Neural Networks

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Bishop PRML Ch. 5

Neural Networks

- Neural networks arise from attempts to model human/animal brains
  - Many models, many claims of biological plausibility
- We will focus on multi-layer perceptrons
  - Mathematical properties rather than plausibility

Applications of Neural Networks

- Many success stories for neural networks, old and new
  - Credit card fraud detection
  - Hand-written digit recognition
  - Face detection
  - Autonomous driving (CMU ALVINN)
  - Object recognition
  - Speech recognition

Outline

Outline

Feed-forward Networks

Network Training

Error Backpropagation

Deep Learning

Feed-forward Networks

- We have looked at generalized linear models of the form:

$$y(\boldsymbol{x}, \boldsymbol{w}) = f\left(\sum_{j=1}^{M} w_j \phi_j(\boldsymbol{x})\right)$$

  for fixed non-linear basis functions $\phi(\cdot)$
  - We now extend this model by allowing adaptive basis functions, and learning their parameters
- In feed-forward networks (a.k.a. multi-layer perceptrons) we let each basis function be another non-linear function of linear combination of the inputs:

$$\phi_j(x) = f\left(\sum_{j=1}^{M} \cdots\right)$$

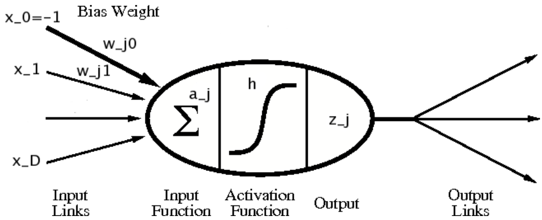
Feed-forward Networks

- Starting with input $\boldsymbol{x} = (x_1, \ldots, x_D)$, construct linear combinations:

$$a_j = \sum_{i=1}^{D} \left(w_{ji}^{(1)} x_i + x_{j0}^{(1)} \right)$$

These a_j are known as activations

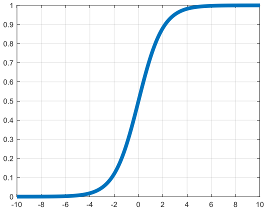
- Pass through an activation function $h(\cdot)$ to get output $z_j = h(a_j)$
    - Model of an individual neuron



from Russell and Norvig, AIMA2e

Activation Functions

- Can use a variety of activation functions
  - Sigmoidal (S-shaped)
    - Logistic sigmoid $1/(1+\exp(-a))$ (useful for binary classification)
    - Hyperbolic tangent $\tanh(\cdot)$
  - Radial basis function $z_j = \sum_i (x_i - w_{ji})^2$
  - Softmax
    - Useful for multi-class classification
  - Identity
    - Useful for regression
  - Threshold
  - ...
- Needs to be differentiable for gradient-based learning (later)
- Can use different activation functions in each unit

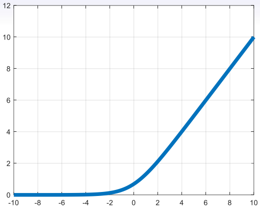


Activation Functions

Common choices of activation functions

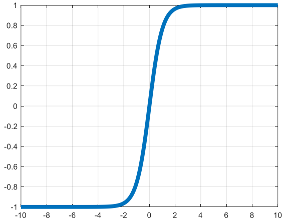
Softplus:

$$\log(1 + e^x)$$



Hyperbolic tangent:

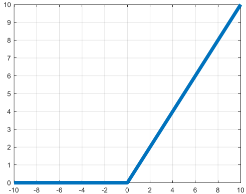
$$\tanh x$$



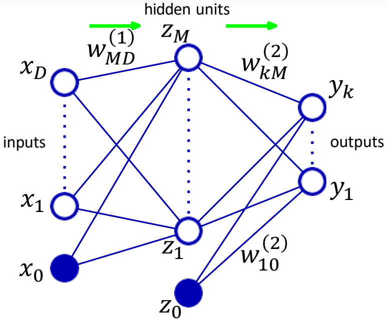
Rectified linear unit (ReLU):

$$\max(0, x)$$

Key feature: easy to differentiate



Feed-forward Networks



- Connect together a number of these units into a feed-forward network (DAG)
- Above shows a network with one layer of hidden units
- Implements function:

$$y_k(x, w) = h\left(\sum_{j=1}^{M} w^{(2)}_{kj} h\left(\sum_{i=1}^{D} w^{(1)}_{ij} x_i + w^{(1)}_{j0}\right) + w^{(2)}_{k0}\right)$$

Outline

Network Training

- Given a specified network structure, how do we set its parameters (weights)?
  - As usual, we define a criterion to measure how well our network performs, optimize against it
- For regression, training data are $(x_n, t_n), t_n \in \mathbb{R}$
  - Squared error naturally arises:

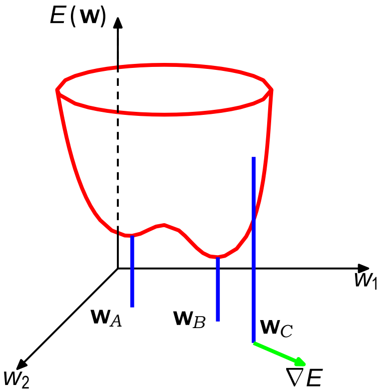
$$E(w) = \sum_{n=1}^{N} \{y(x_n, w) - t_n\}^2$$

- For binary classification, this is another discriminative model, ML:

$$p(\boldsymbol{t}|\boldsymbol{w}) = \prod_{n=1}^{N} y_n^{t_n}\{1 - y_n\}^{1-t_n}$$

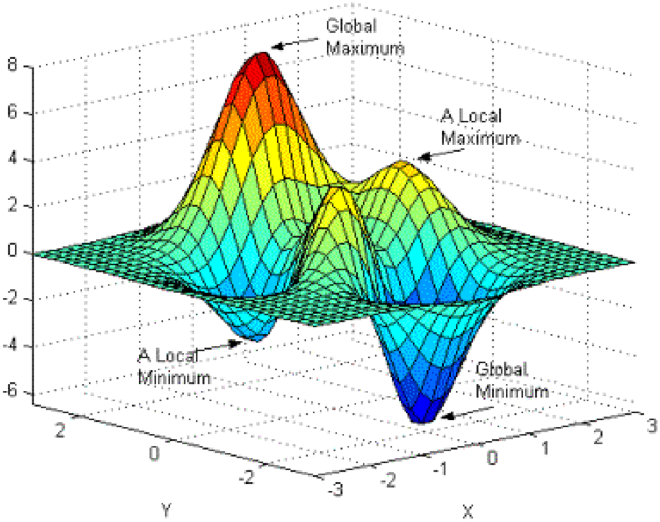
$$E(w) = -\sum_{n=1}^{N} \{t_n \ln(y_n) + (1 - t_n)\ln(1 - y_n)\}$$

Parameter Optimization



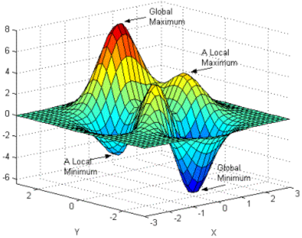
- For either of these problems, the error function $E(\boldsymbol{w})$ is nasty
  - Nasty = non-convex
  - Non-convex = has local minima

A Non-Convex function



Aside: Optimization Program

$$\begin{aligned} &\text{minimize} && f(x) \\ &\text{subject to} && g_i(x) \leq 0, i = 1, \dots, n \\ & && h_j(x) = 0, j = 1, \dots, m \end{aligned}$$

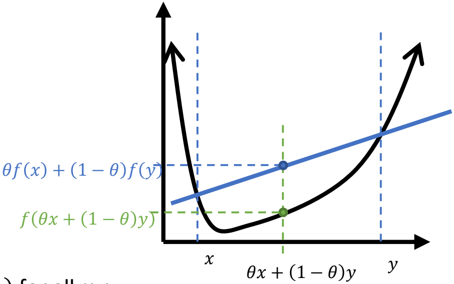


- Very difficult to solve in general
  - Trade-offs to consider: computation time, solution optimality

- Easy cases:
  - Find global optimum for **linear program**: f, g_i, h_j are linear
  - Find global optimum for **convex program**: f, g_i are convex, h_j is linear
  - Find local optimum for **nonlinear program**: f, g_i, h_j are differentiable

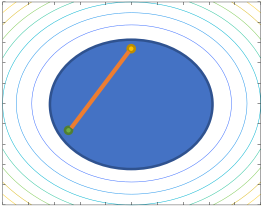
- Neural Networks: Nonlinear and unconstrained

Convex Functions

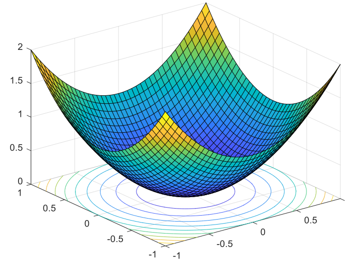


- **Convex function**
  $f(\theta x + (1-\theta)y) \leq \theta f(x) + (1-\theta) f(y)$ for all $x, y \in \mathbb{R}^n$, for all $\theta \in [0,1]$

- Sublevel sets of convex functions, $\{x: f(x) \leq C\}$, are convex
  - **Convex shape** $\mathcal{C}$:
    $x_1, x_2 \in \mathcal{C}, \theta \in [0,1] \Rightarrow \theta x_1 + (1-\theta) x_2 \in \mathcal{C}$



Convex Functions



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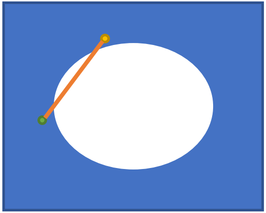
- Sublevel sets of convex functions, $\{x : f(x) \leq C\}$, are convex
  - **Convex shape** $\mathcal{C}$:

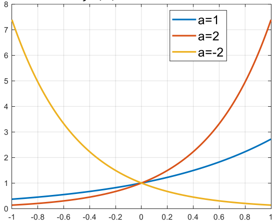
    $x_1, x_2 \in \mathcal{C}, \theta \in [0,1] \Rightarrow \theta x_1 + (1-\theta)x_2 \in \mathcal{C}$
  - *Super*level sets of convex functions are *not* convex!



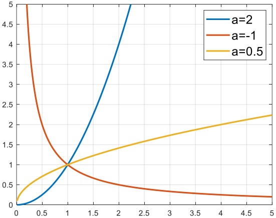
Common Convex Functions on $\mathbb{R}$

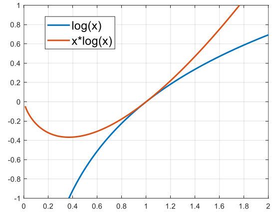
- $f(x) = e^{ax}$ is convex for all $x, a \in \mathbb{R}$
- $f(x) = x^a$ is convex on $x > 0$ if $a \geq 1$ or $a \leq 0$; concave if $0 < a < 1$
- $f(x) = \log x$ is concave
- $f(x) = x \log x$ is convex for $x > 0$ (or $x \geq 0$ if defined to be 0 when $x = 0$)

$f(x) = e^{ax}$



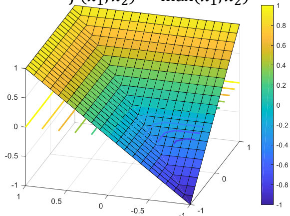
$f(x) = x^a$





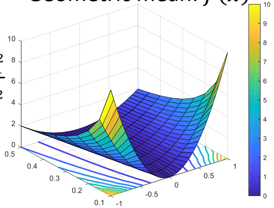
Common Convex Functions on $\mathbb{R}^n$

$f(x_1, x_2) = \max(x_1, x_2)$

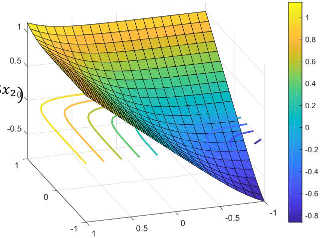


- $f(x) = Ax + b$ is convex for any A, b
- Every norm on $\mathbb{R}^n$ is convex
- $f(x) = \max(x_1, x_2, \ldots, x_n)$ is convex
- $f(x) = \frac{x_1^2}{x_2}$ (for $x_2 > 0$)
- Log-sum-exp softmax: $f(x) = \frac{1}{k}\log\left(e^{kx_1} + e^{kx_2} + \cdots + e^{kx_n}\right)$
- Geometric mean: $f(x) = \left(\prod_{i=1}^{n} x_i\right)^{\frac{1}{n}},\ x_i > 0$

$f(x) = \dfrac{x_1^2}{x_2}$



$f(x) = \dfrac{1}{5}\log(e^{5x_1} + e^{5x_2})$



Descent Methods

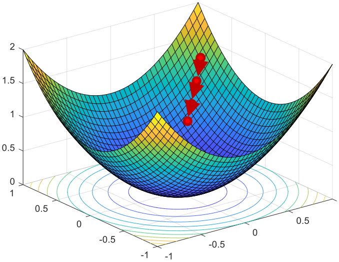
- The typical strategy for optimization problems of this sort is a descent method:

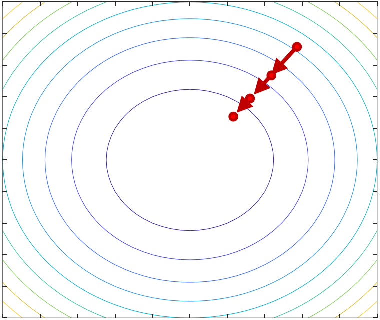
$$\boldsymbol{w}^{(\tau+1)} = \boldsymbol{w}^{(\tau)} + \Delta\boldsymbol{w}^{(\tau)}$$

- As we've seen before, these come in many flavours
  - Gradient descent $\nabla E\left(w^{(\tau)}\right)$
  - Stochastic gradient descent $\nabla E_n\left(\boldsymbol{w}^{(\tau)}\right)$
  - Newton-Raphson (second order)
- All of these can be used here, stochastic gradient descent is particularly effective
  - Redundancy in training data, escaping local minima

Numerical Solution: Gradient Methods

- Start from x^0 and construct a sequence x^k such that $x^k \to x^*$
  - Calculate x^{k+1} from x^k by "going down the gradient"
  - Unconstrained case: $x^{k+1} = x^k - \alpha^k \nabla f(x),\ \alpha^k > 0$





Numerical Solution: Gradient Methods

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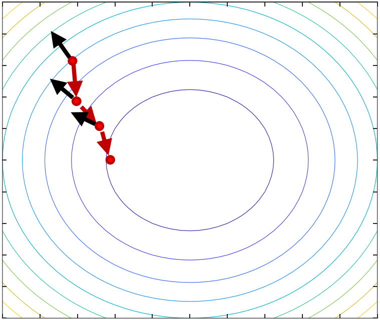
  - Calculate x^{k+1} from x^k by "going down the gradient"
  - Unconstrained case: $x^{k+1} = x^k - \alpha^k \nabla f(x),\ \alpha^k > 0$

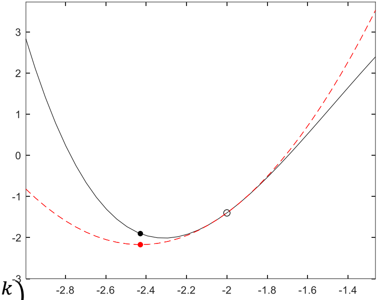
- More generally, $x^{k+1} = x^k + \alpha^k d^k$ for some d such that

$$\nabla f\left(x^k\right) \cdot d^k < 0$$

- Tuning parameters: descent direction d^k, and step size α^k



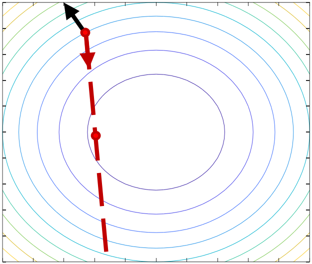
Descent Direction

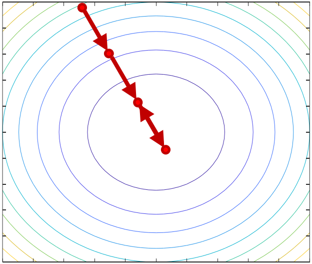


- Steepest descent: $d^k = -\nabla f\left(x^k\right)$
  - $x^{k+1} = x^k - \alpha^k \nabla f(x)$
  - Simple but sometimes leads to slow convergence
- Newton's method: $d^k = \left(\nabla^2 f\left(x^k\right)\right)^{-1} \nabla f\left(x^k\right)$
  - Minimize the quadratic approximation:
    $$f^k(x) = f\left(x^k\right) + \nabla f\left(x^k\right)^\top \left(x - x^k\right) + \frac{1}{2}\left(x - x^k\right)^\top \nabla^2 f\left(x^k\right)\left(x - x^k\right)$$
  - Set gradient to zero to obtain next iterate
    $$\nabla f^k(x) = \nabla f\left(x^k\right) + \nabla^2 f\left(x^k\right)\left(x - x^k\right) = 0$$
    $$\Rightarrow x^{k+1} = x^k - \left(\nabla^2 f\left(x^k\right)\right)^{-1} \nabla f\left(x^k\right)$$
  - Fast convergence, but matrix inverse required
  - Alternatively, use an algorithm to minimize a quadratic function

