MACM 201 Homework 10 (Quiz Nov. 27)

Textbook problems:

Section	Question
12.1	2
12.1	8
12.1	12

Instructor quesions:

1. Prove that a tree $T = (V, E)$ with $|V| \geq 2$ has exactly two leaf vertices if and only if it is a path.

2. Prove that a connected graph $G = (V, E)$ with $|V| \geq 3$ has at least three vertices v so that $G - v$ is connected unless G is a path. Hint: consider a spanning tree.

3. Prove by induction that for every tree $T = (V, E)$ there is a function $f : E \to \{-1, 1\}$ with the property that for every vertex $v \in V$ with incident edges $e_1, e_2, \ldots, e_k$ we have $|f(e_1) + f(e_2) + \ldots + f(e_k)| \leq 1$

Orienting a graph. Let $G = (V, E)$ be a graph (not a multigraph). To *orient* G, we turn this graph into a directed graph by replacing each edge $\{u, v\}$ with the directed edge (u, v) or (v, u) (i.e. we orient this edge so that it goes from u to v or vice-versa).

Degree in directed graphs. If $D = (V, E)$ is a directed graph and $v \in V$, the *outdegree* of v, denoted $\deg^+(v)$ is the number of edges in D that are incident with v but directed away from it. Similarly the *indegree* of v, denoted $\deg^-(v)$ is the number of edges incident with v but directed toward it.

4. Prove by induction that every tree $T = (V, E)$ has an orientation with the property that every vertex $v \in V$ satisfies $|\deg^+(v) - \deg^-(v)| \leq 1$

5. Prove that every graph $G = (V, E)$ has an orientation with the property that every vertex $v \in V$ satisfies $|\deg^+(v) - \deg^-(v)| \leq 1$. Hint: use induction on $|E|$ and the previous problem. What can you do when G has a cycle C?