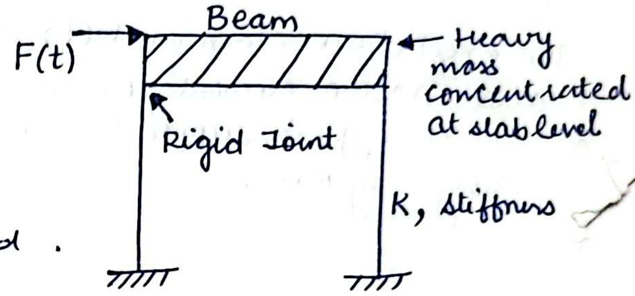
VIBRATION ANALYSIS

The vibration analysis of a real system consists of the following steps:

1. Mathematical modelling of a real system
2. Formulation of governing equation
3. solution of the governing equation of motion
4. Interpretation of results.

Mathematical Modelling of SINGLE DEGREE OF FREEDOM (SDOF) SYSTEM

Let us consider a simple portal frame as shown in figure. while developing a mathematical model, some assumptions are made to simplify the analysis. They are,



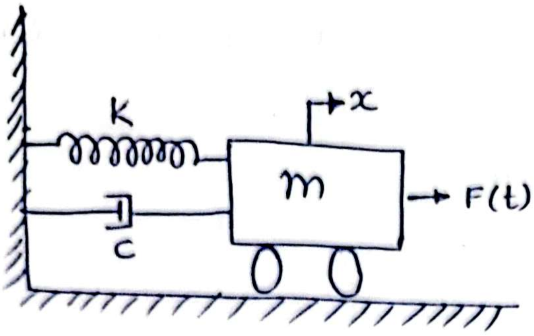
(i) The total mass of a portal frame is assumed to act at the slab level, since the masses of columns are very less when compared to that of slab, i.e masses of columns are ignored.

(ii) The beam/slab is assumed as infinitely rigid, so that the stiffness of the structure is provided only by the column ie, flexibility of slab/beam is ignored.

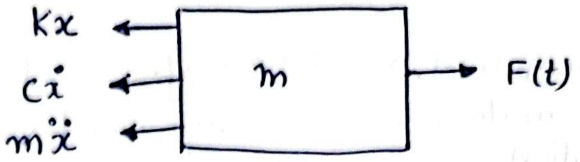
(iii) Since, the beams are usually built monolithically within the columns the beam column joint can be assumed to be rigid as without any rotations at joint.

By these assumptions, the possibility of lateral displacement or deformation is due to only rigid beam/slab. The model resulting from all the above mentioned assumptions is called as 'shear building model'.

The portal frame of figure shown above under the influence of lateral load F(t) can be represented mathematically as the response of SDOF system as shown in figure below.



(a) Equivalent SDOF system
(Spring-mass system)



(b) Free body Diagram

The mass, the spring and the damper (or dashpot) are called inactive or passive elements. The excitation element $F(t)$ is called 'active element' through which energy is supplied to the vibratory system.

where, F_s = spring force = kx
F_D = Damping force = $c\dot{x}$
F_t = Inertia force = $m\ddot{x}$
where; c = damping coefficient.

Considering the force body of mass, the forces acting on it are;

$$F(t) - kx - c\dot{x}$$

According to Newton's 2nd Law of motion, these forces are equal to inertial force ($m\ddot{x}$).

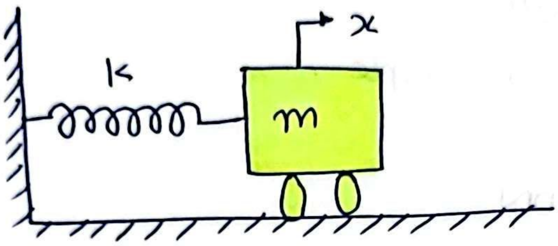
Thus,
$$m\ddot{x} = F(t) - kx - c\dot{x}$$

or
$$\boxed{m\ddot{x} + c\dot{x} + kx = F(t)}$$

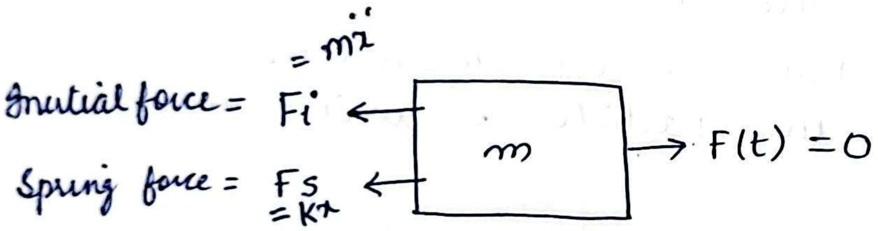
where;
$m\ddot{x}$ = inertial force
kx = restoring force or spring force
$c\dot{x}$ = The damping force.
$F(t)$ = The exciting force.

Free Vibration of Undamped SDOF System

Incase of Free vibration of undamped SDOF system, a structure vibrate in the absence of the external excitation, and in undamped system, the frictional forces or damping is also neglected as shown.
(In this case, the vibration is influenced only by the initial conditions such as the given displacement and at time $t=0$ when the motion of the system is initiated. This undamped SDOF system is known as undamped oscillator.)



Free vibration of undamped SDOF system.



$c\dot{x} = 0$

Free body diagram

Since, the spring used in the system is assumed to behave linearly, the deformation is directly proportional to the force, i.e

$$F_s \propto x$$

or $F_s = kx$, where k = spring constant (stiffness of spring)

According to Newton's second law, $F = ma = m\ddot{x}$

Hence, inertial force, $F_i = m\ddot{x}$

Thus, equation of motion of such an undamped free vibration system can be written as,

$$F_i + F_s = 0$$

or $$\boxed{m\ddot{x} + kx = 0}$$

DERIVATION OF EQUATION OF MOTION

The governing differential equation ~~is for~~ describing the motion is known as equation of motion.
There are various methods to derive the equation of a vibratory system. These are :

1. Simple Harmonic Motion (SHM) method
2. Newton's method
3. Energy method
4. Rayleigh's method
5. D' Alembert's principle.

SIMPLE HARMONIC MOTION

For a particle in a rectilinear motion, if its acceleration is always proportional to the distance of the particle from a fixed point on the path, and is directed towards the fixed point, then the particle is said to have SHM.
SHM is the simplest form of periodic motion.

In differential equation form, SHM is represented as;

$$\ddot{x} \propto -x \quad \text{——} (1)$$

where; x = rectilinear displacement.

we know that; $\frac{dx}{dt} = \dot{x}$ = velocity of particle

$\frac{d^2x}{dt^2} = \ddot{x}$ = acceleration of the motion

(-ve sign indicates that direction of motion of a particle towards a fixed point which is opposite to the direction of motion.

Equation, (1) can be written as

$$\ddot{x} = -\omega_n^2 x$$

$$\Rightarrow \boxed{\ddot{x} + \omega_n^2 x = 0}$$ This is known as equation of mo[tion]

This is a second order linear differential equation).
where; ω_n^2 = constant of proportionality.

Newton's Second Law of Motion

The equation of motion is just another form of Newton's second law of motion which states that, "the rate of ~~rate~~ change of momentum is directly proportional to the applied unbalanced force in the direction of force applied".

Consider a spring-mass system as shown, which is assumed to move only along the vertical direction. So, it has only one degree of freedom, because its motion is described by a single coordinate x.
(And a massless spring of constant stiffness or spring factor k is shown in figure).

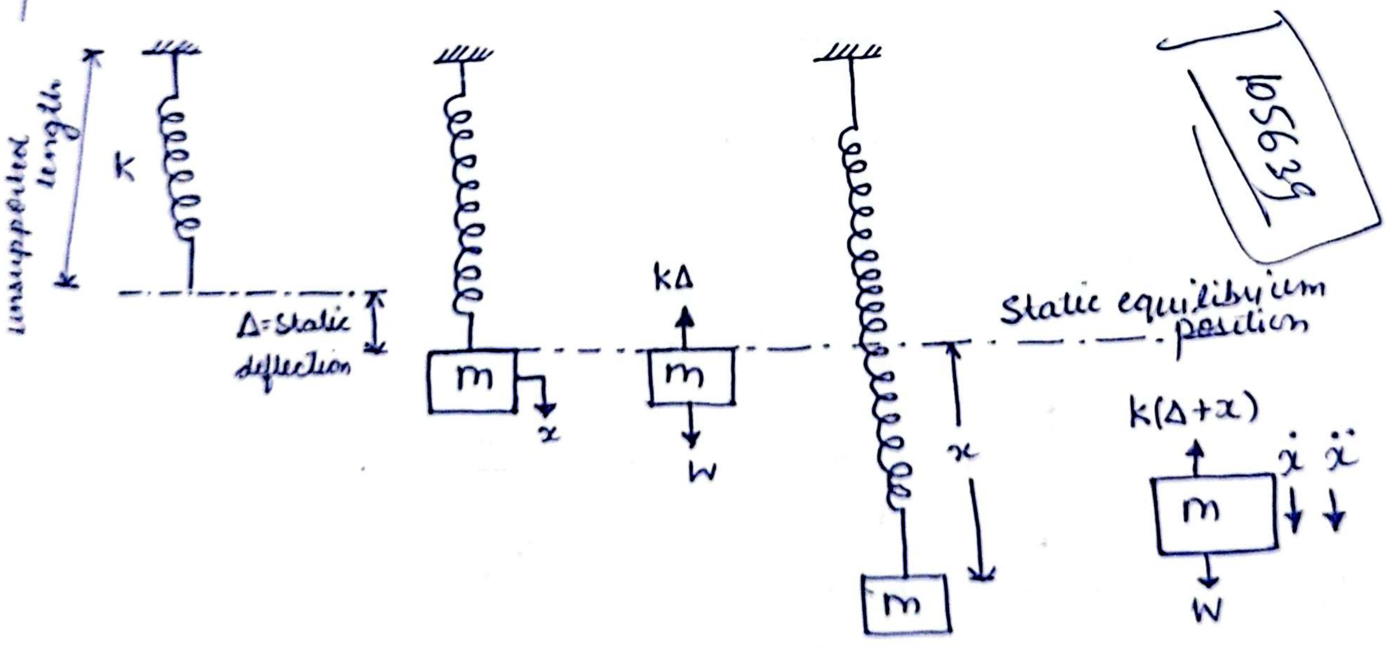
As, stiffness is defined as the "load required to produce unit deformation".

