Midterm 2, MA 265 Section: Spring 2016

Name:

PUID:

Exam B:

No calculators may be used on this exam.

Problem Number	Possible Points	Points Earned
I	80pts	
II	20pts	
Total Points	100pts	

I (80pts)

Answer the following questions by clearly circling your answers. No partial credit is awarded for the questions in Part I

(1) Given the matrices

$$\mathbf{A} = \begin{pmatrix} 0 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 5 \end{pmatrix}, \ \mathbf{B} = \begin{pmatrix} 1 & 3 & 7 \\ 0 & 0 & 5 \\ 0 & 0 & 4 \end{pmatrix} \tag{1}$$

which one of the matrix operations below yields an invertible matrix (circle all that apply)
(a) $\mathbf{A} \cdot \mathbf{B}$
(b) $\mathbf{A} \cdot \mathbf{B}^T$
(c) $\mathbf{A}^T \cdot \mathbf{B}$
(d) $\mathbf{A} + \mathbf{B}$
(e) None of the above

(2) Let

$$\mathbf{v}_1 = \begin{pmatrix} 1 & 1 & 2 & 3 \end{pmatrix}, \mathbf{v}_2 = \begin{pmatrix} 2 & 3 & 4 & 5 \end{pmatrix}, \mathbf{v}_3 = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}, \mathbf{v}_4 = \begin{pmatrix} 3 & 4 & 4 & 4 \end{pmatrix}.$$

be vectors in $\mathbb{R}^{1\times 4}$ spanning the subspace W. Which of the following subset forms a basis for W ? (circle all that apply)
(a) $\{\mathbf{v}_1, \mathbf{v}_2\}$.
(b) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4\}$.
(c) $\left\{\begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 1 & 1 \end{pmatrix}\right\}$.
(d) $\left\{\begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 2 \end{pmatrix}\right\}$.
(e) $\left\{\begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 2 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & -1 \end{pmatrix}\right\}$.

(3) Let W be the vector space spanned by the vectors:

$$\mathbf{u}_1 = \begin{pmatrix} 1 & 0 & 1 & 0 \end{pmatrix}, \mathbf{u}_2 = \begin{pmatrix} -1 & 1 & 1 & 1 \end{pmatrix}, \mathbf{u}_2 = \begin{pmatrix} 1 & 1 & 1 & 0 \end{pmatrix}, \mathbf{u}_2 = \begin{pmatrix} 0 & 2 & 2 & 3 \end{pmatrix}.$$

Apply the Gram-Schmidt process to the vectors $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$, $\mathbf{u}_4$ (in this order) to find an orthonormal basis $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, $\mathbf{v}_4$ of W. What is $\mathbf{v}_3$.

(a) $\mathbf{v}_3 = \left(\, {}^1\!/_2 \quad {}^1\!/_2 \quad -{}^1\!/_2 \quad {}^1\!/_2 \, \right)$
(b) $\mathbf{v}_3 = \left(\, {}^1\!/\sqrt{12} \quad {}^3\!/\sqrt{12} \quad -{}^1\!/\sqrt{12} \quad -{}^1\!/\sqrt{12} \, \right)$
(c) $\mathbf{v}_3 = \left(\, {}^1\!/_4 \quad {}^3\!/_4 \quad -{}^1\!/_4 \quad {}^1\!/_4 \, \right)$
(d) $\mathbf{v}_3 = \left(\, \frac{3-2\sqrt{2}}{4} \quad {}^1\!/_4 \quad \frac{1-\sqrt{2}}{2} \quad {}^1\!/_4 \, \right)$
(e) $\mathbf{v}_3 = \left(\, 0 \quad 0 \quad 0 \quad 1 \, \right)$

(4) Suppose $\mathbf{A}$ is 5×3 matrix such that $\text{Rank}(\mathbf{A}) = 3$. Which of the following statement is TRUE?

(a) The rank of $\mathbf{A}^T$ is 5
(b) The nullity of $\mathbf{A}$ is 2
(c) $\mathbf{A} \cdot \mathbf{x} = \mathbf{0}$ has a nontrivial solution
(d) The rows of $\mathbf{A}$ are linearly dependent
(e) The columns of $\mathbf{A}$ are linearly dependent

(5) What is the determinant of the matrix

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 2 & 3 & 0 \end{pmatrix}$$

(a) 0
(b) 3
(c) 4
(d) 12

(6) What is the dimension of the null space of the matrix below

$$\mathbf{A} = \begin{pmatrix} 4 & 2 & 0 \\ 2 & 0 & 6 \\ 0 & 6 & 1 \end{pmatrix}$$

(a) 0
(b) 1

(c) 2
(d) 3

(7) The row echelon form of a matrix $\mathbf{A}$ yields the following matrix

$$\begin{pmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

then we can conclude that the row space of $\mathbf{A}$ is of dimension
(b) 0
(c) 1
(d) 2
(e) 3
(f) None of the above

(8) Which among the statements below are true about $n \times n$ matrix $\mathbf{Q}$ with orthonormal columns? (circle all that apply)

(a) $\mathbf{Q} \cdot \mathbf{Q}^T = \mathbf{I}$.
(b) $\mathbf{Q}^{-1} = \mathbf{Q}^T$.
(c) $\mathbf{Q} \cdot \mathbf{Q} = \mathbf{I}$.
(c) $\mathbf{Q} + \mathbf{Q}^T = 2\mathbf{I}$.
(d) None of the above.

II (20pts)

Answer the following questions by showing details of your work.

Let

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & -1 & 2 \\ 1 & -1 & 0 & 0 \\ 4 & -5 & 1 & 2 \end{pmatrix}$$

(a) Find a basis for the null space of $\mathbf{A}$ and the rank of $\mathbf{A}$.

Answer:

(b) Let $\mathbf{q}_1 = \frac{\sqrt{2}}{2}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{q}_2 = \begin{pmatrix} x_1 \\ 0 \\ x_2 \end{pmatrix}$ and $\mathbf{q}_3 = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$. Solve for x_1,x_2,y_1,y_2,y_3 so as to ensure that set of vectors below

$$\{\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3\}$$

form an orthonormal basis for $\mathbb{R}^{3\times 1}$.
Answer:

(c) Let $\mathbf{A}$ denote the matrix which collects the column vectors $\mathbf{A}$. Find a basis for the colum space of $\mathbf{A}$ formed by taking a subset of the columns of $\mathbf{A}$.

Answer: