Time Series Modeling

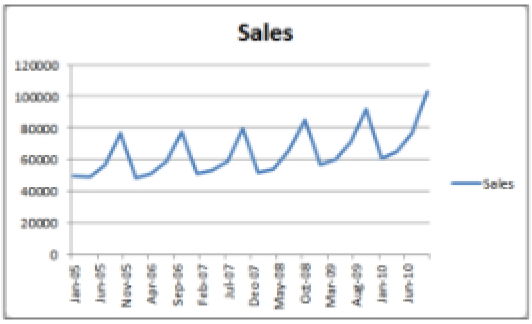
Definition

Time Series can be defined as a set of measurements of certain variable made at **regular time intervals**.

Time acts as an independent variable for estimation

A time series defined by the values Y1, Y2.. of a variable Y at times t1, t2, t3.. is given by :

$$Y = F(t)$$

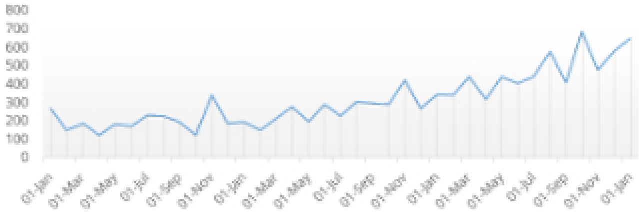


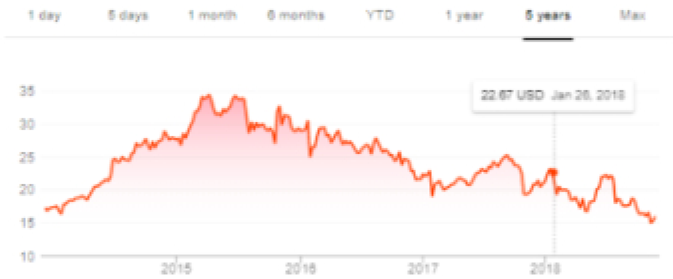
Series of monthly sales data

Applications

Notice that all these datasets include time

Time Series Pattern Types





Uptrend

Smartphone sales for a 3 year period

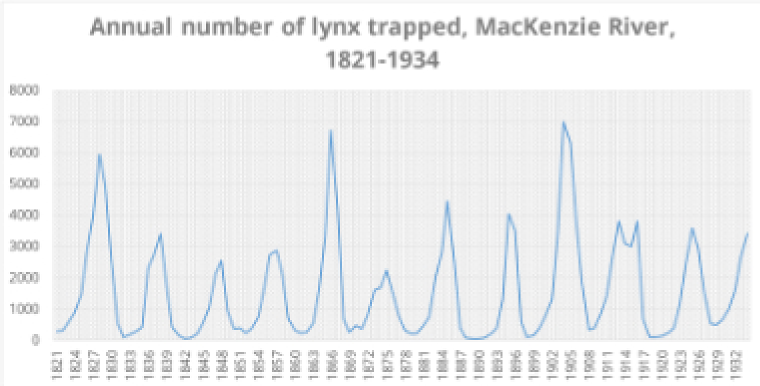
Downtrend

Stock Market price for a wall street company

Trend

A trend is a long-term increase or decrease in time series data

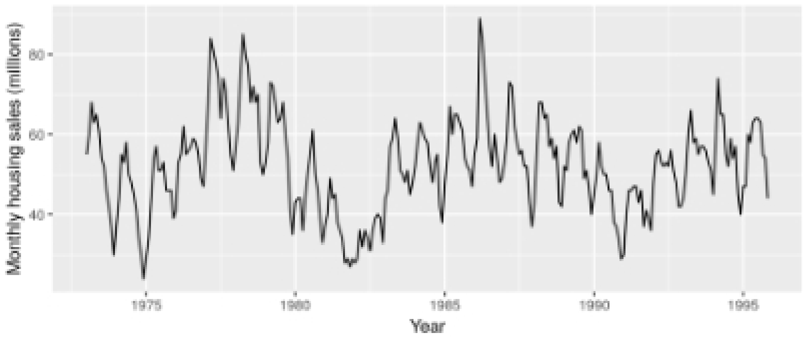
Time Series Pattern Types (Contd.)



Seasonal

- When factors such as the time of the year or the day of the week affect the dependent variable, repetitive patterns are observed in the time series
- Seasonality is always of a fixed and known frequency

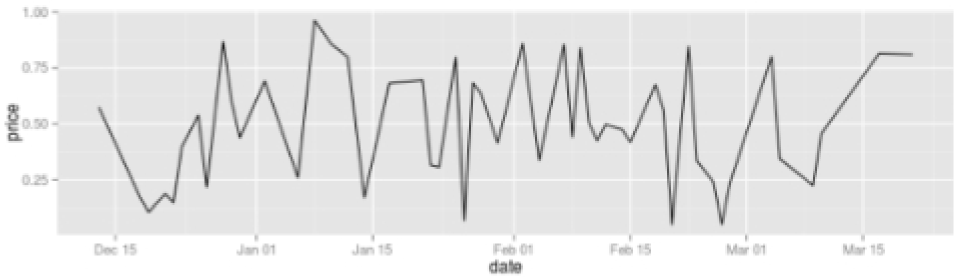
Time Series Pattern Types (Contd.)



Cyclic

- Unlike seasonal patterns, cyclic patterns exhibit rise and fall that are not of fixed period
- Duration is at least 2 years

Time Series Pattern Types (Contd.)

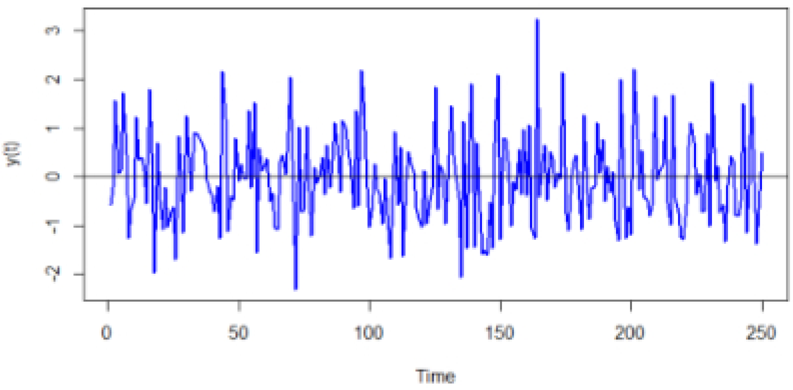


Irregular

- Irregular patterns might occur due to random or unforeseen events
- They are often of short duration and non-repeating

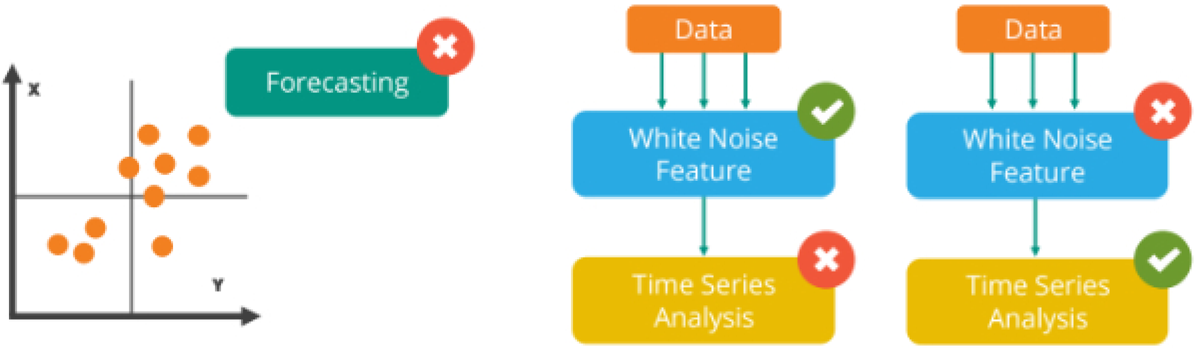
White Noise

A white noise series is one with a zero mean, a constant variance, and no correlation between its values at different times.



Since values are uncorrelated, the adjacent values do not help to forecast future values

White Noise (Contd.)



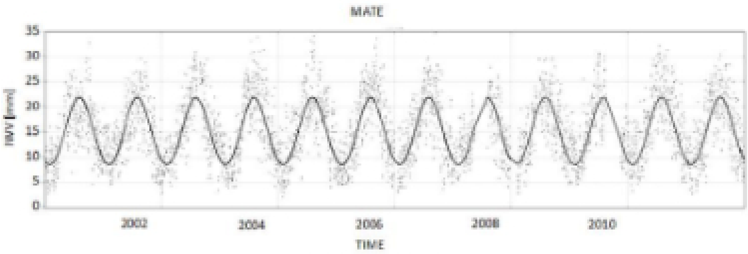
Example: Stock prices of companies may vary daily and time series become uncorrelated

Stationarity

Criterion to classify a series as stationary

Time Invariant (constant)

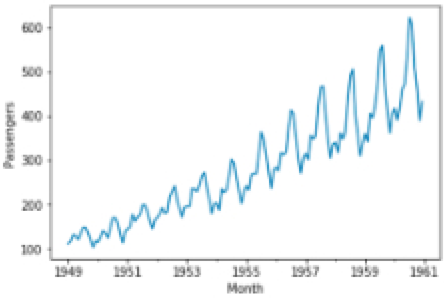
- Mean
- Variance
- Covariance



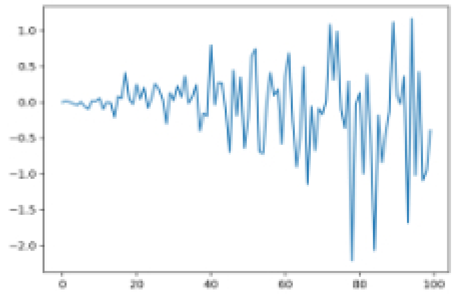
Stationary series

ℹ The time series should be stationary to build the model

Non-Stationary Series

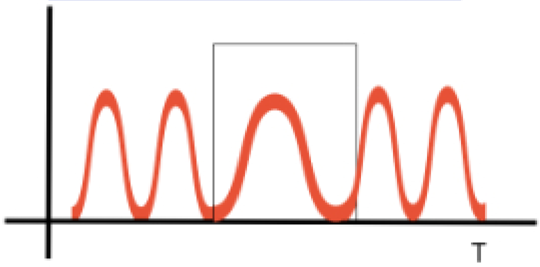


Increasing trend or non-constant **mean**



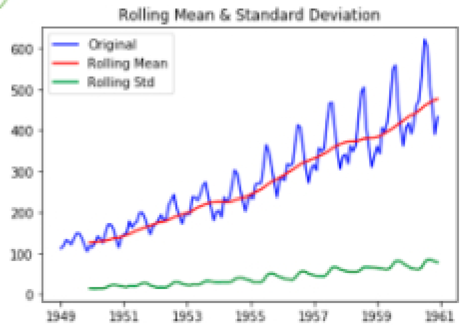
Non-constant **variance**

Co-variance is not constant with time



Stationarity Check

1 Rolling Statistics (Visual)



Plot the moving average or moving variance to check if it varies with time.

Notice the mean and variance **increase** constantly

2 Dickey Fuller test (Statistical)

```
Test Statistic                  0.815369
p-value                         0.991880
#Lags Used                     13.000000
Number of Observations Used   130.000000
Critical Value (1%)            -3.481682
Critical Value (5%)            -2.884042
Critical Value (10%)           -2.578770
dtype: float64
```

Null Hypothesis = TS is non-stationary

If 'Test Statistic' < 'Critical Value', Reject the null hypothesis

Removal of Non-Stationarity

Differencing

Decomposition

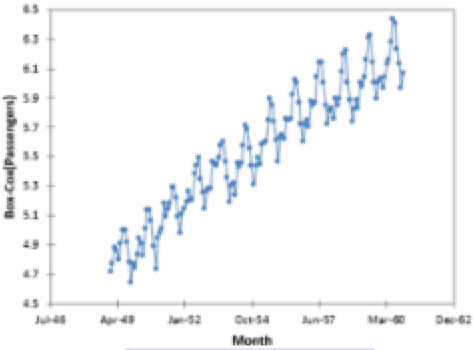
Getting a TS perfectly stationary is desirable but not practical, so it is made as close as possible using these **statistical techniques**

Differencing

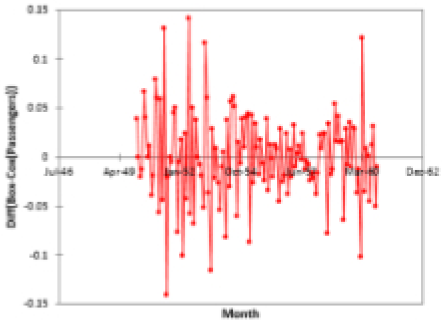
Differencing is performed by subtracting the previous observation from the current observation.

$$\Delta y_t = y_t - y_{t-1}$$

Δy_t is the difference between two successive values
Y_t is the value of y at t and Y_{t-1} is the value preceding Y_t



Non-stationary series



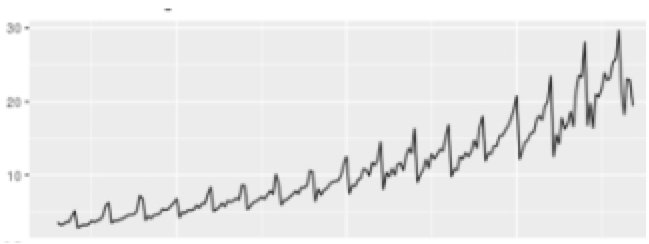
On differencing the series on left

Decomposition

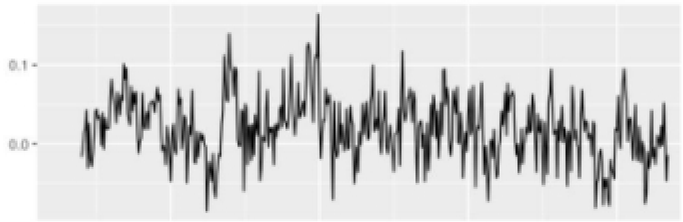
Detrending or de-seasonalizing eliminates the trend and seasonality respectively.

Decomposition is performed on the original series by regressing the series on time and taking the residuals from the regression.

$$\boxed{y_t = \mu + \beta t + \epsilon_t}$$



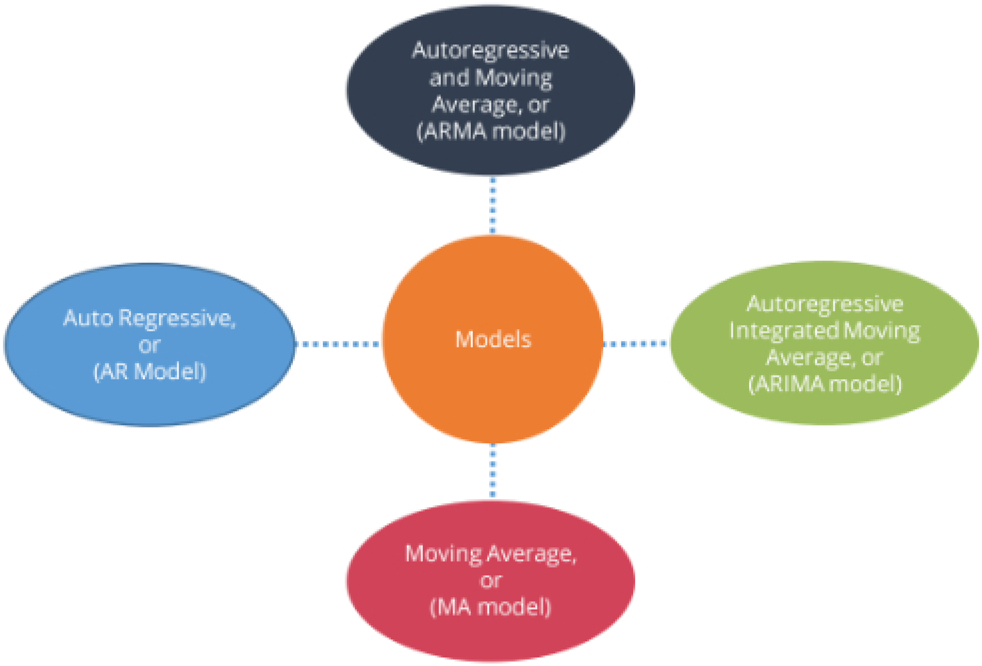
Seasonality with increasing trend



Seasonally decomposed series

You can also use techniques like **transformation** which penalize higher values more than lower values. Example: square root, cube root, log.

Time Series Models



Auto Regressive (AR) Model

In an AR model, you predict future values based on a weighted sum of past values.

