Quiz 4 - MACM 201 - *Solutions*

[4 pts]   State the quadratic formula for solving

$$ar^2 + br + c = 0$$

Solution:

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

[4 pts] For the recurrence

$$x_n - 9x_{n-1} + 20x_{n-2} = 0$$

(a) Find the characteristic equation.

(b) Find all real numbers r so that $x_n = r^n$ is a solution to the recurrence.

(c) Find infinitely many solutions.

Solution:

(a) $r^2 - 9r + 20 = 0$

(b) To do this, we find all solutions to the characteristic equation. We have

$$0 = r^2 - 9r + 20 = (r - 5)(r - 4)$$

So $r = 4$ and $r = 5$ are the two real numbers for which $x_n = r^n$ is a solution to our recurrence.

(c) More generally, for any real numbers C, D, the sequence $x_n = C4^n + D5^n$ is a solution.