Step Size

- Recall $x^{k+1} = x^k + \alpha^k d^k$, with $\nabla f\left(x^k\right)^\top d^k < 0$
- Line search: choose $\alpha^k = \min_{\alpha \geq 0} f\left(x^k + \alpha^k d^k\right)$
  - Requires minimization
- Constant step size: $\alpha^k = \alpha$
  - May not converge
- Diminishing step size: $\alpha^k \to 0$
  - Still need to explore all regions $\sum \alpha^k = \infty$
  - For example: $\alpha^k = \frac{\alpha^0}{k}$





Numerical Solution: Second Order Methods

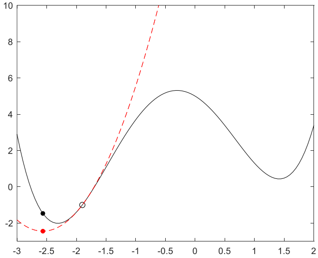
$$\text{minimize} \quad f(x) \quad \longrightarrow \quad \underset{d_x}{\text{minimize}} \quad (\boldsymbol{r^k})^\top d_x + \frac{1}{2} d_x^\top \boldsymbol{B_k} d_x$$

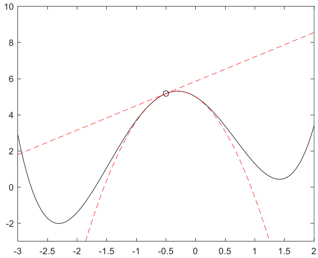
where $d_x := x - x^k$,

- Quadratize $f(x)$:

$$r^k = \nabla f(x_k)$$
$$B_k = \mathrm{H} f(x_k)$$

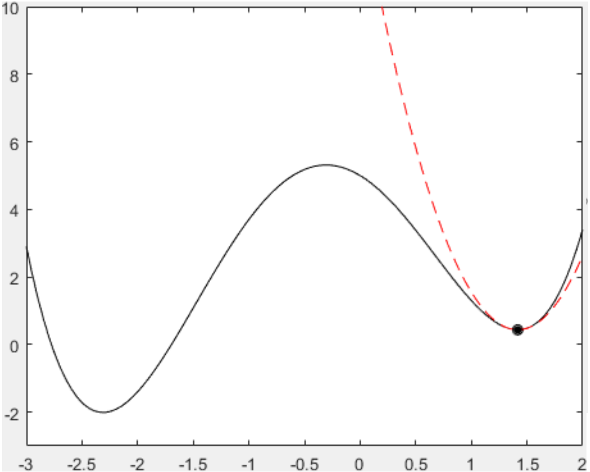
  - Convexify if needed, eg. by removing negative eigenvalues





Example

$$\text{minimize } 0.5x^4 + 0.8x^3 - 3x^2 - 2x + 5$$
$$\text{subject to } -3 \le x \le 2$$



Computing Gradients

- The function $y(\boldsymbol{x}_n, \boldsymbol{w})$ implemented by a network is complicated
  - It isn't obvious how to compute error function derivatives with respect to weights
- Numerical method for calculating error derivatives, use finite differences:

$$\frac{\partial E_n}{\partial w_{ji}} \approx \frac{E_n(w_{ji} + \epsilon) - E_n(w_{ji} - \epsilon)}{2\epsilon}$$

- How much computation would this take with W weights in the network?
  - $O(|W|)$ per partial derivative (evaluation of E_n)
  - $O(|W|^2)$ total per gradient descent step (there are $|W|$ partial derivatives)

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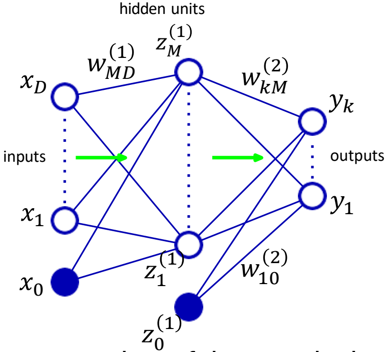


- Connect together a number of these units into a feed-forward network (DAG)
- Above shows a network with one layer of hidden units
- Implements function:

$$y_{(n),k}(x_n, w) = h\left(\sum_{j=1}^{M} w_{kj}^{(2)} \underbrace{h\left(\overbrace{\sum_{i=1}^{D} w_{ji}^{(1)} x_{(n),i} + w_{j0}^{(1)}}^{a_{(n),j}}\right)}_{z_{(n),j}} + w_{k0}^{(2)}\right)$$

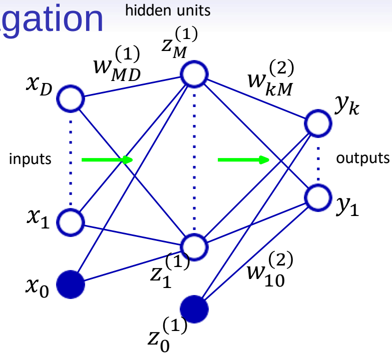
Error Backpropagation

- Backprop is an efficient method for computing error derivatives $\frac{\partial E_n}{\partial w_{ji}^{(m)}}$
  - $O(W)$ to compute derivatives wrt all weights
- First, feed training example x_n forward through the network, storing all activations a_j
- Calculating derivatives for weights connected to output nodes is easy
  - e.g. For linear output nodes $y_k = \sum_i w_{ki}^{(L)} z_{(n),i}^{(L-1)}$:

$$\frac{\partial E_n}{\partial w_{ki}^{(L)}} = \frac{\partial}{\partial w_{ki}^{(L)}} \frac{1}{2}\left(y_{(n),k} - t_{(n),k}\right)^2 = \left(y_{(n),k} - t_{(n),k}\right) z_{(n),i}^{(L-1)}$$

- For hidden layers, propagate error backwards from the output nodes

Error Backpropagation



$y_{(n),k}, E_n$:

- n: data point
- k: component

$w_{ji}^{(m)}$:

- m: layer
- j: index matching output
- i: index matching input

$$E(w) = \frac{1}{2}\sum_{n=1}^{N}\sum_{k}\left(y_{(n),k} - t_{(n),k}\right)^2, \qquad y_{(n),k} = \sum_i w_{ki}^{(L)} z_{(n),i}^{(L-1)}$$

$$E_n(w) = \frac{1}{2}\sum_{k}\left(y_{(n),k} - t_{(n),k}\right)^2$$

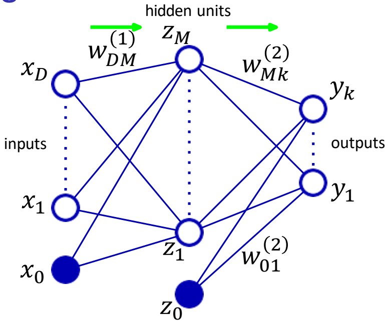
$$\frac{\partial E_n}{\partial w_{ki}^{(L)}} = \frac{\partial}{\partial w_{ki}^{(L)}}\frac{1}{2}\sum_{k'}\left(y_{(n),k'} - t_{(n),k'}\right)^2 = \left(y_{(n),k} - t_{(n),k}\right) z_{(n),i}^{(L-1)} \qquad (*)$$

Chain Rule for Partial Derivatives

- A "reminder"
- For $f(x, y)$, with f differentiable wrt x and y, and x and y differentiable wrt u:

$$\frac{\partial f}{\partial u} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial u}$$

Error Backpropagation



- We can write

$$\frac{\partial E_n}{\partial w_{ji}^{(m)}} = \frac{\partial}{\partial w_{ji}^{(m)}} E_n\left(a_{(n),1}^{(m)}, a_{(n),2}^{(m)}, \dots, a_{(n),D}^{(m)}\right)$$

- Using the chain rule:

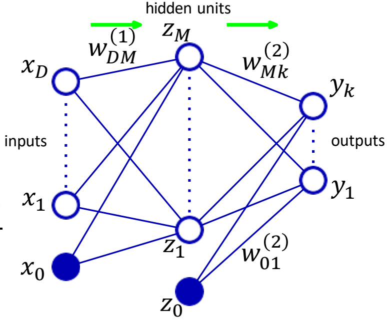
$$\frac{\partial E_n}{\partial w_{ji}^{(m)}} = \frac{\partial E_n}{\partial a_{(n),j}^{(m)}} \frac{\partial a_{(n),1}^{(m)}}{\partial w_{ji}^{(m)}} + \sum_{k \neq j} \frac{\partial E_n}{\partial a_{(n),k}^{(m)}} \frac{\partial a_{(n),k}^{(m)}}{\partial w_{ki}^{(m)}}$$

where $\sum_k(\cdots)$ runs over all other nodes k in the same layer (m)

- Since $a_{(n),k}^{(m)}$ does not depend on $w_{ji}^{(m)}$, all terms in the summation go to 0:

$$\frac{\partial E_n}{\partial w_{ji}^{(m)}} = \frac{\partial E_n}{\partial a_{(n),j}^{(m)}} \frac{\partial a_{(n),j}^{(m)}}{\partial w_{ji}^{(m)}}$$

Error Backpropagation cont.



- Introduce error $\delta_{(n),j}^{(m)} := \frac{\partial E_n}{\partial a_{(n),j}^{(m)}}$

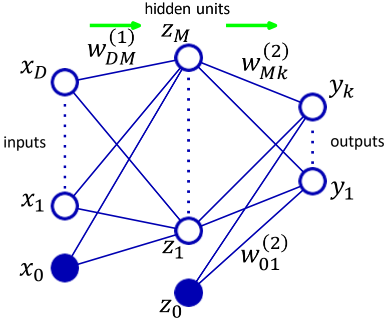
$$\frac{\partial E_n}{\partial w_{ji}^{(m)}} = \delta_{(n),j}^{(m)} \frac{\partial a_{(n),j}^{(m)}}{\partial w_{ji}^{(m)}}$$

- Other factor is

$$\frac{\partial a_{(n),j}^{(m)}}{\partial w_{ji}^{(m)}} = \frac{\partial}{\partial w_{ji}^{(m)}} \sum_k w_{jk}^{(m)} z_k^{(m-1)} = z_i^{(m-1)}$$

$$\frac{\partial E_n}{\partial w_{ji}^{(m)}} = \delta_{(n),j}^{(m)} z_i^{(m-1)}$$

Error Backpropagation cont.



- Error $\delta^{(m)}_{(n),j}$ can also be computed using chain rule:

$$\delta^{(m)}_j := \frac{\partial E_n}{\partial a^{(m)}_{(n),j}} = \sum_k \underbrace{\frac{\partial E_n}{\partial a^{(m+1)}_{(n),k}}}_{\delta^{(m+1)}_k} \frac{\partial a^{(m+1)}_{(n),k}}{\partial a^{(m)}_{(n),j}}$$

where $\sum_k(\cdots)$ runs over all nodes k in the layer **after**.

$$a^{(m+1)}_{(n),k} = \sum_i w^{(m+1)}_{ki} z^{(m)}_{(n),i} = \sum_i w^{(m+1)}_{ki} h^{(m)}\left(a^{(m)}_{(n),i}\right)$$

$$\frac{\partial a^{(m+1)}_{(n),k}}{\partial a^{(m)}_{(n),j}} = w^{(m+1)}_{kj} (h^{(m)})' \left(a^{(m)}_{(n),j}\right)$$

$$\delta^{(m)}_{(n),j} = \sum_k \delta^{(m+1)}_{(n),k} w^{(m+1)}_{kj} (h^{(m)})' \left(a^{(m)}_{(n),j}\right) = (h^{(m)})' \left(a^{(m)}_{(n),j}\right) \sum_k \delta^{(m+1)}_{(n),k} w^{(m+1)}_{kj}$$

Error Backpropagation cont.

- Error $\delta^{(m)}_{(n),j}$ can also be computed using chain rule:

$$\delta^{(m)}_{(n),j} := \frac{\partial E_n}{\partial a^{(m)}_{(n),j}} = \sum_k \underbrace{\frac{\partial E_n}{\partial a^{(m+1)}_{(n),k}}}_{\delta_k} \frac{\partial a^{(m+1)}_{(n),k}}{\partial a^{(m)}_{(n),j}}$$

  where $\sum_k(\cdots)$ runs over all nodes k in the layer **after**.
- Eventually:

$$\delta^{(m)}_{(n),j} = \left(h^{(m)}\right)' \left(a^{(m)}_{(n),j}\right) \sum_k \delta^{(m+1)}_{(n),k} \, w^{(m+1)}_{kj}$$

  - A weighted sum of the later error “caused” by this weight

Error Backpropagation cont.

- Eventually:

$$\delta^{(m)}_{(n),j} = \left(h^{(m)}\right)'\left(a^{(m)}_{(n),j}\right)\sum_k \delta^{(m+1)}_{(n),k}\, w^{(m+1)}_{jk}$$

where $\sum_k(\cdots)$ runs over all nodes k in the layer **after**.

- Above recursion relation needs last set of errors: $\delta_j^{(L)}$

$$\frac{\partial E_n}{\partial w^{(m)}_{ji}} = \delta^{(m)}_{(n),j} z_i^{(m-1)}$$ (by definition)

$$\frac{\partial E_n}{\partial w^{(L)}_{ji}} = \delta^{(L)}_{(n),j} z^{(L-1)}_{(n),i} = \left(y_{(n),j} - t_{(n),j}\right) z^{(L-1)}_{(n),i}$$ (from before $(*)$)

$$\delta^{(L)}_{(n),j} = y_{(n),j} - t_{(n),j}$$ (by comparison)

Summary

$O(|W|)$

Output Definition / forward propagation

$$y_{(n),k}(x_n, w) = h^{(m+1)}\left(\sum_{j=1}^{M} w_{jk}^{(m+1)} h^{(m)}\left(\sum_{i=1}^{D} w_{ij}^{(m)} z_{(n),i}^{(m-1)} + w_{0j}^{(m)}\right) + w_{k0}^{(m+1)}\right)$$

- Save $\boldsymbol{z}, \boldsymbol{a}$

Gradient computation / backpropagation

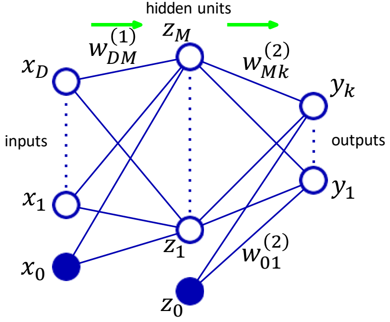
- Last layer: $\dfrac{\partial E_n}{\partial w_{ik}^{(L)}} = \left(y_{(n),k} - t_{(n),k}\right) z_{(n),i}^{(L-1)}$
- Previous layers: Define $\delta_{(n),j}^{(m)} := \dfrac{\partial E_n}{\partial a_{(n),j}^{(m)}}$

  Starting from last layer,

$$\delta_{(n),j}^{(L)} = y_{(n),j} - t_{(n),j}$$

Recursion: $\dfrac{\partial E_n}{\partial w_{ij}^{(m)}} = \delta_{(n),j}^{(m)} z_{(n),i}^{(m-1)}$,

where $\delta_{(n),j}^{(m)} = \left(h^{(m)}\right)'\left(a_{(n),j}^{(m)}\right) \sum_k \delta_k^{(m+1)} w_{jk}^{(m+1)}$



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Gradient computation / backpropagation

Goes through one layer of weights

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$O(|W|)$

Starting from last layer,

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Recursion: $\dfrac{\partial E_n}{\partial w_{ij}^{(m)}} = \delta_{(n),j}^{(m)} z_{(n),i}^{(m-1)}$,

Goes through one layer of weights

where $\delta_{(n),j}^{(m)} = \left(h^{(m)}\right)'\left(a_{(n),j}^{(m)}\right) \sum_k \delta_k^{(m+1)} w_{jk}^{(m+1)}$

Tensorflow Playground

- https://playground.tensorflow.org

Outline

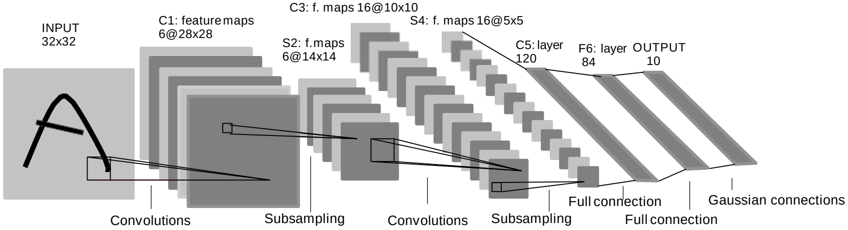
Deep Learning

- Collection of important techniques to improve performance:
  - Multi-layer networks
  - Convolutional networks, parameter tying
  - Hinge activation functions (ReLU) for steeper gradients
  - Momentum
  - Drop-out regularization
  - Sparsity
  - Auto-encoders for unsupervised feature learning
  - ...
- Scalability is key, can use lots of data since stochastic gradient descent is memory-efficient, can be parallelized

Hand-written Digit Recognition

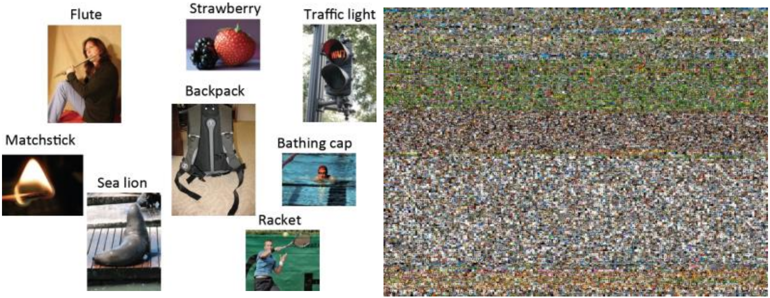
- MNIST - standard dataset for hand-written digit recognition
    - 60000 training, 10000 test images

LeNet-5, circa 1998



- LeNet developed by Yann LeCun et al.
    - Convolutional neural network
        - Local receptive fields (5x5 connectivity)
        - Subsampling (2x2)
        - Shared weights (reuse same 5x5 "filter")
        - Breaking symmetry

ImageNet



- ImageNet - standard dataset for object recognition in images (Russakovsky et al.)
  - 1000 image categories, $\approx$1.2 million training images (ILSVRC 2013)

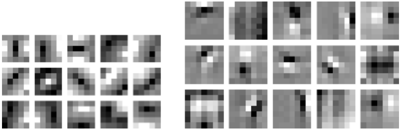
GoogLeNet, circa 2014

- GoogLeNet developed by Szegedy et al., CVPR 2015
- Modern deep network
- ImageNet top-5 error rate of 6.67% (later versions even better)
- Comparable to human performance (especially for fine-grained categories)

ResNet, circa 2015

- ResNet developed by He et al., ICCV 2015
- 152 layers
- ImageNet top-5 error rate of 3.57%
- Better than human performance (especially for fine-grained categories)

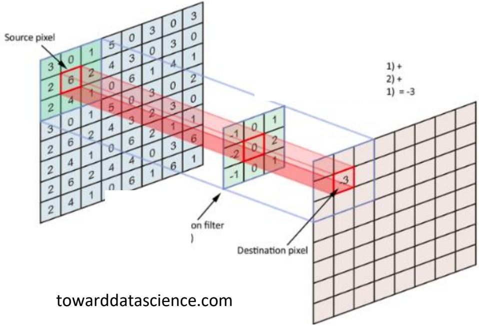
Key Component 1: Convolutional Filters



- Share parameters across network
- Reduce total number of parameters
- Provide translation invariance, useful for visual recognition

Common Operations

- Fully connected (dot product)
- Convolution
  - Translationally invariant
  - Controls overfitting
- Pooling (fixed function)
  - Down-sampling
  - Controls overfitting
- Nonlinearity layer (fixed function)
  - Activation functions, e.g. ReLU



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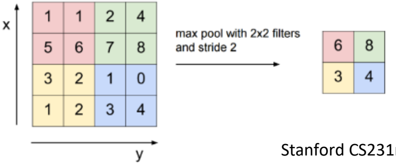
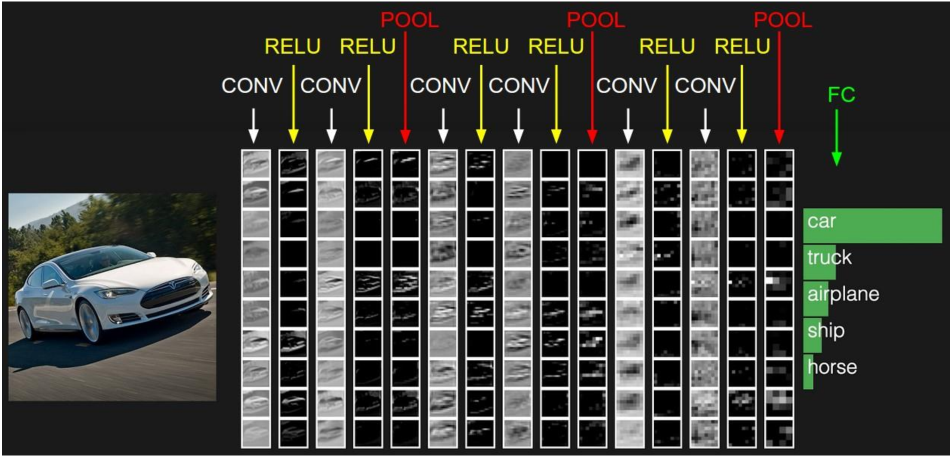
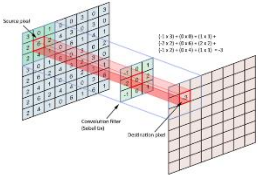


Stanford CS231n

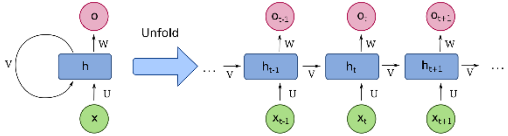
Example: Small VGG Net From Stanford CS231n



Neural Network Architectures



- Convolutional neural network (CNN)
  - Has translational invariance properties from convolution
  - Common used for computer vision
- Recurrent neural network RNN
  - Has feedback loops to capture temporal or sequential information
  - Useful for handwriting recognition, speech recognition, reinforcement learning
  - Long short-term memory (LSTM): special type of RNN with advantages in numerical properties
- Others
  - General feedforward networks, variational autoencoders (VAEs), conditional VAEs



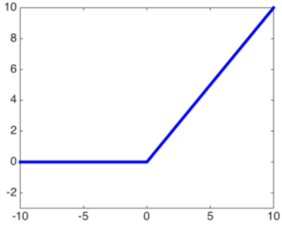
Training Neural Networks

- Data preprocessing
  - Removing bad data
  - Transform input data (e.g. rotating, stretching, adding noise)

- Training process (optimization algorithm)
  - Choice of loss function (eg. L1 and L2 regularization)
  - Dropout: randomly set neurons to zero in each training iteration
  - **Learning rate** (step size) and other hyperparameter tuning

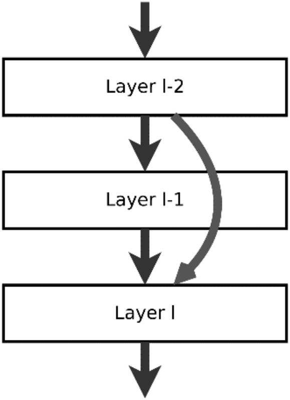
- Software packages: efficient gradient computation
  - Caffe, Torch, Theano, TensorFlow

Key Component 2: Rectified Linear Units (ReLUs)



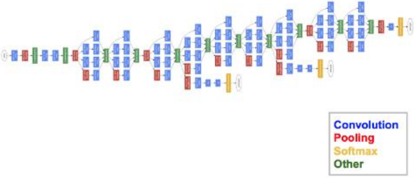
- Vanishing gradient problem
  - If derivatives very small, no/little progress via stochastic gradient descent
  - Occurs with sigmoid function when activation is large in absolute value
- ReLU: $h(a_j) = \max(0, a_j)$
- Non-saturating, linear gradients (as long as non-negative activation on some training data)
- Sparsity inducing

Key Component 3: Many, Many Layers



- **ResNet**: ≈152 layers ("shortcut connections")
- GoogLeNet: ≈27 layers ("Inception" modules)
- VGG Net: 16-19 layers (Simonyan and Zisserman, 2014)
- Supervision: 8 layers (Krizhevsky et al., 2012)

Key Component 3: Many, Many Layers



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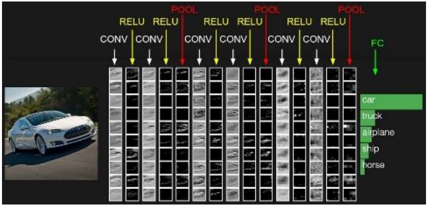

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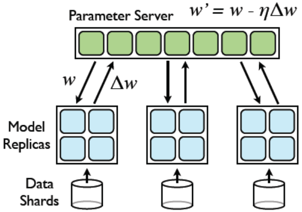
Key Component 4: Momentum

- Trick to escape plateaus / local minima
- Take exponential average of previous gradients

$$\overline{\frac{\partial E_n}{\partial w_{ji}}}^{\tau} = \overline{\frac{\partial E_n}{\partial w_{ji}}}^{\tau} + \alpha \overline{\frac{\partial E_n}{\partial w_{ji}}}^{\tau-1}$$

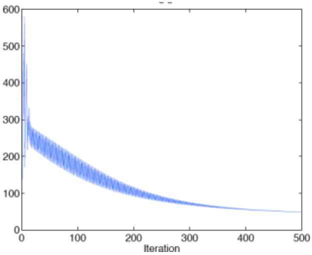
- Maintains progress in previous direction

Key Component 5: Asynchronous Stochastic Gradient Descent



- Big models won't fit in memory
- Want to use compute clusters (e.g. 1000s of machines) to run stochastic gradient descent
- How to parallelize computation?
- Ignore synchronization across machines
  - Just let each machine compute its own gradients and pass to a server storing current parameters
  - Ignore the fact that these updates are inconsistent
  - Seems to just work (e.g. Dean et al. NIPS 2012)

Key Component 6: Learning Rate Schedule



- How to set learning rate η?:

$$\boldsymbol{w}^{\tau} = \boldsymbol{w}^{\tau-1} + \eta \nabla \boldsymbol{w}$$

- Option 1: Run until validation error plateaus. Drop learning rate by x%
- Option 2: Adagrad, adaptive gradient. Per-element learning rate set based on local geometry (Duchi et al. 2010)

Key Component 7: Data Augmentation

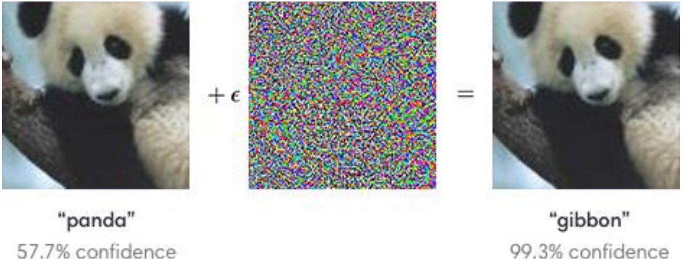
- Augment data with additional synthetic variants (10x amount of data)
- Or just use synthetic data, e.g. Sintel animated movie (Butler et al. 2012)

Key Component 8: Data and Compute

- Get lots of data (e.g. ImageNet)
- Get lots of compute (e.g. CPU cluster, GPUs)
- Cross-validate like crazy, train models for 2-3 weeks on a GPU
- Researcher gradient descent (RGD) or Graduate student descent (GSD): get 100s of researchers to each do this, trying different network structures

Challenges

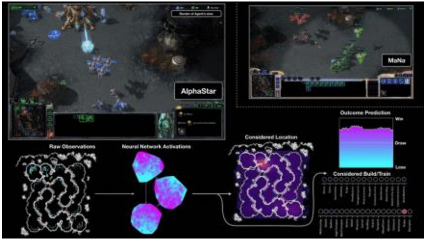
Interpretability:



Challenges

Data efficiency:

- ImageNet: 14 million images, 20000 categories
- AlphaStar: 200 years of gameplay



Challenges

- Problem formulation (what are you trying to predict?)
- Choice of model and optimization algorithm
- Data collection, post-processing
- Feature selection
- ...

More information

- https://sites.google.com/site/deeplearningsummerschool
- http://tutorial.caffe.berkeleyvision.org/
- ufldl.stanford.edu/eccv10-tutorial
- http://www.image-net.org/challenges/LSVRC/2012/supervision.pdf
- Prof. Oliver Schulte's CMPT880: Deep Learning
- Project ideas
  - Long short-term memory (LSTM) models for temporal data
  - Learning embeddings (word2vec, FaceNet)
  - Structured output (multiple outputs from a network)
  - Zero-shot learning (learning to recognize new concepts without training data)
  - Transfer learning (use data from one domain/task, adapt to another)

Conclusion

- Readings: Ch. 5.1, 5.2, 5.3
- Feed-forward networks can be used for regression or classification
  - Similar to linear models, except with adaptive non-linear basis functions
  - These allow us to do more than e.g. linear decision boundaries
- Different error functions
- Learning is more difficult, error function not convex
  - Use stochastic gradient descent, obtain (good?) local minimum
- Backpropagation for efficient gradient computation