Adaptive Control of Nonlinear TRMS Model by Using Gradient Descent Optimizers

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***Abstract*— This study demonstrates an application of direct gradient descent control for adaptively control of a nonlinear stable system models. The approach is based on utilization of gradient descent optimization techniques for the synthesis of control signals to control a specific plant model. In a former work, gradient descent optimizers were designed by considering a first degree instant input-output relation model assumption of the controlled system and this can allow model-independent adaptive control of a class of plant models that can approximate to first order stable plant dynamics. The current study is an extension of this scheme for the purpose of nonlinear adaptive control. Here, we consider a higher degree polynomial assumption of instant input-output relations of the controlled system to obtain gradient descent optimizers that can be applied for adaptive control of a class of nonlinear systems. For evaluation of control performance of gradient descent optimizers, it is applied for the control of nonlinear TRMS model and the results are compared with performance of conventional PID control.**

***Index Terms*—Gradient descent method, control, nonlinear systems, TRMS**

I. Introduction

Intelligent control, which is implementing computational intelligence and optimization techniques, promises more efficient and smart control of complicated real world systems. Intelligent control systems can exhibit the capability of adapting itself for the changing dynamics of real world conditions [1-2]. Due to increasing complexity of systems and rising demand of efficiency and adaptation skills in daily living environments, it is getting more and more substantial for the practical control systems to exhibit robust performance under the change of operating conditions such as ageing of system components, altering environmental conditions, hardware failures, unpredictable disturbances and noise [2]. Classical control structures are static and model-dependent and therefore adaptation skill of these systems is restricted by limitations of mathematical modelling techniques. To decrease model dependency of controllers and increase adaptation capability of control systems, intelligent control systems should employ machine learning, computational intelligence and optimization techniques to improve control performance in real world applications. Main objective of intelligent control systems should be (i) reducing dependency to mathematical model of the controlled system, (ii) increasing adaptation skills of control system to maintain robust control performance, (iii) increasing control efficiency and optimality by employing learning and optimization techniques. These three objectives of intelligent control lead to emergence of new generation of control systems, which can be more effective, efficient and robust in real-world control performance compared to classical counterparts.

Main trends in intelligent control topic can be summarized as: Artificial neural network were employed to implement intelligent control aspects [3-6], metaheuristic methods are utilized to perform auto-tuning of control systems [7-9], model based adaptive control approaches were studied [10-13] and model predictive control are developing topic of intelligent control [14]. Some of these methods aim auto-tuning of controller parameters of a classical control structure to maintain control performance under varying conditions. However, this online controller parameter tuning attempts without ensuring system stability may come out the risk of transient instability and instantaneous control performance degradations while updating the controller coefficients. On the other hand, some of these methods can perform an improved control performance for a determined mathematical model of controlled system. Such a model dependency may affect control performance depending on

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the consistency of mathematical modelling in term of representing real world systems. To eliminate these two weaknesses, instead of tuning conventional controller parameters for a definite plant model, an intelligent control system should directly synthesize control signals with respect to very recent representation of the controlled system [3]. In this sense, model predictive control approach presents advantage of determining an optimal control signal by considering near future predictions of control system states. Although this approach can provide successful results, model predictive controllers may not be a low-cost solution in term of computation complexity for the control of highly complex real system. Therefore, in the present study, we investigated control signal generation according to an estimate of the current state of input-output relations of the controlled systems, which can significantly reduce computation burden.

The current study illustrates an application of gradient descent optimizer bank that is constructed for fourth-degree polynomial assumption of instant input-output relation of the controlled system. Gradient descent optimization is one of famous nonlinear programming techniques, which finds application in computer engineering such as neural networks [15-16], model extracting [17-19], adaptive control [2,3,10-11] and prediction [20]. In a previous work, adaptive control approach based on gradient descent optimizer design was applied for control of closed loop control of an experimental electrical rotor [2]. This discrete time controller structure was composed of two gradient descent optimizers; one was referred to as control optimizer that generates control signal, and the other was adaptation optimizer that estimates state of the first degree instant input-output relation of controlled system [2]. In the present study, we derive continuous time gradient descent optimizer bank (collection) for fourth-degree polynomial instant input-output relation assumption and these optimizers are numerically implemented for the control of a nonlinear TRMS model. Results are compared with the results of a well-tuned PID controller and advantage and disadvantages of the method are discussed, briefly.

II. THEORETICAL BACKGROUND

A. Preliminaries for Adaptive Control by Gradient Descent Optimizers

This section presents a brief summary of adaptive control structure that was implemented by gradient descent optimizers in [2]. Figure 1 shows a basic schematic of this control structure.

The first degree instant input-output relation of a plant function for the moment t is expressed as [2],

$$y_s(t) = k(t)u(t) \,. \tag{1}$$

The error function for gradient descent optimizers is taken the squared difference between reference and system output:

$$E = \frac{1}{2}e(t)^2 = \frac{1}{2}(r(t) - y_s(t))^2 \tag{2}$$

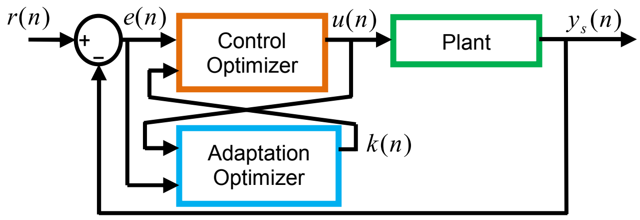

Figure 1. Block diagram of gradient descent optimizer control [2]

The control optimizer, which generates the control signals to minimize error function $E(t)$, is written as,

$$\frac{\partial u}{\partial t} = -\eta_c \frac{\partial E}{\partial u} \,, \tag{3}$$

The adaptation optimizer, which estimates $k(t)$ parameter of the first degree input-output relation $y_s(t) = k(t)u(t)$ to minimize error function $E(t)$, was written as,

$$\frac{\partial k}{\partial t} = -\eta_a \frac{\partial E}{\partial k} \,, \tag{4}$$

When equations (3) and (4) are solved numerically in discrete time to have an iterative solutions with a sampling of $t = nT_s$, the update rules of gradient descent optimizers for a first degree input-output relation model was obtained in [2]

$$u(n+1) = u(n) + \eta_c k(n)e(n) \,, \tag{5}$$

$$k(n+1) = k(n) + \eta_a u(n)e(n) \,, \tag{6}$$

where parameter η_c is the learning rate for control optimizer expressed by equation (5) and parameter η_a is the learning rate for the adaptation optimizer that is expressed by equation (6).

B. Continuous Time Gradient Descent Optimizers for Forth Degree Instant Input-Output Relation Class

The fourth-degree instant input-output relation of a plant function at the instant t can be expressed in polynomial form as

$$y_s(t) = k_1(t)u(t) + k_2(t)u(t)^2 + k_3(t)u(t)^3 + k_4(t)u(t)^4 \,, \tag{7}$$

where $k_1(t)$, $k_2(t)$, $k_3(t)$ and $k_4(t)$ are first, second, third and fourth-degree dynamics adaptation gains, respectively. These high order adaptation gains are particularly proposed for estimation of nonlinear components that are effective in instantaneous input-output relations of the plant function.

The dynamics of control optimizer to synthesize the control signals, which can minimize the error function $E(t)$, can be written as

$$\frac{\partial u}{\partial t} = -\eta_c \frac{\partial E}{\partial u} \,, \tag{8}$$

The dynamics of adaptation optimizers, which estimate $k_1(t)$, $k_2(t)$, $k_3(t)$ and $k_4(t)$ to minimize $E(t)$, can be written as

$$\frac{\partial k_1}{\partial t} = -\eta_{a1} \frac{\partial E}{\partial k_1} \,, \tag{9}$$

$$\frac{\partial k_2}{\partial t} = -\eta_{a2} \frac{\partial E}{\partial k_2}, \tag{10}$$

$$\frac{\partial k_3}{\partial t} = -\eta_{a3} \frac{\partial E}{\partial k_3}, \tag{11}$$

$$\frac{\partial k_4}{\partial t} = -\eta_{a4} \frac{\partial E}{\partial k_4}, \tag{12}$$

where coefficients η_{ai}, $i = \{1,2,3,4\}$ are the learning rates of adaptation optimizers. By considering equation (7), one can write

$$\frac{\partial y_s}{\partial u} = \sum_{i=1}^{4} i.k_i(t).u(t)^{i-1} \text{ and } \frac{\partial y_s}{\partial k_i} = u(t)^i. \tag{13}$$

Then, by considering equation (2), one writes $\frac{\partial E}{\partial u} = -\frac{\partial y_s}{\partial u} e$ and $\frac{\partial E}{\partial k_i} = -\frac{\partial y_s}{\partial k_i} e$ and solutions of optimizers in continuous time can be obtained as

$$u(t) = \eta_c \int (k_1(t) + 2k_2(t)u(t) + 3k_3(t)u(t)^2 + 4k_4(t)u(t)^3) e(t)dt \tag{14}$$

$$k_1(t) = \eta_{a1} \int u(t)e(t)dt \tag{15}$$

$$k_2(t) = \eta_{a2} \int u(t)^2 e(t)dt \tag{16}$$

$$k_3(t) = \eta_{a3} \int u(t)^3 e(t)dt \tag{17}$$

$$k_4(t) = \eta_{a4} \int u(t)^4 e(t)dt \tag{18}$$

These solutions can be implemented in a closed loop control system by using control error $e(t)$ and control signal $u(t)$. Figure 2 shows block diagram for the implementation of this control structure in a closed loop control system.

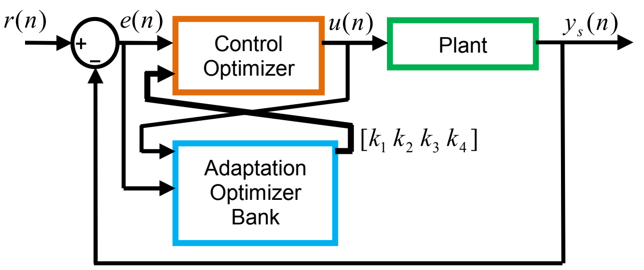

Fig. 2. Block diagram of gradient descent optimizer control for fourth-degree instant input-output relation

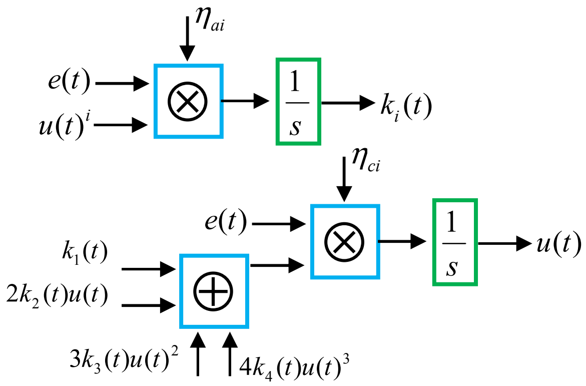

Fig. 3. Block diagram for implementation of adaptation optimizers (upper block diagram) and control optimizer (bottom block diagram)

Figure 3 shows a block diagram that illustrates implementation of adaptation optimizers and the control optimizer by using the integrator operator $\frac{1}{s}$.

The following section presents a numerical study that is illustrating an application of this control structure for the control of a nonlinear model.

III. NUMERICAL STUDY

Figure 4 shows a TRMS rotor control test platform that was produced by Feedback Instruments Inc. for experimental research studies [21,22]. It allows vertical and horizontal position control of twin-rotor system [21]. We used gradient descent optimizers for the control of hovering action. Here, the pitch rotor angle θ was controlled by the gradient descent optimizers. We used nonlinear Matlab/Simulink model of this system to perform multi-input multi-output (MIMO) control simulations of the electrical rotor in Figure 5. Since hovering control of the pitch rotor is a level control type problem, which mostly requires integrator control actions, the gradient descent optimizers are inherently suitable for controlling the hover action of the pitch rotor. However, we observed that the gradient descent optimizers are not so successful in control of higher order time-delay or unstable plants.

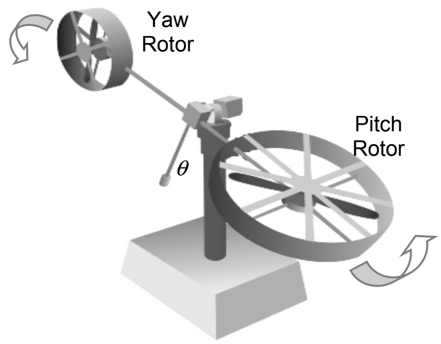

Fig.4. TRMS rotor control platform [22]

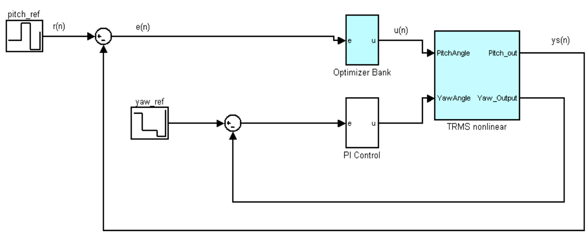

Fig.5. Simulink simulation model for TRMS control simulations

Figure 5 shows the Simulink simulation model that was used for control of nonlinear TRMS model. Pitch rotor is controlled by the proposed optimizer bank and yaw rotor is controlled by a conventional PI controller. Cross coupling occurring between rotors causes a disturbance effect between rotor control systems. Our control objective is a smooth and energy efficient control of the nonlinear multi-rotor system model.

In the Simulink simulations, unit time increment (sampling period) of numerical solution was set to $\Delta t = 10^{-3}$ sec. The learning rate coefficients for all optimizers were identical and set to 50 times of unit time increment. So, $\eta_c = \eta_{ai} = 50\Delta t = 5.10^{-2}$, $i = \{1,2,3,4\}$. Increasing the learning rate coefficients can speed up the optimizers to settle faster to the set-point. However too high values of these coefficients lead to instability of control systems. Initial values of optimizer outputs were set to zero, therefore optimizers cannot increase their outputs over the value of zero because of multiplication with zero of output states in Eqs. (14)-(18). To allow start up of optimizers, we added $h\,e(t)$ term only to adaptation parameter $k_1(t)$. This initially allows a non-zero build-up of $k_1(t)$ depending on $e(t)$ and it stimulates growing others afterward. Here, coefficient h is the growing rate and configured to very low values to decrease impact of $h\,e(t)$ on $k_1(t)$ when $k_1(t)$ approximates to its consistent operating ranges. Other outputs can start up by non-zero values of $k_1(t)$. Figure 6 show block diagram of $k_1(t)$ optimizer that involves the initialization term $h.e(t)$. Accordingly, equation (15) is modified as

$$k_1(t) = \int (\eta_{a1} u(t) + h)\, e(t)\, dt \qquad (19)$$

The coefficient h was set to 10^{-3} in the simulation that provides a build up of all optimizer in appropriate periods.

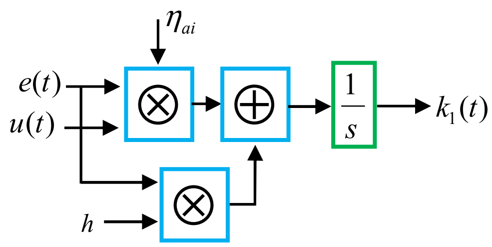

Fig. 6. Block diagram for $k_1(t)$ adaptation optimizer with initialization term

Figure 7 shows simulation results of pitch rotor control. After a build-up phase of optimizers, the proposed gradient descent optimizer bank can perform set-point tracking control of the nonlinear model of TRMS rotor. It can settle to set-points without high overshoots and reject the disturbances caused from yaw rotor motions as shown in the subfigure. Temporal evolutions of adaptation gains and the control output are shown in Figure 8. It is noteworthy in the figures that, at sharp changes of reference signal, contributions of adaptation gains to control signal, for instance by $k_3(t)u(t)^3$ and $k_4(t)u(t)^4$, increase. This indicates that sharp changes in input-output relation of nonlinear model are mainly handled by higher-degree polynomial terms.

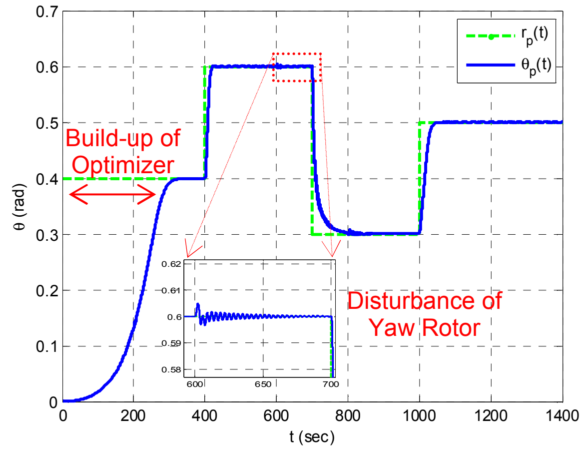

Fig. 7. Simulation results obtained for pitch rotor control of gradient descent optimizer bank

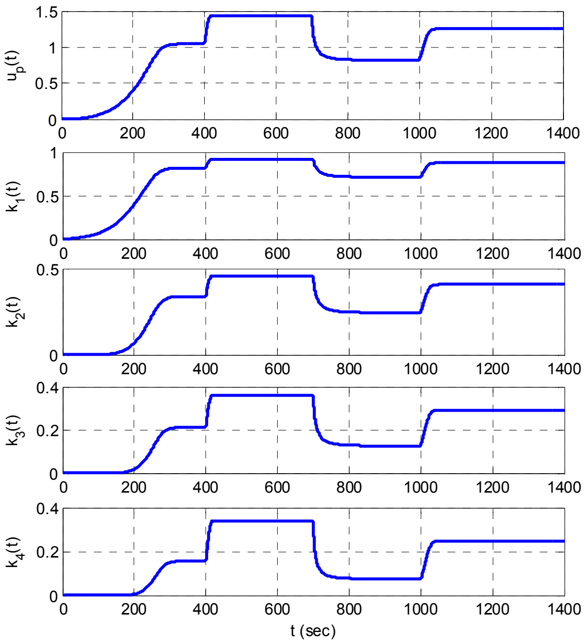

Fig. 8. Temporal evolution of optimizer outputs during simulation

Figure 9 shows control errors of pitch and yaw control loops. Figure demonstrates high error at the build-up phases of optimizers at the beginning. Effects of disturbances associated with cross coupling between rotors can also be observed in this figure by correspondence between slight error rises. Figure 10 shows simulation results of yaw rotor by a PI control. In the second simulation, we substituted gradient descent optimizer bank with a conventional PID controller tuned by Feedback Instruments and performed the same simulation with the same reference signal and conditions. Simulation results are shown in Figure 11 and 12. Although the PID controller is very fast in response, it leads to high overshoots and cross coupling disturbances on yaw rotor. Rotor control applications require smooth and energy efficient control [8]. Therefore, minimization of control signal amplitude beside the reduction of control errors is necessary for optimal control of limited energy systems.

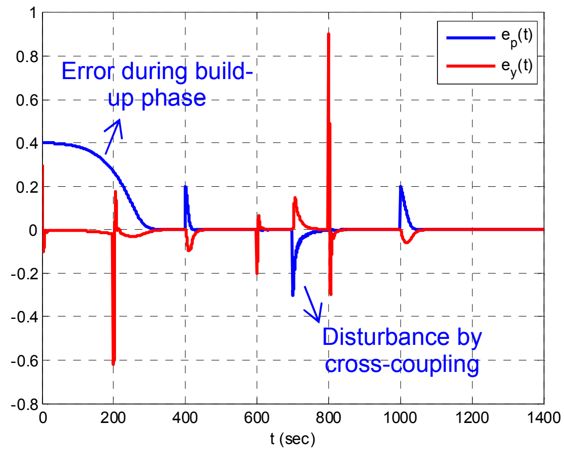

Fig. 9. Control errors for pitch (e_p) and yaw (e_y) control loops

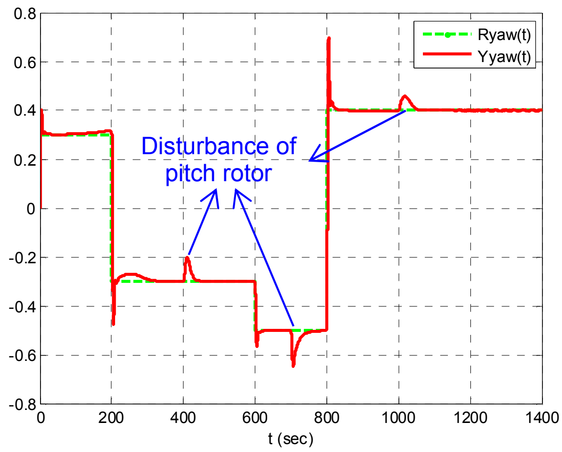

Fig. 10. Simulation results obtained for PI control of yaw rotor in case of gradient descent optimizer bank control of pitch rotor

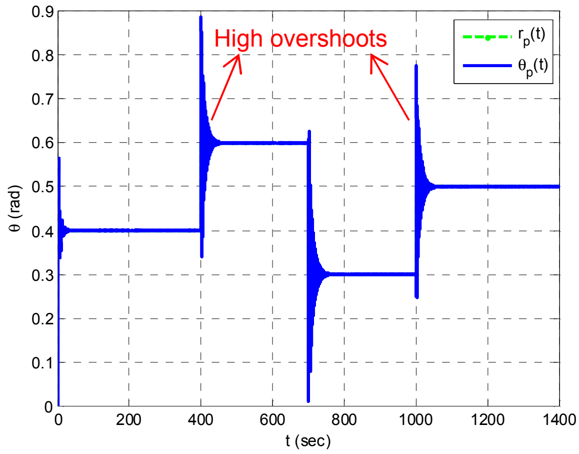

Fig. 11. Simulation results obtained for PID control of pitch rotor

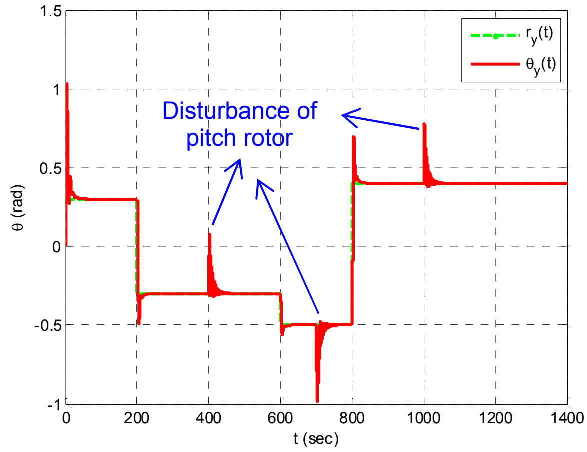

Fig. 12. Simulation results obtained for PI control of yaw rotor in case of PID control of pitch rotor

Table 1 shows performance parameters that can be used to evaluate optimality of control actions for the proposed optimizers and the PID controller. Although the optimizer bank leads to higher mean square error (MSE), energy of the control signal, which is calculated by

$$E_u = \int_0^t u(t)^2 \, dt \,, \tag{20}$$

is one third of the control signal energy of the PID controller. On the other hand, high overshoots in response of PID control also increase MSE of yaw control due to disturbances of cross coupling effect among rotors and it also increases energy consumption of yaw control. This result indicates that smooth rotor control can improve energy efficiency of rotor control actions and provide satisfactory control performance.

Table 1. Comparisons of performance parameters

Controllers	*MSE Pitch*	*MSE Yaw*	E_u *Pitch*	E_u *Yaw*
Optimizer Bank	$10\ 10^{-4}$	$16\ 10^{-4}$	$1.6\ 10^{6}$	$2.0\ 10^{5}$
PID Controller	$6.3\ 10^{-4}$	$23\ 10^{-4}$	$4.8\ 10^{6}$	$2.4\ 10^{5}$

IV. Conclusions

This paper illustrated an application of gradient descent optimization control for the adaptive control of nonlinear model of TRMS test platform. Adaptation capability and independence of plant modeling are two major objectives of fundamental intelligent control methods. In this manner, to reduce dependence of controller design to plant modeling, a fourth-degree polynomial was used to approximate instant input-output relation of a plant function. Thus, the proposed algorithm estimates only current state of instantaneous input-output relation of the plant. Even though the disadvantages, it can significantly reduce complexity of the proposed method because very past and future states of plants are not considered in calculations of current control action. Some important remarks can be summarized as,

(i) Although the gradient descent controller bank responses slower to step reference input compared to responses of conventional PID controllers, it can provide energy efficient and smoother control almost without overshoots. This asset may be very valuable for the certain type control applications that involve a limited-energy MIMO system with strong cross coupling effects. For certain type of stable systems, high-degree polynomial assumption of instant input-output relation of the controlled system can reduce need for the plant modeling.
(ii) Direct synthesis of control signals can be more effective solution for adaptive control compared to online tuning of conventional controller coefficients. System stability during adjustment process of controller coefficients may not be guaranteed to avoid short-term instability cases.
(iii) A major drawback of the presented gradient descent controller banks is that it is effective for stable plant models that are approximating the first order system dynamics. Our test shows us that unstable plant models cannot be controlled by the proposed optimizers. Future study should address control of instable and time-delay systems.

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