MATH 308 D200, Fall 2019

5. Slack variables and first look at tableaus

(based on notes from Dr. J. Hales, Dr. L. Stacho, and Dr. L. Godyyn)

Dr. Masood Masjoody

SFU Burnaby

First example again

Maximize the value $f(x, y) = x + y$ subject to constraints

(1) $y - x \leqslant 1$
(2) $x + 6y \leqslant 15$
(3) $4x - y \leqslant 10$
(4) $x \geqslant 0$
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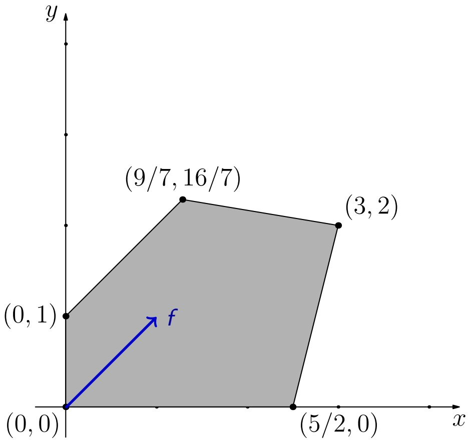

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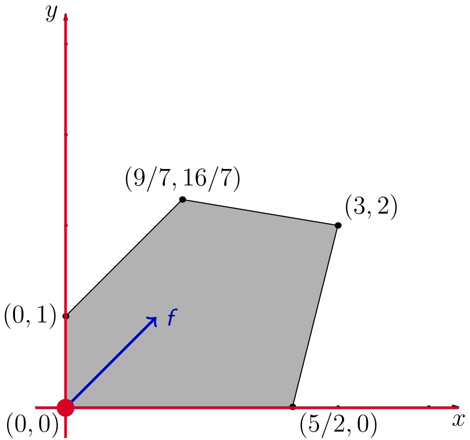

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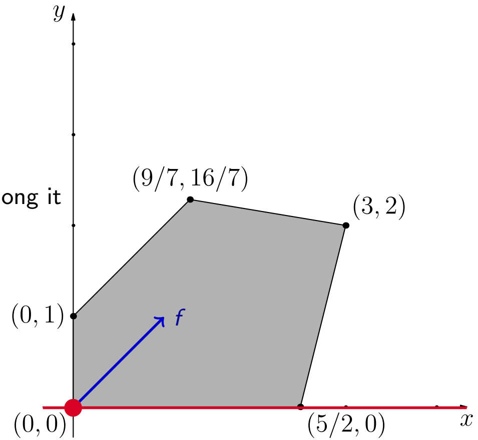

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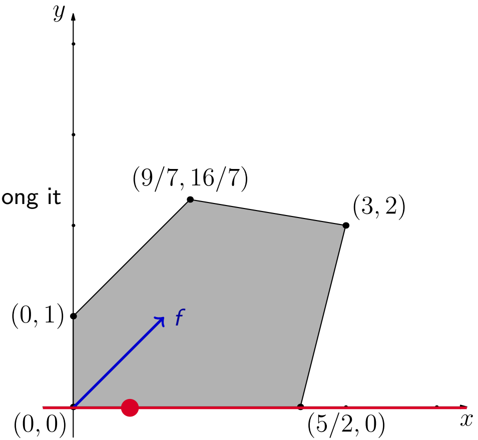

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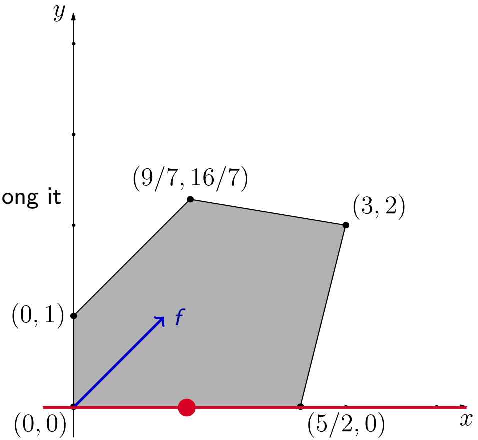

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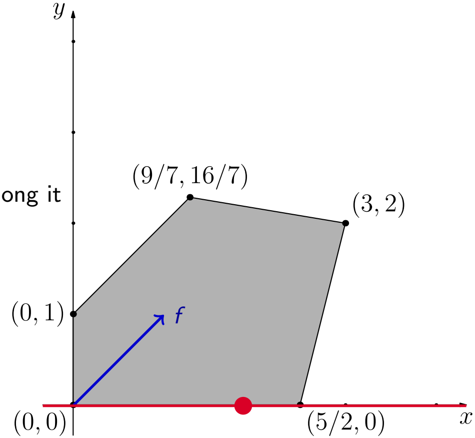

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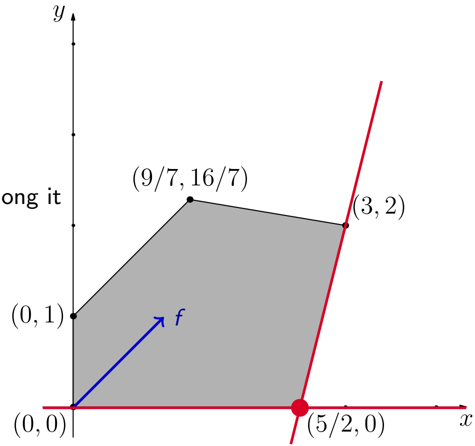

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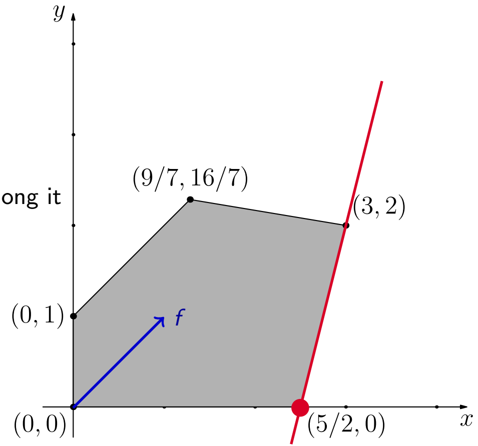

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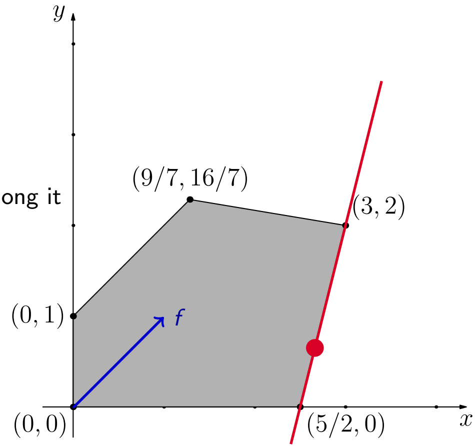

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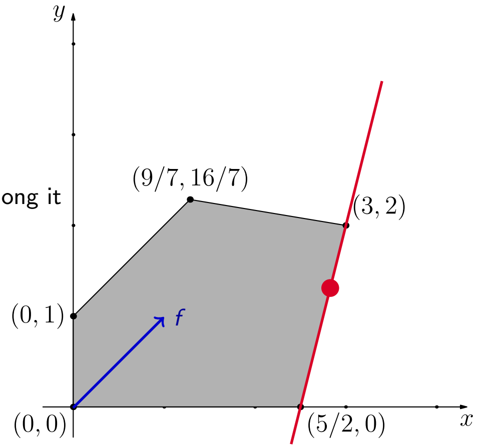

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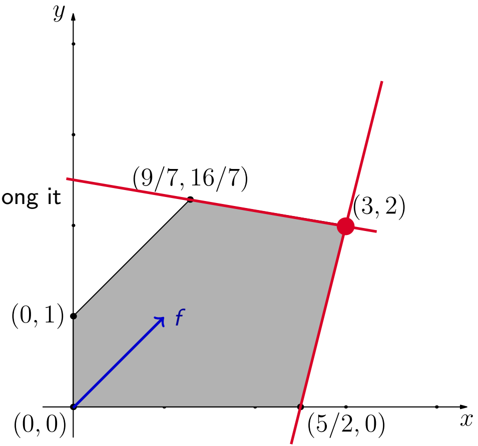

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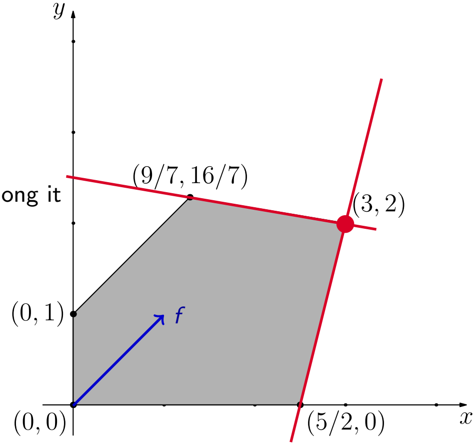

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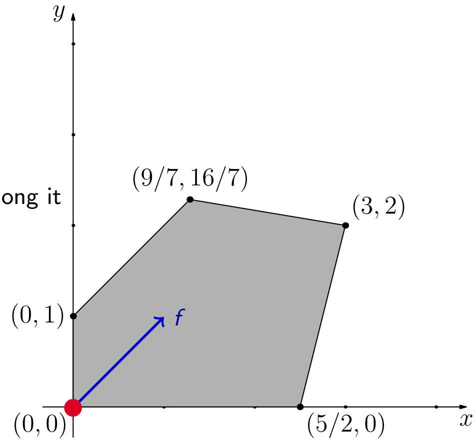

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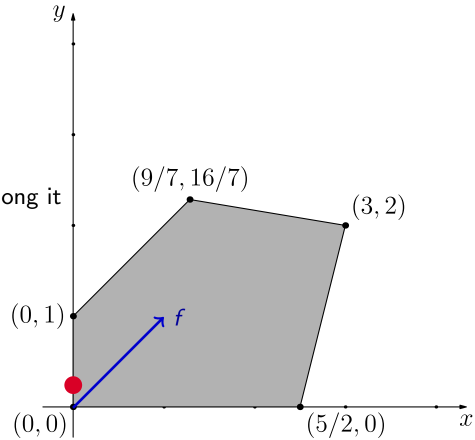

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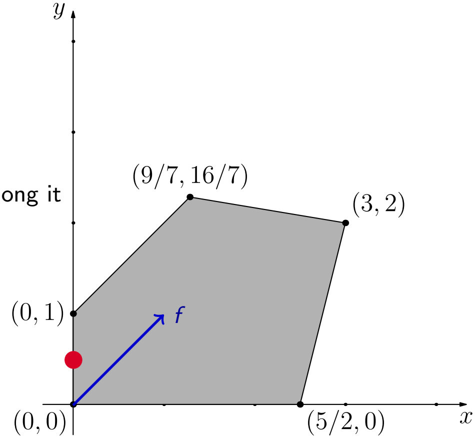

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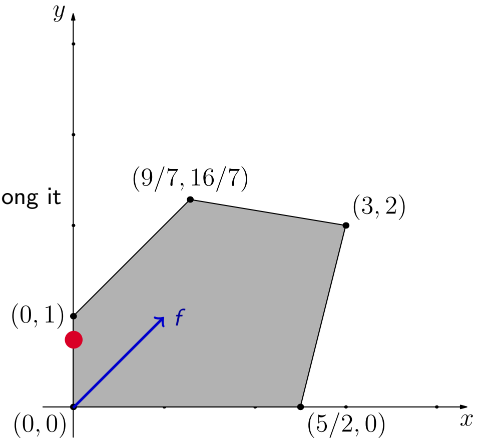

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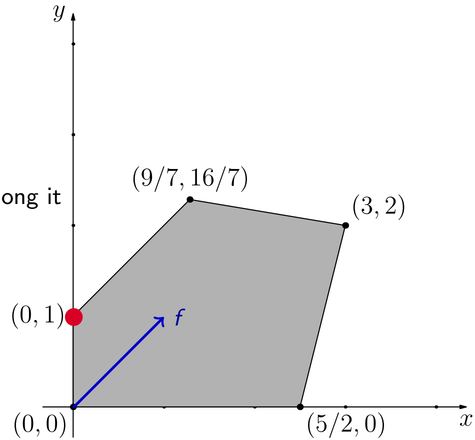

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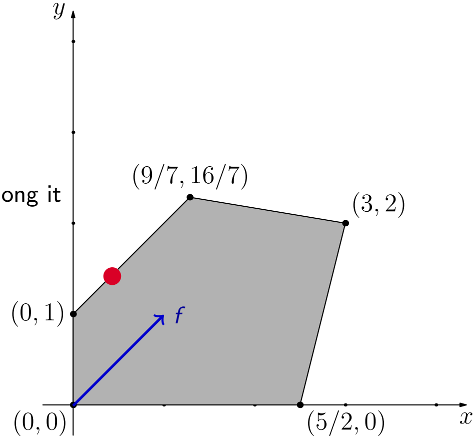

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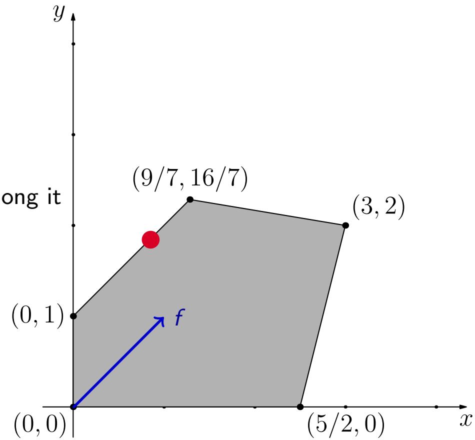

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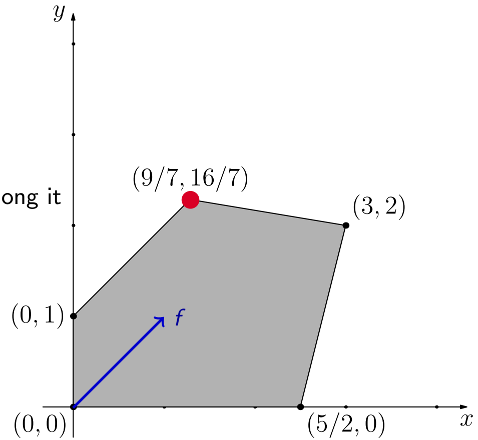

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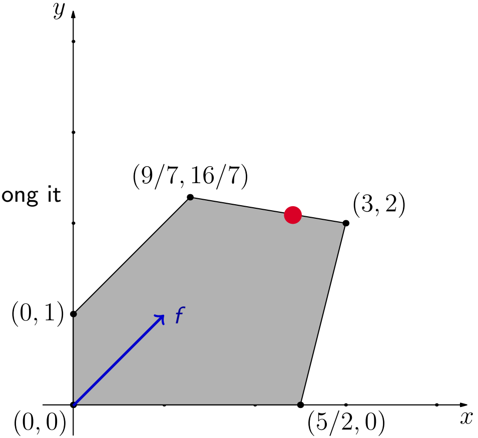

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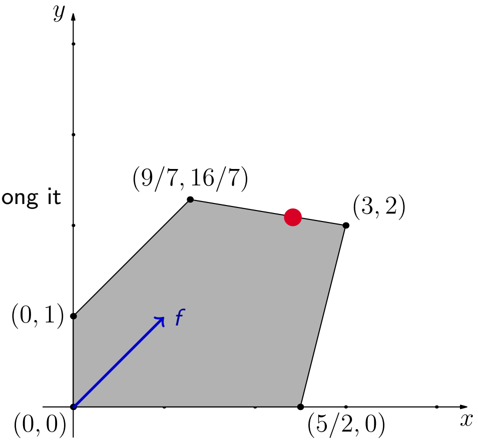

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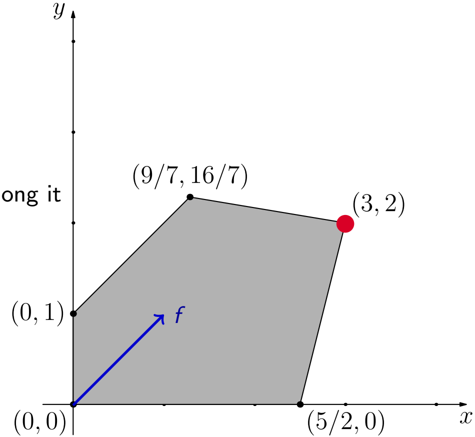
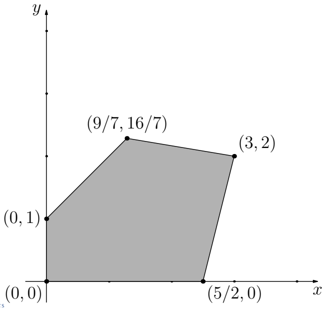


Slack variables

We introduce "slack variables" t_1, t_2, t_3 (one for each main constraint): $x, y,$

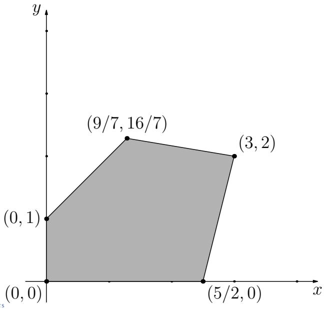
$$\begin{array}{rrl@{\qquad\qquad}rl}
 & \max\ f(x,y) &= x+y-0 & f(x,y) &= x+y-0 \\
(1) & -x+y &\leqslant 1 & -x+y &= 1 \\
(2) & x+6y &\leqslant 15 & x+6y &= 15 \\
(3) & 4x-y &\leqslant 10 & 4x-y &= 10 \\
 & x,y &\geqslant 0 & &
\end{array}$$



Slack variables

We introduce "slack variables" t_1, t_2, t_3 (one for each main constraint): $x, y, t_1, t_2, t_3 \geq 0$
At all times 2 of the 5 variables are independent, the other 3 are dependent.

$$
\begin{array}{llll}
 & \max\ f(x,y) = x + y - 0 & \qquad & f(x,y) = x + y - 0 \\
(1) & -x + y \leqslant 1 & & -x + y + t_1 = 1 \\
(2) & x + 6y \leqslant 15 & & x + 6y + t_2 = 15 \\
(3) & 4x - y \leqslant 10 & & 4x - y + t_3 = 10 \\
 & x, y \geqslant 0 & &
\end{array}
$$



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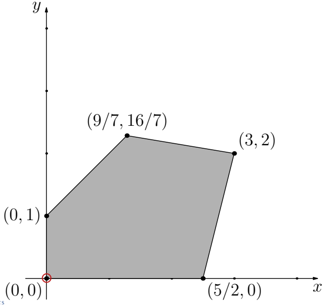
At all times 2 of the 5 variables are independent, the other 3 are dependent.
Initially: x, y are independent; ineqs. $\{x, y \geq 0\}$ are tight; extreme point is $(x, y) = (0, 0)$

max	$f(x,y) = x + y - 0$	$f(x,y) = x + y - 0$
(1)	$-x + y \leqslant 1$	$-x + y + t_1 = 1$
(2)	$x + 6y \leqslant 15$	$x + 6y + t_2 = 15$
(3)	$4x - y \leqslant 10$	$4x - y + t_3 = 10$
	$x, y \geqslant 0$	$x = 0$
		$y = 0$

x, y are independent variables

t_1, t_2, t_3 are dependent variables



Slack variables

We introduce "slack variables" t_1, t_2, t_3 (one for each main constraint): $x, y, t_1, t_2, t_3 \geq 0$
At all times 2 of the 5 variables are independent, the other 3 are dependent.
To move $(0,0) \mapsto (5/2,0)$, $x \geq 0$ will go "slack" (so x becomes dependent),
and (3) will go "tight" ($t_3 \to 0$ and t_3 becomes independent)

	max $f(x,y) = x + y - 0$	$f(x,y) = x + y - 0$
(1)	$-x + y \leqslant 1$	$-x + y + t_1 = 1$
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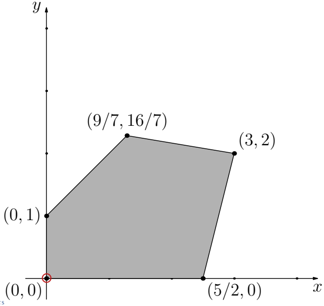

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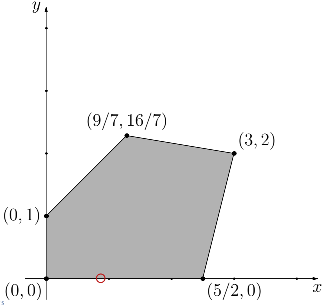
(1)	$-x + y \leqslant 1$	$-x + y + t_1 = 1$
(2)	$x + 6y \leqslant 15$	$x + 6y + t_2 = 15$
(3)	$4x - y \leqslant 10$	$4x - y + t_3 = 10$
	$x, y \geqslant 0$	$x > 0$
		$y = 0$

x, y are independent variables

t_1, t_2, t_3 are dependent variables



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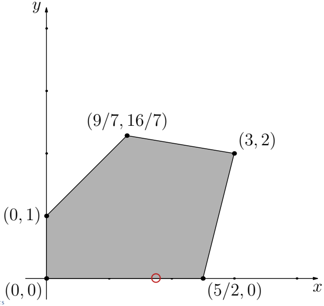
\begin{array}{lcc}
 & \max\ f(x,y) = x + y - 0 & f(x,y) = x + y - 0 \\
(1) & -x + y \leqslant 1 & -x + y + t_1 = 1 \\
(2) & x + 6y \leqslant 15 & x + 6y + t_2 = 15 \\
(3) & 4x - y \leqslant 10 & 4x - y + t_3 = 10 \\
 & x, y \geqslant 0 & x > 0 \\
 & & y = 0
\end{array}
$$

x, y are independent variables

t_1, t_2, t_3 are dependent variables



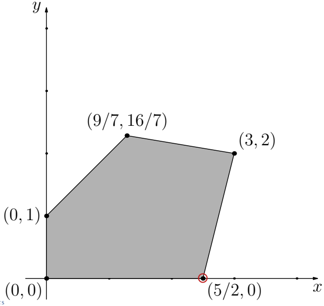
Slack variables

We introduce "slack variables" t_1, t_2, t_3 (one for each main constraint): $x, y, t_1, t_2, t_3 \geq 0$
At all times 2 of the 5 variables are independent, the other 3 are dependent.
To move $(0,0) \mapsto (5/2, 0)$, $x \geq 0$ will go "slack" (so x becomes dependent),
and (3) will go "tight" ($t_3 \to 0$ and t_3 becomes independent)

	$\max\ f(x,y) = x + y - 0$	$f(x,y) = x + y - 0$
(1)	$-x + y \leqslant 1$	$-x + y + t_1 = 1$
(2)	$x + 6y \leqslant 15$	$x + 6y + t_2 = 15$
(3)	$4x - y \leqslant 10$	$4x - y + t_3 = 10$ $(t_3 = 0)$
	$x, y \geqslant 0$	$x > 0$
		$y = 0$

x, t_3 are independent variables

t_1, t_2, y are dependent variables



We introduce "slack variables" t_1, t_2, t_3 (one for each main constraint): $x, y, t_1, t_2, t_3 \geq 0$
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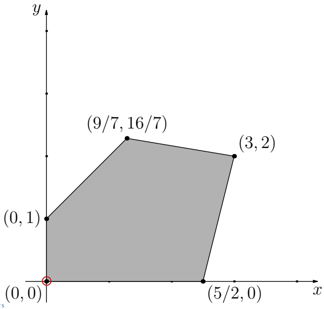
To move $(0,0) \mapsto (5/2, 0)$, $x \geq 0$ will go "slack" (so x becomes dependent),
and (3) will go "tight" ($t_3 \to 0$ and t_3 becomes independent)

	max $f(x,y) = x + y - 0$	$f(x,y) = x + y - 0$
(1)	$-x + y \leqslant 1$	$-x + y + t_1 = 1$
(2)	$x + 6y \leqslant 15$	$x + 6y + t_2 = 15$
(3)	$4x - y \leqslant 10$	$4x - y + t_3 = 10$
	$x, y \geqslant 0$	

x, y are independent variables

t_1, t_2, t_3 are dependent variables



Slack variables

We introduce "slack variables" t_1, t_2, t_3 (one for each main constraint): $x, y, t_1, t_2, t_3 \geq 0$
At all times 2 of the 5 variables are independent, the other 3 are dependent.

$$
\begin{array}{llll}
 & \max\ f(x,y) = x + y - 0 & f(x,y) = x + y - 0 & x + y - 0 = f(x,y) \\
(1) & -x + y \leqslant 1 & -x + y + t_1 = 1 & -x + y - 1 = -t_1 \\
(2) & x + 6y \leqslant 15 & x + 6y + t_2 = 15 & x + 6y - 15 = -t_2 \\
(3) & 4x - y \leqslant 10 & 4x - y + t_3 = 10 & 4x - y - 10 = -t_3 \\
 & x, y \geqslant 0 & &
\end{array}
$$

x, y are independent variables

t_1, t_2, t_3 are dependent variables

Slack variables

We introduce "slack variables" t_1, t_2, t_3 (one for each main constraint): $x, y, t_1, t_2, t_3 \geq 0$
At all times 2 of the 5 variables are independent, the other 3 are dependent.

$$
\begin{array}{llll}
 & \max\ f(x,y) = x + y - 0 & f(x,y) = x + y - 0 & x + y - 0 = f(x,y) \\
(1) & -x + y \leqslant 1 & -x + y + t_1 = 1 & -x + y - 1 = -t_1 \\
(2) & x + 6y \leqslant 15 & x + 6y + t_2 = 15 & x + 6y - 15 = -t_2 \\
(3) & 4x - y \leqslant 10 & 4x - y + t_3 = 10 & 4x - y - 10 = -t_3 \\
 & x, y \geqslant 0 & &
\end{array}
$$

x, y are independent variables

t_1, t_2, t_3 are dependent variables

Write the data into a tableau (Tucker tableau):

$$
\begin{array}{cc|c|l}
x & y & -1 & \\
\hline
 & & & = -t_1 \\
 & & & = -t_2 \\
 & & & = -t_3 \\
\hline
 & & & = f \\
\hline
\end{array}
$$

Slack variables

We introduce "slack variables" t_1, t_2, t_3 (one for each main constraint): $x, y, t_1, t_2, t_3 \geq 0$
At all times 2 of the 5 variables are independent, the other 3 are dependent.

$$
\begin{array}{lrcl}
 & \max\ f(x,y) = x + y - 0 & \qquad f(x,y) = x + y - 0 & \qquad x + y - 0 = f(x,y) \\
(1) & -x + y \leqslant 1 & \qquad -x + y + t_1 = 1 & \qquad -x + y - 1 = -t_1 \\
(2) & x + 6y \leqslant 15 & \qquad x + 6y + t_2 = 15 & \qquad x + 6y - 15 = -t_2 \\
(3) & 4x - y \leqslant 10 & \qquad 4x - y + t_3 = 10 & \qquad 4x - y - 10 = -t_3 \\
 & x, y \geqslant 0 & &
\end{array}
$$

x, y are independent variables

t_1, t_2, t_3 are dependent variables

Write the data into a tableau (Tucker tableau):

$$
\begin{array}{cc|c|l}
x & y & -1 & \\
\hline
-1 & 1 & 1 & = -t_1 \\
1 & 6 & 15 & = -t_2 \\
4 & -1 & 10 & = -t_3 \\
\hline
 & & & = f \\
\hline
\end{array}
$$

Slack variables

We introduce "slack variables" t_1, t_2, t_3 (one for each main constraint): $x, y, t_1, t_2, t_3 \geq 0$
At all times 2 of the 5 variables are independent, the other 3 are dependent.

$$
\begin{array}{lll}
\max \ f(x,y) = x + y - 0 & f(x,y) = x + y - 0 & x + y - 0 = f(x,y) \\
(1) \quad -x + y \leqslant 1 & -x + y + t_1 = 1 & -x + y - 1 = -t_1 \\
(2) \quad x + 6y \leqslant 15 & x + 6y + t_2 = 15 & x + 6y - 15 = -t_2 \\
(3) \quad 4x - y \leqslant 10 & 4x - y + t_3 = 10 & 4x - y - 10 = -t_3 \\
\qquad x, y \geqslant 0 & &
\end{array}
$$

x, y are independent variables

t_1, t_2, t_3 are dependent variables

Write the data into a tableau (Tucker tableau):

x	y	-1	
-1	1	1	$= -t_1$
1	6	15	$= -t_2$
4	-1	10	$= -t_3$
1	1	0	$= f$

Slack variables

We introduce "slack variables" t_1, t_2, t_3 (one for each main constraint): $x, y, t_1, t_2, t_3 \geq 0$
At all times 2 of the 5 variables are independent, the other 3 are dependent.

$$\begin{array}{lrl}
 & \max\ f(x,y) = x + y - 0 \\
(1) & -x + y \leqslant 1 \\
(2) & x + 6y \leqslant 15 \\
(3) & 4x - y \leqslant 10 \\
 & x, y \geqslant 0
\end{array}
\qquad
\begin{array}{l}
f(x,y) = x + y - 0 \\
-x + y + t_1 = 1 \\
x + 6y + t_2 = 15 \\
4x - y + t_3 = 10
\end{array}
\qquad
\begin{array}{r}
x + y - 0 = f(x,y) \\
-x + y - 1 = -t_1 \\
x + 6y - 15 = -t_2 \\
4x - y - 10 = -t_3
\end{array}$$

x, y are independent variables

t_1, t_2, t_3 are dependent variables

Write the data into a tableau (Tucker tableau): ($*$ indicates pivot entry)

x	y	-1	
-1	1	1	$= -t_1$
1	6	15	$= -t_2$
4^*	-1	10	$= -t_3$
1	1	0	$= f$

Slack variables

We introduce "slack variables" t_1, t_2, t_3 (one for each main constraint): $x, y, t_1, t_2, t_3 \geq 0$
At all times 2 of the 5 variables are independent, the other 3 are dependent.

max	$f(x,y) = x + y - 0$	$f(x,y) = x + y - 0$	$x + y - 0 = f(x,y)$
(1)	$-x + y \leqslant 1$	$-x + y + t_1 = 1$	$-x + y - 1 = -t_1$
(2)	$x + 6y \leqslant 15$	$x + 6y + t_2 = 15$	$x + 6y - 15 = -t_2$
(3)	$4x - y \leqslant 10$	$4x - y + t_3 = 10$	$4x - y - 10 = -t_3$
	$x, y \geqslant 0$		

x, y are independent variables

t_1, t_2, t_3 are dependent variables

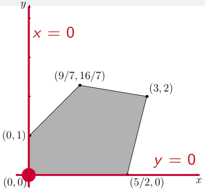
Write the data into a tableau (Tucker tableau): ($*$ indicates pivot entry)

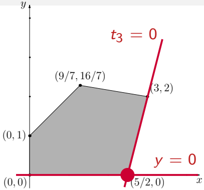
x	y	-1	
-1	1	1	$= -t_1$
1	6	15	$= -t_2$
4^*	-1	10	$= -t_3$
1	1	0	$= f$

$\xrightarrow{\text{Challenge: swap } x \text{ and } t_3}$

t_3	y	-1	
			$= -t_1$
			$= -t_2$
			$= -x$
			$= f$

Swapping two variables: $x \longleftrightarrow t_3$:





$$x + y - 0 = f(x, y)$$

(1) $\quad -x + y - 1 = -t_1$

(2) $\quad x + 6y - 15 = -t_2$

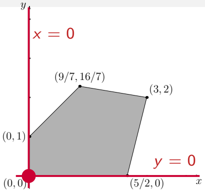
(3) $\quad 4x - y - 10 = -t_3$

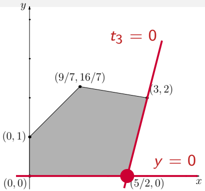
x	y	-1	
-1	1	1	$= -\,t_1$
1	6	15	$= -\,t_2$
4^*	-1	10	$= -\,t_3$
1	1	0	$= f$

Challenge:

swap x and t_3 $\longrightarrow$

t_3	y	-1	
			$= -t_1$
			$= -t_2$
			$= -x$
			$= f$





$$x + y - 0 = f(x, y)$$

(1) $-x + y - 1 = -t_1$

(2) $x + 6y - 15 = -t_2$

(3) $4x - y - 10 = -t_3$ $\qquad x - y/4 - 10/4 = -t_3/4$

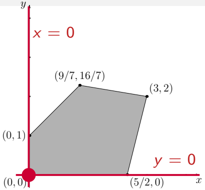
x	y	-1	
-1	1	1	$= -t_1$
1	6	15	$= -t_2$
4^*	-1	10	$= -t_3$
1	1	0	$= f$

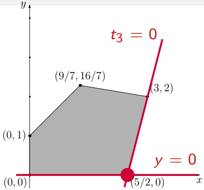
Challenge:

$\xrightarrow{\text{swap } x \text{ and } t_3}$

t_3	y	-1	
			$= -t_1$
			$= -t_2$
			$= -x$
			$= f$

Swapping two variables: $x \longleftrightarrow t_3$:





$$x + y - 0 = f(x, y)$$

(1) $\quad -x + y - 1 = -t_1$

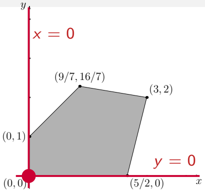
(2) $\quad x + 6y - 15 = -t_2$

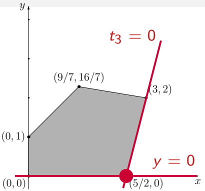
(3) $\quad 4x - y - 10 = -t_3 \qquad x - y/4 - 10/4 = -t_3/4 \qquad t_3/4 - y/4 - 10/4 = -x$

x	y	-1	
-1	1	1	$= -t_1$
1	6	15	$= -t_2$
4^*	-1	10	$= -t_3$
1	1	0	$= f$

Challenge:
swap x and t_3 $\longrightarrow$

t_3	y	-1	
			$= -t_1$
			$= -t_2$
			$= -x$
			$= f$





$$x + y - 0 = f(x, y)$$

(1) $-x + y - 1 = -t_1$

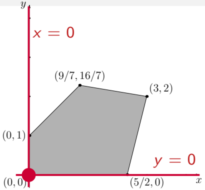
(2) $x + 6y - 15 = -t_2$

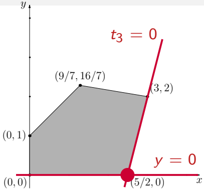
(3) $4x - y - 10 = -t_3$ $\qquad x - y/4 - 10/4 = -t_3/4 \qquad t_3/4 - y/4 - 10/4 = -x$

x	y	-1	
-1	1	1	$= -t_1$
1	6	15	$= -t_2$
4^*	-1	10	$= -t_3$
1	1	0	$= f$

Challenge:
swap x and t_3 $\longrightarrow$

t_3	y	-1	
			$= -t_1$
			$= -t_2$
1/4	-1/4	5/2	$= -x$
			$= f$





$$x + y - 0 = f(x,y)$$

(1) $-x + y - 1 = -t_1$

(2) $x + 6y - 15 = -t_2$ $\qquad x = -t_3/4 + y/4 + 5/2$

(3) $4x - y - 10 = -t_3$ $\qquad x - y/4 - 10/4 = -t_3/4$ $\qquad t_3/4 - y/4 - 10/4 = -x$

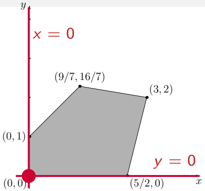
x	y	-1	
-1	1	1	$= -t_1$
1	6	15	$= -t_2$
4^*	-1	10	$= -t_3$
1	1	0	$= f$

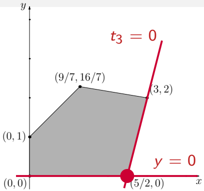
Challenge:

$\xrightarrow{\text{swap } x \text{ and } t_3}$

t_3	y	-1	
			$= -t_1$
			$= -t_2$
1/4	-1/4	5/2	$= -x$
			$= f$

Swapping two variables: $x \longleftrightarrow t_3$: Solve for x and substitute everywhere





$$x + y - 0 = f(x, y)$$

(1) $-x + y - 1 = -t_1$

(2) $x + 6y - 15 = -t_2$ $\qquad x = -t_3/4 + y/4 + 5/2 \qquad -t_3/4 + 25y/4 - 25/2 = -t_2$

(3) $4x - y - 10 = -t_3$ $\qquad x - y/4 - 10/4 = -t_3/4 \qquad t_3/4 - y/4 - 10/4 = -x$

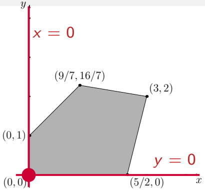
x	y	-1	
-1	1	1	$= -t_1$
1	6	15	$= -t_2$
4^*	-1	10	$= -t_3$
1	1	0	$= f$

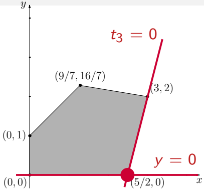
Challenge:

$\xrightarrow{\text{swap } x \text{ and } t_3}$

t_3	y	-1	
			$= -t_1$
			$= -t_2$
1/4	-1/4	5/2	$= -x$
			$= f$

Swapping two variables: $x \longleftrightarrow t_3$: Solve for x and substitute everywhere





$$x + y - 0 = f(x, y)$$

(1) $-x + y - 1 = -t_1$

(2) $x + 6y - 15 = -t_2$ $\quad x = -t_3/4 + y/4 + 5/2 \quad -t_3/4 + 25y/4 - 25/2 = -t_2$

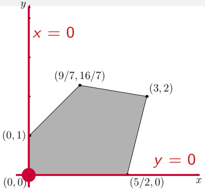
(3) $4x - y - 10 = -t_3$ $\quad x - y/4 - 10/4 = -t_3/4 \quad t_3/4 - y/4 - 10/4 = -x$

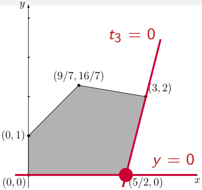
x	y	-1	
-1	1	1	$= -t_1$
1	6	15	$= -t_2$
4^*	-1	10	$= -t_3$
1	1	0	$= f$

Challenge:

swap x and t_3 $\longrightarrow$

t_3	y	-1	
			$= -t_1$
-1/4	25/4	25/2	$= -t_2$
1/4	-1/4	5/2	$= -x$
			$= f$





$$x + y - 0 = f(x, y)$$

(1) $-x + y - 1 = -t_1$ $\quad -x = t_3/4 - y/4 - 5/2$

(2) $x + 6y - 15 = -t_2$ $\quad x = -t_3/4 + y/4 + 5/2$ $\quad -t_3/4 + 25y/4 - 25/2 = -t_2$

(3) $4x - y - 10 = -t_3$ $\quad x - y/4 - 10/4 = -t_3/4$ $\quad t_3/4 - y/4 - 10/4 = -x$

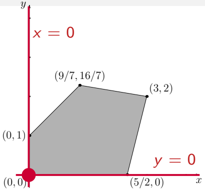
x	y	-1	
-1	1	1	$= -t_1$
1	6	15	$= -t_2$
4^*	-1	10	$= -t_3$
1	1	0	$= f$

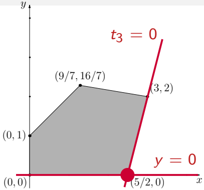
Challenge:

$\xrightarrow{\text{swap } x \text{ and } t_3}$

t_3	y	-1	
			$= -t_1$
-1/4	25/4	25/2	$= -t_2$
1/4	-1/4	5/2	$= -x$
			$= f$

Swapping two variables: $x \longleftrightarrow t_3$: Solve for x and substitute everywhere





$$x + y - 0 = f(x, y)$$

(1)	$-x + y - 1 = -t_1$	$-x = t_3/4 - y/4 - 5/2$	$t_3/4 + 3y/4 - 7/2 = -t_1$
(2)	$x + 6y - 15 = -t_2$	$x = -t_3/4 + y/4 + 5/2$	$-t_3/4 + 25y/4 - 25/2 = -t_2$
(3)	$4x - y - 10 = -t_3$	$x - y/4 - 10/4 = -t_3/4$	$t_3/4 - y/4 - 10/4 = -x$

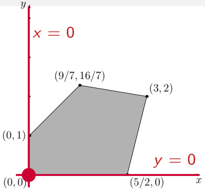
x	y	-1	
-1	1	1	$= -t_1$
1	6	15	$= -t_2$
4^*	-1	10	$= -t_3$
1	1	0	$= f$

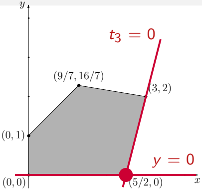
Challenge:

$\xrightarrow{\text{swap } x \text{ and } t_3}$

t_3	y	-1	
			$= -t_1$
-1/4	25/4	25/2	$= -t_2$
1/4	-1/4	5/2	$= -x$
			$= f$

Swapping two variables: $x \longleftrightarrow t_3$: Solve for x and substitute everywhere





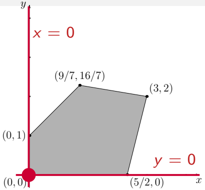
$$x + y - 0 = f(x,y)$$

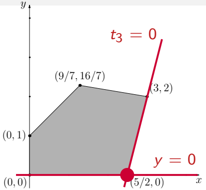
(1)	$-x + y - 1 = -t_1$	$-x = t_3/4 - y/4 - 5/2$	$t_3/4 + 3y/4 - 7/2 = -t_1$
(2)	$x + 6y - 15 = -t_2$	$x = -t_3/4 + y/4 + 5/2$	$-t_3/4 + 25y/4 - 25/2 = -t_2$
(3)	$4x - y - 10 = -t_3$	$x - y/4 - 10/4 = -t_3/4$	$t_3/4 - y/4 - 10/4 = -x$

x	y	-1	
-1	1	1	$= -t_1$
1	6	15	$= -t_2$
4^*	-1	10	$= -t_3$
1	1	0	$= f$

Challenge:
$\xrightarrow{\text{swap } x \text{ and } t_3}$

t_3	y	-1	
1/4	3/4	7/2	$= -t_1$
-1/4	25/4	25/2	$= -t_2$
1/4	-1/4	5/2	$= -x$
			$= f$





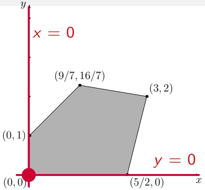
$$
\begin{array}{llll}
 & x + y - 0 = f(x,y) & & -t_3/4 + 5y/4 + 5/2 = f \\
(1) & -x + y - 1 = -t_1 & -x = t_3/4 - y/4 - 5/2 & t_3/4 + 3y/4 - 7/2 = -t_1 \\
(2) & x + 6y - 15 = -t_2 & x = -t_3/4 + y/4 + 5/2 & -t_3/4 + 25y/4 - 25/2 = -t_2 \\
(3) & 4x - y - 10 = -t_3 & x - y/4 - 10/4 = -t_3/4 & t_3/4 - y/4 - 10/4 = -x
\end{array}
$$

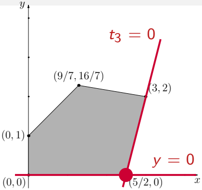
x	y	-1	
-1	1	1	$= -t_1$
1	6	15	$= -t_2$
4^*	-1	10	$= -t_3$
1	1	0	$= f$

Challenge:
swap x and t_3 $\longrightarrow$

t_3	y	-1	
1/4	3/4	7/2	$= -t_1$
-1/4	25/4	25/2	$= -t_2$
1/4	-1/4	5/2	$= -x$
			$= f$

Swapping two variables: $x \longleftrightarrow t_3$: Solve for x and substitute everywhere



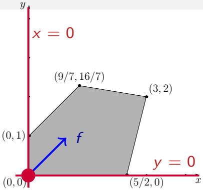


	$x + y - 0 = f(x,y)$		$-t_3/4 + 5y/4 + 5/2 = f$
(1)	$-x + y - 1 = -t_1$	$-x = t_3/4 - y/4 - 5/2$	$t_3/4 + 3y/4 - 7/2 = -t_1$
(2)	$x + 6y - 15 = -t_2$	$x = -t_3/4 + y/4 + 5/2$	$-t_3/4 + 25y/4 - 25/2 = -t_2$
(3)	$4x - y - 10 = -t_3$	$x - y/4 - 10/4 = -t_3/4$	$t_3/4 - y/4 - 10/4 = -x$

x	y	-1	
-1	1	1	$= -t_1$
1	6	15	$= -t_2$
4^*	-1	10	$= -t_3$
1	1	0	$= f$

$$\xrightarrow{\text{Challenge: swap } x \text{ and } t_3}$$

t_3	y	-1	
1/4	3/4	7/2	$= -t_1$
-1/4	25/4	25/2	$= -t_2$
1/4	-1/4	5/2	$= -x$
-1/4	5/4	-5/2	$= f$

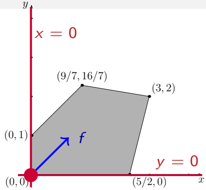


$$\max\ f(x,y) = x + y - 0$$

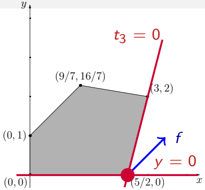
$$\begin{array}{ll} (1) & -x + y \leqslant 1 \\ (2) & x + 6y \leqslant 15 \\ (3) & 4x - y \leqslant 10 \\ & x, y \geqslant 0 \end{array}$$

$$\begin{array}{c} \begin{array}{ccc} x & y & -1 \end{array} \\ \begin{array}{|cc|c|}\hline -1 & 1 & 1 \\ 1 & 6 & 15 \\ 4 & -1 & 10 \\ \hline 1 & 1 & 0 \\ \hline \end{array} \end{array} \begin{array}{l} \\ = -t_1 \\ = -t_2 \\ = -t_3 \\ = f \end{array}$$

Two pivots of the Simplex Algorithm

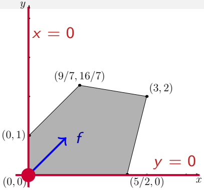


$\rightarrow$

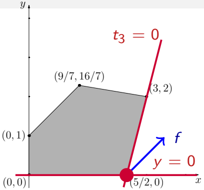


x	y	-1	
-1	1	1	$= -t_1$
1	6	15	$= -t_2$
4^*	-1	10	$= -t_3$
1	1	0	$= f$

First pivot: $x \leftrightarrow t_3$



$\rightarrow$



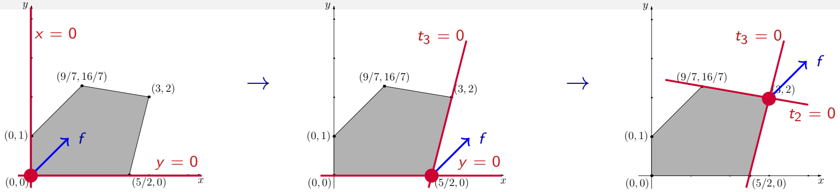
x	y	-1	
-1	1	1	$= -t_1$
1	6	15	$= -t_2$
4^*	-1	10	$= -t_3$
1	1	0	$= f$

$\rightarrow$

t_3	y	-1	
1/4	3/4	7/2	$= -t_1$
-1/4	25/4	25/2	$= -t_2$
1/4	-1/4	5/2	$= -x$
-1/4	5/4	-5/2	$= f$

First pivot: $x \leftrightarrow t_3$

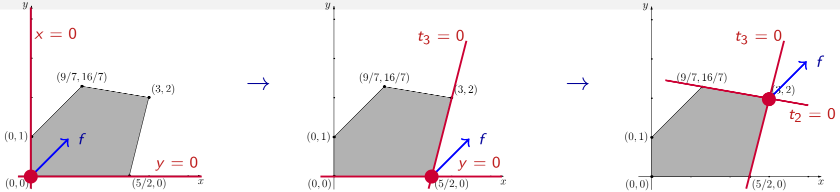
Two pivots of the Simplex Algorithm



$$\begin{array}{cc|c|l} x & y & -1 & \\ \hline -1 & 1 & 1 & = -t_1 \\ 1 & 6 & 15 & = -t_2 \\ 4^* & -1 & 10 & = -t_3 \\ \hline 1 & 1 & 0 & = f \end{array} \quad \rightarrow \quad \begin{array}{cc|c|l} t_3 & y & -1 & \\ \hline 1/4 & 3/4 & 7/2 & = -t_1 \\ -1/4 & 25/4^* & 25/2 & = -t_2 \\ 1/4 & -1/4 & 5/2 & = -x \\ \hline -1/4 & 5/4 & -5/2 & = f \end{array}$$

First pivot: $x \leftrightarrow t_3$ Second pivot: $y \leftrightarrow t_2$

Two pivots of the Simplex Algorithm



x	y	-1	
-1	1	1	$= -t_1$
1	6	15	$= -t_2$
4^*	-1	10	$= -t_3$
1	1	0	$= f$

$\rightarrow$

t_3	y	-1	
1/4	3/4	7/2	$= -t_1$
-1/4	$25/4^*$	25/2	$= -t_2$
1/4	-1/4	5/2	$= -x$
-1/4	5/4	-5/2	$= f$

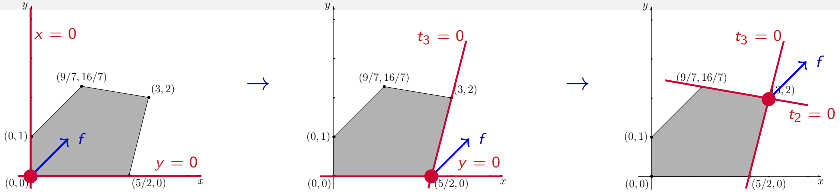
$\rightarrow$

t_3	t_2	-1	
7/25	-3/25	2	$= -t_1$
-1/25	4/25	2	$= -y$
6/25	1/25	3	$= -x$
-1/5	-1/5	-5	$= f$

First pivot: $x \leftrightarrow t_3$

Second pivot: $y \leftrightarrow t_2$

Two pivots of the Simplex Algorithm



x	y	-1	
-1	1	1	$= -t_1$
1	6	15	$= -t_2$
4^*	-1	10	$= -t_3$
1	1	0	$= f$

→

t_3	y	-1	
1/4	3/4	7/2	$= -t_1$
-1/4	$25/4^*$	25/2	$= -t_2$
1/4	-1/4	5/2	$= -x$
-1/4	5/4	-5/2	$= f$

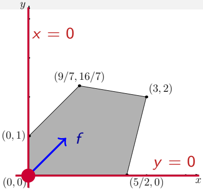
→

t_3	t_2	-1	
7/25	-3/25	2	$= -t_1$
-1/25	4/25	2	$= -y$
6/25	1/25	3	$= -x$
-1/5	-1/5	-5	$= f$

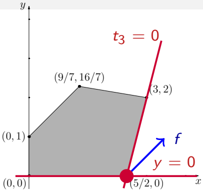
First pivot: $x \leftrightarrow t_3$ Second pivot: $y \leftrightarrow t_2$

An optimum solution found (since both main entries in last row are $\leqslant 0$)

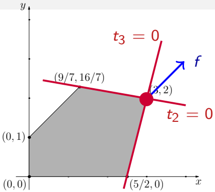
Two pivots of the Simplex Algorithm



$\rightarrow$



$\rightarrow$



x	y	-1	
-1	1	1	$= -t_1$
1	6	15	$= -t_2$
4^*	-1	10	$= -t_3$
1	1	0	$= f$

$\rightarrow$

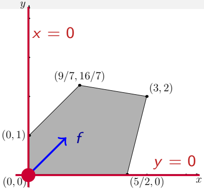
t_3	y	-1	
1/4	3/4	7/2	$= -t_1$
-1/4	$25/4^*$	25/2	$= -t_2$
1/4	-1/4	5/2	$= -x$
-1/4	5/4	-5/2	$= f$

$\rightarrow$

t_3	t_2	-1	
7/25	-3/25	2	$= -t_1$
-1/25	4/25	2	$= -y$
6/25	1/25	3	$= -x$
-1/5	-1/5	-5	$= f$

First pivot: $x \leftrightarrow t_3$ Second pivot: $y \leftrightarrow t_2$

An optimum solution found (since both main entries in last row are $\leqslant 0$)
This optimum solution is $(x, y) = (2, 3)$



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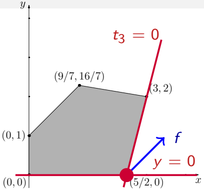
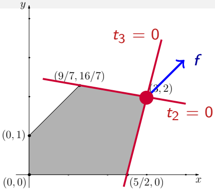


→



	x	y	-1	
	-1	1	1	$= -t_1$
	1	6	15	$= -t_2$
	4^*	-1	10	$= -t_3$
	1	1	0	$= f$

→

	t_3	y	-1	
	1/4	3/4	7/2	$= -t_1$
	-1/4	$25/4^*$	25/2	$= -t_2$
	1/4	-1/4	5/2	$= -x$
	-1/4	5/4	-5/2	$= f$

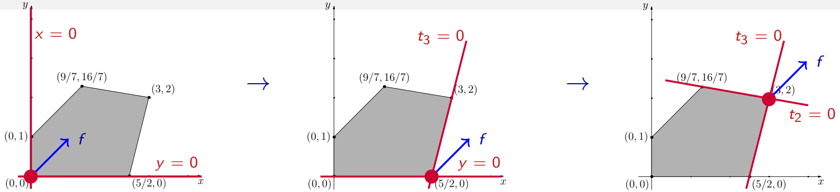
→

	t_3	t_2	-1	
	7/25	-3/25	2	$= -t_1$
	-1/25	4/25	2	$= -y$
	6/25	1/25	3	$= -x$
	-1/5	-1/5	-5	$= f$

First pivot: $x \leftrightarrow t_3$ Second pivot: $y \leftrightarrow t_2$

An optimum solution found (since both main entries in last row are $\leqslant 0$)
This optimum solution is $(x, y) = (2, 3)$, since $(t_3, t_2) = (0, 0)$

Two pivots of the Simplex Algorithm



x	y	-1	
-1	1	1	$= -t_1$
1	6	15	$= -t_2$
4^*	-1	10	$= -t_3$
1	1	0	$= f$

$\rightarrow$

t_3	y	-1	
1/4	3/4	7/2	$= -t_1$
-1/4	$25/4^*$	25/2	$= -t_2$
1/4	-1/4	5/2	$= -x$
-1/4	5/4	-5/2	$= f$

$\rightarrow$

t_3	t_2	-1	
7/25	-3/25	2	$= -t_1$
-1/25	4/25	2	$= -y$
6/25	1/25	3	$= -x$
-1/5	-1/5	-5	$= f$

First pivot: $x \leftrightarrow t_3$ Second pivot: $y \leftrightarrow t_2$

An optimum solution found (since both main entries in last row are $\leqslant 0$)
This optimum solution is $(x, y) = (2, 3)$, since $(t_3, t_2) = (0, 0)$
The optimum value is $f(x, y) = (-1) \cdot (-5) = 5$