Quiz 5 - MACM 201 - *Solutions*

[4 pts] Find a particular solution to the recurrence

$$x_n + x_{n-1} + 20x_{n-2} = 4 \cdot 5^n$$

Solution: Since the right hand side is exponential in n with base 5, we try a solution of the form $x_n = C5^n$. Plugging this into our equation gives

$$C5^n + C5^{n-1} + 20C5^{n-2} = 4 \cdot 5^n$$

Now dividing through by 5^{n-2} gives us

$$25C + 5C + 20C = 4 \cdot 25$$

and so $50C = 100$ and $C = 2$. We conclude that $x_n = 2 \cdot 5^n$ is a particular solution to the recurrence.

[4 pts] Find the unique solution to

$$a_n - 5a_{n-1} + 4a_{n-2} = 0 \qquad a_0 = 4 \text{ and } a_1 = 7$$

Note: the characteristic equation is:

$$0 = r^2 - 5r + 4 = (r-4)(r-1)$$

Solution: We can deduce the general solution to the recurrence from the factorization of the characteristic equation. The general solution is

$$a_n = C4^n + D$$

Now we plug in for our initial values and solve for C and D.

$$4 = a_0 = C4^0 + D = C + D$$

$$7 = a_1 = C4^1 + D = 4C + D$$

subtracting the second equation from the first gives $3 = 3C$ so we have $C = 1$ and then $D = 3$. Thus the unique solution is

$$a_n = 4^n + 3$$