CENG 251: ELECTRIC CIRCUIT ANALYSIS AND DESIGN

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Content

- Basic Concepts
- Basic Laws
- Methods of Analysis
- Circuit Theorems
- Capacitors and Inductors
  - First – Order Circuits
  - Second – Order Circuits
  - Laplace Transforms
- Operational Amplifiers
- Passive and Active Filters

Assessment

- Continuous assessment – 40 marks
  - Weekly quizzes and assignments - 20 marks
  - Mid-semester examination – 20 marks

- End of semester examination – 60mks
  - Multiple choice questions – 20 marks
  - Written examination – 40 marks

Further study

- Fundamentals of Electric Circuits (5th Edition): Charles K. Alexander, Matthew N. O. Sadiku, 2013.

Click here for lecture materials

Basic Concepts

Basic Electrical Quantities

Quantity	Unit	Basic equation
Electric current (I): the time rate of change of charge.	amperes (A)	$i \triangleq \frac{dq}{dt}$ q – charge (C) t – time (s)
Voltage / potential difference (V): the energy required to move a unit charge through an element.	volts (V)	$v_{ab} \triangleq \frac{dw}{dq}$ w – energy (J) q – charge (C)
Power (P): the time rate of expending or absorbing energy.	watts (W)	$P = V.I$
Energy (E) the capacity to do work.	joules (J)	$w = \int_{t_0}^{t} p\, dt = \int_{t_0}^{t} vi\, dt$
Resistance (R): the physical property of an element or device that impedes the flow of current.	ohms (Ω)	$R = \frac{V}{I}$

Circuit Elements

- An element is the basic building block of a circuit
- **Circuit analysis** is the process of determining voltages across (or the currents through) the elements of the circuit.
- There are two types of elements, namely (1) passive element and (2) active element
- An active element generates energy, while a passive element does not generate energy.
- The most important active elements are voltage or current sources that generally deliver power to the circuit connected to them.
- There are two kinds of sources: independent and dependent sources.
- An **ideal independent source** provides a specified voltage or current that is completely independent of other circuit elements.
- An ideal dependent (or controlled) source is an active element in which the source quantity is controlled by another voltage or current.
- Dependent sources are usually designated by diamond-shaped symbols as shown in Fig. 1.

- There are four possible types of dependent sources, namely:
    1. A voltage-controlled voltage source (VCVS)
    2. A current-controlled voltage source (CCVS)
    3. A voltage-controlled current source (VCCS)
    4. A current-controlled current source (CCCS)

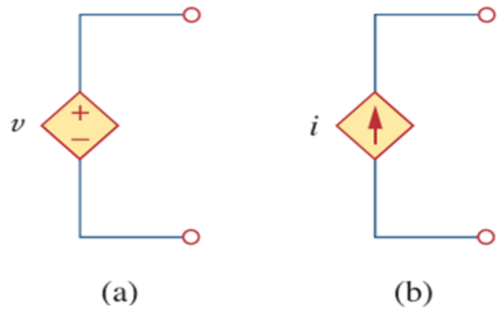
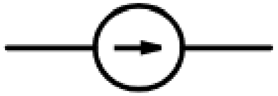
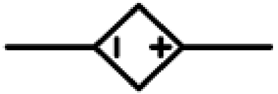


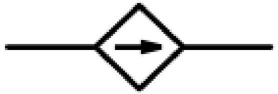
Figure 1
Symbols for: (a) dependent voltage source, (b) dependent current source.

Circuit Elements


Independent voltage source


Independent current source


Dependent voltage source

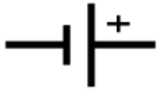

Dependent current source


Diode


Capacitor


Inductor


Resistor


DC voltage source


AC voltage source


Variable resistor

Basic Laws

Ohm's Law

- **Ohm's law** states that the voltage v across a resistor is directly proportional to the current i flowing through the resistor.

$$v \propto i$$
$$v = iR$$

- The resistance of any material with a uniform cross-sectional area A, depends on A and its length l, as shown in Fig. 2.

$$R = \rho \frac{\ell}{A}$$

- Where ρ is the resistivity of the material in ohm-meters.
- Conductance G is a measure of how well an element will conduct electricity, measured in mho (ohm spelled backwards).
- It's the reciprocal of R. $G = \frac{1}{R}$, unit ℧, SI unit is siemens (S).

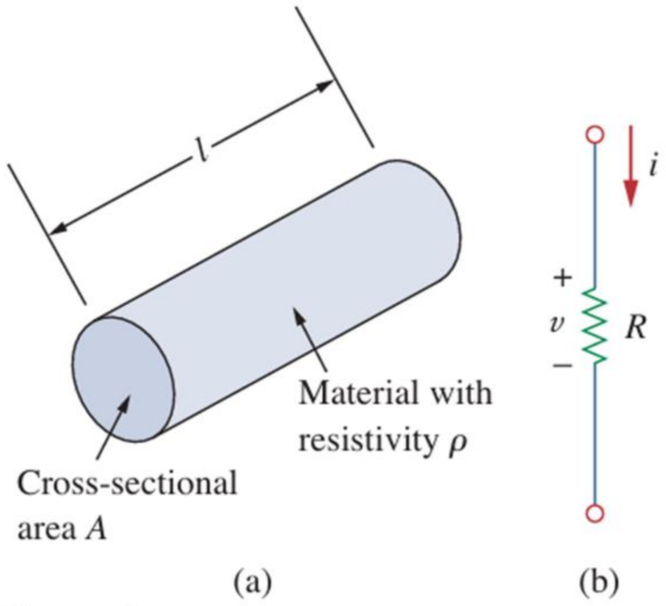
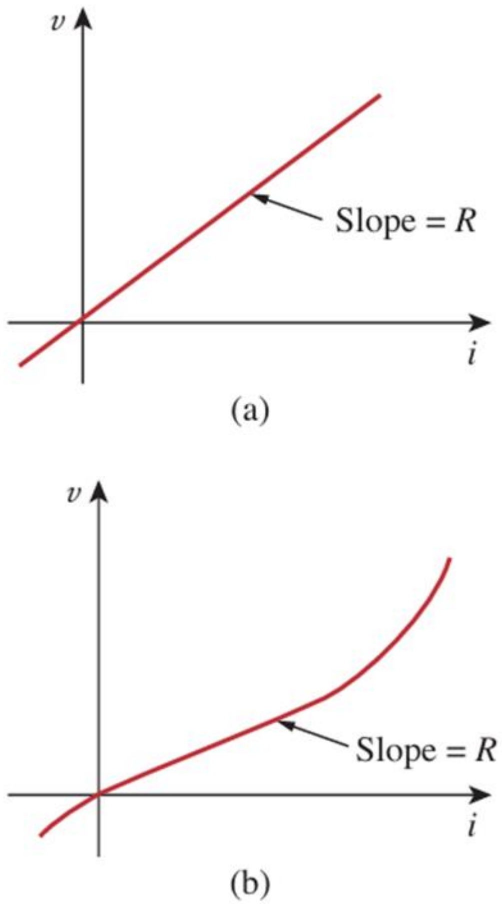


Figure 2

(a) Resistor, (b) Circuit symbol for resistance.

Ohm's Law

- It should be noted that not all resistors obey Ohm's law.
- A resistor that obeys Ohm's law is known as a linear resistor and has i-v characteristics as shown in Figure (a).
- A nonlinear resistor does not obey Ohm's law. Its resistance varies with current and its i-v characteristic is typically shown in Figure (b).



The i-v characteristic of: (a) a linear resistor, (b) a nonlinear resistor.

Branches, Nodes, and Loops

- A **branch** represents a single element.
- A **node** is the point of connection between two or more branches.
- A **loop** is any closed path in a circuit.
- Fig. 3 can be redrawn as Fig. 4 by combining the nodes connected by conducting wires.
- Two or more elements are in **series** if they exclusively share a single node and consequently carry the same current.
- Two or more elements are in **parallel** if they are connected to the same two nodes and consequently have the same voltage across them.

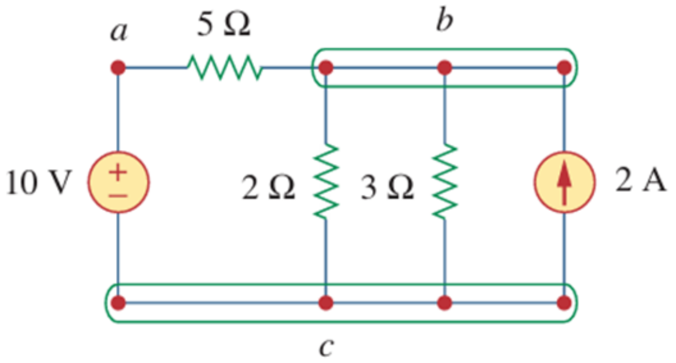


Figure 3
Nodes, branches, and loops.

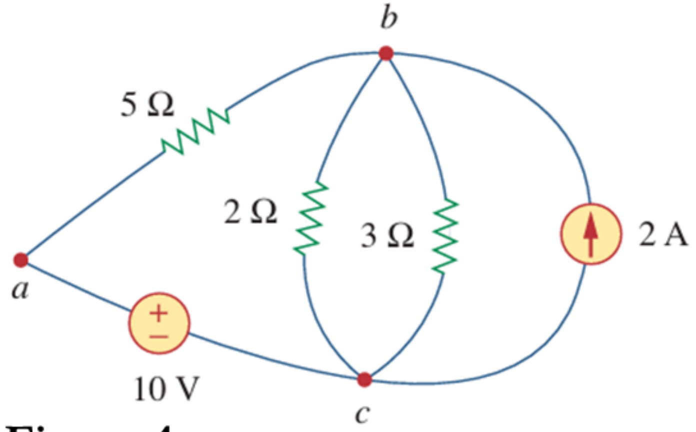
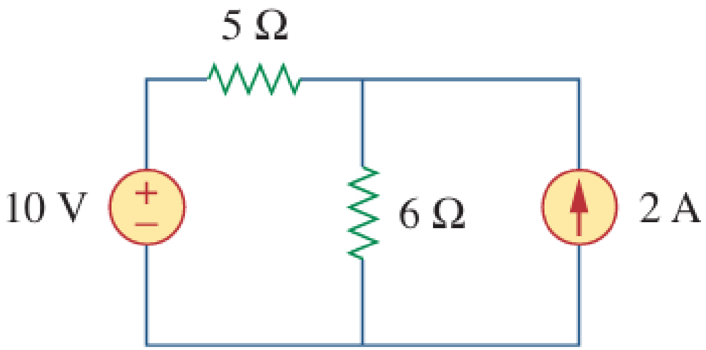


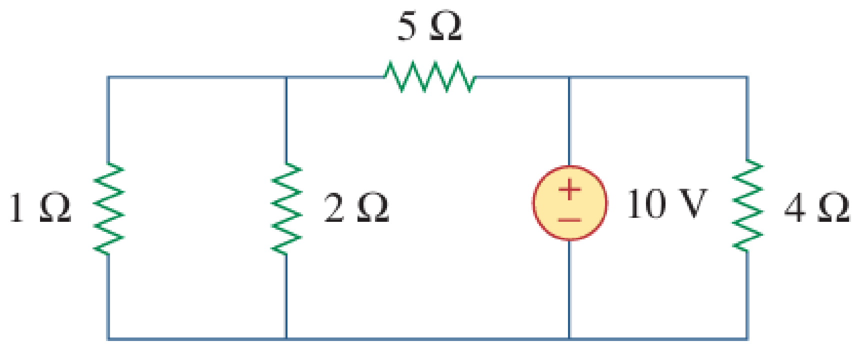
Figure 4
The three-node circuit of Fig. 2.10 is redrawn.

Problem

- Determine the number of branches and nodes in the circuits shown below. Identify which elements are in series and which are in parallel.



