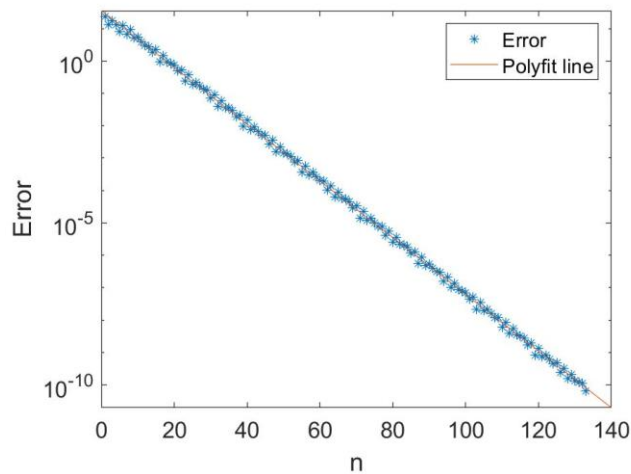


Computing Assignment 5 MACM 316

(a)



(b)

For $(a, b) = (0.2, 0.2)$ the fitting is $-0.0875n + 1.5508$, $\log(\text{error}) = -0.0875n + 1.5508$,

We can find the value of ρ by following steps: $\text{error}(n) = c \cdot \rho^{-n}$ gives

$\log(\text{error}) = \log(c) - n \cdot \log(\rho)$, we have $\log(\rho) = 0.0875$ therefore $\rho = 10^{0.0875}$

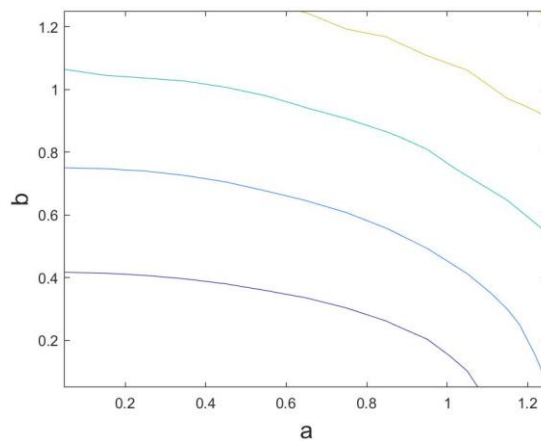
$\text{error} = 10^{(-0.0875n)} \cdot 10^{1.5508} = 10^{1.5508} \cdot (10^{(0.0875)})^{-n}$, c is $10^{1.5508}$, ρ is $10^{0.0875}$.

when $(a, b) = (0.5, 0.5)$, ρ is $10^{0.2292}$, when $(a, b) = (0.7, 1)$, ρ is $10^{0.4218}$.

(c)

When tolerance is fixed, we can easily identify the fact that the fewer steps the algorithm takes to reach tolerance, the faster the convergence rate.

(d)



(e)

When a -value and b -value go up, ρ -value increases. The shape of the level curves generated is like a part of a circle. When ρ goes up the circle gets larger in proportional to the growth rate of ρ value.

Code

```
err_grid = (linspace(-1,1,10000))';
tol = 1e-10;

a = 0.7; b = 1.0;
f = @(x) 1./((x-a).^2+b.^2);

f_grid = f(err_grid);
errvals = [];
err = 1; % set the error initially to 1
n = 0; % initialize n

while err > tol
    n = n+1;

    x = (linspace(0,1,n+1))';
    x = cos(pi*x);
    w = (-1).^((0:n)');
    w(1) = 1/2; w(n+1) = w(n+1)/2;

    y = f(x);
    p = baryinterp(x,w,y,err_grid);
    err = max(abs(p - f_grid));
    errvals = [errvals err];

end

figure(1);
semilogy(1:n,errvals,'*');
set(gca,'FontSize',14);
xlabel('n','fontsize',16);
ylabel('Error','fontsize',16);

p = polyfit(1:n, log10(errvals), 1);
xx = linspace(0,140,1000);
yy = polyval(p, xx);

hold on;
plot(xx,10.^(yy) )
legend(["Error", "Polyfit line"])

err_grid = (linspace(-1,1,10000))';
tol = 1e-10;
a_range = 0.05:0.1:1.25;
b_range = 0.05:0.1:1.25;
[A,B] = meshgrid(a_range,b_range);
```

```
rho = zeros(size(A));

i=1;
for b=b_range
    j=1;
    for a=a_range
        f = @(x) 1./((x-a).^2+b.^2); % Define f(x)
        f_grid = f(err_grid);
        errvals = [];
        err = 1; % set the error initially to 1
        n = 0; % initialize n
        while err > tol
            n = n+1;
            x = (linspace(0,1,n+1))';
            x = cos(pi*x);
            w = (-1).^((0:n)');
            w(1) = 1/2; w(n+1) = w(n+1)/2;
            y = f(x);
            p = baryinterp(x,w,y,err_grid);
            err = max(abs(p - f_grid));
            errvals = [errvals err];
        end
        p = polyfit(1:n, log10(errvals), 1);
        rho(i,j) = 10^(-p(1));
        j = j + 1;
        s = sprintf("%d,%d",i,j);
        disp(s);
    end
    i = i + 1;
end
contour(A,B, rho);
xlabel('a','fontsize',14);
ylabel('b','fontsize',14);
```