Quiz 2 - MACM 201 - *Solutions*

[4 pts] Consider the word SUCCESS

(a) In how many ways can the letters from this word be arranged?

(b) For the arrangements in part (a), how many have all three S's together?

Solution:

(a) The word success has 3 S's, 2 C's, 1 U, and 1 E for a total of 7 letters. The number of ways to rearrange these letters is

$$\binom{7}{3,2,1,1} = \frac{7!}{3!2!}$$

(b) If all three S's are together, then we may represent SSS by the letter X. Now have a word with 1 X, 2 C's, 1 U, and 1 E so the total number of words of this type is

$$\binom{5}{1,2,1,1} = \frac{5!}{2!}$$

[4 pts] Consider integer solutions to $x_1 + x_2 + x_3 \leq 15$

(a) How many satisfy $x_1, x_2, x_3 \geq 0$?

(b) How many satisfy $x_1, x_2, x_3 \geq 3$?

Solution:

(a) Define $x_4 = 15 - (x_1 + x_2 + x_3)$. Now $x_1, \ldots, x_4$ are nonnegative variables satisfying the equation

$$x_1 + x_2 + x_3 + x_4 = 15.$$

In particular, this means that (x_1, x_2, x_3, x_4) is a sequence of nonnegative integers with sum 15. This gives us a correspondence between the nonnegative integer solutions to the original equation and nonnegative integer sequences with length 4 and sum 15. We deduce that the number of solutions to the original equation is $|SNI_{4,15}| = \binom{15+3}{3} = \binom{18}{3}$.

(b) Define $y_i = x_i - 3$ for $1 \leq i \leq 3$ and note that $y_1, y_2, y_3 \geq 0$ and $y_1 + y_2 + y_3 = 6$. Now setting $y_4 = 6 - (y_1 + y_2 + y_3)$ we have that (y_1, y_2, y_3, y_4) is a sequence of nonnegative integers with sum 6. We deduce that the number of solutions is $|SNI_{4,6}| = \binom{6+3}{3} = \binom{9}{3}$.