1 EM Waves & Dielectrics

(1) INTRODUCTION

Electrostatics is a branch of physics which deals with study of charges at rest while in electrodynamics we study electric effects of moving charges. Similarly in magnetostatics, we study magnetic effects of constant currents and in magnetodynamics, magnetic effects of varying currents are studied. Earlier it was assumed that electric and magnetic processes are independent of each other. However, it was proved by Maxwell later on that electric and magnetic processes are actually related to each other. The branch of physics in which both electric and magnetic effects are studied together is called electromagnetism.

(2) SOME IMPORTANT DEFINITIONS

We shall describe briefly a few definitions in electrostatics.

1. The region in space, in which a scalar quantity has unique value or magnitude at every point is called scalar field. For example temperature and electric potential. The scalar quantity is in general a function of the position co-ordinates.

2. The region in space in which a vector quantity has unique value (magnitude and direction) at every point is called vector field. For example gravitational field, electric field & magnetic field.

3. We define a differential operatcr

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z} \qquad ...(1)$$

This operator is called 'Del' operator. It has no meaning in itself. But it acquires meaning, when it is applied on some quantity which is a function of position co-ordinates.

For example if V (x, y, z) is a scalar defined as

$$V(x, y, z) = x^3 + y^3 + x^2 yz$$

then

$$\vec{\nabla} V = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(x^3 + y^3 + x^2 yz)$$

$$= \hat{i}(3x^2 + 0 + 2xyz) + \hat{j}(0 + 3y^2 + x^2 z) + \hat{k}(0 + 0 + x^2 y)$$

$$= \hat{i}(3x^2 + 2xyz) + \hat{j}(3y^2 + x^2 z) + \hat{k}(x^2 y) \qquad ...(2)$$

Similarly if a vector quantity is defined as $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then its dot product with $\vec{\nabla}$ is calculated as follows :

$$\vec{\nabla}\cdot\vec{r} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right).(x\hat{i} + y\hat{j} + z\hat{k})$$

$$= \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(z) = 1 + 1 + 1 = 3 \qquad ...(3)$$

Similarly cross product of $\vec{r}$ with $\vec{\nabla}$ is calculated as follows

$$\vec{\nabla}\times\vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial(z)}{\partial y} - \frac{\partial(y)}{\partial z}\right) - \hat{j}\left(\frac{\partial(z)}{\partial x} - \frac{\partial(x)}{\partial z}\right) + \hat{k}\left(\frac{\partial(y)}{\partial x} - \frac{\partial(x)}{\partial y}\right)$$

$$= \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(0-0) \qquad ...(4)$$

$$\therefore \qquad \vec{\nabla}\times\vec{r} = 0$$

4. "Gradient of a scalar field function is defined as operation of $\vec{\nabla}$ on scalar field function."

i.e. if V is some scalar field function then its gradient is given by

$$\text{grad (V)} = \vec{\nabla} V$$

Physical Significance of Gradient. The gradient of a scalar field at a point is equal to maximum rate of change of function w.r.t. position of that point and is directed along the direction in which this maximum rate of change of scalar field function occurs.

Figure (1) shows a scalar field function V (x, y, z) having two equifield surfaces. Thus value of function everywhere on surface I is V and for all points on surface II is V + dV.

If we move from point P on surface I to points A or B on surface II, then change in V in both cases is dV.

However distance PA is least. Hence rate of change of V is $\left(\frac{dV}{dr}\right)$ which is maximum along $\hat{r}$ (a unit vector along PA).

Thus grad V = $\left(\frac{dV}{dr}\right)_{max} \hat{r}$

It can be shown that grad V is given by

$$\text{grad V} = \vec{\nabla} V \qquad ...(5)$$

Note that although V is a scalar function, however its gradient is a vector quantity. *e.g.* in equation (2) this fact is easily demonstrated. There we have actually found gradient of scalar field function V (x, y, z).

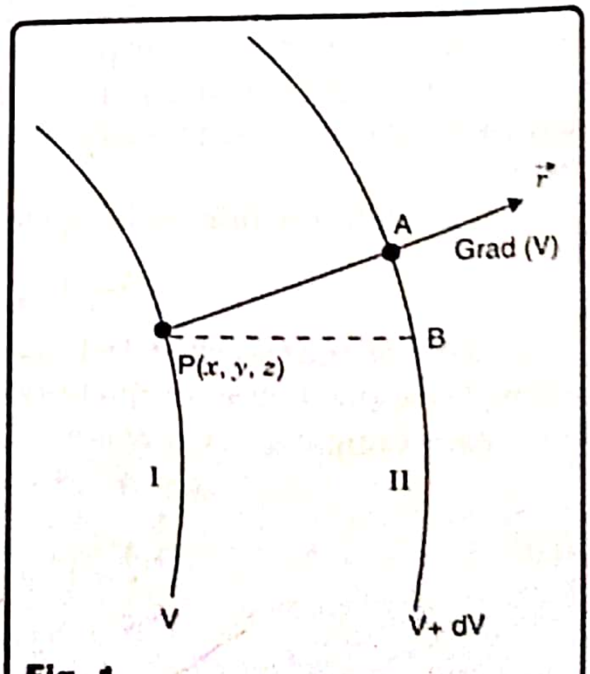

Fig. 1.

5. "The divergence of a vector field at a point is defined as its dot product with $\vec{\nabla}$. *i.e.* if $\vec{E}$ is any vector, then its divergence at a point is given as

$$\text{div}(\vec{E}) = \vec{\nabla}.\vec{E} \qquad ...(6)$$

Physical Significance of Divergence. The divergence of a vector field at any point gives maximum flux (of the vector field) per unit volume of a closed surface drawn around that point such that volume of closed surface approaches zero.

If $\vec{E}$ is a vector field function then

$$\text{div}\,\vec{E} = \lim_{V \to 0} \frac{1}{V}(\phi) = \lim_{V \to 0} \frac{1}{V} \iint \vec{E}.d\vec{S}$$

Although $\vec{E}$ is a vector function, but its divergence is always a scalar (see equation (3) as an example.)

6. "The curl of a vector field at a point is defined as its cross product with $\vec{\nabla}$." *i.e.* if $\vec{E}$ is any vector field then its curl at a point is given as

$$\text{curl}\,(\vec{E}) = \vec{\nabla} \times \vec{E} \qquad ...(7)$$

Physical Significance of Curl. The magnitude of curl of a vector field at a point gives maximum circulation per unit area of a closed path drawn around that point such that area of closed path approaches zero.

Let $\vec{E}$ is a vector field function existing in the region. Consider any hypothetical closed path C as shown in fig. 2. $\vec{E}$ may or may not be uniform in the region. Then circulating power (or circulation) of vector field is defined as closed line integration of $\vec{E}$ along closed path C.

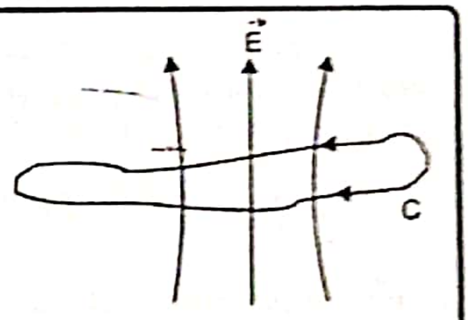

Fig. 2.

Thus circulation = $\oint \vec{E}.d\vec{l}$.

Hence $$|\text{curl}\,\vec{E}| = \lim_{S \to 0} \frac{1}{S} \oint \vec{E}.d\vec{l}$$

where S is area of the closed path.

One example we have already given in equation (4)

Note that curl of a vector field is also a vector quantity.

7. Some important mathematical results are given below :

(*a*) $\vec{\nabla}.\vec{r} = 3$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

(*b*) $\vec{\nabla} \times \vec{r} = 0$

(*c*) $\vec{\nabla} \times (\phi\vec{A}) = (\vec{\nabla}\phi) \times \vec{A} + \phi\vec{\nabla} \times \vec{A}$ where ϕ is some scalar

(*d*) $\vec{\nabla}.(\vec{\nabla} \times \vec{A}) = 0$ *i.e.* div (curl $\vec{A}$) = 0 [where $\vec{A}$ is any vector]

(*e*) $\vec{\nabla} \times \vec{\nabla}\phi = 0$ *i.e.* curl (grad ϕ) = 0

(*f*) $\vec{\nabla}.\vec{\nabla}\phi = \nabla^2\phi$

(*g*) $\vec{\nabla}\left(\frac{1}{r^n}\right) = -\frac{n\vec{r}}{r^{n+2}}$

(*h*) $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla}.\vec{A}) - \nabla^2\vec{A}$

(*i*) $\vec{\nabla}\left(\frac{1}{r}\right)=\frac{-\vec{r}}{r^3}$ where $r=\sqrt{x^2+y^2+z^2}$

(*j*) $\vec{\nabla}\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)=\frac{-(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$ where $\vec{r}'=x'\hat{i}+y'\hat{j}+z'\hat{k}$

(*k*) $\vec{\nabla}'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)=\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$ where $\vec{\nabla}'=\hat{i}\frac{\partial}{\partial x'}+\hat{j}\frac{\partial}{\partial y'}+\hat{k}\frac{\partial}{\partial z'}$

(*l*) $\vec{\nabla}\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)=-\vec{\nabla}'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)$

8. **Stokes theorem.** It states that line integration of a vector field function around a closed path is equal to surface integration of the curl of vector field taken over an open surface bounded by the closed path.

i.e. for any vector field $\vec{E}$,

$$\oint_C \vec{E}\,.\,d\vec{l}=\iint_S(\vec{\nabla}\times\vec{E})\,.\,d\vec{S} \quad ...(5)$$

where C represents closed path and S represents area of the closed path.

