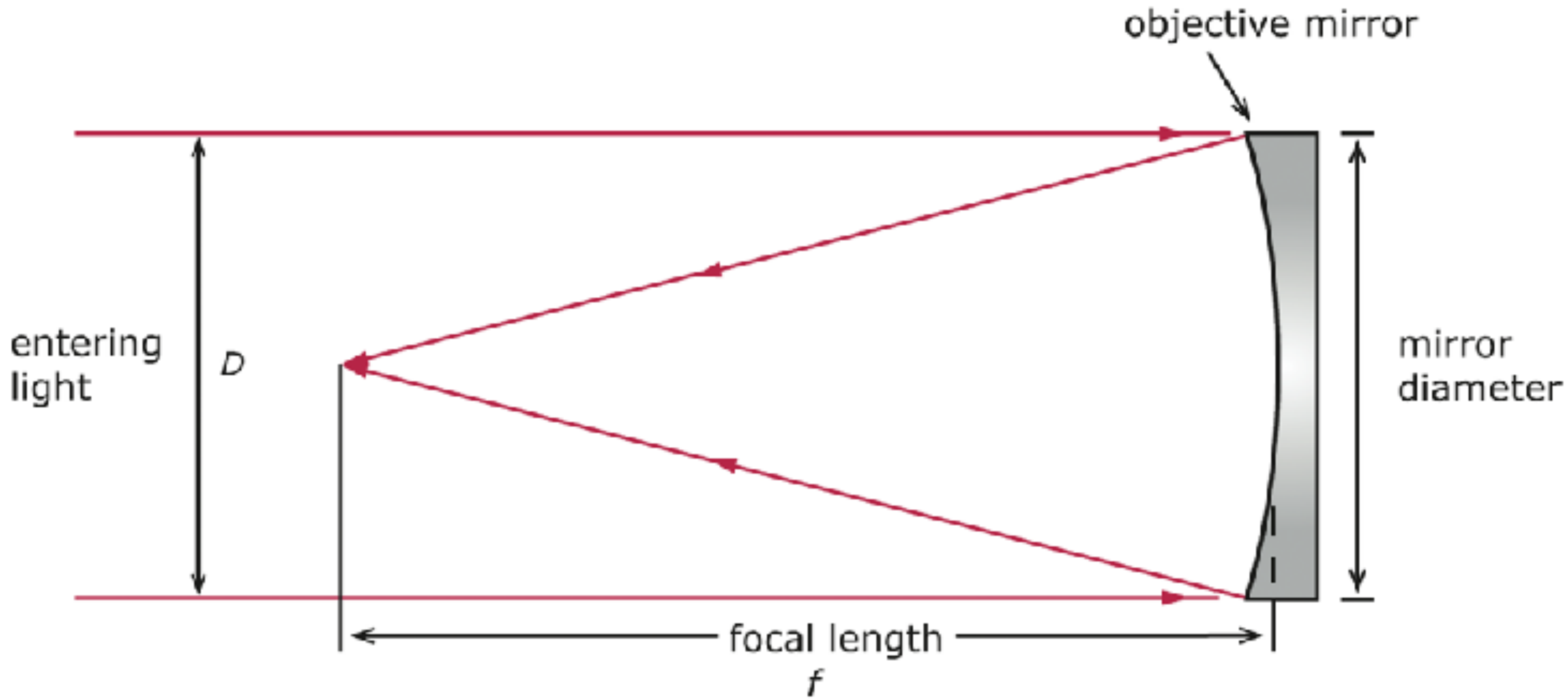
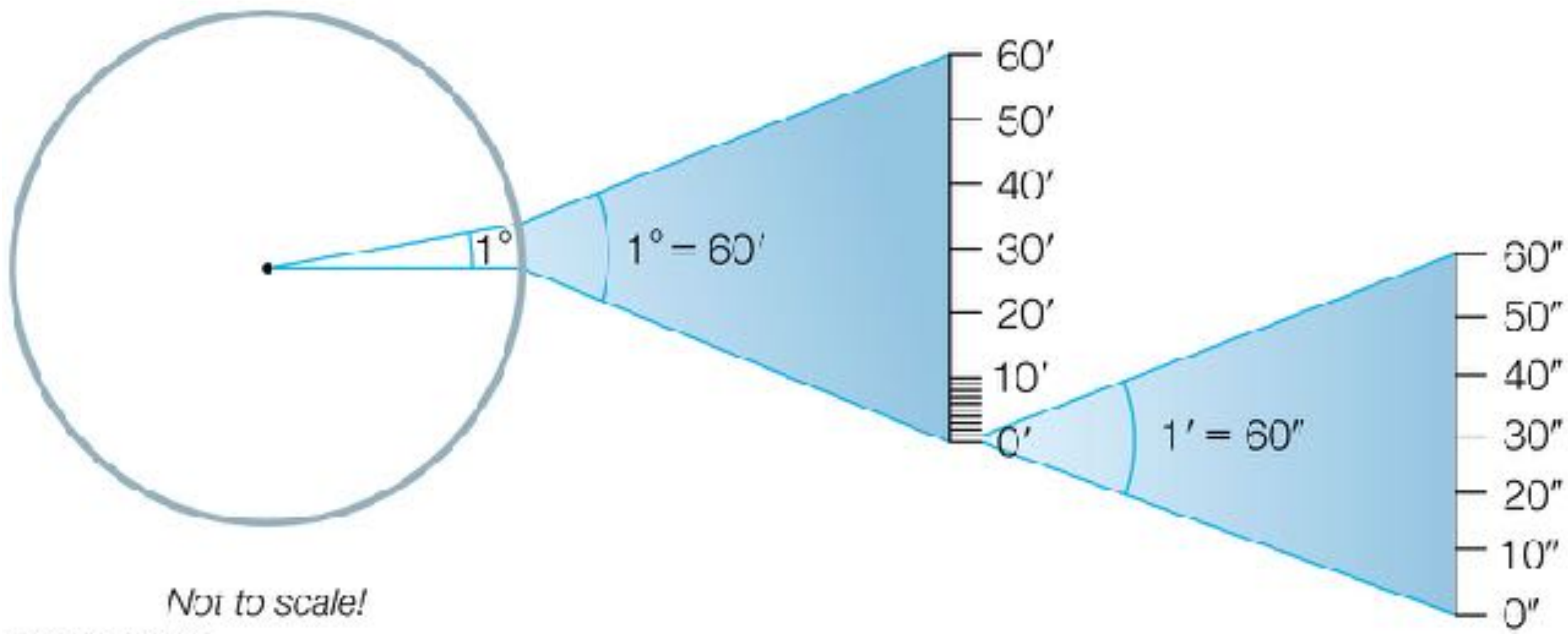
Astronomy 503
Observational Techniques

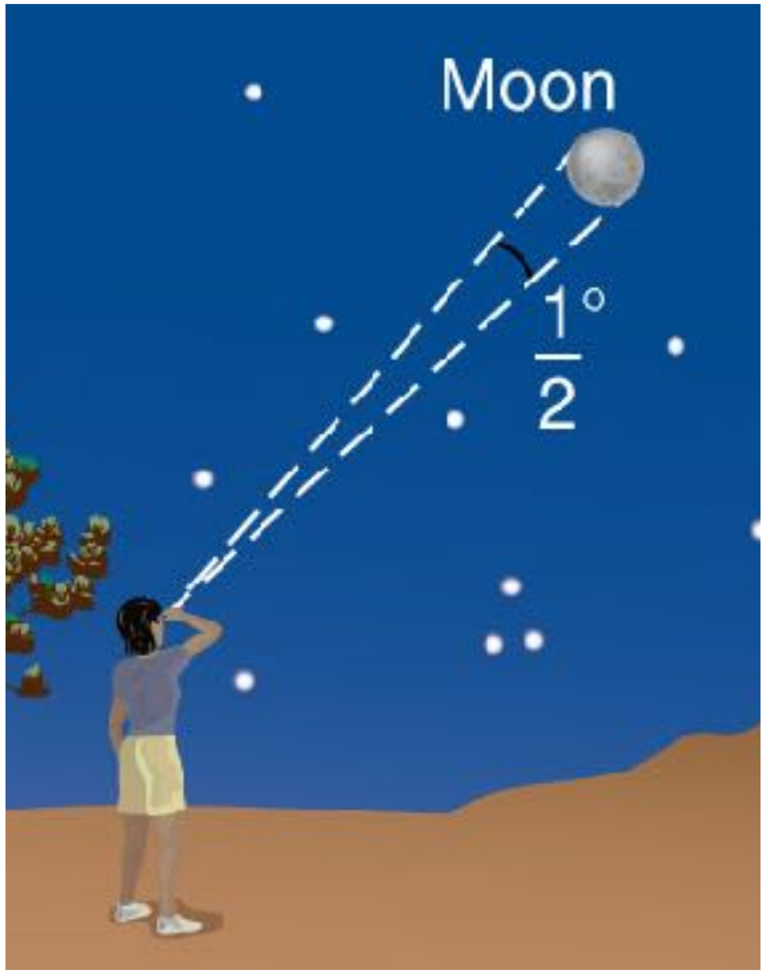


Prof. Gautham Narayan

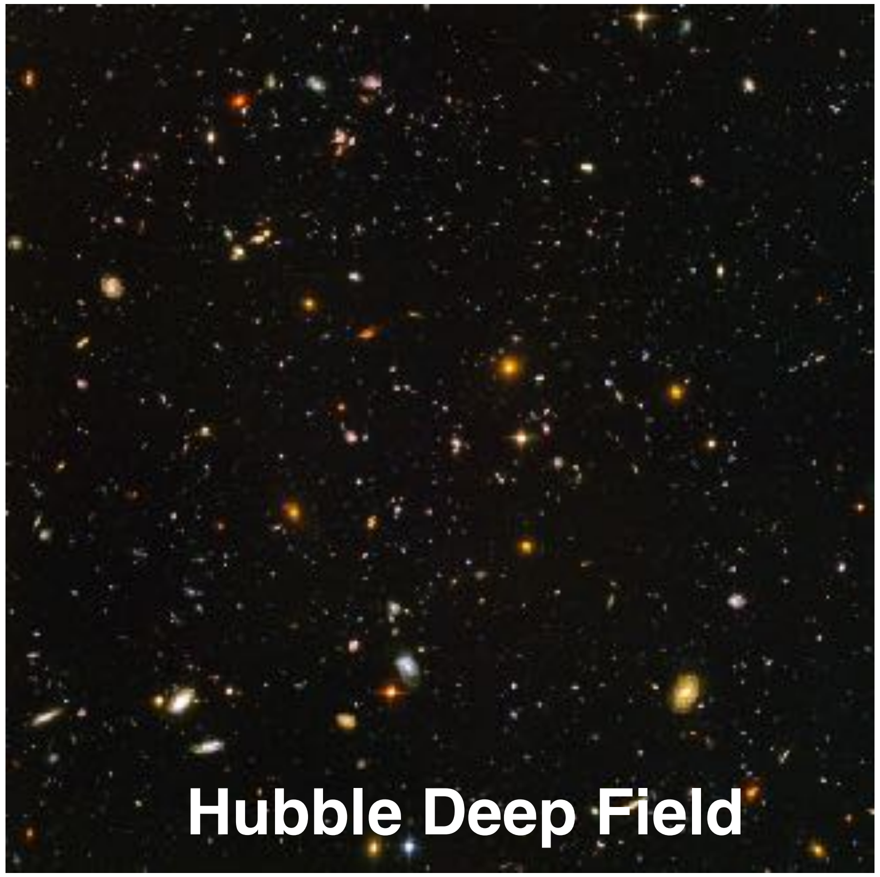
Lecture 07: Optics

Resolution and Angular Sizes:
degrees, arcminutes, arcseconds



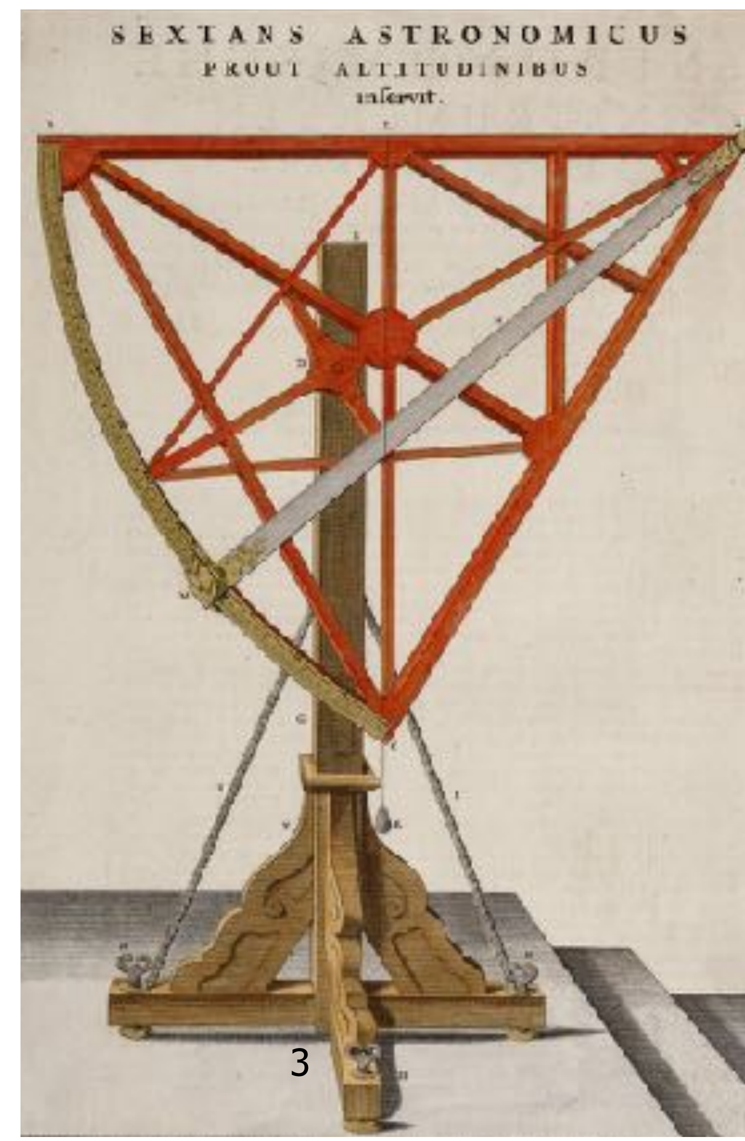


- A circle has 360°
- The Sun and Moon are ½°
- An arcmin is 1/60th of a degree
- 20/20 human vision is 1' (arcmin) resolution
- Good resolution for an optical telescope on the ground is 1" (arcsec)
- Hubble/JWST resolution in space is 0.05-0.1"



Tycho Brahe (1546—1601)
the master observer

- Danish Nobleman, later exiled to Bohemia
- Lost his nose in a sword duel over who was the better mathematician. (Subsequently wore a brass prosthetic nose.)
- Observed positions of 1000 stars and the planets using a custom-made dedicated observatory with state-of-the-art equipment
  - Uraniborg – “Castle of the Heavens”
- Conducted more precise observations than had ever been done
  - Accurate to ~1 arcminute
  - x10 better than had ever been accomplished before
  - No telescopes!
- Witnessed a supernovae in 1572
- Correctly saw that the Moon orbited the Earth and tried to develop a “geo-heliocentric” system.
- Remained a Geocentrist all his life
- Johannes Kepler was his assistant and apprentice
- Died (or was murdered with mercury) after massive banquet in Prague of kidney failure, because he didn’t want to leave.
  - Observers go hard.



Charles Messier

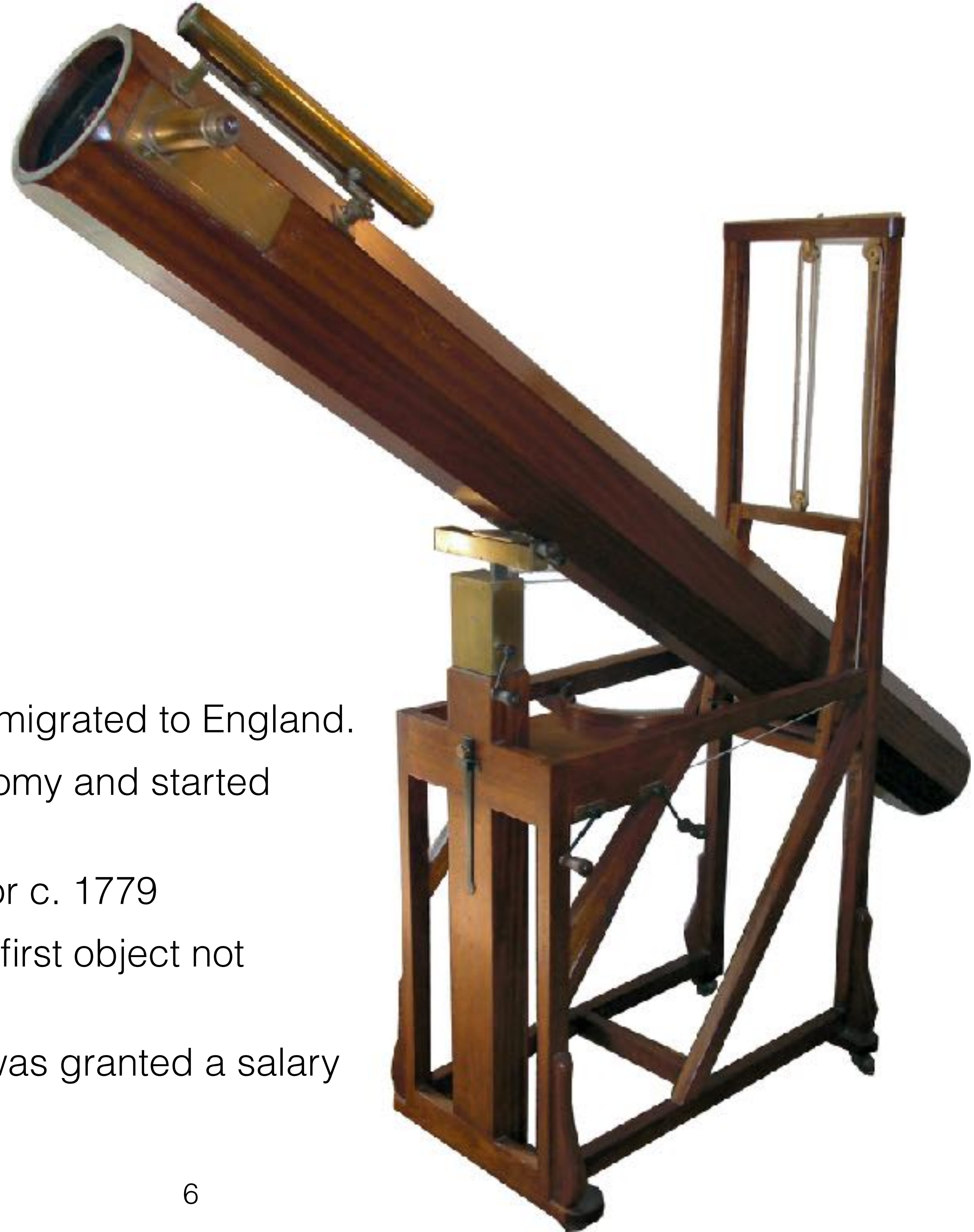
- 1730–1817
- Devoted his life to astronomy after witnessing Haley’s comet.
- Worked for the French Naval Observatory, used a 4” refracting telescope.
- By 1789 he had found between 13–21 comets.
- Made the first catalog of the deep sky.

New General Catalog = NGC

- New General Catalog of Nebulae and Clusters of Stars
- Started in 1888 by John Dreyer as a followup to Herschel's General Catalog
- 7,840 sources

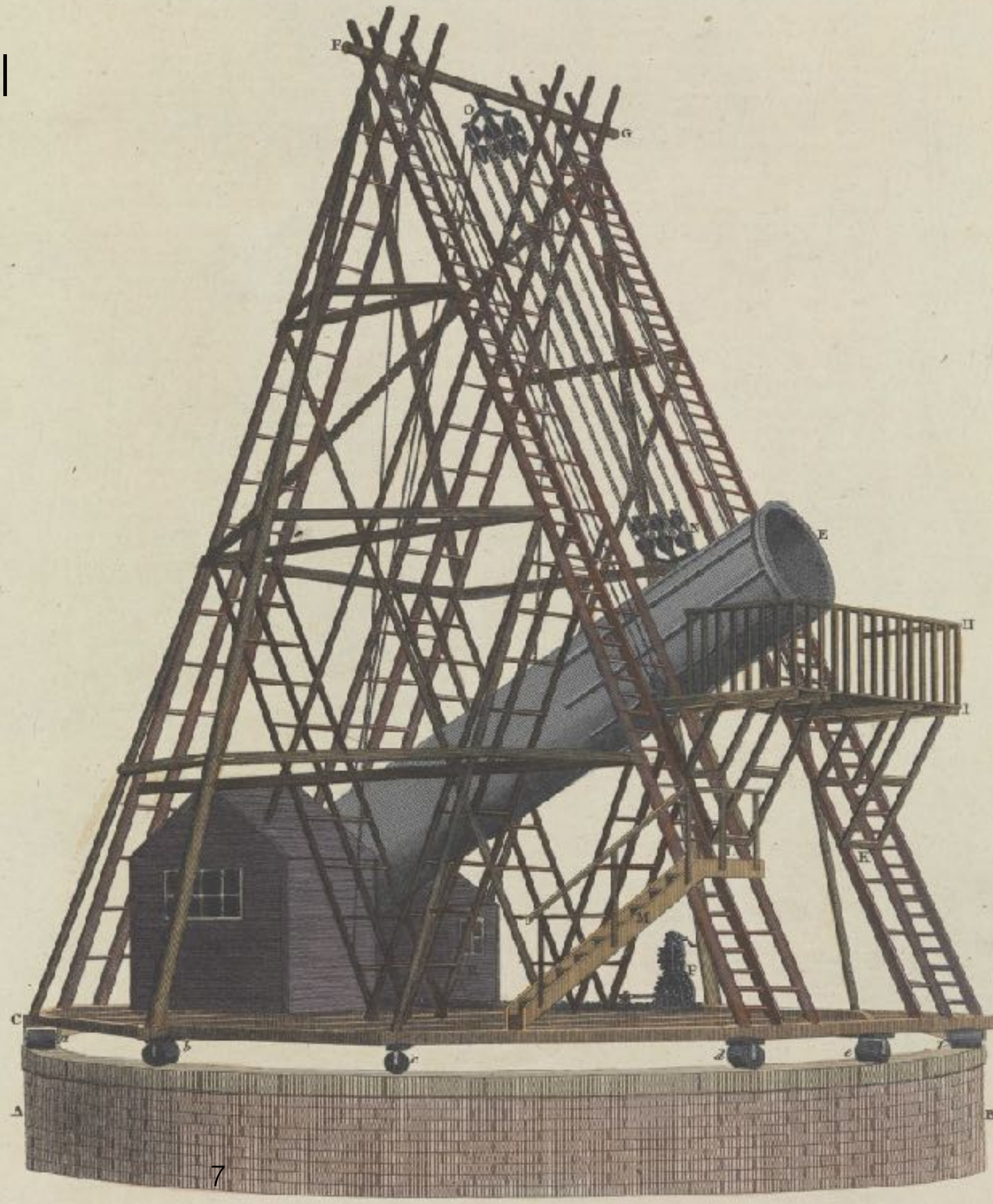
Reflectors: Herschel

- William Herschel 1738—1822
- Was a German musician who emigrated to England.
- Became fascinated with astronomy and started building his own telescopes.
- 6.2” (16cm) Newtonian Reflector c. 1779
- Discovered Uranus (!) in 1781 (first object not known to the ancients)
- Instantly became famous and was granted a salary from the king.
- Discovered infrared radiation.



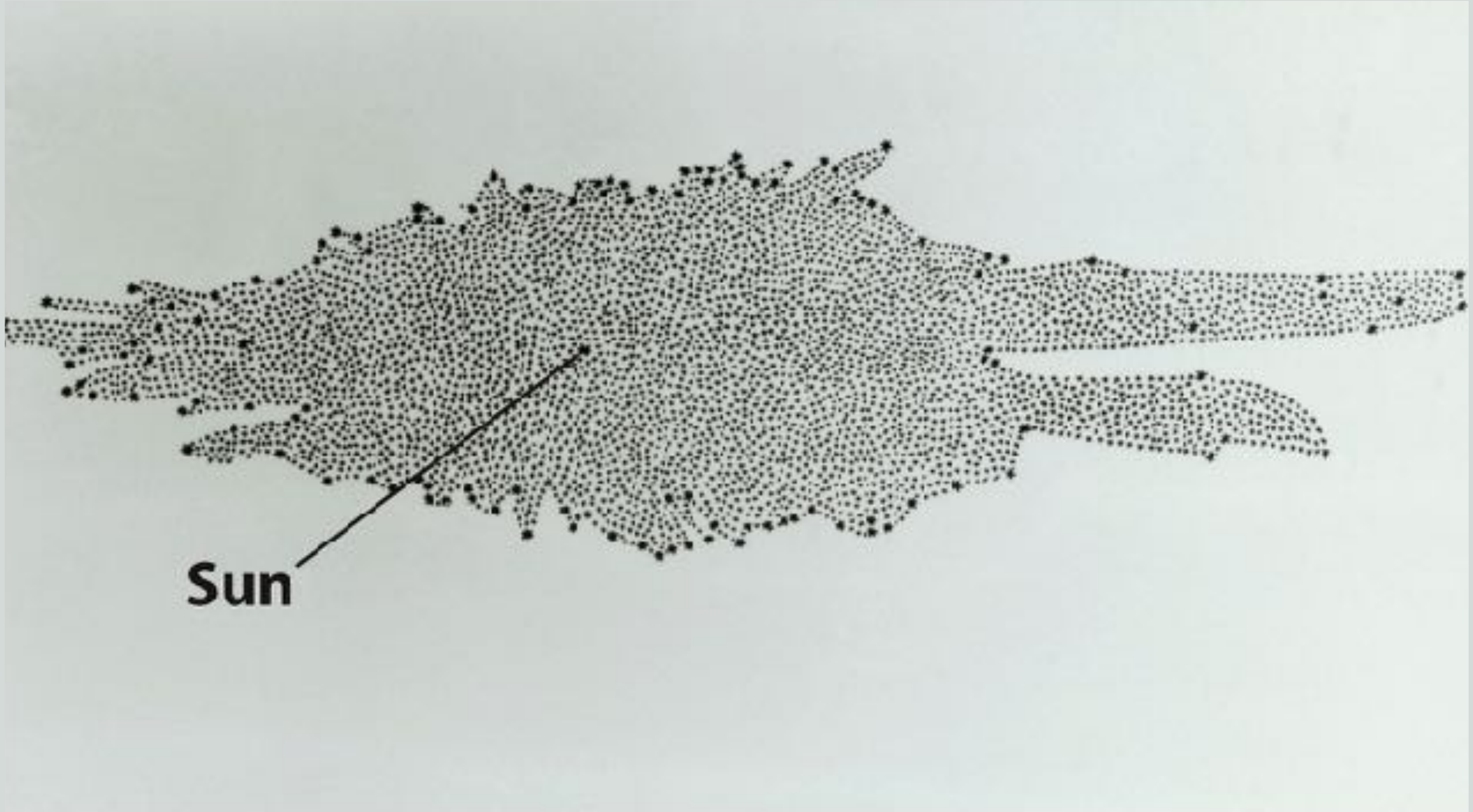
Reflectors: Herschel

- Herschel's 40-foot telescope 47-inch diameter (1.2m) completed in 1789
- Largest in the world until 1845
- Herschel used to map the catalog the density of stars in the Milky Way and our place in the Universe.
- Proved to be cumbersome and unwieldy



Early attempts to measure the Galaxy came up short

- From 1783 to 1790 William & Caroline Herschel counted stars in 683 different directions in the sky to measure the Galaxy.
- Assumed that all stars had the same intrinsic luminosity and, thus, fainter stars were farther away.
- Concluded that it has a disk shape
  - ~15,000 light years across
  - Sun near the center



Herschels' map of the Milky Way (the Sun is approximately in the center)

Annual parallax

Distance: d

Radius of Earth's orbit: a

$a = 1.496 \times 10^{11}$ m (1 AU)

Parallax angle: θ_{par}

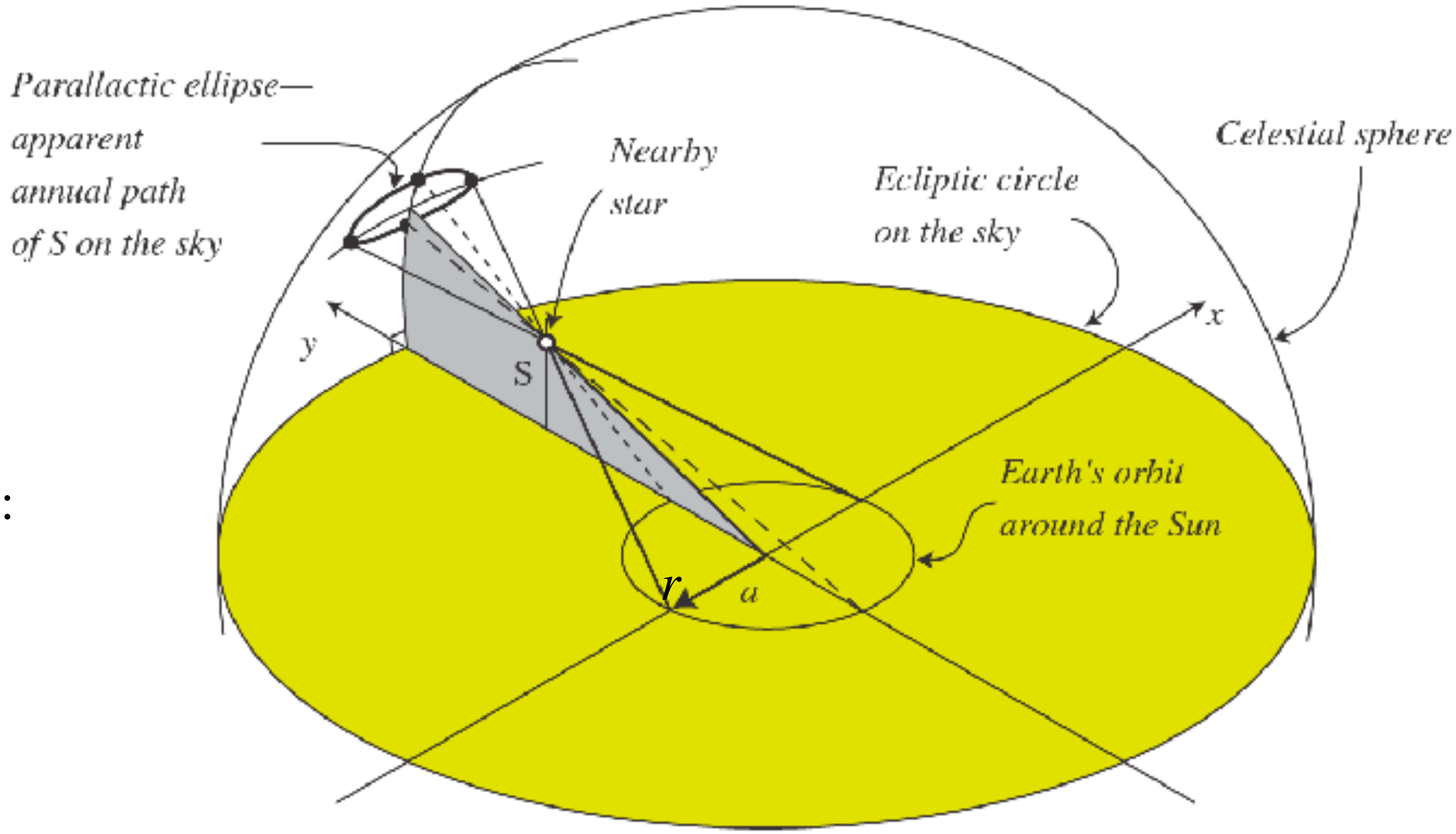
Use small angle approximation:

$$\tan\theta_{par} = \frac{a}{d}$$

$$\theta_{par} \approx \frac{a}{d}, \text{ for } \theta_{par} \ll 1 \text{ rad}$$

For θ_{par} in arcsec, d in parsec

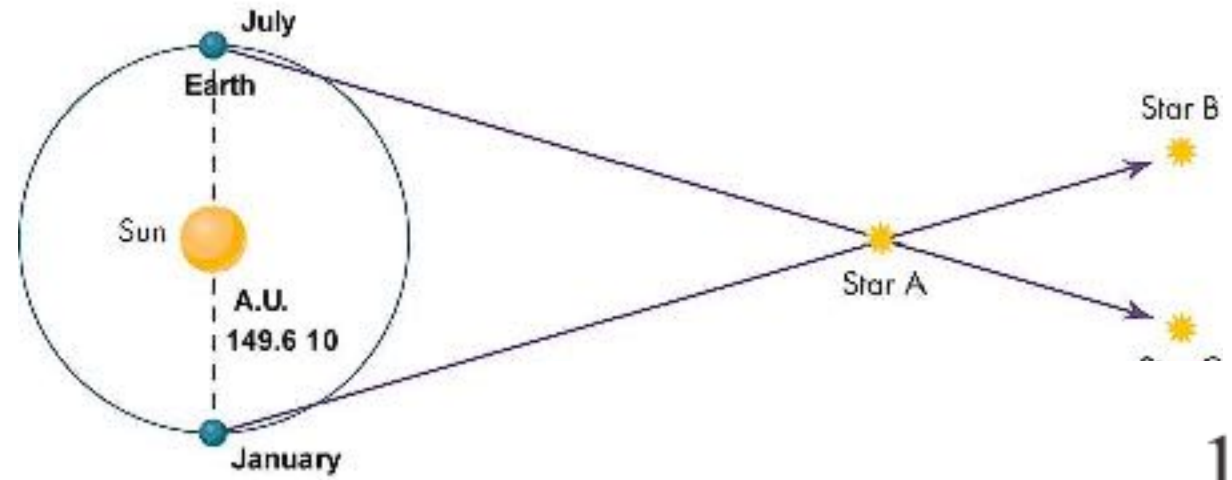
(1pc = 206,265 AU)



1 pc = 3.26 ly

$$p[\text{arcsec}] = \frac{a[\text{au}]}{r[\text{pc}]} \quad p[\text{arcsec}] = \frac{1}{r[\text{pc}]}$$

Stellar Parallax



$$p[\mathrm{arcsec}] = \frac{1}{r[\mathrm{pc}]}$$

1 pc = 3.26 ly

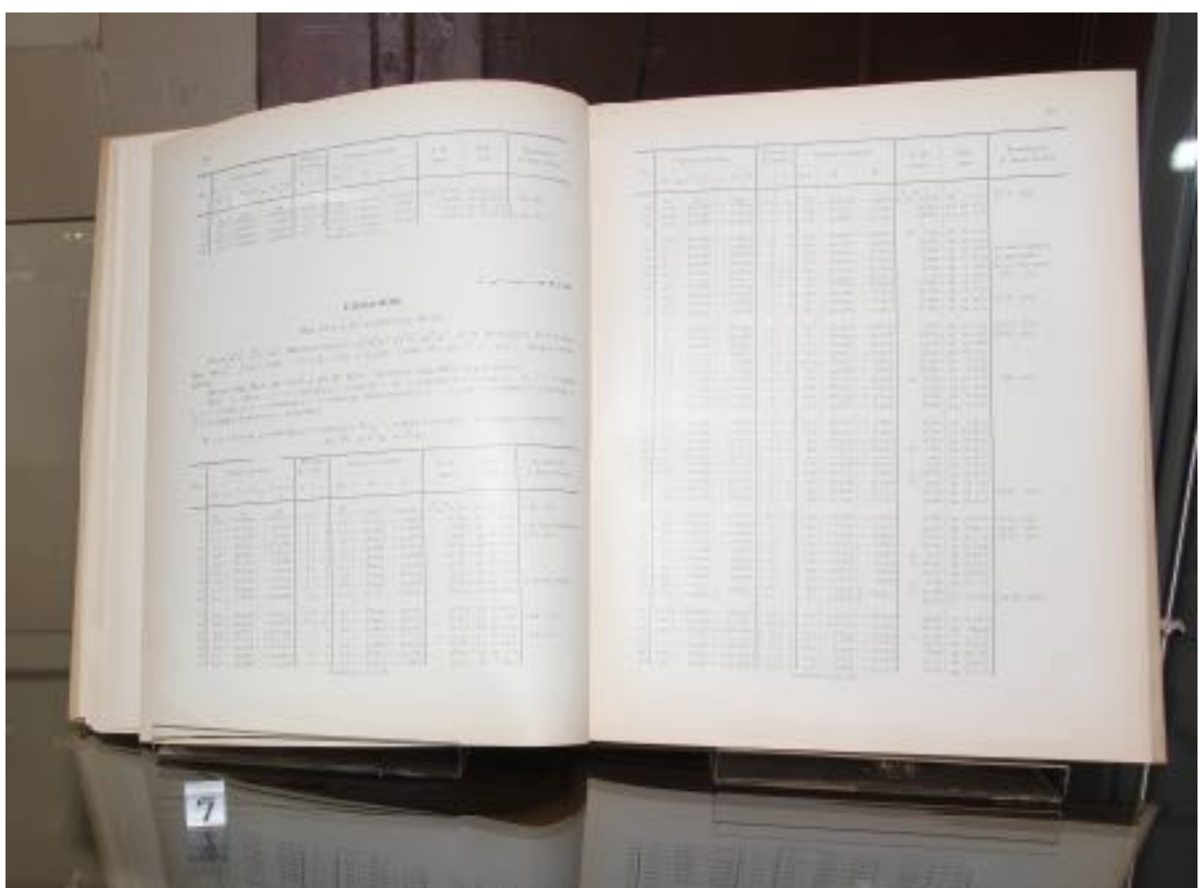
- The angles involved in measuring stellar parallax are very small and thus extraordinarily difficult to measure.
- The nearest star to the Sun (and also the star with the largest parallax), Proxima Centauri, has a parallax of 0.77 arcsec and is 4.25 ly (1.3 pc) away.
- This angle is approximately that subtended by an object 2 centimeters in diameter located 5.3 kilometers away.
- The lack of apparent stellar parallax was part of the evidence against the Copernican model.
- For centuries astronomers tried (and failed) to measure parallax, but still succeeded in setting lower limits and even demonstrating conclusively that the Earth was moving around the Sun.
- Eventually, in 1843, Bessel and Struve independently measured the distance to 61 Cygni (0.3”, 3.5ly) and Vega (0.13”, 7.7pc). Each used instruments designed and built by Fraunhofer.

Carte du Ciel
"Map of the Sky"

- Initiated in 1887 by Paris Observatory director Amedee Mouchez
- entire sky mapped to 11th magnitude
- map the positions of millions of stars
- 20 observatories agreed to participate
- 22,000 photographic plates
- First step, typically ~6 min exposures, to 11.5 mag
- second step, 3x20 min exposures, to 14 mag
- continues to ~1940

Participating observatories and stars measured in the context of the Astrographic Catalogue[1]

Observatory (Zone)	Declination From	Declination To	Epoch	No. of stars
Greenwich	+90°	+65°	1892–1905	179,000
Vatican	+64°	+55°	1895–1922	256,000
Catania	+54°	+47°	1894–1932	163,000
Helsingfors	+46°	+40°	1892–1910	159,000
Potsdam	+39°	+32°	1893–1900	108,000
Hyderabad north	+39°	+36°	1928–1938	149,000
Uccle	+35°	+34°	1939–1950	117,000
Oxford 2	+33°	+32°	1930–1936	117,000
Oxford 1	+31°	+25°	1892–1910	277,000
Paris	+24°	+18°	1891–1927	253,000
Bordeaux	+17°	+11°	1893–1925	224,000
Toulouse	+10°	+05°	1893–1935	270,000
Algiers	+04°	−02°	1891–1911	200,000
San Fernando	−03°	−09°	1891–1917	225,000
Tacubaya	−10°	−16°	1900–1938	312,000
Hyderabad south	−17°	−23°	1914–1929	293,000
Córdoba	−24°	−31°	1909–1914	309,000
Perth	−32°	−37°	1902–1919	229,000
Perth/Edinburgh	−38°	−40°	1903–1914	139,000
Cape Town	−41°	−51°	1897–1912	540,000
Sydney	−52°	−64°	1892–1948	430,000
Melbourne	−65°	−90°	1892–1940	218,000

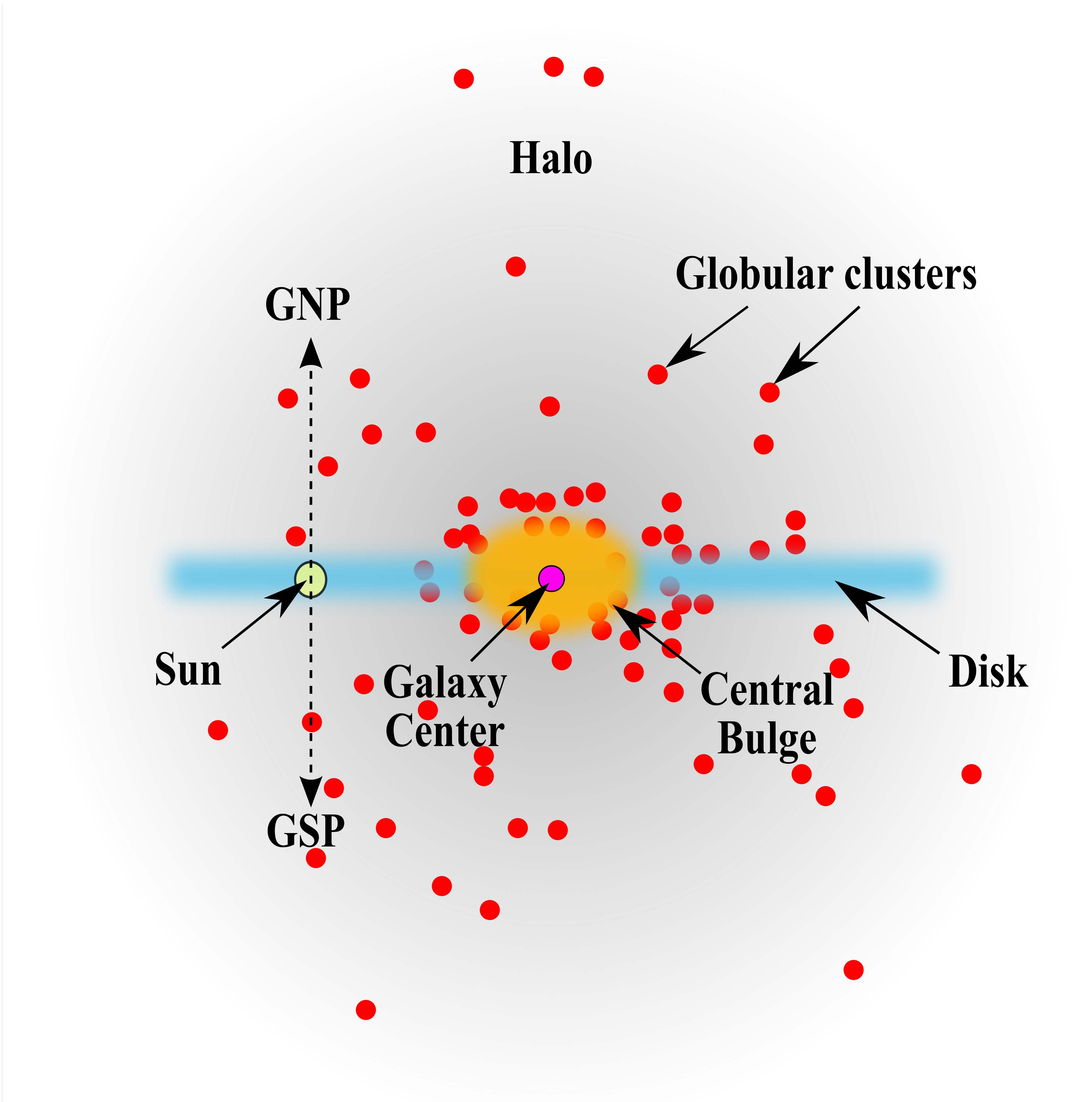



Carte du Ciel
data acquisition

data reduction

Harlow Shapley

- Henrietta Swan Leavitt discovered the luminosity-period relation of Cepheid variable stars at Harvard in 1912.
- Harlow Shapley received his PhD from Princeton in 1914 working with Henry Norris Russell on understanding Cepheid variable stars and showing that they weren't binary star systems (spectroscopy!).
- He then went to work at the Mt. Wilson Observatory on the Hale 60-inch reflector — then the largest, most powerful reflector in the world.
- In 1915 he measured the distance to a single Cepheid variable star using parallax, and was thus able to calibrate their distance scale.
- Next, he started using Cepheid variable stars to measure the distances to globular clusters.
- In 1918 he measured the size of the Milky Way, increasing the size of the known Universe by x10, and showed that the Sun was *not* at the center of the Universe.
- Shapley became famous and became the director of the Harvard Observatory in 1921,



Halo
Globular clusters
GNP
Sun
Galaxy Center
Central Bulge
Disk
GSP

Historical Development

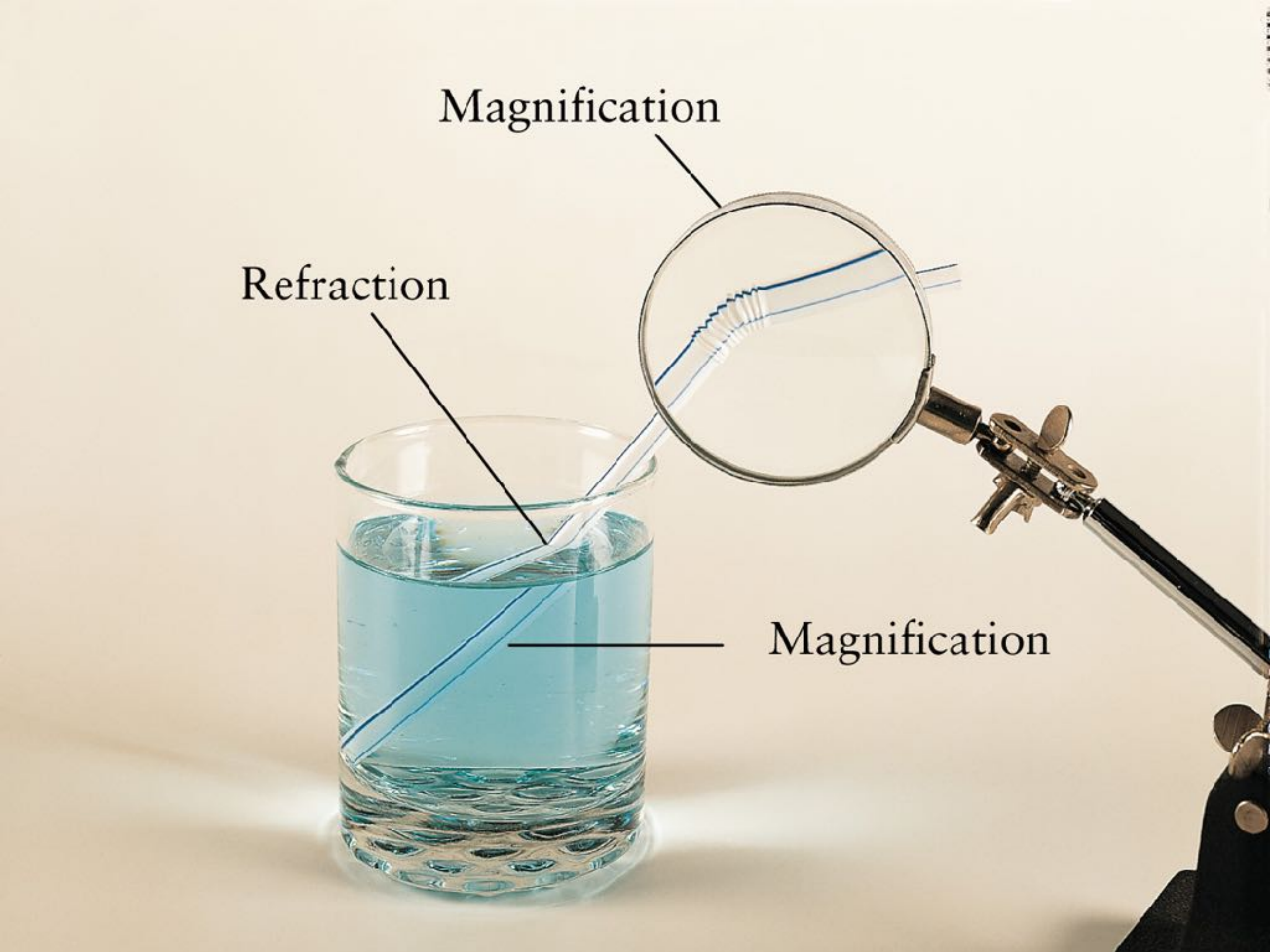
- 1608: patent application by Hans Lippershey (c1570-1619), a German-born Dutch citizen, of a device with a convex and a concave lens in a tube with a magnification of about 3 times.
- 1609: Galileo Galilei (1564-1642) made his own version of the telescope and increased the magnification to 8 and 20 times.
- From the early Galilean telescope of 1.52-1.83 m (5 - 6 ft) in length, astronomical telescopes attained lengths of 4.57-6.10 m (15 - 20 ft) by the middle of the 17th century.
- 1656: Christiaan Huygens (1629-1695) made a telescope 7 meters (23 ft) long; its objective lens had an aperture of about 10 cm (~4 inches), it magnified about 100 times, and its field of view was 17 arc-minutes.
- 1672: Sir Isaac Newton showed that white light is a mixture of colored light and that every color has its own degree of refraction.
- Consequently, any curved lens will decompose white light into the colors of the spectrum, each of which will come to a focus at a different point on the optical axis. This effect, which became known as **chromatic aberration**, causes the image of a star to be surrounded by circles of different colors.
- Thus lenses had limitations for astronomy, but telescopes with long focal lengths helped to reduce these effects.

Basic principles of optical systems

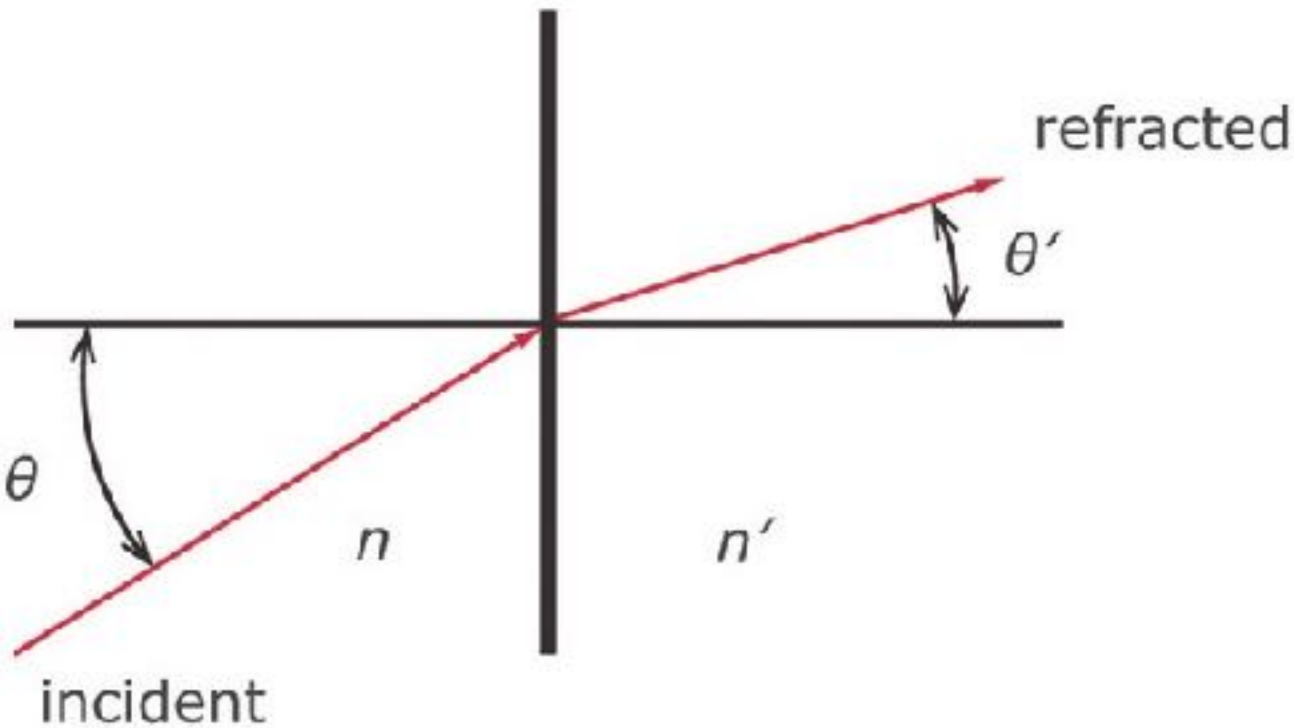
- ***Geometrical optics:***
  - Treats electromagnetic radiation as light rays ($\lambda <<$ size of optical elements)
  - Allows ray tracing analysis
- ***Optical axis:***
  - Line through centered optical elements.
- ***Geometrical path:***
  - The actual physical distance traveled by a light ray on path ***s***.
- ***Optical path:***
  - The geometrical path multiplied by the local refractive index ***n:***

$$\int_a^b n(s)\, ds$$

  - ***A focusing system: the paths of the rays from a point on an object to its matching image point all have the same optical path length.***

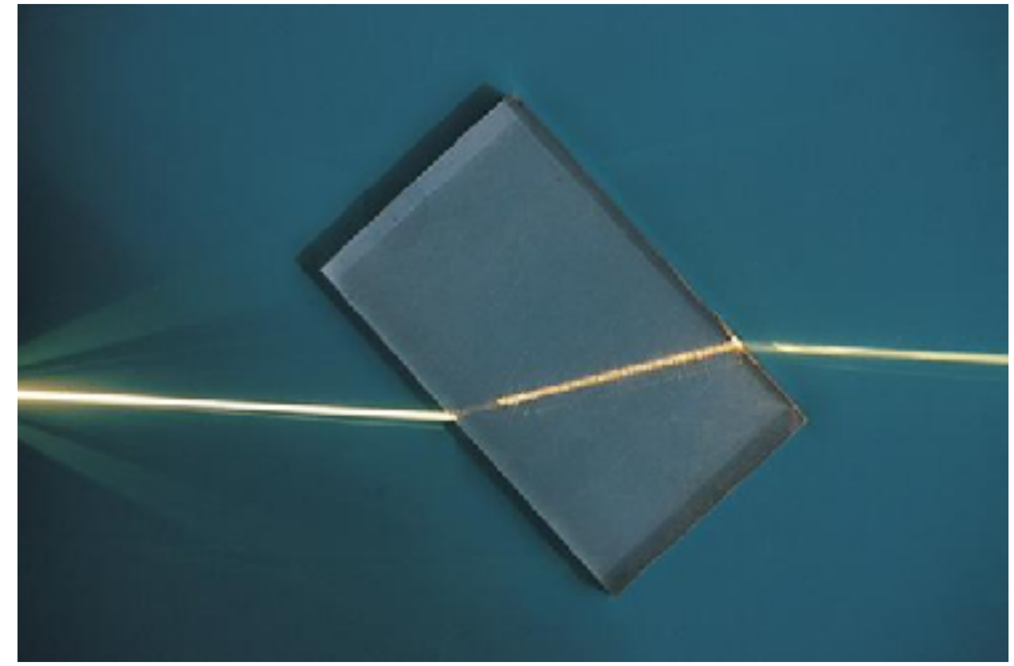

Magnification
Refraction
Magnification

Refraction and Snell's law



lower index higher index

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

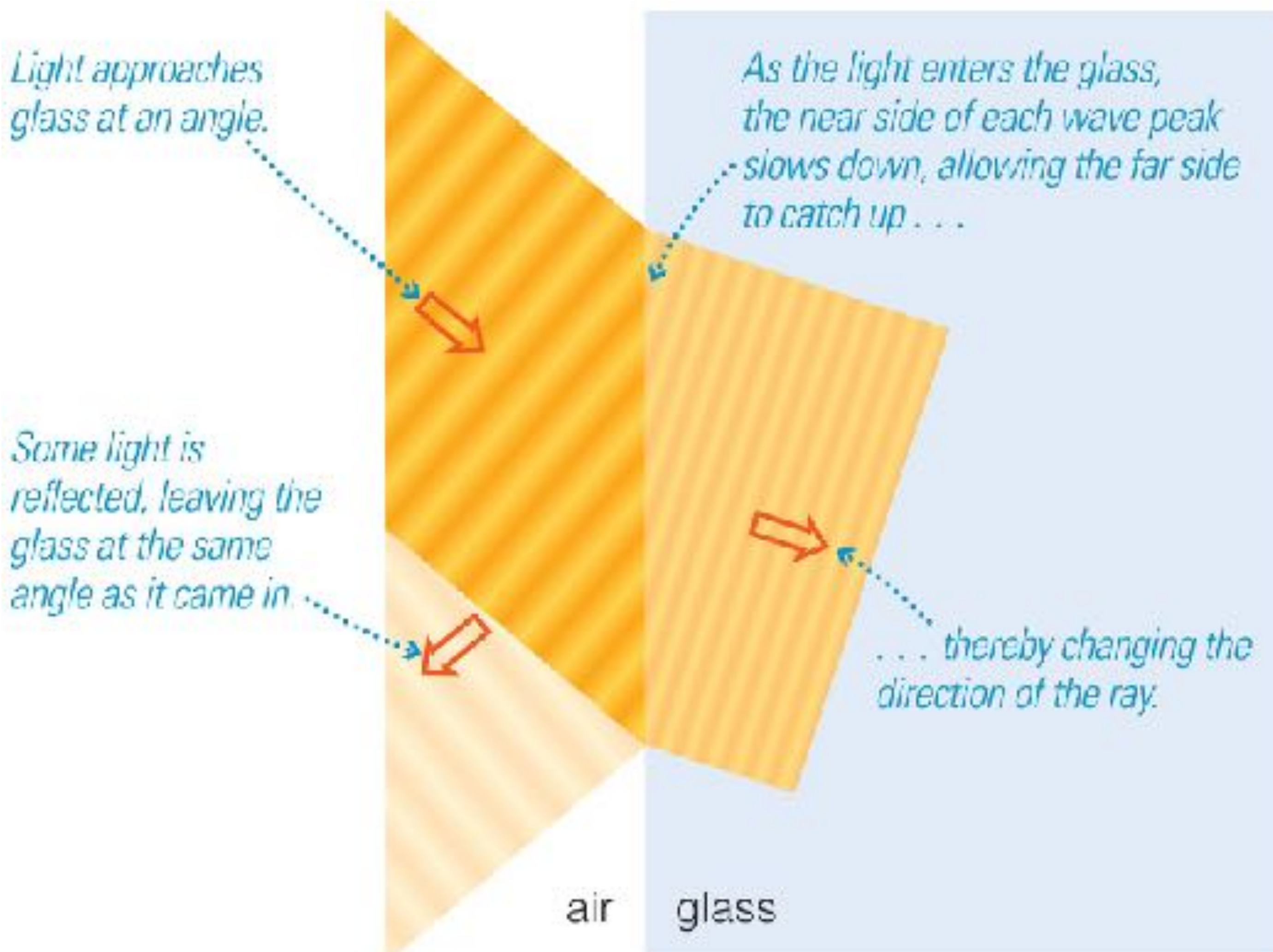


condition for total internal reflection:

$$\theta_C = \sin^{-1}\left(\frac{n_1}{n_2}\right)$$

Refraction

Material	n (λ = 588 nm)	$dn/d\lambda$ (μm^{-1})	ν_D
Air (STP)	1.00029	2×10^{-5}	85
Water	1.33	0.017	114
Calcium fluoride	1.435	0.027	95
Fused quartz	1.458	0.040	68
Borosilicate crown glass (BK7)	1.517	0.047	64
Flint glass (F2)	1.620	0.100	36
Dense flint glass (SF4)	1.756	0.161	28
Diamond	2.42	0.250	33



index of refraction used for Fresnel's equations and Snell's law

$$n(\lambda) = \frac{c}{v(\lambda)}$$

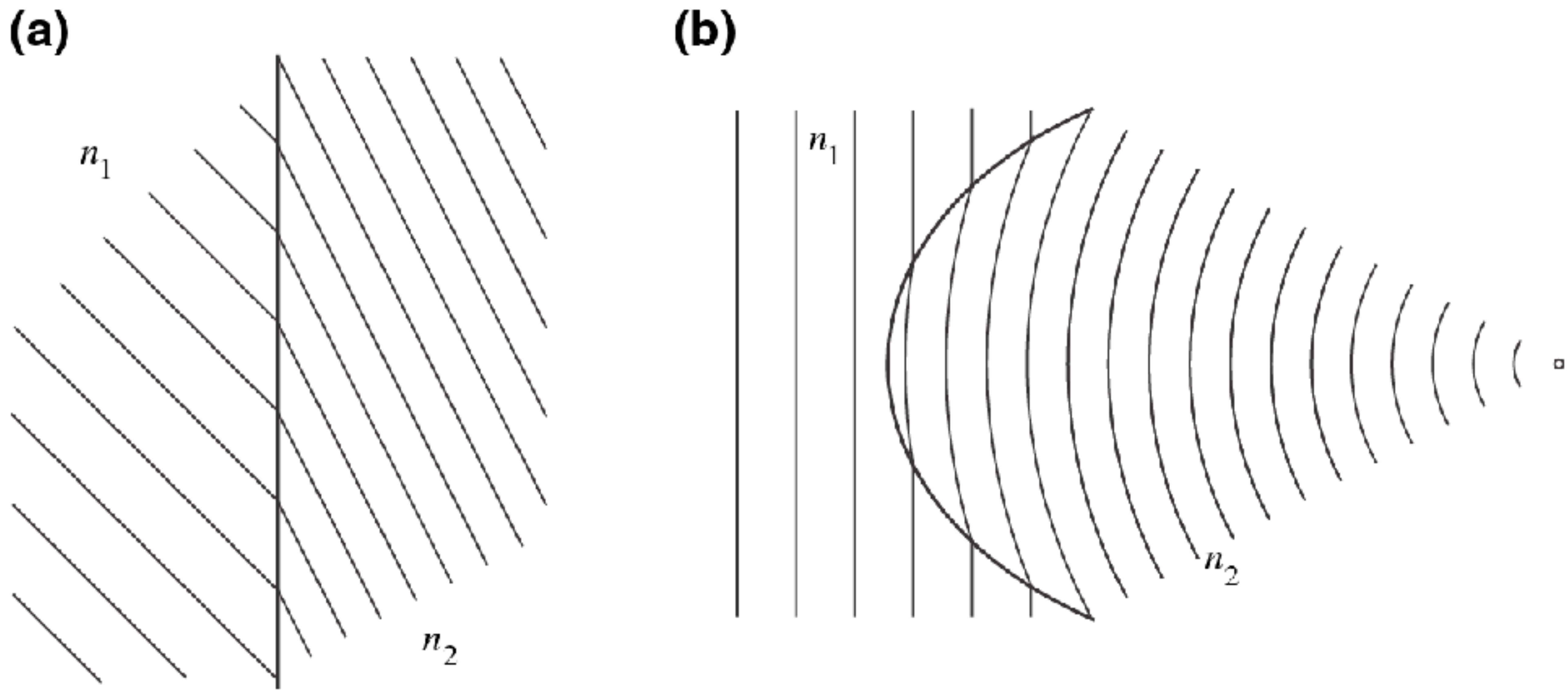
dielectric constant and relative permittivity used for Maxwell's equations:

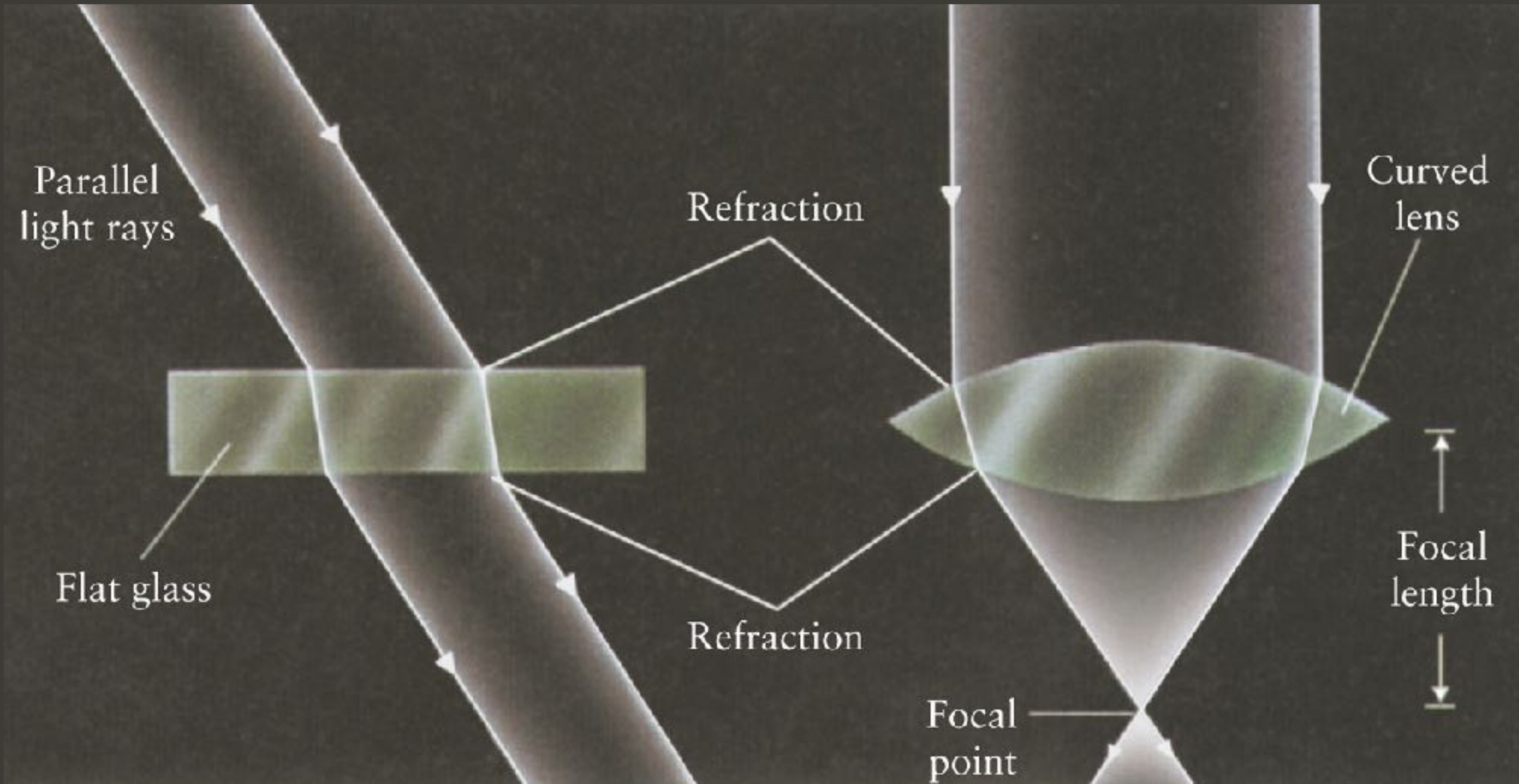
$$n=\sqrt{\epsilon}$$

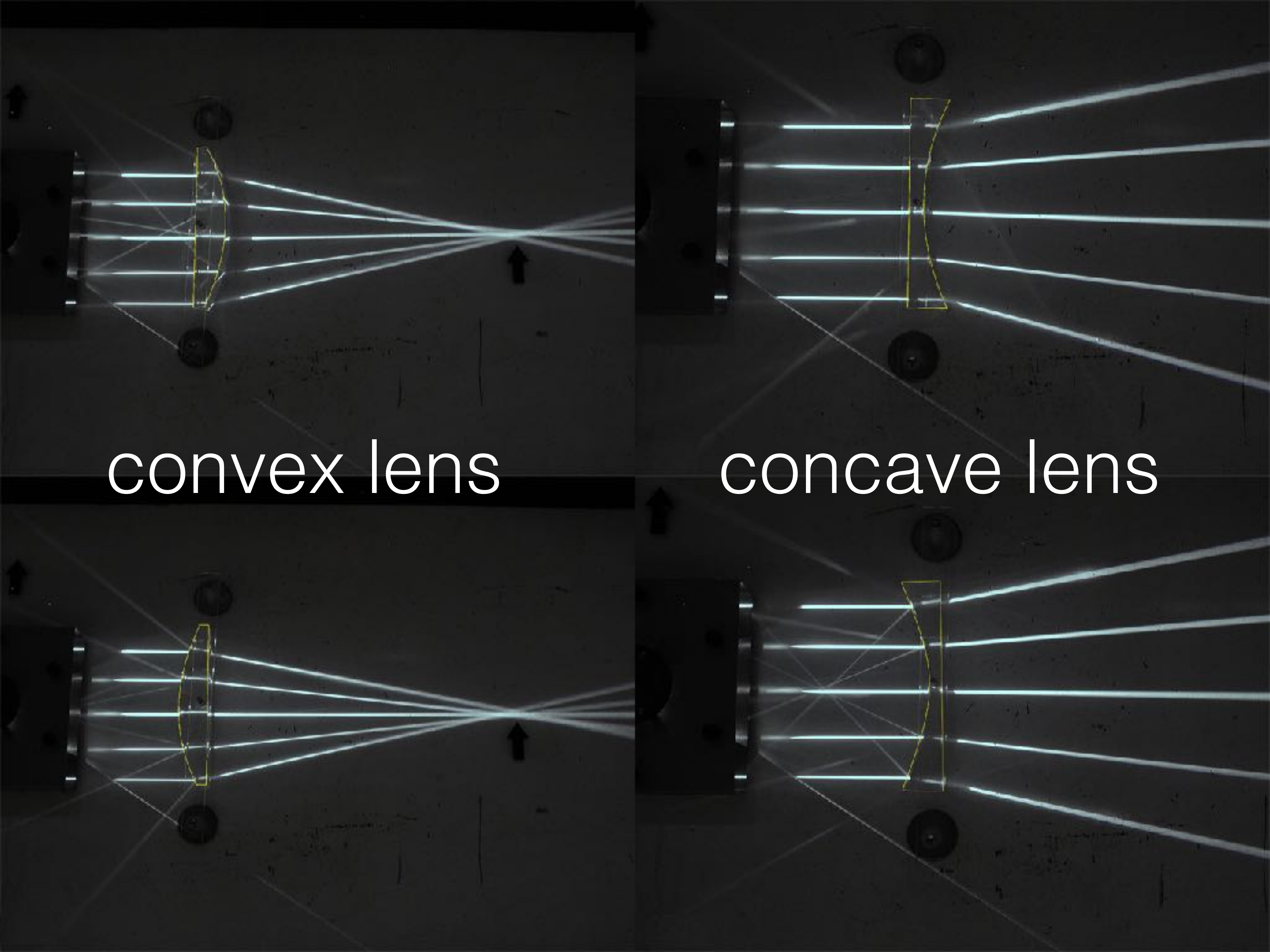
Fresnel's Equations

Reflection from a surface

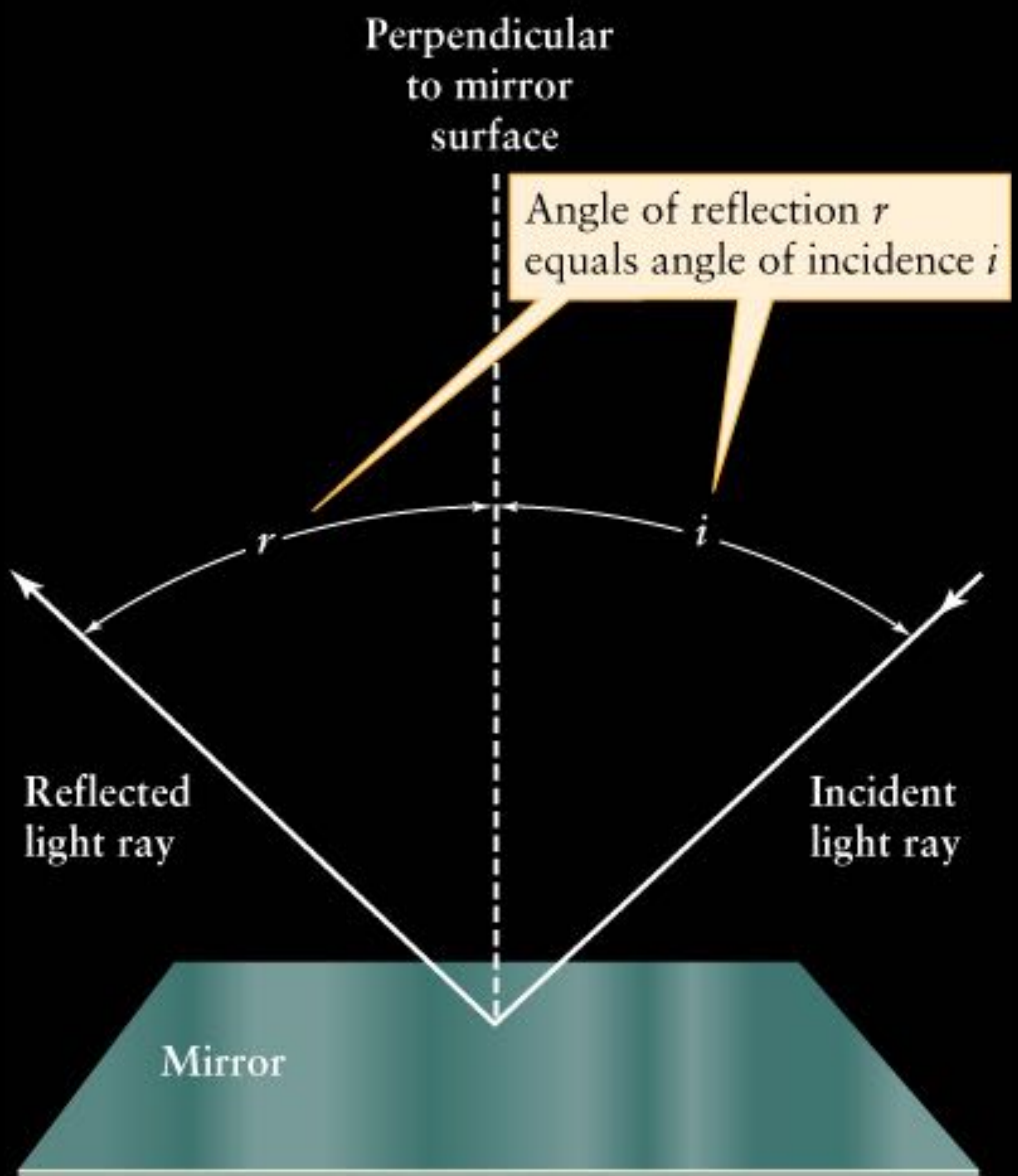
$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$


(a)
n_1
n_2
(b)
n_1
n_2

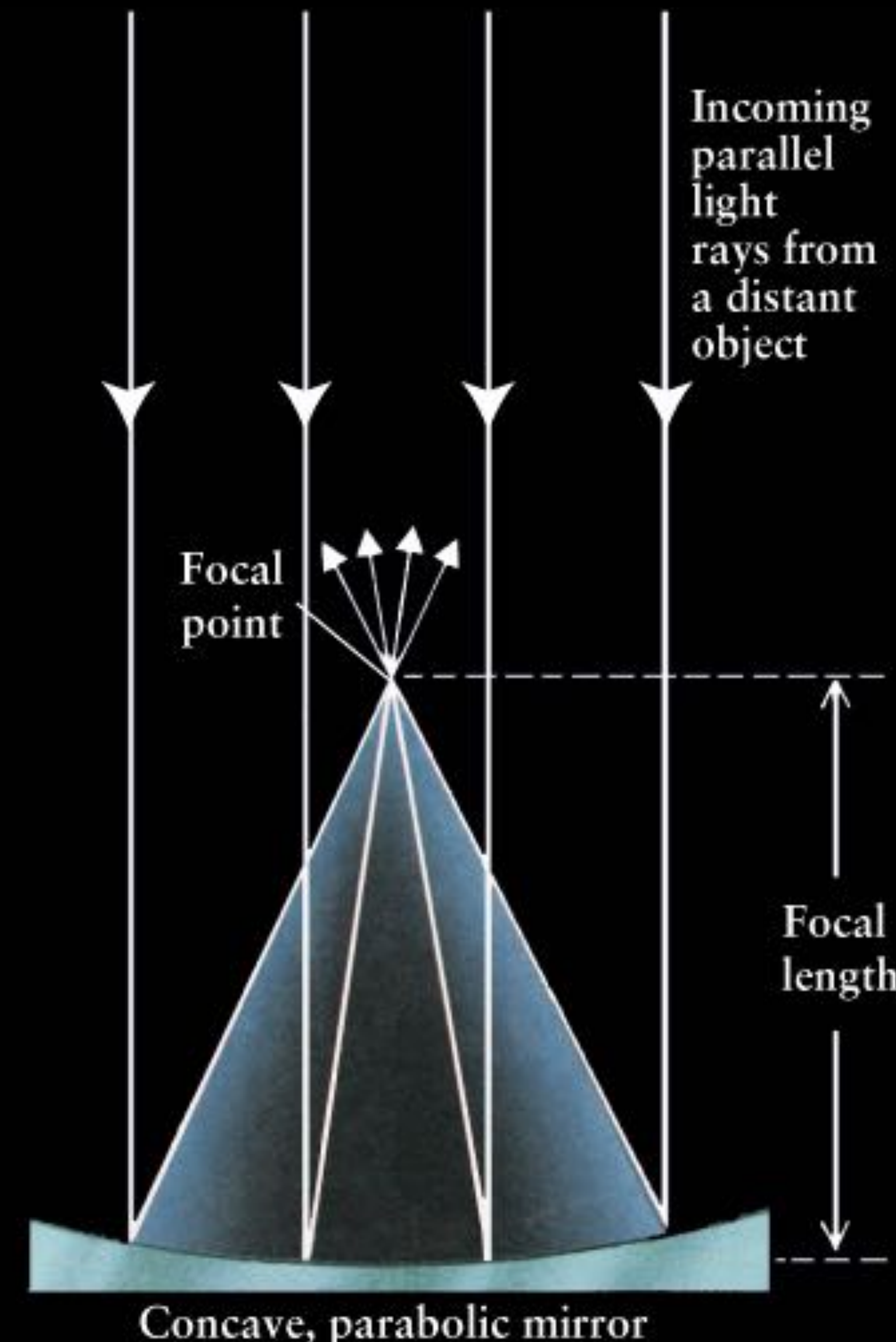

Parallel
light rays
Refraction
Curved
lens
Flat glass
Refraction
Focal
length
Focal
point


convex lens
concave lens

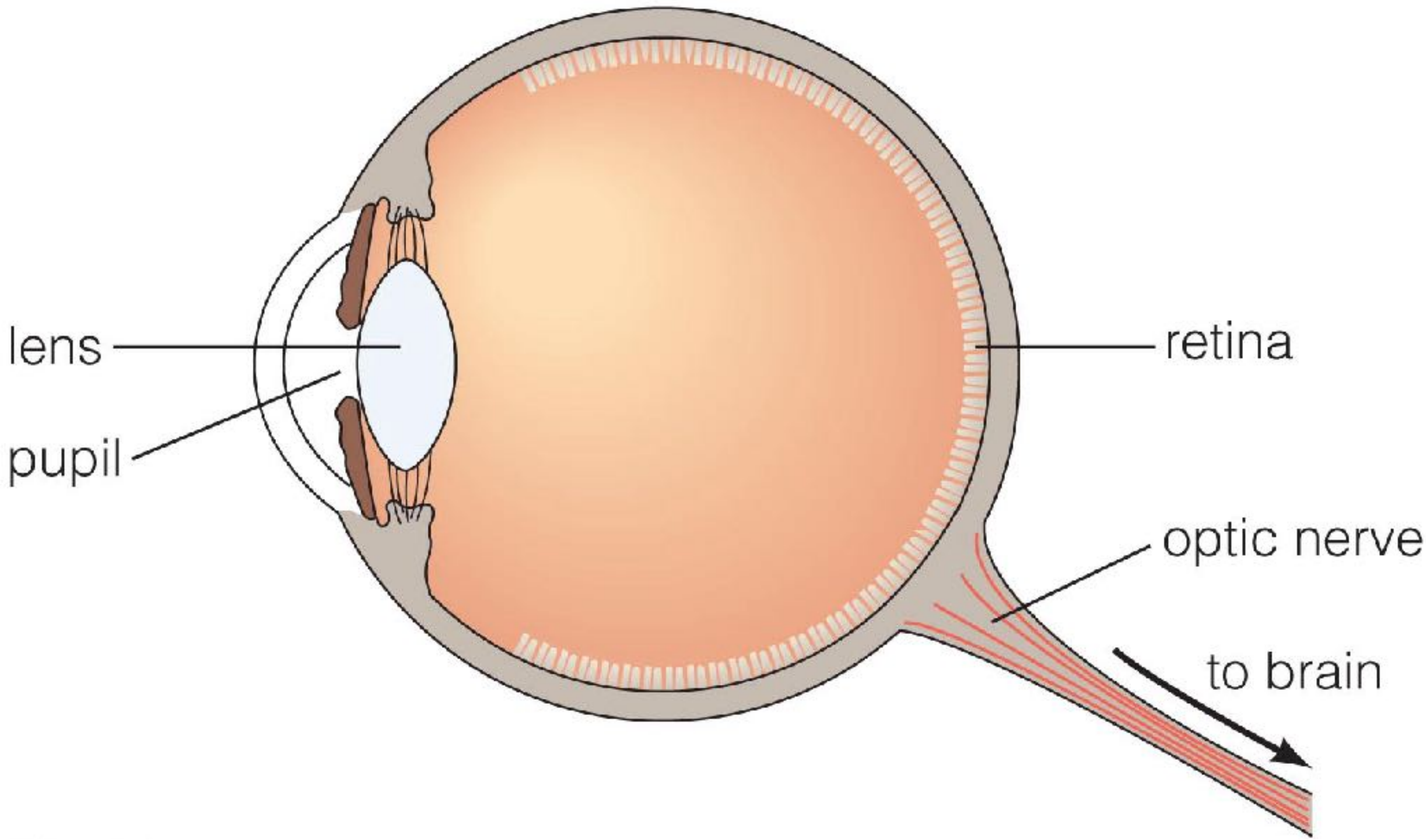
mirrors



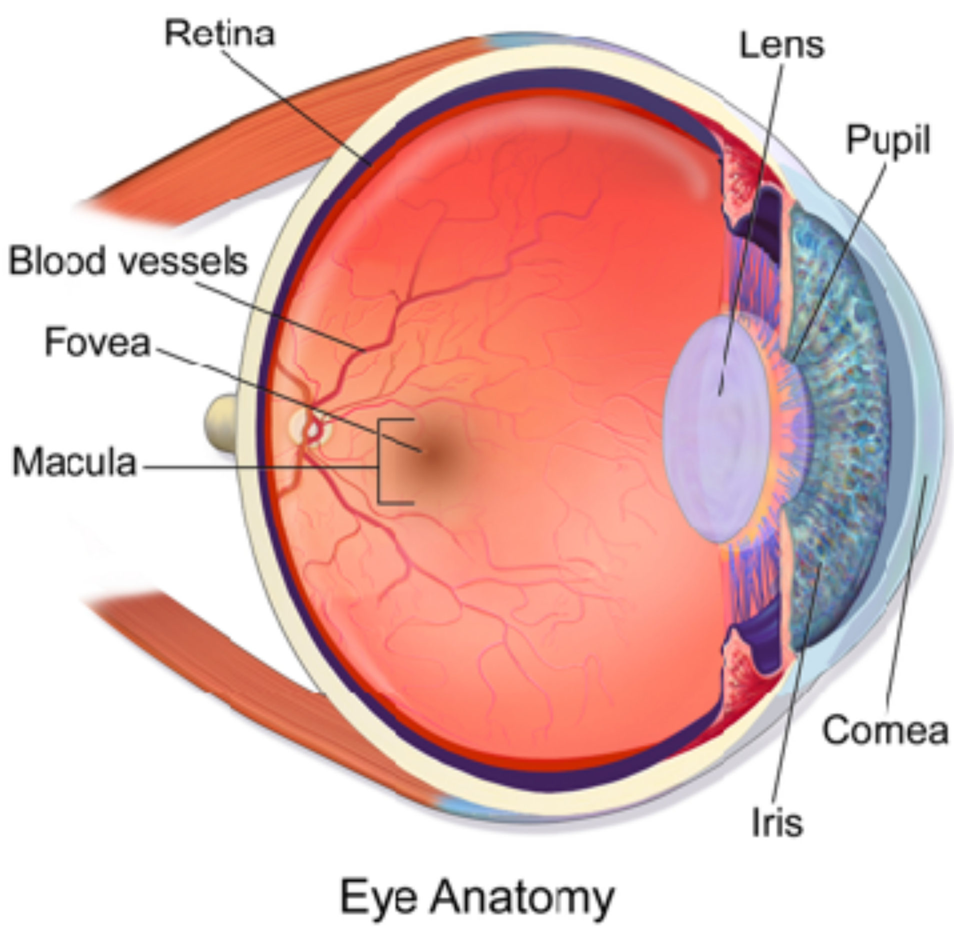
Reflection off of a flat surface

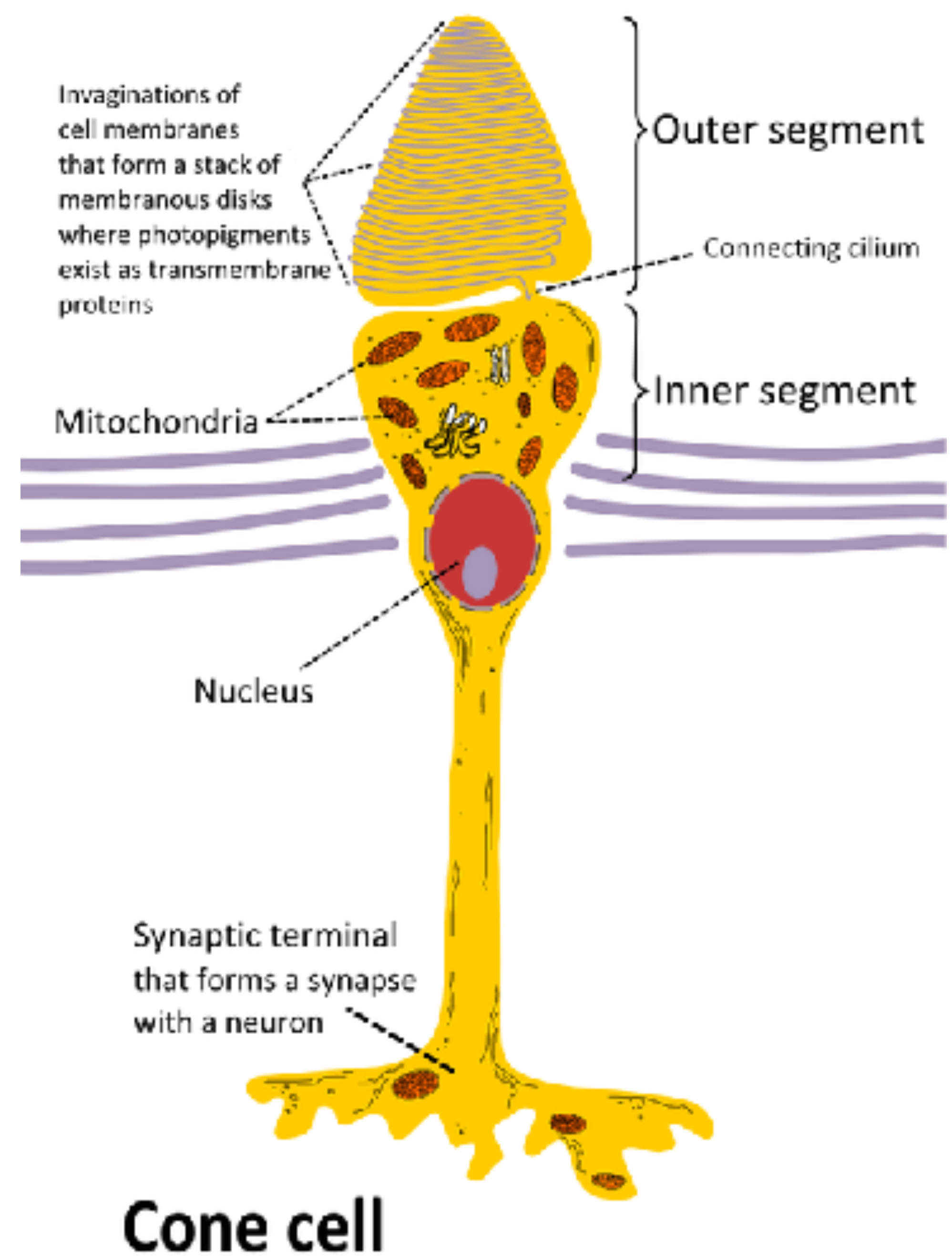


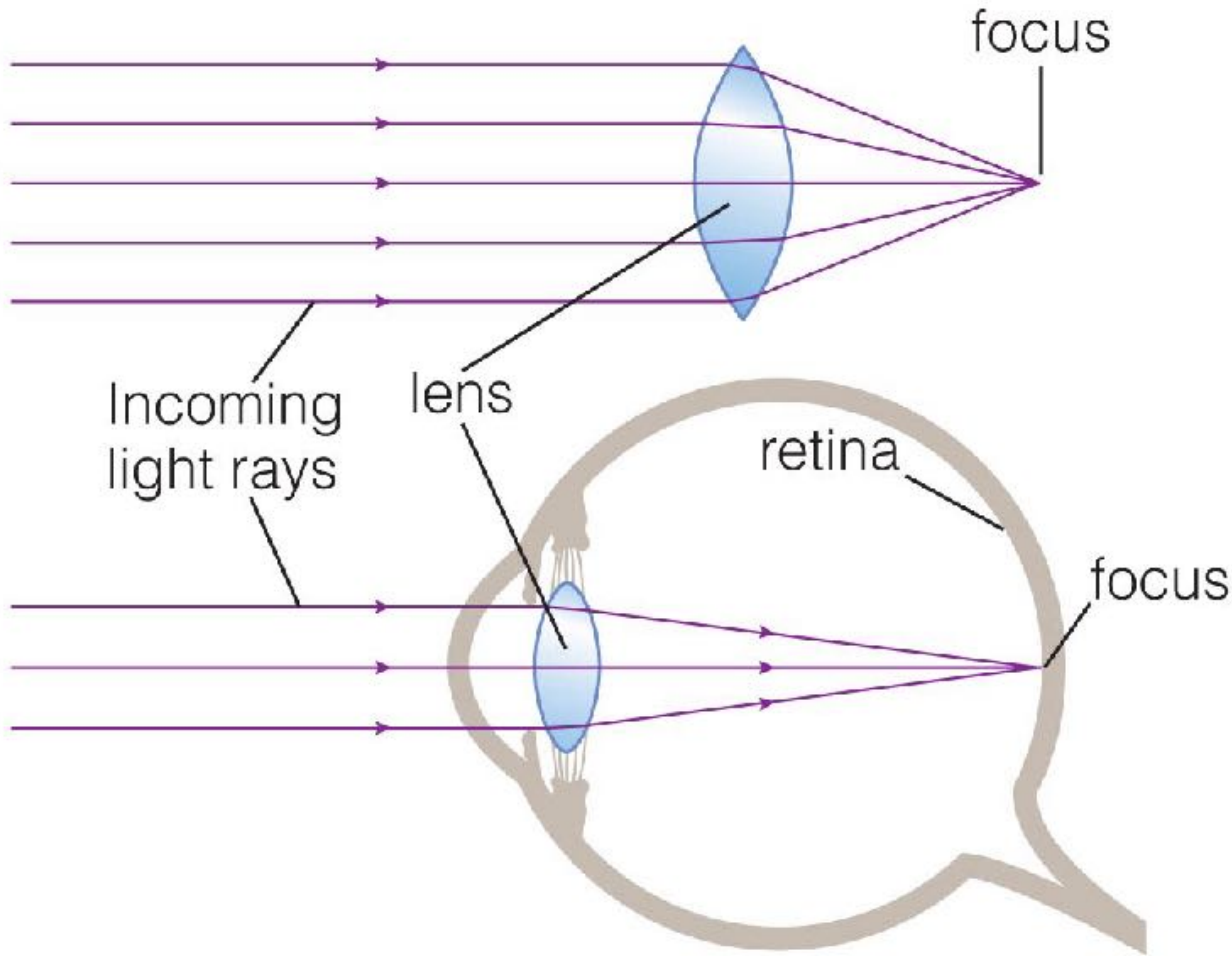
Reflection off of a curved surface

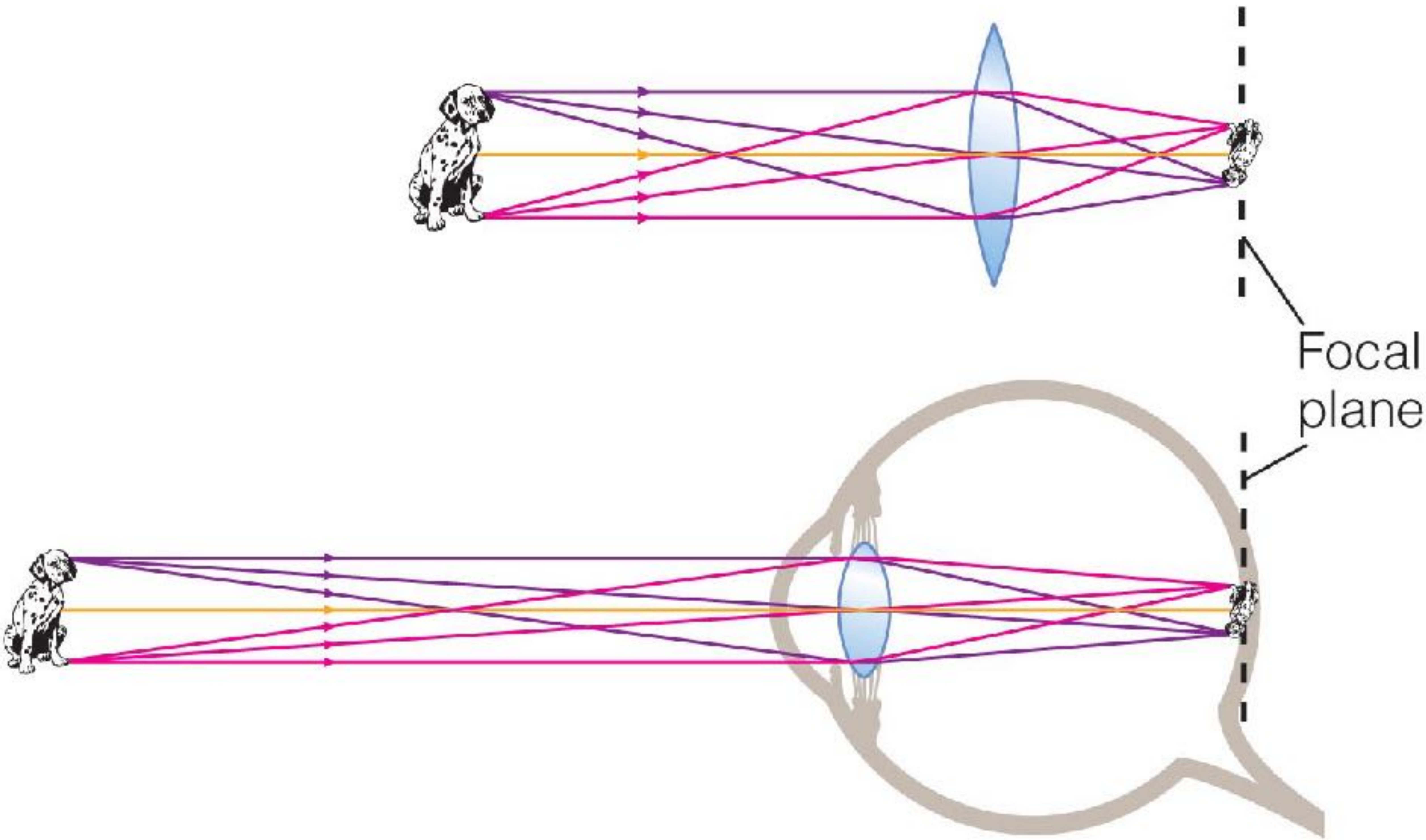

lens
pupil
retina
optic nerve
to brain

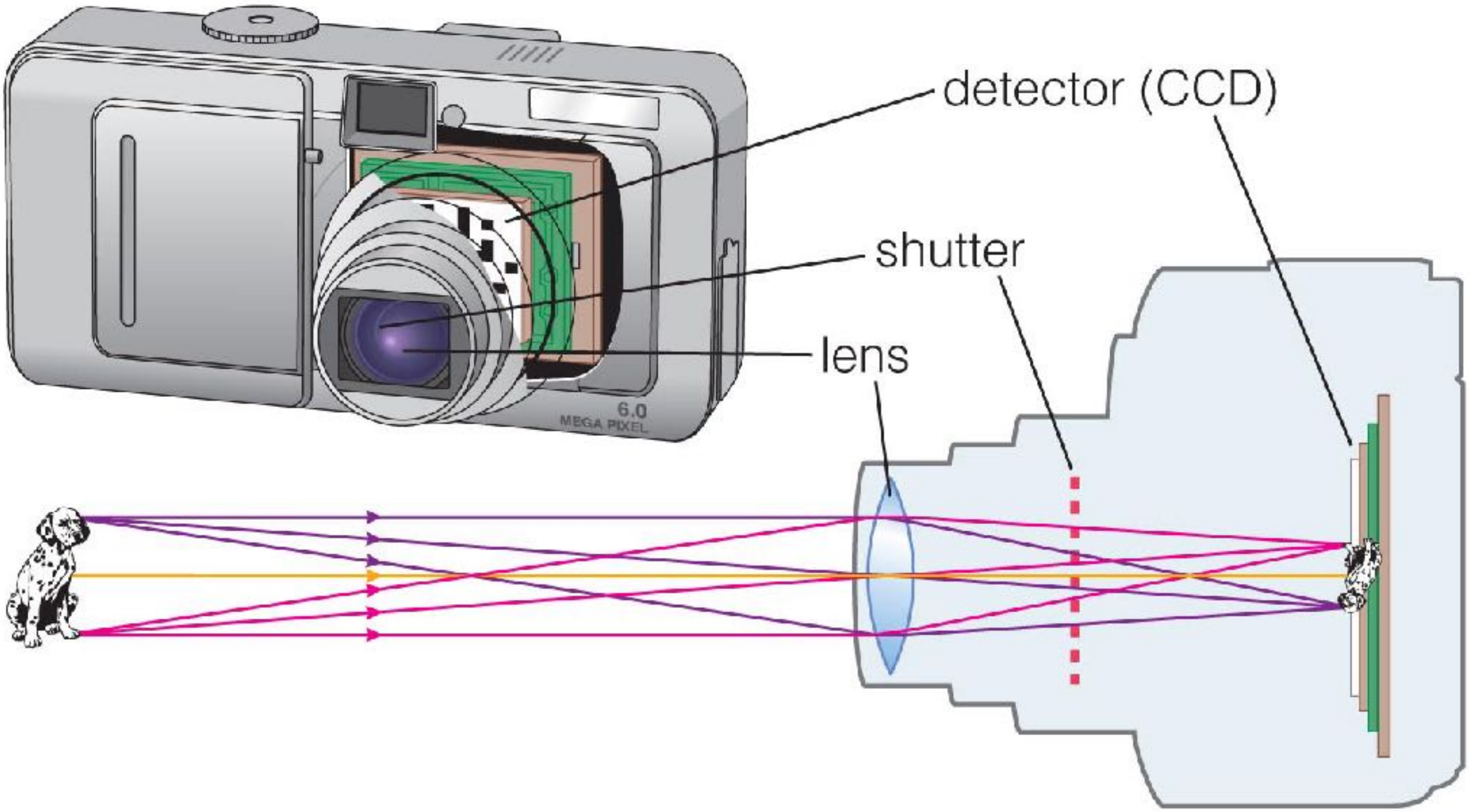
Human Eye


Retina
Lens
Pupil
Blood vessels
Fovea
Macula
Cornea
Iris
Eye Anatomy

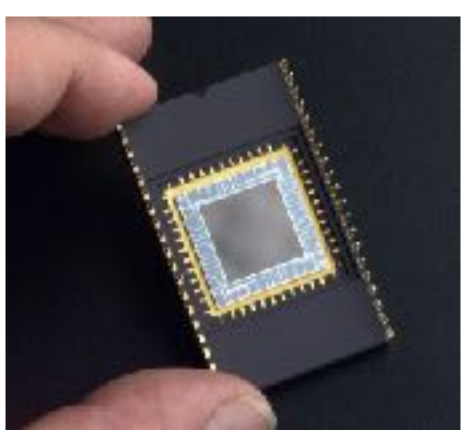

Invaginations of cell membranes that form a stack of membranous disks where photopigments exist as transmembrane proteins
Outer segment
Connecting cilium
Inner segment
Mitochondria
Nucleus
Synaptic terminal that forms a synapse with a neuron
Cone cell


focus
Incoming
light rays
lens
retina
focus


Focal
plane

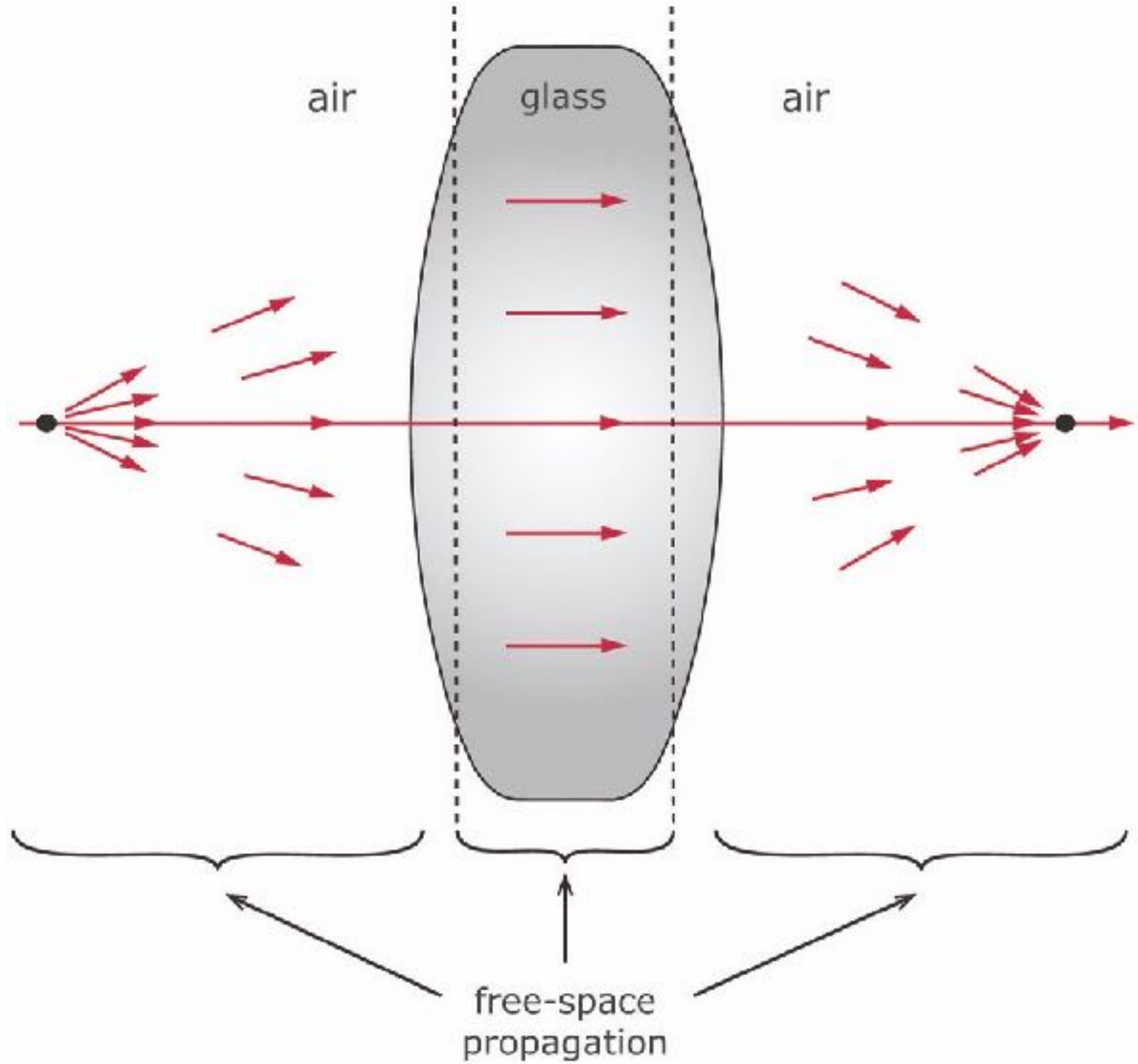

detector (CCD)
shutter
lens
6.0
MEGA PIXEL

Optical Detector Comparison

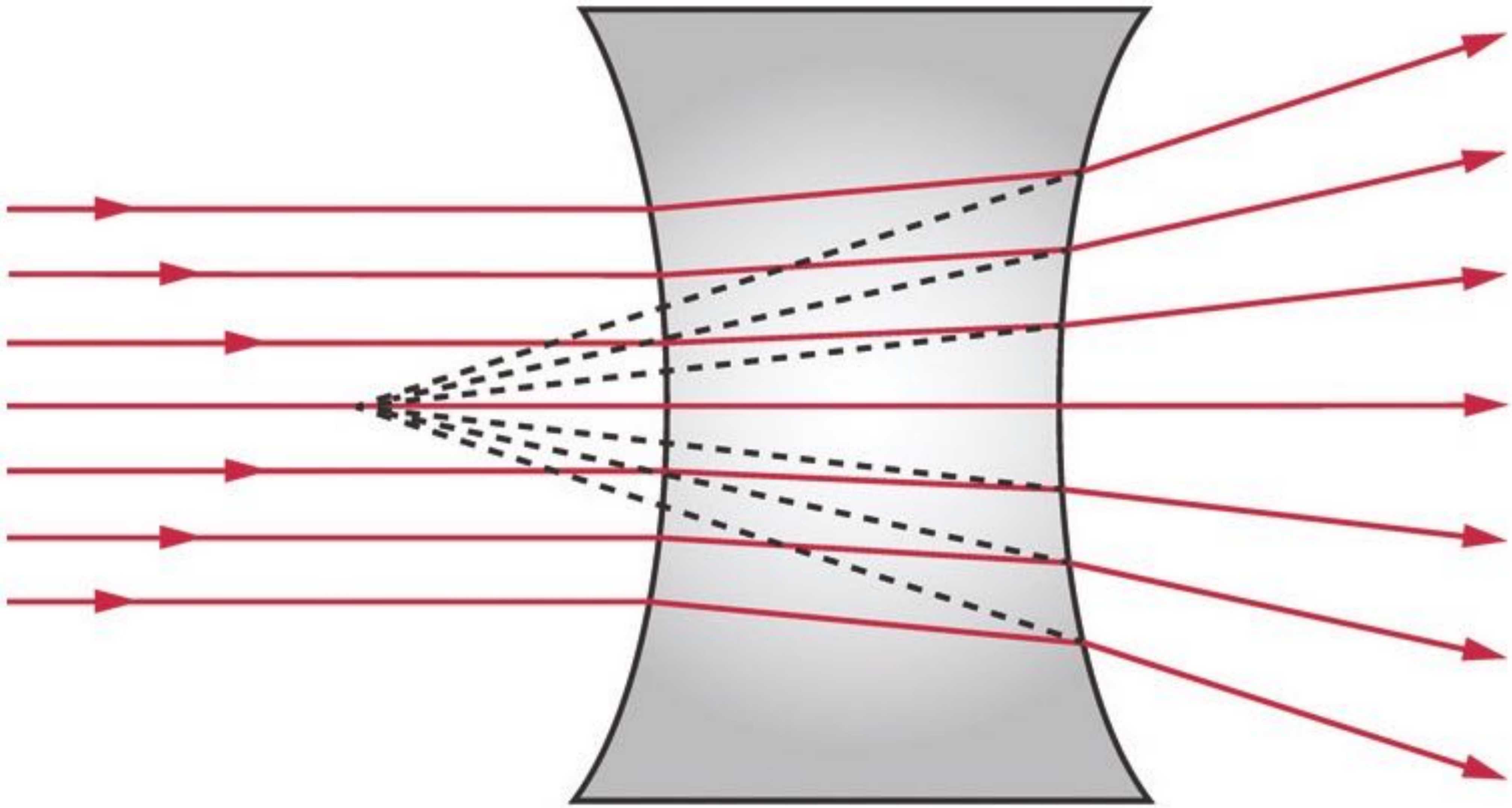


Detector	physical size	resolution elements	QE	Integrate?	Readout time	linear?	precision
Human Eye	12 mm x 12 mm	1M	6%	no	instant	no	2%
Photograph	356 mm x 356 mm	1.3G	3%	yes	fast	no	8%
CCD	61 mm x 61 mm	16M	70%	yes	slow	yes	0.05%

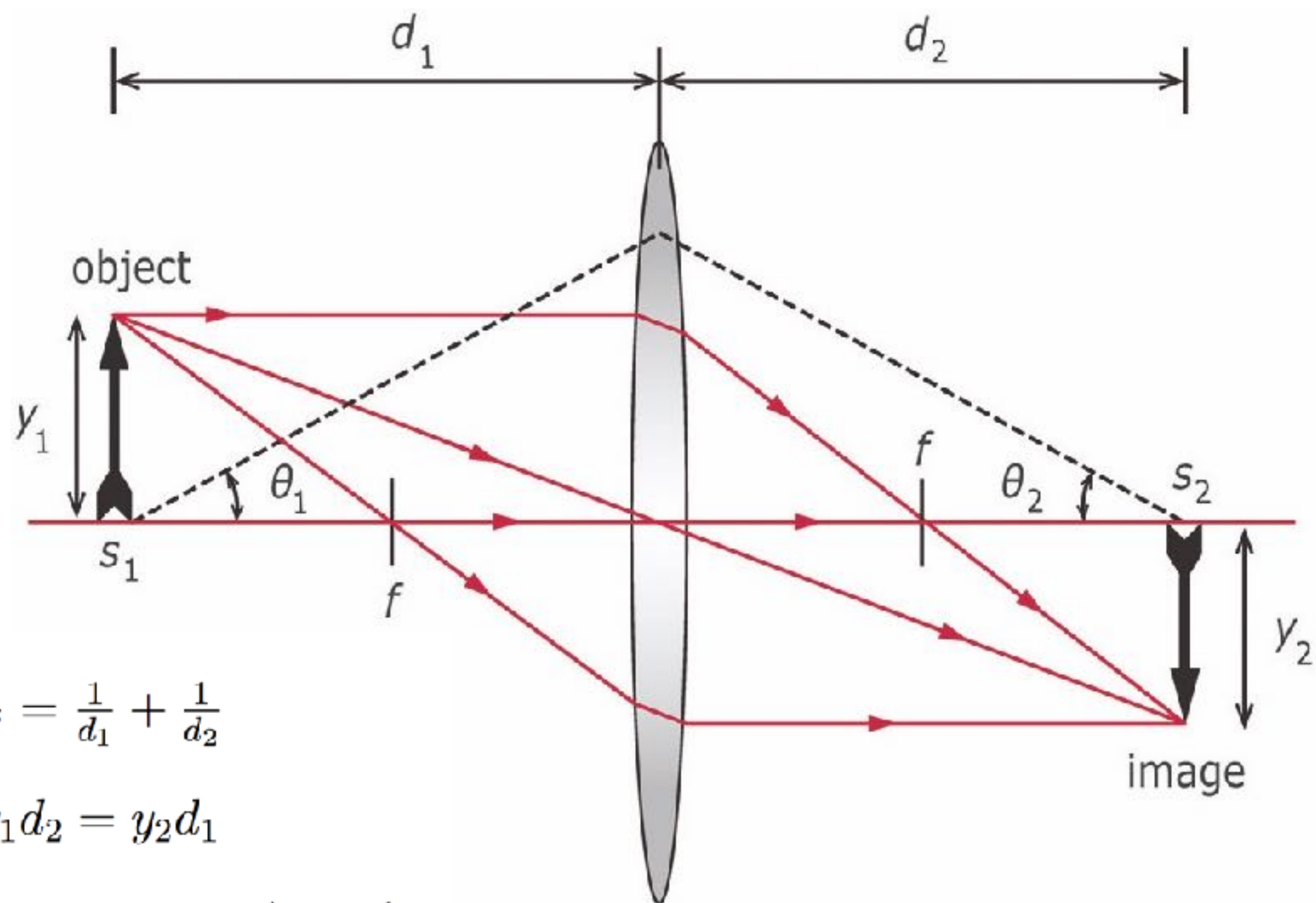
BUT … can build a mosaic array of CCDs …



The operation of a simple lens.
It operates by using a transparent material with an index of refraction greater than the surrounding air.



A negative lens with concave surfaces forms a virtual image
— it looks like light is coming from a point but it is not



$$\frac{1}{f} = \frac{1}{d_1} + \frac{1}{d_2}$$

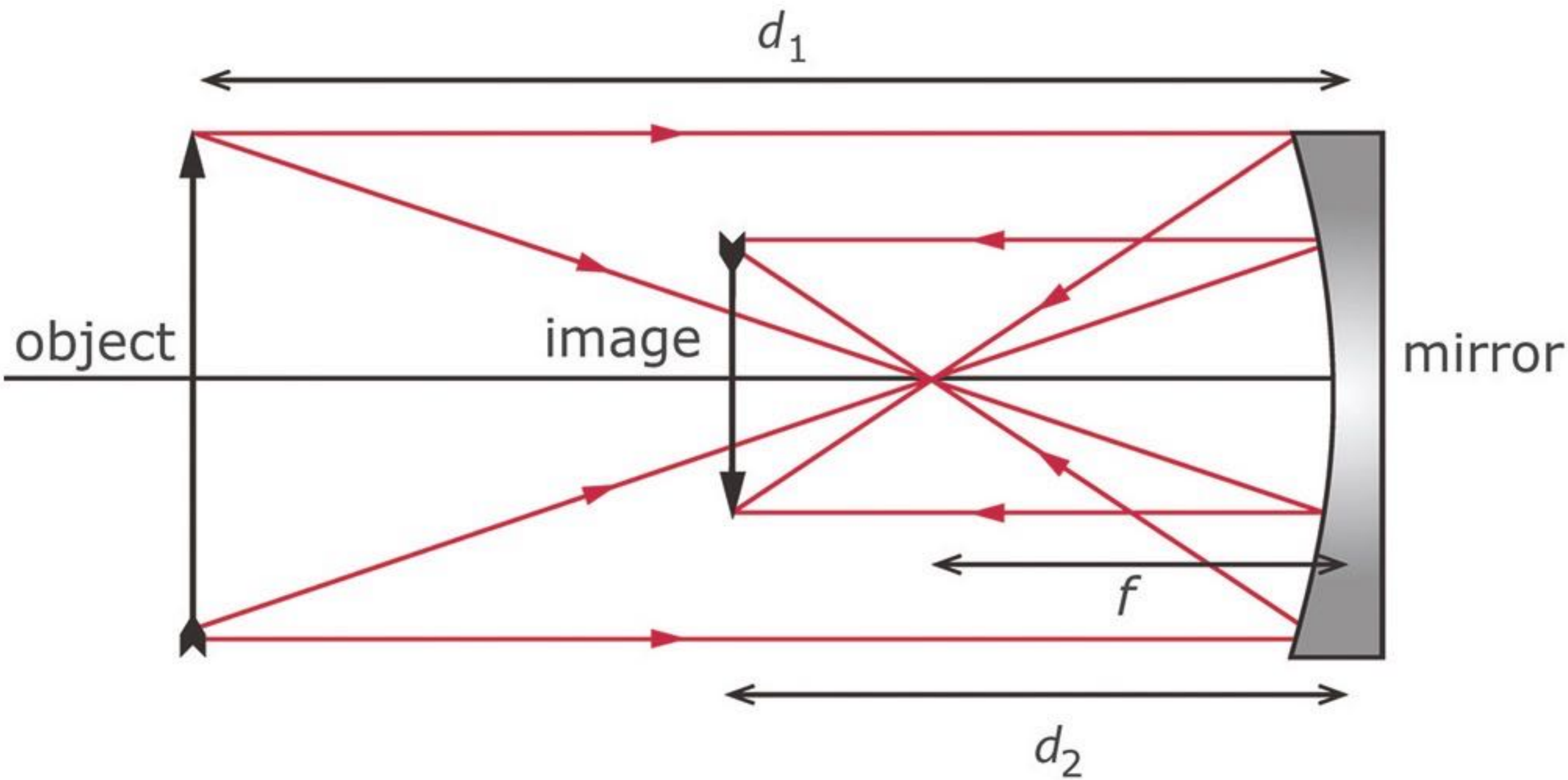
$$y_1 d_2 = y_2 d_1$$

$$\frac{1}{f} = (n-1)(\frac{1}{R_1} - \frac{1}{R_2})$$

Image formation by a positive lens.

R is the radius of curvature. + = convex, - = concave

Thin lens formula.

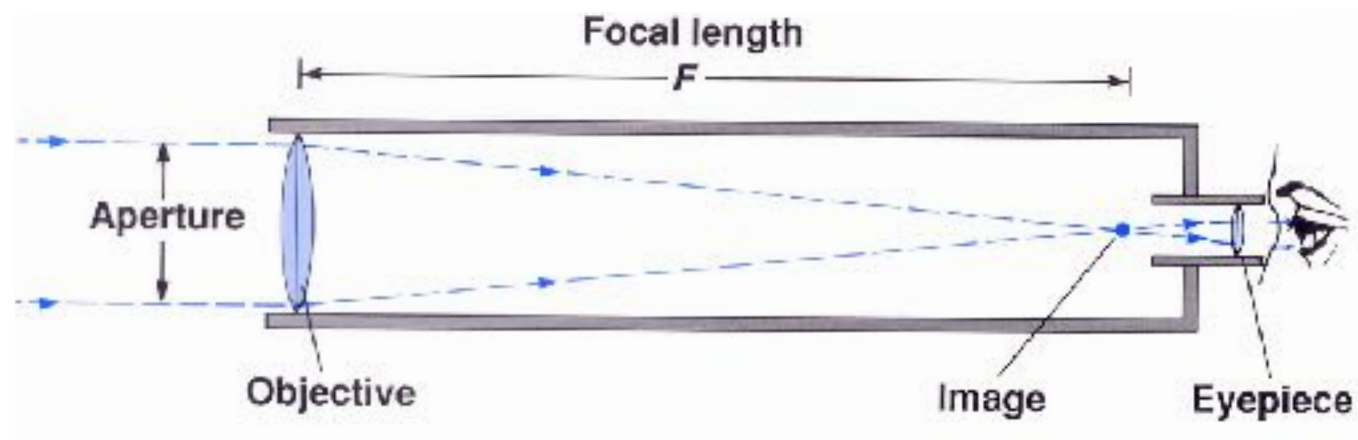


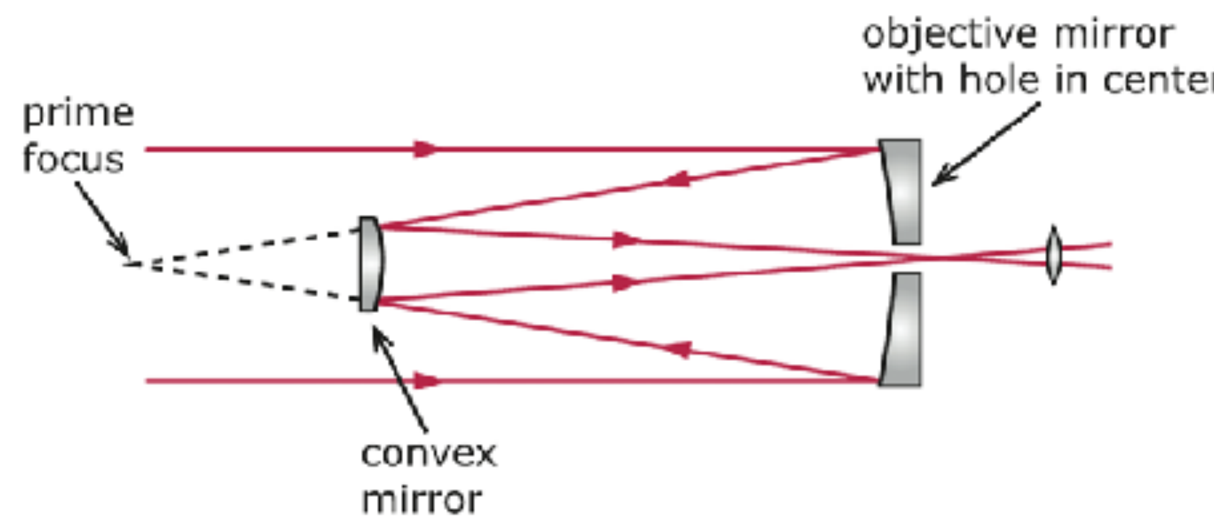
A concave mirror has image properties similar to those of a positive lens

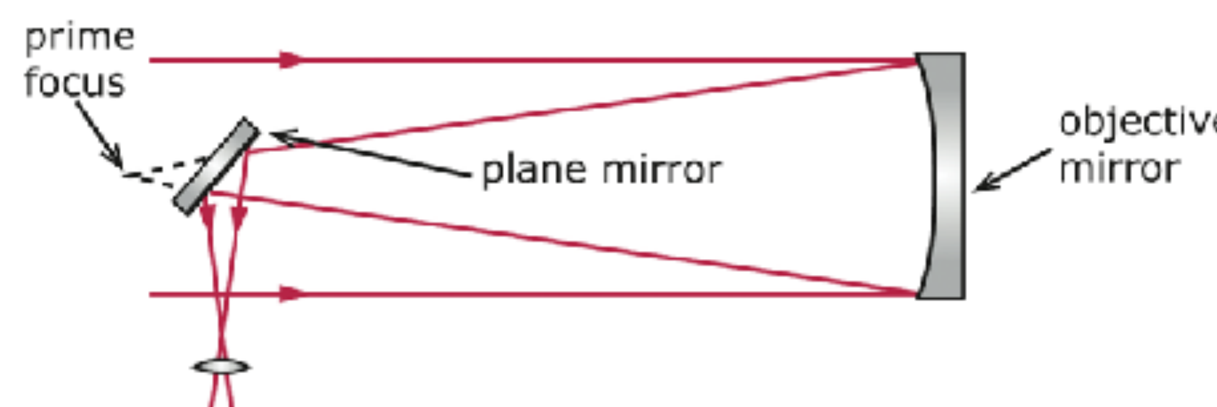
Telescope designs

fall into one of 3 basic types:

- **Refractive** (uses lenses);
- **Reflective** (uses mirrors);
- Hybrid (a combination of mirrors and lenses).

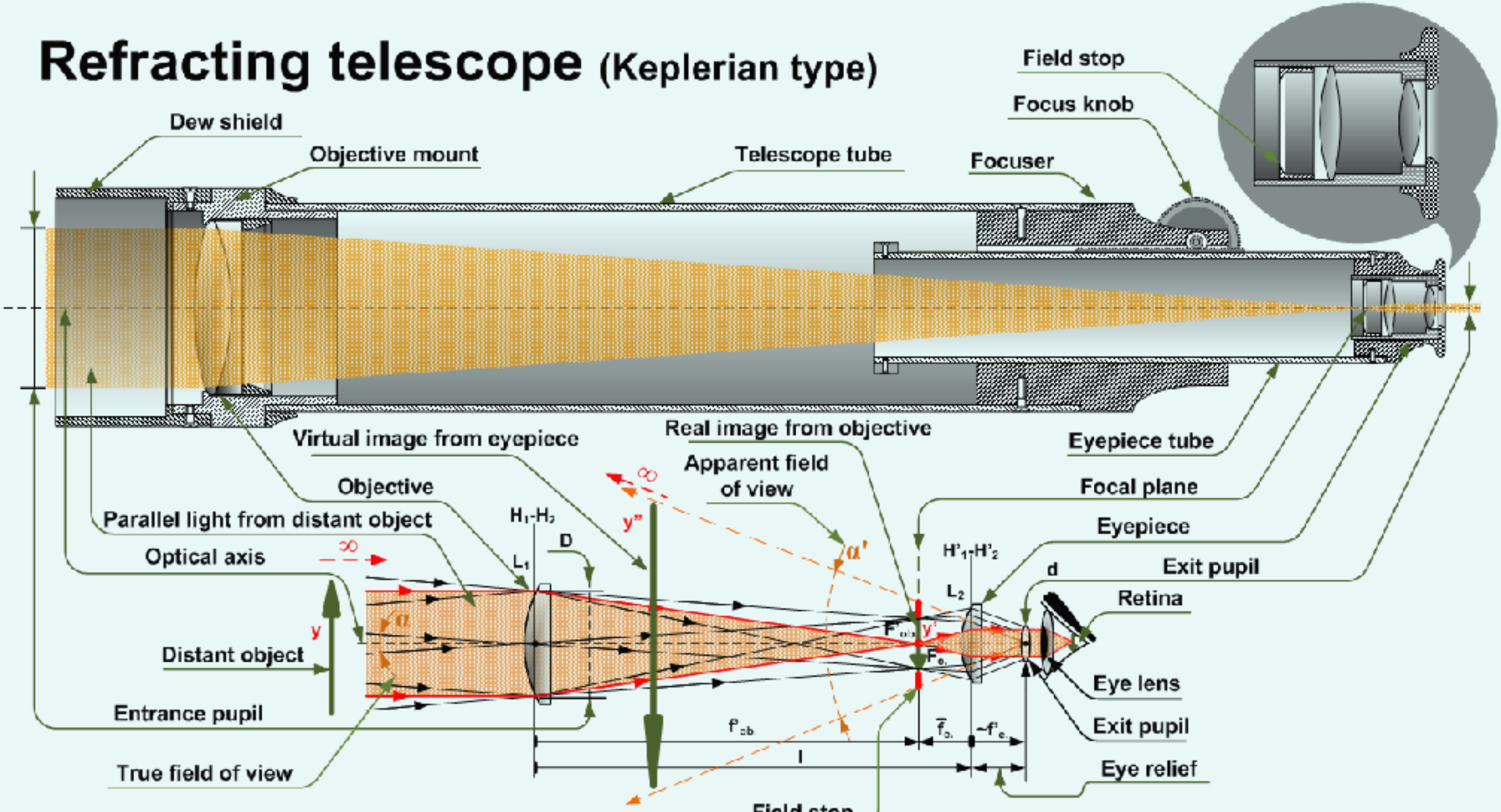






Newtonian telescope

Refracting telescope (Keplerian type)



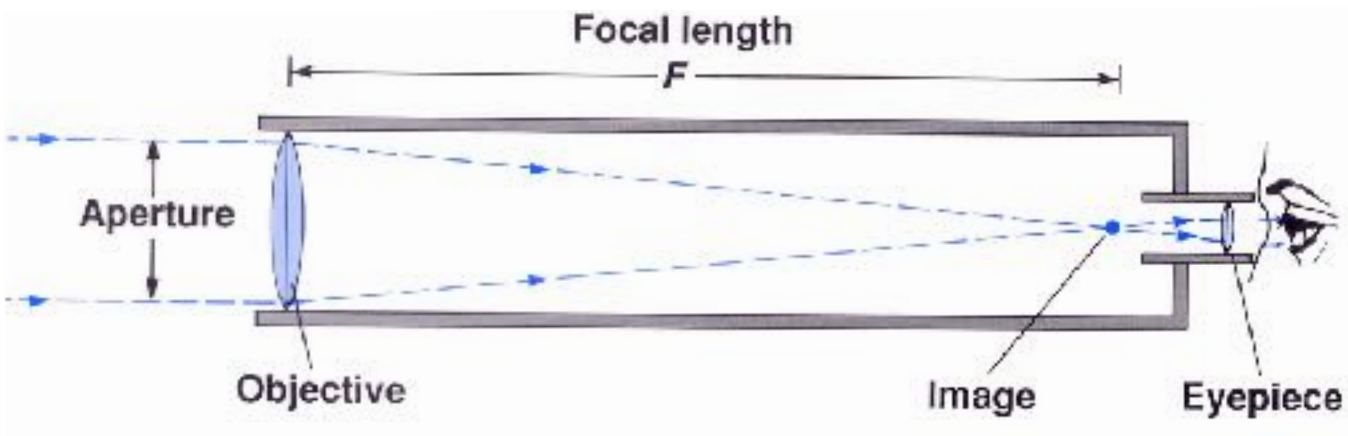
Telescope magnification: $M = \frac{tg\alpha'}{tg\alpha} = \frac{f'ob.}{fe.} = \frac{D}{d}$

Simple example

Simple example

focal length F = 4572 mm

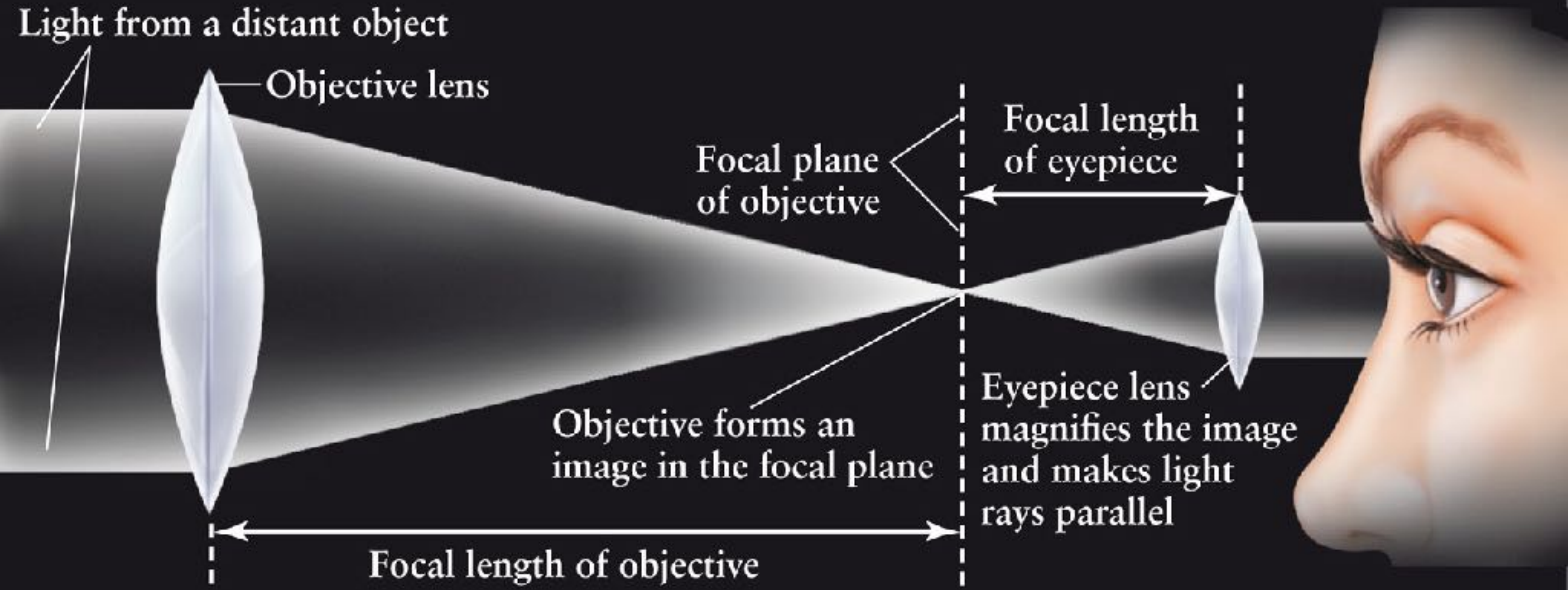
objective aperture diameter D = 305 mm



f/# = F/D = “f/14.5”

magnification = focal length / eyepiece length
= 4571mm/15mm = 300

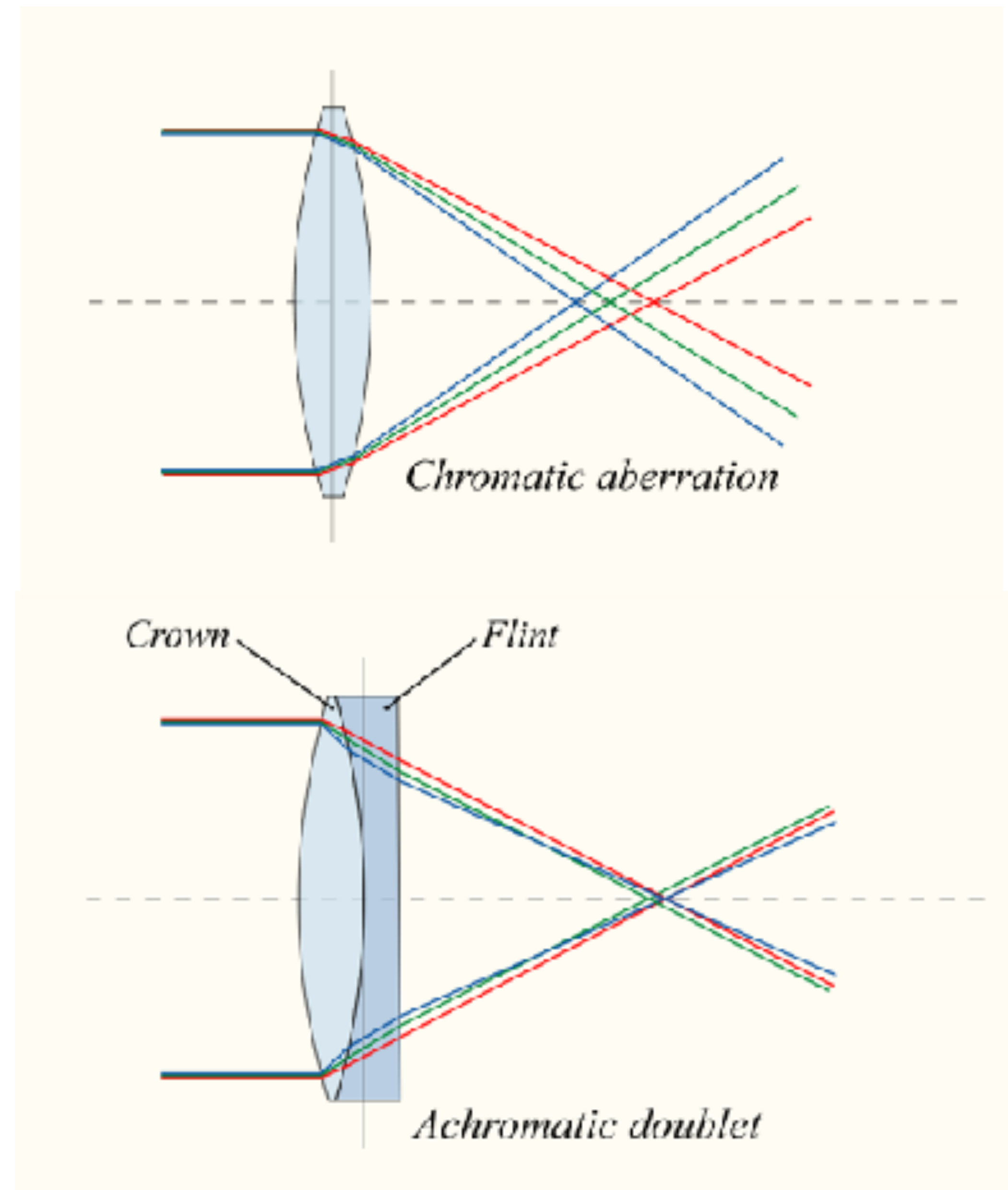
field of view (“FOV”) =
= opening angle of eyepiece / mag
= 82°/300 = 0.3°



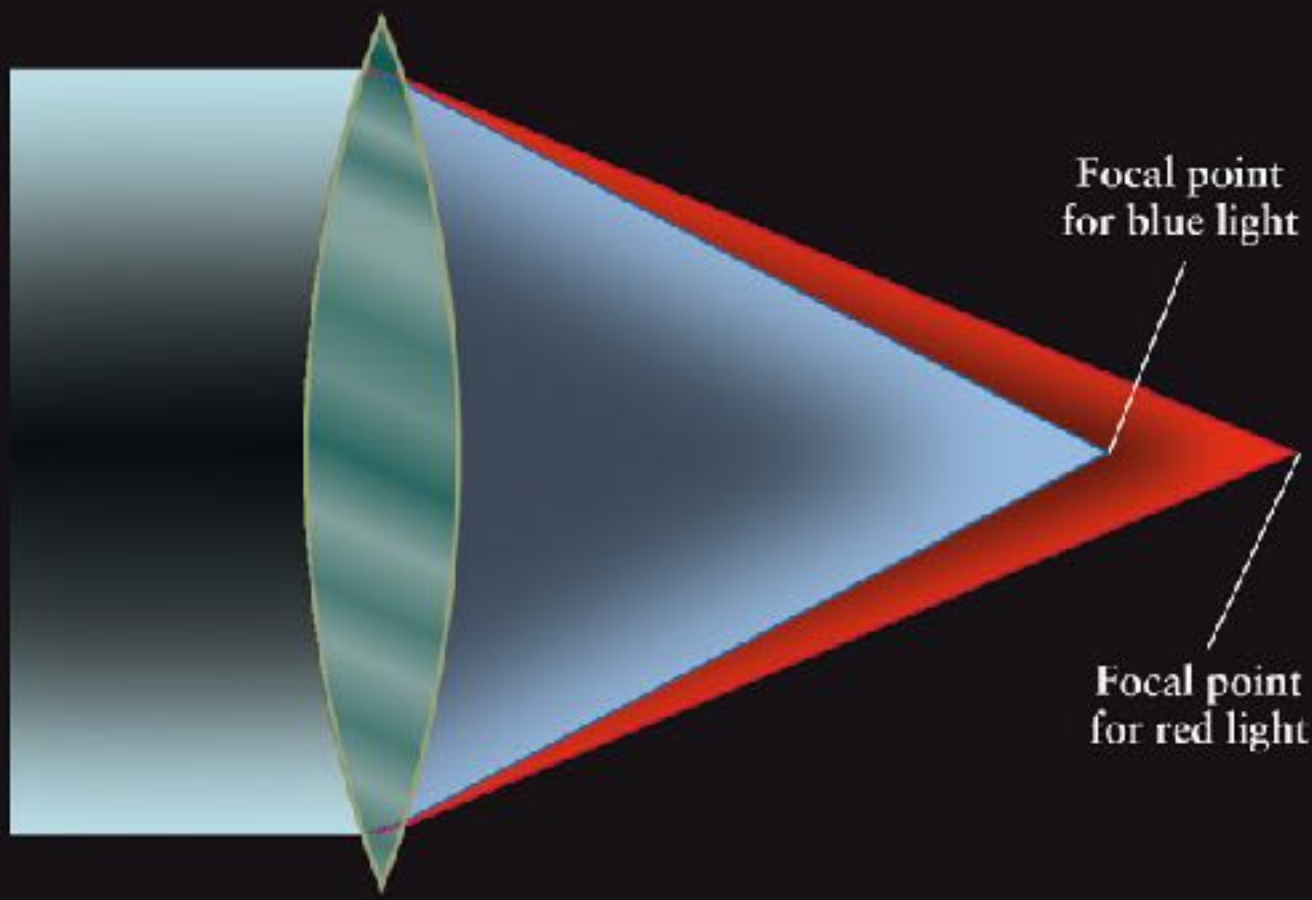
Light from a distant object
Objective lens
Focal plane
of objective
Focal length
of eyepiece
Objective forms an
image in the focal plane
Eyepiece lens
magnifies the image
and makes light
rays parallel
Focal length of objective

Limits to refracting telescopes

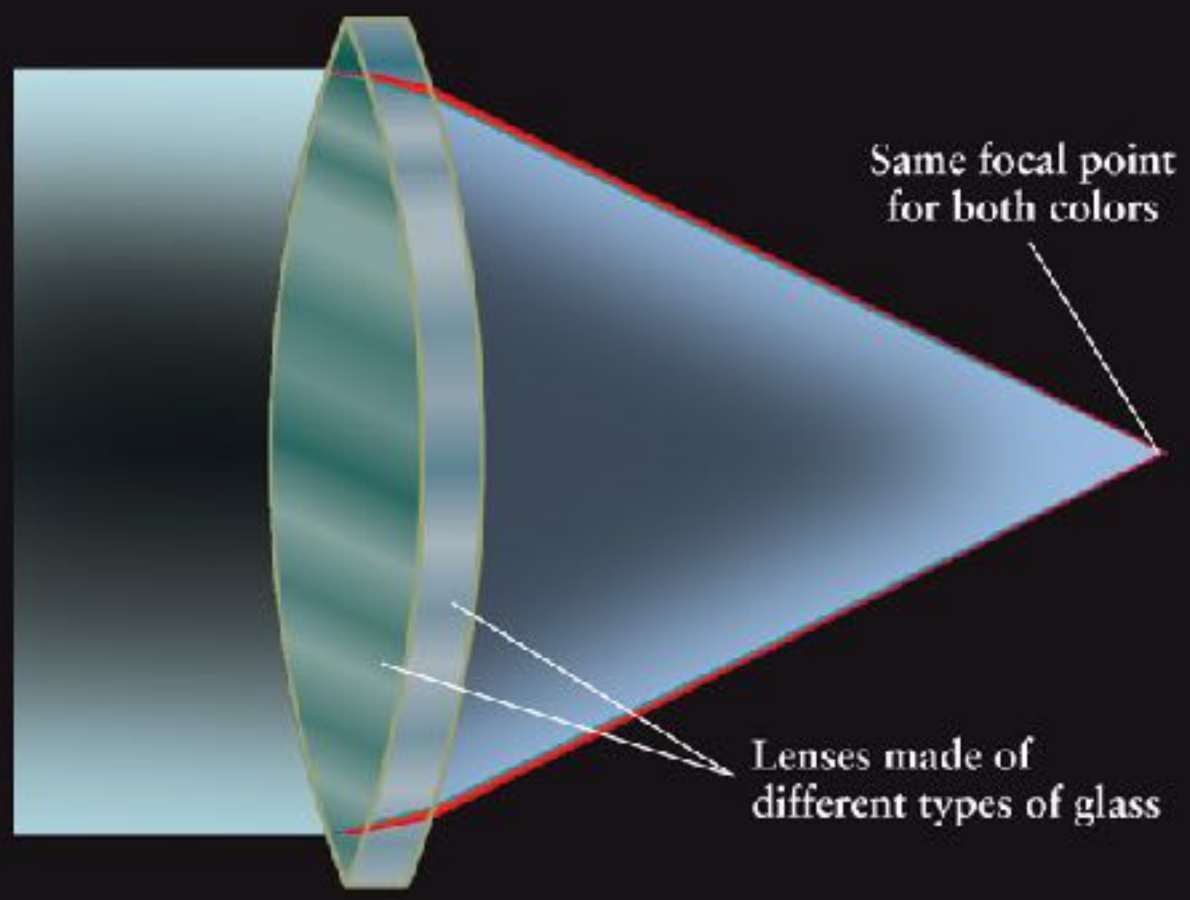
- Problems with refractors:
  - Chromatic aberration – limits to the extent it can be corrected perfectly. Focus can be off by inches for a typical 12” refractor !
  - Reflection losses at each surface. Optics need to be anti-reflection coated with a broad band.
  - Transmission losses (especially UV). Light must pass through the lens, so can’t have defects (e.g. air bubbles).
  - Long focal lengths (to mitigate chromatic aberration) require large domes.
  - Heavy convex lenses, hung on the end of the optical tube make for a difficult mechanical problem – gravitational support and deformation issues.



lenses usually require an additional lens to correct for chromatic aberration



The problem: chromatic aberration

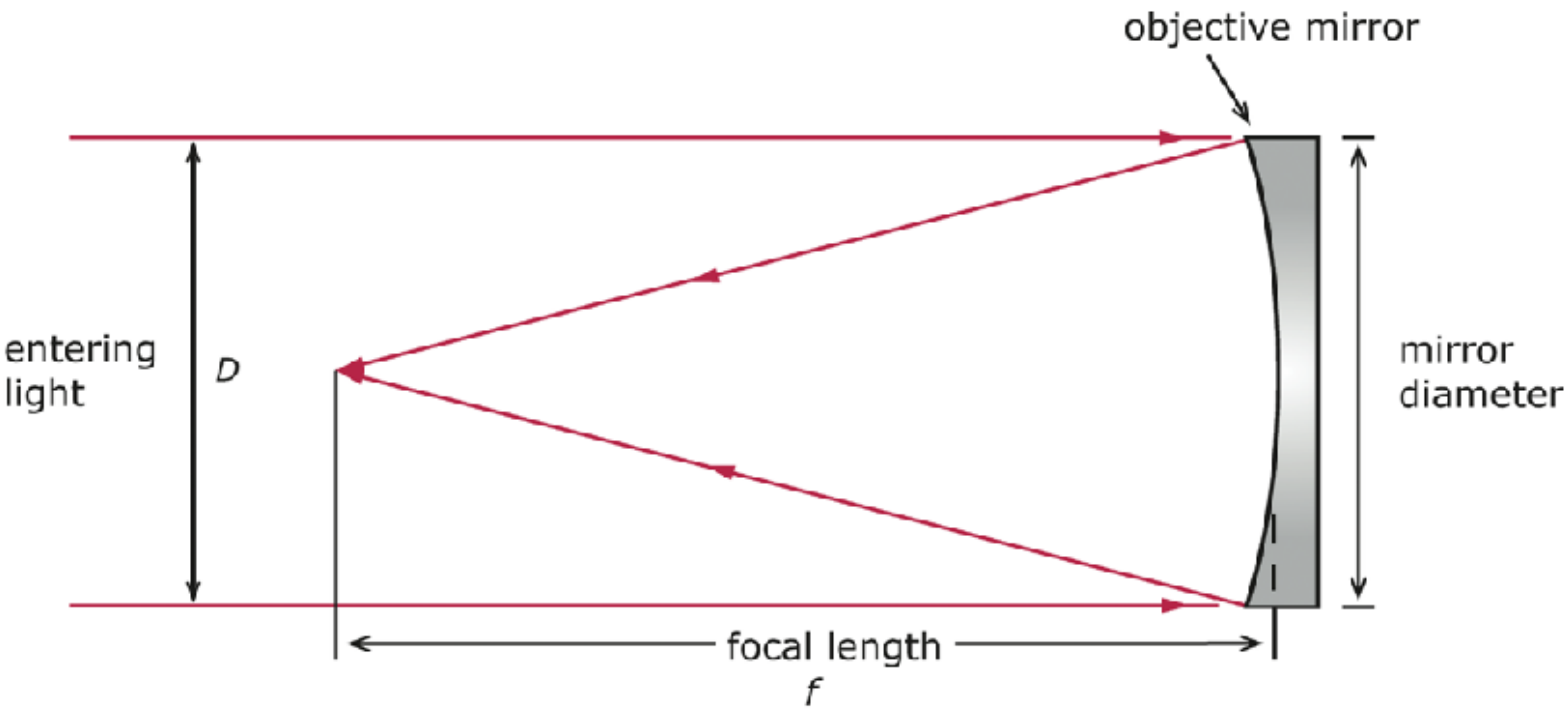


The solution: use two lenses

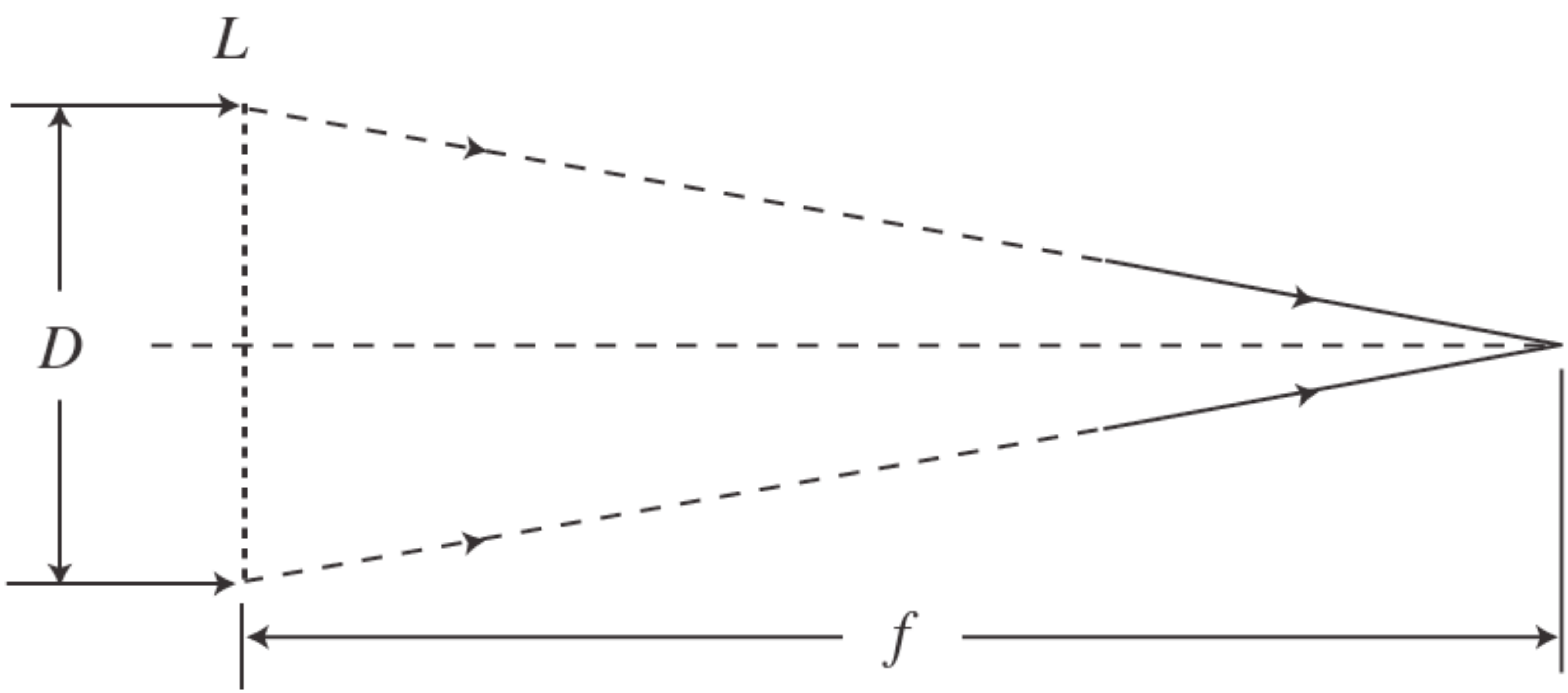
Refractors: Great 9-inch Dorpat Refractor

- Built in 1824 by Joseph von Fraunhofer (who started the field of astrophysics with his spectroscopic work in 1814), it was installed at Tartu (Dorpat) Observatory in Estonia.
- The 9.6” was the first modern achromatic refractor and the largest refractor in the world.
- Also developed the first modern (German) equatorial mount with clock drives
- While reflectors were much bigger (47”!), the refractors were much better.
- Ushered in the era of the Great Refractors but Fraunhofer died at age 39.

