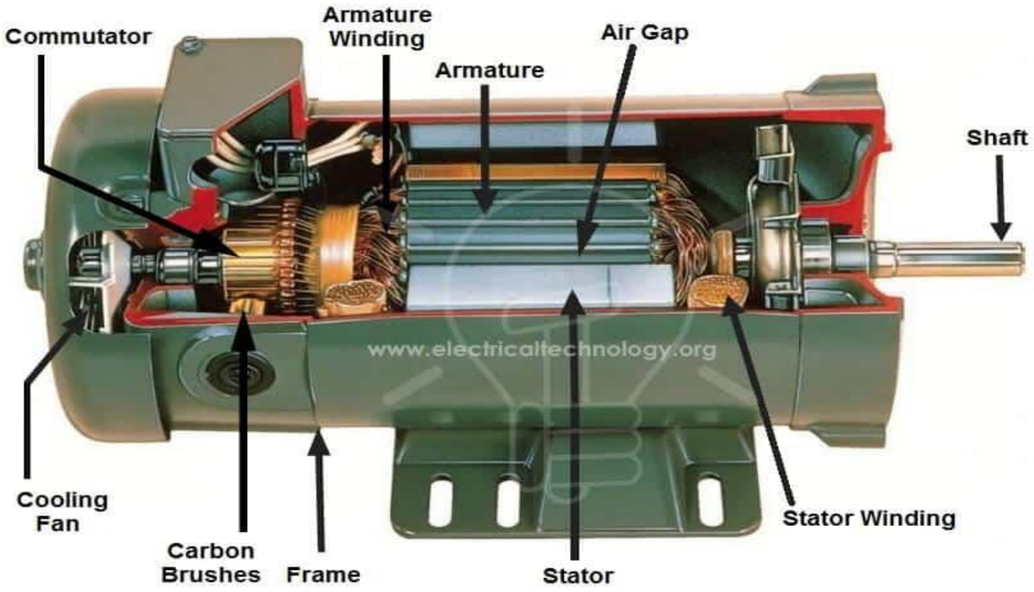
Induced current opposing magnetic field – **LENZ'S LAW**

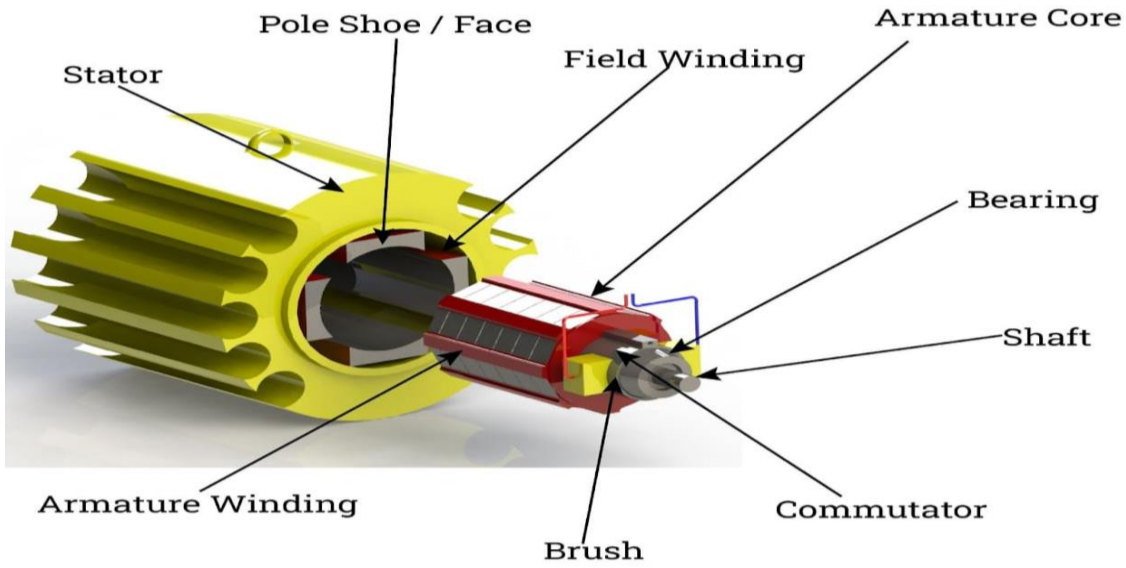


Click here for a video on Lenz's law

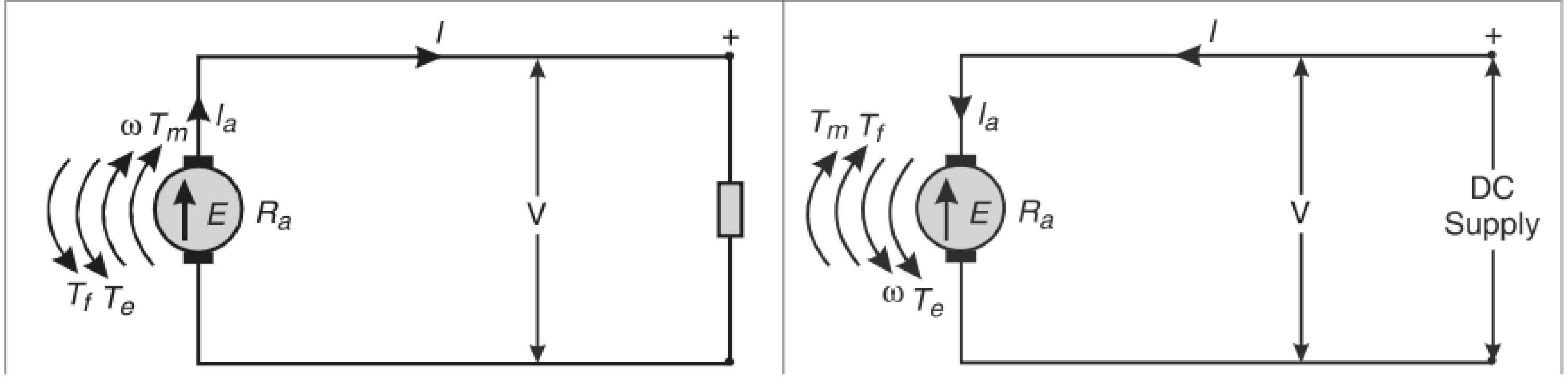
DC machines

- A DC machine is an electro-mechanical energy conversion device.
- When it converts mechanical power into DC electrical power, it is known as a DC generator.
- On the other hand, when it converts DC electrical power into mechanical power it is known as a DC motor.
- Applications where large quantity of DC power is required are in chemical and metal extraction plants, for electroplating and electrolysis processes etc. At such places DC generators are used to deliver power.
- DC motors have several applications in industry and every day use; toys, windshield wipers, small appliances, power tools, electric vehicles, robotics, industrial automation, etc.





Comparing DC generator and motor operation



Generator action	Motor action
1. In generating action, the rotation is due to mechanical torque, therefore, T_m and ω are in the same direction.	1. In motoring action, the rotation is due to electromagnetic torque, therefore, T_e and ω are in the same direction.
2. The frictional torque T_f acts in opposite direction to rotation ω.	2. The frictional torque T_f acts in opposite direction to rotation ω.
3. Electromagnetic torque T_e acts in opposite direction to mechanical torque T_m so that $\omega T_m = \omega T_e + \omega T_f$.	3. Mechanical torque T_m acts in opposite direction to electromagnetic torque T_e so that $\omega T_e = \omega T_m + \omega T_f$.
4. In generating action, an emf is induced in the armature conductors which circulates current in the armature when load is connected to it. Hence, e and i both are in the same direction.	4. In motoring action, current is impressed to the armature against the induced emf (e), therefore current flows in opposite direction to that of induced emf.
5. In generator action, $E > V$	5. In motor action, $E < V$
6. In generating action, the torque angle θ is leading.	6. In motoring action, the torque angle θ is lagging.
7. In generating action, mechanical energy is converted into electrical energy.	7. In motoring action, electrical energy is converted into mechanical energy.

Armature winding of a dc machine

Armature winding

- The armature winding are the insulated conductors housed in the armature slot.

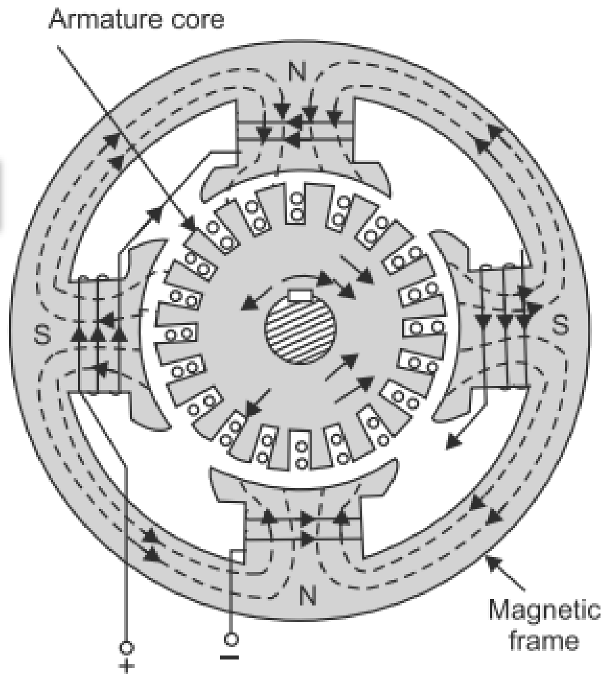
NB: DC motors and DC generators are collectively referred to as DC machine

- It is the place where one form of power is converted to the other form i.e., in case of generator, mechanical power is converted into electrical power and in case of motor, electrical power is converted into mechanical power.
- There are two types of armature windings:

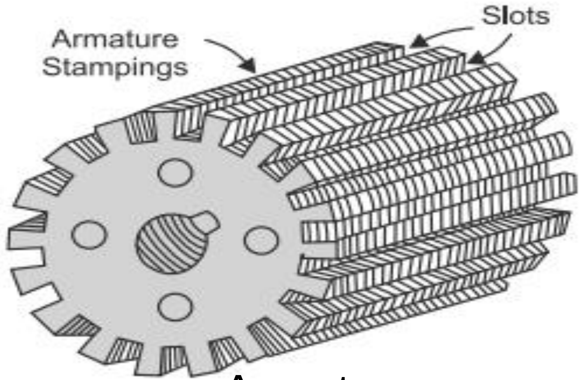
(i) Lap winding: In this winding, that the number of parallel paths (A) is equal to number of poles. In this case, the number of brushes is equal to the number parallel paths. Out of which half the brushes are positive and the remaining (half) are negative.

(ii) Wave winding: In this winding, the parallel paths are only two irrespective of the number of poles. In this case, is equal to two i.e., number of parallel paths.

[Click here for further study on armature windings]



Magnetic circuit of a DC machine



Armature core

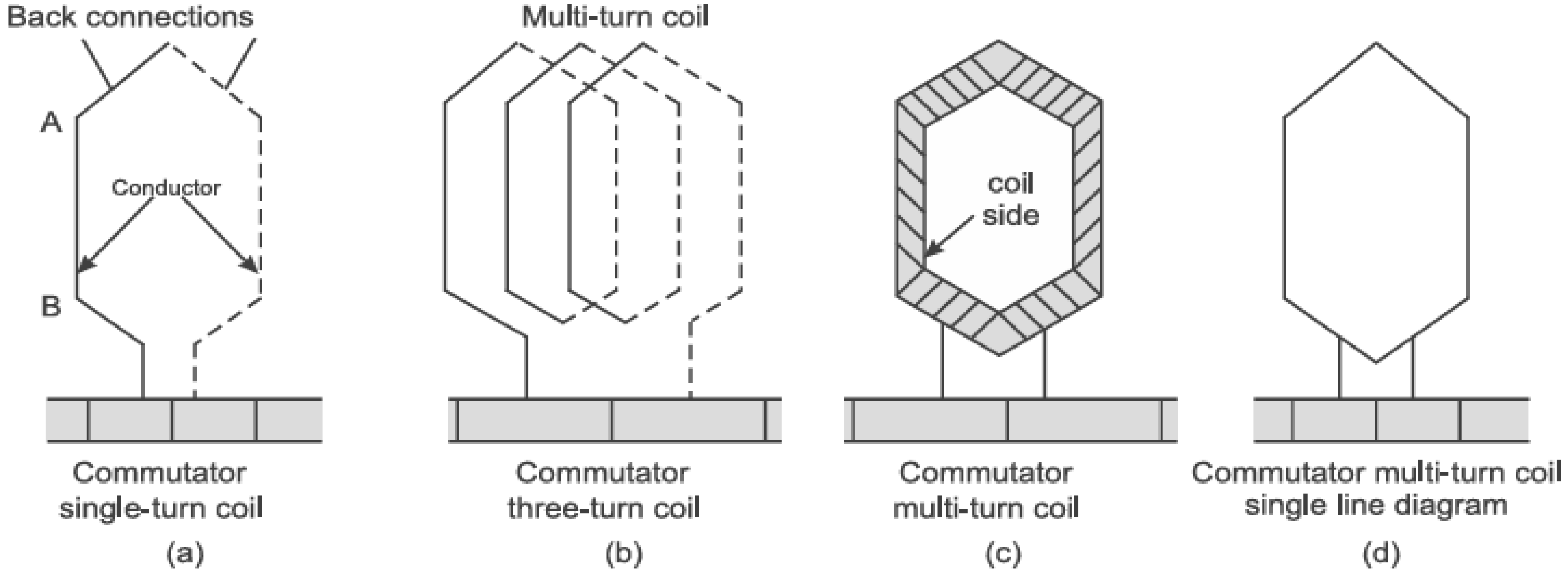
Armature winding of a dc machine

Conductor: The length of wire embedded in armature core and lying within the magnetic field. Number of conductor is represented by $\boldsymbol{Z}$.

Turn: Two conductors lying in a magnetic field connected in series at the back.

$$Z = number\ of\ turns\ \times 2$$

Coil: A coil may be a single-turn coil having only two conductors, as shown in Fig (a), or it may be a multi-turn coil having more than two conductors as shown in Fig (b). In Fig. 4.17(b), a three-turn coil is shown. The bunch of three conductors may be wrapped by the cotton tape, as shown in Fig (c), before placing it in the armature slot. A multi-turn coil can be represented by single line diagram as shown in Fig (d).



Emf and torque equation of a DC machine

Emf equation

P = number of poles

Ø = Flux per pole

Z = Number of conductors

A = Number of parallel paths

N = speed in rpm

- Total flux cut by one conductor

$$= P\emptyset$$

- Time in seconds taken in one revolution

$$= \frac{60}{N}$$

- Per Faraday's law, induced emf per conductor

$$e = \frac{\emptyset}{t} = \frac{P\emptyset}{60/N} = \frac{P\emptyset N}{60}$$

- Total emf for all conductors

$$\boldsymbol{E = \frac{P\emptyset N}{60} \times \frac{Z}{A}}$$

Where $\frac{Z}{A}$ is the number of conductors per parallel path

Torque equation

I_a = Armature current

ω = speed in rad/s = $\frac{2\pi}{60}N$

- The torque generated in a dc machine is due to electromagnet effect, hence it's called electromagnetic torque, T_e.
- The electrical power in the armature is equal to the electromagnetic torque, hence

$$EI_a = T\omega$$

$$\frac{P\emptyset NZ}{60A} \times I_a = T \times \frac{2\pi}{60}N$$

$$\boldsymbol{T = \frac{P\emptyset Z I_a}{2\pi A}}$$

DC generator

- A DC generator is an **electro-mechanical** energy conversion device (electrical machine) that converts mechanical energy or power (ωT) into DC electrical energy or power (EI).
- There are some applications where large quantity of DC power is required (such as in chemical and metal extraction plants, for electroplating and electrolysis processes etc.), at such places DC generators are used to deliver power.
- DC generators share the same internal composition and design with DC motors. The difference lies in the peripheral systems that support them.
- Motors often require control circuits for speed regulation, while generators require protection circuits for overvoltage and overcurrent.

Working principle

- The basic principle of a DC generator is electro-magnetic induction i.e., “*When a conductor cuts across the magnetic field, an emf is induced in it.*”
- The direction of the induced emf is determined by the **Fleming’s Right Hand Rule**.

DC generator

- In a DC generator, maximum emf is induced when the conductor cuts the magnetic field lines at 90^o, any deviation from this decreases the emf.
- Since the conductor rotates, it only cuts the magnetic field at 90^o twice in its revolution, and 0^o twice when its perfectly parallel with the field.
- At 0^o, no emf is induced.
- Hence, the emf is said to be dynamically induced as seen in Figure (a), where $e = Blv \sin \theta$

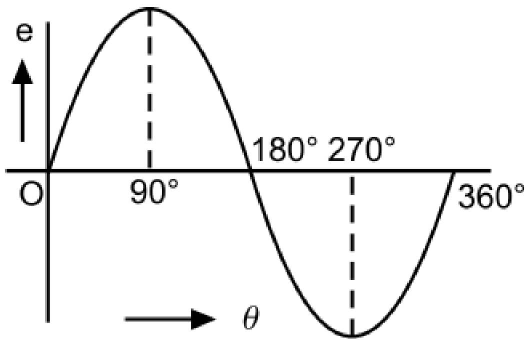
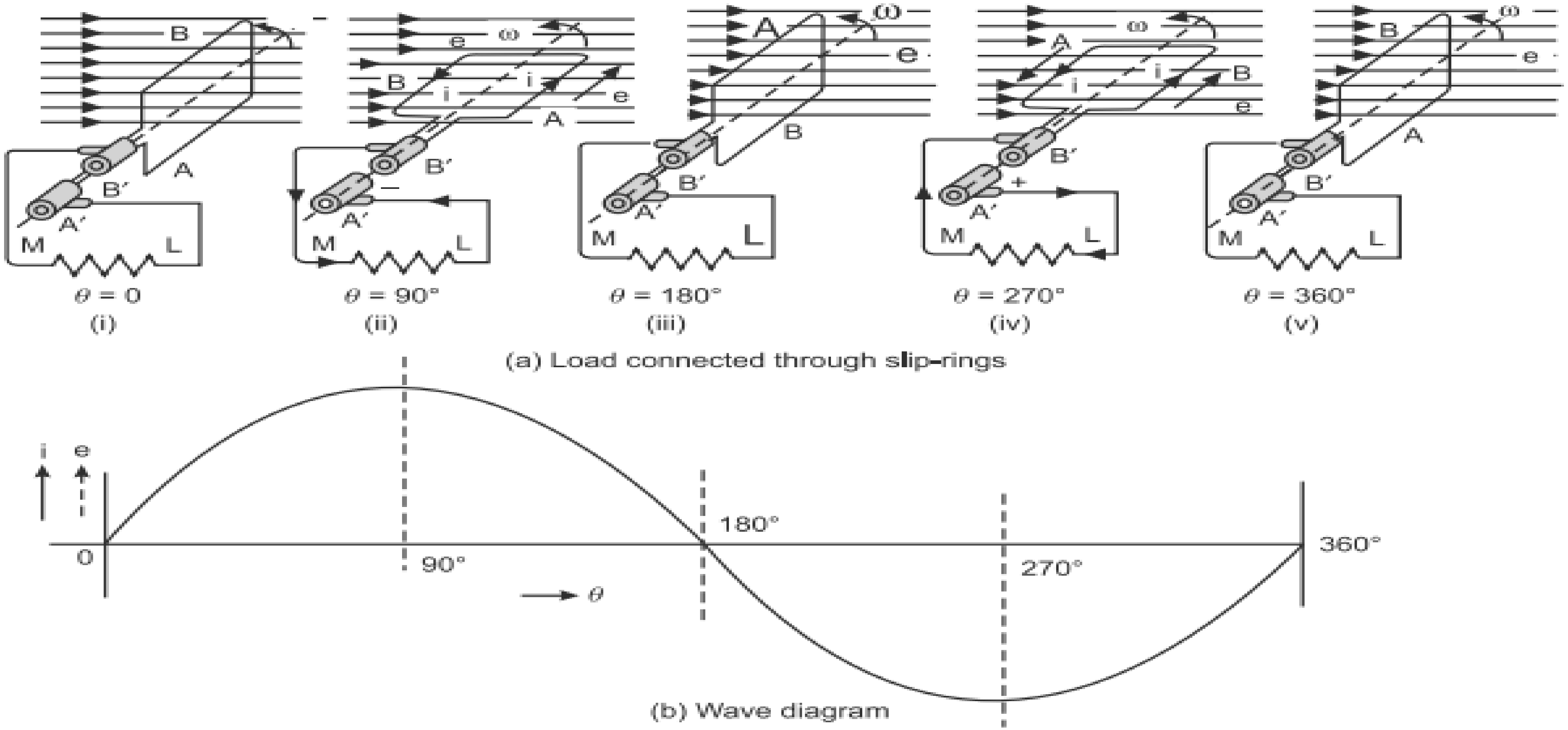


Figure (a): Wave shape of induced emf

How a DC generator works

- For simplicity, consider only one coil AB placed in the strong magnetic field. The two ends of the coil are joined to slip rings A’ and B’ respectively. Two brushes rest on these slip rings as shown in Figure (a).



Generated emf for external circuit connected through slip rings

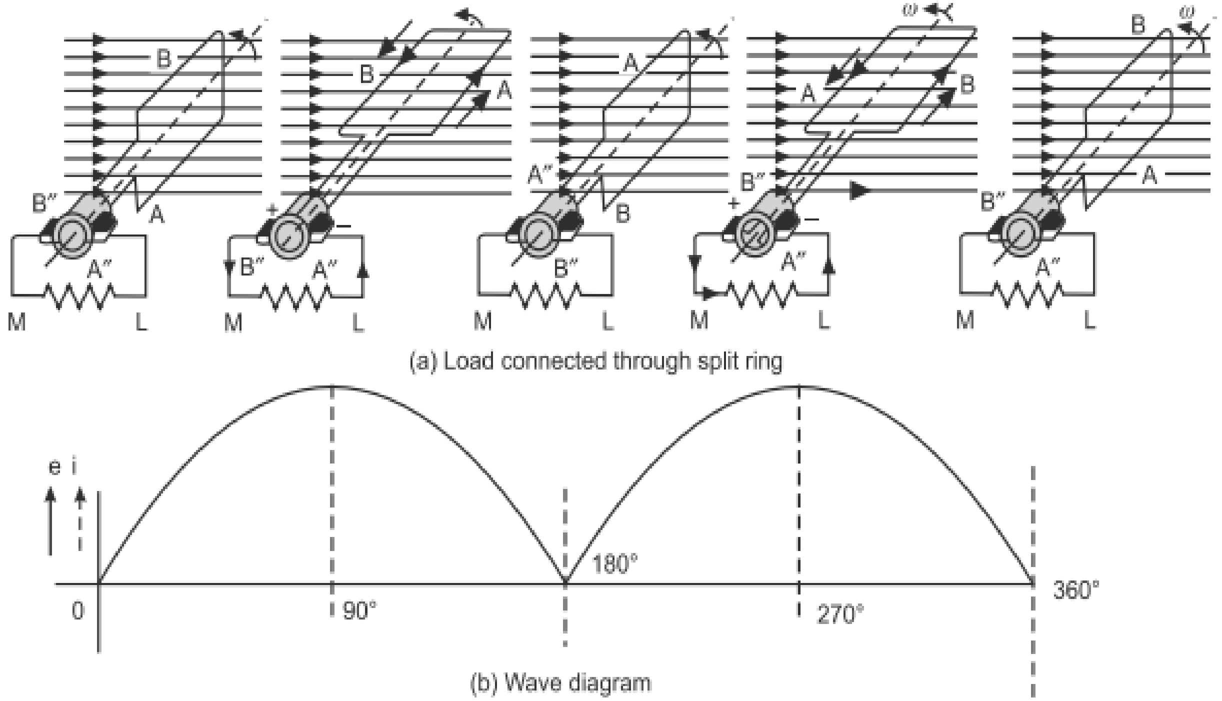
DC generator

Function of commutator

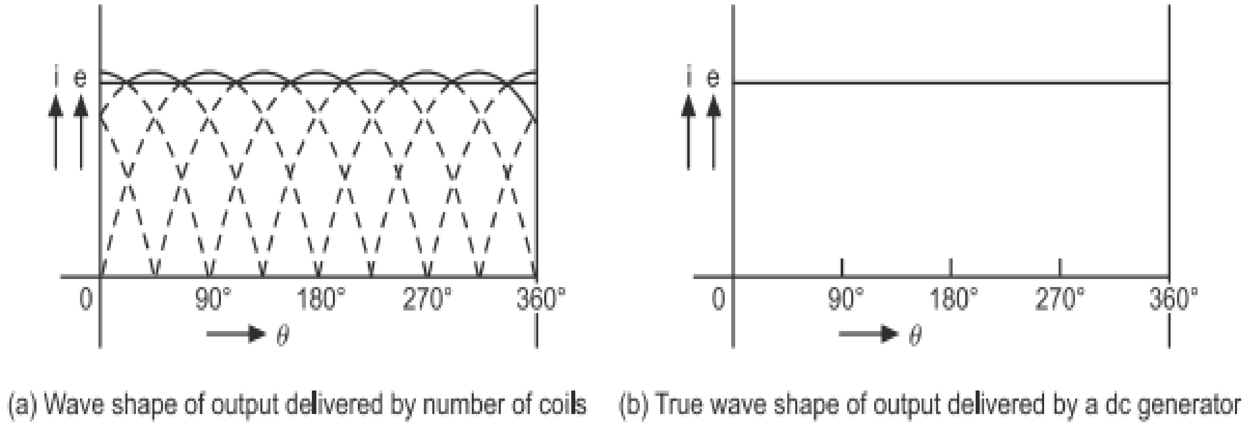
- The commutator is to converts the alternating current produced in the armature into direct current in the external circuit.

