

MACM 201 - Discrete Mathematics

5 Graph Isomorphism

Department of Mathematics

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Isomorphism

Definition

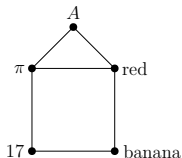
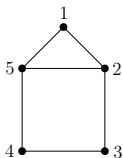
Two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are *isomorphic* if there exists a bijection $f : V_1 \rightarrow V_2$ satisfying:

$$E_2 = \{\{f(u), f(v)\} \mid u, v \in E_1\}.$$

Equivalently, G_1 and G_2 are isomorphic if the same drawing is valid for both graphs (but with different labels on the vertices).

Note: By the definition, any two paths on the same number of vertices are isomorphic. The same is true for any two cycles on the same number of vertices.

Example The following graphs are isomorphic

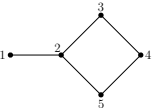


Counting subgraphs

Problem 1. How many subgraphs of K_n are isomorphic to K_t ?

Problem 2. Let K_4^- be the graph obtained from K_4 by deleting an edge. How many subgraphs of K_n are isomorphic to K_4^- ?

Problem 3. How many subgraphs of K_{n_1, n_2} are isomorphic to $K_{3,4}$?

Problem 4. Let H be the graph . How many subgraphs of $K_{n,n}$ are isomorphic to H ?