REPORT ON INEVSTMENT PORTFOLIO

MVPT AND CAPM APPLICATION TO INVESTMENT STRATEGY

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UNIVERSITY OF KENT
CANTERBURY

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INTRODUCTION

In this assessment I worked with 3 stocks from different sectors but the same market, which is NASDAQ as well as using the treasury bills from the USA to calculate my risk-free rate.

The New Germany Fund Inc. (Stock symbol - GF)

The new Germany fund inc. is a closed-end investment management company, mostly investing in the German middle-market. Such sectors invested in include, construction and engineering, food products, metals and mining, pharmaceuticals, real estate, internet software, etc. their investment advisor is Deutsche asset management international GmbH. [1]

American airlines holding Inc. (Stock symbol - AAL)

They are a holding company, primarily focusing on airline transport for cargo and passengers operating both locally and internationally. They travel to places such as London, Chicago, Dallas, Phoenix, Sydney, and Tokyo. Also, they have subsidiaries such as Envoy Aviation Group Inc., American airlines, Piedmont Airlines Inc., etc. It is headquartered in America and founded in the year 2013. I picked this stock based on my middle name. [2]

The Lion Electric Company (Stock symbol - LEV)

Lion electric is a manufacturer of zero-emission electric medium and heavy-duty vehicles. They are in the farm & heavy construction machinery industry. Such include school buses, bucket trucks and garbage trucks. This company is headquartered in Canada and founded in the year 2011. I chose this stock based on the first letter of my surname. [3]

As you may see all stocks are unique and do not belong to the same industry.

In the assessment, my findings were based on the CAPM and MPVT theories. I will elaborate further in the next section.

Treasury Bills (T Bills)

Treasury bills are bonds that are issued by the Government. They tend to be less risky, and people tend to invest in them. I used their return rate as my risk-free rate for my safest stock to invest in. I took my quarterly values from the Federal reserve bank of St. Louis. [4]

National Association of Securities Dealers (NASDAQ)

It is an American stock exchange just like the London Stock Exchange of the UK. This is the market against my stocks are compared against to.

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MEAN VARIANCE PORTFOLIO THEORY (MVPT)

MVPT is a theory that helps maximize returns with a lower level of risk in your portfolio. Below are some of the metrics I used and what they mean.

Daily return:

This value explains how much the stock dividends changes daily. If it is positive the investor will gain, if it is negative, you make a loss. The daily return was calculated using the following formula:

$$Daily\ Return\ of\ stock = Ln(Present\ dividend\ paid) - Ln(Previous\ dividend\ paid)$$

$$Ln = natural\ logarithm$$

Average returns:

This is a measure of how much an investor should gain over a year from investing in the stocks GF, AAL and LEV. It's a ratio so has no units but can be represented in a percentage. The table below shows the daily average return of the stocks we chose from 30 October,2020 to 29th October,2021. As one might notice, investing in AAL will provide quite a higher return compared to GF and LEV. Also, the 2nd highest average is that of treasury bills, which makes sense as its supposed to be a fall back on.

	AVERAGE RETURN
GF	0.001055112
AAL	0.002153357
LEV	0.001586533
TBILL	0.001679009

$$Average\ return = \frac{Total\ of\ daily\ returns}{Number\ of\ days}$$

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Variance and Standard deviation of returns:

The variance of the stocks measures their variability in the market and the standard deviation measures their volatility to risk. I preferred to use standard deviation compared to variance in my further calculations due to the possibility of skewness from the squared.

	AVERAGE RETURN	VARIANCE	STANDARD DEVIATION
GF	0.001055112	0.000136217	0.01167121
AAL	0.002153357	0.000982602	0.031346485
LEV	0.001586533	0.002468148	0.049680463
TBILL	0.001679009	0.001037689	0.032213174

Looking at the table of results, stock LEV has the greatest volatility compared to GF, AAL and the treasury bill. This implies you stand a chance of higher risk investing in stock LEV compared to the other 2 stocks. Also, AAL gave higher returns than others, this could be as its an airline and during this period, people travelled back to their respective homes to be safe during the Covid-19 pandemic.

The formulas used can be seen below:

$$Variance\ of\ returns = \frac{\sum(daily\ returns - average\ returns)^2}{total\ number\ of\ days - 1}$$

$$Standard\ deviation\ of\ returns = \left(\frac{\sum(daily\ returns - average\ returns)^2}{total\ number\ of\ days - 1}\right)^{1/2}$$

Covariance and correlation coefficients:

Covariance measures the joint variability of two assets. The variance-covariance matrix and correlation table of the portfolio can be seen below:

VARIANCE-COVARIANCE MATRIX

	GF	AAL	LEV
GF	0.0001362	0.0000648	0.0001306
AAL	0.0000648	0.0009826	0.0002199
LEV	0.0001306	0.0002199	0.0024681

CORRELATION MATRIX

	GF	AAL	LEV
GF	1.0000000	0.1875512	0.2139395
AAL	0.1875512	1.0000000	0.1408192
LEV	0.2139395	0.1408192	1.0000000

As you may see, GF and AAL has the highest covariance value compared to combinations of AAL and Lev and LEV and GF. This implies GF & AAL will provide the greatest gain for diversification. Also, the GF&GF, LEV &LEV and AAL& AAL values are equal to their respective variances as the laws of variances show.

Now, looking at the correlation table, we can measure the strength in relationship between the stocks. One thing is for sure, the less correlated the stocks the better for the portfolio as this tends to reduce the chance of loss. As you may see LEV& GF have a greater correlation compared to GF & AAL and AAL & LEV, this implies a downturn in the returns of LEV might have an impact on GF returns. The best thing to do with the portfolio of LEV &AAL is to short sell (borrow) one of them and take a long position (lend) on the other.

$$Correlation\ between\ asset\ x\ and\ asset\ y, \qquad Corr_{x,y} = \frac{Cov_{x,y}}{\sigma_x \sigma_y}$$

$$Covariance\ between\ asset\ x\ and\ asset\ y, \qquad Cov_{x,y} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{n}$$

Expected return and Standard deviation of Portfolio

From using the expected return and standard deviation portfolio equations and using the weights of the 3 stocks, GF, AAL and LEV. I got the following values.

PORTFOLIO METRICS	
EXPECTED RETURN OF PORTFOLIO	0.001055112
VARIANCE OF PORTFOLIO	0.000136217
STANDARD DEVIATION OF PORTFOLIO	0.01167121

The portfolio is expected to have a return of 0.1055% whilst taking on a risk of 1.167%. The weights were 0 for Lev and AAL, but 1 for GF.

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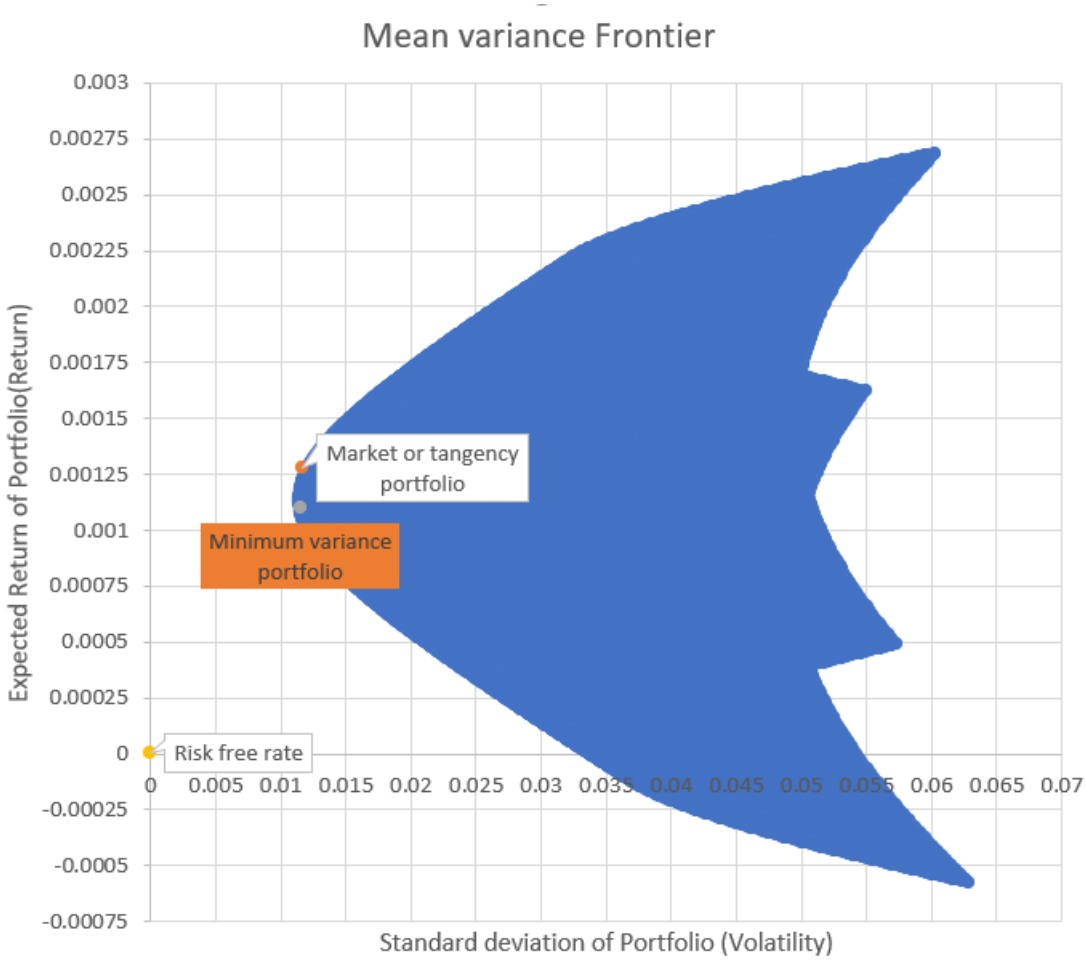
Mean variance Frontier & Efficient frontier:



This is a graph of standard deviation against expected return of the portfolio. The graph produced is a bullet shaped curve, I used a data table to produce several data points for different combinations of weights of assets AAL&LEV assuming the weight of GF is 1- (sum of AAL & LEV). Any value on the data table for standard deviation is a standard deviation and that for the expected return data table is a return value. Note we use the expected return and Standard deviation of the portfolio formula to produce the data table.

After doing this a graph of standard deviation against expected return is plotted. As you may see the mean variance frontier graph is bullet shaped indicating every increase in volatility leads to a relatively smaller increase in returns. **(Law of diminishing marginal return of risk)**. This can be shown by the values used for the scatter plot, you will notice. We have very small returns compared to standard deviation values. Also, we have a curved shape as the correlation of our assets are almost 0.

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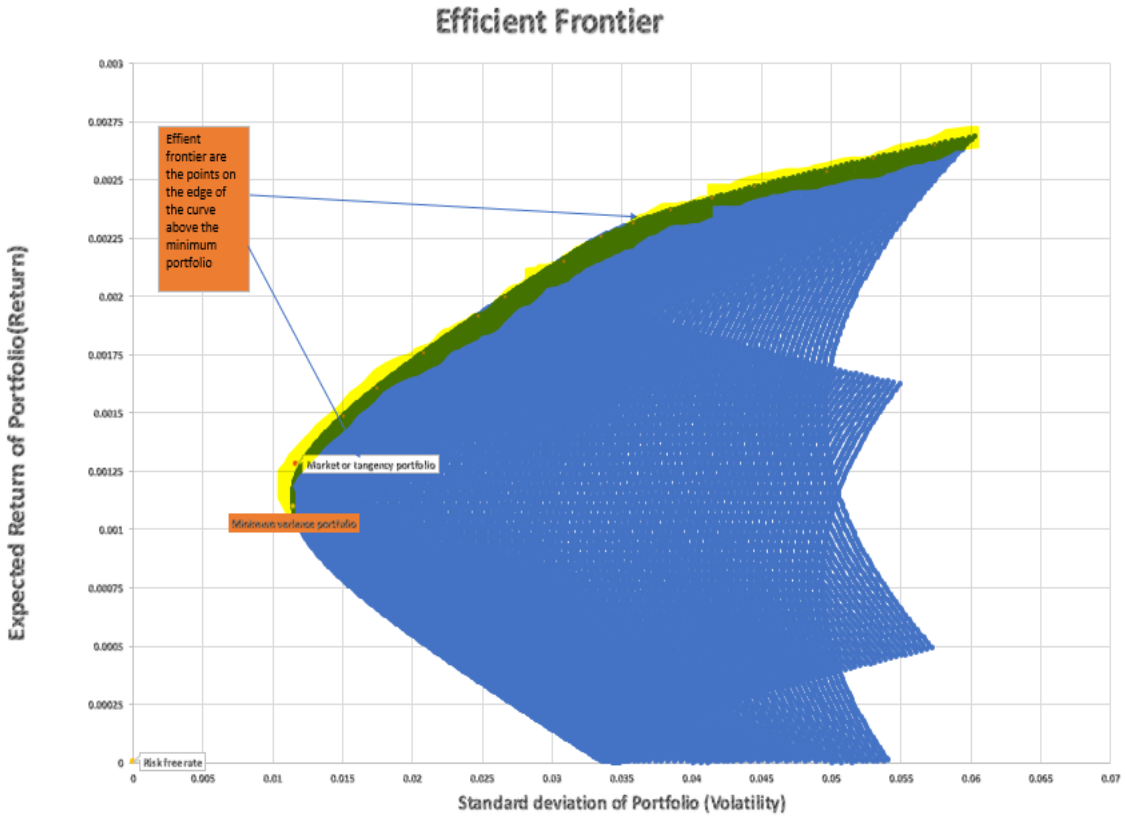

The image above shows the region of the efficient frontier. It starts right from the global minimum variance value. Any point on this region represents possible dominant portfolios, any point below is an inefficient portfolio (offer less value when invested in) and at a higher risk/volatility/standard deviation. We would like to invest in any portfolio on this region because it gives the maximum expected return for a given level of standard deviation.

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Assumptions for MVPT

In every model there are some assumptions that needs to be implied and followed. Under the MVPT theory we assume the following:

- Investors are risk-hating (i.e., risk averse) – We assume investors do not like to take risks, so will do their best to invest in the portfolio with the lowest or least risk.
- Investments are influenced by values expected return and variance of the portfolio, Hence, investors do not like portfolios that are risky and, those that offer low returns.
- Most importantly, values of expected returns, standard deviations. Variance and covariances are known. This can be seen from the Excel assessment, that these values were calculated theoretically.

Global minimum Variance Portfolio

This measure for the capital asset pricing model measures their portfolio with the lowest volatility under frontier. Also, this is the puts value with the lowest risk on the efficient once frontier.

For my portfolio of GF, AAL and LEV my global minimum variance was 0.011448 or 1.148%.

This is the point where the curve starts to change shape. Note, we would not like to trade at this point but rather at the tangency portfolio or market portfolio. As we get more returns from the tangency portfolio.

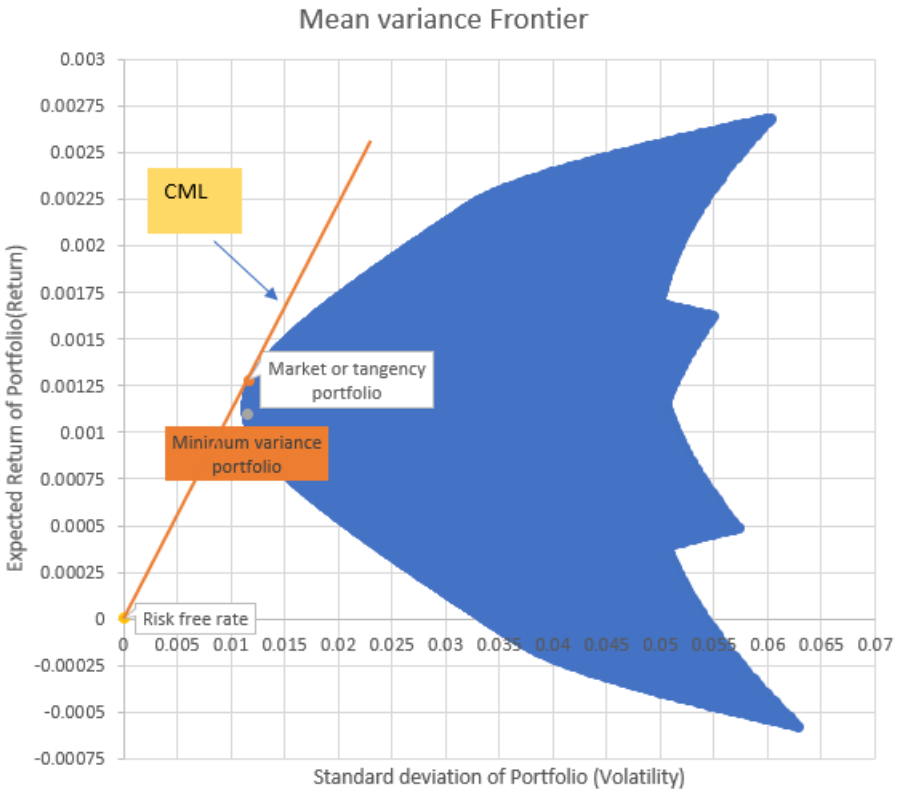
CAPITAL ASSET PRICING MODEL (CAPM)

From the report, I found out about the following metrics which form the CAPM model.

Tangency portfolio (also known as market portfolio):

This is the point that tells the optimal portfolio allocation to be used to gain the optimal return at the most reasonable amount of risk. So, applying this to my calculation, my tangency portfolio occurred at a standard deviation of 1.217423% and expected return of 0.1281247%, the slope of this point is known as the Sharpe ratio.

Joining a line from the risk-free rate to the tangency portfolio, creates a tangent to the efficient frontier. This is the point where investors want to invest in. Also, note, the capital market line (CML), is a new efficient frontier, as any portfolio on it gives a better return and substantial risk, than anywhere else on the graph. You could have many capital markets lines but the one which passes through the tangency portfolio, is a unique one, as it considers the return of the risk-free asset.



Note the slope of the CML is fixed and it is sometimes called the Sharpe ratio. The higher the Sharpe ratio the better. In this case it has a value of 0. 109384. Our value is under 1, hence, sub-optimal. Which seems right, as we get very low returns for quite a large rate of risk, the graph above proves this. Hence, explaining why investors like to invest in treasury bills as it lies on the capital market line, with the lowest risk value of 0.

Every investor wants to achieve a portfolio with weights, standard deviation and returns at this point. The tangency portfolio has the higher Sharpe ratio compared to all other portfolio combinations.

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Assumptions for which economy is in equilibrium

1. Investors are non-satiated: they always want more. Every investor is assumed to want higher returns on their investment, no matter what.
2. The market is perfect: This implies the market has only the three stocks, GF, AAL and LEV. Also, meaning assets can leave and enter market without cost charges, market has no influence on stocks and people are price takers. No one tries to change the price of stocks; they leave it as it is.
3. The market is frictionless: This implies we trade in a market with no taxes, transaction costs, etc
4. Investors measure returns, Covariance, variance and standard deviation in the same units and currency.
5. Any amount of assets can be borrowed or loaned at the rate of the risk-free asset (i.e., Treasury bill). This is shown by the SML graph as the left bottom half represents lending and the right bit is borrowing. There are no restrictions.
6. Most importantly, investors are rational, they make decisions based on the value of expected returns, variance, and standard deviation.

Expected return and Standard deviation of market portfolio when Economy is in Equilibrium

From using the expected return and standard deviation portfolio equations and using the weights of the 3 stocks, GF, AAL and LEV. I got the following values.

MARKET PORTFOLIO METRICS	
EXPECTED RETURN OF MARKET PORTFOLIO	0.1281%
STANDARD DEVIATION OF MARKET PORTFOLIO	1.1671%

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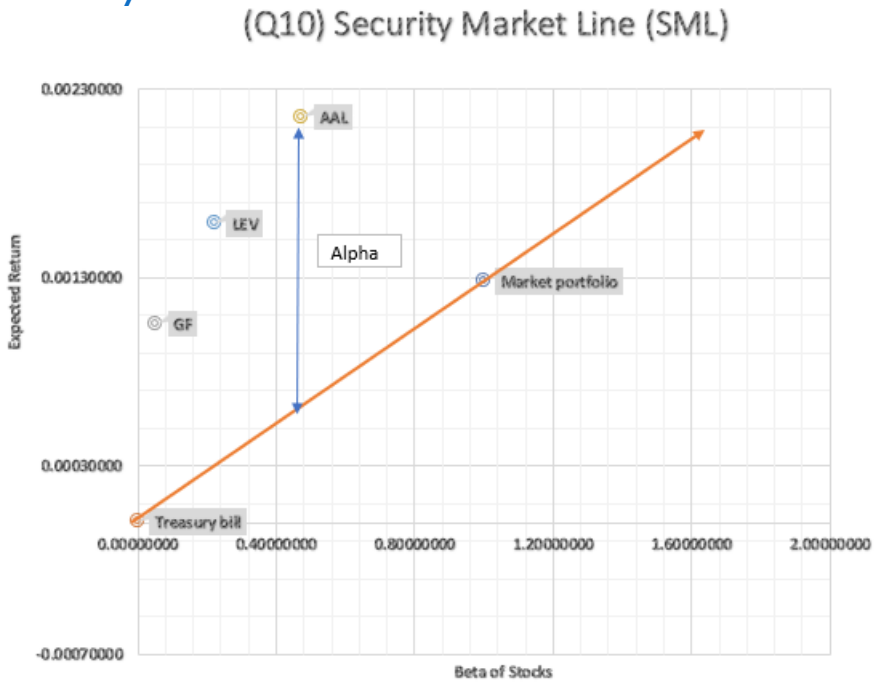
Security Market Line



This Security market line (SML) describes their relationship between expected return of the portfolio and the beta of each asset provided they are in equilibrium. The graph slopes to the right. beta can be calculated using the formula below. From this security market line, we could calculate the Jensen's alpha. This is the distance from a point on the security market line to any portfolio outside the line. Take note alpha is a measure of performance return and the beta of stocks.

Systematic risk is the risk out of our control such as, inflation rates, business control cycle, etc.

Beta is the sensitivity of an asset return to the market, so in my case, the NASDAQ market. So, the treasury bill has no reaction to the market because it is risk free, but the market portfolio (tangency portfolio) as it moves directly with market.

So, risk averse investors will invest in the treasury bill. Using beta can help set your risk tolerances, to know how much risk you are willing to take.

But beware a beta of 0 does not mean there is no risk involved, it could be the stock is a new asset and has no data to calculate its beta.

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Tangency Portfolio Calculation Using Covariance and Risk-Free Rate

VARIANCE-COVARIANCE MATRIX WITH RISK PREMIUM				
	GF	AAL	LEV	Risk Premium
GF	0.0001362	0.0000648	0.0001306	0.001050509
AAL	0.0000648	0.0009826	0.0002199	0.002148754
LEV	0.0001306	0.0002199	0.0024681	0.00158193

$$Covariance\ Variance\ Matrix\ = \begin{bmatrix} 0.0001362 & 0.0000648 & 0.0001306 \\ 0.0000648 & 0.0009862 & 0.0002199 \\ 0.0001306 & 0.00021999 & 0.0024681 \end{bmatrix}$$

$$weight\ of\ GF(wg) + weight\ of\ AAl(wa) + weight\ of\ LEV(Wl) = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$Expected\ Return = [0.00155112 \quad 0.002153357 \quad 0.001586533]$$

$$E(Ri) = Wg * E(Rg) + Wa * E(Ra) + Wl * E(Rl) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

But $Expected\ return\ of\ an\ asset,\ \bar{R} = R_f + \beta(R_m - R_f) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$

And $\beta = \frac{\sigma_{jk}}{\sigma_x{}^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$

Put ……..(4) into …….(3)

This gives,

$$\frac{E(R_k) - R_f}{\sigma_{jk}} = \frac{E(R_k) - R_f}{\sigma_x{}^2}$$

$$\frac{E(R_k) - R_f}{\sigma_{jk}} = constant$$

$$Let\ constant = \frac{1}{M}$$

$$\sigma_{jk} = [E(R_k) - R_f] * M \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

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We will now use the covariance and risk premiums to find our value of M and the weights of the stocks:

VARIANCE-COVARIANCE MATRIX				
	GF	AAL	LEV	Risk Premium
GF	0.0001362	0.0000648	0.0001306	0.001050509
AAL	0.0000648	0.0009826	0.0002199	0.002148754
LEV	0.0001306	0.0002199	0.0024681	0.00158193