9. **Gauss divergence theorem.** It states that the surface integration of a vector field function over a closed surface is equal to volume integration of the divergence of the function taken over the volume of the closed surface.

Thus for any vector field $\vec{E}$

$$\oiint_S \vec{E}\,.\,d\vec{s}=\iiint_V(\vec{\nabla}\,.\,\vec{E})\,dV \quad ...(6)$$

where V is the volume of the closed surface S.

(3) DIFFERENTIAL FORM OF GAUSS LAW IN VACUUM

Fig. (3) shows a closed surface S in vacuum enclosing net charge Q.

Let $\vec{E}$ is electric field at any point A on the surface. The total charge enclosed is related to volume charge density ρ by

$$Q=\iiint\rho\,dV \quad ...(7)$$

According to Gauss law in integral form, we have

$$\oiint\vec{E}\,.\,d\vec{S}=\frac{Q}{\varepsilon_0}=\iiint\frac{\rho}{\varepsilon_0}\,dV$$

Apply Gauss divergence theorem, we get

$$\iiint\vec{\nabla}\,.\,\vec{E}\,dV=\iiint\frac{\rho}{\varepsilon_0}\,dV$$

where V is the volume bounded by the surface S.

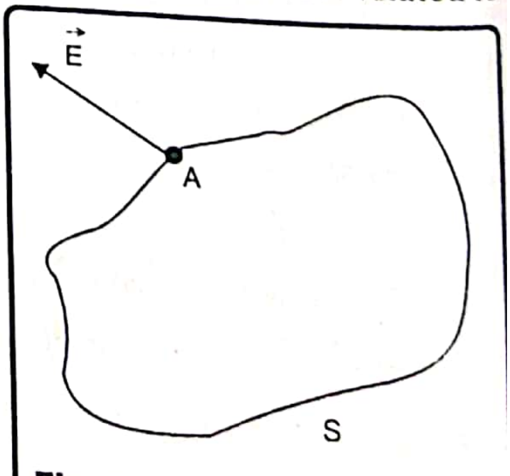

Fig. 3.

or $$\iiint\left(\vec{\nabla}.\vec{E}-\frac{\rho}{\varepsilon_0}\right)dV=0$$

This integration is zero independent of the volume of the surface *i.e.* independent of the limits of integration. Hence the integrand itself should be zero.

Thus $$\vec{\nabla}.\vec{E}-\frac{\rho}{\varepsilon_0}=0 \quad \text{or} \quad \vec{\nabla}.\vec{E}=\frac{\rho}{\varepsilon_0} \qquad ...(8)$$

This equation gives differential form of Gauss law in vacuum. It is a point equation *i.e.* its solution gives the value of electric field at that point.

However for electrostatics, the electric field is conservative. Hence it is equal to the negative of the gradient of electric potential at that point.

i.e. $$\vec{E}=-\vec{\nabla}V \qquad ...(9)$$

Put in (8), we get

$$\vec{\nabla}.(-\vec{\nabla}V)=\frac{\rho}{\varepsilon_0} \quad \text{or} \quad \nabla^2 V=\frac{-\rho}{\varepsilon_0} \qquad ...(10)$$

This equation is called Poisson's equation of electrostatics.

In a charge free region, the surface will enclose no charge

i.e. $\rho = 0$ Put in (10), we get

$$\nabla^2 V=0 \qquad ...(11)$$

This equation is called Laplace equation of electrostatics. The equation is very useful in solving many problems of physics *e.g.* solution of hydrogen atom in spherical polar co-ordinates. Any scalar function satisfying Laplace equation is called Harmonic function. The solution of Laplace equation in spherical Polar co-ordinates are called Spherical harmonics.

(4) DIELECTRICS

These are the materials, which do not conduct electricity but when placed in strong electric field, allow the passage of electric field lines through them.

Dielectric materials are of two kinds.

(*i*) **Polar Dielectrics.** Those dielectrics, whose each molecule possesses non zero dipole moment in the absence of external electric field are called Polar Dielectrics. In such molecules the centre of positive and negative charges are at different points. These molecules are usually unsymmetrical and various atoms forming molecule donot have same electronegativity value so that –ve charge in the molecule resides near more electronegative region of molecule and positive charge resides near more electropositive region of the molecule.

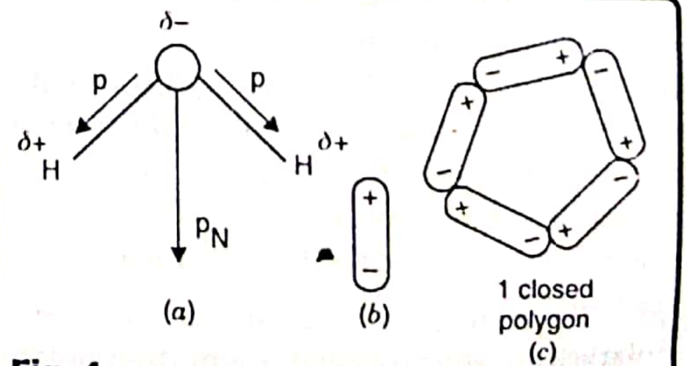

Fig. 4.

Fig. (4(*a*)) shows structure of water molecule. Its shape is V-like. Since O-atom is more electronegative than H-atom. So electron pair in both covalent bonds shifts slightly towards O-atom. So that O-atom acquires partially negative charge and H-atoms acquire partially positive charge. Thus every bond has dipole moment p and net dipole moment p_N is given by parallelogram law of vectors, obviously p_N is non zero as dipoles p & p are not in opposite directions. Other examples of polar molecules are HCl, NH_3 etc.

A polar molecule is usually represented, as shown in fig. 4(*b*). However in the absence of external electric field various molecules within the sample are randomly oriented so that they form closed loops, as shown in fig. 4(*c*). Hence although on microscopic level dipole of each molecule is non zero, but due to random distribution of dipoles, the net dipole moment of the sample is zero when no external electric field is applied.

(*ii*) **Non Polar Dielectrics.** A dielectric is said to be non polar if dipole moment of each molecule of the sample is zero when no external electric field is applied.

Such molecules are symmetrical and are made from atoms in such a way that atoms of equal electronagetivity cancel each other's effect in the molecule. Hence centres of +ve and negative charge lie at the same point. A non polar molecule is represented as shown in the figure (5). Examples of non polar molecules are H_2, Cl_2, CH_4 etc. Obviously the net dipole moment of the sample of non polar dielectric will also be zero when no external electrif field is applied.

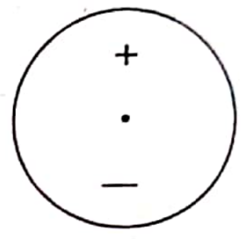

Fig. 5.

Thus in the absence of external electric field the polar and non polar dielectrics differ only at microscopic level.

(5) POLARISATION OF A DIELECTRIC

When a sample of dielectric is placed in external electric field then net dipole moment of the whole sample becomes non zero and bounded charges are produced at its surface. This process is called polarisation of dielectric.

(*a*) **Polarisation of Polar Dielectric.** When no external electric field is applied across a polar dielectric then its dipoles are randomly oriented so that net dipole moment of the sample is zero.

However when this sample is placed in external electric field, then each dipole experiences torque whose tendency is to align the dipoles in the direction of applied external electric field.

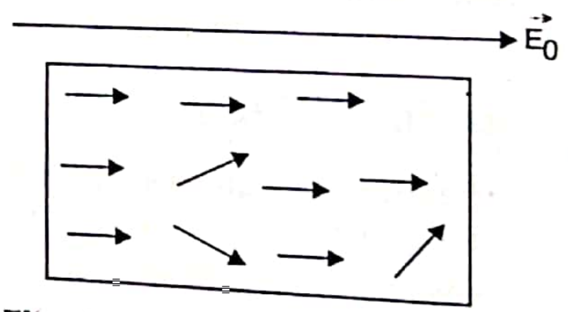

Fig. 6.

Since $\tau = p\,E_0 \sin\theta$

and θ is different for different dipoles. Thus more torque is required to align dipoles, which initially make large angle with E_0 and less torque is required for aligning dipole which initially make small angle with electric field. Thus different dipoles may take different time to align in the direction of field. However with increase in magnitude of applied electric field, more and more dipoles will start aligning in the direction of applied electric field. Thus dipole moment of the sample goes on increasing. However, a stage will be reached, when all dipoles will be oriented in the direction of applied electric field. In that case material is said to be saturately polarised. If electric field is now further increased, no increase in dipole moment occurs.

Since material is polarised because of reorientation of dipoles. Hence this kind of polarisation is also called orientation polarisation.

(*b*) **Polarisation of Non Polar Dielectric.** The dipole moment of each molecule of non polar molecule is zero initially as shown in fig. (5). Hence polarisation is not possible because of torque as $\tau = p\,E\sin\theta = 0$ ($\because\ p = 0$)

Thus mechanism of polarisation of non polar dielectric is different from that of polar dielectric.

When sample of non polar molecule is placed in external electric field, then the centre of positive charge experiences force $\vec{F} = q\,\vec{E}_0$ in the direction of electric field and centre of –ve charge experiences force $\vec{F}_1 = -q\,\vec{E}_0$ opposite to the direction of electric field (see fig. (7)).