Commutator action

- Now, consider that the two ends of the coil are connected to only one slip ring split into two parts (segment) i.e., A'' and B''. Each part is insulated from the other by a mica layer. Two brushes rest on these parts of the ring as shown in the figure below.
- The magnitude of emf induced in the coil at various instants will remain the same as shown in the previous slide, however, the flow of current in the external load will become unidirectional as been below



Generated emf for external circuit connected through split ring



Wave shape of output delivered by a DC generator

DC generator

Equivalent circuit of a DC generator

$$\boldsymbol{E_g = V + I_a R_a}$$

$$I_a = I_L$$

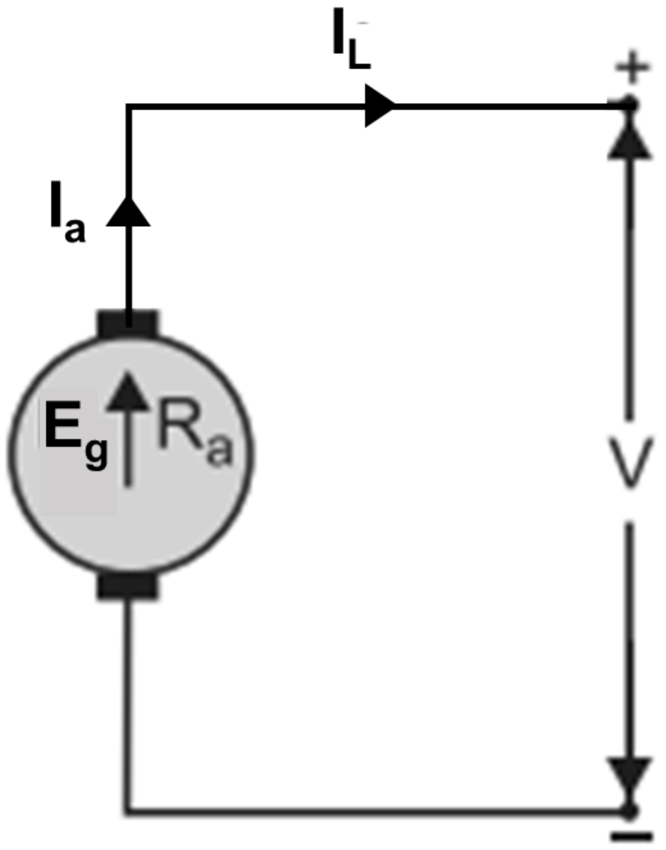
- Where E_g is the generated voltage

I_a is the armature current

I_L is the load current

V is the terminal voltage

R_a is the armature resistance



DC generator

Problems

1. A 12 kW, six-pole DC generator develops an emf of 240 at 1500 rpm. The armature has a lap connected winding. The average flux density over the pole pitch is 1.0 T. The length and diameter of the armature is 30 cm and 25 cm respectively. Calculate flux per pole, total number of active armature conductors. Power generated in the armature and torque developed when the machine is delivering 50 A current to the load.
2. The useful flux of a four-pole, lap wound DC generator is 0.07 Wb. Its armature carries 220 turns and each turn has 0.004 ohms resistance, if its armature current is 50 A, running at a speed of 900 rpm, determine its terminal voltage.
3. A DC generator is connected to a 220 V DC mains. The current delivered by the generator to the mains is 100A. The armature resistance is 0.1 ohm. The generator is driven at a speed of 500 rpm Calculate (i) the induced emf (ii) the electromagnetic torque (iii) the mechanical power input to the armature neglecting iron, winding and friction losses, (iv) Electrical power output from the armature, (v) armature copper loss.