Equation for the auto regressive model :

$$Y_t = c + \varphi_1 Y_{t-1} + \varphi_2 y_{t-2} + \cdots + \varphi_p Y_{t-p} + e_t$$

Y_t is the function of different past values of the same variable
e_t is the error term
c is a constant
φ_1 to φ_p are the parameters

AR(1) is a model whose current value is based on the preceding value
AR(2) is based on the preceding two values

Day	Price	
1	21	y_{t-p}
2	22	.
3	23	.
4	24	.
5	23	.
6	26	.
7	27	.
8	27	.
9	29	y_{t-3}
10	30	y_{t-2}
11	32	y_{t-1}
12	?	y_t

Moving Average (MA) Model

MA model is used to forecast time series if Y_t depends only on the random error terms.

Equation for the MA model :

$$Y_t = \mu + \varphi_1 E_{t-1} + \varphi_2 E_{t-2} + \cdots + \varphi_p E_{t-p}$$

Y_t is the function of different past error terms
μ is the mean of the series
E_t is the error term
φ_1 to φ_p are the parameters

The error terms here are assumed to be white noise processes with mean zero and constant variance.

Year	Units	Moving Avg
1994	2	—
1995	5	3
1996	2	3
1997	2	3.67
1998	7	5
1999	6	—

ARMA Model

ARMA model is used to forecast time series using both the past values and the error terms.

Equation for the ARMA model :

$$Y_t = c + \varphi_1 Y_{t-1} + \varphi_2 y_{t-2} + \cdots + \varphi_p Y_{t-p} + e + \mu + E_t + \varphi_1 E_{t-1} + \varphi_2 E_{t-2} + \text{---------} + \varphi_p E_{t-p}$$

Autoregressive part $\oplus$ Moving Average part $=$ ARMA

It is referred as ARMA (p, q), where p is autoregressive terms and q is moving average terms

ARIMA Model

ARIMA model predicts a value in a response time series as a linear combination of its own past values, past errors, also current and past values of other time series.

ARIMA (p, d, q)

- p is the order of the autoregressive part
- d is the order of the differencing
- q is the order of the moving-average process

If no differencing is done (d = 0), the models are usually referred to as ARMA(p, q) models

ACF and PACF

Autocorrelation refers to the way the observations in a time series are related to each other.

Autocorrelation Function (ACF)

ACF is the coefficient of correlation between the value of a point at a current time and its value at lag p, that is, correlation between Y(t) and Y(t-p)

ACF will identify the order of MA process

Partial Autocorrelation Function (PACF)

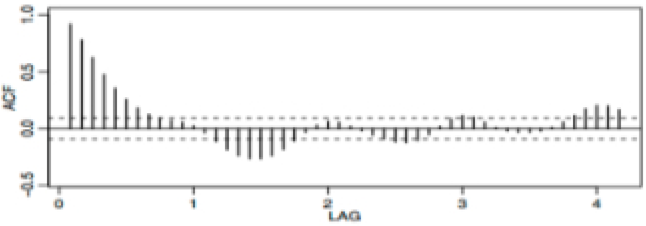
PACF is similar to ACF, but the intermediate lags between t and t-p are removed, that is, correlation between Y(t) and Y(t-p) with p-1 lags excluded.

PACF will identify the order of AR process

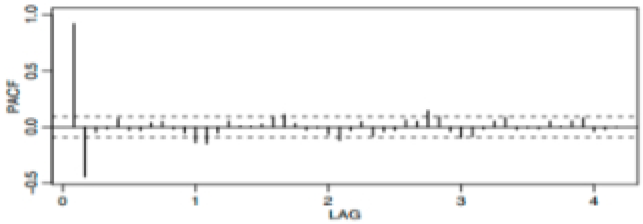
ACF and PACF are used to determine the value of p and q

Characteristics of ACF and PACF

MODEL	ACF	PACF
AR(p)	Spikes decay towards zero	Spikes cutoff to zero
MA(q)	Spikes cutoff to zero	Spikes decay towards zero
ARMA(p,q)	Spikes decay towards zero	Spikes decay towards zero



ACF "decays" to zero



PACF "cuts off" to zero after the 2nd lag

Steps in Time Series Forecasting

Step 01	Visualize the time series – check for trend, seasonality, or random patterns
Step 02	Stationarize the series using decomposition or differencing techniques
Step 03	Plot ACF / PACF and find (p, d, q) parameters
Step 04	Build ARIMA model
Step 05	Make predictions using final ARIMA model