Practice Midterm 2, MA 265 Section: Spring 2015

Name:

PUID:

Exam B:

No calculators may be used on this exam.

Problem Number	Possible Points	Points Earned
I	80pts	
II	20pts	
Total Points	100pts	

I (80pts)

Answer the following questions by clearly circling your answers. No partial credit is awarded for the questions in Part I

(1) Let $\mathbf{U}$, $\mathbf{V}$, $\mathbf{W}$, $\mathbf{X}$ denote $n \times n$ matrices. Which expression below is always equal to $\left(\mathbf{U} \cdot (\mathbf{V} \cdot \mathbf{W})^T \cdot \mathbf{X}\right)^T$?

(a) $\mathbf{U}^T \cdot \mathbf{V} \cdot \mathbf{W} \cdot \mathbf{X}^T$
(b) $\mathbf{U}^T \cdot \mathbf{W} \cdot \mathbf{V} \cdot \mathbf{X}^T$
(c) $\mathbf{X}^T \cdot \mathbf{W} \cdot \mathbf{V} \cdot \mathbf{U}^T$
(d) $\mathbf{X}^T \cdot \mathbf{V} \cdot \mathbf{W} \cdot \mathbf{U}^T$
(e) None of the above.

(2) The product of matrices

$$\begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 & 5 \end{pmatrix}$$

has rank equal to

(a) 0
(b) 1
(c) 2
(d) 3
(e) None of the above.

(3) Let $\mathbf{v}_1$,$\mathbf{v}_2$,$\mathbf{v}_3$ denote three vectors in a vector space. Which of the following statement is FALSE ?

(a) If $\mathbf{v}_3$ is in the $\mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$, then $\mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2\} = \mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.
(b) If $\mathbf{v}_1$,$\mathbf{v}_2$,$\mathbf{v}_3$ are linearly independent, then $\mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2\} \neq \mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.
(c) If $\mathbf{v}_1$,$\mathbf{v}_2$,$\mathbf{v}_3$ are linearly dependent, then $\mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2\} = \mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.
(d) If $\mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2\} \neq \mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, then $\mathbf{v}_3$ is not in $\mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$.
(e) If $\mathbf{v}_3$ is in the $\mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$, then $\mathbf{v}_3 - \mathbf{v}_2$ is in $\mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$.

(4) Which of the following sets of vectors are linearly independent ?

(a) $\{\begin{pmatrix} 1 & 3 & 5 \end{pmatrix}, \begin{pmatrix} 5 & 7 & 11 \end{pmatrix}, \begin{pmatrix} 11 & 13 & 17 \end{pmatrix}, \begin{pmatrix} 17 & 19 & 1 \end{pmatrix}\}$.
(b) $\{\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 3 & 6 & 9 \end{pmatrix}, \begin{pmatrix} 11 & 13 & 17 \end{pmatrix}\}$.
(c) $\{\begin{pmatrix} 3 & 5 & 0 \end{pmatrix}, \begin{pmatrix} 13 & 17 & 0 \end{pmatrix}, \begin{pmatrix} 7 & 11 & 0 \end{pmatrix}\}$.
(d) $\{\begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 5 & 6 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 7 & 8 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 11 \end{pmatrix}\}$.
(e) None of the above.

(5) On $\mathbb{R}^{3\times 1}$, with the standard inner product, the angle between $\begin{pmatrix} 2 & 1 & 2 \end{pmatrix}^T$ and $\begin{pmatrix} 1 & 1 & 0 \end{pmatrix}^T$ is

(a) $\pi/2$
(b) $\pi/4$
(c) $\pi/6$
(d) $\pi/12$
(e) None of the above.

(6) What is the dimension of the column space of the matrix below

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & 1 \end{pmatrix}$$

(a) 0
(b) 1
(c) 2
(d) 3

(7) Let P_3 denote the vector space of degree 3 or less, with standard addition of polynomials and standard scalar multiplication as operations. Which of the following subsets of P_3 are subspaces of P_3 ?
(i) T= {all polynomials of degree 2 or less}.
(ii) U= {all polynomials in $g(x)$ in P_3 such that $g(0) = 3$}.

(iii) V= $\{$all polynomials in $g(x)$ in P_3 such that $g(3) = 0\}$.

(a) (i) only
(b) (i) and (ii)
(c) (iii) only
(d) (i) and (iii)
(e) (ii) and (iii)

(8) Let V denote the vector space $\mathbb{R}^{3\times 1}$ with standard vector addition and scalar multiplication. Which of the following subsets of V are subspaces

(i) S= $\left\{ \text{all vectors } \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ in } V \text{ such that } x + y > z \right\}$.

(ii) W= $\left\{ \text{all vectors } \begin{pmatrix} t \\ u \\ v \end{pmatrix} \text{ in } V \text{ such that } 2u = 3v \right\}$.

(iii) U= $\left\{ \text{all vectors } \begin{pmatrix} a \\ b \\ c \end{pmatrix} \text{ in } V \text{ such that } abc = 0 \right\}$.

(a) (iii) only
(b) (i), (ii), and (iii)
(c) (i) and (iii)
(d) (i) and (ii)
(e) (ii) only

II 20pts)

(a) Use the Gram–Schmidt process to find the orthonormal basis corresponding to the basis

$$\mathcal{B} = \left\{ \mathbf{u}_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{u}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \mathbf{u}_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \mathbf{u}_4 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \right\}$$

show all your work in detail and clarity:

Answer:

(b) Let $\mathbf{v} = \begin{pmatrix} 1 \\ x \\ 2 \end{pmatrix}$, find the value of paramater x which ensure that

(i) $\mathbf{v}$ is orthogonal to $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

Answer:

(ii) $\mathbf{v}$ is parallel to $\begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix}$

Answer: