MATH 308 D200, Fall 2019

10. Simplex algorithm for minimum tableau (based on notes from Dr. J. Hales, Dr. L. Stacho, and Dr. L. Godyyn)

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Canonical Forms for Minimization LP Problem

Canonical Minimization LP

minimize
$$c_1x_1 + c_2x_2 + \cdots + c_nx_n - d = g(x_1, x_2, \dots, x_n)$$

subject to $a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \geqslant b_1$
 $a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \geqslant b_2$
 \vdots
 $a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \geqslant b_m$
 $x_1, x_2, \dots, x_n \geqslant 0$

Add Slack variables

minimize
$$c_1x_1 + c_2x_2 + \cdots + c_nx_n - d = g(x_1, x_2, \dots, x_n)$$

subject to $a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 + t_1$
 $a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 + t_2$
 \vdots
 $a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m + t_m$
 $x_1, x_2, \dots, x_n, t_1, t_2, \dots, t_m \geqslant 0$

Definition (Canonical Slack Form)

The linear programming problem

minimize
$$g(x_1, x_2, \dots, x_n) = c_1 x_1 + c_2 x_2 + \dots + c_n x_n - d$$
subject to
$$a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n - b_1 = t_1$$

$$a_{21} x_1 + a_{22} x_2 + \dots + a_{2n} x_n - b_2 = t_2$$

$$\vdots$$

$$a_{m1} x_1 + a_{m2} x_2 + \dots + a_{mn} x_n - b_m = t_m$$

$$x_1, x_2, \dots, x_n, t_1, t_2, \dots, t_m \ge 0$$

is said to be canonical slack minimization linear programming problem. The variables t_1, t_2, \ldots, t_m are said to be slack variables.

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Tucker Tableau of the Canonical Slack Minimization LP Problem

Given the canonical slack minimization LP

minimize
$$g(x_1, x_2, \dots, x_n) = c_1 x_1 + c_2 x_2 + \dots + c_n x_n - d$$
 subject to
$$a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n - b_1 = t_1$$

$$a_{21} x_1 + a_{22} x_2 + \dots + a_{2n} x_n - b_2 = t_2$$

$$\vdots$$

$$a_{m1} x_1 + a_{m2} x_2 + \dots + a_{mn} x_n - b_m = t_m$$

$$x_1, x_2, \dots, x_n, t_1, t_2, \dots, t_m \geqslant 0$$

Its Minimum Tucker tableau is

x_1	a ₁₁	a 21		a_{m1}	<i>c</i> ₁
<i>X</i> ₂	a ₁₂	a 22		a_{m2}	c ₂
:	:	٠.,	:	:	:
Xn	a _{1n}	a_{2n}		a_{mn}	Cn
-1	b_1	<i>b</i> ₂		b_m	d
	$= t_1$	$=t_{2}$		= t	= 0

The independent (nonbasic) variables are listed to the West. The dependent (basic) variables are listed to the South.

Tucker Tableau of the Canonical Slack Minimization LP Problem

Given the canonical slack minimization LP

minimize
$$g(x_1, x_2, \dots, x_n) = c_1 x_1 + c_2 x_2 + \dots + c_n x_n - d$$
subject to
$$a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n - b_1 = t_1$$

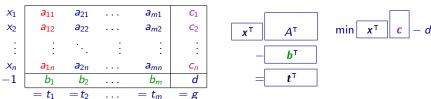
$$a_{21} x_1 + a_{22} x_2 + \dots + a_{2n} x_n - b_2 = t_2$$

$$\vdots$$

$$a_{m1} x_1 + a_{m2} x_2 + \dots + a_{mn} x_n - b_m = t_m$$

$$x_1, x_2, \dots, x_n, t_1, t_2, \dots, t_m \ge 0$$

Its Minimum Tucker tableau is



The independent (nonbasic) variables are listed to the West. The dependent (basic) variables are listed to the South.