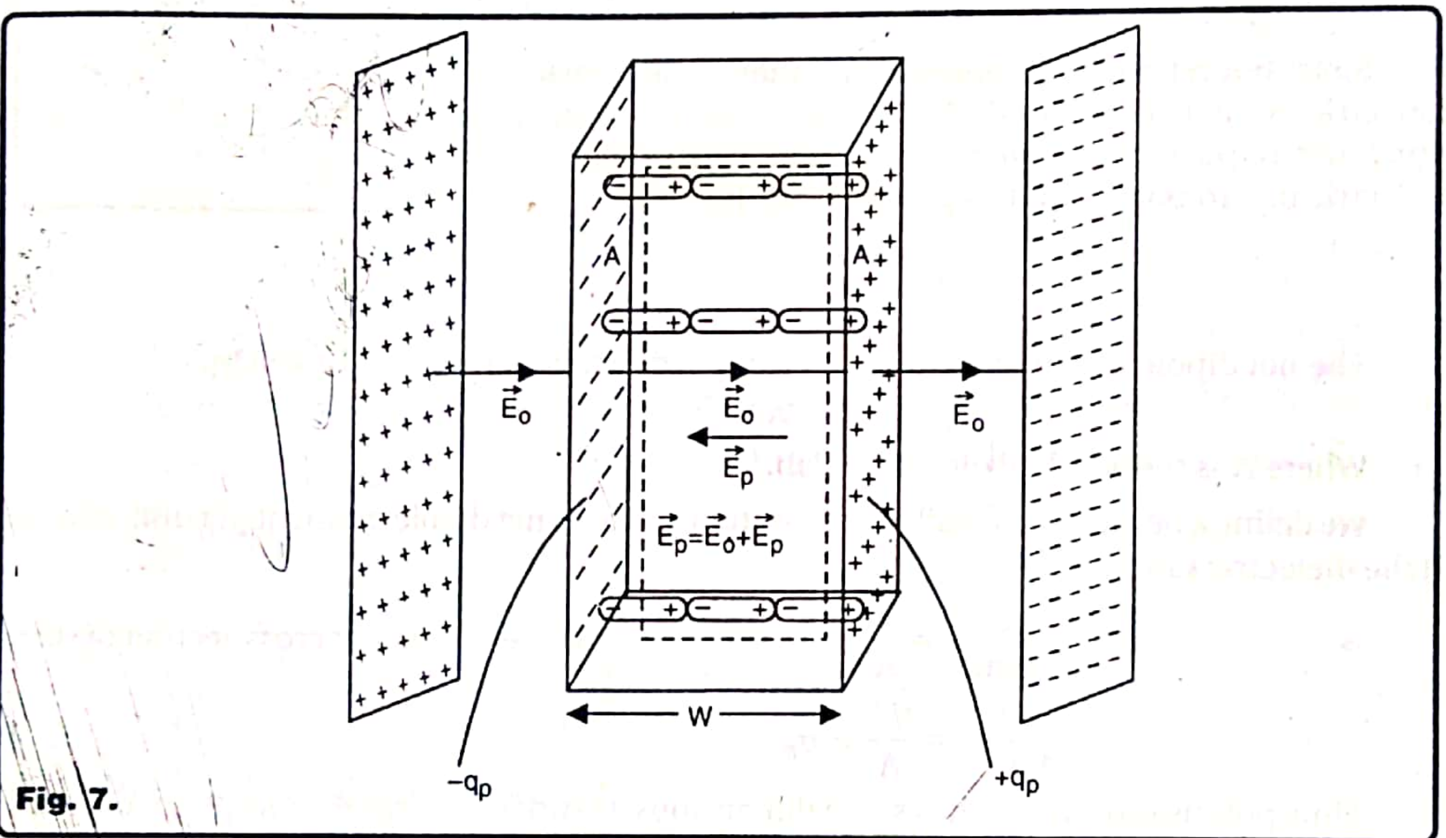

Fig. 7.

Thus centres of +ve and negative charges get separated and each molecule acquires a dipole moment $\vec{p}$ in the direction of applied electric field. Thus net dipole moment of sample becomes non zero and material is thus polarised. This kind of polarisation is called "Electronic Polarisation."

We know that an electric dipole is itself a source of electrifield. Thus as every molecule becomes slightly polar, it modifies the electric field existing inside the dielectric. Thus net electric field inside dielectric becomes.

$$\vec{E} = \vec{E}_0 + \vec{E}_p \qquad \text{...(12)}$$

Thus each molecule now experiences electric field $\vec{E}$ instead of $\vec{E}_0$. Here $\vec{E}_p$ is the electric field produced because of polarisation of dielectric.

