



### Objectives of this assignment:

- to explore time complexity and “real time”

### What you need to do:

1. Implement the **Merge-Sort** algorithm to sort an array. (See Appendix for the Merge-Sort algorithm)
2. Collect the execution time  $T(n)$  as a function of  $n$
3. Plot the functions  $T(n)/\log_2(n)$ ,  $T(n)/n.\log_2(n)$ , and  $T(n)/n^2.\log_2(n)$  on the same graph.
4. In Module 4 (next module), we will establish that the running time  $T(n)$  of Merge-Sort is  $\Theta(n.\log(n))$ . Discuss  $T(n)$  in light of the graph you plotted above.

### Objective:

The objective of this programming assignment is to design and implement in Java the Merge-Sort algorithm presented in the lecture to sort a list of numbers. We are interested in exploring the relationship between the time complexity and the “real time”. For this exploration, you will collect the execution time  $T(n)$  as a function of  $n$  and plot the functions  $T(n)/\log_2(n)$ ,  $T(n)/n.\log_2(n)$ , and  $T(n)/n^2.\log_2(n)$  on the same graph. Finally, discuss your results.

### Program to implement

```
collectData()
    Generate an array G of HUGE length L (as huge as your language allows)
    with random values capped at 0xffffffff.
    for n = 5,000 to L (with step 1,000)
        copy in Array A n first values from Array G

        Start timing // We time the sorting of Array A of length n
        Merge-Sort(A,0,n-1)
        Store the value n and the values  $T(n)/\log_2(n)$ ,  $T(n)/n.\log_2(n)$ , and
         $T(n)/n^2.\log_2(n)$  in a file F where  $T(n)$  is the execution time
```

### Data Analysis

Use any plotting software (e.g., Excel) to plot the values  $T(n)/\log_2(n)$ ,  $T(n)/n.\log_2(n)$ , and  $T(n)/n^2.\log_2(n)$  in File F as a function of  $n$ . File F is the file produced by the program you implemented. Discuss your results based on the plots. (**Hint:** is  $T(n)$  closer to  $K.\log_2(n)$ ,  $K.n.\log_2(n)$ , or  $K.n^2.\log_2(n)$  where  $K$  is a constant?)

### Report

- Write a report that will contain, explain, and discuss the plot. The report should not exceed one page.
- In addition, your report must contain the following information:
  - whether the program works or not (this must be just ONE sentence)
  - the directions to compile and execute your program
- Good writing is expected.
- Recall that answers must be well written, documented, justified, and presented to get full credit.

### What you need to turn in:

- Electronic copy of your source program
- Electronic copy of the report (including your answers) (standalone). Submit the file as a Microsoft Word or PDF file.

### Grading

- Program is worth 30% if it works and provides data to analyze
- Quality of the report is worth 70% distributed as follows: good plot (25%), explanations of plot (10%), discussion



and conclusion (35%).

Appendix: Merge-Sort Algorithm.

At this stage, you do NOT need to understand Merge-Sort (It will be presented and explained in Module 4)).

Implement Merge-Sort exactly the way it is described below. Replace the infinity value ( $\infty$ ) with 0xffffffff.

MERGE-SORT( $A, p, r$ )

```
1  if  $p < r$ 
2       $q = \lfloor (p + r) / 2 \rfloor$ 
3      MERGE-SORT( $A, p, q$ )
4      MERGE-SORT( $A, q + 1, r$ )
5      MERGE( $A, p, q, r$ )
```

MERGE( $A, p, q, r$ )

```
1   $n_1 = q - p + 1$ 
2   $n_2 = r - q$ 
3  let  $L[1..n_1 + 1]$  and  $R[1..n_2 + 1]$  be new arrays
4  for  $i = 1$  to  $n_1$ 
5       $L[i] = A[p + i - 1]$ 
6  for  $j = 1$  to  $n_2$ 
7       $R[j] = A[q + j]$ 
8   $L[n_1 + 1] = \infty$ 
9   $R[n_2 + 1] = \infty$ 
10  $i = 1$ 
11  $j = 1$ 
12 for  $k = p$  to  $r$ 
13     if  $L[i] \leq R[j]$ 
14          $A[k] = L[i]$ 
15          $i = i + 1$ 
16     else  $A[k] = R[j]$ 
17          $j = j + 1$ 
```