

MACM 201 Homework 9 (Quiz Nov. 20)

Textbook problems:

Section	Question
11.3	2
11.3	16
11.3	18
11.3	30

Instructor questions:

1. Let $G = (V, E)$ be a graph with n vertices and $n - 1$ edges. Prove that G either has a vertex of degree 1 or a vertex of degree 0.
2. Let G be a connected graph with exactly two vertices of odd degree, say u and v . Prove that there is a walk from u to v . Hint: construct a new graph G^+ from G by adding an edge incident to u and v .
3. Let $T = (V, E)$ be a rooted tree with the property that every internal vertex has exactly 2 children. If there are k internal vertices, prove that $|V| = 2k + 1$. Hint: induction.
4. Let $T = (V, E)$ be a rooted tree with the property that every internal vertex has an even number of children. Prove that there exists a function $f : V \rightarrow \{-1, 1\}$ with the property that for every internal vertex v with children v_1, v_2, \dots, v_{2k} the sum $f(v_1) + f(v_2) + \dots + f(v_{2k}) = 0$. Hint: induction.