Structural Dynamics.

Introduction to Theory Of Vibrations

Vibration : Vibration is the motion of a particle or a body or a system of concentrated bodies having been displaced from a position of equilibrium, appearing as an oscillation.
Thus, any motion which repeats itself after an interval of time is called vibration or oscillation.

Sources Of Vibrations.

Structures are subjected to vibratory motion by earthquakes, blasts, wind, moving loads, machines and flow of water

The pattern of variation of dynamic forces with respect to time mainly consists of following two types -

1. Periodic vibrations
2. Random vibrations.

1. Periodic Vibrations : Periodic vibration are those, which develop due to unbalanced rotating machine parts, self excited aerodynamic vibrations in chimneys and pipe line structures caused by steady flow etc. These periodical motions can be resolved into sinusoidally varing components

2. Random Motions : These motion have complicated non periodic time history. Vibrations caused by earthquakes and quarry blasts are classified as complicated time history vibrations.

For the analysis of events of random motions, statistical techniques may be applied. Examples may be loads imposed by rough rides in vehicles and aerodynamic buffeting (pushing side to side) problems. Some loads when suddenly applied cause transient (or temporary) motion / vibration

Inorder to determine the seismic forces on a structure, it is necessary to study its vibration characteristics on which effects of earthquake ground motion depend.

BASIC CONCEPTS OF VIBRATION.

All bodies having mass and elasticity are capable to vibrate. The mass is inherent in the body and elasticity causes relative motion among its parts. When body particles are displaced by the application of external force, the internal forces in the form of elastic energy present in the body, try to bring it to its original position. At equilibrium position, the whole of the elastic energy is converted into kinetic energy. and the body continues to move in the opposite direction because of it. The whole of the kinetic energy is again converted into elastic or strain energy due to which the body returns to the equilibrium position.

In this way, vibratory motion is repeated continuously and interchange of energy takes place. Thus, any motion which repeats itself after an interval of time is called **'vibration'** or **'oscillation'**.

DYNAMIC LOADING

Dynamics is concerned with the study of forces and motions, which are time dependent. Dynamic is simply defined as time-varying.

Thus, A dynamic load is any load whose magnitude, direction and position vary with respect to time.

Hence, the structural response to a dynamic load is also time-varying.

- when a structure is subjected to dynamic load, it starts vibrating. The structure develop significant number of inertial force and also significant amount of mechanical energy is stored as kinetic energy.

- Generally, vibrations are undesirable for structures because they produce increased stresses and energy losses. Hence, vibrations can be harmful and should be avoided. They can occur in many directions and can be the result of interaction of many objects.

- The motion of vibrating system is governed by the laws of motion, and in particular by Newton's second law of motion ($F=ma$).

Analysis of structural response to a dynamic loading can be done by the following two different approaches :

(i) Deterministic Analysis
(ii) Non-Deterministic Analysis.

(i) Deterministic Analysis : In this method, the structural response i.e displacement, acceleration, velocity, stress etc., are completely known precisely as a function of time. Hence, this method requires perfect control over all the variables that influence the properties and loadings. This method is also called "prescribed dynamic loading".

(ii) Non-Deterministic Analysis : In this method; the time variation of vibration is not completely known. It provides only statistical information about the response from the statistically defined loading. This method is also known as 'random dynamic loading'.

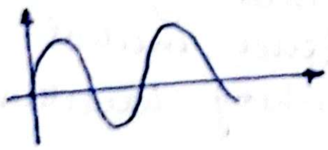
TYPES OF PRESCRIBED OR DETERMINISTIC LOADING

'Periodic Loading'

- In which, the loads exhibit the same time-variation successively for a large number of cycles.
- The simplest form of periodic loading which is a sinusoidal vibration which is termed as 'simple harmonic'.

(By means of Fourier series, any periodic loading can be represented as the sum of a series of simple harmonic components.)

Examples:

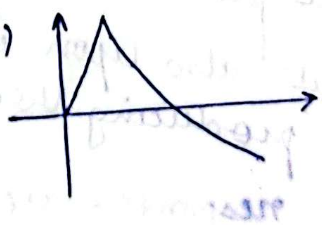
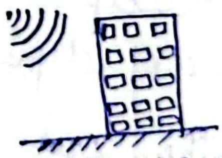
(a) 
Rotating machinery in building

(b) 
Propeller forces at step of ship

'Aperiodic Or Non-Periodic'

- In which, the loading does not exhibit the same time variation successively; (i.e it may be either short-duration or long-duration impulsive loadings).

Examples:

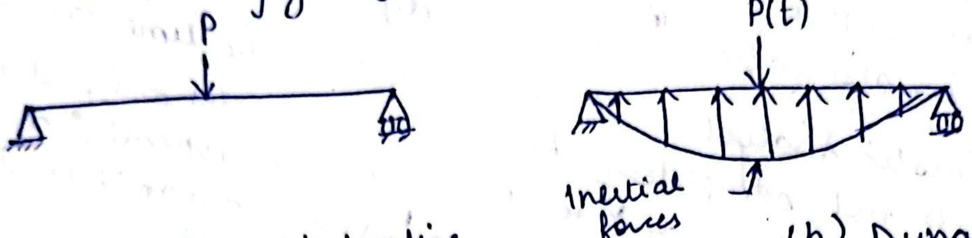
(a)  
BOMB BLAST LOADING

(b)  
LOADING HISTORY
Earthquake Load on Water tank

COMPARISON OF STATIC AND DYNAMIC LOADING

A structural dynamic problem differs from its static loading by the following aspects :

(i) In a static problem, load is constant with respect to time as shown in figure (a). On the other hand, the dynamic problem is the time-varying in nature. Because both loading and its responses varies with respected to time as shown in figure (b).



(a) Static Loading (b) Dynamic Loading.

(ii) static problem has only one response, i.e displacement. But the dynamic problem has mainly three responses, such as displacement, velocity and acceleration.

(iii) static problem has only one solution whereas a dynamic problem has an infinite number of solutions which are time dependent in nature. Thus, dynamic analysis is more complex and time-consuming than static analysis.

(iv) In static problem, the response can be calculated by the principles of force or static equilibrium whereas in case of dynamic problem the responses depend not only upon the load but also upon inertial forces which oppose the acceleration producing them.

Thus, the total responses are calculated by including inertial forces along with the static equilibrium. Hence, inertial forces are the most important distinguishing characteristic of a structural dynamic problem.

Causes Of Dynamic Effects.

Various natural and man-made sources may influence the dynamic effects in structure. The most common types of causes are –

(a) Initial conditions

(b) Applied forces

(c) Support motions.

(a) Initial conditions – Initial conditions such as velocity & displacement ~~procedure~~ produce dynamic effect in the system.

Example: Consider a lift moving up or down with an initial velocity. When the lift is suddenly stopped, the cabin begins to vibrate up and down since it possesses initial velocity.

(b) Applied forces: Sometimes vibration in the system is produced due to the application of external forces.

Example:

(i) A building subjected to a bomb blast or wind forces.

(ii) Machine foundation.

(c) Support motions: Structures are often subjected to vibration due to the influence of support motions.

Example: Earthquake motion.

BASIC DEFINITIONS.

Mass - Dynamically, it is the property that describes how an unrestricted ~~unrestrain~~ body resists the application of an external force. Mass is obtained by dividing the weight of body by the acceleration of gravity. Unit of mass is given in kilograms (kgs).

Stiffness - It is defined as the force required to produce unit deformation. It is an elastic property that describes the level of resisting force that results that results when a body undergoes a change in length. The unit for stiffness is N/m.

. **Periodic Motion** - A motion which repeats itself after equal interval of time.

. **Time period** (T) (or Natural period) - It is defined as the time required to complete one cycle of free vibration. It is expressed in seconds. $(T = \frac{1}{\nu})$

. ~~Natural~~ **Frequency** - Frequency is defined as the number of cycles per with time. Its SI unit is Hertz.

6. **Amplitude** - The maximum displacement of a vibrating body from its mean position is called 'amplitude'.

7. **Free vibration** - The vibration which persists in a structure after the force causing the motion has been removed is known as Free Vibration. No external force act on them.

It takes place when a system oscillates under the action of forces inherent in the system itself.

Example -: Oscillation of a simple pendulum.

8. **Forced Vibration** - The vibration which is maintained in a structure by steady periodic force acting on the structure is known as Forced vibration. When the excitation is oscillatory, the system is forced to vibrate at the excitation frequency. The behaviour of a system under forced vibration depends on the type of excitation.

Fundamental mode of vibration: The fundamental mode of vibration of a structure is the mode having the lowest natural frequency.

10. Damping : Damping is the resistance to the motion of a vibrating body.

The vibrations associated with this resistance are known as damped vibration.