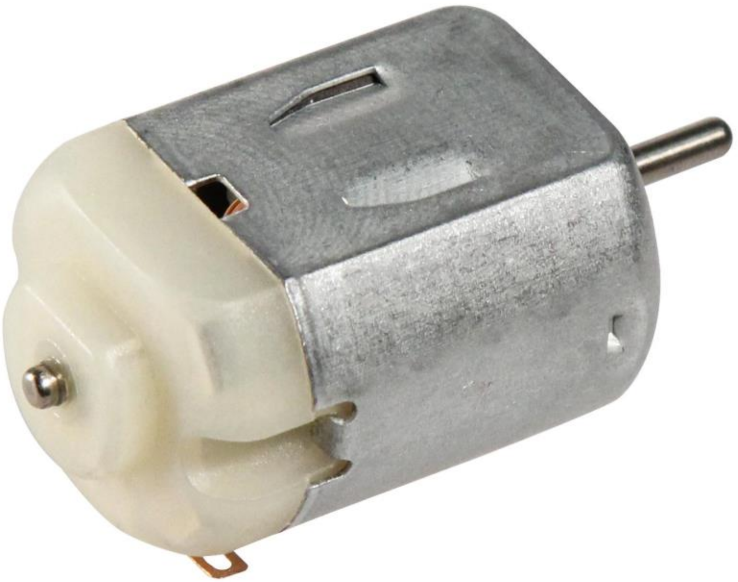
DC generator

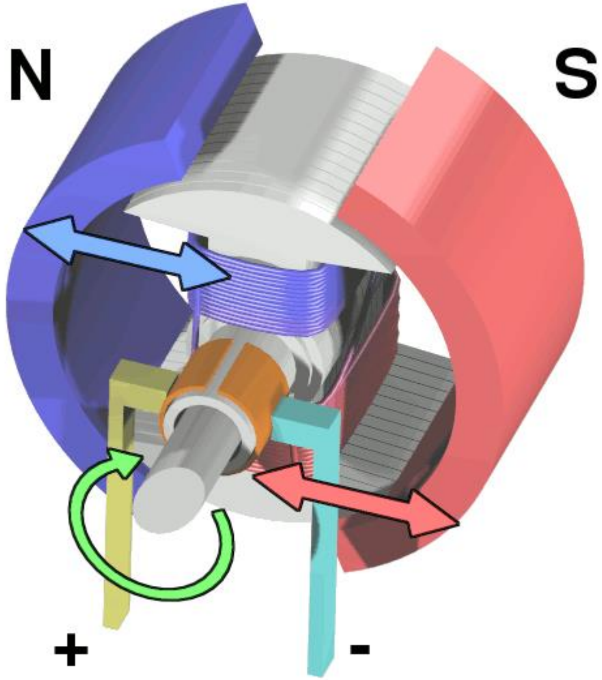
Assignment 1 – (Deadline Mon, Jan 20, 2025, 23:59)

1. A four-pole, lap wound DC machine is having 500 conductors on its armature and running at 1000 rpm. The flux per pole is 30 m Wb Calculate the voltage induced in the armature winding. What will be induced emf if the armature is rewound for wave winding?
2. A 4-pole machine has an armature with 90 slots and 8 conductors per slot, the flux per pole is 0.05 Wb and runs at 1200 rpm. Determine induced emf if winding is (i) lap connected (ii) wave connected.
3. A wave wound armature of a six-pole generator has 51 slots. Each slot contains 20 conductors. The voltage required to be generated is 250 V. What would be the speed of coupled prime mover if flux per pole is 0.07 Wb. If the armature is rewound as lap wound machine and run by same prime mover, what will be the generated voltage.
4. The armature of a four-pole 200 V, lap wound generator has 400 conductors and rms of 300 rpm. Determine the useful flux per pole. If the number of turns in each field coil is 900, what is the average induced emf in each field coil on breaking its connection if the magnetic flux set-up by it dies away completely in 0.1 second?

DC motor

- A DC motor is an **electro-mechanical** energy conversion device (electrical machine) that converts DC electrical energy or power (EI) into mechanical energy or power (ωT).
- DC motors have extensive applications in industrial automation, transportation, robotics, aerospace, home appliances, medical devices, etc.

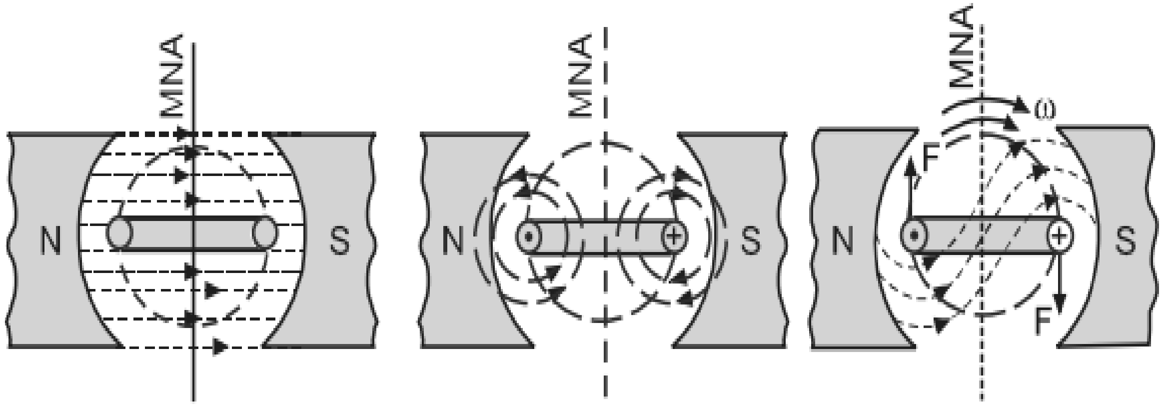




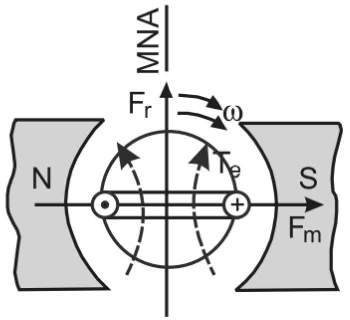
Working principle

- The operation of a DC motor is based on the principle that when a current carrying conductor is placed in a magnetic field, a mechanical force is experienced by it.
- The direction of this force is determined by **Fleming's Left Hand Rule** and its magnitude is given by the relation: $F = B\ i\ l$ (newton)