A Prime Focus Telescope

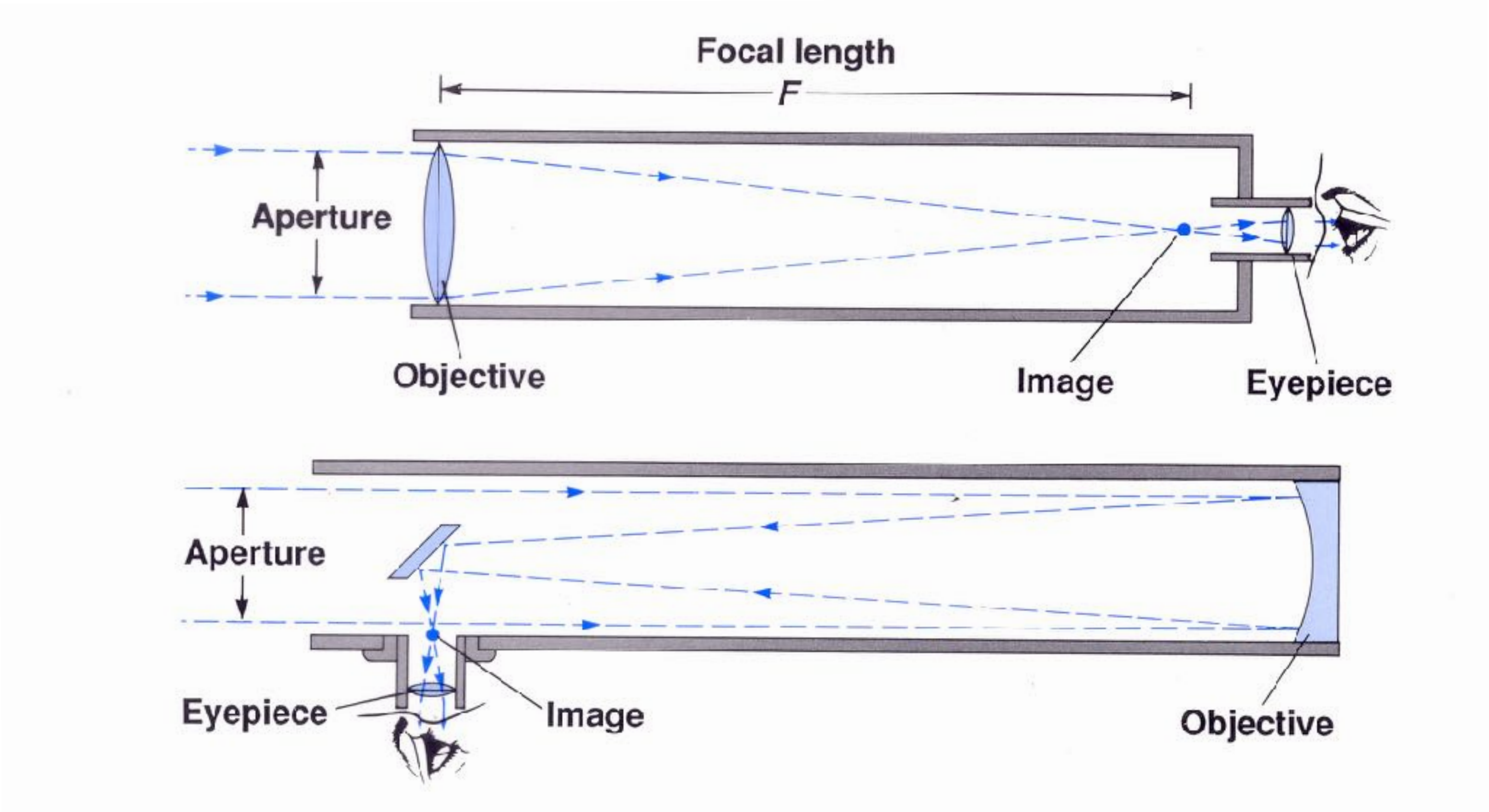


equivalent single element refractive telescope



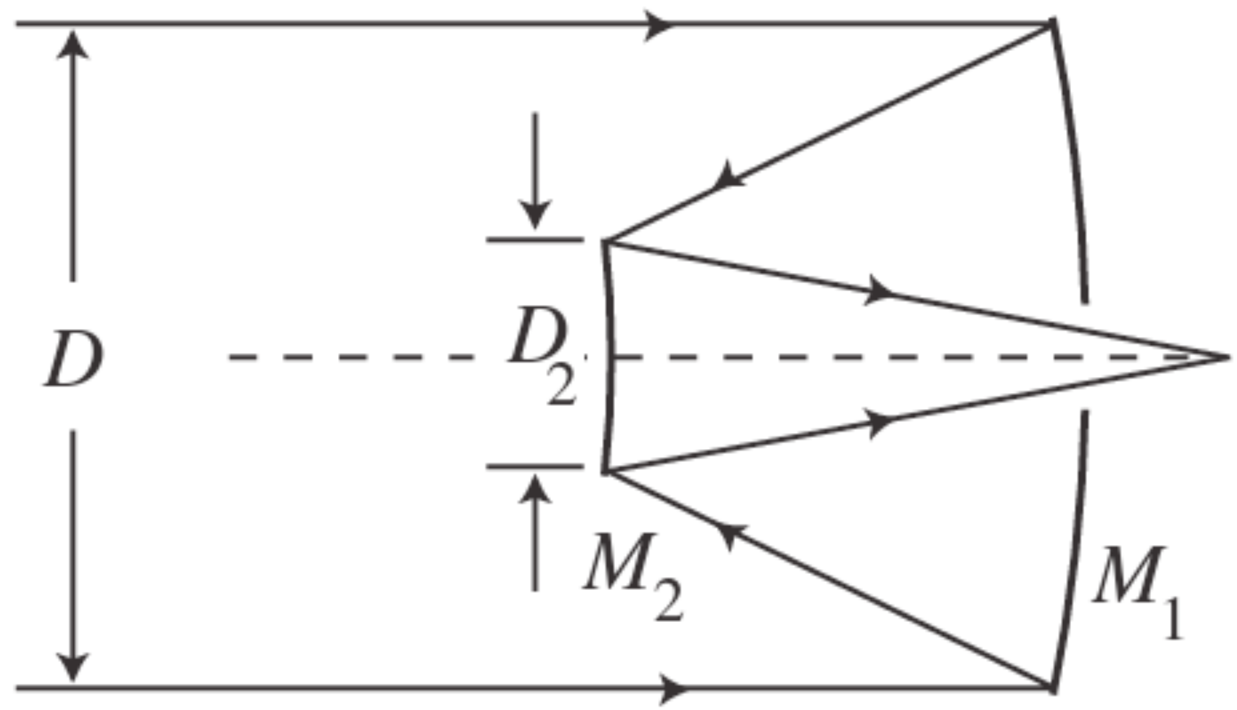
focal ratio = "f/#" = $\dfrac{f}{D} = \dfrac{\{\text{focal length}\}}{\{\text{diameter of entrance aperture}\}}$

Refractor

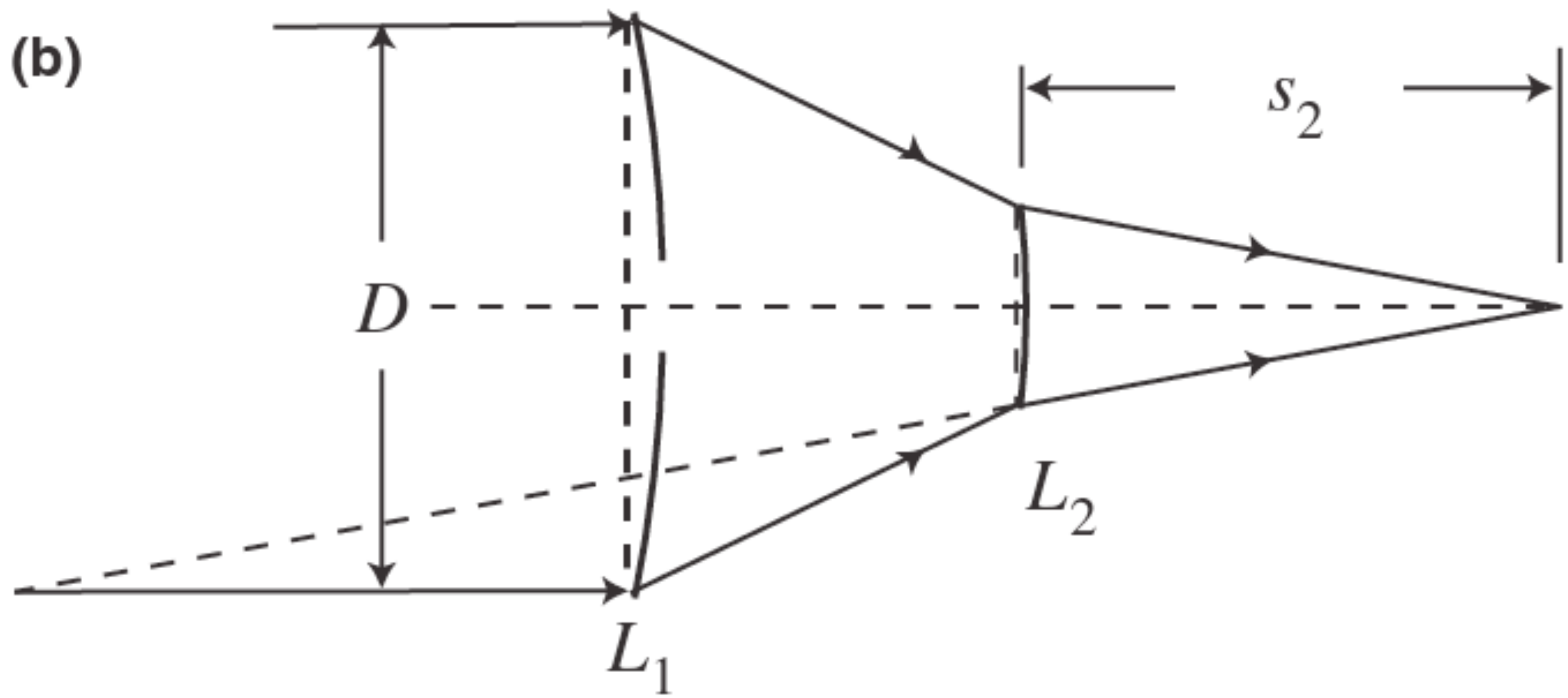


Reflector

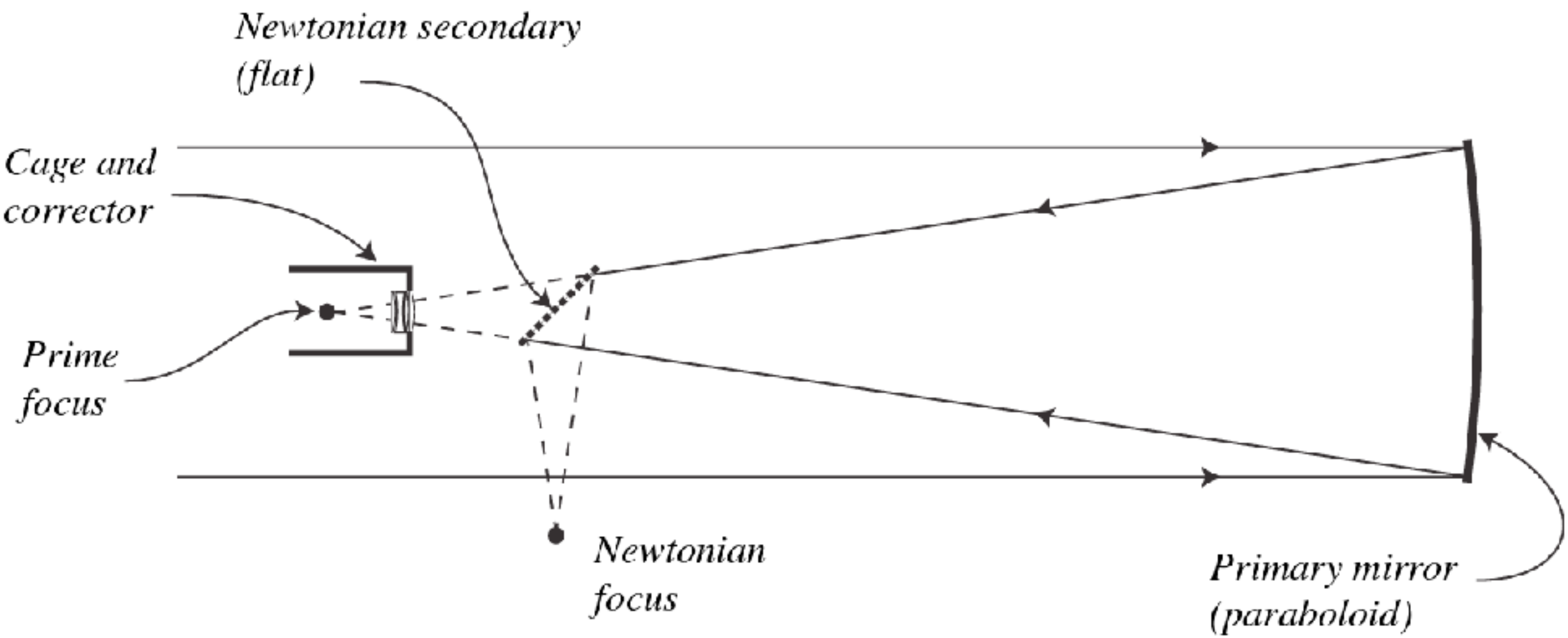
reflective telescope



equivalent refractive telescope



Newtonian Telescope



Mirrors: I

Why didn't they just build telescopes out of mirrors?

At the time, they only had **speculum**, which was an expensive and heavy alloy made of copper and tin.

The surface was difficult to cast, difficult to polish, and tarnished quickly, meaning it had to be be re-polished constantly.

The resulting surfaces reflect only ~50% of the light.

Nevertheless, people did make the largest telescopes with these, but the logistics of such large, cumbersome, and inefficient structures hampered further development.

William Herschel's 1.2 meter speculum primary mirror from the 200 foot telescope

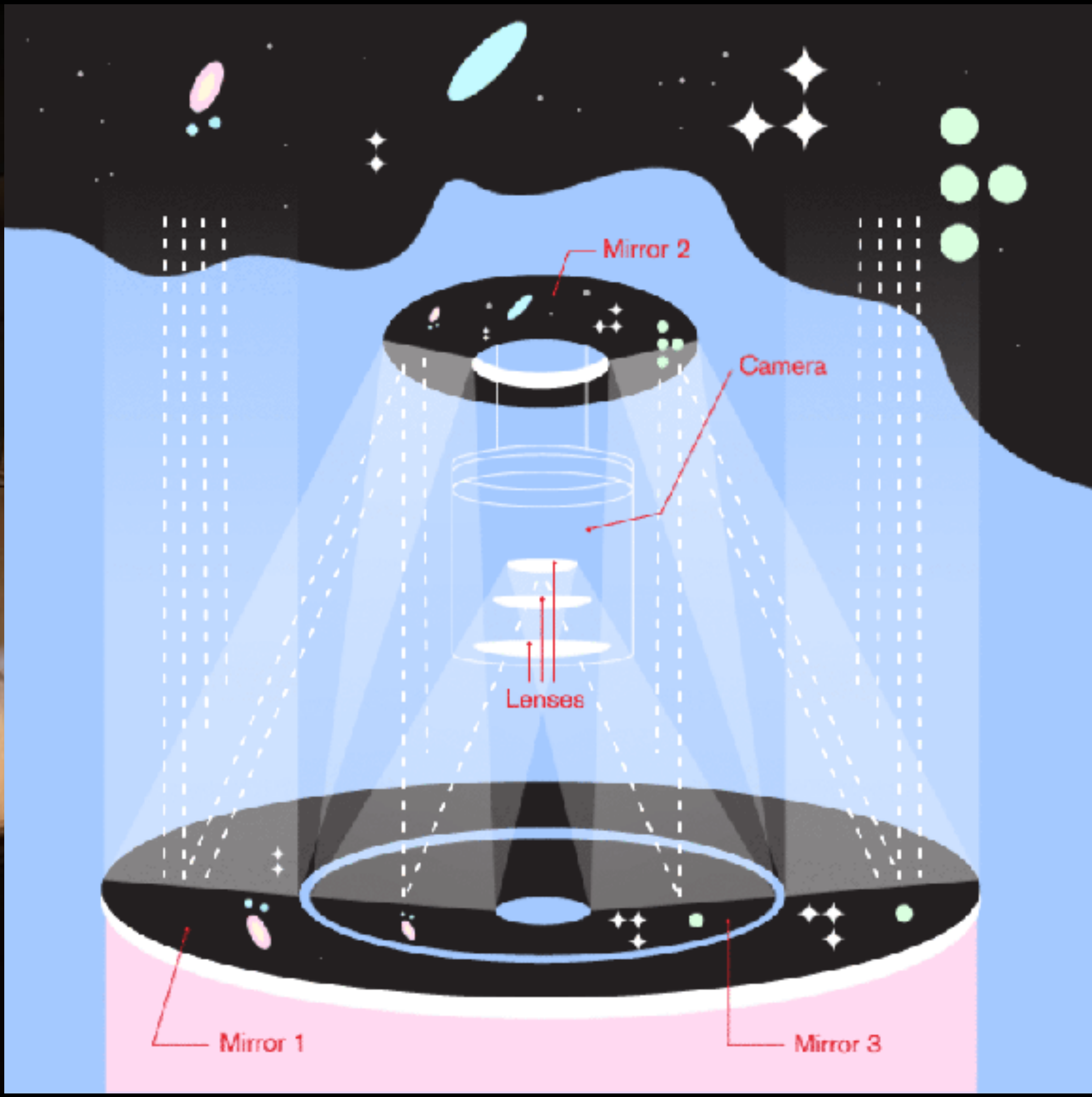
Mirrors: II
Silvered glass

- Towards the end of the 19th century, advances in chemistry allowed astronomers to deposit a thin layer of silver on glass to make high-quality mirrored surfaced.
- Glass could be easily figured into the right shape and polished to a high degree of accuracy.
- The first such telescopes were built by amateur astronomer Dr. Andrew Common in the 1870-80s, who eventually built a 36” reflector.
- This technique culminated in the Hale 60” telescope on Mt. Palomar in 1908.
- Later, in 1930, aluminum was able to be deposited. This is now the standard modern practice.


82” Struve Telescope primary mirror circa 1935

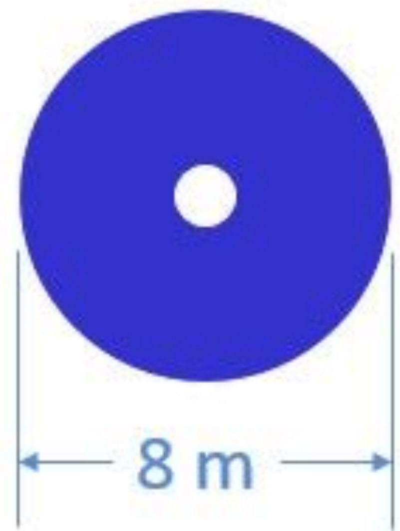
What's special about Rubin Observatory

- Rubin isn't going to have the biggest telescope
  - We want more area to collect more light and see fainter, farther sources
  - … but, we want to also cover as much area on the sky as possible - a "synoptic survey"

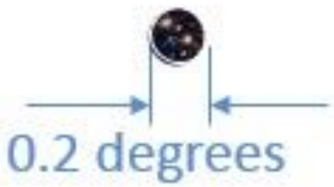


8m Class Telescope

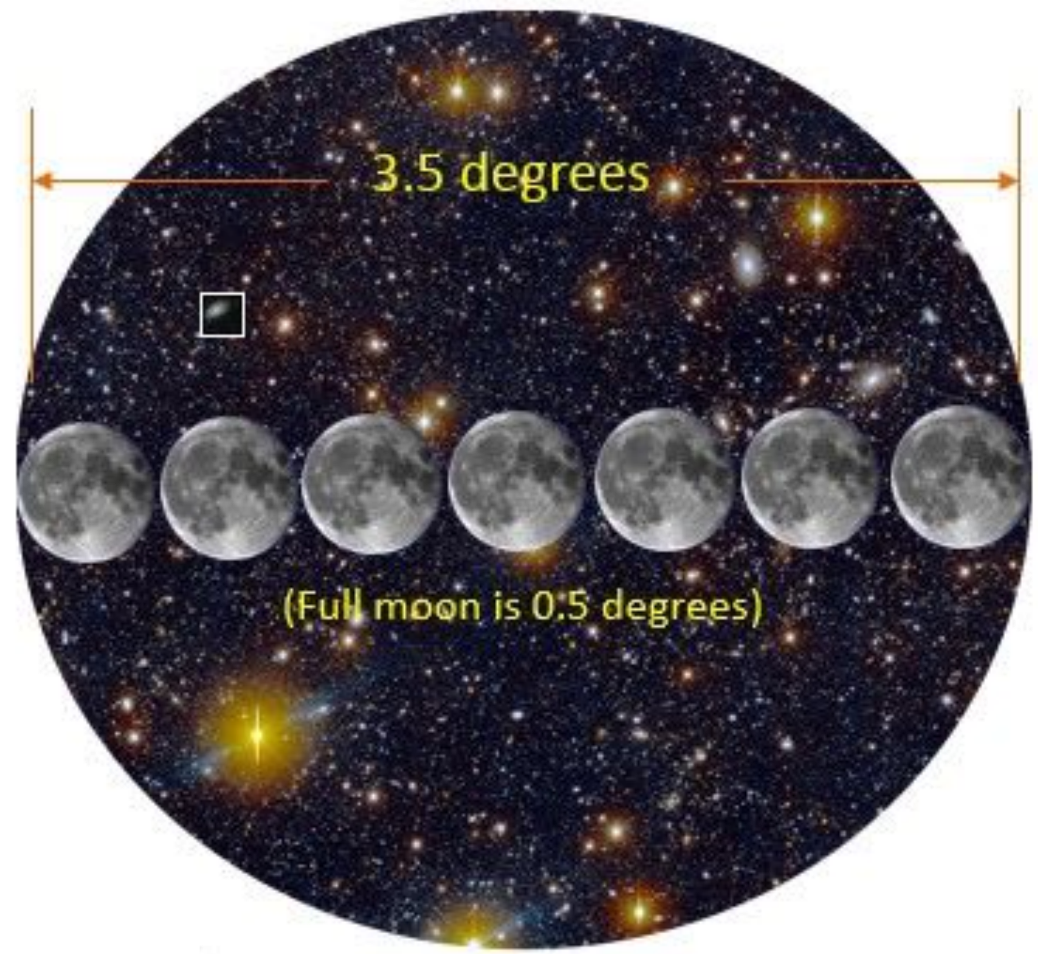
Primary Mirror Diameter



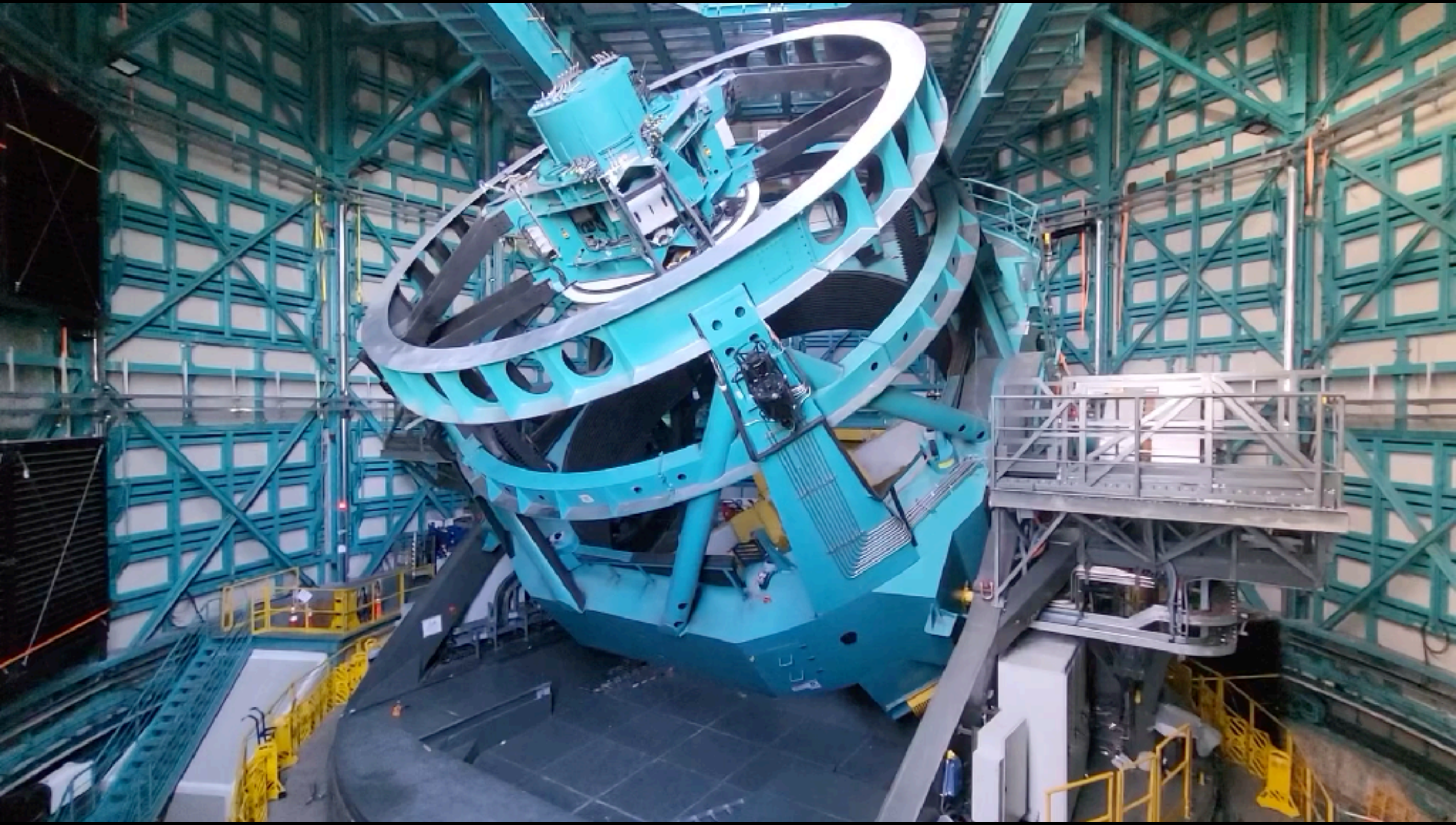
Field of View

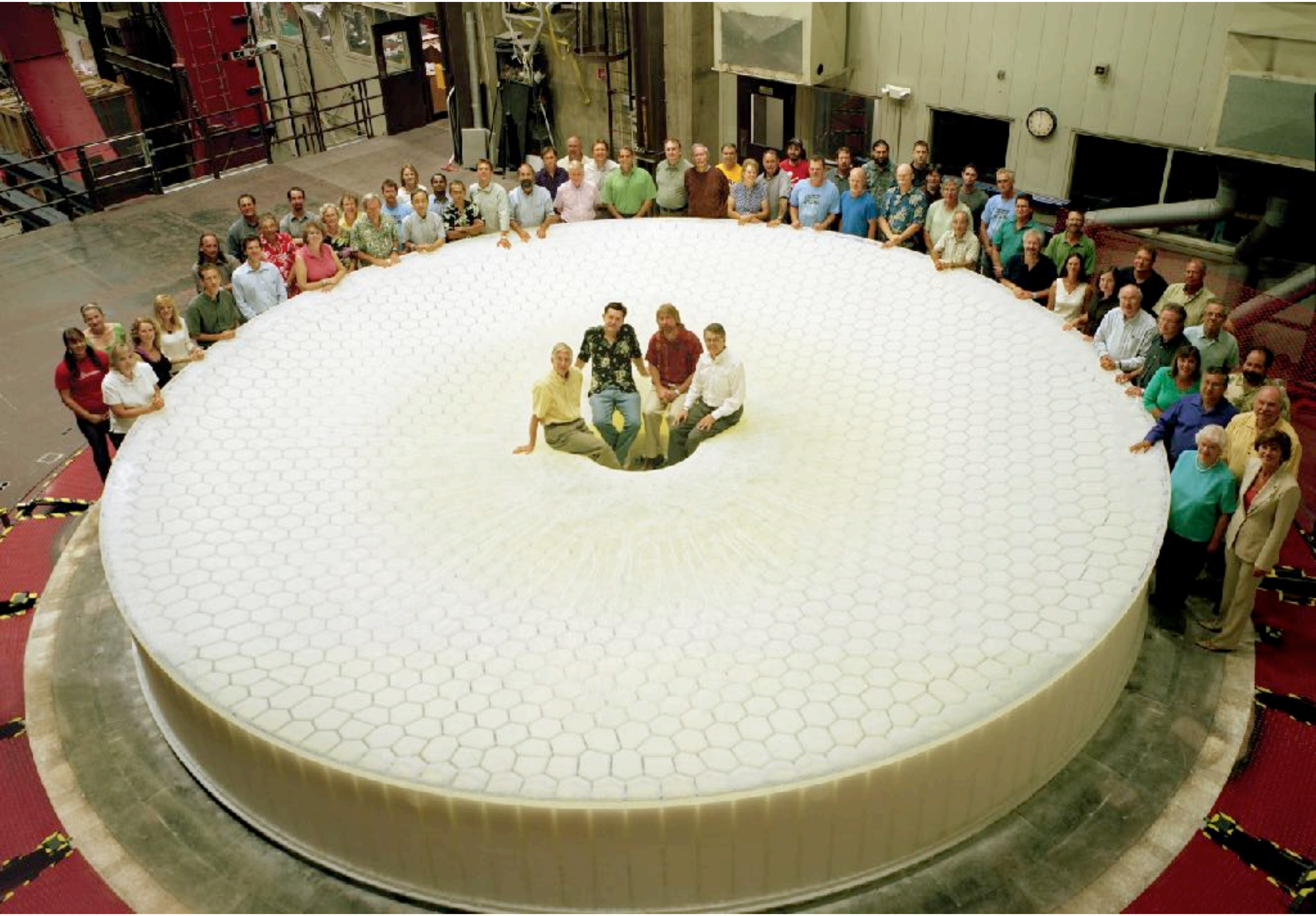


LSST



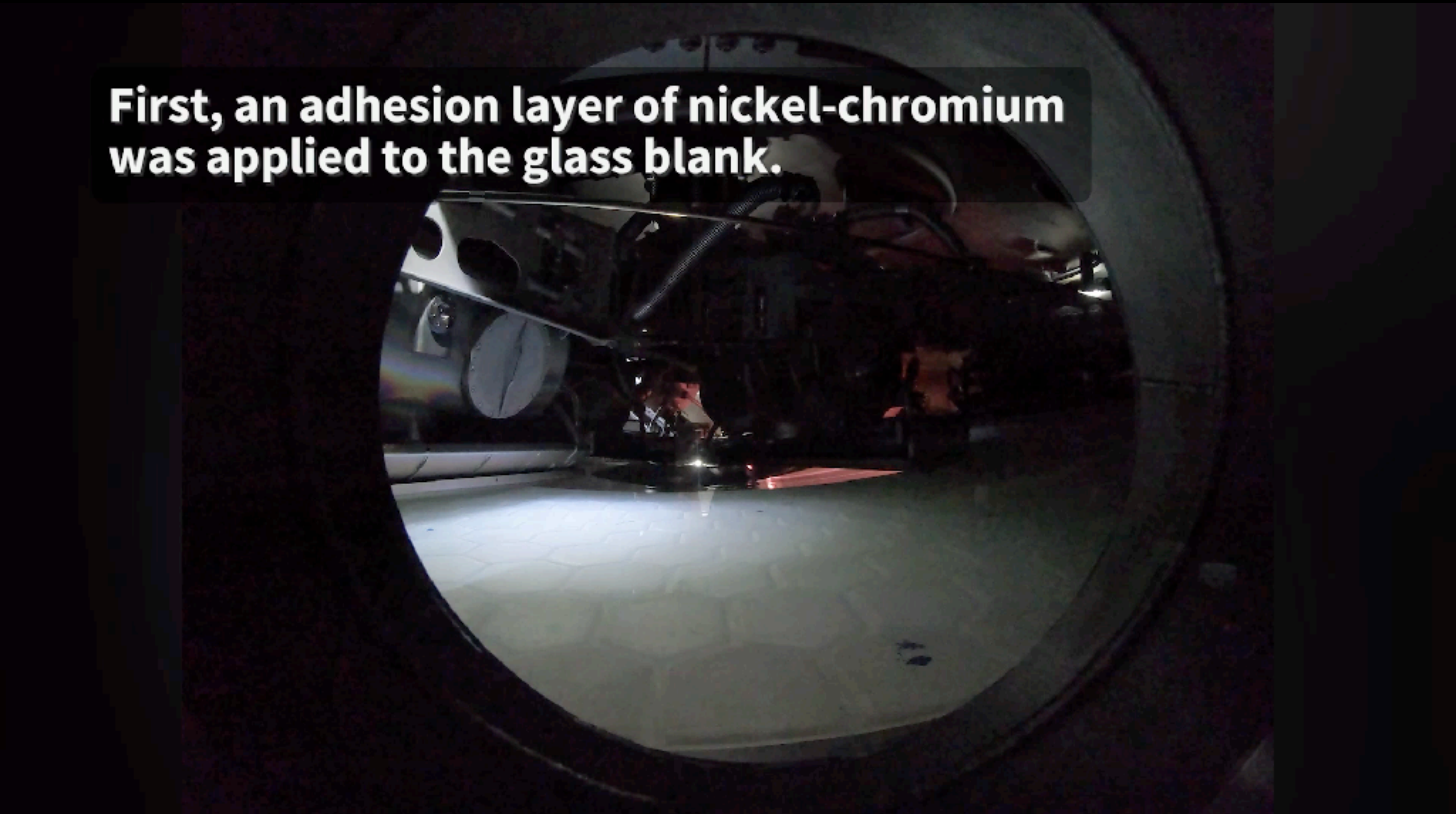
Z. Ivezic,

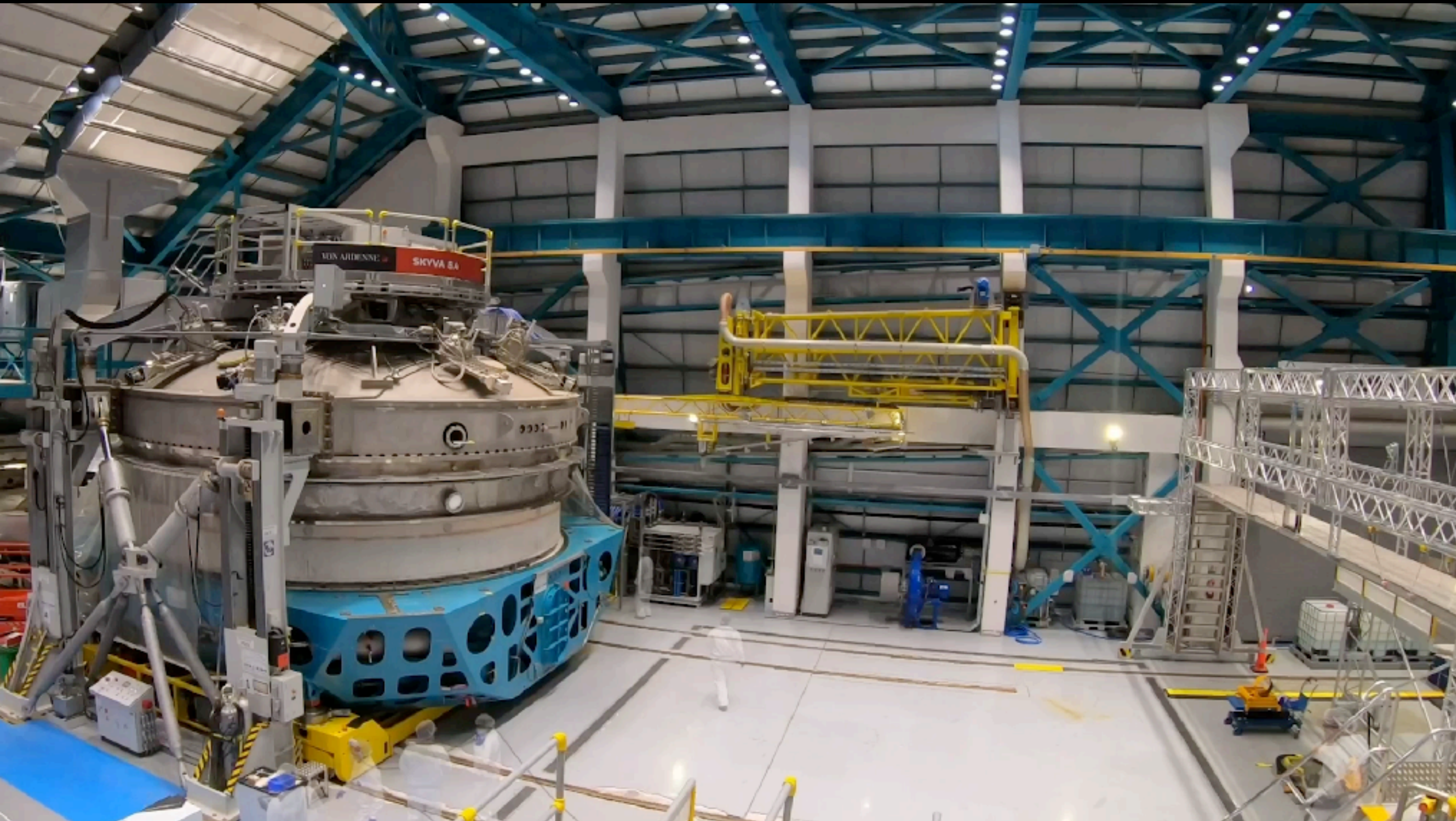




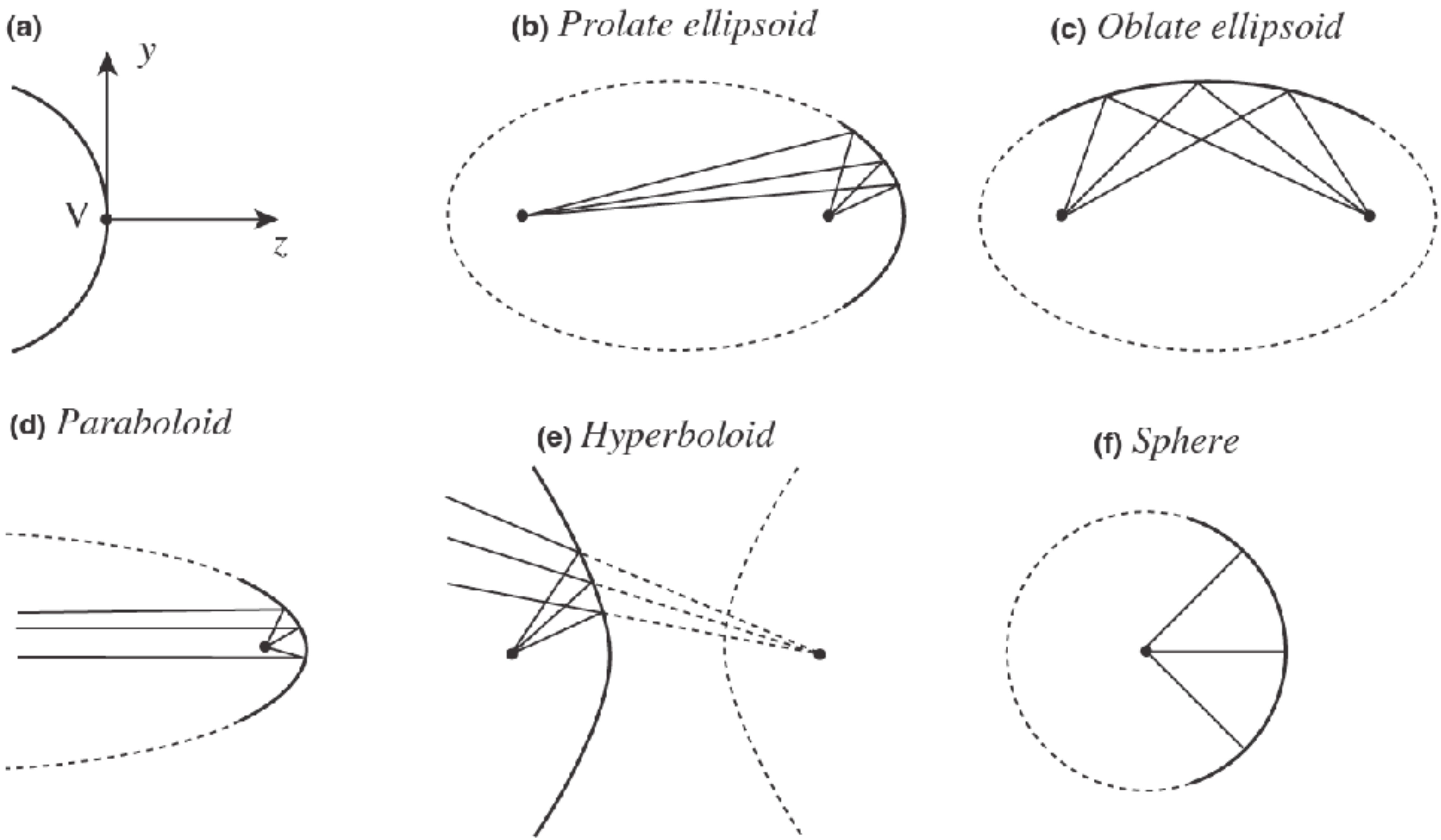

Túnel
Puclaro
VON ARDENNE
60


KONECRANES

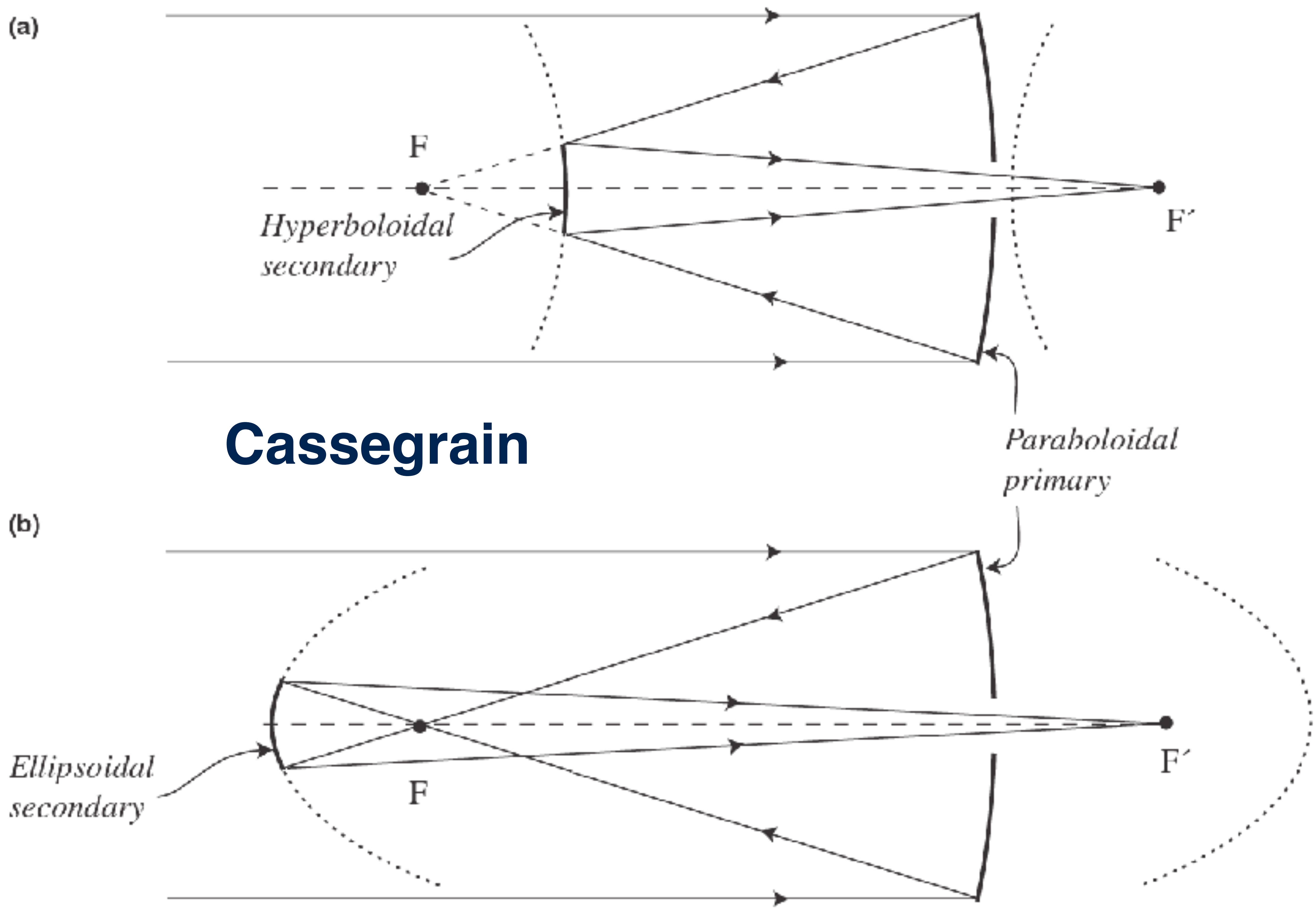

First, an adhesion layer of nickel-chromium was applied to the glass blank.


SKYVA SA

conic sections of mirrors



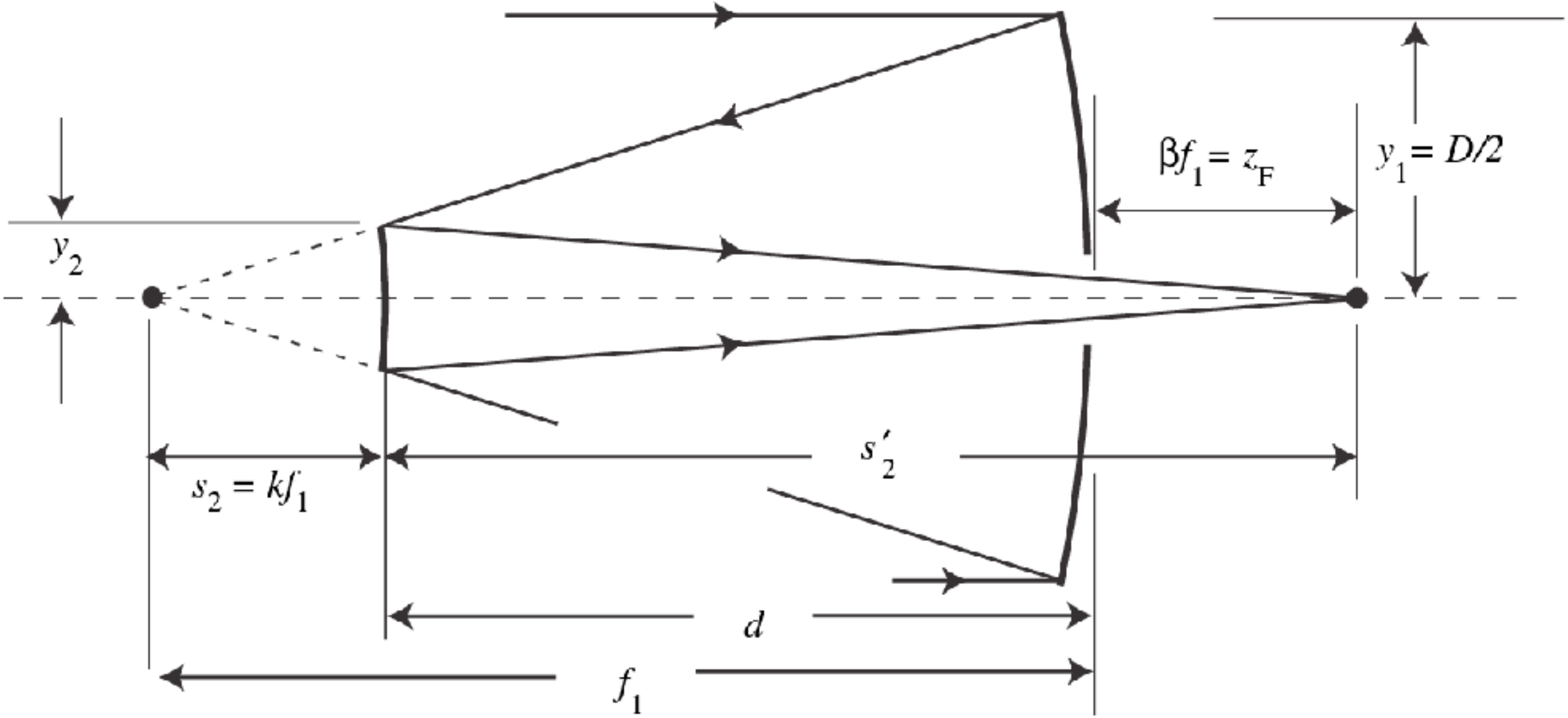
Most common telescopes



Cassegrain

Gregorian

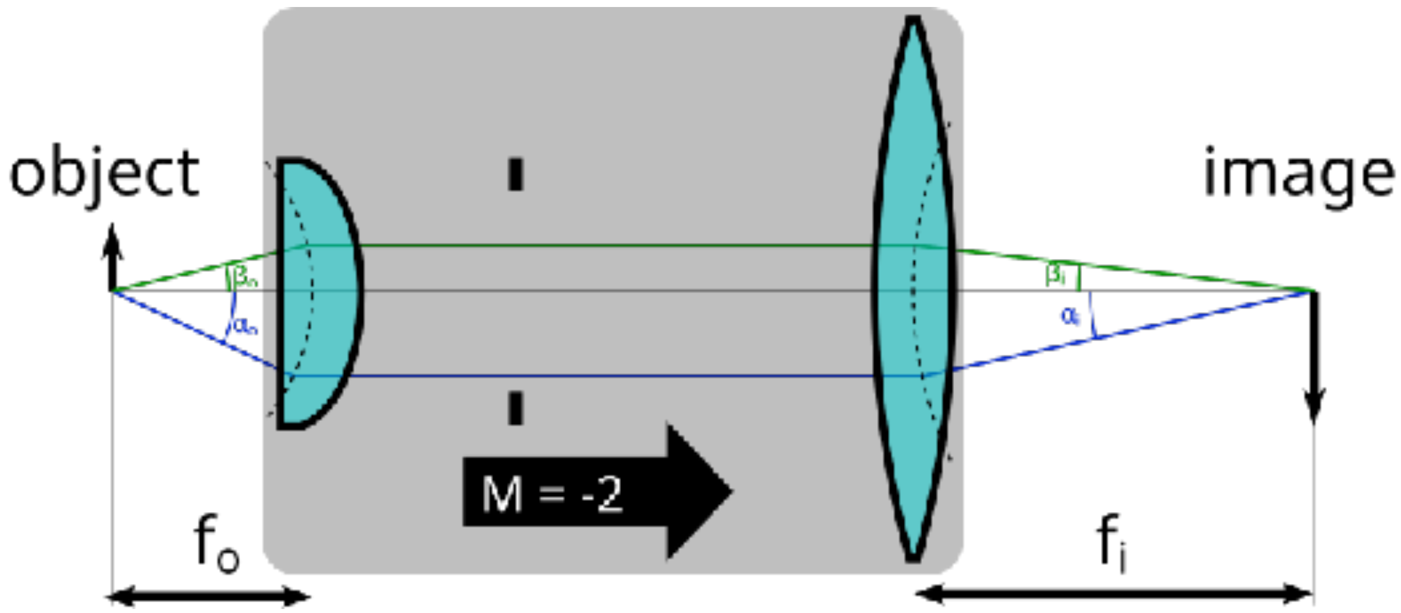
Parameters for a two-mirror telescope



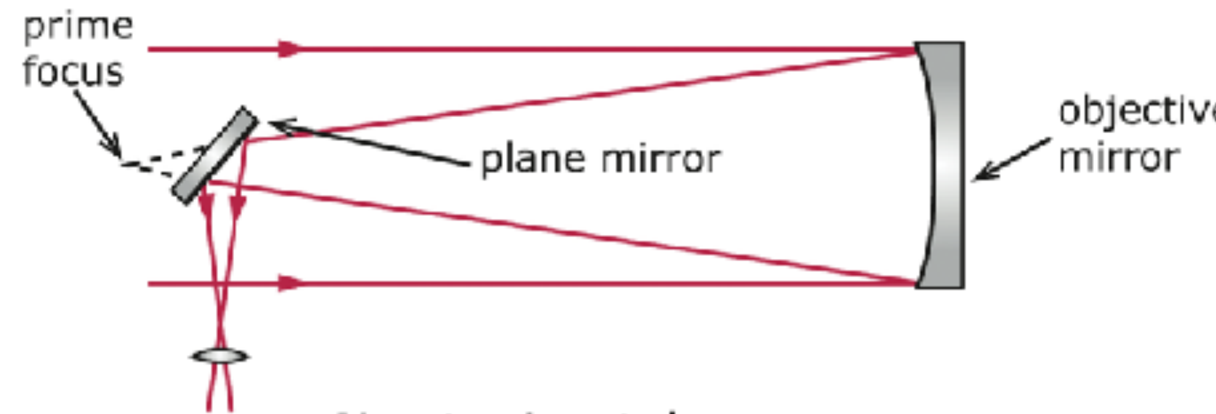
Basic telescope types

These are all based on Fermat's Principle.

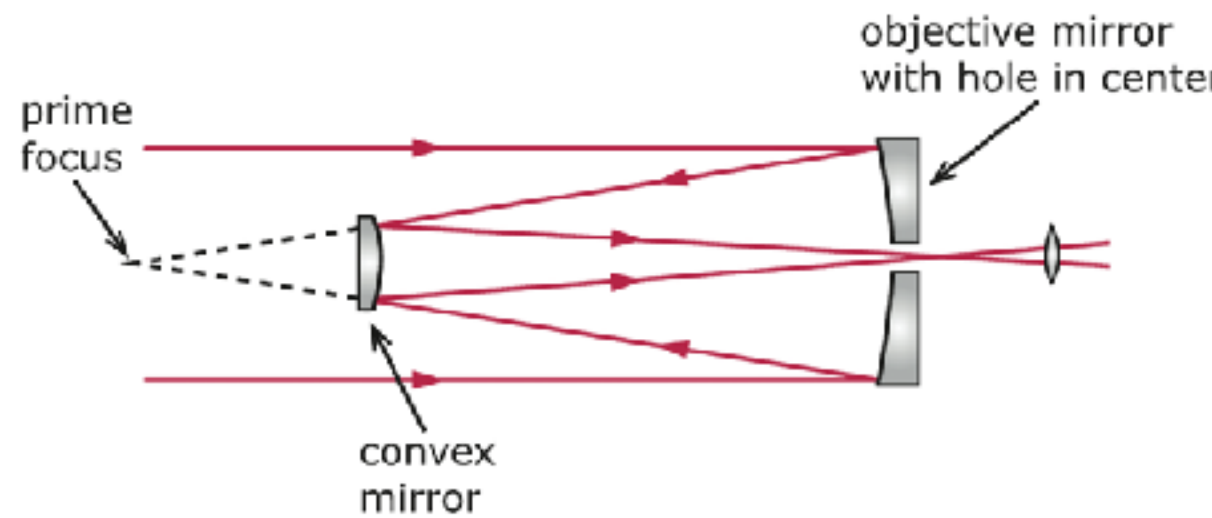
The Ritchey-Cretien looks like the Cassegrain, but abandons the conic sections and instead is optimized around the Abbe Sine Condition:



$$\frac{\sin \alpha_o}{\sin \alpha_i} = \frac{\sin \beta_o}{\sin \beta_i} = |M|$$



Newtonian telescope



Cassegrain telescope

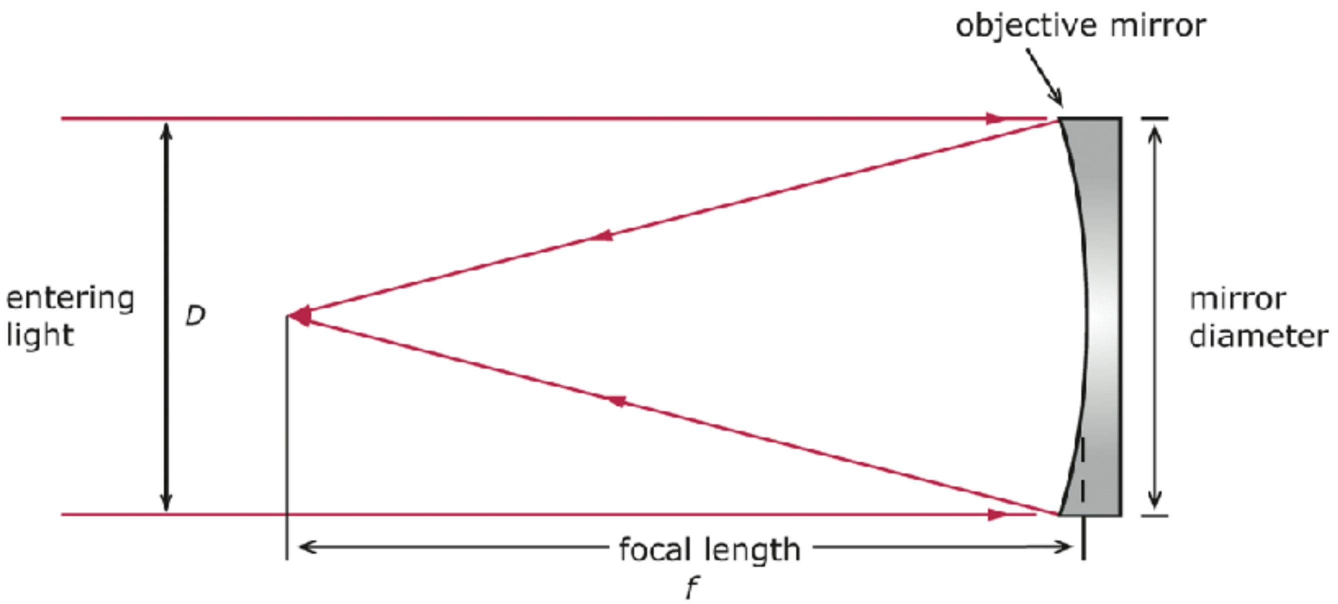
Gregorian telescope

Diffraction limit

$$\theta \simeq 1.22\frac{\lambda}{D}$$

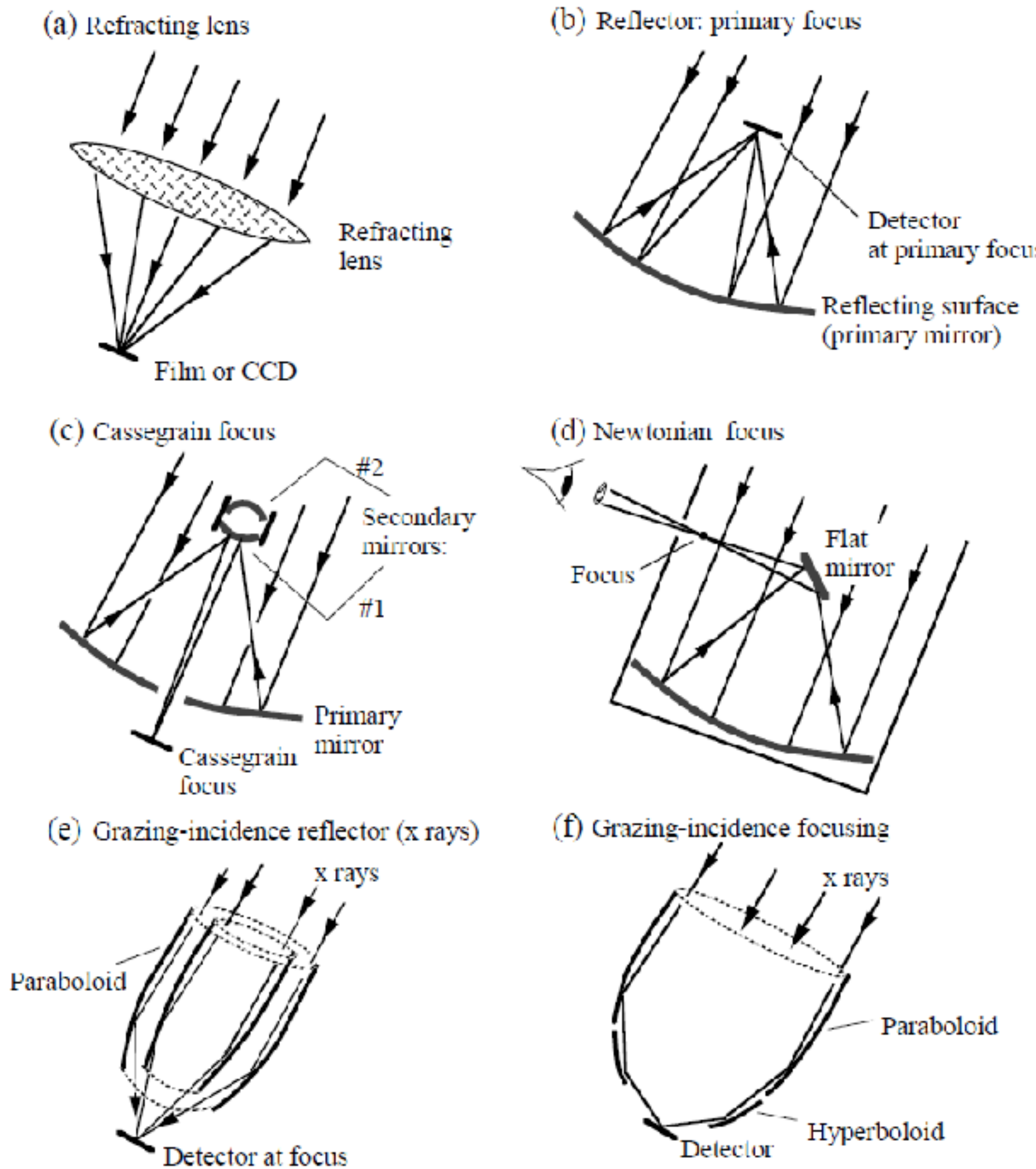
focal length

f-number = f/D

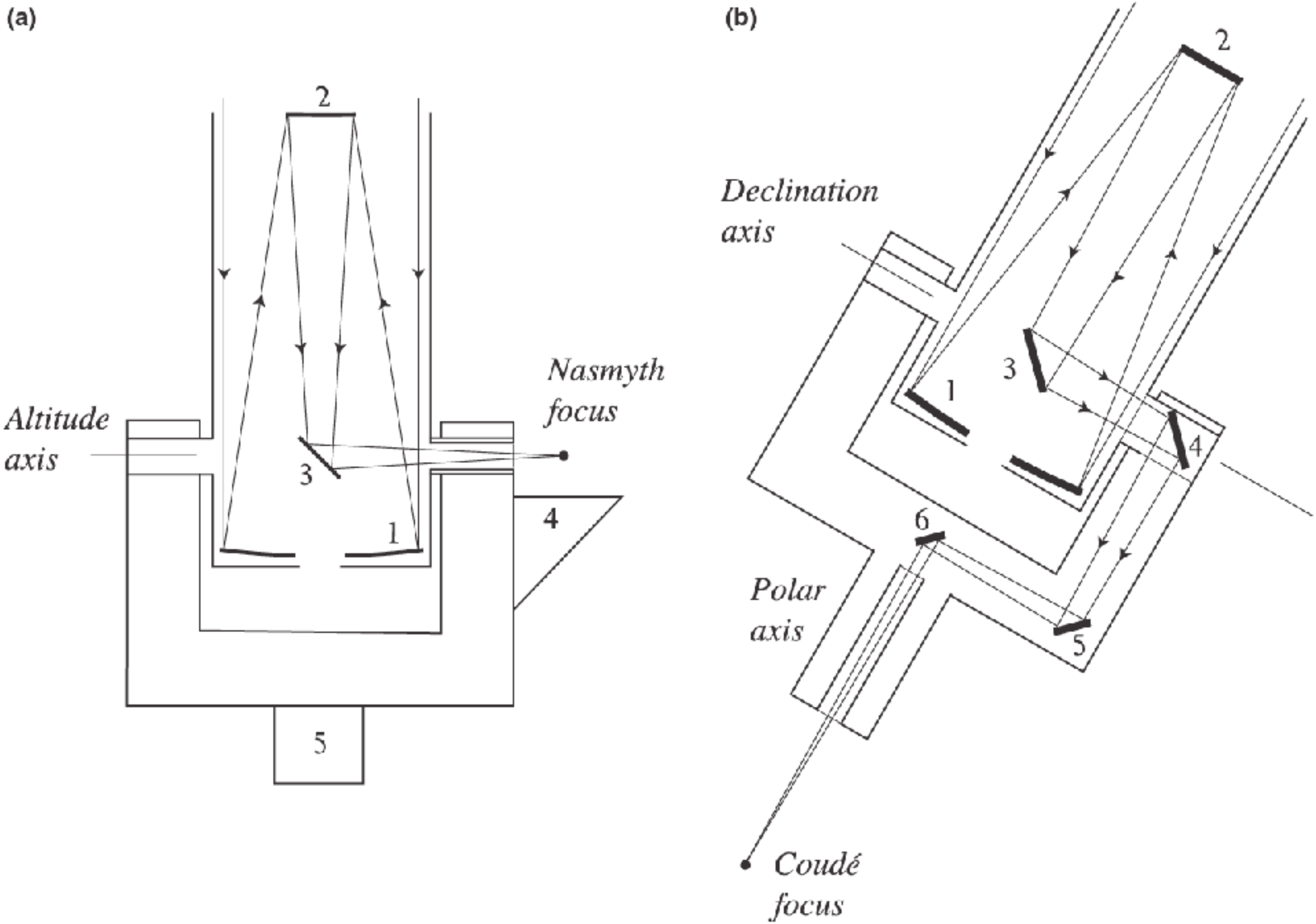


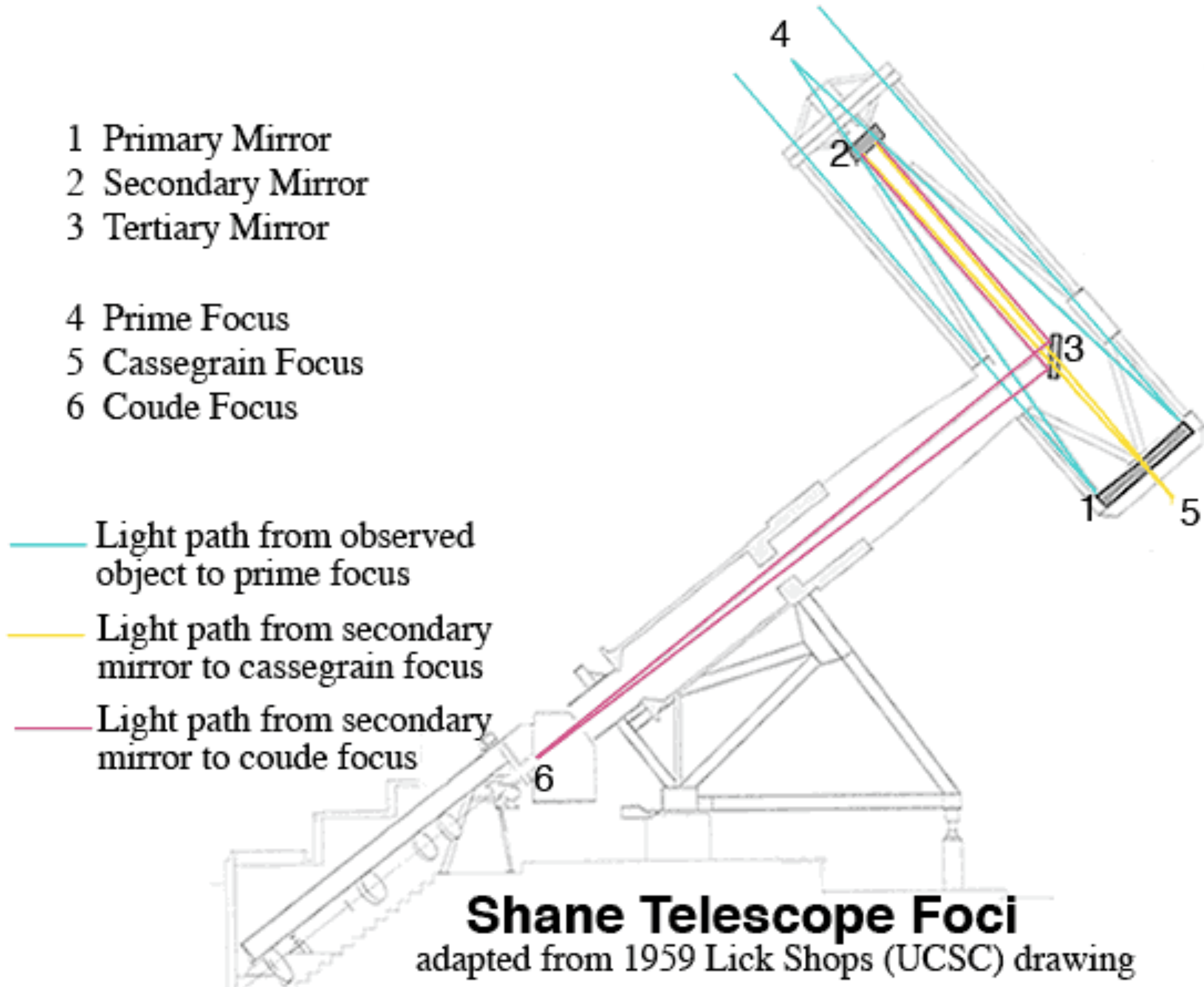
optical accuracy requirements

RMS < λ/10


(a) Refracting lens
Refracting lens
Film or CCD
(b) Reflector: primary focus
Detector
at primary focus
Reflecting surface
(primary mirror)
(c) Cassegrain focus
#2
Secondary
mirrors:
#1
Primary
mirror
Cassegrain
focus
(d) Newtonian focus
Flat
mirror
Focus
(e) Grazing-incidence reflector (x rays)
x rays
Paraboloid
Detector at focus
(f) Grazing-incidence focusing
x rays
Paraboloid
Hyperboloid
Detector

Other foci

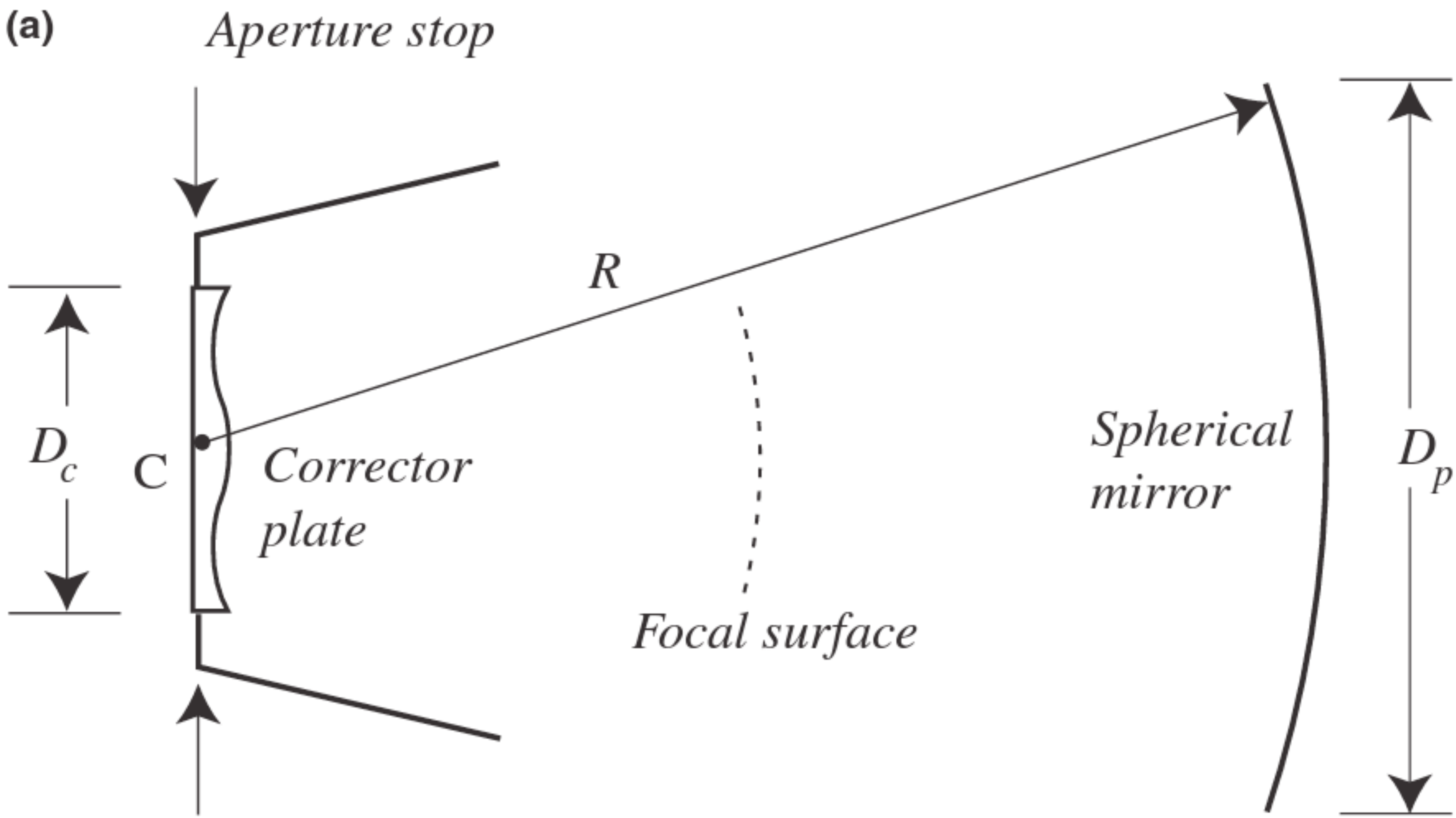

(a)
2
Nasmyth
focus
Altitude
axis
3
4
1
5
(b)
2
Declination
axis
3
1
4
6
Polar
axis
5
Coudé
focus



Shane Telescope Foci

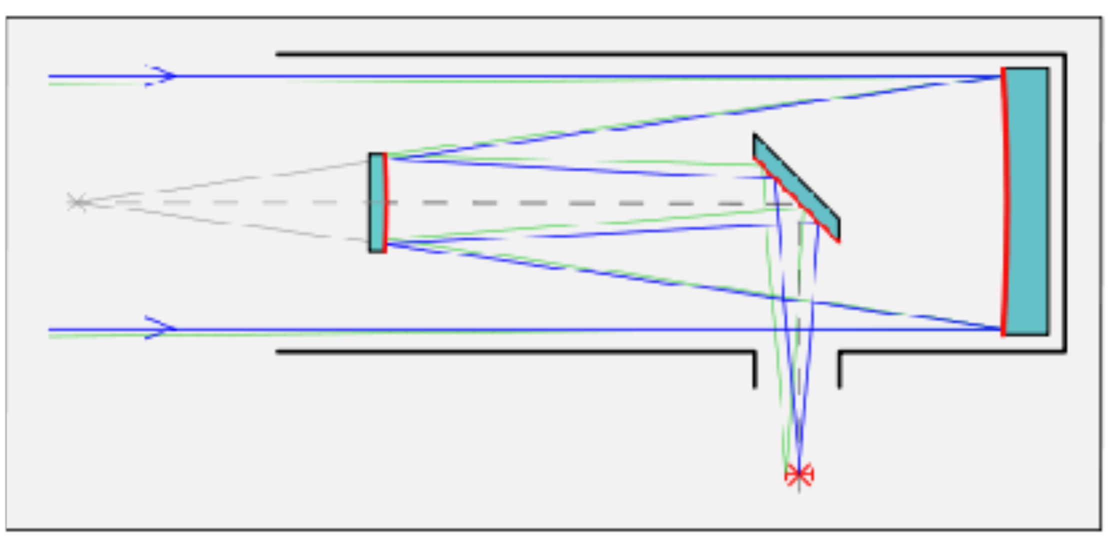
adapted from 1959 Lick Shops (UCSC) drawing

Schmidt Telescope

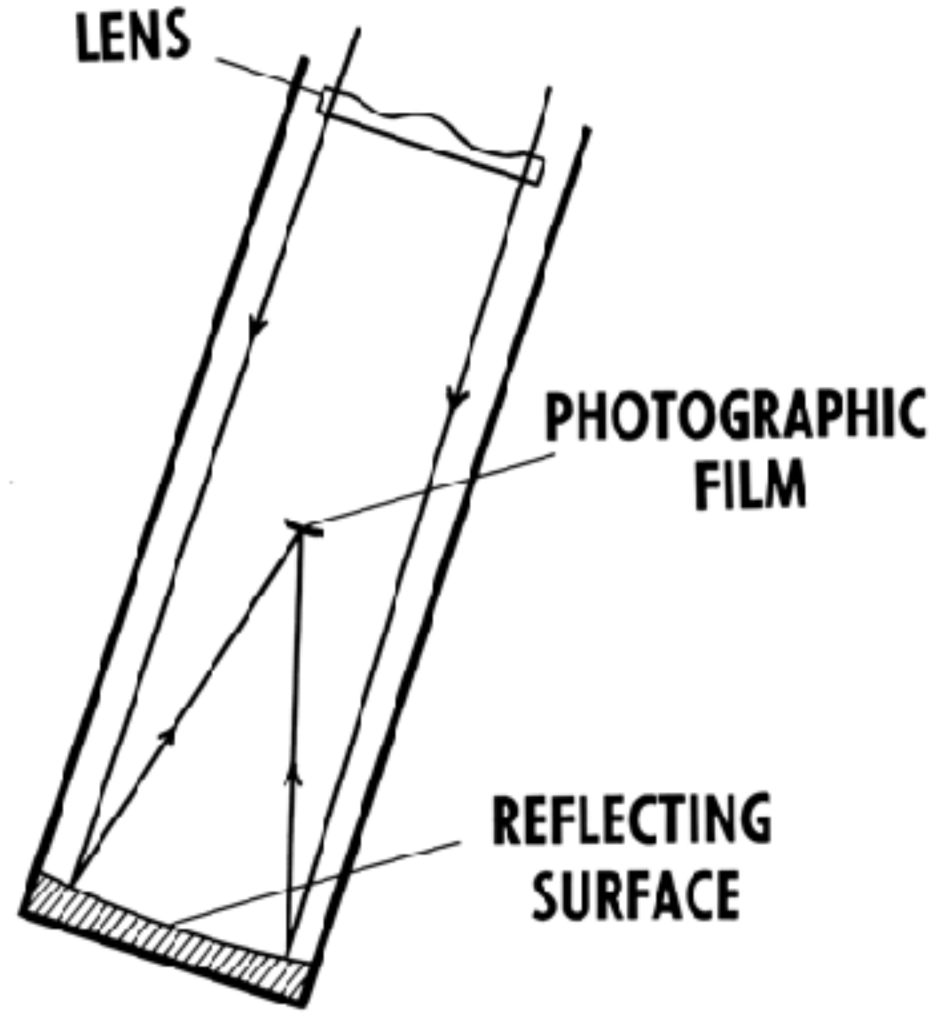


Hybrids, correctors, and tertiaries

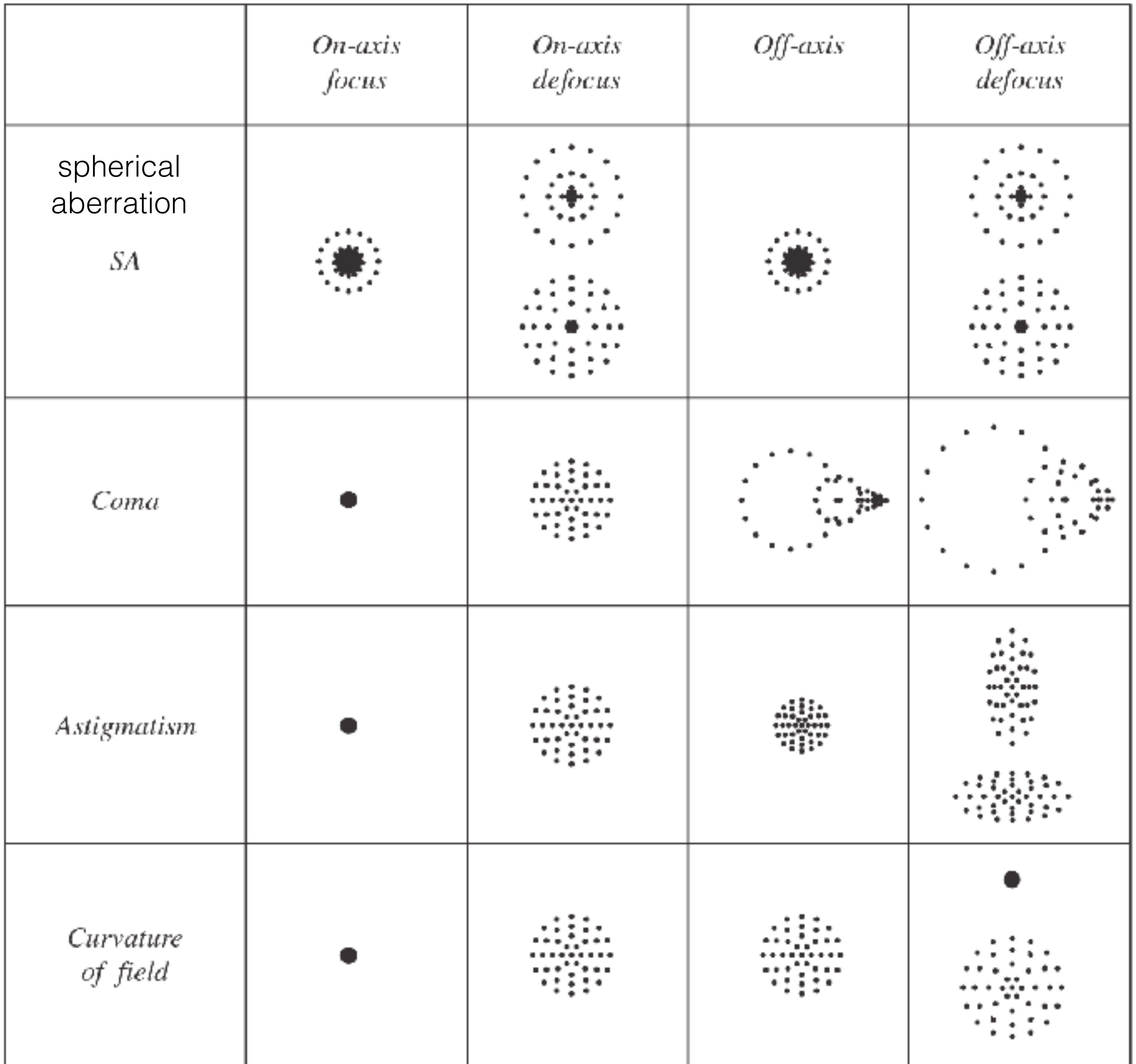
- The ***Schmidt telescope*** has a primary mirror with a spherical surface and a spherical aberration corrector.
  - Focal surface is significantly curved.
- To make use of the Prime Focus of a large telescope for wide-field imaging requires additional optics in front of the camera.
  - ***Prime focus correctors*** are high performance refractive elements for this purpose.
- ***Tertiary mirrors***:
  - Nasymth (alt-az) and coude (equatorial).
  - Stationary focal point.



