

## Quiz 7 - MACM 201 - *Solutions*

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[4 pts] Express the generating function associated with the following sequence in rational form.

$$0, 0, 3, -3, 3, -3, 3, -3, \dots$$

*Solution:* The associated generating function is

$$(3x^2 - 3x^3 + 3x^4 - 3x^5 + \dots) = 3x^2(1 - x + x^2 - x^3 + \dots) = 3x^2 \frac{1}{1 - (-x)} = \frac{3x^2}{1 + x}$$

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[4 pts] Find the coefficient of  $x^n$  in the following GF.

$$\frac{5x^3}{1 - 2x}$$

Answer in the form  $[x^n] \frac{5x^3}{1 - 2x} = \underline{\hspace{2cm}}$

*Solution:* We have

$$\frac{5x^3}{1 - 2x} = 5x^3 \frac{1}{1 - 2x} = 5x^3 \sum_{m=0}^{\infty} (2x)^m = \sum_{m=0}^{\infty} 5 \cdot 2^m x^{m+3}$$

Now setting  $n = m + 3$  (so  $m = n - 3$ ) we obtain the equivalent expression

$$\sum_{m=0}^{\infty} 5 \cdot 2^m x^{m+3} = \sum_{n=3}^{\infty} 5 \cdot 2^{n-3} x^n$$

and we conclude

$$[x^n] \frac{5x^3}{1 - 2x} = \begin{cases} 0 & \text{if } n \leq 2 \\ 5 \cdot 2^{n-3} & \text{if } n \geq 3 \end{cases}$$