Midterm 2, MA 265 Section: Fall 2014

Name:

PUID:

Exam B:

No calculators may be used on this exam.

Problem Number	Possible Points	Points Earned
I	80pts	
II	20pts	
Total Points	100pts	

I (80pts)

Answer the following questions by clearly circling your answers. No partial credit is awarded for the questions in Part I

(1) Given the matrices

$$\mathbf{A} = \begin{pmatrix} 0 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 5 \end{pmatrix}, \ \mathbf{B} = \begin{pmatrix} 1 & 3 & 7 \\ 0 & 0 & 5 \\ 0 & 0 & 4 \end{pmatrix} \tag{1}$$

which one of the matrix operations below yields an invertible matrix (circle all that apply)
(a) $\mathbf{A} \cdot \mathbf{B}$
(b) $\mathbf{A} \cdot \mathbf{B}^T$
(c) $\mathbf{A}^T \cdot \mathbf{B}$
(d) $\mathbf{A} + \mathbf{B}$
(e) None of the above

(2) Which of the expression below describes the last column vector derived from the Gram-Schmidt process

$$\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4\} \overset{\text{G.S.}}{\rightarrow} \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$$

(a) $\mathbf{u}_4 = \mathbf{a}_4 - \left(\mathbf{a}_4^T \cdot \mathbf{u}_1\right) \mathbf{u}_1 - \left(\mathbf{a}_4^T \cdot \mathbf{u}_2\right) \mathbf{u}_2 - \left(\mathbf{a}_4^T \cdot \mathbf{u}_3\right) \mathbf{u}_3$
(b) $\mathbf{u}_4 = \mathbf{a}_4 - \left(\mathbf{a}_4^T \cdot \mathbf{a}_1\right) \mathbf{u}_1 - \left(\mathbf{a}_4^T \cdot \mathbf{a}_2\right) \mathbf{u}_2 - \left(\mathbf{a}_4^T \cdot \mathbf{a}_3\right) \mathbf{u}_3$
(c) $\mathbf{u}_4 = \mathbf{a}_4 - \left(\mathbf{a}_4^T \cdot \mathbf{u}_1\right) \mathbf{a}_1 - \left(\mathbf{a}_4^T \cdot \mathbf{u}_2\right) \mathbf{a}_2 - \left(\mathbf{a}_4^T \cdot \mathbf{u}_3\right) \mathbf{a}_3$
(d) $\mathbf{u}_4 = \mathbf{a}_4 - \left(\mathbf{a}_4^T \cdot \mathbf{a}_1\right) \mathbf{a}_1 - \left(\mathbf{a}_4^T \cdot \mathbf{a}_2\right) \mathbf{a}_2 - \left(\mathbf{a}_4^T \cdot \mathbf{a}_3\right) \mathbf{a}_3$

(3) Let V denote the vector space of 3×3 matrices endowed with the usual matrix addition and the usual product of matrices with numbers. Which ones of the subsets of V below are not subspaces of V? (circle all that apply)

(a) The set of diagonal 3×3 matrices.
(b) The set of 3×3 matrices for which $\mathbf{A} \cdot \mathbf{A}^T \neq 0$.
(c) The set of 3×3 matrices for which $\mathbf{A}^{-1} = \mathbf{A}^T$

(d) The set of 3×3 matrices for which $\mathbf{A} + 2\mathbf{A}^T = \mathbf{0}_{n\times n}$
(e) The set of 3×3 matrices for which $2\mathbf{A} + \mathbf{A}^T = \mathbf{I}$

(4) Which statements below are true (circle all that apply)

(a) Every vector in the null space of a matrix $\mathbf{A}$ is necessarily orthogonal to every vector in the rowspace of $\mathbf{A}$.
(b) There are vector spaces of dimension 0.
(c) The row rank can sometimes differ from the column rank.
(d) Orthonormal set of vectors in a basis can sometimes be linearly dependent

(5) What is the determinant of the matrix

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 2 & 3 & 0 \end{pmatrix}$$

(a) 0
(b) 3
(c) 4
(d) 12

(6) What is the dimension of the null space of the matrix below

$$\mathbf{A} = \begin{pmatrix} 4 & 2 & 0 \\ 2 & 0 & 6 \\ 0 & 6 & 1 \end{pmatrix}$$

(a) 0
(b) 1
(c) 2
(d) 3

(7) The row echelon form of a matrix $\mathbf{A}$ yields the following matrix

$$\begin{pmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

then we can conclude that the row space of $\mathbf{A}$ is of dimension
(b) 0
(c) 1
(d) 2
(e) 3
(f) None of the above

(8) Which among the statements below are true about $n \times n$ matrix $\mathbf{Q}$ with orthonormal columns? (circle all that apply)

(a) $\mathbf{Q} \cdot \mathbf{Q}^T = \mathbf{I}$
(b) $\mathbf{Q}^{-1} = \mathbf{Q}^T$
(c) $\mathbf{Q} \cdot \mathbf{Q} = \mathbf{I}$
(c) $\mathbf{Q} + \mathbf{Q}^T = 2\mathbf{I}$
(d) None of the above

II (20pts)

Answer the following questions by showing details of your work.

(a) Use the Gram-Schmidt process to turn the basis below into an orthonormal basis

$$\left\{ \mathbf{a}_1 = \begin{pmatrix} 1/3 \\ 2/3 \\ 2/3 \end{pmatrix}, \mathbf{a}_2 = \begin{pmatrix} -2/3 \\ -1/3 \\ 2/3 \end{pmatrix}, \mathbf{a}_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\} \to \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$$

Answer:

(b) Use the orthogonality property of the matrix $\mathbf{U}$ derived in (a) to solve for the vector $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ in the equation

$$\mathbf{U} \cdot \mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Answer:

(c) Let $\mathbf{A}$ denote the matrix which collects the column vectors given in (a). Find a basis for a basis for the row space of $\mathbf{A}$ as a subset of the rows of $\mathbf{A}$.

Answer: