Name and Username (abc123): _____

Submit a plain text file (username_hw7.txt) to BBLearn with your solutions.

1. (20 points) Prove by induction that for every integer $n \in \mathbb{N}_{>0}$ it follows that $(\text{F } n) = \frac{n(n+1)(2n+1)}{6}$

```
( define  (F n)
   ( if  (= n  1)
         1
         (+  (* n n)  (F (− n  1)))))))
```

2. (20 points) Prove by induction that for every integer $n \in \mathbb{N}_{>0}$ it follows that $(\text{G } n) = 2^{n+1} - 2$

```
( define  (G n)
   ( if  (= n  1)
         2
         (+  (expt  2  n)  (G (− n  1)))))))
```

3. (20 points) Prove by induction that for every integer $n \in \mathbb{N}_{>0}$ it follows that $(\text{H } n) = \frac{n^2(n+1)^2}{4}$

```
( define  (H n)
   ( if  (= n  1)
         1
         (+  (expt  n  3)  (H (− n  1)))))))
```

4. (20 points) Prove by induction that for every integer $n \in \mathbb{N}_{>0}$ it follows that $(\text{K } n) = \frac{n(n+1)(2n+7)}{6}$

```
( define  (K n)
   ( if  (= n  1)
         3
         (+  (* n  (+ n  2))  (K (− n  1)))))))
```

5. (20 points) Prove by induction that for every integer $n \in \mathbb{N}_{>0}$ it follows that $(\text{M } n) \leq 2 - \frac{1}{n}$

   Note: If you want to experiment in Racket the following function will help. (exact−>inexact (/ 1 2)).

```
( define  (M n)
   ( if  (= n  1)
         1
         (+  (/  1  (* n n))  (M (− n  1)))))))
```