- It is a phenomenon in which the vibrational energy of the system is gradually reduced or the amplitude of vibration is slowly decreased. Unit of damping is N/m/s. (or $\frac{Ns}{m}$)

11. Resonance : When the frequency of external force is equal to the natural frequency of the vibrating system, the amplitude of vibration becomes excessively large. This phenomenon is called resonance.

The failure of major structures such as bridges and buildings largely is due to resonance.

TYPES OF VIBRATIONS

The following are the different types of vibrations in a structural dynamic problems.

(i) Free and Forced vibration -

The vibration which persists in a structure after the force causing the motion has been removed is known as free vibration.

The vibration which is maintained in a structure by steady periodic force acting on the structure is known as forced vibration.

(ii) Linear and Non-linear Vibration -

If the basic components of a vibrating system, namely the spring, the mass and the damper behave in a linear manner, the resulting vibrations caused are known as linear vibration. (These are governed by linear differential equation. They should obey the law of superposition.)

On the other hand, if any of the basic components of a vibrating system behave in a non-linear manner, the resulting vibration is called non-linear vibration.

(In this case, the governing differential equation is also non-linear. It does not follow the law of superposition.)

When a damper or damping element is attached to the vibratory system, the motion of the system will be opposed by it and the energy of the system will be dissipated in friction. This type of vibration is called damped vibration.

On the other hand, the vibration generated by the system having no damping element is known as undamped vibration.

(iv) Deterministic and Random Vibrations:

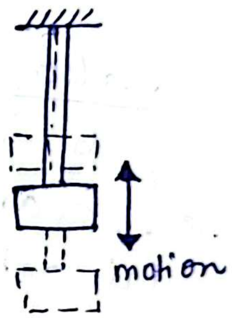
If the amount of excitation (force or motion) acting on vibrating system is completely known precisely, the resulting vibrations are called as deterministic vibrations.

Contrary to it, when the amount of excitation is not completely known, the resulting vibrations are known as non-deterministic vibrations or random vibrations.

(Random vibration analysis is used to analyse the earthquake excitation of buildings and structures.)

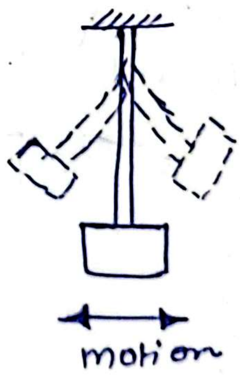
(v) Longitudinal, Transverse and Torsional vibration:

- If the mass of the vibratory system moves up and down parallel to the axis of the shaft, the vibrations created are known as longitudinal vibrations.



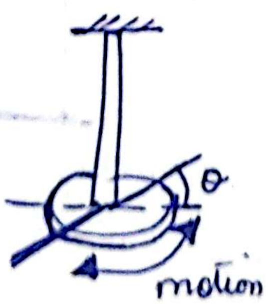
. Longitudinal vibration.

- When the particles of the body or shaft move perpendicular to the axis of the shaft, the vibrations created are known as tranverse vibrations.



. Transverse vibration.

- Torsional vibration – If the shaft gets alternately twisted and untwisted on account of vibratory motion of the suspended discs, such vibrations are called torsional vibrations.



. Torsional vibrations.

In ideal systems, the free vibrations continue indefinitely as there is no damping. The amplitude of vibration decays continuously because of damping (in a real system) and vanishes ultimately. Such vibration in a real system is called <u>transient vibration</u>.

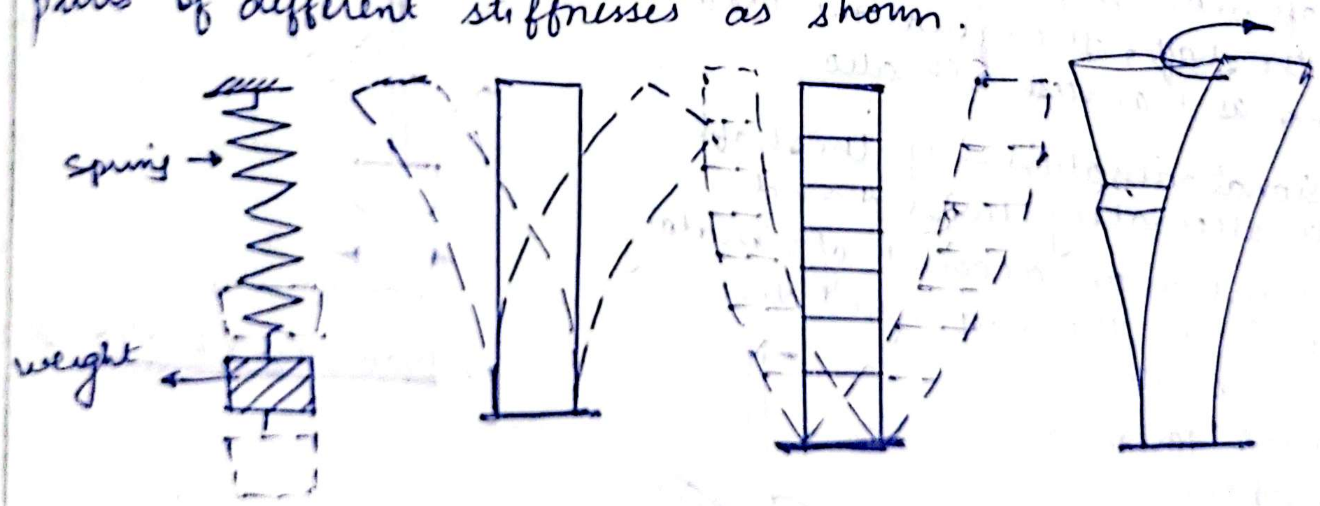
- Whatever may be the cause of vibrations, a structure vibrates in one of the following four deformation or a combination of these of. The line diagrams of these deformations during vibrations are -

(a) <u>Extensional vibrations</u> — This type of vibrations develops in vertical direction by a rigid block tied with a spring, one end of which is tied with a fixed support as shown.

(b) <u>Bending vibrations</u> - This type of vibrations develop in tall vertical structures as that in chimney stack as shown.

(c) <u>Shear vibrations</u> - This type of vibrations develop in multi storey buildings having columns stiffened by rigid floors, beams at different levels. These vibrations are horizontal vibrations.

(d) <u>Torsional vibrations</u> - This type of vibrations develop due to Twisting of a building having its supporting parts of different stiffnesses as shown.



(a) Extensional (b) Bending (c) Shearing (d) Torsional

Response of the System

Response is defined as the magnitude and distribution of the resulting force and displacements in a system due to vibration.

- The motion due to initial condition is generally known as <u>free response</u>. When the motion is due to applied forces, it is known as <u>forced response</u>.

Degree Of freedom

The number of independent coordinates required to describe the position of a vibrating mass is known as <u>degree of freedom</u> of that mass.

The position of a vibrating mass in space with respect to its position of equilibrium can be described by six coordinates, three translation along the three orthogonal axes x, y, z and three rotational about the same axes i.e x, y, z.

However, a mass may have freedom to move only in certain directions and may be constrained in other directions. Thus, no. of independent coordinates required to describe the position of a vibrating mass is known as degree of freedom (DOF)

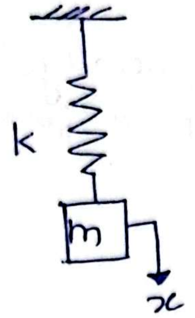
- Each degree of freedom is having corresponding natural frequency. Therefore, a structure possesses as many natural frequencies as it has the degrees of freedom. For each natural frequency, the structure has its own way of vibration. The vibrating shape is known as <u>characteristic shape</u> or <u>mode of vibration</u>.

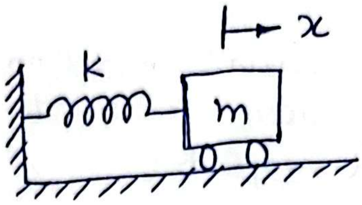
Depending on the independent coordinates required to describe the motion, the vibratory system is divided into the following categories:

(a) Single Degree Of Freedom system (SDOF system)
(b) Multiple Degree Of Freedom system (MDOF system)
(c) Continuous system.

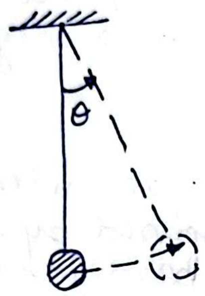
• If a single coordinate is sufficient to define the position or geometry of the mass of the system at any instant of time, it is known as <u>single degree of freedom</u> (SDOF) system.

