MATH 308 D200, Fall 2019

11. Cycling and Anticycling rules

(based on notes from Dr. J. Hales, Dr. L. Stacho, and Dr. L. Godyyn)

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Relationship between slack variables and distance of basic solution form corresponding constraint

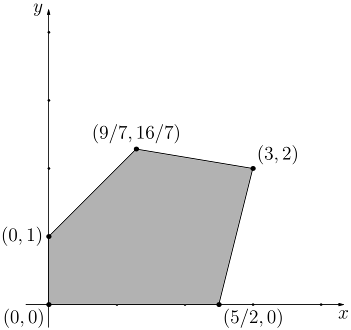
Lemma

The ratio b_i/a_{ij}, for $a_{ij} > 0$ represents the distance of the current BS from constraint i.

t_3	y	-1	
1/4	3/4	7/2	$= -t_1$
-1/4	25/4*	25/2	$= -t_2$
1/4	-1/4	5/2	$= -x$
-1/4	5/4	-5/2	$= f$

$\longrightarrow$

t_3	t_2	-1	
7/25	-3/25	2	$= -t_1$
-1/25	4/25	2	$= -y$
6/25	1/25	3	$= -x$
-1/5	-1/5	-5	$= f$



Cycling Phenomenon

Apply SA for MBFT to the following MBFT tableau:

x_1	x_2	x_3	x_4	-1	
$1/4^*$	-8	-1	9	0	$= -t_1$
1/2	-12	-1/2	3	0	$= -t_2$
0	0	1	0	1	$= -t_3$
3/4	-20	1/2	-6	0	$= f$

t_1	x_2	x_3	x_4	-1	
4	-32	-4	36	0	$= -x_1$
-2	4^*	3/2	-15	0	$= -t_2$
0	0	1	0	1	$= -t_3$
-3	4	7/2	-33	0	$= f$

t_1	t_2	x_3	x_4	-1	
-12	8	8^*	-84	0	$= -x_1$
-1/2	1/4	3/8	-15/4	0	$= -x_2$
0	0	1	0	1	$= -t_3$
-1	-1	2	-18	0	$= f$

t_1	t_2	x_1	x_4	-1	
-3/2	1	1/8	-21/2	0	$= -x_3$
1/16	-1/8	-3/64	$3/16^*$	0	$= -x_2$
3/2	-1	-1/8	21/2	1	$= -t_3$
2	-3	-1/4	3	0	$= f$

t_1	t_2	x_1	x_2	-1	
2^*	-6	-5/2	56	0	$= -x_3$
1/3	-2/3	-1/4	16/3	0	$= -x_4$
-2	6	5/2	-56	1	$= -t_3$
1	-1	1/2	-16	0	$= f$

x_3	t_2	x_1	x_2	-1	
1/2	-3	-5/4	28	0	$= -t_1$
-1/6	$1/3^*$	1/6	-4	0	$= -x_4$
1	0	0	0	1	$= -t_3$
-1/2	2	7/4	-44	0	$= f$

x_3	x_4	x_1	x_2	-1	
-1	9	1/4	-28	0	$= -t_1$
-1/2	3	1/2	-12	0	$= -t_2$
1	0	0	0	1	$= -t_3$
1/2	-6	3/4	-20	0	$= f$

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x_1	x_2	x_3	x_4	-1	
1/4	-8	-1	9	0	$= -t_1$
1/2	-12	-1/2	3	0	$= -t_2$
0	0	1	0	1	$= -t_3$
3/4	-20	1/2	-6	0	$= f$

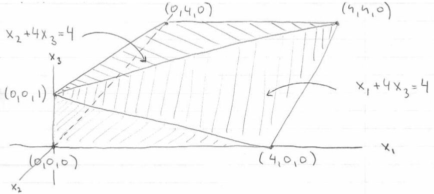
- Notice how these tableaux differ from tableaux we saw before.
- The value of the objective function has not improved.
- The value of slack variable represents a (non Euclidean) distance of the current basic solution from the corresponding hyperplane.

Example

Maximize: $f(x_1, x_2, x_3) = x_1 + x_2 + 4x_3$

subject to:

$x_1 + 4x_3 \leq 4 \rightarrow t_1$
$x_2 + 4x_3 \leq 4 \rightarrow t_2$
$x_1, x_2, x_3 \geq 0$



Cycling Phenomenon - terminology

Definition (Degenerate Basic Solution)

Basic solution with one or more dependent variables equal to zero is called degenerate BS.

Definition (Degenerate Simplex Iteration)

Simplex iterations that do not change the basic solution are called degenerate pivots.

Definition (Cycling of the Simplex Algorithm)

We say that the simplex algorithm cycles if the same tableau appears in two different iterations.

In practice cycling is a very rare phenomenon. In fact, constructing an LP problem on which the SA may cycle is difficult.

Theorem

If the SA fails to terminate, then it must cycle.

Proof.

□

Simplex Algorithm Anticycling Rules

Order all variables, both independent and dependent; the ordering is not important but it must not change during the algorithm. Any pivot entry is uniquely determined by a pivot row and a pivot column.

Rule #1 (Determination of pivot row). Whenever there is more than one possible choice of pivot row in accordance to the SA, choose the row corresponding to the variable that appears nearest the beginning of the list.

Rule #2 (Determination of pivot column). Whenever there is more than one possible choice of pivot column in accordance to the SA, choose the column corresponding to the variable that appears nearest the beginning of the list.

Theorem (Bland, 1977)

The simplex algorithm terminates if the pivot entries are determined by the anticycling rules above.

Example

Apply the anticycling rules to the LP problem:
Order of variables: x_1, x_2, x_3, x_4, t_1, t_2, t_3

t_1	t_2	x_1	x_2	-1	
2	-6	-5/2	56	0	$-x_3$
1/3	-2/3	-1/4	16/3	0	$-x_4$
-2	6	5/2	-56	1	$-t_3$
1	-1	1/2	-16	0	f

t_1	t_2	t_3	x_2	-1	
0	0	1	0	1	$-x_3$
2/15	-1/15	1/10	-4/15	1/10	$-x_4$
-4/5	12/5	2/5	-112/5	2/5	$-x_1$
7/5	-11/5	-1/5	-24/5	-1/5	f

x_4	t_2	t_3	x_2	-1	
0	0	2/15	0	2/15	$-x_3$
15/2	-1/2	3/4	-2	3/4	$-t_1$
6	2	1	-24	1	$-x_1$
-21/2	-3/2	-5/4	-2	-5/4	f