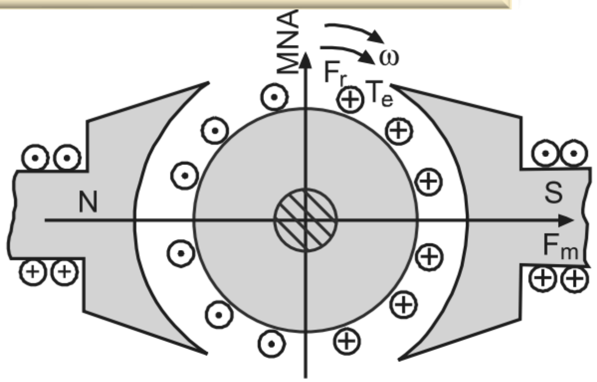
Where, B is the magnetic flux density in Tesla (T), i is the current in the conductor and l is the length of the conductor



(a) Main field (b) Field due to current carrying coil (c) Resultant field



Position of main field (F_m) and rotor field



Motor action

the direction of flow of current in each conductor or coil side must be reversed when it passes through the magnetic neutral axis (MNA). A commutator is used to achieve this.

[How a DC motor works]

DC motor

Function of a Commutator

- The function of a commutator in DC motors is to reverse the direction of flow of current in each armature conductor when it passes through the M.N.A. to obtain continuous torque.

Back EMF

- Due to the armature conductors (coil) cutting across the magnetic field of the main poles, a back emf is induced. “Back” because **the direction of this emf is opposite that of the applied voltage** in the coils.
- The back EMF plays a crucial role in DC motor operation.
  - **Speed regulation:** The back EMF helps regulate the motor's speed by opposing the applied voltage.
  - **Current limitation:** The back EMF reduces the current flowing through the coil, preventing excessive currents and potential damage.
  - **Efficiency:** The back EMF contributes to the motor's efficiency, as it reduces energy losses due to excessive currents.

DC motor

- **Shaft torque:** This is the actual torque seen at the output of the motor after windage and frictional losses. Hence $T_{sh}\,\omega = P_e - P_{losses}$, where P_e is the electrical power, and P_{losses} is the power losses.
- **Brake horse power (BHP):** In the case of motors, the mechanical power (H.P.) available at the shaft is known as brake horsepower.

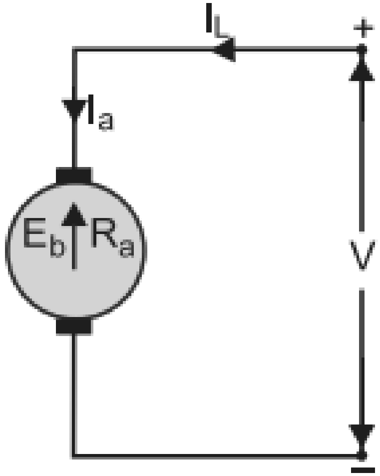
$$BHP = \frac{T_{sh}\,\omega}{745.7}$$

- Hence, BHP is shaft power (P_{sh}) converted to units of horse power.

DC motor

$$E_b = \frac{P\Phi NZ}{60\,A}$$

$E_b \propto \Phi N$ since $\frac{PZ}{60\,A}$ are constants



The supply voltage (V) is always greater than the induced or back emf (E_b) (i.e., $V > E_b$). Therefore, current is always supplied to the motor from the mains and the relation among the various quantities will be; $\mathbf{E_b = V - I_a R_a}$**.**

Hence the armature current is: $\boldsymbol{I_a = \frac{V - E_b}{R_a}}$

- When mechanical load applied on the motor increases, its speed decreases which reduces the value of E_b. As a result the value ($V - E_b$) increases which consequently increases I_a. Hence, motor draws extra current from the mains. Thus, the back emf regulates the input power as per the extra load.

DC motor

Work example

A 50 HP, 400 V, 4 pole, 1000 rpm, DC motor has flux per pole equal to 0.027 Wb. The armature having 1600 conductors is wave connected. Calculate the gross torque when the motor takes 70 ampere.

Solution

Torque developed, $\mathrm{T} = \dfrac{\boldsymbol{P\emptyset Z I_a}}{\boldsymbol{2\pi A}}$

Where, P = 4 poles; Ø = 0.027 Wb; I_a = 70A; A = 2 (wave connected)

$$\mathrm{T} = \frac{\mathbf{4 \times 0.027 \times 1600 \times 70}}{\mathbf{2\pi \times 2}} = \mathbf{963}\ \boldsymbol{Nm}$$

Assignment

- The induced emf in a DC machine is 200 V at a speed of 1200 rpm. Calculate the electromagnetic torque developed at an armature current of 15 A.
- A four-pole DC motor has a wave-wound armature with 594 conductors. The armature current is 40 A and flux per pole is 7.5 m Wb. Calculate H.P. of the motor when running at 1440 rpm.
- A DC motor has 6-poles with lap wound armature. What will be its brake horse power when it draws a current of 340 A and rotates at 400 rpm. The flux per pole is 0.05 Wb and the armature carries 864 turns, Neglect mechanical losses.