(a)



(b)

Kirchhoff's Laws

- Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.
- The sum of the currents entering a node is equal to the sum of the currents leaving the node.

$$i_1 + i_3 + i_4 = i_2 + i_5$$

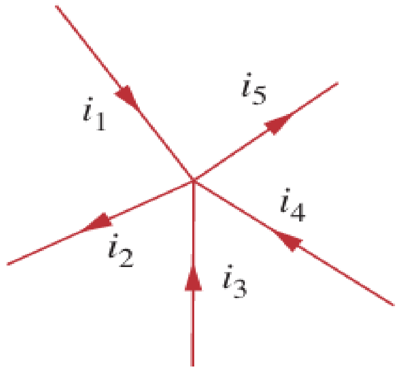


Figure 2.16
Currents at a node illustrating KCL.

- In Fig. 2.17, the total current entering the closed surface is equal to the total current leaving the surface.

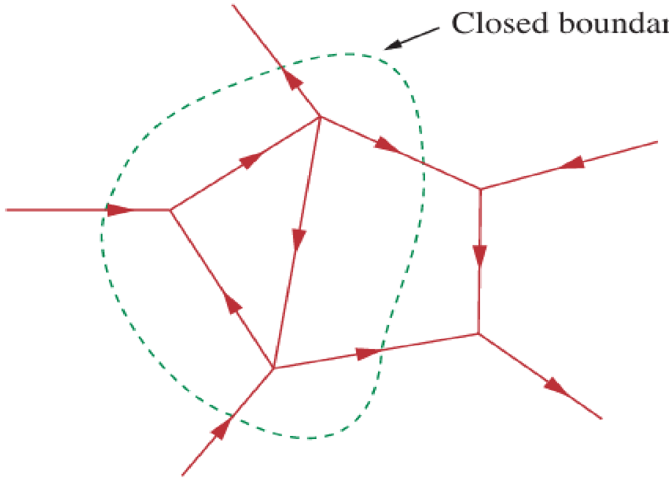


Figure 2.17
Applying KCL to a closed boundary.

- KCL can be applied to current sources in parallel such that

$$I_T = I_1 - I_2 + I_3$$

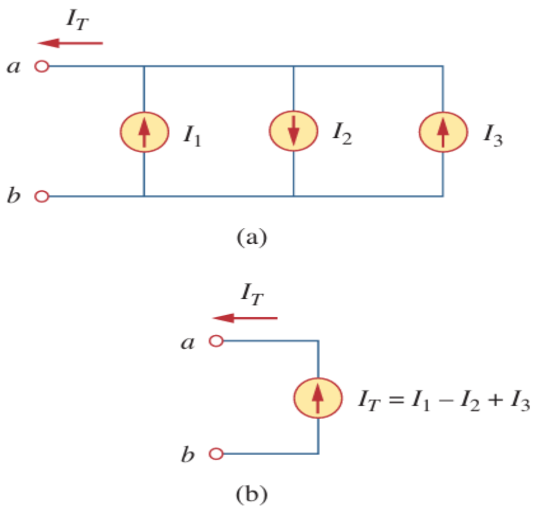


Figure 2.18
Current sources in parallel: (a) original circuit, (b) equivalent circuit.

Kirchhoff's Laws

- Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.
- Consider the circuit in Fig. 2.19. The sign on each voltage is the polarity of the terminal encountered first as we travel around the loop.

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$
$$v_1 + v_4 = v_2 + v_3 + v_5$$

- In other words,

$$Sum\ of\ voltage\ drops\ =\ Sum\ of\ voltage\ rises$$

- In Fig. 2.20, KVL can be applied to voltage sources in series such

$$V_{ab} = V_1 + V_2 - V_3$$

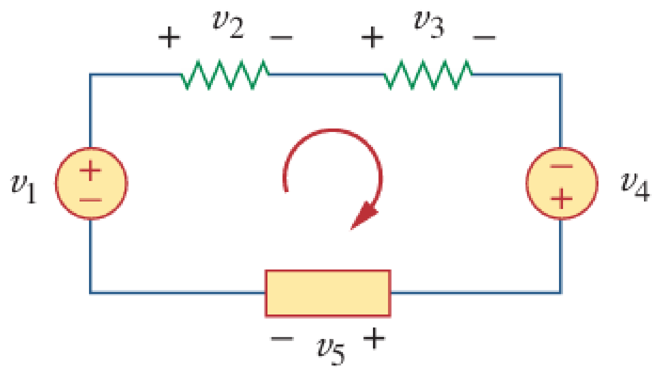


Figure 2.19
A single-loop circuit illustrating KVL.