Mathematically, it can be represented as

$$k = \frac{W}{\Delta} \quad — ①$$

where; Δ = static deflection of spring

After attaching a load w to the spring as in figure, the spring elongates or displaces from its equilibrium position vertically downwards. This position is called as "equilibrium position".



SPRING-MASS SYSTEM

FBD

from equ. 1;

$$w = k\Delta \quad \text{—— (2)}$$

from the equilibrium position, the load w is pulled down a little, by some force and then the pulling force is removed. The load w will continue to execute vibrations up and down which is called a free vibration,

$\therefore$ Restoring force in x-direction;

$$= w - k(\Delta + x)$$

$$= w - k\Delta - kx$$

$$= \cancel{k\Delta} - \cancel{k\Delta} - kx \qquad (\text{as}, w = k\Delta)$$

$$= -kx$$

According to newton's law; $(F = ma)$

$$m\ddot{x} = -kx$$

$$\Rightarrow m\ddot{x} + kx = 0$$

$$\Rightarrow \boxed{\ddot{x} + \frac{k}{m}x = 0} \qquad (\text{dividing by } m) \quad \text{—— (3)}$$

Also, from equation of motion (SHM);

we have; $\ddot{x} + \omega_n^2 x = 0$

On comparing, we get;

$$\omega_n = 2\pi\nu = \frac{2\pi}{T}$$

$$\omega_n^2 = \frac{k}{m}$$

$$\boxed{\omega_n = \sqrt{\frac{k}{m}}} \quad \text{or} \quad \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

$$\text{or} \quad \boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

$$\text{or} \Rightarrow \boxed{f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

The constant of proportionality ω_n is known as the natural frequency or angular velocity. It is measured in rad/sec.

ENERGY METHOD

In this method, it is assumed that the system is to be a conservative one.

In a conservative system, the total sum of the energy is constant at all time. (For an undamped system, since there is no friction or damping force, the total energy of the system is partly potential and partly kinetic).

Acc. to law of conservation of energy;

Total energy = constant

i.e. K.E + P.E = constant

The time rate of change of total energy will be zero;

i.e. $\frac{d}{dt}(KE + PE) = 0$

as; $KE = \frac{1}{2}mv^2 = \frac{1}{2}m\dot{x}^2$

$k_d = \frac{k}{\delta}$

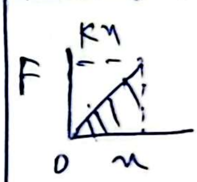
& $PE = \frac{1}{2}kx^2$ (elastic PE)

mgh

$\frac{k}{\delta} \cdot x$

{(PE) grav. = mgh

So; $\frac{d}{dt}\left(\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2\right) = 0$



hence; $\left(\frac{1}{2}m \times 2\dot{x} \times \ddot{x}\right) + \left(\frac{1}{2}k \times 2x \times \dot{x}\right) = 0$

$P.E = \frac{1}{2} x \times kx$

$= \frac{1}{2}kx^2$

or, $m\dot{x}\ddot{x} + kx\dot{x} = 0$

On dividing by $\dot{x}$; $\boxed{m\ddot{x} + kx = 0}$

or $\boxed{\ddot{x} + \frac{k}{m}x = 0}$

This is the same equation as obtained by Newton's method.

Rayleighs Method.

In this method, it is assumed that the max KE at the equilibrium position is equal to the max. potential energy at the extreme position.

