Group Coursework Submission Form

Specialist Masters Programme

Please list all names of group members: (Surname, first name) 1. Shubham Borda 2. Ziyang Huang 3. Grace Laryea	4. Sean McCallion 5. **GROUP NUMBER:** 7
MSc in: Quantitative Finance, Financial Mathematics	
Module Code: SMM270	
Module Title: FOUNDATIONS OF ECONOMETRICS	
Lecturer: PROF GIOVANNI URGA	**Submission Date: 01/12/2023**

Declaration:

By submitting this work, we declare that this work is entirely our own except those parts duly identified and referenced in my submission. It complies with any specified word limits and the requirements and regulations detailed in the coursework instructions and any other relevant programme and module documentation. In submitting this work, we acknowledge that we have read and understood the regulations and code regarding academic misconduct, including that relating to plagiarism, as specified in the Programme Handbook. We also acknowledge that this work will be subject to a variety of checks for academic misconduct.

We acknowledge that work submitted late without a granted extension will be subject to penalties, as outlined in the Programme Handbook. Penalties will be applied for a maximum of five days lateness, after which a mark of zero will be awarded.

Marker's Comments (if not being marked on-line):

Deduction for Late Submission of assignment:

For Students:
Once marked please refer to Moodle for your final coursework grade, including your Peer Assessment grade.

Question 1 – Data transformation

With the excel file given to us, we uploaded the file onto Oxmetrics and aggregated the daily data into quarterly data. Subsequently, we transformed the quarterly data by taking the natural logarithm and naming the series as requested: lp1q=ln (EURO STOXX50), lp2q=ln (HANG SENG), lp3q=ln(CAC40), and lp4q=ln(NIKKEI225).

Question 2 – Stationarity vs non-stationary time series

Part a: All the charts lp1q, lpq2, lpq3 and lp4q[1] possess an upward sloping trend as determined by the green lines explaining we expect the quarterly lognormal prices to rise over time (i.e., deterministic trend). To determine if the stocks are trend stationary, we will detrend the series to confirm, as eyeballing is not sufficient.

From the charts of the residuals, the detrended series pattern looks almost as a re-scaled version of the original series lp1q, lpq2, lpq3 and lp4q. So, from this we can infer all the series are non-stationary, hence differencing needs to be performed, even though it modifies the error term.

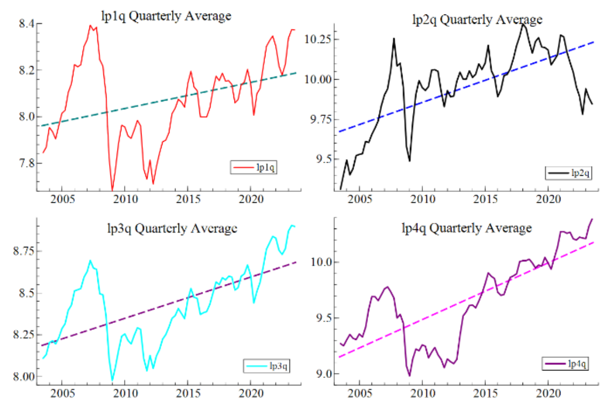


Figure 1 - Series plots

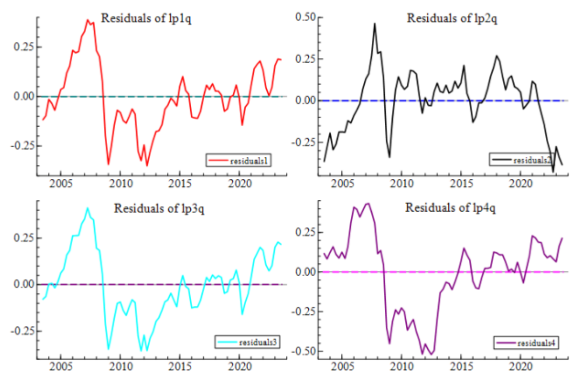


Figure 2 - Series Residual pots

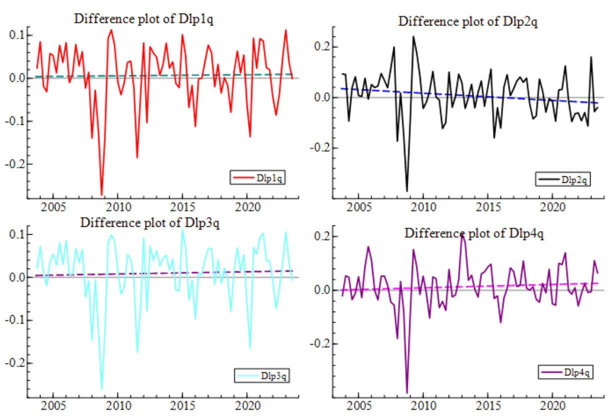


Figure 3 - Differenced Series plots

Moreover, the first order differenced plots of the series show that, the series are stationary by visual inspection even though series seem to possess some drift, thus we can conclude that series lp1q, lpq2, lpq3 and lp4q are difference stationary.

Part b: To further investigate the difference stationarity, a unit root test must be run in Oxmetrics. Based on the random walk, we obtain the following time-series equation:

$$\Delta Y_t = \propto + \beta t + \theta Y_{t-1} + \varepsilon_t$$

and hypothesis,

$$H_0: \theta = 0, Y_t \sim I(1)\ a\ unit\ root$$
$$H_1: \theta \neq 0\ , Y_t \sim I(0)\ , possesses\ stationarity$$

With critical values of $-3.47 *$ [2] and $-4.085 **$ compared at 5% and 1% significance level. Results are as summarised below,

Variable	t-value (calculated)	t-value versus Critical value	Reject/Not enough evidence to reject H_0	Order of integration	Stationary/ Unit root
Lp1q	-2.1789	Larger	Not enough evidence	1	Unit root
Dlp1q	-3.7839*	Smaller	Reject	0	Stationary
Lp2q	-2.0484	Larger	Not enough evidence	1	Unit root
Dlp2q	-4.3138**	Smaller	Reject	0	Stationary

[1] Lp1q = EURO STOXX50, Lp2q = HANG SENG, Lp3q = CAC40, Lp4q = NIKKEI225

[2] * Represents 5% critical value and ** represents 1% critical value

Lp3q	-1.9989	Larger	Not enough evidence	1	Unit root
Dlp3q	-3.5856*	Smaller	Reject	0	Stationary
Lp4q	-1.912	Larger	Not enough evidence	1	Unit root
Dlp4q	-3.465	Smaller	Reject	0	Stationary
DDlp4q	-5.477**	Smaller	Reject	0	Stationary

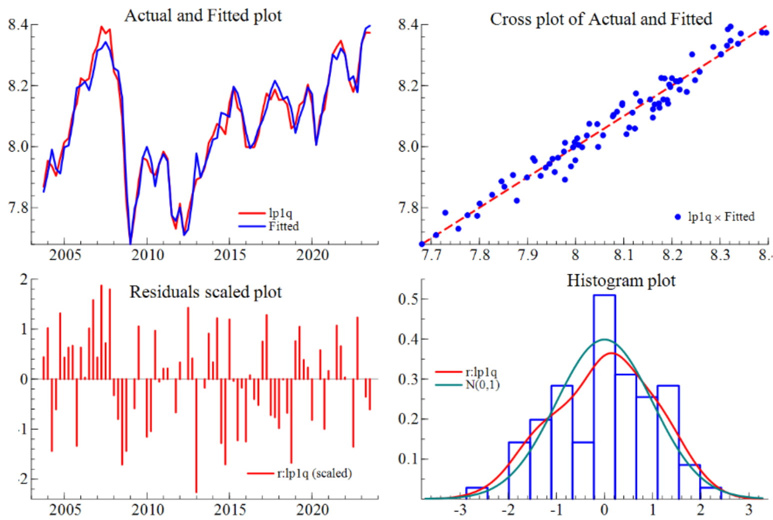
According to Holden and Perman,[3] if a timeseries variable viewed in isolation has a unit root, then we need to take the first difference of the variable in order to obtain the stationary series, which is demonstrated in the table above, as we took the first difference of lp1q, lp2q, lp3q and will have to take further differences for lp4q, as the first order difference still has a unit root based on lag 5. From further differencing we obtain stationarity at the 2nd order of differencing for each variable based on looking at the t-adf values, but when we observe the values of Dlp4q, we notice at lag 5, the t-adf is -3.465 with AIC of -4.942, with other values being significant. So, we can observe it closer and choose the best t-adf based on the lowest AIC values. In that case we choose Dlp4q at lag 2 or 1, since they have an AIC of -5.014, and both have t-adf less than -3.47 and -4.09 respectively. In conclusion, Dlp4q is stationary by further observation. Moreover, we do not have sufficient evidence to reject the null hypothesis for lp1,2,3,4q variables as other tests can be performed to counteract the unit root test.

Question 3 - Model specifications and misspecification tests

Part a: As from the inference previously, $lp1q$ and $lp4q$ are all the same integrated order of $I(1)$. From running an $ADL(1)$ model we have an equation of:

$$lp1q = a_0 + a_1\, lp1q\_1 + b_0 lp4q + b_1 lp4q\_1$$

So, we now test the time series model and notice it is slightly mis-specified from the $ADF\ test, with\ AR\ 1-5\ test$: $F(5,71) = 2.7698\ [0.0242] *$, so, we re-run the model by inserting 9 dummies from 2009 (1) to 2023 (1) which absorbed the noise from residuals of 1.5, forming a well specified model, as below.




```
AR 1-5 test:        F(5,62)   =  0.66235 [0.6533]
ARCH 1-4 test:      F(4,72)   =  0.48098 [0.7496]
Normality test:     Chi^2(2)  =   1.5638 [0.4575]
Hetero test:        F(6,64)   =  0.76730 [0.5984]
RESET23 test:       F(2,65)   =   1.3544 [0.2653]
```

Also, the plot on the right, as well as residuals not being identically and independently distributed (iid) as seen from the normal plot, as well as showing serial correlation in the quantile plot.
Due to this we impose restrictions on the model by nesting the models in the table below.

Nested Model	Restrictions	Test statistic	Reject/Not Reject
Static model	$a1= b_1=0$	F (2,76) = 128.40 [0.0000] **	Reject
Autoregressive model	$b_0=b_1=0$	F (2,76) = 38.401 [0.0000] **	Reject
Leading indicator model	$a_1= b_0=0$	F (2,76) = 137.16 [0.0000] **	Reject
Difference data model	$a_1=1$ & $b_0=- b_1$	Chi^2(2) = 3.0843 [0.2139]	Not Reject
Distributed lag model	$a_1=0$	F (1,76) = 255.70 [0.0000] **	Reject
Partial adjustment model	$b_1=0$	F (1,76) = 46.101 [0.0000] **	Reject
Static model with AR (1) errors	$a_1+ b_0+ b_1=0$	Chi^2(1) = 590.67 [0.0000] **	Reject
Autoregressive error	$a_1* b_0+ b_1=0$	Chi^2(1) = 0.75079 [0.3862]	Not Reject
Error correction	$a_1+ b_0+ b_1=1$	Chi^2(1) = 2.1966 [0.1383]	Not Reject
Dead-start model	$b_0=0$	F (1,76) = 67.067 [0.0000] **	Reject

Part b: From the table, 3 models were not rejected, the difference data model, autoregressive model and the error correction model as their test statistics were less than the significance levels. To find the best model, we can compare the chi square value of each model and take the one with the greatest value.

[3] 3.4 Error Correction Mechanism (ECM) – Holden and Perman

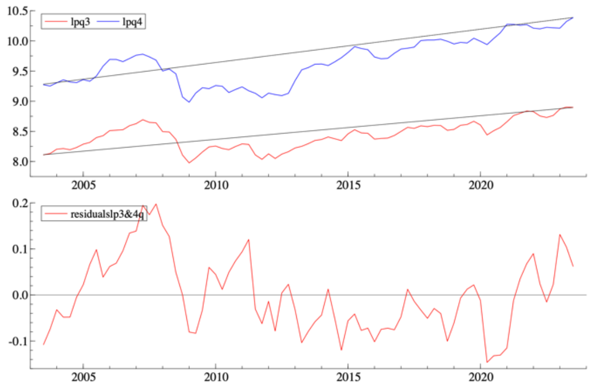
Nested Model	Test Statistic	Restriction
Autoregressive error	Chi^2(1) = 0.75079 [0.3862]	$a_1 * b_0 + b_1 = 0$
Error correction	Chi^2(1) = 2.1966 [0.1383]	$a_1 + b_0 + b_1 = 1$
Difference data model	Chi^2(2) = 3.0843 [0.2139]	$a_1 = 1$ & $b_0 = -b_1$

Based on the chi square calculations, the best model is the difference data model as it has the greatest value $3.0843 > 2.1966 > 0.75079$. Additionally, the difference data model has two parameters compared to the autoregressive error and error correction that only have one parameter. We can then assume that the difference data model indicates the best model has the lowest AIC or BIC because it has more parameters, thus the model is more parsimonious and has the best fit to the data. With $\boldsymbol{AIC = ln\hat{\sigma} + \frac{2k}{T}}$ *and* $\boldsymbol{BIC/Schwartz = ln\hat{\sigma}^2 + \frac{k}{T} lnT}$, $where\ k = number\ of\ parameters$, thus the numerator is larger which then implies that the AIC and BIC will be smaller in value. Moreover, an ADL (4,4) will be a better model as its specified and satisfies all conditions, without the use of dummies, due to missing factors in the ADL (1,1).
The Specified Model of lp1q and lp4q is at the end of the report.

Question 4 - Cointegration vs spurious regression

Part a: Based on Q2, we found that $lp3q$ and $lp4q$ are $I(1)$. Since they have the same order of integration, we must test whether $lp3q$ and $lpq4$ cointegrate or spuriously correlate. There exists two ways of identifying whether they are cointegrated or spuriously correlated:

- ADF unit root test
- 2-step Engle & Granger (covered in 4c)



Before testing, let us have a look at the graphs (on the right). From the graph, there's a clear distinction that $lpq3$ and $lpq4$ are stationary and hence cointegrated. As $lpq3$ rises, so does $lpq4$ and vice versa. Additionally, graphing the residuals of $lp3q$ and $lp4q$ can further demonstrate the existence of cointegration, as the values of the residuals are close to zero. Furthermore, the $Adj. R^2 = 0.866$ is statistically significant and further emphasises on cointegration. This indicates that a large proportion of the variance in the dependent variable, $lp3q$, is explained by the independent variable, $lp4q$, in the model. This suggests that the model fits the data well. However, to statistically check for stationarity we should run an ADF unit root test, which will help in identifying if there exists cointegration between $lp3q$ and $lp4q$

```
EQ( 6) Modelling lpq3 by OLS
       The dataset is: /Users/seanmccallion/Desktop/Bayes/Coursework
       The estimation sample is: 2004(2) - 2023(3)

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant              3.15883     0.2361     13.4  0.0000   0.7020
lpq4                 0.546234    0.02437     22.4  0.0000   0.8686

sigma               0.0818553  RSS              0.509222673
R^2                  0.868602  F(1,76) =      502.4 [0.000]**
Adj.R^2              0.866873  log-likelihood       85.5544
no. of observations        78  no. of parameters          2
mean(lpq3)            8.44609  se(lpq3)            0.224344

AR 1-5 test:      F(5,71)   =   33.461 [0.0000]**
ARCH 1-4 test:    F(4,70)   =   14.921 [0.0000]**
Normality test:   Chi^2(2)  =   5.6086 [0.0605]
Hetero test:      F(2,75)   =   2.4193 [0.0959]
RESET23 test:     F(2,74)   =   4.9114 [0.0099]**
```

Testing for the ADF unit root test, the hypothesis is not rejected given that the calculated $t - value = -2.663$ is greater than the 5% ($t - value = -2.90$) and 1% ($t - value = -3.52$) significance level. Having found that our 5 series are $I(0)$, we have evidence of cointegration and conclude that the relationship.

$$lp3q_t = \beta_0 + \beta_1 lp4q_t + u_t$$

between $lp3q$ and $lp4q$ is cointegrated.

```
Unit-root tests
The dataset is: /Users/seanmccallion/Desktop/Bayes/Coursework/QuarterlyData
The sample is: 2005(1) - 2023(3) (81 observations and 1 variables)

residualslp3&4q: ADF tests (T=75, Constant; 5%=-2.90 1%=-3.52)
D-lag    t-adf     beta Y_1    sigma  t-DY_lag  t-prob       AIC  F-prob
  5     -2.070     0.82200  0.04703    0.3909  0.6971    -6.025
  4     -2.048     0.82992  0.04674    -1.814  0.0740    -6.050  0.6971
  3     -2.685     0.78414  0.04750    0.4243  0.6726    -6.030  0.1900
  2     -2.702     0.79478  0.04722   -0.6529  0.5159    -6.054  0.3180
  1     -3.199*    0.77602  0.04703     1.971  0.0526    -6.075  0.4115
  0     -2.663     0.81960  0.04795                      -6.049  0.1774
```

For a deeper approach on whether the statement remains, the Engle & Granger 2-step process will be computed and be covered in the next page.

Part b: In time series analysis, cointegration and spurious regression are two separate concepts that are essential to comprehending the relationship between variables, especially in financial economics. Ferson et al. (2003a), Ferson et al. (2003b), and Hendry (2004) all provide useful insights into these concepts.

Cointegration refers to a statistical property exhibited by non-stationary time series variables. Many financial time series exhibit non-stationarity, which suggests that their mean or variance is not constant over time. When two or more time series are cointegrated, it means that, despite their individual random walks (which possibly make them non-stationary), they have a long-term equilibrium relationship that binds them together. As it implies predictability and a long-term relationship between financial variables—like asset prices or interest rates—that may appear to drift apart in the near term, this idea is essential to the study of financial economics.

In the context of econometric modelling, Hendry (2004) highlights the significance of cointegration, especially in figuring out the dynamic relationships between economic variables. For instance, even if two stock prices appear to move independently, cointegration implies that they are linked in the long run. Hendry (2004) discusses how this concept aids in our understanding of economic relationships. He makes the point that failing to recognise this relationship could lead to incorrect inferences about the collective behaviour of economic variables, particularly when non-stationary processes are included in the models.

Example of cointegration in the real world: Economies frequently expand in conjunction with their energy usage. A nation's GDP and total energy consumption may cointegrate over time, meaning that any significant divergence between them is temporary and will eventually return to equilibrium.

Spurious regression, on the other hand, arises when a statistical analysis mistakenly shows a significant association between non-stationary time series data, generally due to their trending nature over time. This false conclusion occurs even though there is no true causal relationship between the variables. Hendry (2004) highlights that these regressions are misleading, particularly when used to non-stationarity economic variables.
Ferson et al. (2003a) expand on spurious regressions and discuss how spurious regression bias can mislead analysts, particularly when predicting stock returns using lagged variables such as dividend yields or interest rates. They warn that variables with high autocorrelation are especially prone to producing spurious regression results, which are frequently exacerbated by inexperienced data mining practises. The combination of naïve data mining and spurious regression can be especially deceptive.

Example of spurious regression in the real world: Consider regressing the ice cream sales annually against the total number of shark attacks. Both may exhibit an increasing trend over time (possibly because of population growth or other factors). The two variables are unrelated, and their correlation is coincidental, which is a classic example of spurious regression. A false regression might imply a significant relationship.

In conclusion, it is critical for financial econometricians to comprehend the distinction between spurious regression and cointegration. Spurious regression acts as a warning against overinterpreting statistical relationships where none may exist, while cointegration offers a framework for finding significant long-term relationships in non-stationary financial data. Making this distinction is essential when creating models for risk management, economic forecasting, and investment strategies. Hendry and Ferson et al.'s observations highlight the significance of thorough statistical analysis in understanding the intricate dynamics of financial markets.

