1: Analytical Solution:

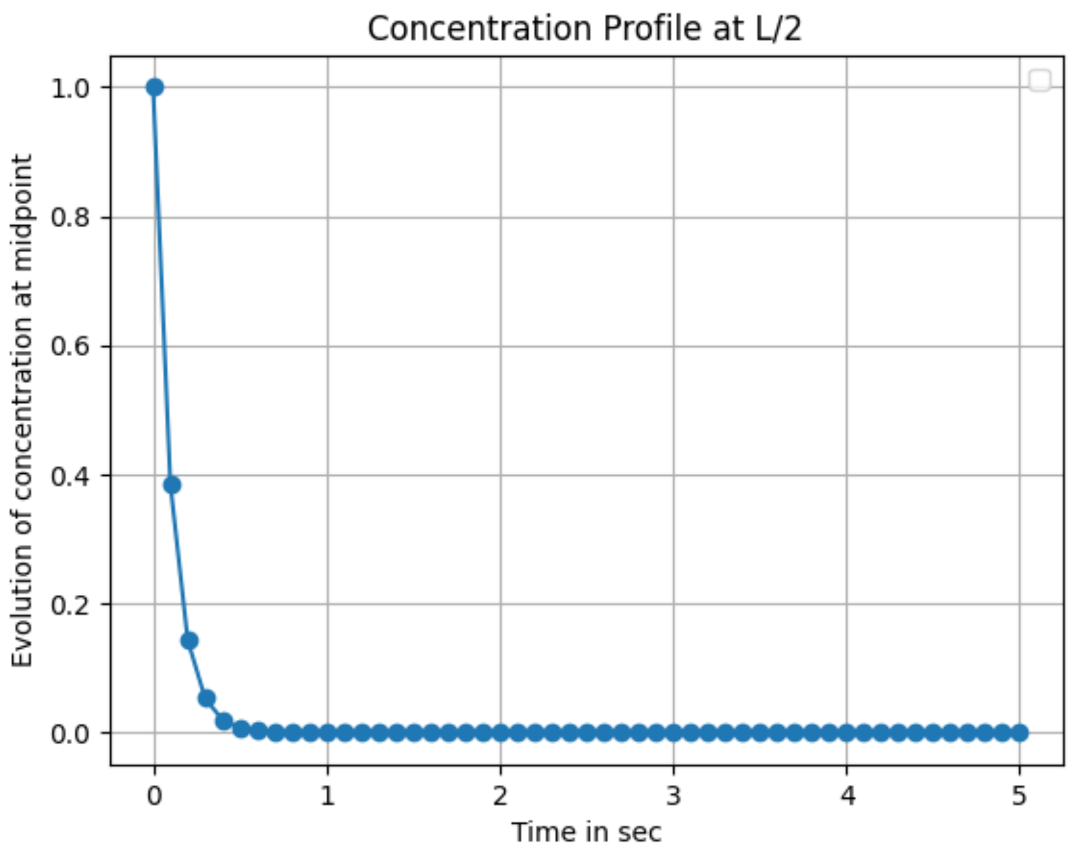
$$C(x,t) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi}{L}x\right) \exp\left(-D\left(\frac{\pi n}{L}\right)^2 t\right)$$

$$A_n = \frac{2}{L}\int_0^L f(\xi)\cdot \sin\left(\frac{n\pi}{L}\cdot \xi\right) d\xi$$

$$f(\xi) = 4\xi(1-\xi)$$

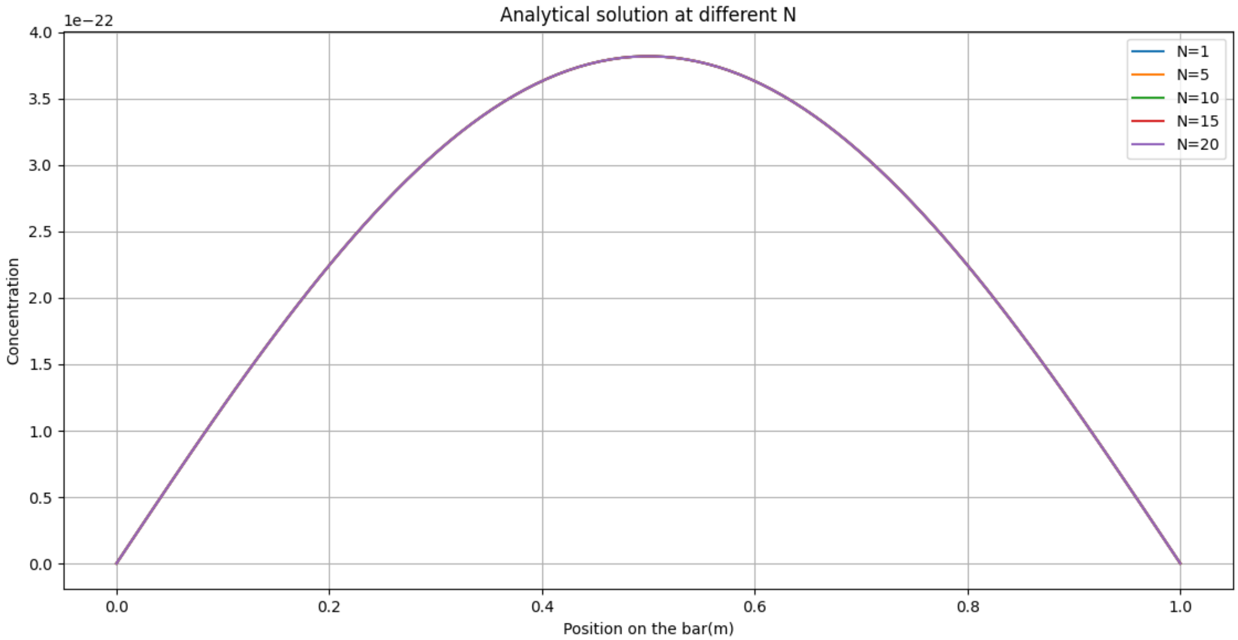
$$A_n = \frac{16}{\pi^3 n^3}\left(1-(-1)^n\right)$$ [integrating using uv rule]

$$C(x,t) = \sum_{n=1}^{\infty} \frac{16}{\pi^3 n^3}\left(1-(-1)^n\right)\cdot \sin\left(\frac{n\pi}{L}x\right)\cdot \exp\left(-D\left(\frac{\pi n}{L}\right)^2 t\right)$$



Part 1: evolution of concentration at L/2 with time

Part 2: C(x,t) vs x with varying 'N' values

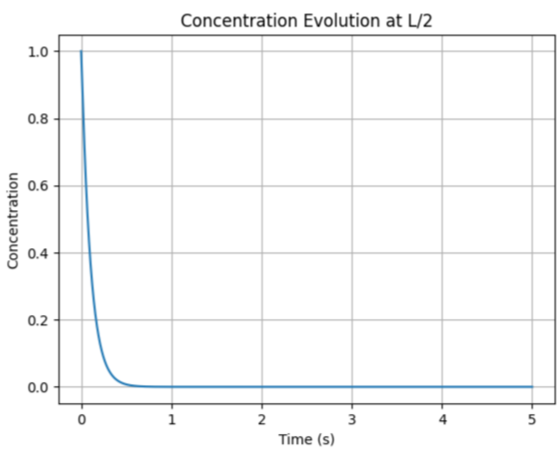

Analytical solution at different N
1e−22
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Concentration
0.0
0.2
0.4
0.6
0.8
1.0
Position on the bar(m)
N=1
N=5
N=10
N=15
N=20

The concentration curves at $N = 1, 5, 10, 15, 20$ are converging.

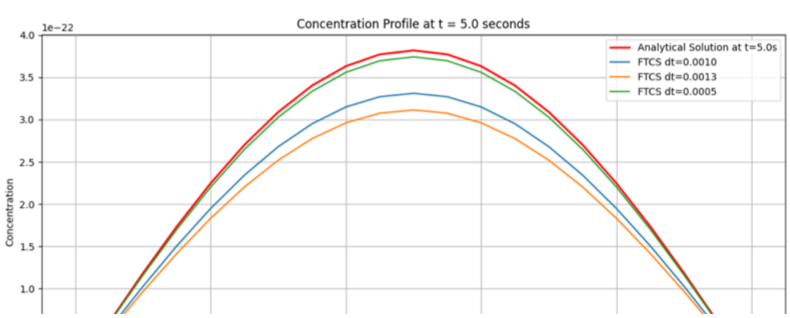
→ The fourier solution has an exponential term that's decaying rapidly due to diffusivity coefficient, $D = 1\ m/s$ being very large and due to the n^2 term in the $\exp(-D n^2 \pi^2 t)$

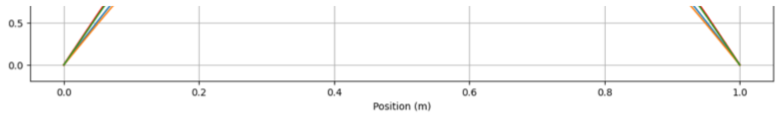
1b FTCS Solution

Part 1: time evolution of concentration



Part 2:

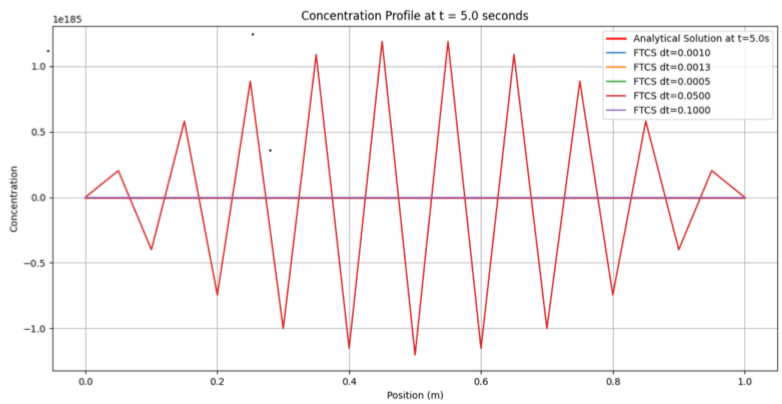




when $\Delta t \leq \frac{1}{2}\left(\frac{\Delta x}{D}\right)^2 \leq 0.00125 \rightarrow$ we get a stable solution

when $\Delta t > 0.00125$, the FTCS solution becomes numerically unstable.

$\rightarrow$ Plot at $\Delta t > 0.00125 \rightarrow$ at $\Delta t = 0.05$ and 0.2



1C <u>Crank-Nicolson method:</u>

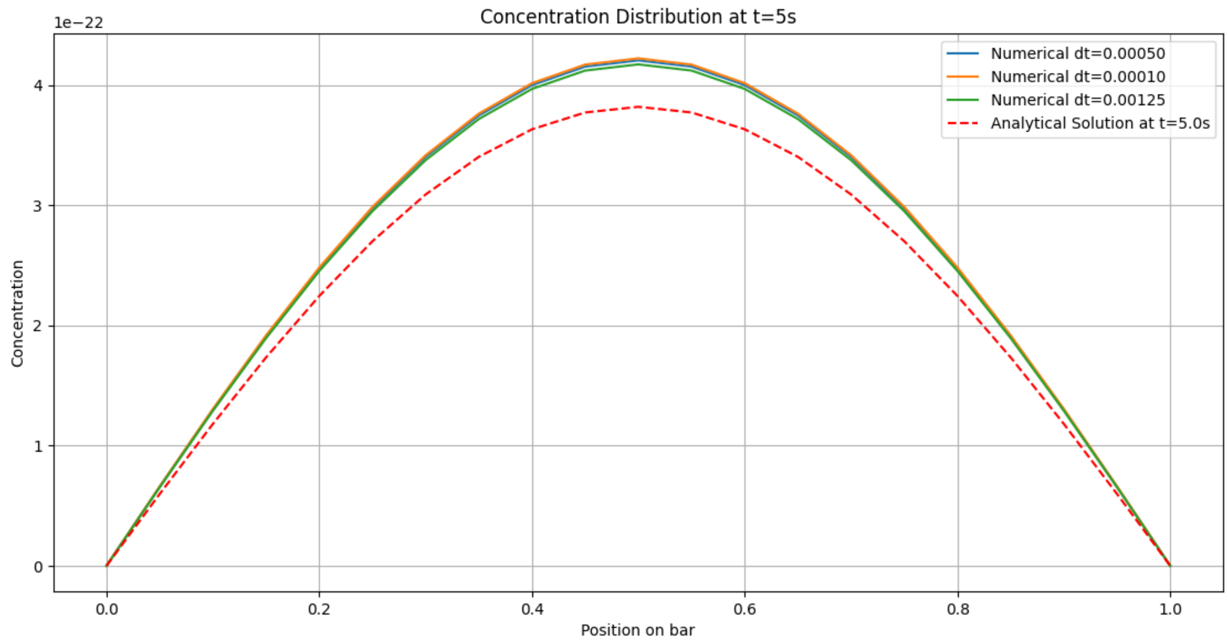
(unconditionally stable)

Tridiagonal matrix algorithm:

$$\boxed{C_x^{t+1} - C_x^t = \frac{\alpha}{2}\left[\left(C_{x+1}^{t} - 2C_x^{t} + C_{x-1}^{t}\right) + \left(C_{x+1}^{t+1} - 2C_x^{t+1} + C_{x-1}^{t+1}\right)\right]}$$

Crank Nicholson at diff Δt

Analytical Solution and [illegible]



Observations:

1. Crank Nicholson method took longer than analytical and FCTS method.
2. When large values of dt were chosen, the curves were unstable (oscillations).