Quiz 3 - MACM 201 - *Solutions*

[4 pts]   How many subgraphs of K_{100} are isomorphic to

(a) $K_{1,4}$?

(b) $K_{3,8}$?

Solution:

(a) You can select a $K_{1,4}$ uniquely by dividing the vertices into a set of 1 vertex, a set of 4 vertices, and then the remaining 95. So our answer is $\binom{100}{1,4,95}$. Alternately, you can select a set of 5 vertices and then choose which of the 5 to be incident to all others. This gives us the (equal) answer $\binom{100}{5}5$.

(b) You can select a $K_{3,8}$ uniquely by dividing the vertices into a set of 3 vertices, a set of 8 vertices, and then the remaining 89. So our answer is $\binom{100}{3,8,89}$. Alternately, you can select the 11 vertices to be used in your subgraph, and then select which 3 element subset will form the small side in the bipartite graph. This gives the equal answer of $\binom{100}{11}\binom{11}{3}$.

[4 pts] Up to isomorphism, find all graphs with 4 vertices and 3 edges.
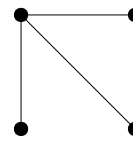
Solution: There are just 3 such graphs:
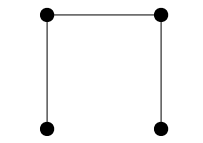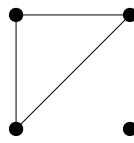
| 4 vertex path | 3 vertex cycle + isolated vertex | $K_{1,3}$ |