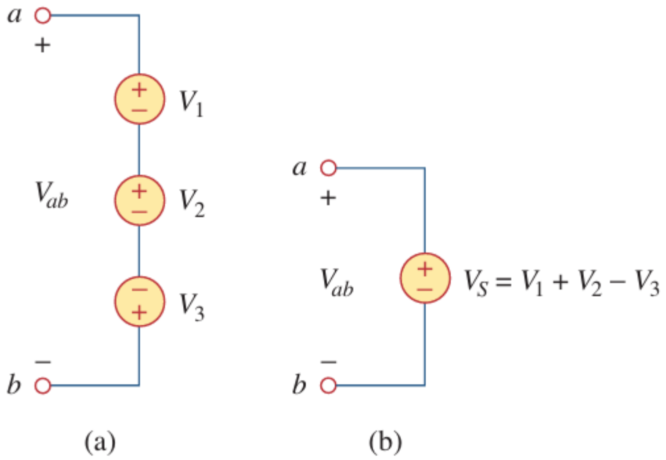
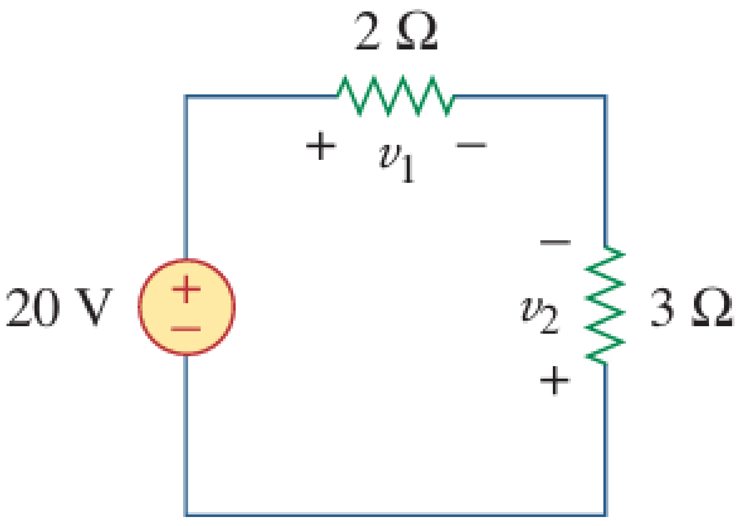


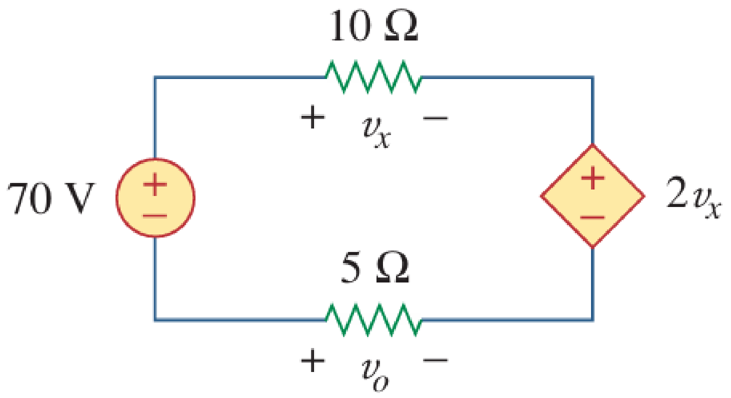
Figure 2.20
Voltage sources in series: (a) original circuit, (b) equivalent circuit.

Problems

For the circuits below, find voltage v_1 and v_2.

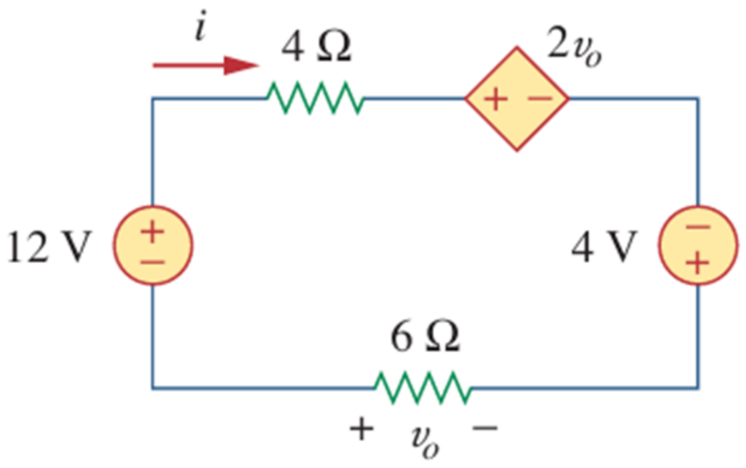


(a)



For the circuits below, find voltage v_x and v_o.

Determine v_o and i in the circuit below



(b)

Series Resistors and Voltage Division

- For two resistors in series as shown in Fig. 2.29

$$R_{eq} = R_1 + R_2$$

- The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.
- Per the voltage divider rule, the source voltage v is divided among the resistors in direct proportion to their resistances.

$$v_1 = \frac{R_1}{R_1 + R_2} v$$

$$v_2 = \frac{R_2}{R_1 + R_2} v$$

- In general, if a voltage divider has N resistors (R_1, R_2, R_3 . . . , R_N) in series with the source voltage v, the nth resistor (R_n) will have a voltage drop of

$$v_n = \frac{R_n}{R_1 + R_2 + R_3 \ldots + R_N} v$$

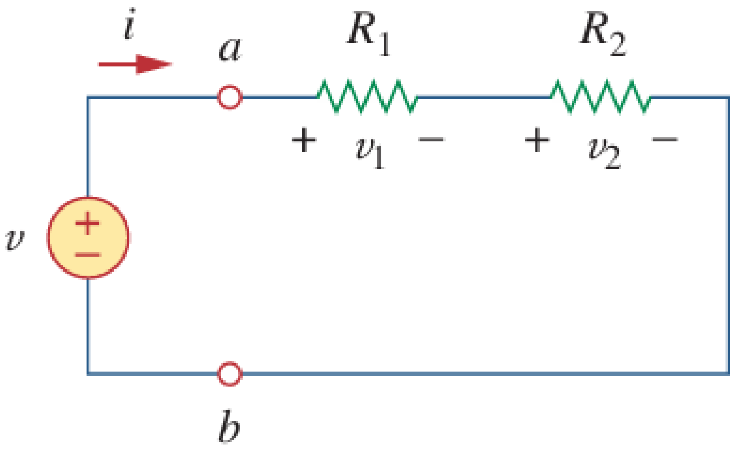


Figure 2.29
A single-loop circuit with two resistors in series.

Parallel Resistors and Current Division

- For two resistors in parallel as shown in Fig. 2.31.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

- In the general case of a circuit with N resistors in parallel. The equivalent resistance is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

- Per the current divider rule,

$$i_1 = \frac{R_2}{R_1 + R_2} i$$

$$i_2 = \frac{R_1}{R_1 + R_2} i$$

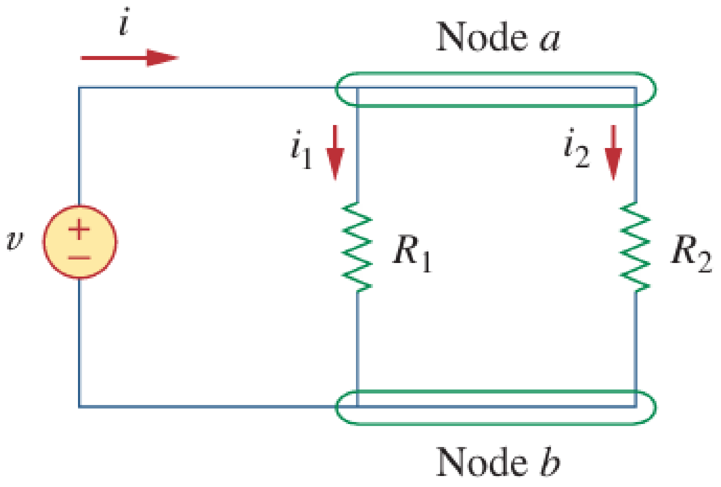
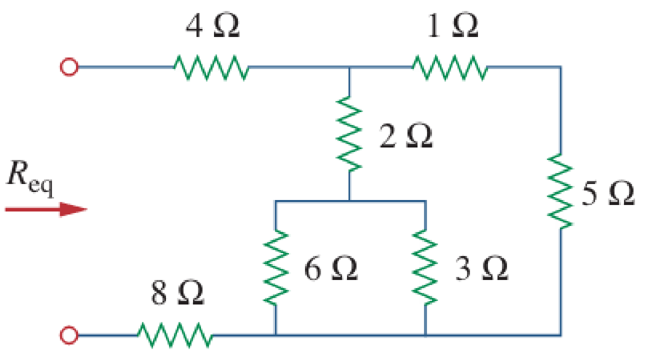
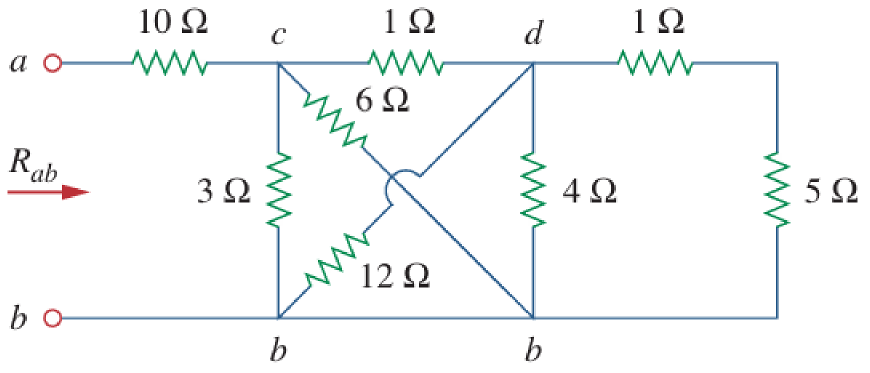


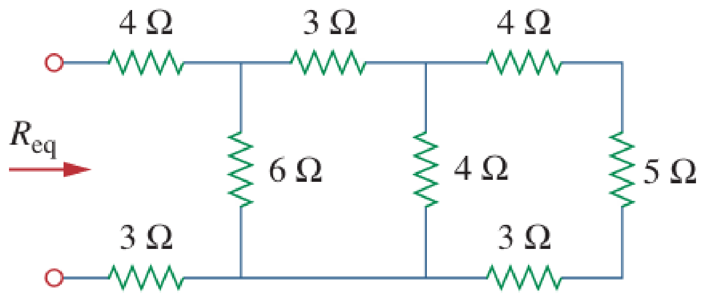
Figure 2.31
Two resistors in parallel.

Problems

- Find the equivalent resistances of the circuits shown below







Methods of Analysis

Nodal Analysis

- **<u>Nodal Analysis with only current sources</u>**

Steps to Determine Node Voltages:

1. Select a node as the reference node. Assign voltages v_1, $v_2, \ldots, v_{n-1}$ to the remaining $n-1$ nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the $n-1$ nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

- Current flows from a higher potential to a lower potential in a resistor.

$$i = \frac{v_{\text{higher}} - v_{\text{lower}}}{R}$$

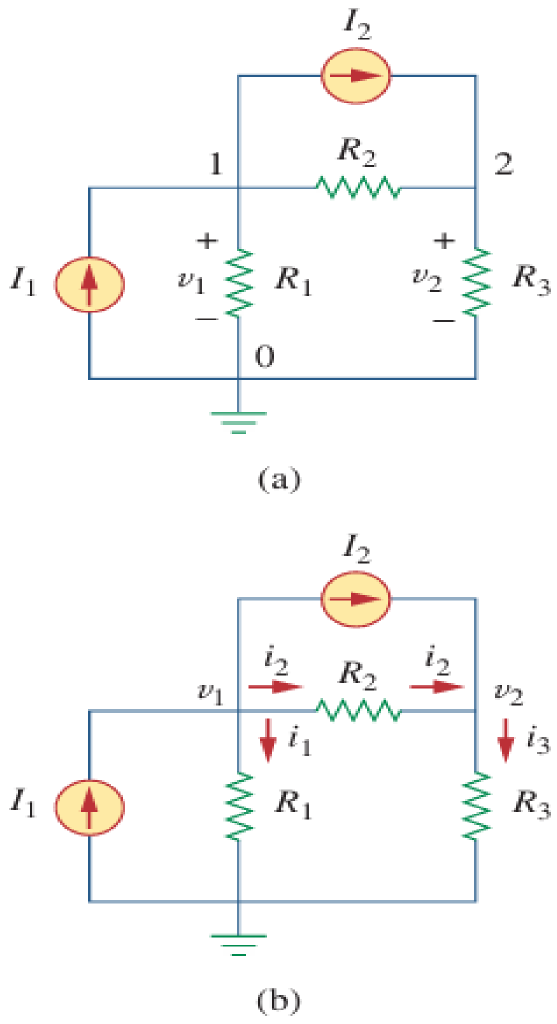


Figure 3.2
Typical circuit for nodal analysis.

At node 1,

$$I_1 = I_2 + i_1 + I_2$$

$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$$

At node 2,

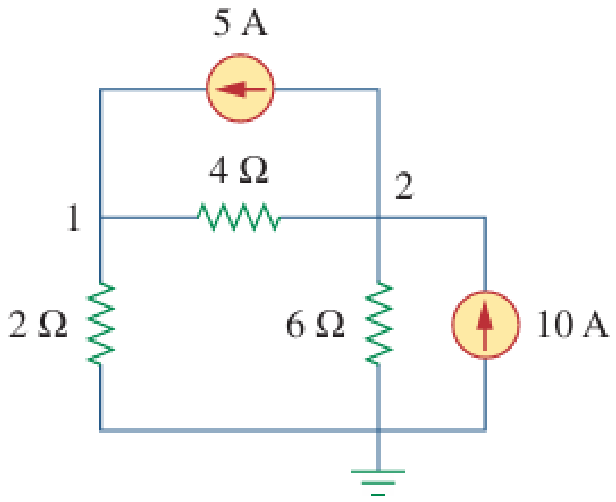
$$I_2 = i_3 + i_3$$

$$I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3}$$

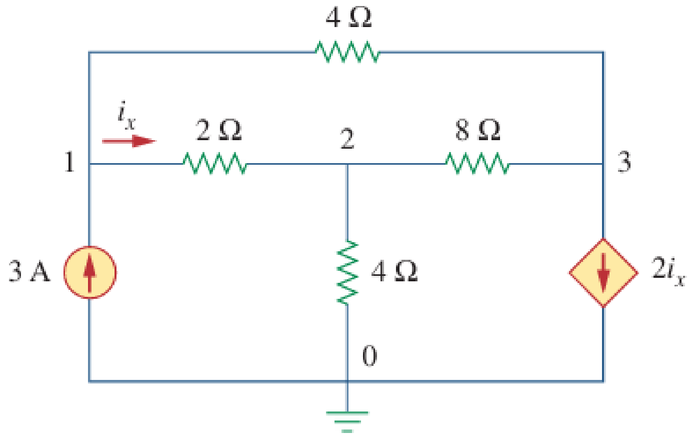
Nodal Analysis

- **Problems**

Calculate the node voltages in the circuits (a) and (b)



(a)



(b)

- **Assignment** – (Deadline: Thursday Jan 16, 2025, 23:59)

Determine the voltage at the three non-reference nodes