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Definition (Negative Transposition)

The negative transposition of the minimum tableau

Definition (Negative Transposition)

The negative transposition of the minimum tableau

is the maximum tableau

Definition (Negative Transposition)

The negative transposition of the minimum tableau

$$\min \begin{bmatrix} x^{\mathsf{T}} & \mathbf{c} \\ -d = g(x) \end{bmatrix}$$
$$\begin{bmatrix} x^{\mathsf{T}} & A^{\mathsf{T}} \\ -\mathbf{b}^{\mathsf{T}} \end{bmatrix} = \begin{bmatrix} \mathbf{t}^{\mathsf{T}} \\ \end{bmatrix}$$

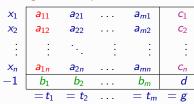
is the maximum tableau

$$\max \boxed{-c^{\mathsf{T}}} \boxed{x} - (-d) = -g(x)$$

$$\boxed{-A} \boxed{x} - \boxed{-b} = \boxed{-t}$$

Definition (Negative Transposition)

The negative transposition of the minimum tableau



min
$$x^{\mathsf{T}} c - d = g(x)$$

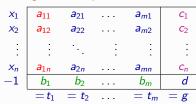
 $x^{\mathsf{T}} A^{\mathsf{T}} - b^{\mathsf{T}} = t^{\mathsf{T}}$
 $x, t \ge 0$

is the maximum tableau

$$\max -c^{\mathsf{T}}x - (-d) = -g(x)$$
$$-Ax - (-b) = -t$$
$$x, t \ge 0$$

Definition (Negative Transposition)

The negative transposition of the minimum tableau



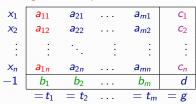
$$\min \mathbf{c}^{\mathsf{T}} \mathbf{x} - \mathbf{d} = \mathbf{g}(\mathbf{x})$$
$$\mathbf{x}^{\mathsf{T}} \mathbf{A}^{\mathsf{T}} - \mathbf{b}^{\mathsf{T}} = \mathbf{t}^{\mathsf{T}}$$
$$\mathbf{x}, \mathbf{t} \ge 0$$

is the maximum tableau

$$\max -c^{\mathsf{T}}x - (-d) = -g(x)$$
$$-Ax - (-b) = -t$$
$$x, t \ge 0$$

Definition (Negative Transposition)

The negative transposition of the minimum tableau



min
$$c^{\mathsf{T}}x - d = g(x)$$

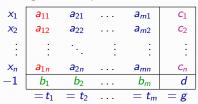
 $(Ax)^{\mathsf{T}} - b^{\mathsf{T}} = t^{\mathsf{T}}$
 $x, t \ge 0$

is the maximum tableau

$$\max -c^{\mathsf{T}}x - (-d) = -g(x)$$
$$-Ax - (-b) = -t$$
$$x, t \ge 0$$

Definition (Negative Transposition)

The negative transposition of the minimum tableau



$$\min c^{\mathsf{T}}x - d = g(x)$$

$$Ax - b = t$$

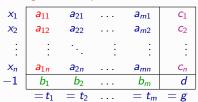
$$x, t \ge 0$$

is the maximum tableau

$$\max -c^{\mathsf{T}}x - (-d) = -g(x)$$
$$-Ax - (-b) = -t$$
$$x, t \ge 0$$

Definition (Negative Transposition)

The negative transposition of the minimum tableau



$$\min c^{\mathsf{T}}x - d = g(x)$$

$$Ax - b = t$$

$$x, t \ge 0$$

is the maximum tableau

$$\max \quad -\mathbf{c}^{\mathsf{T}}\mathbf{x} + d = -\mathbf{g}(\mathbf{x})$$
$$-A\mathbf{x} - (-\mathbf{b}) = -\mathbf{t}$$
$$\mathbf{x}, \mathbf{t} \ge 0$$

Definition (Negative Transposition)

The negative transposition of the minimum tableau



min
$$c^{\mathsf{T}}x - d = g(x)$$

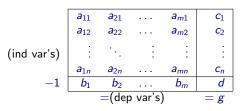
 $Ax - b = t$
 $x, t \ge 0$

is the maximum tableau

$$\max \quad -c^{\mathsf{T}}x + d = -g(x)$$
$$Ax - b = t$$
$$x, t \ge 0$$

SA for Minimum Tableaux

Method 1: (See Chapter 2, §7 P. 54)



- 1. We have minimum Tucker tableau.
- 2. Take the negative transposition of the tableaux to obtain a maximum tableau.
- **3.** Apply SA for maximum tableaux.
- **4.** $\min g = -\max(-g)$.

Apply the simplex algorithm to the minimum tableau

	$= t_1$	$= t_2$	= g
-1	1000	800	0
<i>X</i> ₂	40	20	500
<i>x</i> ₁	20	25	300