Hypothesis Testing

Hypothesis Testing

- Hypothesis testing can be used to determine whether a statement about the value of a population parameter should or should not be rejected.
- The null hypothesis, denoted by H_0 , is a tentative assumption about a population parameter
- The alternative hypothesis, denoted by Ha, is the opposite of what is stated in the null hypothesis
- The hypothesis testing procedure uses data from a sample to test the two competing statements indicated by H0 and Ha.

Developing Null and Alternative Hypotheses

- It is not always obvious how the null and alternative hypotheses should be formulated
- Care must be taken to structure the hypotheses appropriately so that the test conclusion provides the information the researcher wants
- The context of the situation is very important in determining how the hypotheses should be stated
- In some cases it is easier to identify the alternative hypothesis first. In other cases the null is easier
- Correct hypothesis formulation will take practice

Developing Null and Alternative Hypotheses

Alternative Hypothesis as a Research Hypothesis

•Many applications of hypothesis testing involve an attempt to gather evidence in support of a research hypothesis

• In such cases, it is often best to begin with the alternative hypothesis and make it the conclusion that the researcher hopes to support

• The conclusion that the research hypothesis is true is made if the sample data provide sufficient evidence to show that the null hypothesis can be rejected

Developing Null and Alternative Hypotheses

Alternative Hypothesis as a Research Hypothesis

- Example: A new manufacturing method is believed to be better than the current method.
- Alternative Hypothesis:
  - The new manufacturing method is better.
- Null Hypothesis:
  - The new method is no better than the old method.

Developing Null and Alternative Hypotheses

- Alternative Hypothesis as a Research Hypothesis
- Example: A new bonus plan, that is developed in an attempt to increase sales
- Alternative Hypothesis:
  - The new bonus plan increase sales
- Null Hypothesis:
  - The new bonus plan does not increase sales

Developing Null and Alternative Hypotheses

- Alternative Hypothesis as a Research Hypothesis
- Example:
  - A new drug is developed with the goal of lowering Cholesterol-level more than the existing drug
- Alternative Hypothesis:
  - The new drug lowers Cholesterol-level more than the existing drug
- Null Hypothesis:
  - The new drug does not lower Cholesterol-level more than the existing drug

Developing Null and Alternative Hypotheses

- Null Hypothesis as an Assumption to be Challenged
- Example:
  - The label on a milk bottle states that it contains 1000 ml
- Null Hypothesis:
  - The label is correct. $\mu \geq 1000$ ml
- Alternative Hypothesis:
  - The label is incorrect. $\mu < 1000$ ml

Null and Alternative Hypotheses about a Population Mean μ

- The equality part of the hypotheses always appears in the null hypothesis
- In general, a hypothesis test about the value of a population mean μ must take one of the following three forms (where μ_0 is the hypothesized value of the population mean)

$H_0: \mu \geq \mu_0$	$H_0: \mu \leq \mu_0$	$H_0: \mu = \mu_0$
$H_a: \mu < \mu_0$	$H_a: \mu > \mu_0$	$H_a: \mu \neq \mu_0$
One-tailed (lower-tail)	One-tailed (upper-tail)	Two-tailed

Null and Alternative Hypotheses

- A major hospital in Chennai provides one of the most comprehensive emergency medical services in the world
- Operating in a multiple hospital system with approximately 10 mobile medical units, the service goal is to respond to medical emergencies with a mean time of 8 minutes or less
- The director of medical services wants to formulate a hypothesis test that could use a sample of emergency response times to determine whether or not the service goal of 8 minutes or less is being achieved.

Null and Alternative Hypotheses

H_0: $\mu \leq 8$ — The emergency service is meeting the response goal; no follow-up action is necessary.

H_a: $\mu > 8$ — The emergency service is not meeting the response goal; appropriate follow-up action is necessary.

where: μ = mean response time for the population of medical emergency requests

Type I and Type II Errors

	Population Condition	
Conclusion	H0 True ($\mu \leq 8$)	H0 False ($\mu > 8$)
Accept H0 (Conclude $\mu \leq 8$)	Correct Decision	Type I Error (Incorrect Acceptance)
Reject H0 (Conclude $\mu > 8$)	TypeII Error (Incorrect Rejection)	Correct Decision

One-Tailed Tests About a Population when (σ) Known

- Example: The mean response times for a random sample of 30 Pizza Deliveries is 32 minutes
- The population standard deviation is believed to be 10 minutes.
- The pizza delivery services director wants to perform a hypothesis test, with α =0.05 level of significance, to determine whether the service goal of 30 minutes or less is being achieved.

One-Tailed Tests About a Population Mean: σ Known

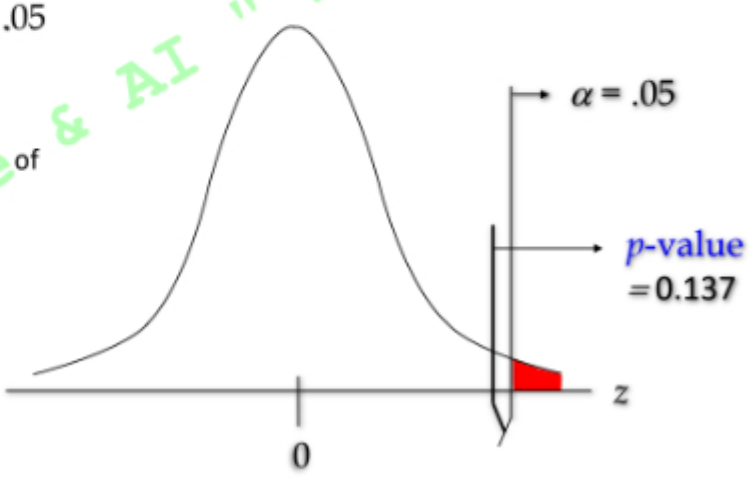
1. Develop the hypotheses. $H_0: \mu \leq 30$, $H_a: \mu > 30$

2. Specify the level of significance. $\alpha = .05$

3. Compute the value of the test statistic.

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{32 - 30}{10 / \sqrt{30}} = 1.09$$

```
In [8]: 1-stats.norm.cdf(1.09)
Out[8]: 0.1378565720320355
```



One-Tailed Tests About a Population Mean: σ Known

p –Value Approach

4. Compute the p –value.

   For z = 1.09, p–value = = 0.137

5. Determine whether to reject H0.

- Because p–value = 0.137 > $\alpha = .05$, we do not reject H0.
- There are not sufficient statistical evidence to infer that Pizza delivery services is not meeting the response goal of 30 minutes.

Two-Tailed Tests About a Population Mean: σ Known

- Example: Milk Carton
- Assume that a sample of 30 milk carton provides a sample mean of 505 ml.
- The population standard deviation is believed to be 10 ml.
- Perform a hypothesis test, at the 0.03 level of significance, population mean 500 ml and to help determine whether the filling process should continue operating or be stopped and corrected.

Two-Tailed Tests

1. Determine the hypotheses.
2. Specify the level of significance.
3. Compute the value of the test statistic.

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{505 - 500}{10 / \sqrt{30}} = 2.74$$

H_0: $\mu = 500$
H_a: $\mu \neq 500$
$\alpha = .03$

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In [9]: 1-stats.norm.cdf(2.74)
Out[9]: 0.003071959218650444
```

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In [10]: (1-stats.norm.cdf(2.74))*2
Out[10]: 0.006143918437300888
```

Two-Tailed Tests About a Population Mean: σ Known

p –Value Approach

4. Compute the p –value.
   - For $z = 2.74$, p–value = 2(1 - .9969) = .0061

5. Determine whether to reject H0.
   - Because p–value = .0062 < α = .03, we reject H_0.

There is no sufficient statistical evidence to infer that the null hypothesis is true (i.e. the mean filling quantity is not 500 ml)