MACM 316 – Homework 6

- Problems are not to be submitted. The related quiz will be given in class.
- Feel free to use Canvas discussions but please keep in mind that these forums are open.

Barycentric form

1. Recall that the barycentric form of the interpolating polynomial is

$$P(x) = \frac{\sum_{j=0}^{n} \frac{f(x_j)w_j}{x-x_j}}{\sum_{j=0}^{n} \frac{w_j}{x-x_j}},$$

where the weights w_j are defined by

$$w_j := \left(\prod_{\substack{k=0 \\ k \neq j}}^{n} (x_j - x_k) \right)^{-1}.$$

   a) Suppose that $n = 2$ and $x_0 = 0$, $x_1 = 1$ and $x_2 = 2$. Show that

$$w_0 = \frac{1}{2}, \quad w_1 = -1, \quad w_2 = \frac{1}{2}.$$

   b) Suppose that $n = 3$ and $x_0 = 0$, $x_1 = 1$, $x_2 = 2$ and $x_3 = 3$. Show that

$$w_0 = -\frac{1}{6}, \quad w_1 = \frac{1}{2}, \quad w_2 = -\frac{1}{2}, \quad w_3 = \frac{1}{6}.$$

   c) For any n, let $x_j = j, \quad j = 0, \ldots, n$. Show that

$$w_j = \frac{(-1)^{n-j}}{j!(n-j)!}, \quad j = 0, \ldots, n.$$

   Use this formula to check your answers to a) and b).

2. In class, the Chebyshev nodes were given as

$$w_0 = \frac{1}{2}, \quad w_n = \frac{(-1)^n}{2}, \quad w_j = (-1)^j, \; j = 1, \ldots, n-1.$$

   a) Show that the definition of the weights as in question 1 gives the following

$$w_0 = \frac{2}{3}, \quad w_1 = -\frac{4}{3}, \quad w_2 = \frac{4}{3}, \quad w_3 = -\frac{2}{3}.$$

   b) Why does this not contradict the statement from class?

3. Let $x_0 = 0$, $x_1 = 1$, $x_2 = 2$ and $x_3 = 3$ abd let $P(x)$ be the interpolating polynomial of some function $f(x)$ with the following barycentric form

$$P(x) = \frac{\frac{-1}{x} + \frac{3}{x-1} + \frac{-3}{x-2} + \frac{1}{x-3}}{\frac{-1/6}{x} + \frac{1/2}{x-1} + \frac{-1/2}{x-2} + \frac{1/6}{x-3}}.$$

   a) Determine $f(0)$, $f(1)$, $f(2)$ and $f(3)$.
   b) Rewrite P in monomial form.

A. Cubic splines

Section 3.5, Ex 3

Section 3.5, Ex 5

Section 3.5, Ex 7

Section 3.5, Ex 11

Section 3.5, Ex 14

B. Least squares

Section 8.1, Ex 3

Section 8.1, Ex 5

Section 8.1, Ex 7