The following are the equations to be solved, we derived it from the table above

$$(0) = (0.0001362Wg) + (0.0000648Wa) + (0.00001306Wl) + (-0.001050509M)$$
$$(0) = (0.0000648Wg) + (0.0009826Wa) + (0.0002199Wl) + (-0.002148754M)$$
$$(0) = (0.0001306Wg) + (0.0002199Wa) + (0.0024681Wl) + (-0.00158193M)$$

$$(1) = (Wg) + (Wa) + (Wl)$$

Let's solve this using a matrix, as there are 4 unknowns,

So let A*Wi=B

So $B*A^{-1}=Wi$

$$A = \begin{bmatrix} 0.0001362 & 0.0000648 & 0.0001306 & -0.001050509 \\ 0.0000648 & 0.0009862 & 0.0002199 & -0.002148754 \\ 0.0001306 & 0.00021999 & 0.0024681 & -0.00151893 \end{bmatrix}$$

We now compute the inverse matrix of A;

$$A^{-1} = \begin{bmatrix} 2333.71 & -872.7028 & -364.3394 & 0.78629762 \\ -1846.673 & 952.5912 & -67.59998 & 0.198603656 \\ -487.038 & -79.88837 & 431.9394 & 0.015099082 \\ -823.8745 & -64.29031 & 2.295518 & 0.116095189 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Then $B*A^{-1}= Wi$

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$$[Wi] = \begin{bmatrix} \begin{bmatrix} 2333.71 & -872.7028 & -364.3394 & 0.78629762 \\ -1846.673 & 952.5912 & -67.59998 & 0.198603656 \\ -487.038 & -79.88837 & 431.9394 & 0.015099082 \\ -823.8745 & -64.29031 & 2.295518 & 0.116095189 \end{bmatrix} \end{bmatrix} * \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix}$$

$$[Wi] = [0.786297, 0.198604, 0.015099, 0.116095]$$

Solutions	
Wg	0.786297
Wa	0.198604
Wl	0.015099
M	0.116095

These are the results, so the market portfolio or tangency portfolio occurs when we allocate weights of 78.62%, 19.86% and 11.61% to GF, AAL and LEV respectively.

The gradient of the line is $1/0.116095 = 861.3\%$, which is equivalent to the market risk premium.

CONCLUSIONS

To conclude, from finding the portfolio expected return and standard deviation, I decided to check whether my portfolio is fully diversified. I carried out the following test as below.

$$\boldsymbol{Average\ of\ covariance} = \frac{\boldsymbol{CovGF,AAL + CovGF,LEV + CovLEV,AAL}}{\mathbf{3}}$$

$$\boldsymbol{Average\ of\ covariance} = \frac{\mathbf{0.0000648 + 0.0001306 + 0.0002199}}{\mathbf{3}}$$

$$\boldsymbol{Average\ covariance} = \mathbf{0.00013433}$$

$$\boldsymbol{Portfolio\ variance} = \mathbf{0.000136217}$$

So, the values are not entirely the same, hence my portfolio is not fully diversified.

On the other hand, the CAPM and MVPT have a few limitations that may offset the values calculated above. They are some of the unrealistic assumptions on being in s perfect market, no limit to the amount of money invested, and the idea markets are frictionless. To counteract this I will suggest, using the Arbitrage pricing model to get a more realistic idea about the Expected returns and Volatility values.

Also, a new efficient frontier emerges as soon as you draw the Capital market line tangent to the market/tangency portfolio and passing though the risk-free rate. This gives us combinations of portfolios with better returns than that on the efficient frontier. So, all investors would like to hold investments on that line.

AAL, LEV and GF are under-priced, but AAL is way under-priced compared to all the stocks. I will advise lending these stocks than borrowing them.

The CML shows the portfolio has a risk of 10.938% compared to the market risk, which was measured by the SML graph of 0.1277%, this means investing in the market portfolio is a wise decision.

Our optimal, market and tangency portfolio will always be at but changes once we change the type of assets and the time.

Our optimal portfolio to invest in will have to have weights of 78.62%, 19.86% and 11.61% to GF, AAL and LEV respectively. On the other hand, I will suggest, these results can be reliable provided we understand what beta, the return and other metric mean.

Finally, the Covid-19 pandemic took place during this period, so it may be possible these values have been affected by an economic downturn. I will suggest keeping an eye on the stock market and using the CAPM and MVPT for longer periods of time to give an accurate analysis.

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