The motion is assumed to be SHM, then.

$$x = A \sin \omega_n t \quad \text{—— } ①$$

where, x = displacement of the system from its mean position after time t,

A = maximum displacement of the system from equilibrium position to extreme position

x is max., when $\sin \omega_n t = 1$

$$\therefore \quad \boxed{x_{max} = A}$$

On diff. w.r.t t, we get, (equ.①)

$$\text{velocity} = \dot{x} = A \omega_n \cos \omega_n t$$

velocity $\dot{x}$ is max. only when $\cos \omega_n t = 1$

$$\therefore \quad \boxed{\dot{x}_{max} = A\omega_n}$$

So, max KE at equ. position $= \frac{1}{2} m v^2$

$$= \frac{1}{2} m \dot{x}^2_{max}$$

$$= \frac{1}{2} m (A\omega_n)^2$$

$$= \frac{1}{2} m A^2 \omega_n^2 \quad \text{—— } ②$$

Max PE at the extreme position; $= \frac{1}{2} k x_{max}^2$

$$(PE)_{max} = \frac{1}{2} k (A)^2 \quad \text{--- (3)}$$

from equ. (2) and (3), as $(KE)_{max} = (PE)_{max}$

$$\cancel{\frac{1}{2}} m \omega_n^2 \cancel{A^2} = \cancel{\frac{1}{2}} k \cancel{A^2}$$

$$\therefore \quad \omega_n^2 = \frac{k}{m}$$

$$\text{or} \quad \omega_n = \sqrt{\frac{k}{m}}$$

ω_n is natural frequency of the system.

$$\text{or } f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\text{or } T = 2\pi\sqrt{\frac{m}{k}}$$

Note: k is in N/m
m is in kg and
ω_n is in rad/s.

D' Alembert's Principle.

It gives the solution of a dynamic problem using the methods of statics;

Acc. to Newton's Law,

$$F = ma$$

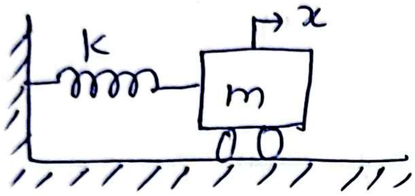
$$\text{or} \quad F - ma = 0 \quad \text{--- (1)}$$

(The above equation is in the form of an equation of motion of force equilibrium in which the sum of a number of force terms equal to zero.)

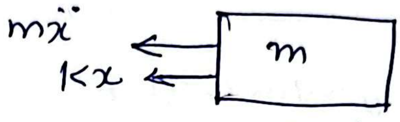
D'Alembert's principle states that, 'a system may be in dynamic equilibrium by adding to the external forces, an imaginary force, which is commonly known as the inertia force'.

According to this principle, the transformation of a problem in dynamics may be reduced to one in statics.

Consider a spring-mass system as shown in figure,



(a) Spring-Mass system



(FBD)
Dynamic equilibrium.

Using D'Alembert's principle, to bring the body to a dynamic equilibrium position, the inertia force $m\ddot{x}$ is to be added in the direction opposite to the direction of motion.

Equilibrium equation is, $\Sigma F_x = 0$

$$-m\ddot{x} - kx = 0$$

$$-(m\ddot{x} + kx) = 0$$

$$m\ddot{x} + kx = 0$$

or $$\ddot{x} + \frac{k}{m}x = 0$$

Let, $\omega_n^2 = \frac{k}{m}$

$\Rightarrow$ $$\boxed{\omega_n = \sqrt{\frac{k}{m}}}$$

SOLUTION OF THE EQUATION OF MOTION

The governing differential equation of motion is

$$m\ddot{x} + kx = 0$$

It is in the form of homogeneous second order linear equation. There are five differential solutions for the above equation of motion.

1. $x = A \cos \omega_n t$
2. $x = B \sin \omega_n t$
3. $x = A \cos \omega_n t + B \sin \omega_n t$
4. $x = A \sin(\omega_n t + \phi)$,
5. $x = A \cos(\omega_n t - \phi)$

where, A and B are constants depending on their initial condition of the motion.

Solution NO. 1

One solution for the above equation (problem) is $x = A \cos \omega_n t$ — ①

To determine the constant A, let us use the initial condition by assuming that at $t = 0$, the displacement $x = x_0$

Putting this in equ ①, we get.

$$x_0 = A \cos(\omega_n \times 0)$$

$$\therefore \quad x_0 = A$$

Hence, the solution is $\boxed{x = x_0 \cos \omega_n t}$ — ②

Solution No. 2.

$$x = B \sin \omega_n t \quad \text{— ③}$$

Initial condition is used to determine the constant B.

The assumed initial conditions are;

(i) At time $t = 0$; $x = x_0$ and

(ii) At time $t = 0$; $\dot{x} = \dot{x}_0$

$\left(\cancel{x = B \times 0} \quad \cancel{\dot{x} = B\omega_n \cos\omega_n t} \right)$

Apply differentiating equ ③ w.r.t time,

$$\dot{x} = B\omega_n \cos \omega_n t \quad \text{— ④}$$

Applying initial condition; (At $t = 0$, $\dot{x} = \dot{x}_0$)

$$\dot{x}_0 = B\omega_n$$

$$\Rightarrow \quad B = \frac{\dot{x}_0}{\omega_n}$$

Substituting in equation (3); we get

$$\boxed{x = \frac{\dot{x}_0}{\omega_n} \sin \omega_n t} \quad \text{--- (5)}$$

Solution No.3

$$x = A \cos \omega_n t + B \sin \omega_n t \quad \text{--- (6)}$$

Since this differential equation is linear, The superposition of the above two solutions is also a solution.
The general solution for this second order differential equation is

$$\boxed{x = x_0 \cos \omega_n t + \frac{\dot{x}_0}{\omega_n} \sin \omega_n t} \quad \text{--- (7)}$$

Solution No.4.

$$x = A \sin(\omega_n t + \phi) \quad \text{--- (8)}$$

By expanding sine terms:

$$x = A \sin \omega_n t \cos \phi + A \cos \omega_n t \sin \phi \quad \text{--- (9)}$$

But the general solution is,

$$\left\{ \text{using, } \sin(A+B) = \sin A \cos B + \cos A \sin B \right\}$$

$$= x_0 \cos \omega_n t + \frac{\dot{x}_0}{\omega_n} \sin \omega_n t$$

On comparing equ(9) with above general solution; we get.

$$x_0 = A \sin \phi \quad \text{--- (10)}$$

and $$\frac{\dot{x}_0}{\omega_n} = A \cos \phi \quad \text{--- (11)}$$

Squaring and adding equ. (10) & (11), we get.

$$A^2 \sin^2 \phi + A^2 \cos^2 \phi = x_0^2 + \frac{\dot{x}_0^2}{\omega_n^2}$$

$$\Rightarrow A^2 (\sin^2 \phi + \cos^2 \phi) = x_0^2 + \frac{\dot{x}_0^2}{\omega_n^2}$$

$$\Rightarrow A^2 = x_0^2 + \frac{\dot{x}_0^2}{\omega_n^2}$$

Thus, $\boxed{\text{Amplitude; } A = \sqrt{x_0^2 + \frac{\dot{x}_0^2}{\omega_n^2}}}$

Dividing equation (10) by (11), we get.

$$\frac{A \sin \phi}{A \cos \phi} = \frac{x_0}{\dot{x}_0 / \omega_n}$$

$$\Rightarrow \quad \tan \phi = \frac{x_0 \omega_n}{\dot{x}_0}$$

Hence, the phase angle, $\boxed{\phi = \tan^{-1} \frac{x_0 \omega_n}{\dot{x}_0}}$ — (12).

Solution NO. 5

$$x = A \cos (\omega n t - \phi') \quad \text{— (13)}$$

By expanding the cosine term, we get.

$$x = A \cos \omega n t \cos \phi' + A \sin \omega n t \sin \phi' \quad \text{— (14)}.$$

(using, $\cos (A - B) = \cos A \cos B + \sin A \sin B$)

The general solution is;

$$x = x_0 \cos \omega_n t + \frac{\dot{x}_0}{\omega n} \sin \omega n t$$

By comparing equ (14) with the above general solution, we get;

$$x_0 = A \cos \phi' \quad \text{— (15)}$$

$$\frac{\dot{x}_0}{\omega n} = A \sin \phi' \quad \text{— (16)}$$

By squaring and adding equ. (15) and (16), we get.

$$x_0^2 + \frac{\dot{x}_0^2}{\omega_n^2} = A^2 \cos^2 \phi + A^2 \sin^2 \phi = A^2 (\sin^2 \phi + \cos^2 \phi).$$

$$\text{or} \quad A^2 = x_0^2 + \frac{\dot{x}_0^2}{\omega n^2}$$

$\Rightarrow$ Amplitude; $\boxed{A = \sqrt{x_0^2 + \frac{\dot{x}_0^2}{\omega_n^2}}}$ — (17).

Similarly, dividing eqn. (16) by (15), we get

$$\frac{A\sin\phi'}{A\cos\phi'} = \frac{\dot{x}_0/\omega_n}{x_0}$$

$$\text{or} \quad \tan\phi' = \frac{\dot{x}_0}{x_0\omega_n}$$

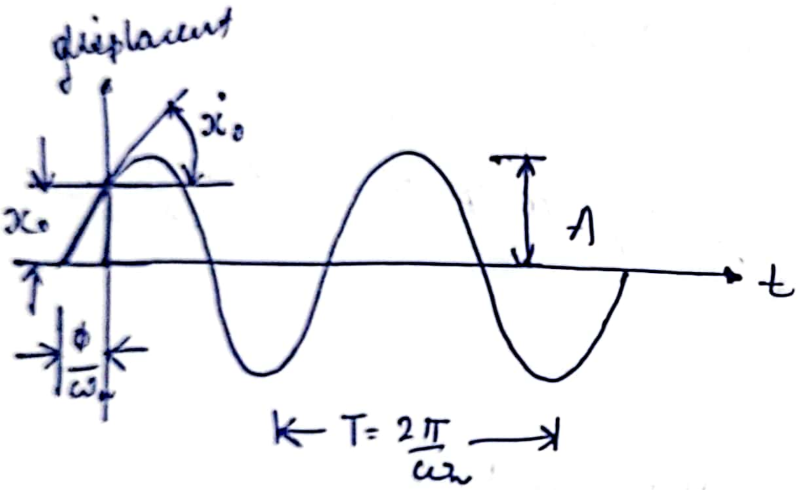
$$\Rightarrow \text{Phase angle}, \quad \boxed{\phi' = \tan^{-1}\left(\frac{\dot{x}_0}{x_0\omega_n}\right)} \quad (18).$$

Phase and Phase Angle.

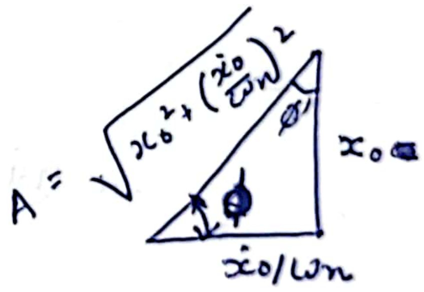
$$\tan\phi = \frac{x_0}{(\dot{x}_0/\omega_n)}$$

$$\phi = \tan^{-1}\left(\frac{x_0\omega_n}{\dot{x}_0}\right)$$

$$A = \sqrt{x_0^2 + \left(\frac{\dot{x}_0^2}{\omega_n^2}\right)}$$



Undamped free-vibration response.



Definition of angle α

where: A = Amplitude of motion
ϕ (or ϕ') = phase angle.

Example 1. A harmonic motion has a time period of 0.2s and an amplitude of 0.4cm. Find the max. velocity and acceleration.

Solution:

$$T = 0.2s$$
$$A = 0.4cm$$

frequency, $\omega_n = \frac{2\pi}{T} = \frac{2\pi}{0.2} = 31.42$ rad/s

Max. velocity $= A\,\omega_n = 0.4 \times 31.42 = 12.57$ cm/sec

Max. acceleration $= A\,\omega_n^2 = 0.4 \times (31.42)^2 = 394.89$ cm/s²

Example 2. A harmonic motion has a max. velocity of 6 m/s & it has a frequency of 12 cps, Determine its amplitude, its period and its max. acceleration.

Solution:

max. velocity; $\dot{x}_{max} = 6$ m/s

Frequency; $f = 12$ cps

$$\boxed{T = \frac{1}{f}}$$

$\omega_n = \frac{2\pi}{T} = 2\pi f$

Time period; $T = \frac{1}{12} = 0.083$ sec.

Amplitude; $A = ?$

Max. velocity; $\dot{x}_{max} = A\,\omega_n$

$$6 = A \times \omega_n$$
$$6 = A \times \left(\frac{2\pi f}{\cancel{T}}\right)$$
$$A = \frac{6 \times \cancel{0.0833}}{2\pi \times 12} \cong 0.0796\text{ m}$$

or $\boxed{A \cong 79.6\text{ mm}}$

max acceleration; $\ddot{x} = A\,\omega_n^2$

$$= 79.6 \times \left(\frac{2\pi f}{\cancel{T}}\right)^2$$
$$= 79.6 \times \left(\frac{2\pi \times 12}{\cancel{0.0833}}\right)^2$$
$$= 452587.42\text{ mm/s}^2$$

or $\cong \boxed{452.52\text{ m/s}^2}$

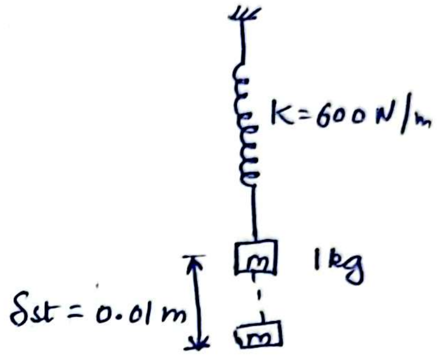
CamScanner

Q1. A mass of one kg is suspended by a spring having a stiffness of 600 N/m. The mass is displaced downward from its equilibrium position by a distance of 0.01 m. Find :

(a) Equation of motion of the system
(b) Natural frequency of the system.
(c) The response of the system as a function of time
(d) Total energy of the system.

Sol: Given details :

$$m = 1\,kg$$
$$K = 600\,N/m$$
$$\delta_{st} = 0.01\,m$$



(a) The equation of motion is given by ;

$$m\ddot{x} + Kx = 0$$
$$1\ddot{x} + 600x = 0$$

or $\boxed{\ddot{x} + 600x = 0}$

(b) The natural frequency is given by :

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{600}{1}} = \sqrt{6} \times 10 = 24.49 \text{ rad/s} \quad = 2\pi f$$

or $$f = \frac{\omega_n}{2\pi} = \frac{24.49}{2\pi} = 3.898 \text{ Hz}$$

(c) Response of the system is given by :

$$x = A \sin(\omega_n t + \phi)$$

where, Amplitude ; $$A = \sqrt{x_0^2 + \left(\frac{\dot{x}_0}{\omega_n}\right)^2} = \sqrt{(0.01)^2 + 0}$$

$$A = 0.01\,m$$

Phase angle ; $$\phi = \tan^{-1}\left[\frac{x_0 \omega_n}{\dot{x}_0}\right] = \tan^{-1}\left[\frac{0.01 \times 24.49}{0}\right]$$

$$\phi = \tan^{-1}(\infty) = 90° = \frac{\pi}{2} \qquad (\tan 90° = \tfrac{1}{0})$$

$\therefore$ Response ; $\boxed{x = 0.01 \sin\left(24.49t + (\pi/2)\right)}$

(d) The total energy is equal to the max. K.E or max. P.E.

We know that ;

$$PE_{max} = \frac{1}{2}kx^2 = \frac{1}{2} \times 600 \times (0.01)^2$$

$$= 0.03 \ \cancel{N/m} \ Nm$$

$$KE_{max} = \frac{1}{2}mv^2 = \frac{1}{2}m(\dot{x})^2_{max} = \frac{1}{2}m(A\omega_n)^2$$

$$= \frac{1}{2} \times 1 \times (24.49 \times 0.01)^2$$

$$= 0.03 \ \cancel{N/m} \ Nm$$

$$\therefore \boxed{\text{Total energy} = 0.03 \ \cancel{N/m}} \ Nm$$

Q2. A system vibrating with a natural frequency of 6 Hz start with an initial amplitude (x_0) of 2cm and an initial velocity ($\dot{x}_0$) of 25 cm/s. Determine the (i) natural period.

(ii) amplitude

(iii) max. velocity

(iv) max. acceleration

(v) phase angle.

Also write the equation of motion of a vibrating system.

Solution: Given details :

$f = 6$ Hz

$x_0 = 2$ cm

$\dot{x}_0 = 25$ cm/s

(i) The natural period ; $T = \frac{1}{f} = \frac{1}{6} = 0.167$ s

(ii) amplitude of motion , $A = \sqrt{x_0^2 + \left(\frac{\dot{x}_0}{\omega_n}\right)^2}$

where; $\omega_n = 2\pi f = 2\pi(6) = 37.7$ rad/s

$$\therefore \quad A = \sqrt{2^2 + \left(\frac{25}{37.7}\right)^2} = 2.11 \text{ cm}$$

(iii) max velocity ; $\dot{x}_{max} = A\omega_n$

$$= 2.11 \times 37.7 = 79.55 \text{ cm/s}$$

(iv) max. acceleration of a system ;

$$\ddot{x}_{max} = A\,\omega_n^2$$
$$= 2.11 \times 37.7^2$$
$$= 2998.92\ cm/s^2$$
$$= 29.98\ m/s^2$$

(v) Phase angle ; $\phi = \tan^{-1}\left[\dfrac{x_0 \omega_n}{x_0}\right]$

$$= \tan^{-1}\left[\frac{2 \times 37.7}{25}\right]$$
$$\phi = \tan^{-1}[3.016]$$
$$\phi = 71.66^\circ$$
$$= 71.66 \times \frac{\pi}{180}\ rad.$$
$$\phi = 1.25\ rad.$$

Thus, the equation of motion is ; $x = A \sin(\omega_n t + \phi)$

$$\boxed{x = 2.11 \sin(37.7t + 1.25)}$$

Q3. A vertical cable 3m long has a cross-sectional area of $4 cm^2$ supports a weight of 50kN. What will be the natural period and natural frequency of the system? $E = 2.1 \times 10^6\ kg/cm^2$

Sol: Given details : Area = $4 cm^2$
W = 50kN ; L = 3m = 300cm

as W = mg

$$\therefore m = \frac{W}{g} = \frac{50 \times 10^3}{9.81} = 5096.8\ kg$$

$$E = 2.1 \times 10^6\ kg/cm^2$$

$K = \frac{P}{\delta}$

$K = \frac{AE\delta}{L\delta}$

Stiffness ; $K = \dfrac{AE}{L} = \dfrac{4 \times 2.1 \times 10^6}{300}\ \dfrac{cm^2 \times kg/cm^2}{cm}$

$E = \frac{\sigma}{\epsilon}$

$E = \frac{PL}{A\delta L}$

$\delta L = \frac{PL}{AE}$

1kg = 9.81 N

$$k = 28000\ kg/cm = 28000 \times \frac{9.81}{10^{-2}}$$
$$= 27.468 \times 10^6\ N/m$$

Natural frequency; $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{27.468 \times 10^6}{5096.8}}$

$= 73.41 \text{ rad/s}$

Natural period; $T = \frac{2\pi}{\omega_n} = \frac{2\pi}{73.41}$ $\left(\omega_n = \frac{2\pi}{T}\right)$

$T = 0.085 \text{ sec}$

also; Frequency $\Rightarrow f = \frac{1}{T} = \frac{1}{0.085} = 11.76 \text{ Hz}$

or 11.76 cps.

Q4. A one kg mass is suspended by a spring having a stiffness of 1N/mm. Determine the natural frequency and static deflection of the spring.

Solution: Given details:

$k = 1 N/mm = \frac{1N}{10^{-3} m} = 1000 N/m$

$m = 1 kg$

- Natural frequency; $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1000}{1}}$

$\omega_n = 31.62 \text{ rad/s}$

$f = \frac{\omega_n}{2\pi} = 5.03 \text{ Hz}$

- Static deflection $= \delta_{st} = ?$

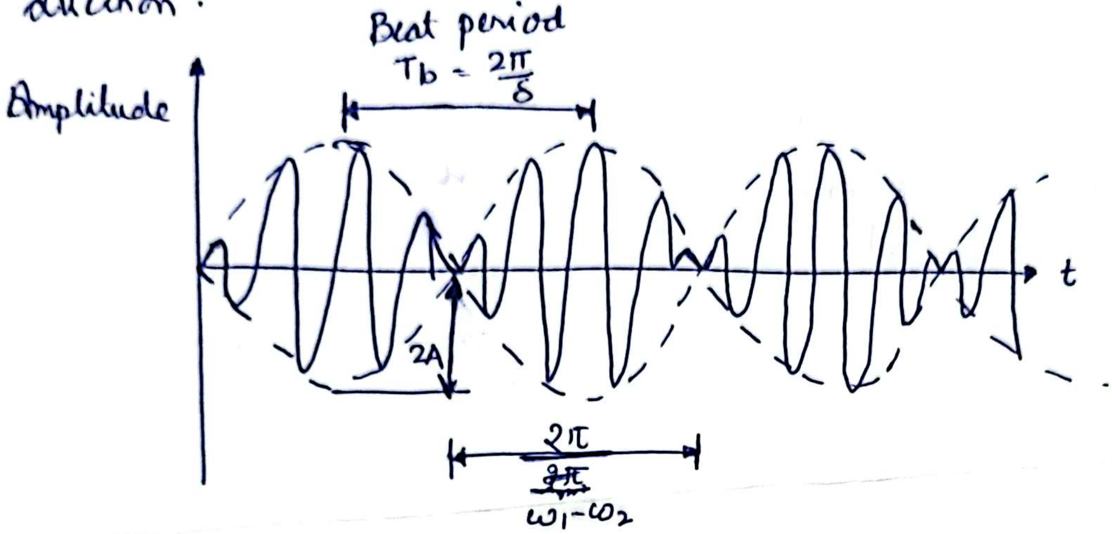
$\Rightarrow \omega_n = \sqrt{\frac{g}{\delta_{st}}}$ or $\omega_n^2 = \frac{g}{\delta_{st}}$

$\Rightarrow \delta_{st} = \frac{g}{\omega_n^2} = \frac{9.81}{(31.62)^2} = 0.00981 \text{ m}$

$= \boxed{9.81 \text{ mm}}$

When two SHMs pass through the same point in a medium simultaneously, the resultant displacement at that point is the sum of the displacements due to two components of motion. This superposition of motion is called <u>interference</u>.

- The phenomenon of beat occurs as a result of interference between two waves of slightly different frequencies moving along the same straight line in the same direction.



Beating phenomenon.

Consider that at a particular time, the two wave motions are in the same phase. At this stage, the resultant ~~displacement~~ amplitude of vibration will be maximum. On the other hand, when two motions are not in phase with each other, they produce minimum amplitude of vibration.

Again after some time, the ~~mot~~ two motions are in phase and produce maximum amplitude and then minimum amplitude. This process goes on repeating and the resultant amplitude continuously keeps on changing from maximum to minimum, with a frequency equal to the difference between the frequencies. This phenomenon is known as 'beat' or 'beating phenomenon' as shown in figure above.

(The frequency of beats i.e ($\Delta\omega$) should be small in order to experience the phenomenon)

PHASE DIFFERENCE

Phase difference between two SHMs indicates how much the two motions are out of step with each other or by how much angle or how much time one is ahead of the other.

- When two sinusoidal motion are out of phase then phase difference $(\Delta \omega) = 180^\circ$. When two sinusoidal motions are in phase, then phase difference $(\Delta \omega) = 0$.