MACM 201 Homework 4 - *Solutions*

1. For each recurrence below, find the order and determine if it is linear. If it is linear, determine if it is homogeneous.

   (a) $5x_n - 7x_{n-3} + 5x_{n-5} = n^2$

   (b) $10x_{n+3} - 4x_{n+1} = x_n^2$

   (c) $11x_n - 3x_{n-1} + 2x_{n-4} = 0$ (typo corrected from original version)

   (d) $11x_n - 5x_{n-2} + 3x_{n-5}x_{n-6} = 0$

   Solution:

   (a) This is a 5th order recurrence since x_{n-5} appears in the calculation of x_n. It is linear since the coefficient of each x_i term is a constant, but nonhomogeneous thanks to the n^2 term.

   (b) This is a 3rd order recurrence since x_n appears in the calculation of x_{n+3}. It is nonlinear thanks to the presence of the x_n^2 term.

   (c) This is a 4th order recurrence since x_{n-4} appears in the calculation of x_n. It is linear since the coefficient of each x_i term is a constant, and it is homogeneous since we have a sum of constants times x_i terms equalling 0.

   (d) This is a 6th order recurrence since x_{n-6} appears in the calculation of x_n. It is nonlinear since we have a product of x_{n-5} and x_{n-6}.

2. What is the set of all solutions to the recurrence $x_n = 7x_{n-1}$?

   Solution: . The general solution is $x_n = C7^n$.

3. For each recurrence below, find the characteristic equation and use this to find all real numbers r so that $x_n = r^n$ is a solution.

   (a) $x_n - 6x_{n-1} + 5x_{n-2} = 0$

   (b) $2x_n - 9x_{n-1} + 5x_{n-2} = 0$

   (c) $x_n - 6x_{n-1} + 9x_{n-2} = 0$

   (d) $x_n + 3x_{n-1} + 4x_{n-2} = 0$

Solution:

(a) The characteristic equation is $r^2 - 6r + 5 = 0$ and this factors as

$$0 = r^2 - 6r + 5 = (r-5)(r-1)$$

so we have two real roots $1, 5$ and this gives us $x_n = 1^n = 1$ and $x_n = 5^n$ as solutions to our recurrence.

(b) The characteristic equation is $2r^2 - 9r + 5 = 0$ and this requires the quadratic formula to solve. The roots are

$$r = \frac{9 \pm \sqrt{41}}{4}.$$

This gives solutions to our recurrence of $x_n = (\frac{9+\sqrt{41}}{4})^n$ and $x_n = (\frac{9-\sqrt{41}}{4})^n$.

(c) The characteristic equation is $r^2 - 6r + 9 = 0$ and this factors as

$$0 = r^2 - 6r + 9 = (r-3)^2$$

so we have one real root of 3 and we find $x_n = 3^n$ as a solution to our recurrence.

(d) The characteristic equation is $r^2 + 3r + 4 = 0$ and the quadratic formula gives us

$$r = \frac{-3 \pm \sqrt{-7}}{2}.$$

Here there are no real number solutions, so there is no real number r so that $x_n = r^n$ is a solution.

4. For parts (a)-(c) in the previous problem, describe infinitely many solutions to the given recurrence.

Solution:

(a) $x_n = C + D5^n$ is a solution for every C, D.

(b) $x_n = C(\frac{9+\sqrt{41}}{4})^n + D(\frac{9-\sqrt{41}}{4})^n$ is a solution for every C, D

(c) $x_n = C3^n$ is a solution for every C.