The dipole moment of every molecule is found to be directly proportional to the net electric field $\vec{E}$.

i.e. $\vec{p} \propto \vec{E}$ or $\vec{p} = \varepsilon_0 \alpha\,\vec{E}$ ...(13)

Here the constant of proportionality α is called Atomic/Molecular Polarizability. It has units of volume *i.e.* m^3.

We now turn our attention to figure (7). The charges induced the surface of Dielectric are uncompensated and cannot cancel. While induced charges inside the sample (inside dotted region) cancel each other's effect. Thus bound charges appear only on the surface of dielectric.

Let $-q_p$ is net negative bound charge developed on left face

$+q_p$ is net +ve bound charge developed on right face.

W = Width of dielectric slab.

σ_p = surface charge density on each face of dielectric slab

$$= \frac{q_p}{A} \quad \text{...(51)}$$

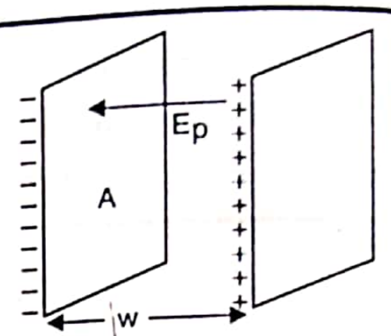

Fig. 8.

Since bound charges cancel each other's effect inside the dielectric, we may consider dielectric slab to be a system of two equal and oppositely charged plates. Thus electric field inside dielectric due to polarisation is given as (See fig. (8))

$$E_p = \frac{\sigma_p}{\varepsilon_0} \quad \text{...(15)}$$

The net dipole moment acquired by sample due to polarisation is given by

$$p_N = (q_p)\,(W)$$

Where W is the width of dielectric slab.

We define a new vector $\vec{P}$ called polarisation Vector as net dipole moment per unit volume of the dielectric sample.

$$\Rightarrow \quad P = \frac{p_N}{\text{Volume}} = \frac{p_N}{AW} \qquad \text{where A = area of cross section of slab.}$$

$$= \frac{(q_p)(W)}{AW} = \frac{q_p}{A} = \sigma_p \quad \text{...(16)}$$

Thus polarisation vector has same dimensions as surface charge density. From (53) and (15), we get

$$E_p = \frac{P}{\varepsilon_0} \quad \text{...(17)}$$

The polarisation vector P is also found to be proportional to net electric field.

i.e. $P \propto E$ or $P = \chi_e \varepsilon_0 E$ ...(18)

Here χ_e is called electric susceptibility of the dielectric. It is a dimensionless parameter, which is always positive. For given value of E, if χ_e is large then value of P will also be large.

Let n = number of molecules (dipoles) per unit volume of the sample.

Since all dipoles are always oriented in same direction for non polar dielectric. Hence P & p are related by obvious relation.

$$P = np$$

$$\chi_e \varepsilon_0 E = n\,(\alpha\, \varepsilon_0 E) \quad \text{...(19)}$$

or $\chi_e = n\alpha$ ...(20)

Equation (20) gives relation between electric susceptibility and molecular polarizability.

Substitute (18) in (17), we get

$$E_p = \chi_e E \quad \text{...(21)}$$

Also $\vec{E}_0$ and $\vec{E}_p$ are directed in opposite direction. Thus equation (12), in magnitude can be written as

$$E = E_0 - E_p = E_0 - \chi_e E \quad \text{or} \quad E(1+\chi_e) = E_0$$

or $$1 + \chi_e = \frac{E_0}{E}$$

or $$K = 1 + \chi_e \qquad ...(22) \left(\because K = \frac{E_0}{E}\right)$$

where $K = \frac{E_0}{E}$ is dielectric constant of the material.

Equation (22) gives relation between electric susceptibility and dielectric constant or between susceptibility and permittivity.

(*c*) **Polarisation of Non Homogeneous Dielectric.** Consider a dielectric material composed of both types of dielectrics. Then on placing such material in external electric field both orientation as well as Electronic polarisation will take place. In this case the bound charges developed in the bulk of dielectric will not cancel each other's effect because of non uniformity. Thus there will be induced volume charge density ρ_p instead of surface charge density (σ_p). The volume charge density of induced charges is related to the polarisation vector $\vec{P}$ by the relation.

$$\rho_p = -\vec{\nabla} . \vec{P} \qquad ...(23)$$

The proof of this relation is beyond of the scope of this book.

The net induced bound charge is given by

$$q_p = \iiint \rho_p \, dV \quad \text{or} \quad q_p = -\iiint \vec{\nabla} . \vec{P} \, dV \qquad ...(24)$$

(6) DIELECTRIC STRENGTH

We know that due to polarisation of dielectric in the external field, the separation between centres of opposite charges is produced, which increases further, with increase in the electric field. If we continue to increase external electric field, then a stage is reached, when the outer electons get separated from the dielectric molecule. Thus suddenly a number of electrons are produced and dielectric starts behaving a conductor. This process is called dielectric Breakdown.

"The Dielectric strength may be defined as the maximum value of electric field, that can be applied to the dielectric with out its electric breakdown."

Dielectric strength depends upon, thickness of material, temperature, dielectric constant, humidity and time for which electric field is applied.

(7) GAUSS'S LAW FOR DIELECTRICS

Consider a region in space which contains free charges, polar molecules, as well as non polar molecules.

Let ρ_f is the volume charge density of free charges in the region. Then net free charge in the region is given by

$$q_f = \iiint \rho_f \, dV \qquad ...(25)$$

The electric field due to free charges, will cause the polarisation of dielectric present in the region and bound charges will be produced in region.

Net bound charge produced in the region is given by

$$q_p = -\iiint \vec{\nabla} \cdot \vec{P} \, dV \qquad ...(26)$$

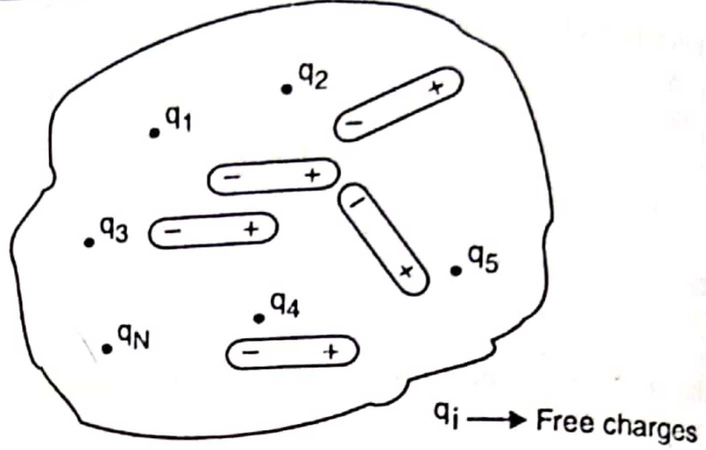

Fig. 9.

Let $\vec{E}$ is electric field at any point on the surface. Then net charge enclosed by surface is $q_f + q_p$. Hence by Gauss law.

$$\oiint \vec{E} \cdot \vec{ds} = \frac{q_f + q_p}{\varepsilon_0} = \frac{\iiint \rho_f \, dV - \iiint \vec{\nabla} \cdot \vec{P} \, dV}{\varepsilon_0}$$

Apply Gauss divergence theorem on left side, we get

$$\iiint \vec{\nabla} \cdot \vec{E} \, dV = \frac{\iiint \rho_f \, dV - \iiint \vec{\nabla} \cdot \vec{P} \, dV}{\varepsilon_0}$$

or

$$\iiint \vec{\nabla} \cdot (\varepsilon_0 \vec{E} + \vec{P}) \, dV = \iiint \rho_f dV \qquad ...(27)$$

We define a new vector

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

This vector is called electric displacement vector. It has same units as $\vec{P}$.

Thus equation (27) becomes

$$\iiint \vec{\nabla} \cdot \vec{D} \, dV = \iiint \rho_f \, dV$$

or

$$\iiint (\vec{\nabla} \cdot \vec{D} - \rho_f) \, dV = 0 \qquad ...(28)$$

This integration is always zero, independent of the volume of the body *i.e.* limits of integration. Hence the integrand itself should be zero.

i.e. $\vec{\nabla} \cdot \vec{D} - \rho_f = 0$ or $\vec{\nabla} \cdot \vec{D} = \rho_f \qquad ...(29)$

This equation gives Gauss law of Dielectrics. This equation can also be written in integral form as

$$\oiint \vec{D} \cdot \vec{dS} = q_f \qquad ...(30)$$

(By using Gauss divergence theorem to (29) & using (25)

Importance of Electric Displacement Vector

We know $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

If we consider figure (7), we notice that $\vec{E}$ & $\vec{P}$ are in same direction also ε_0 is positive number. Hence $\vec{D}, \vec{E}, \vec{P}$ will be in same direction. Thus we may drop vector notations.