Part c: The Engle & Granger 2- step method, involves,

1. Checking if all variables are $I(0)$, cointegrating regression estimation and testing of residuals.
2. Use of step 1 residuals as one variable in the error correction model.

```
Augmented Dickey-Fuller test for residualslp3&4q; regression of Dresidualslp3&4q on:

                     Coefficient   Std.Error     t-value
residualslp3&4q_1       -0.20241    0.073342     -2.7598
Dresidualslp3&4q_1       0.22014     0.11271      1.9532
Dresidualslp3&4q_2     -0.086896     0.11556    -0.75195

sigma = 0.0462392  DW = 1.978  DW-residualslp3&4q = 0.365  ADF-residualslp3&4q = -2.76**
Critical values used in ADF test: 5%=-1.944, 1%=-2.593
RSS = 0.1603545026 for 3 variables and 78 observations
```

In the first step of the Engle & Granger 2-step, we must store the residuals after the estimation and run the ADF test on this series. Important to note is that the constant from this ADF regression is excluded, as residuals are supposed to be of mean zero. From the calculated results, the null hypothesis $H_0 : \theta = 0$ at both 5% and 1% confidence level is rejected, given that the $t - value = -2.76 < 1\% = -2.593 < 5\% = -1.944$ of the residuals is smaller than the critical values. This signifies that the residuals are stationary, and there exists a valid cointegrating relationship.

In the second step, the estimation for the Error Correction Model (ECM) is as follows:

$$\Delta lp3q_t = \varphi_1 \Delta lp4q_t + \alpha(\hat{u}_{t-1}) + v_t$$

```
EQ( 7) Modelling Dlp3q by OLS
       The dataset is: /Users/seanmccallion/Desktop/Bayes/Coursework/Q
       The estimation sample is: 2004(2) - 2023(3)

                     Coefficient  Std.Error  t-value  t-prob Part.R^2
Dlp4q                   0.576548    0.06503     8.87  0.0000   0.5084
residualslp3&4q_1      -0.175731    0.06786    -2.59  0.0115   0.0811

sigma               0.0472287  RSS               0.169521989
R^2                  0.564685  log-likelihood        128.451
no. of observations        78  no. of parameters           2
mean(Dlp3q)        0.00888793  se(Dlp3q)           0.0707762

AR 1-5 test:      F(5,71)   =   1.7180 [0.1416]
ARCH 1-4 test:    F(4,70)   =  0.86871 [0.4872]
Normality test:   Chi^2(2)  =   2.8503 [0.2405]
Hetero test:      F(4,73)   =  0.86939 [0.4865]
RESET23 test:     F(2,74)   =   1.9792 [0.1454]
```

The ECM includes the lagged residuals from step one (representing the error correction term) and the differenced values of the variables to capture short-term dynamics. Additionally, the constant is removed once again, as leaving the constant in the first difference model would mean assuming the presence of a deterministic trend in the levels of the data. The coefficient of the error correction term should be negative and statistically significant, indicating adjustment towards the long-term equilibrium. Give

that the coefficient of the $residualslp3\&4\_1 = -0.175731$ inferring that the calculated result is well-specified as the coefficient is negative.
The $residualslp3\&4\_1$ coefficient tells us the proportion of the disequilibrium of the previous period that is corrected for in the current period and the speed of adjustment back to equilibrium. About 18% of disequilibrium occurring at time $t-1$ is corrected in the following year. Every time there is a deviation from the long-run outcome equilibrium, it takes $\frac{1}{0.175731} = 5.69\ years$ to return to equilibrium.

In Hendry's work it is highlighted that in early econometrics existed problems with nonsense regressions and the challenged posed by unit root tests in time series data. Granger critiqued existing econometric models and emphasizing the need for formal testing of crucial hypotheses and assumptions regarding stationarity. In fact, the Engle & Granger 2-step approach further verifies for stationarity.

Question 5 – Granger Causality

Part a: To prove Granger causality, this holds:

$$Dlpiq_t = \propto + \sum_{i=1}^{n} \beta_{1,i} Dlpxq_{t-i} + \sum_{i=1}^{n} \beta_{2,i}\, Dlp4q_{t-i} + u_t$$

$$H_0: \sum_{i=1}^{n} \beta_{2,i} = 0\ , Dlp4q_t\ fails\ to\ Granger\ cause\ lpxq_t$$

$$H_1: \sum_{i=1}^{n} \beta_{2,i} \neq 0\ , Dlp4q_t\ Granger\ causes\ lpxq_t$$

$$where\ u_t \sim WN\ and\ x = 1,2, and\ 3$$

To choose which lags will be included in the model[4], we run an ADF test on the differenced variables and choose the lags with the lowest AIC.

As we are testing for $Dlp4q$ granger causing on $Dlp1q$, $Dlp2q$ or $Dlp3q$, we now need to run a regression test to see how they relate to $Dlp4q$, with respect to their p-values.

D-lag of Dlp1q	Best Lag	t-adf	AIC
Dlp1q	0	-6.077	-5.266
Dlp2q	3	-5.671	-4.723
Dlp3q	0	-6.080	-5.301

Based on our results from Oxmetrics:

Analysis for Dlp2q	Coefficient	t-value	t-prob	Part R^2
Dlp2q_1	-0.0375341	-2.51	0.0121	0.0012
Dlp2q_2	-0.0141568	-0.833	0.4051	0.0001
Dlp2q_3	-0.0349324	-2.10	0.0360	0.0008
Dlp4q_1	0.00879130	0.557	0.5778	0.0001
Dlp4q_2	-0.00779311	-0.484	0.6281	0.0000
Dlp4q_3	0.00247216	0.175	0.8612	0.0000

All t-probabilities of Dlp4q_t is greater than 0.05, hence we do not accept the null, as values are significant and there is ***we say Dlp4q does not granger cause Dlp2q***.

Analysis for Dlp3q	t-value	t-prob
Dlp4q	9.19	0.0000

As the optimal lag is 0, we say the dlp4q can granger cause dlp3q and dlp1q instantaneously.

Analysis for Dlp1q	t-value	t-prob
Dlp4q	8.87	0.0000

t- probability is less than 0.05 as well as the F- test statistics of 77.47 and 80.93 are significant values, hence we can reject the null hypothesis, for both $Dlp1q$ and $Dlp3q$ and conclude ***Dlp4q granger causes both Dlp1q and Dlp3q.***

Part b: Granger causality test, first introduced in 1969 by Clive Granger is a statistical procedure for determining whether one time series can be used to predict another time series. Hendry[5] discusses Clive Granger's major contribution to the study of causality, known as 'Granger causality' - in a path-breaking paper published in 1969, Granger argued that the previous definition of causality in econometrics was mainly related to the causal interpretation of a system of simultaneous equations, which was usually "instantaneous" causality.

[4] https://www.slideserve.com/BenjoviBenson/unit-root-test-cointegration-test-granger-causality-test
[5] Hendry (2004, Section VI pp. 204-205)

In this definition, the role of the random variable was downplayed, which Granger found unsatisfactory and gave a central place to the random variable and the "arrow of time" in the definition of causation. Granger's definition is based on the concept of a smooth series of predicted variances, where if the joint distribution of a set of observable variables is altered by eliminating the histories of other variables, then this second set of variables is the cause of the first set. It allows the use of statistical methods to test hypotheses of non-causal relationships between economic variables. Because of the relative simplicity of its implementation, this definition quickly became popular and stimulated a great deal of empirical research. It also sparked controversy about whether it was a general definition of causality. Nowadays, "Granger causality" is commonly used as an important concept in empirical economic research and policy analysis.

Thurman and Fisher (1988) used Granger causality tests to solve the "chicken or egg" problem. The researchers used annual time series data for the United States from 1930 to 1983 to analyze egg production and chicken numbers. In this study, they regressed the number of eggs on the lagged number of eggs and the lagged number of chickens, and if the coefficient on the lagged chickens as a whole was significant, then the chickens were considered to be the cause of the eggs. They used one to four lags in their Granger causality tests. The number of lags was the same for eggs and chickens in each equation. The results of the test rejected the hypothesis that eggs are not the Granger cause of chickens. Therefore, the researchers concluded that the egg appeared before the chicken. But this study was purely statistical and did not consider real-world causes - the fact that we would not know whether the chicken or the egg appeared first is a ***philosophical question***. The formulation is as below:

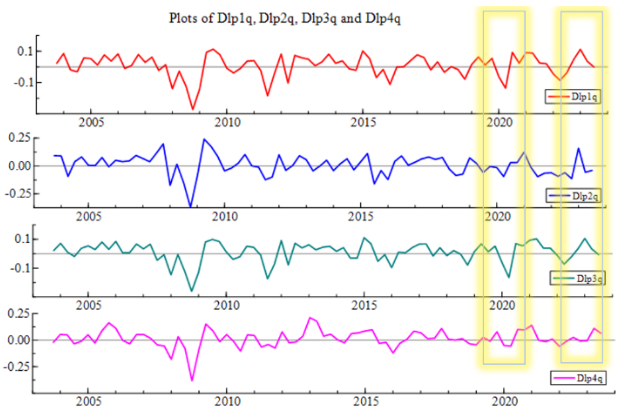
$$\Delta Eggs_t = \propto + \sum_{i=1}^{n} \beta_{1,i} Eggs_{t-i} + \sum_{i=1}^{n} \beta_{2,i}\, Chicken_{t-i} + u_t \,, H_0\!: \; Chickens\ do\ not\ Granger\ cause\ Eggs$$

$$\Delta Chicken_t = \propto + \sum_{i=1}^{n} \beta_{1,i} Chicken_{t-i} + \sum_{i=1}^{n} \beta_{2,i}\, Eggs_{t-i} + u_t \,,\; H_0\!: \; Eggs\ do\ not\ Granger\ cause\ Chickens$$

From an evolutionary point of view, eggs existed before the evolution of chickens, for example, other oviparous animals such as dinosaurs, but dinosaurs can be regarded as the ancestors of chickens, so it is difficult for us to define what is the earliest chicken and what is the earliest egg, and thus the question of whether there was a chicken, or an egg first cannot be resolved.

In conclusion, Granger causality is to test whether a time series can effectively predict the future values of another time series, thus providing statistical evidence of causal relationships between economic variables. The conclusion of "chicken or egg" is based on a specific statistical method and data set and is analyzed primarily from an econometric point of view and does not involve a broader discussion of biology or philosophy. Therefore, when conducting Granger causality tests, the economic logic behind them cannot be set aside, and it is not reasonable to interpret the results solely in terms of statistics.

For our test for **lp4q** granger causing **lp3q**, **lp1q** and **lp2q**, based on Clive Granger's paper, we log transformed the series enabling it look like a linear trend, as data exhibited explosive behavior.[6] Furthermore, if a pair of variables is cointegrated then one of them must Granger-cause the other; see Granger (1986), which is demonstrated by **lp4q** granger causing **lp3q**. Also, **lp4q** granger causes **lp1q**, as we can see towards the last 2 selections in the plot, seems to follow of pattern with **lp4q** compared with **lp2q**, which does not follow the trend. Moreover, the results of **lp4q** granger causing **lp3q** and **lp1q** are consistent with research by Zhang and Huang (2014),[7] but conflicts with Narayan and Smyth (2006),[8] which is expected as there may be discrepancies due to small dataset, and use of different methods. We cannot say results are conclusive as Granger causality implies a prediction and not a causal relationship,

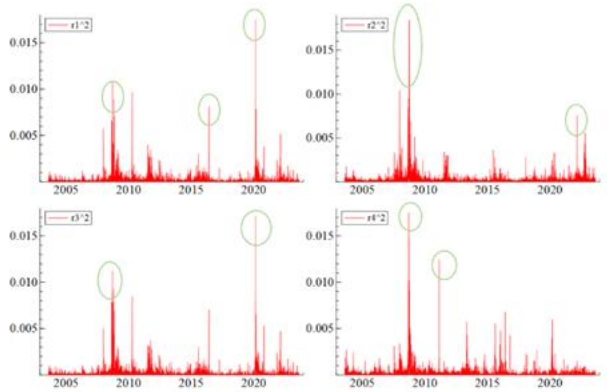


[6] III. Hendry, Equilibrium Correction
[7] Zhang, Z., & Huang, W. (2014). Dynamic Causal Relationships among Euro Stoxx 50, Hang Seng, Nikkei 225, and CAC 40 Stock Indices. *International Journal of Finance & Economics*, *19*(2), 217-231
[8] Narayan, P., & Smyth, R. (2006). Testing for cointegration in the presence of a possible structural break. *Journal of Applied Econometrics*, *21*(2), 221-238.

hence, further research needs to be done into this, as conclusion is based on statistical analysis and ignores real life factors.

Question 6 – (G)ARCH models

Part a: ARCH effect explains the presence of serial correlation of heteroskedacity in variance returns. By visual inspection, the ACF and PACF plots of the time series, show a high correlation over time indicating variance of returns have ARCH effects. To prove this mathematically, we use Lagrange multiplier by running the F-test on the squared 1st order differences of the log variable, assuming they are asymptotically equivalent. After running this process in Oxmetrics all 4 variables Dlp1, Dlp2, Dlp3 and Dlp4 were all misspecified according to normality, Hetero test, RESET, AR, and ARCH tests.



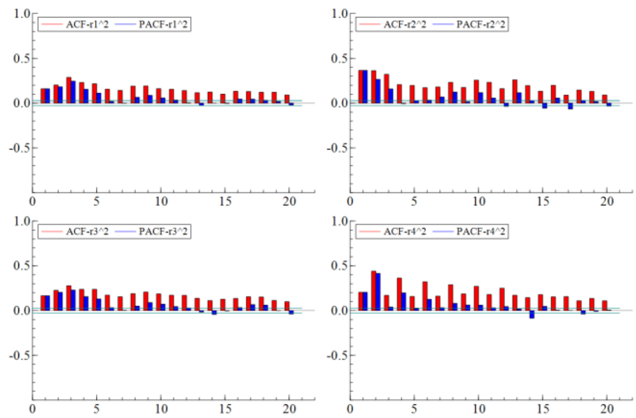
All coefficients are statistically different from 0, so variance of returns is serially correlated, implying there is some existence of historical volatility.

$$H_0: No\ Serial\ correlation\ between\ variances - no\ prescence\ of\ ARCH\ effects$$
$$H_1: Serial\ correlation\ between\ variances - prescence\ of\ ARCH\ effects$$

Model	F-value	Decision
Dlp1^2	F (6,5206) =149.6 [0.000] **	Reject
Dlp2^2	F (6,5206) =239.1 [0.000] **	Reject
Dlp3^2	F (6,5206) =157.75 [0.000] **	Reject
Dlp4^2	F (6,5206) =292.02 [0.000] **	Reject

Due to the very large F-values we can conclude the strong presence of ARCH effects since we have variance present in the series. Moreover, the presence of ARCH effects makes it difficult to interpret time series models so we will have to convert to GARCH model to accommodate for the ARCH effects.

Part b: From running the ADF test and unit roots, our series possess stationarity at the first order difference, so our $Dlpx$, where $x = 1,2,3\ and\ 4$, we use this form to identify the best univariate GARCH representation amongst some selected alternative models, as EGARCH, APARCH and GARCH-M. To find the best ARMA (p, q)-GARCH (p, q) model we assumed, normality (Jacque -Bera), no serial correlation of residuals and no ARCH effects should hold.



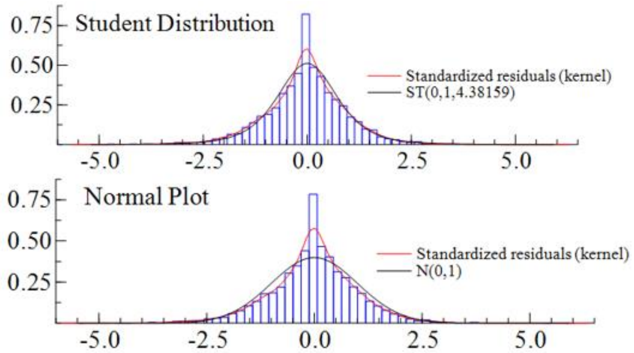
Dlp4: We had 2 models, where ARMA (0,0) – EGARCH (1,1) is a better model compared to ARMA (1,1) – EGARCH (1,1) due to its lower Schwartz of -6.009722. The GED distribution, has the lowest Schwartz value as seen below, making it the best model to choose as normality is rejected in all 4 options (Jacque - Bera) – the $p - values < 0.05$. The models have no ARCH effects and serial correlation of residuals both squared and original. Also, GED is leptokurtic, with student distribution platykurtic.

Dlp4	BIC	Jacque – Bera	Skewness	Excess Kurtosis
Normal	-5.947135	2.2982e-252	1.7009e-30	1.3645e-225
Student distribution	-6.009722	0.00000	1.7180e-29	1.6078e-285
Skewed Student distribution	-6.008217	1.7362e-306	6.7269e-30	2.0939e-280
GED	-6.029210	0.00000	7.7489e-26	0.0000

Dlp3: We compared a ARMA (1,1) – GARCH (1,1) or EGARCH (1,1), but the EGARCH (1,1), gave a better fit to the data, due to lower AIC, as well as catering for leverage effects of theta 1. From the line scatter plots (r3^2), seems to have a very significant spike around the begging of the plot, which will be well catered for by an exponential distribution, as well as showing why we used a GED distribution over a Normal and Student, which were misspecified. The skewed student distribution had a good fit, but the AIC was higher than the GED, possessing no serial correlations of residuals and no ARCH effects.

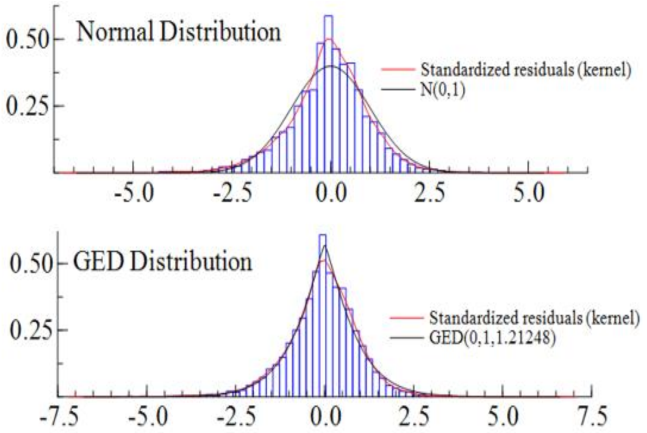
Dlp3	BIC	Jacque – Bera	Skewness	Excess Kurtosis
GED	-6.200767	0.00000	3.8305e-102	0.00000
Skewed Student distribution	-6.206255	0.00000	1.8951e-109	0.00000

Dlp2: Shows us that ARMA (1,1)-EGARCH (1,2) is better than having a ARMA (1,1) - GARCH (3,2) which is overparameterized and has a higher Schwartz. Also, skewed and GED possess serial correlation of squared residuals, which makes them not optimal for the model selection. Even though, the student distribution possessed minor serial correlation up to the 5th lag, we would consider this model as it provides a better fit to the data and smaller Schwartz value compared to the Normal distribution. The RHS[9] density plots show the fits.



	BIC	Jacque – Bera	Skewness	Excess Kurtosis
Normal	-5.950430	4.3882e-165	2.2739e-09	5.2149e-159
Student Distribution	-6.007168	5.4445e-212	1.6852e-08	7.2765e-207

Dlp1: We used a ARMA (1,2) – GARCH (1,2) with a normal distribution, which had a small serial correlation of variance of returns residuals a up to the 5th lag, and the GED to the 5th and the 50th lag but the skewed, and Student distribution all had worse off misspecification. Still, we will choose the GED version. However, ARMA (1,1)-HYGARCH (1,2) model can be used, as well as introducing the factor of conditional variance into our model. This will allow the heterosckedacity to be masked and a more parsimonious model will be generated and no serial correlation of both returns, and variance.



Dlp1	BIC	Jacque – Bera	Skewness	Excess Kurtosis
Normal	-6.144872	2.9503e-247	5.8413e-41	4.6308e-210
GED	-6.208880	1.6399e-261	2.5856e-44	5.2896e-221

As all data modelled were indices, we can assume normality does not exist due to extreme events, e.g., Covid 19 pandemic, accounting for the selection of distributions with fatter tails.

Question 3 - Model specifications and misspecification tests: **Specified Model of lp1q and lp4q**

```
lp1q =  + 0.9043*lp1q_1 + 0.6103 + 0.5623*lp4q - 0.5451*lp4q_1 - 0.1103*2009(1) + 0.07181*2009(4) - 0.1675*2011(3) - 0.1088*2012(2)
(SE)     (0.0432)        (0.230)  (0.0564)      (0.0633)        (0.0396)         (0.0388)           (0.0390)         (0.0391)
        - 0.1083*2013(2) - 0.1074*2020(2) + 0.08693*2021(2) - 0.07052*2022(2) + 0.1166*2023(1)
         (0.0395)         (0.0389)         (0.0393)          (0.0390)          (0.0391)
```

[9] RHS = Right hand side