(Nasmyth/Coude optics; CC)

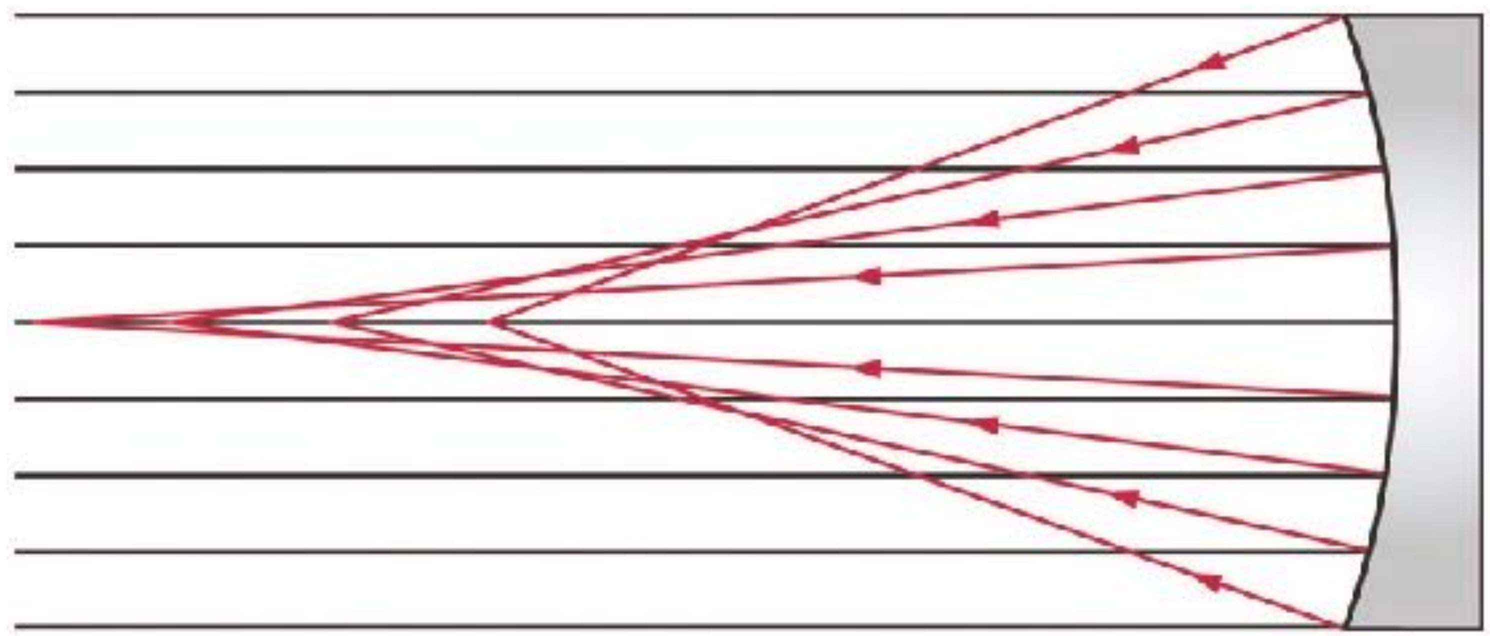


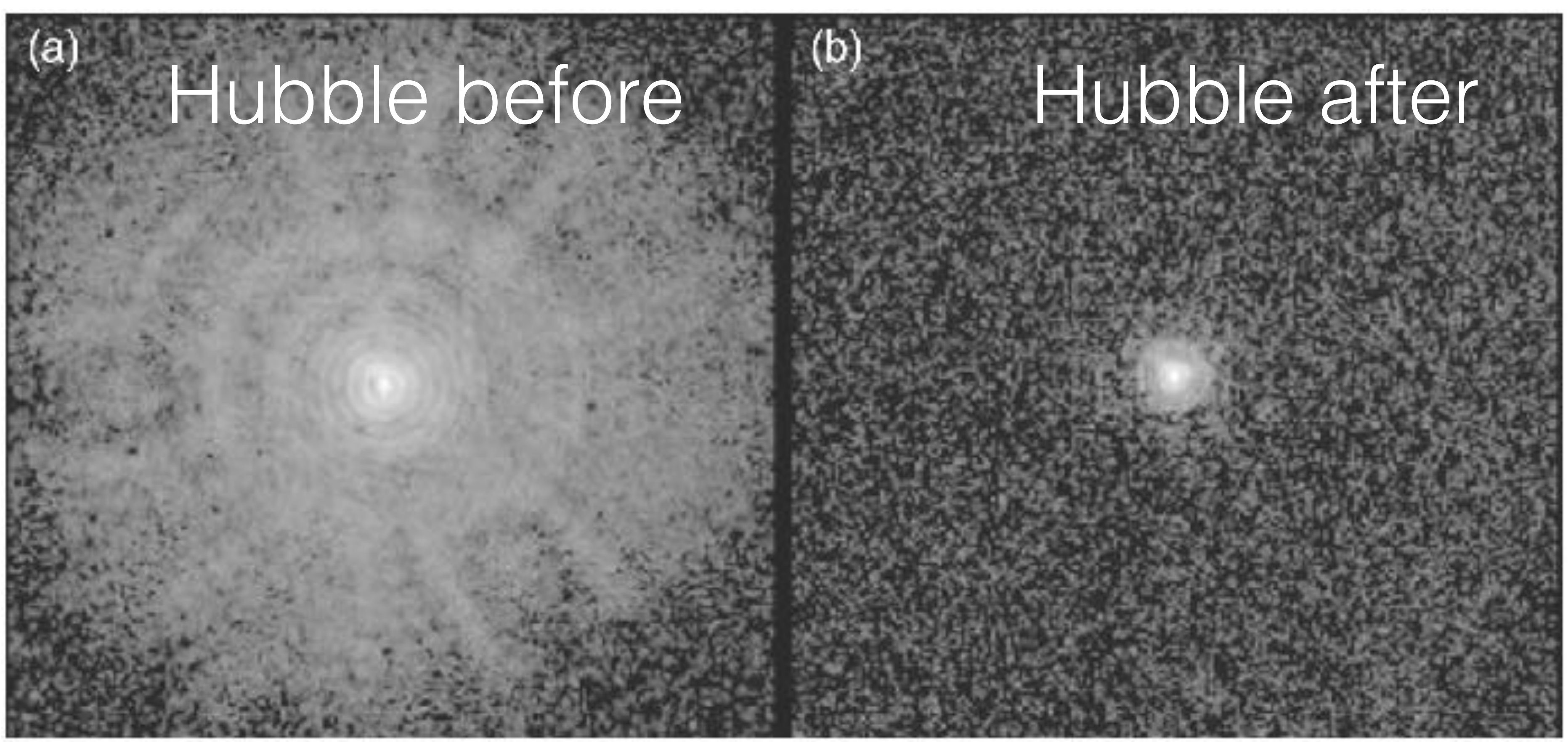
Optical Aberrations



Spherical aberration

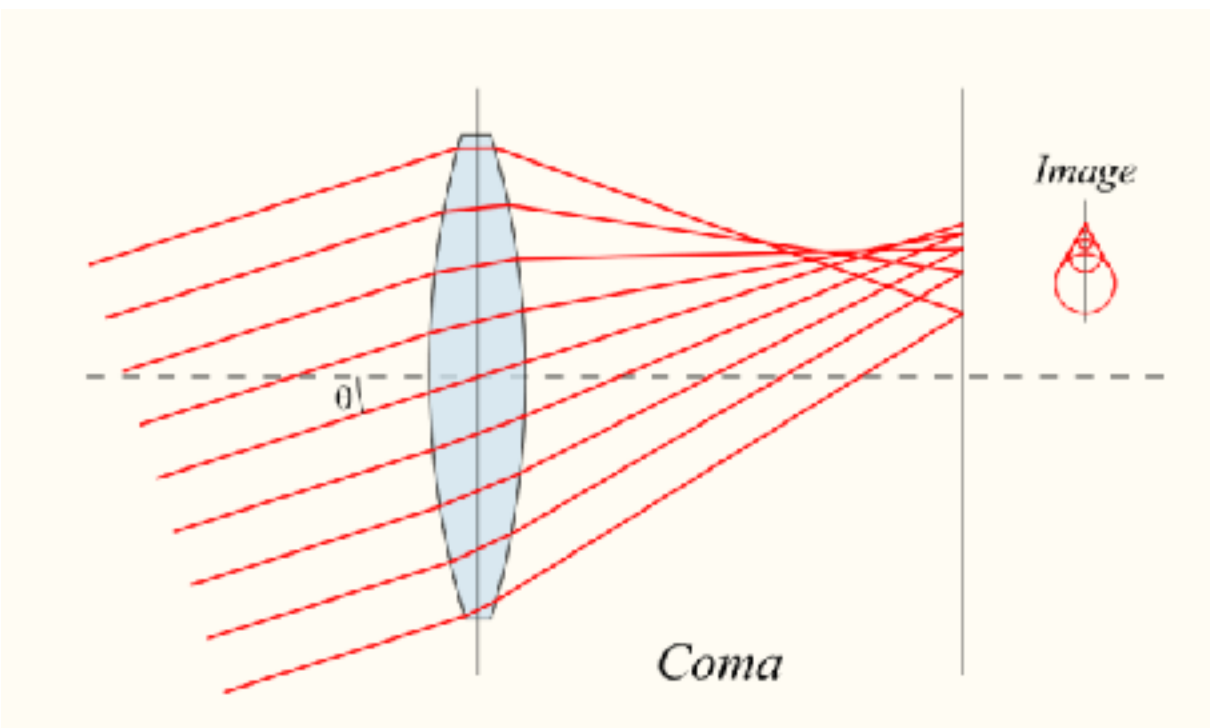
e.g. spherical reflectors
Off-axis rays don’t trace properly to form an image
produces blurred halo around image





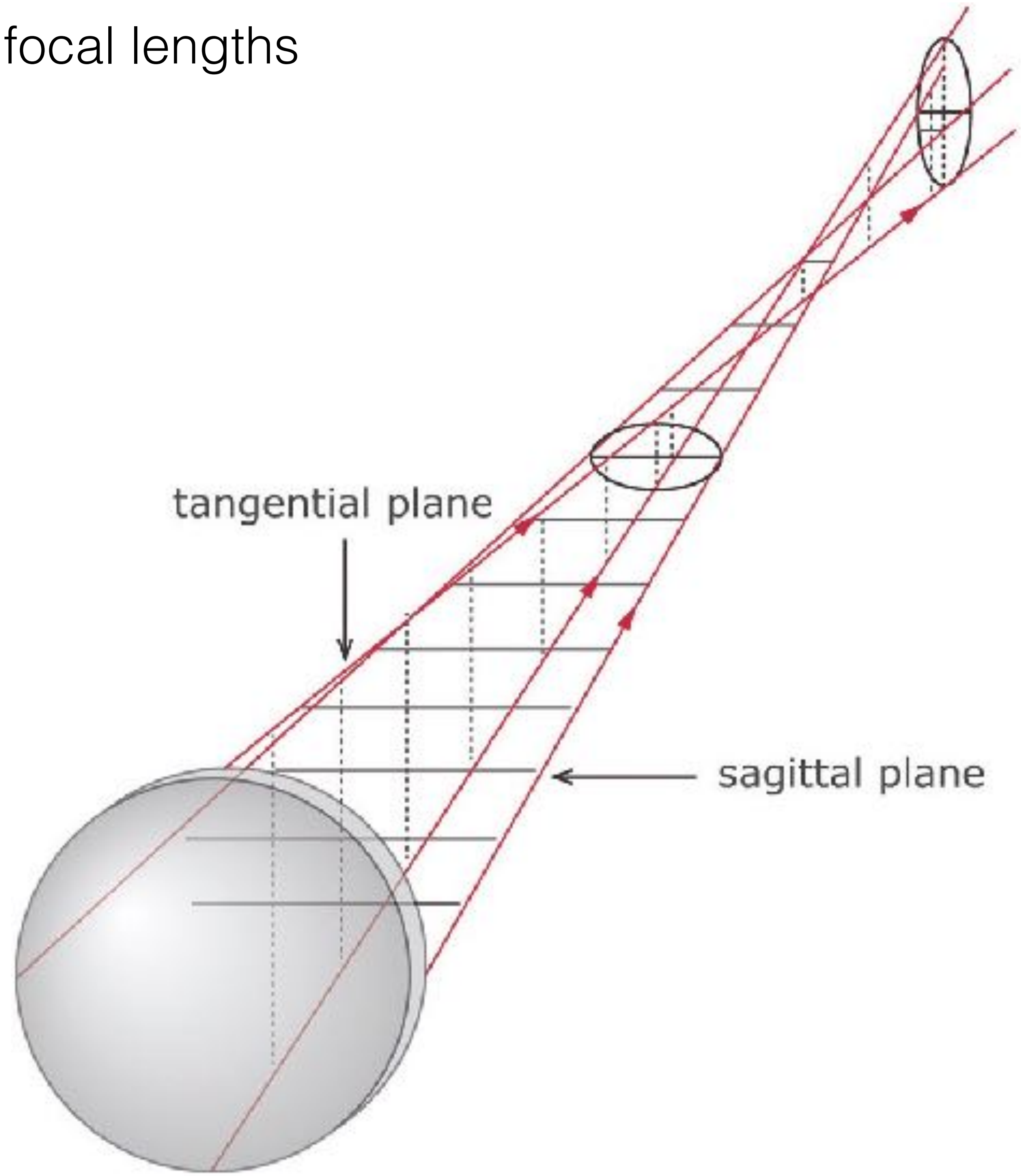
Coma

e.g. paraboloid
Off-axis rays don't trace properly to form an image
produces blurred halo around image



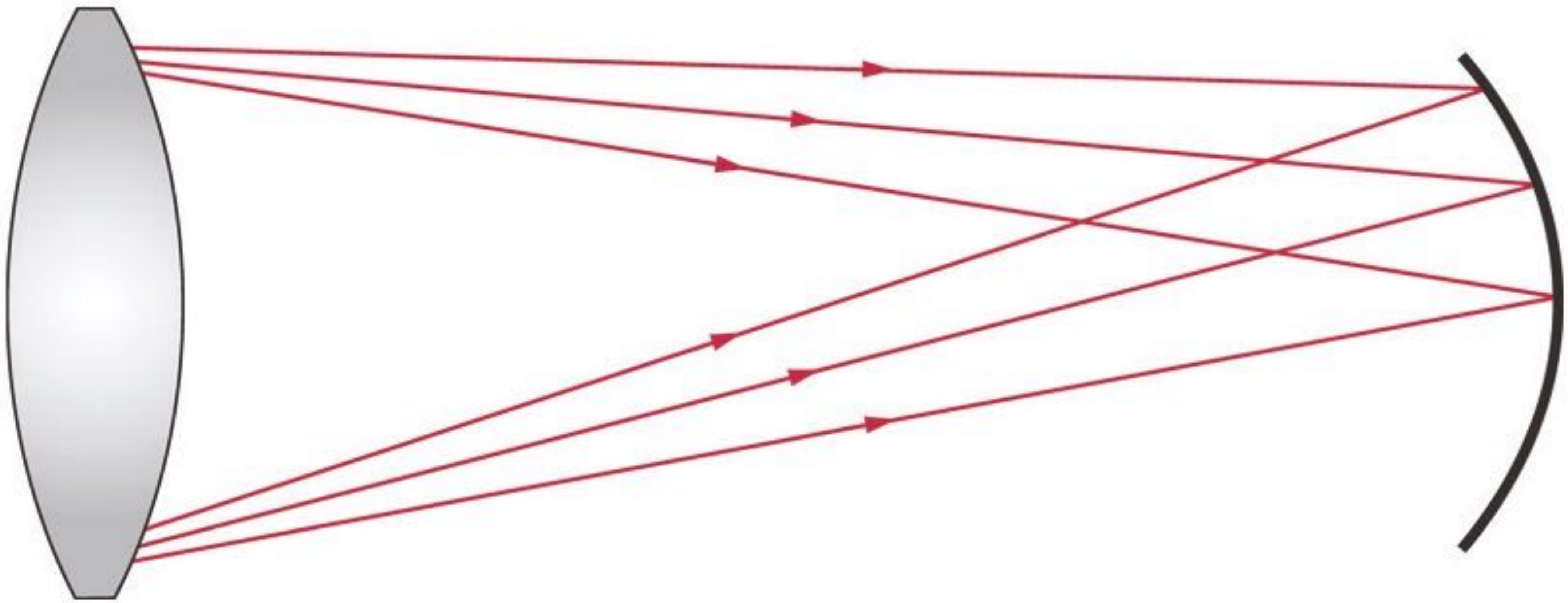
Astigmatism

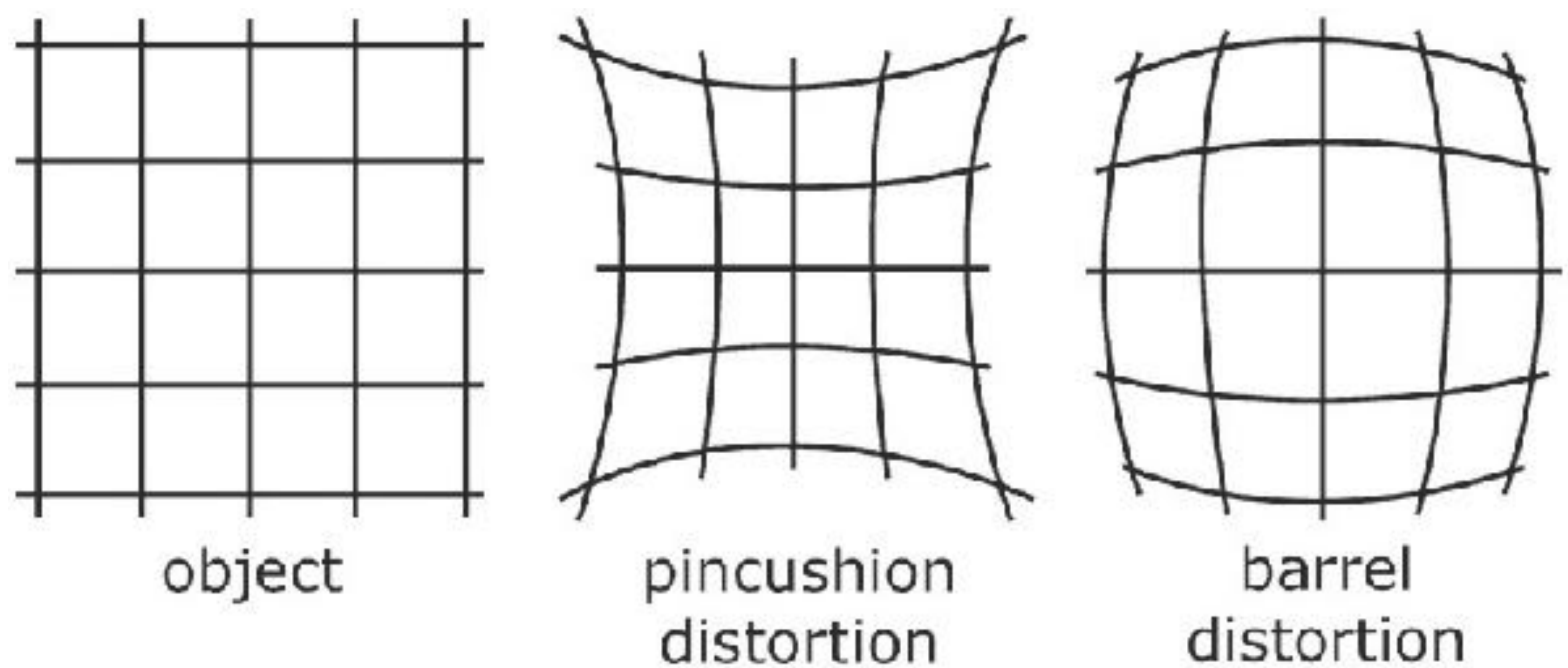
x- and y- planes have different focal lengths



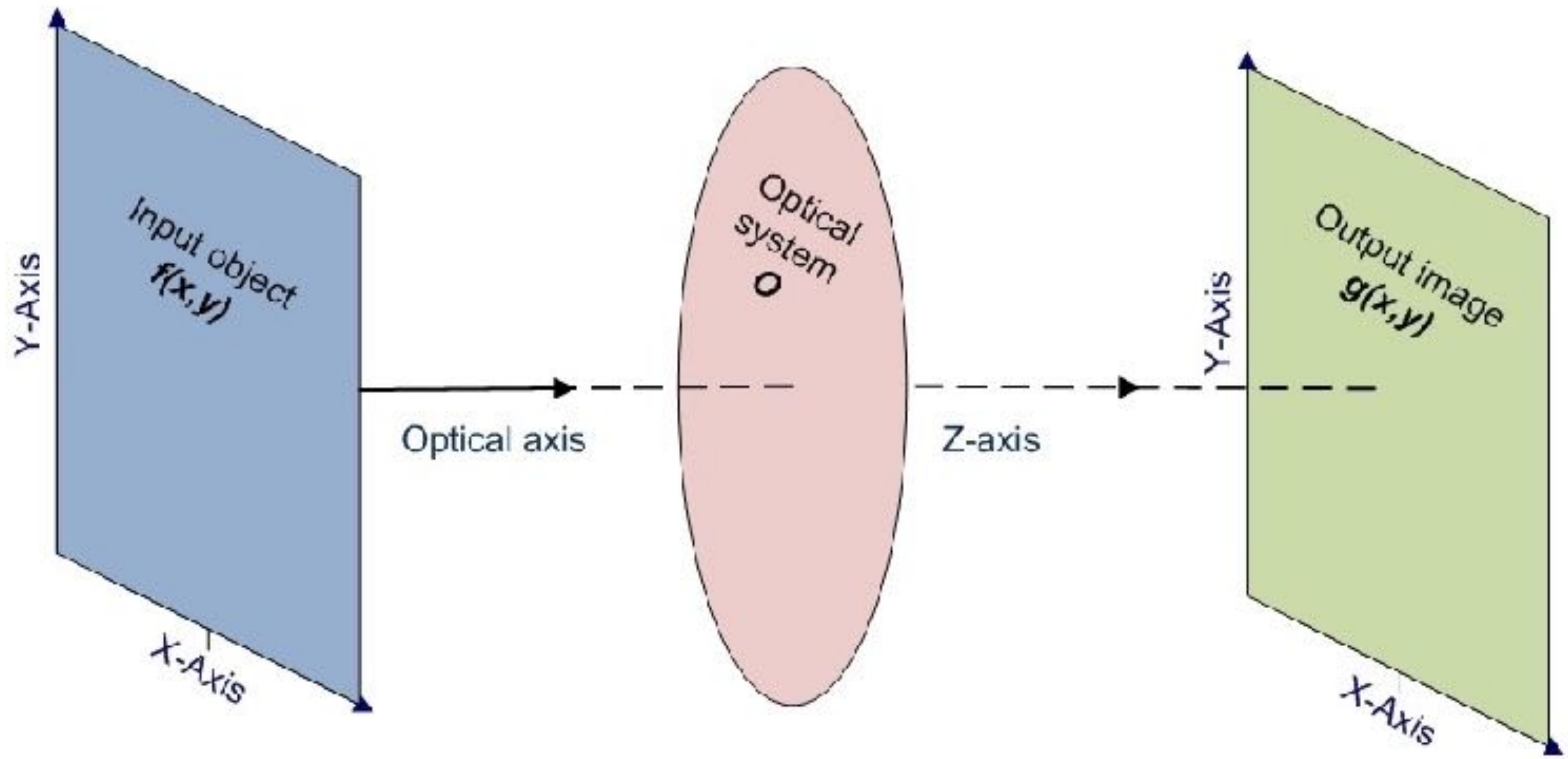
Curvature of field

Focal plane is curved.



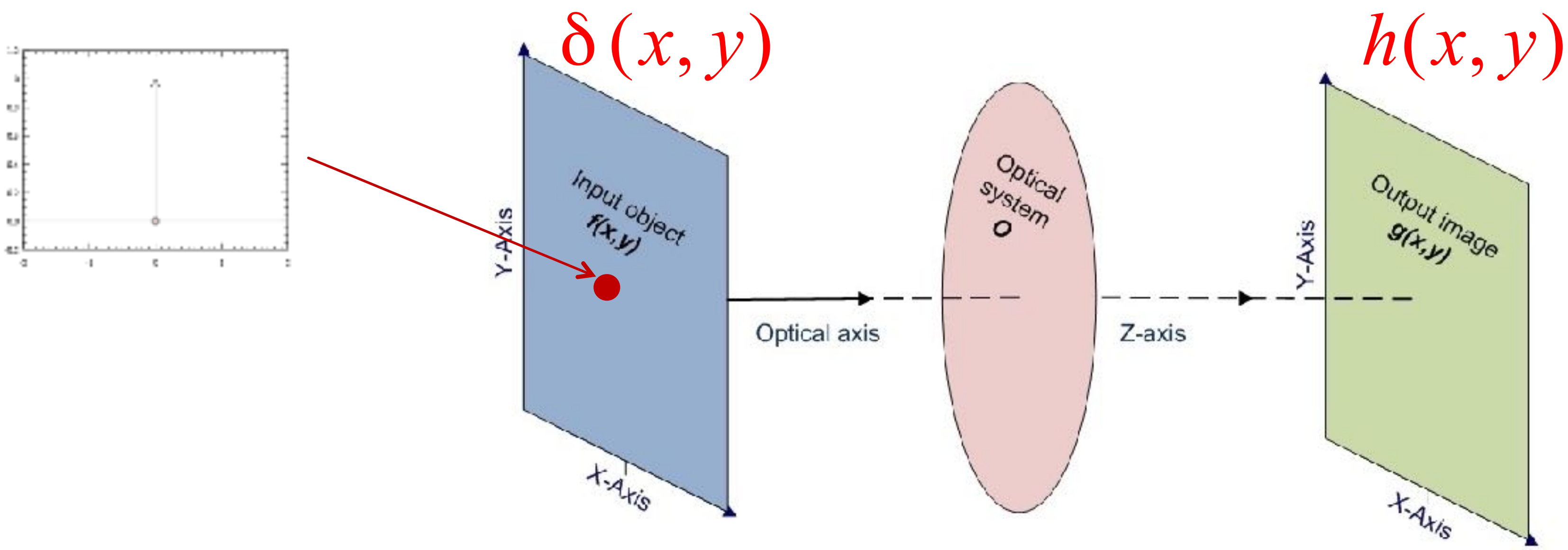


Imaging systems



- Can represent the action of 2-d imaging system by a general ***imaging operator g(x,y)=O(f(x,y))***
- An important component of our overall data model.

The point-spread function (PSF)



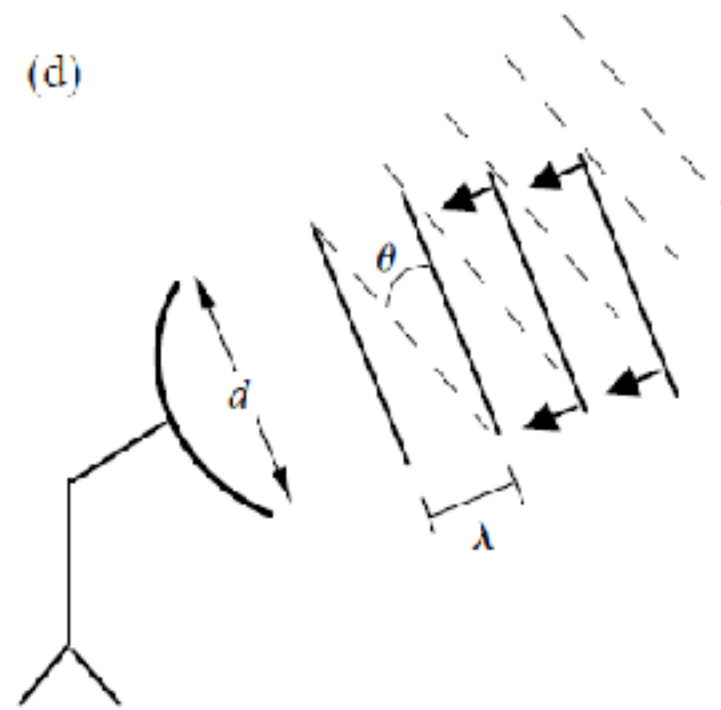
$$g(x, y) = O\{f(x, y)\}$$

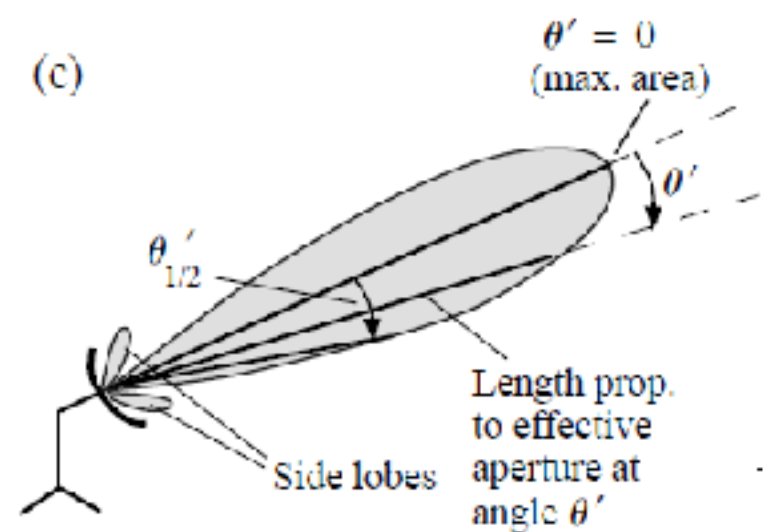
- The response of the imaging operator to a point source impulse function (Dirac delta function) at the origin:

$$h(x, y) = O\{\delta(x, y)\}$$

Telescope angular response function

- In practice, telescopes have finite angular resolution.
- ***Why is the PSF not a delta function ?***
  - ***Diffraction:***
    - Due to the wave nature of electromagnetic radiation
    - ***Not*** included in the ***geometric optics approximation*** (i.e. the *light ray* model)
  - ***Propagation effects*** (e.g. refractive turbulence along incident path).
  - ***Optical errors***
  - ***Scattering***
- We will denote as ***P(θ',ϕ',ν),*** where ***(θ',ϕ')*** are measured relative to the optical axis (here assumed the same as pointing direction):
  - ***Caution: - need to take care with units of response function:***
    - ***Acting on electric field of incident EM wave, E, or***
    - ***Acting on intensity of incident EM wave |E|² (power pattern)***

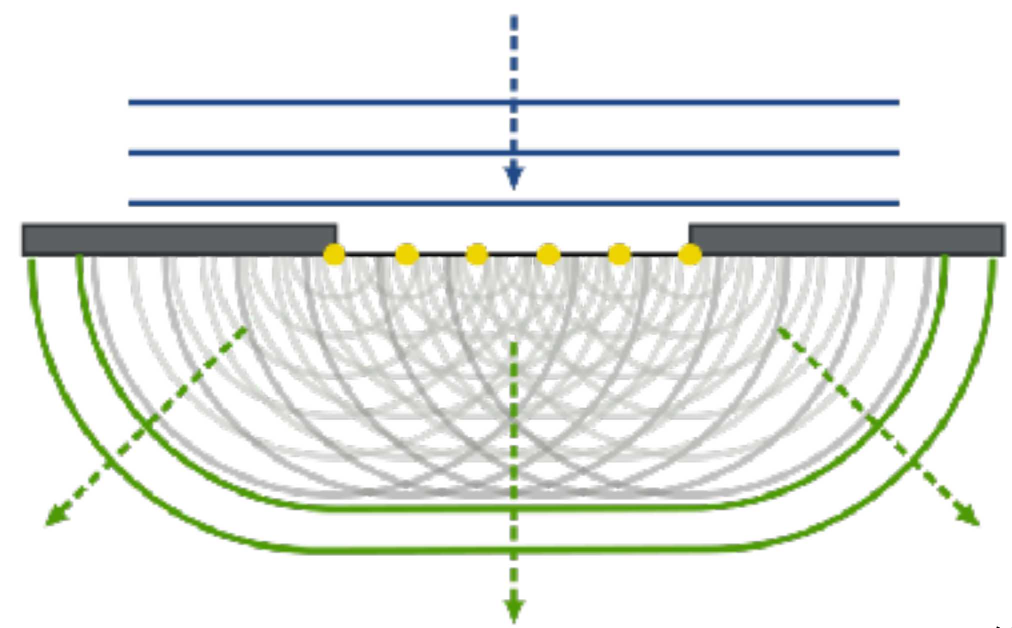




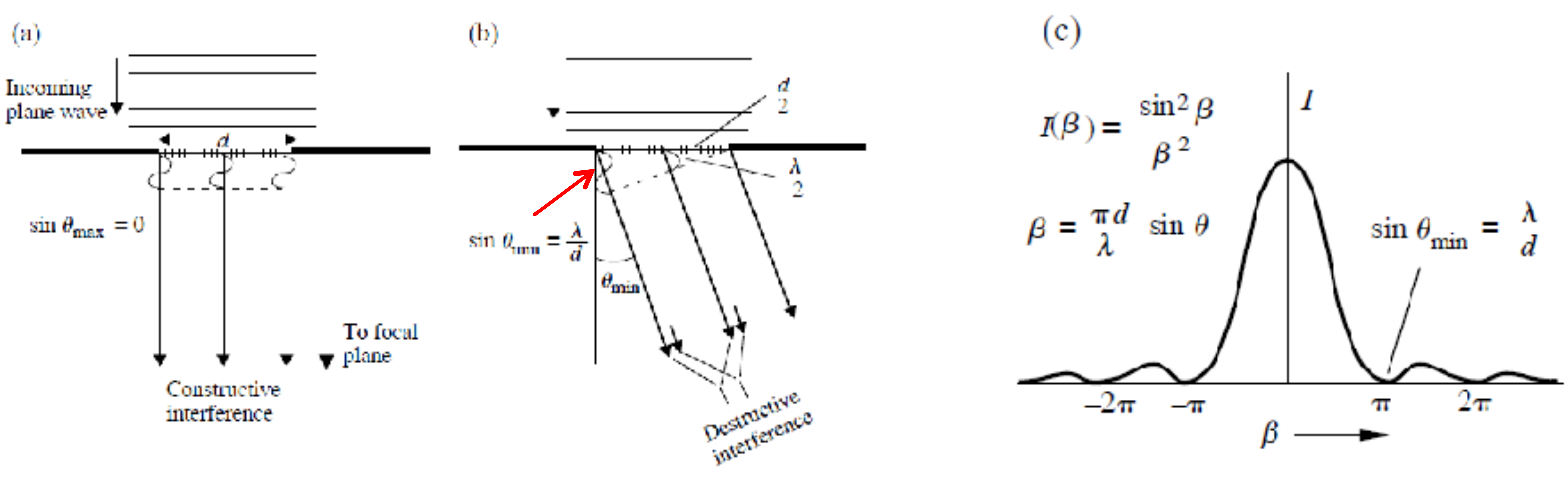
(Bradt 2004)

Diffraction – an intuitive approach

- Diffraction places the intrinsic limit on angular resolution
- The ***Huyghens-Fresnel*** principle:
  - *Each point on an advancing wavefront becomes the source of a secondary Huyghens wavelet that radiates spherically.*



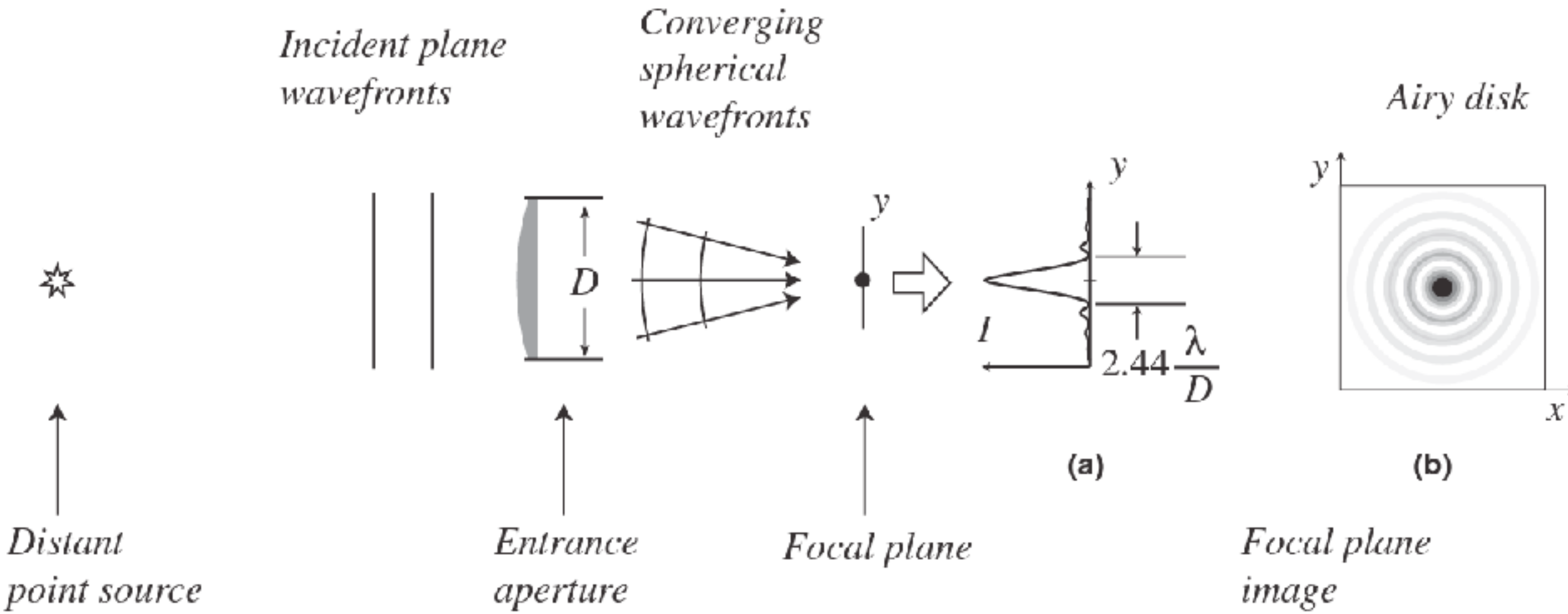
(CC)



(Bradt 2004)

(Bradt 2004)

Diffraction limit



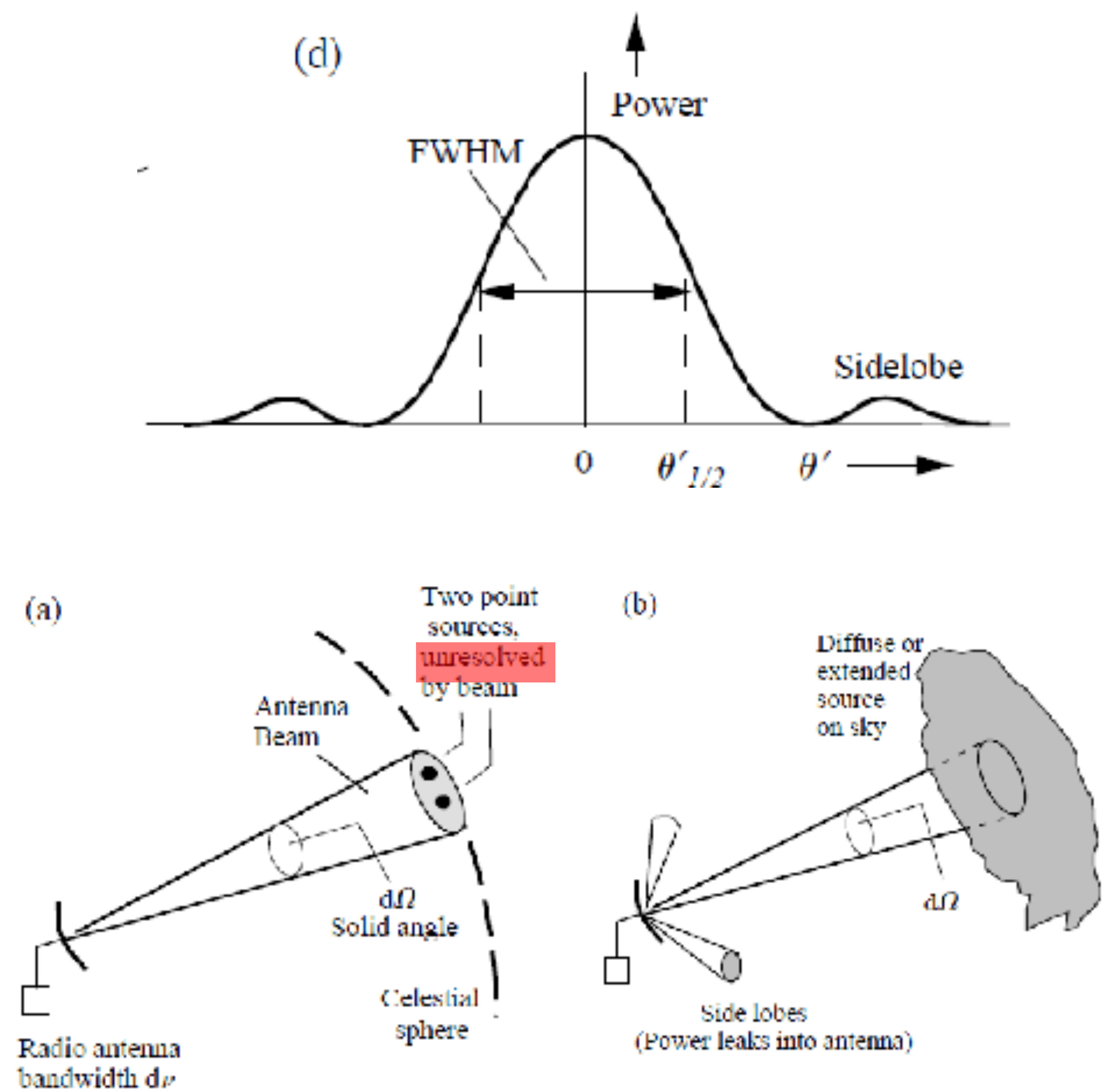
$$\theta \simeq 1.22\frac{\lambda}{D}$$

optical accuracy requirements

RMS < λ/10

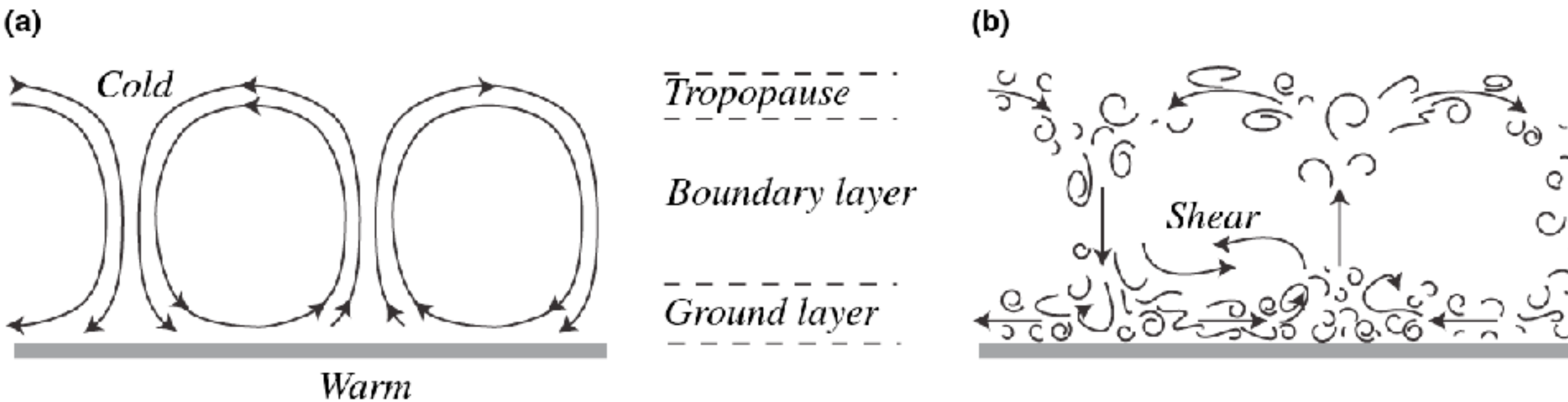
PSF properties

- A beam pattern (or PSF) has some key qualitative properties:
  - ***Beam solid angle*** – the total solid angle of the beam.
  - ***Main lobe*** (or main beam) – central lobe of the response pattern.
  - ***Angular resolution*** – a measure of the angular width of the main lobe;
    - Also, the minimum ***angular separation*** that can be resolved between two point sources.
  - ***Sidelobes*** – angular response lobes outside the main lobe.
  - ***Efficiency***
    - The ratio of the main lobe solid angle to the total beam solid angle.



(Bradt 2004)

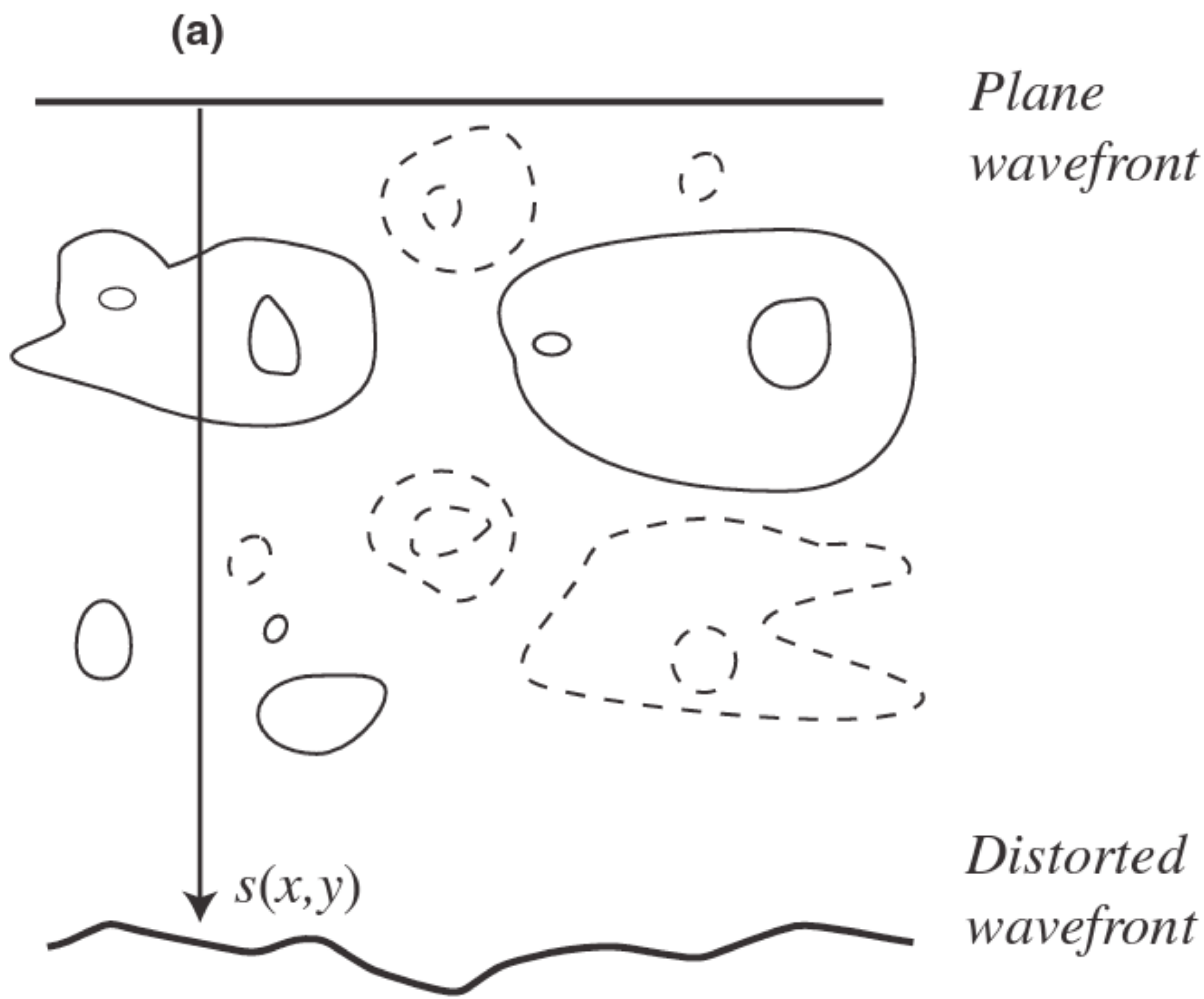
Atmosphere



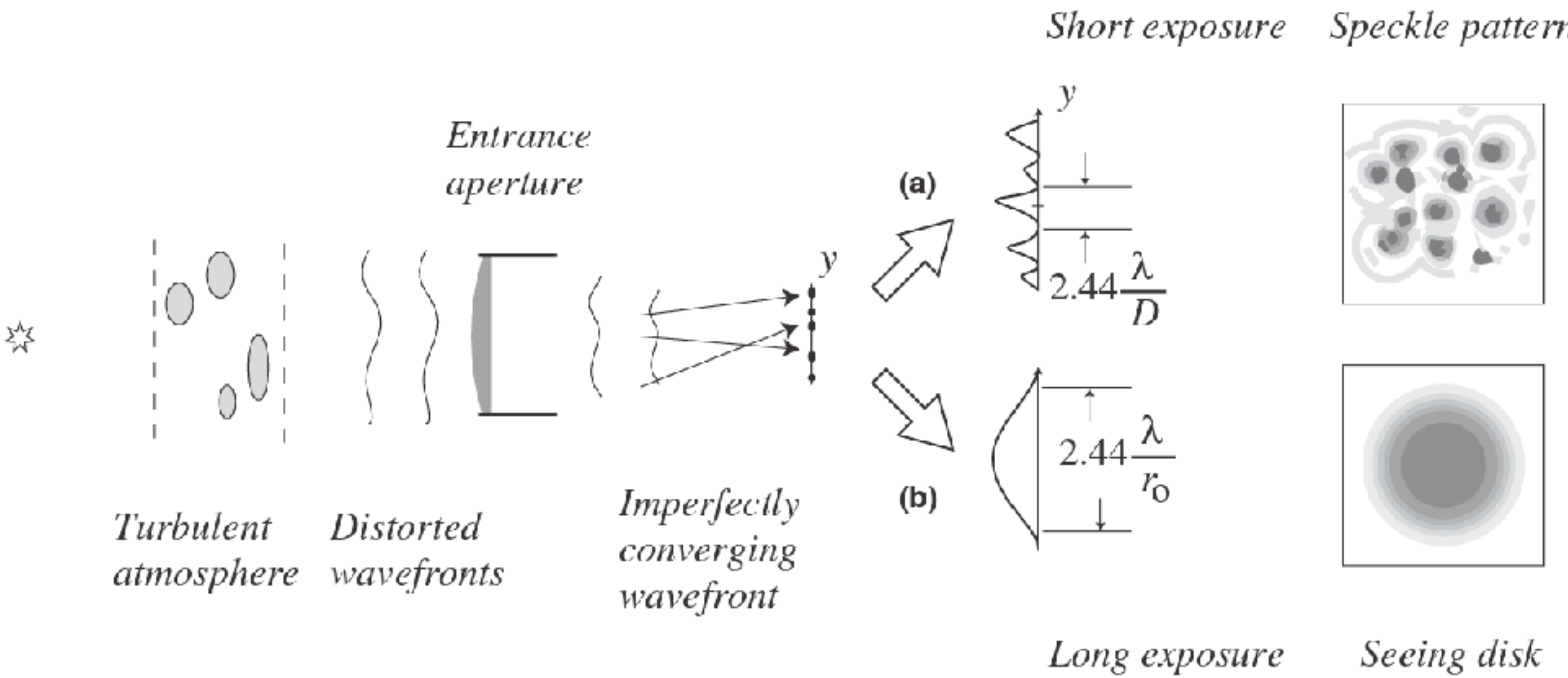
no turbulence

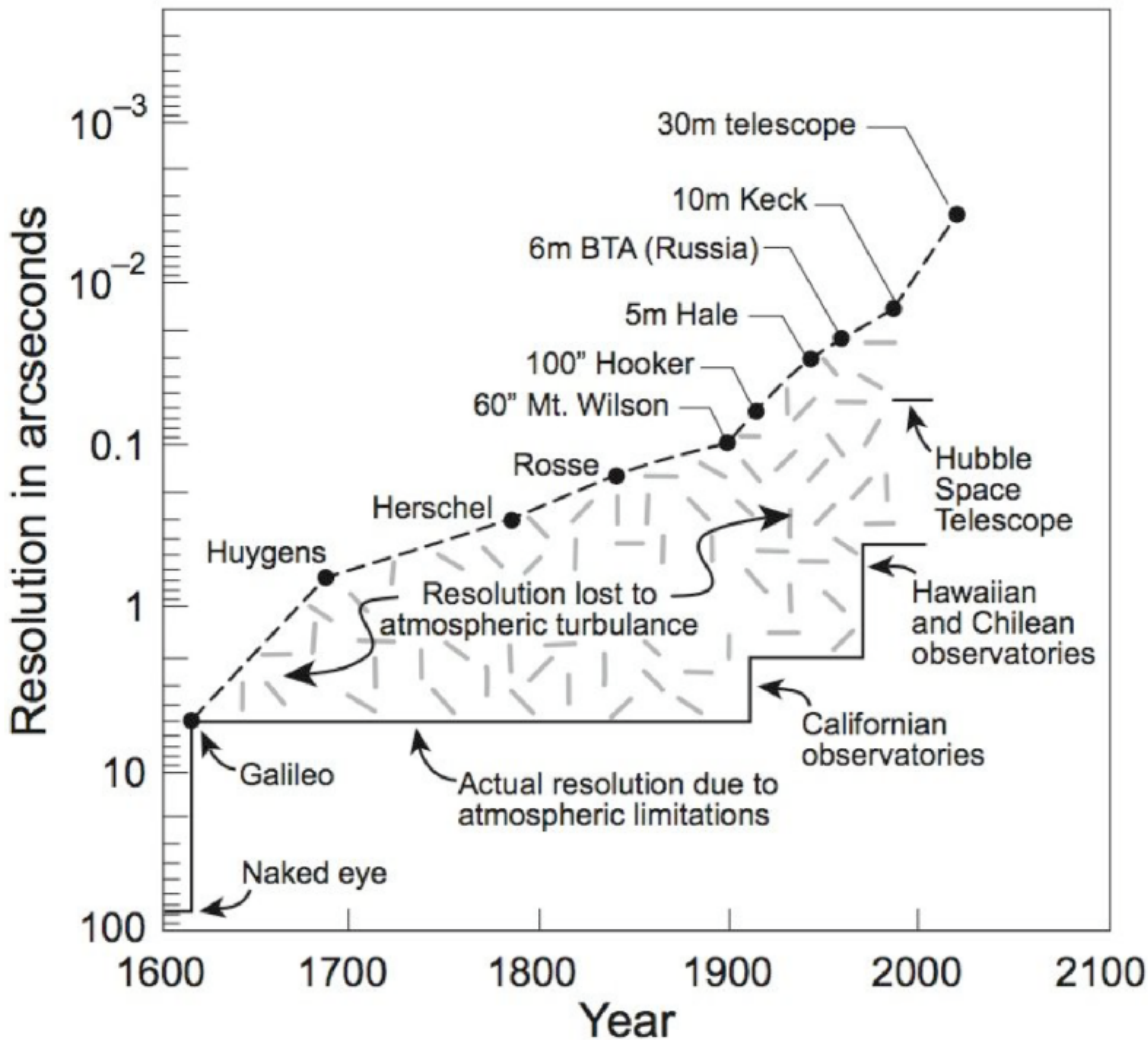
turbulence

~1 km height

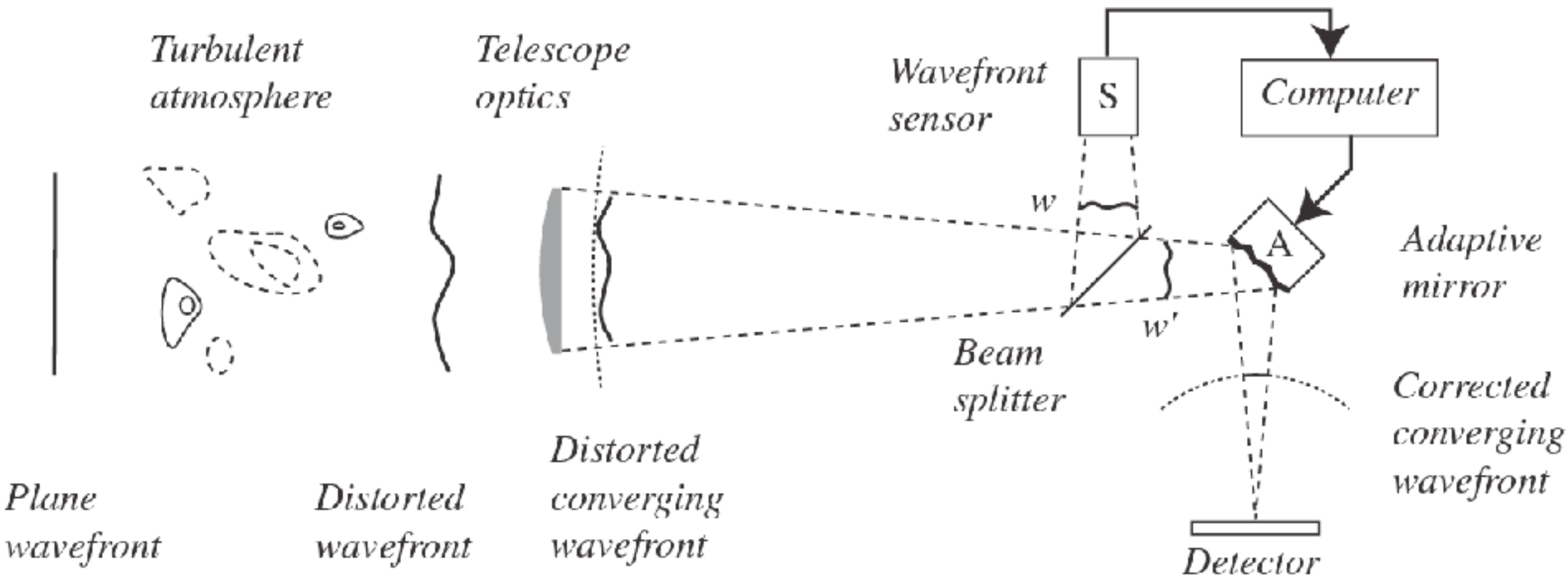

(a)
Plane
wavefront
s(x,y)
Distorted
wavefront

seeing

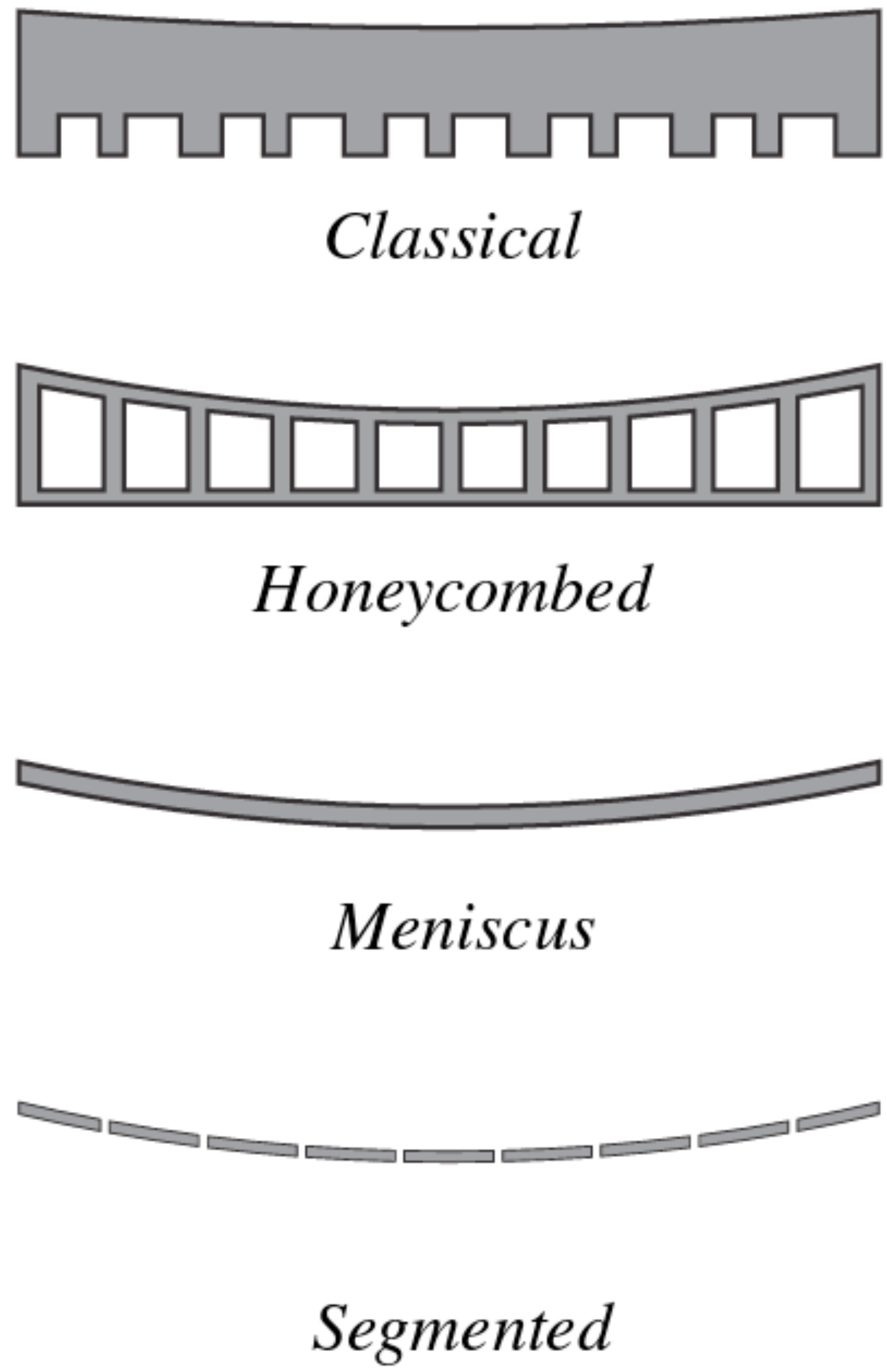

Short exposure
Speckle pattern
Entrance aperture
(a)
y
y
2.44 λ/D
2.44 λ/r0
(b)
Turbulent atmosphere
Distorted wavefronts
Imperfectly converging wavefront
Long exposure
Seeing disk



30m telescope
10m Keck
6m BTA (Russia)
5m Hale
100" Hooker
60" Mt. Wilson
Rosse
Herschel
Huygens
Resolution lost to atmospheric turbulance
Hubble Space Telescope
Hawaiian and Chilean observatories
Californian observatories
Galileo
Actual resolution due to atmospheric limitations
Naked eye
Resolution in arcseconds
10^{-3}
10^{-2}
0.1
1
10
100
1600
1700
1800
1900
2000
2100
Year


Turbulent atmosphere
Telescope optics
Wavefront sensor
S
Computer
w
A
Adaptive mirror
w'
Beam splitter
Corrected converging wavefront
Distorted converging wavefront
Plane wavefront
Distorted wavefront
Detector

Modern mirrors



Two-dimensional Fourier transforms

A direct generalization from 1-d:

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(\xi,\eta)\, e^{-i2\pi(x\xi + y\eta)} d\xi\, d\eta$$

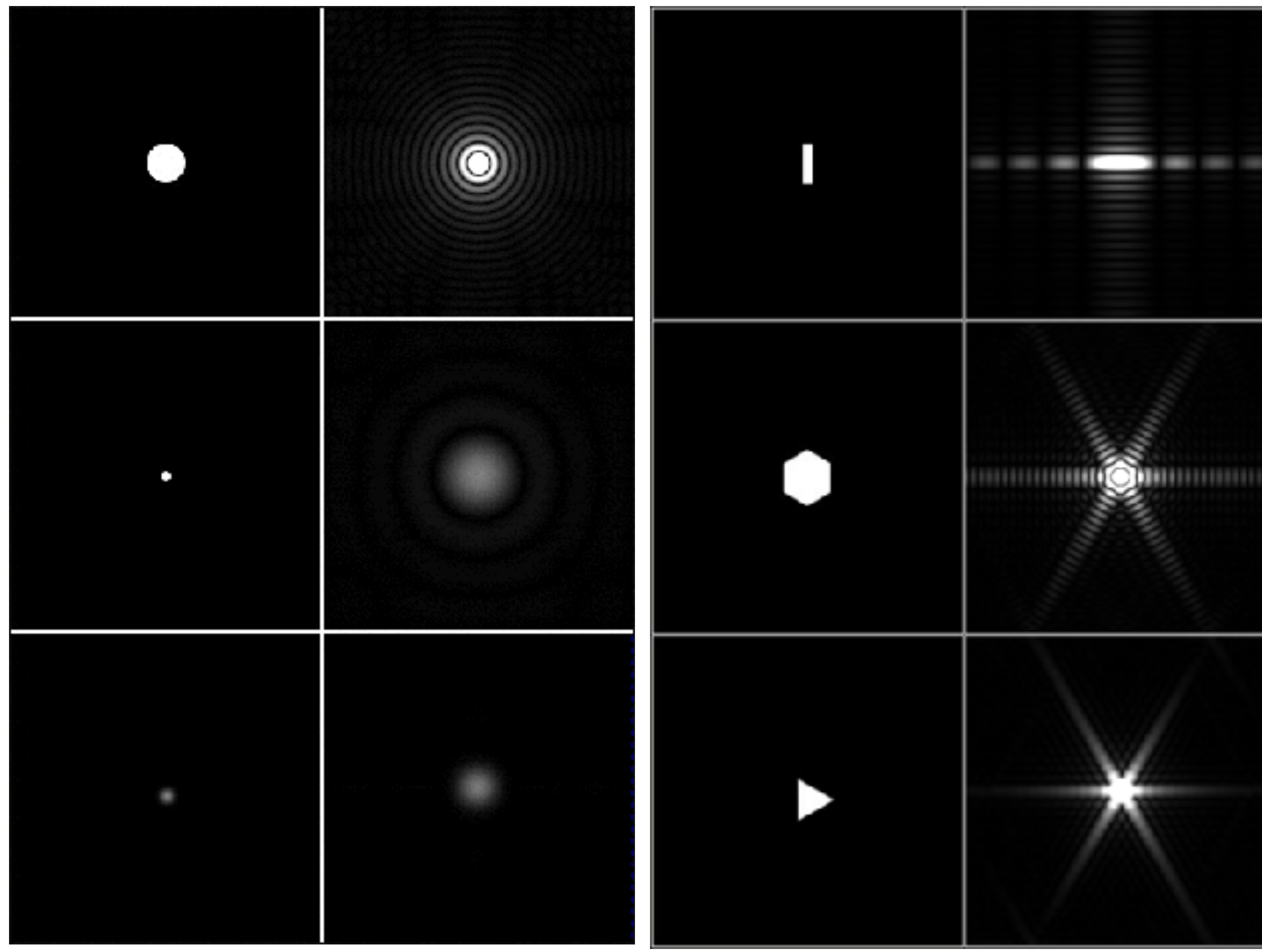
$$F(\xi,\eta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, e^{+i2\pi(x\xi + y\eta)} dt\, ds$$

In vector form:

$$\mathrm{f}(\vec{x}) = \int\int_{-\infty}^{\infty} \mathrm{F}(\vec{s})\ e^{i2\pi\vec{s}\cdot\vec{x}}\ d^2\vec{s}$$

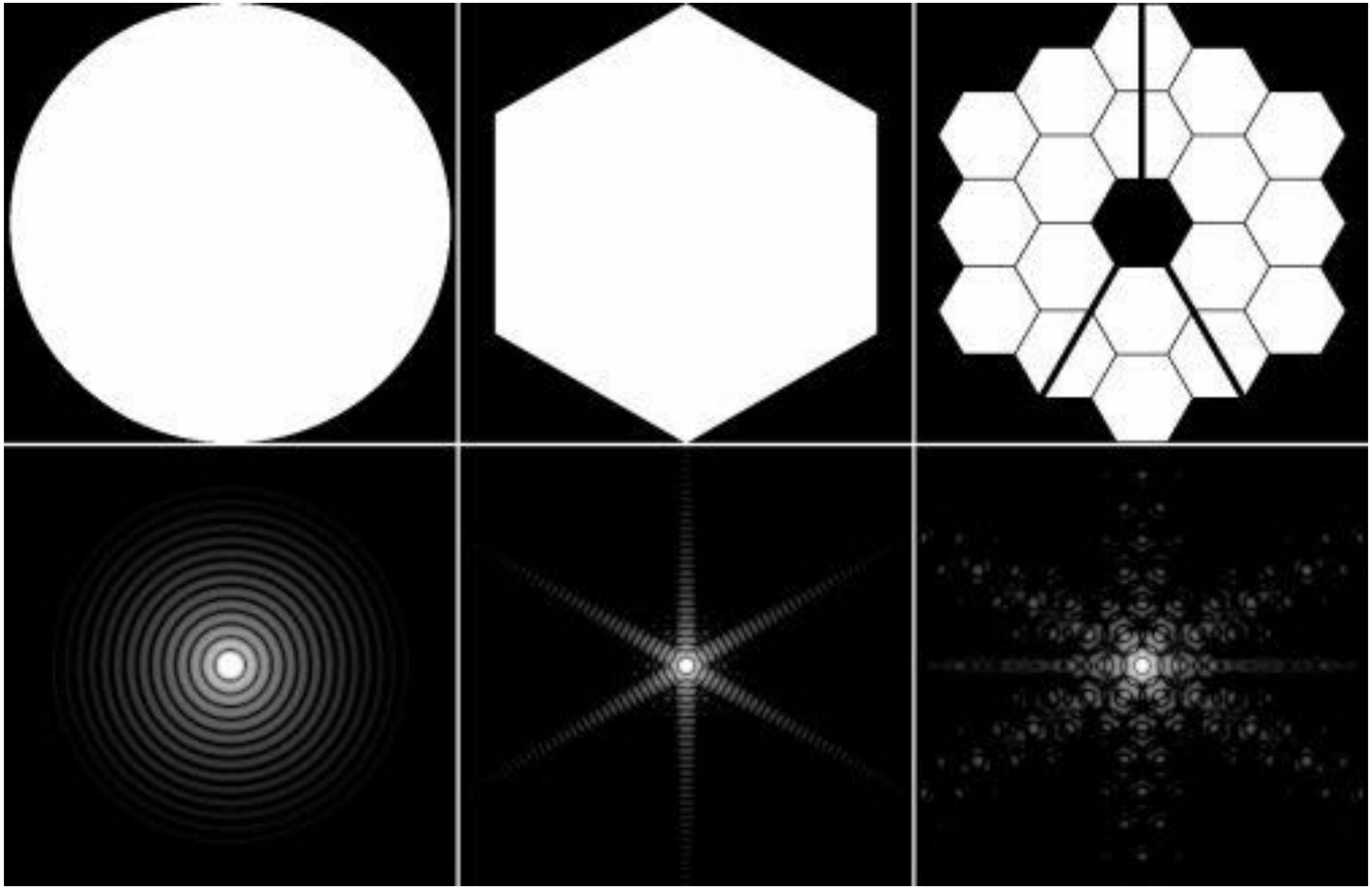
$$\mathrm{F}(\vec{s}) = \int\int_{-\infty}^{\infty} \mathrm{f}(\vec{x})\ e^{-i2\pi\vec{s}\cdot\vec{x}}\ d^2\vec{x}.$$

Can simplify greatly if separable or symmetric.



(CC)

Sample PSFs



(NASA/JWST)