Examples –

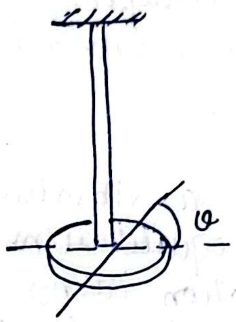




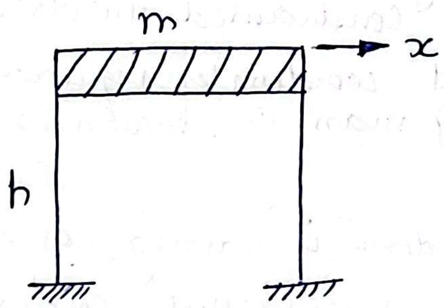
Spring – mass system.



Simple Pendulum



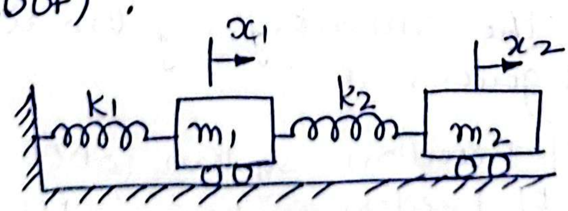
Torsional system



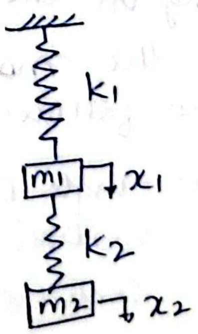
Building frame.

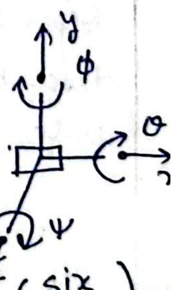
• If more than one independent coordinates are required to completely specify the the position or geometry of different masses of the system at any instant of time, it is called <u>multiple degrees of freedom system</u> (MDOF).

Examples –.

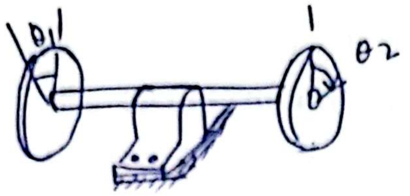


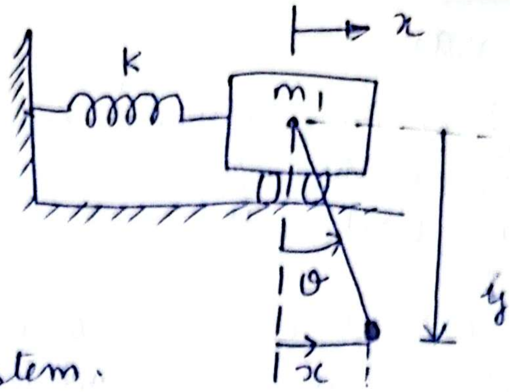
(a)
(2° of freedom)





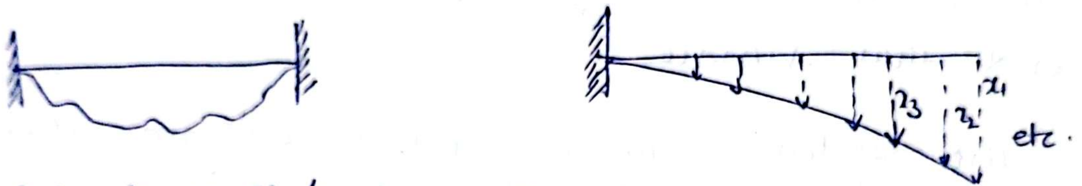
(six DOF)





Examples for MDOF system.

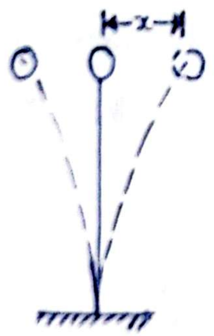
- If the mass of a system may be considered to be distributed over its entire length as shown, in which mass is considered to have infinite degrees of freedom, it is referred to as a continuous system. It is also known as distributed system.



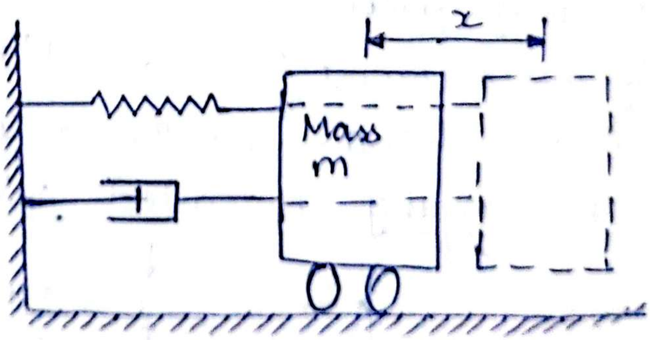
Infinite degrees of freedom system.

SPRING ACTION AND DAMPING.

Imagine a mass 'm' fixed on the top of a column as shown in figure (a) or a mass resting on rollers and fixed to a spring and a damping device dashpot as shown in figure (b).



(a) Mass at top of column



Mass, Spring and Damping

If the mass 'm' is displaced by 'x', the elastic straining of the column in figure (a) or the stretching of the spring in figure (b) will tend to bring the mass back to the original position.

- The force exerted by the column or spring is a function of the displacement 'x' and is known as its '<u>restoring force</u> or <u>spring force</u>'.

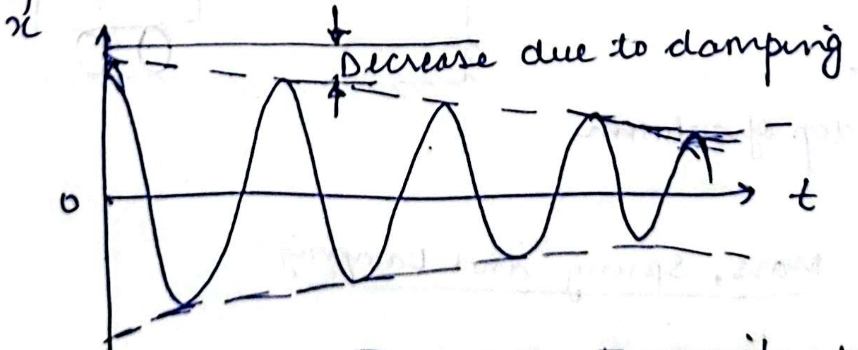
If the displacement 'x' is small, this force could be assumed to be a linear function of 'x'.

This mass 'm' will come back to its original position with a certain velocity and pass to the other side and thus vibrate.

- Incase the system is elastic and there is no loss of energy, the mass will vibrate forever.

But in practice, friction due to air, or friction between the particles of the system or at junctions or due to yielding of the material, etc. energy dissipation will take place, so that vibrations will die out in course of time.

The forces which cause loss of energy are known as 'damping forces'. Figure below shows the variation of displacement 'x' with time. <u>If the damping force is proportional to the velocity motion of the mass, then this damping is called viscous damping.</u>



Damped Free vibration

SIMPLE HARMONIC MOTION (SHM)

The harmonic motion is one of the forms of periodic motion.

'The motion of a body to and fro about a fixed point is called simple Harmonic motion (SHM)'.

The harmonic motion is represented in terms of circular sine and cosine functions. All harmonic motions are periodic in nature but all the periodic motions are not always harmonic.

SHM possesses the following characteristics:

1. The motion is periodic
2. When displaced from the fixed point or the mean position, a restoring force acts on the particle tending to bring it to the mean position.
3. Restoring force on the particle is directly proportional to its displacement.

Consider a harmonic motion of type;

$x = A\sin(\omega n t + \phi)$

$x = A\sin(\omega n t + \phi)$

where; x is the displacement
A is the amplitude
ω is the frequency and
ϕ is phase angle.

The velocity and acceleration are;

$$\frac{dx}{dt} = \text{velocity} = A\cos(\omega n t + \phi) \times \omega n$$

or

$$\boxed{\dot{x} = A\omega n\cos(\omega n t + \phi)}$$

and

$$\frac{d^2x}{dt^2} = \text{acceleration} = -A\omega n\sin(\omega n t + \phi) \times \omega n$$

or

$$\boxed{\ddot{x} = -A\omega n^2\sin(\omega n t + \phi)}$$

for max. value of displacement;

$$\sin(\omega_n t + \phi) = 1$$

$$\therefore \quad x_{max} = A$$

To get max. value of velocity,

$$\cos(\omega_n t + \phi) = 1$$

$$\therefore \quad \dot{x}_{max} = A\omega_n$$

for the max. value of acceleration,

$$\sin(\omega_n t + \phi) = 1$$

$$\therefore \quad \ddot{x}_{max} = -A\omega_n^2 \quad = -\omega_n^2 (x)$$

$$\text{or} \quad \ddot{x} \propto x$$

Thus, acceleration in a SHM is always proportional to its displacement and directed towards a particular fixed point.

The velocity of the vibrating particle is maximum at the mean position of rest and zero at the max. position of vibration. But acceleration of vibrating particle is zero at the mean position of rest and max. at the max. positions of vibration.

The acceleration is always directed towards the mean position of rest and is directly proportion to the displacement of the vibrating particle.

- When a body undergoes SHM, its total energy consists of potential energy and kinetic energy. The velocity and the consequent kinetic energy are maximum at mean position.

Potential energy is zero at mean position. and is max. at the extreme position.

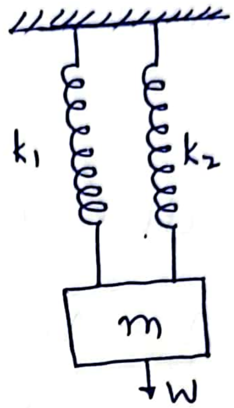
$$\boxed{\text{Kinetic energy} = \frac{1}{2}mv^2}$$

EQUIVALENT STIFFNESS OF SPRING COMBINATIONS

Many systems have more than one spring. The spring may be connected in series or parallel or both. Sometimes, it can be connected in an inclined position also.. They can be replaced by a single spring of the same stiffness as they all show the same stiffness as a whole. This stiffness is called 'equivalent stiffness'.

SPRINGS IN PARALLEL

When two springs are connected in parallel as shown, the springs in a system is subjected to a common deflection and the total load supported is the sum of the individual loads shared by each spring.



Springs in parallel

$$\therefore \quad \Delta = \Delta_1 = \Delta_2$$

where; Δ, Δ_1 and Δ_2 are the static deflection of the springs.

$$W = W_1 + W_2$$

$$\Rightarrow \quad K\Delta = K_1\Delta_1 + K_2\Delta_2$$

(as, $W = K\Delta$ or $K = \frac{W}{\Delta}$)

$$\because \quad \Delta = \Delta_1 = \Delta_2$$

$$\therefore \quad K_e\Delta = K_1\Delta + K_2\Delta$$

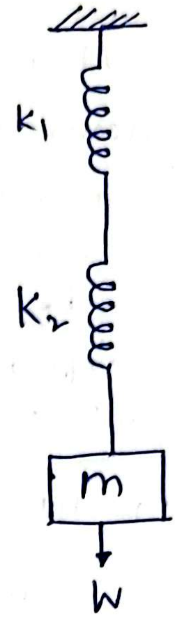
$$\Rightarrow \quad K_e\cancel{\Delta} = \cancel{\Delta}(K_1 + K_2)$$

$$\Rightarrow \quad \boxed{K_e = K_1 + K_2}$$

where; K_e = equivalent stiffness of the system.

Thus, equivalent spring stiffness in case of parallel system is equal to the sum of individual spring stiffness. In general, for n no. of springs in parallel, the equivalent spring stiffness, $\boxed{K_e = \sum_{i=1}^{n} K_i}$

SPRINGS IN SERIES



Springs in series

Consider two linear springs of stiffness K_1 and K_2 arranged in series as shown.
When the springs are connected in series, if they share a common load, the total deflection of the system must be equal to the sum of the deflection of the individual springs.

Then, the total deflection Δ is given by:

$$\Delta = \Delta_1 + \Delta_2$$

$$\frac{W}{K_e} = \frac{W_1}{K_1} + \frac{W_2}{K_2}$$

as $W = K\Delta$

$$\Delta = \frac{W}{K}$$

$$\Rightarrow \frac{\cancel{W}}{K_e} = \cancel{W}\left(\frac{1}{K_1} + \frac{1}{K_2}\right) \qquad (\because W = W_1 = W_2$$

or

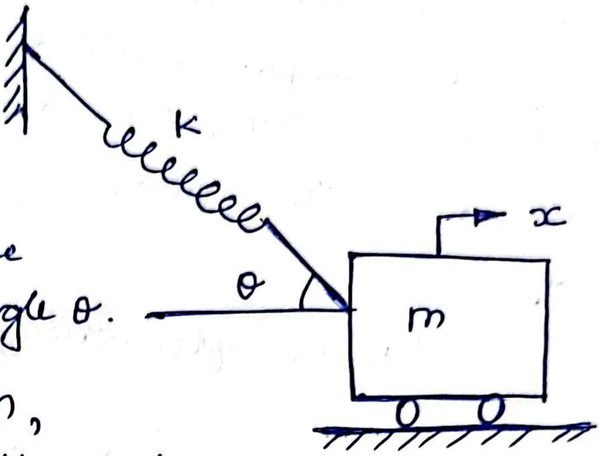
$$\boxed{\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2}}$$

Thus, when springs are connected in series, the reciprocal of equivalent spring stiffness is equal to the sum of the reciprocals of individual spring stiffnesses.
In general, for n number of springs in series, the equivalent stiffness may be obtained from the following expression:
$$\frac{1}{K_e} = \sum_{i=1}^{n} \frac{1}{K_i}$$

NOTE: When springs are connected in parallel, it helps to increase the amount of load required to produce

unit deflection, i.e, the springs become harder.
Whereas, when the springs are connected in series, it helps to reduce the stiffness of the component springs. In other words, the resulting equivalent spring requires less amount of load to produce unit deflection; i.e the spring becomes softer.

Inclined Spring



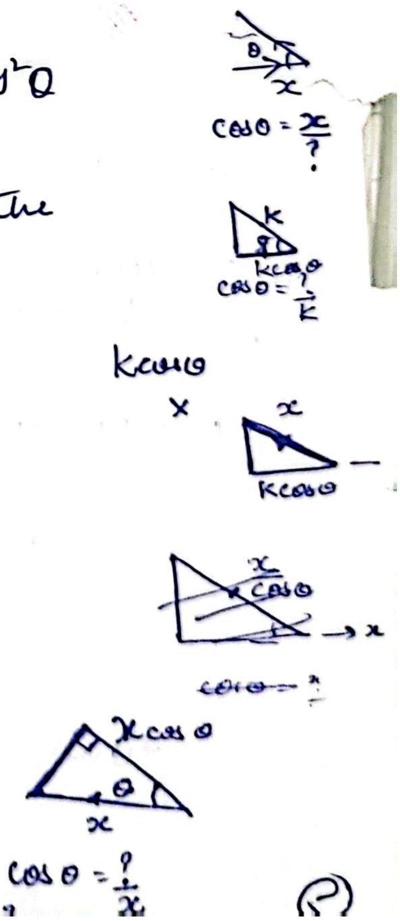
Consider a mass m moving in x direction as shown.
The spring is attached to the direction of motion by an angle θ.

For a displacement x of mass m, the corresponding stretch in the spring is $x\cos\theta$ and therefore the spring force in its own axial direction is $kx\cos\theta$. The component of this force in the direction of motion of mass is $kx\cos^2\theta$.

Hence, effective stiffness of spring $= k\cos^2\theta$

In general, for n no. of inclined springs, the equivalent stiffness may be obtained from;

$$\boxed{k_e = \sum_{i=i}^{n} k_i\cos^2\theta}$$



NATURAL FREQUENCY AND TIME PERIOD.

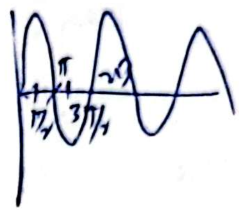
The equation for simple harmonic oscillations is given by,

$$x = A \sin(\omega_n t + \phi)$$

where; ϕ = phase angle.

In general, the particles having a phase difference of integral ~~muplti~~ multiples of 2π will be in same phase. For SHM, the time period is the time interval in which the phase of the vibrating particles changes by 2π.

Hence, after every time period T,



$$\omega_n(t+T) - \omega_n t = 2\pi$$

$$\cancel{\omega_n t} + \omega_n T - \cancel{\omega_n t} = 2\pi$$

$$\omega_n T = 2\pi$$

or $\boxed{T = \frac{2\pi}{\omega_n}}$ or $\boxed{\omega_n = \frac{2\pi}{T} = 2\pi f}$

where: T = natural period or Time period of vibration,

It is defined as the time required to complete one cycle of free vibration.

We know then; $\omega_n = \sqrt{\frac{k}{m}}$

also; $W = k\,\delta_{st}$, $W = mg$

$k = \frac{W}{\delta_{st}}$; $m = \frac{W}{g}$

Hence; $\omega_n = \sqrt{\frac{W/\delta_{st}}{W/g}} = \sqrt{\frac{g}{\delta_{st}}}$ $\Rightarrow \boxed{\omega_n = \sqrt{\frac{g}{\delta_{st}}}}$

$\Rightarrow T = \frac{2\pi}{\omega_n} = \boxed{2\pi\sqrt{\frac{\delta_{st}}{g}}}$