** = Apply Inductive Hypothesis

1.

$$\sum_{k=1}^{k} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Base Case:

$$\sum_{k=1}^{k} k^2 = \frac{1(1+1)(2(1)+1)}{6}$$

$$1^2 = \frac{2 \times 3}{6}$$

$$1 = 1$$

Inductive Proof:

$$\sum_{k=1}^{k} k^2 = \frac{(n+1)(n+1+1)(2(n+1)+1)}{6}$$

$$\sum_{k=1}^{k} k^2 + (n+1)^2 = \frac{(n+1)(n+2)(2n+3)}{6}$$

$$\frac{n(n+1)(2n+1)}{6} + (n+1)^2 = \frac{(n+1)(n+2)(2n+3)}{6} **$$

$$\frac{n(n+1)(2n+1) + 6(n+1)^2}{6} = \frac{(n+1)(n+2)(2n+3)}{6}$$

$$\frac{(n+1)[n(2n+1) + 6(n+1)]}{6} = \frac{(n+1)(n+2)(2n+3)}{6}$$

$$\frac{(n+1)(2n^2 + n + 6n + 6)}{6} = \frac{(n+1)(n+2)(2n+3)}{6}$$

$$\frac{(n+1)(n+2)(2n+3)}{6} = \frac{(n+1)(n+2)(2n+3)}{6}$$

2.

$$\sum_{k=1}^{n} 2^k = 2^{n+1} - 2$$

Base Case:

$$\sum_{k=1}^{1} 2^k = 2^{1+1} - 2$$
$$2^1 = 4 - 2$$
$$2 = 2$$

Inductive Proof:

$$\sum_{k=1}^{n+1} 2^k = 2^{(n+1)+1} - 2$$
$$\sum_{k=1}^{n} 2^k + 2^{n+1} = 2^{n+2} - 2$$
$$2^{n+1} - 2 + 2^{n+1} = 2^{n+2} - 2 \;**$$
$$2 \times 2^{n+1} - 2 = 2^{n+2} - 2$$
$$2 \times 2^{n} \times 2 - 2 = 2^{n+2} - 2$$
$$2^{n+2} - 2 = 2^{n+2} - 2$$

3.

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

Base Case:

$$\sum_{k=1}^{1} k^3 = \frac{1^2(1+1)^2}{4}$$

$$1^3 = \frac{4}{4}$$

$$1 = 1$$

Inductive Proof:

$$\sum_{k=1}^{n+1} k^3 = \frac{(n+1)^2(n+1+1)^2}{4}$$

$$\sum_{k=1}^{n+1} k^3 + (n+1)^2 = \frac{(n+1)^2(n+2)^2}{4}$$

$$\frac{n^2(n+1)^2 + 4(n+2)^3}{4} = \frac{(n+1)^2(n+2)^2}{4}$$

$$\frac{(n+1)^2(n+2)^2}{4} = \frac{(n+1)^2(n+2)^2}{4}$$

4.

$$\sum_{k=1}^{n} \mathrm{k}(\mathrm{k}+2) = \frac{n(n+1)(2n+7)}{6}$$

Base Case:

$$\sum_{k=1}^{1} \mathrm{k}(\mathrm{k}+2) = \frac{1(1+1)(2(1)+7)}{6}$$

$$1(1+2) = \frac{1(2)(9)}{6}$$

$$3 = 3$$

Inductive Proof:

$$\sum_{k=1}^{n+1} \mathrm{k}(\mathrm{k}+2) = \frac{(n+1)(n+1+1)(2(n+1)+7)}{6}$$

$$\sum_{k=1}^{n+1} \mathrm{k}(\mathrm{k}+2) + ((\mathrm{n}+1)(\mathrm{n}+1+2) = \frac{(n+1)(n+2)(2n+9)}{6}$$

$$\frac{n(n+1)(2n+7)}{6} + (\mathrm{n}+1)(\mathrm{n}+3) = \frac{(n+1)(n+2)(2n+9)}{6} **$$

$$\frac{n(n+1)(2n+7) + 6(\mathrm{n}+1)(\mathrm{n}+3)}{6} = \frac{(n+1)(n+2)(2n+9)}{6}$$

$$\frac{(n+1)[n(2n+7) + 6(n+3)]}{6} = \frac{(n+1)(n+2)(2n+9)}{6}$$

$$\frac{(n+1)[2n^2 + 7n + 6n + 18]}{6} = \frac{(n+1)(n+2)(2n+9)}{6}$$

$$\frac{(n+1)[2n^2 + 13n + 18]}{6} = \frac{(n+1)(n+2)(2n+9)}{6}$$

$$\frac{(n+1)[2n^2 + 9n + 4n + 18]}{6} = \frac{(n+1)(n+2)(2n+9)}{6}$$

$$\frac{(n+1)(2n+9)(n+2)}{6} = \frac{(n+1)(n+2)(2n+9)}{6}$$

5.

$$\sum_{k=1}^{n} \frac{1}{k^2} \leq 2 - \frac{1}{\mathrm{n}}$$

Base Case:

$$\sum_{k=1}^{1} \frac{1}{k^2} \leq 2 - \frac{1}{\mathrm{n}}$$

$$\frac{1}{1^2} \leq 2 - \frac{1}{1}$$

$$1 \leq 1$$

Inductive Proof:

$$\sum_{k=1}^{n+1} \frac{1}{k^2} \leq 2 - \frac{1}{\mathrm{n}+1}$$

$$\sum_{k=1}^{1} \frac{1}{k^2} + (\frac{1}{(n+1)^2}) \leq 2 - \frac{1}{\mathrm{n}+1}$$

$$2 - \frac{(\mathrm{n}+1)^2 + n}{n(n+1)^2} \leq 2 - \frac{1}{\mathrm{n}+1} \;**$$

$$2 - \frac{\mathrm{n}^2 + 2n + 1 + n}{n(n+1)^2} \leq 2 - \frac{1}{\mathrm{n}+1}$$

$$2 - \frac{\mathrm{n}^2 + 1 + 2n + 1}{n(n+1)^2} \leq 2 - \frac{1}{\mathrm{n}+1}$$

$$2 - \frac{\mathrm{n}(\mathrm{n}+1)}{n(n+1)^2} - \frac{2\mathrm{n}+1}{n(n+1)^2} \leq 2 - \frac{1}{\mathrm{n}+1}$$

$$2 - \frac{1}{(n+1)} - \frac{2\mathrm{n}+1}{n(n+1)^2} \leq 2 - \frac{1}{\mathrm{n}+1}$$