

# CS 270 - Lab 2

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Week 1

## 1 Introduction

For this lab, you will work in a team of 1-3 students.

**Assign Roles:**

Each member of your group must be assigned a role. No member of the group can have the same position twice in a row.

**Spokesperson:** talks to the instructor, TA, and other teams. Only the spokesperson may raise their hand to ask a question.

**Scribe:** Records the team's answers on the Activity Sheet.

**Developer:** Writes and executes code. Uses computers to calculate answers. The developer is the only group member that may write code during lab. (If in a group of 2, this job is taken by the spokesperson.)

Enter the name of the student in each role

Spokesperson: \_\_\_\_\_

Scribe: \_\_\_\_\_

Developer: \_\_\_\_\_

Score (out of 90): \_\_\_\_\_ Graded By: \_\_\_\_\_

Question 1 : 16 points

## 2 Functional Programming

We will start by reasoning about how the below function works.

```
; Computes x*y using addition  
; Assuming x >=0 and y >= 0  
(define (prod x y)  
  (if (= y 0)  
      0  
      (+ x (prod x (- y 1)))))  
)
```

Answer the following questions about the function.

- (a) (4 points) A **base case** is a condition of the function that **does not** make a recursive call.  
Describe all input values that cause the **base case** to be executed?
  
  
  
  
  
  
  
  
  
  
- (b) (4 points) A **recursive case** is a condition of the function that **does** make a recursive call.  
Describe all input values that cause the **recursive case** to be executed?
  
  
  
  
  
  
  
  
  
  
- (c) (4 points) A recursive function will **terminate** if it reaches the **base case**.  
Explain why executing (prod 5 2) will eventually reach the base case.
  
  
  
  
  
  
  
  
  
  
- (d) (4 points) Give an input for the prod function that will **never** terminate.  
Hint: Think about the program's assumptions. They are not enforced.

### 3 Equational Reasoning

When writing a proof justifying code works correctly, equational reasoning is a crucial skill.

**Equational Reasoning** is a processes of reasoning about code by evaluating and replacing values in code.

The below is an example of using equational reasoning to show that `(prod 4 2)` returns 8.

1.	<code>(= 8 (prod 4 2))</code>	Initial Problem
2.	<code>(= 8 (if (= 2 0) 0 (+ 4 (prod 4 (- 2 1)))))</code>	Replace $x = 4$ and $y = 2$ in def of <code>prod</code>
3.	<code>(= 8 (if #f 0 (+ 4 (prod 4 (- 2 1)))))</code>	Evaluate $2 = 0$ to False
4.	<code>(= 8 (+ 4 (prod 4 (- 2 1))))</code>	Select else because if is False
5.	<code>(= 8 (+ 4 (prod 4 1)))</code>	Evaluate $2 - 1 = 1$
6.	<code>(= 8 (+ 4 (if (= 1 0) 0 (+ 4 (prod 4 (- 1 1)))))</code>	Replace $x = 4$ and $y = 1$ in def of <code>prod</code>
7.	<code>(= 8 (+ 4 (if #f 0 (+ 4 (prod 4 (- 1 1)))))</code>	Evaluate $1 = 0$ to be False
8.	<code>(= 8 (+ 4 (+ 4 (prod 4 (- 1 1)))))</code>	Select else because if is False
9.	<code>(= 8 (+ 4 (+ 4 (prod 4 0))))</code>	Evaluate $1 - 1 = 0$
10.	<code>(= 8 (+ 4 (+ 4 (if (= 0 0) 0 ( ... )))))</code>	Replace $x = 4$ and $y = 0$ in def of <code>prod</code>
11.	<code>(= 8 (+ 4 (+ 4 (if #t 0 ( ... )))))</code>	...
12.	<code>(= 8 (+ 4 (+ 4 0)))</code>	...
13.	<code>(= 8 (+ 4 4))</code>	Evaluate $4 + 0 = 4$
14.	<code>(= 8 8)</code>	Evaluate $4 + 4 = 8$
15.	<code>#t</code>	Evaluate $8 = 8$ to True

Question 2 : 20 points

Answer the following questions about the proof.

- (a) (4 points) A **justification** is a written explanation of what happened on a line of the proof.

Which of the following is the justification on Line 5 telling you?

- ☐ Why Line 5 is a valid follow-up to Line 4  
☐ Why Line 6 is a valid follow-up to Line 5

- (b) (4 points) The equational reasoning on Line 10 has be shorthanded using an Ellipsis `...`.

What should be written to replace the `...` on Line 10 to fully write out the proof?

- (c) (4 points) No Justification has been provided for Line 11. What is the missing justification?

- (d) (4 points) Line 11's equational reasoning has been shorthanded.

What should be written to replace the `...` on Line 11 to fully write out the proof?

- (e) (4 points) No Justification has been provided for Line 12. What is the missing justification?

## 4 Equational Reasoning Proof

Question 3 : 24 points

Complete the following proof by filling in the blanks.

1.  $(= 0 (\text{prod } 0 \ 2))$
2.  $(= 0 (\text{if } (= 2 \ 0) \ 0 \ (+ \ 0 (\text{prod } 0 \ (- \ 2 \ 1)))))$
3.  $(= 0 (\text{if } \mathbf{A} \ ))$
4.  $(= 0 (+ \ 0 (\text{prod } 0 \ (- \ 2 \ 1))))$
5.  $(= 0 (+ \ 0 (\text{prod } 0 \ 1)))$
6.  $(= 0 (+ \ 0 (\text{if } (= 1 \ 0) \ 0 \ (+ \ 0 (\text{prod } 0 \ (- \ 1 \ 1)))))$
7.  $(= 0 (+ \ 0 (\text{if } \#f \ 0 \ (+ \ 0 (\text{prod } 0 \ (- \ 1 \ 1)))))$
8.  $(= 0 (+ \ 0 (+ \ 0 (\text{prod } 0 \ (- \ 1 \ 1))))$
9. **C**
10.  $(= 0 (+ \ 0 (+ \ 0 (\text{if } (= 0 \ 0) \ 0 \ (+ \ 0 (\text{prod } 0 \ (- \ 0 \ 1)))))$
11.  $(= 0 (+ \ 0 (+ \ 0 (\text{if } \#t \ 0 \ (+ \ 0 (\text{prod } 0 \ (- \ 0 \ 1)))))$
12.  $(= 0 (+ \ 0 (+ \ 0 \ 0)))$
13. **D**
14.  $(= 0 \ 0)$
15.  $\#t$

Initial Problem

Replace  $x = 0$  and  $y = 2$  in function def

Evaluate  $2 = 0$  to False

Take condition of if when False

Evaluate  $2 - 1 = 1$

**B**

Evaluate  $1 = 0$  to False

Take third input of if because False

Evaluate  $1 - 1 = 0$

Replace  $x = 0$  and  $y = 0$  in function def

Evaluate  $0 = 0$  to True

Take second input of if because True

**E**

Evaluate  $0 + 0 = 0$

Evaluate  $0 = 0$  to True

(a) (4 points) Fill in blank A

(b) (4 points) Fill in blank B

(c) (4 points) Fill in blank C

(d) (4 points) Fill in blank D

(e) (4 points) Fill in Blank E

(f) (4 points) Propose a solution to improve performance when multiplying by zero. You do not need to rewrite the function, just explain how you would change it.

## 5 Infinite Loop

Question 4 : 20 points

In Section 1, you determined the function had an infinite loop when given a negative number.

- (a) (10 points) Modify the function to work correctly with when the  $y$  input is negative.
- (b) (10 points) Use Equational Reasoning to prove  $(= -2 (\text{prod } 2 -1))$  with your new definition for  $\text{prod}$ . You may evaluate as many simple operations as possible per line. Make sure to distinguish between substitution and evaluation steps.

Question 5 : 5 points

Review the following function

```
(define (f x)
  (if (= x 1)
      x
      (+ x (f (- x 1)))))
```

Use Equational Reasoning to show that `(equal? 3 (f 2))`.

Question 6 : 5 points

**Group Discussion Question:** Some people describe Equational Reasoning as *algebra for code*. Do you agree or disagree? Explain why.