Thus $$D = \varepsilon_0 E + P \quad ...(31)$$

But $$E = E_0 - E_p = E_0 - \frac{P}{\varepsilon_0} \quad \text{(See article 5(}b\text{))}$$

Thus $$D = \varepsilon_0 \left(E_0 - \frac{P}{\varepsilon_0} \right) + P \quad \Rightarrow \quad D = \varepsilon_0 E_0 \quad ...(32)$$

Alternatively we know that $\dfrac{\varepsilon}{\varepsilon_0} = K = \dfrac{E_0}{E}$

Thus $$\varepsilon_0 E_0 = \varepsilon E \quad ...(33)$$

Hence (32) can also be written as

$$D = \varepsilon E \quad ...(34)$$

In vector notation $$\vec{D} = \varepsilon_0 \vec{E}_0 = \varepsilon \vec{E} \quad ...(35)$$

If we give a close look to (33), we observe that quantity $\varepsilon_0 E_0$ is constant *i.e.* if a medium is placed in the region (ε_0 is changed to ε) then electric field modifies itself (charges from E_0 to E) such that product $\varepsilon_0 E_0$ or εE remains same. Thus electric displacement vector is a vector which does not depend on the presence of medium. It actually depends upon the distribution of charge in the region only.

e.g., consider a point charge q. The electric field at a distance r from q is given in vacuum and medium respectively as

$$E_0 = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \quad \text{and} \quad E = \frac{1}{4\pi\varepsilon} \frac{q}{r^2}$$

so that $$\varepsilon_0 E_0 = \varepsilon E = D = \frac{1}{4\pi} \frac{q}{r^2}$$

clearly D is independent of presence of medium and depends only on the distribution of charge.

Thus it is better to write equations of electrostatics in term of $\vec{D}$ instead of $\vec{E}$.

(8) DIFFERENTIAL FORM OF AMPERE'S CIRCUITAL LAW

According to Ampere's circuital law, the line integral of magnetic flux density over a closed path in vaccum is equal to μ_0 times the current threaded by closed path

i.e. $$\oint \vec{B} . \vec{dl} = \mu_0 I \quad ...(36)$$

Let $\vec{J}$ is the current density in the conductor then current threaded by closed path is given as

$$I = \iint \vec{J} . d\vec{S} \quad ...(37)$$

(note the area of closed path is an open area).

Put equation (37) in equation (36), we get

$$\oint \vec{B}.\vec{dl} = \mu_0 \iint \vec{J}.\vec{dS}$$

Apply stokes theorem, we get

$$\iint \vec{\nabla}\times\vec{B}.\vec{dS} = \iint \mu_0 \vec{J}.\vec{dS} \quad \text{or} \quad \iint (\vec{\nabla}\times\vec{B} - \mu_0 \vec{J}).\vec{dS} = 0$$

This integration is zero independent of the area of closed with (*i.e.* limits of integration). Hence integrand itself should be zero.

or $$\vec{\nabla}\times\vec{B} - \mu_0 \vec{J} = 0 \quad \text{or} \quad \vec{\nabla}\times\vec{B} = \mu_0 \vec{J}$$

This equation is differential form of Amperels circuital law for magnetostatics.

(9) EQUATION OF CONTINUITY

Consider a conductor, carrying current I. The conductor is bounded by a closed surface. Let ρ is volume charge density, then total charge enclosed by the conductor is given by

$$q = \iiint \rho dV$$

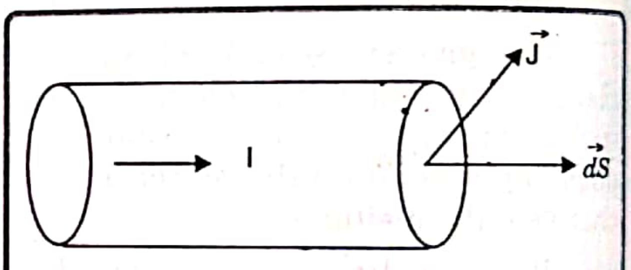

Fig. 10.

Thus current flowing through conductor is given by

$$I = -\frac{\partial q}{\partial t} = \frac{-\partial}{\partial t}\iiint \rho\, dV$$

$$= \iiint\left(\frac{-\partial\rho}{\partial t}\right)dV \quad \text{...(38)}$$

Here negative sign shows the outward flow of charge through the surface of conductor. Also current is given by

$$I = \oiint \vec{J}.\vec{dS} = \iiint \vec{\nabla}.\vec{J}\, dV$$ ...(39) (Using Gauss divergence Theorem)

Equationg (99) and (100), we get

$$\iiint \vec{\nabla}.\vec{J}\, dV = -\iiint \frac{\partial\rho}{\partial t} dV \quad \text{or} \quad \iiint\left(\vec{\nabla}.\vec{J} + \frac{\partial\rho}{\partial t}\right)dV = 0$$

This integration is zero independent of the volume of the conductor (*i.e.* limits of integration). Thus integrand intself should be zero.

Thus $$\vec{\nabla}.\vec{J} + \frac{\partial\rho}{\partial t} = 0 \quad \text{...(40)}$$

This equation is called equation of continuity and is the basic equation for conservation of charge.

If current flowing through conductor is steady then volume charge density remains constant *i.e.* $\frac{\partial\rho}{\partial t} = 0$

Put in (40) we get $\quad \vec{\nabla}.\vec{J} = 0$

Thus if current is steady, then divergence of current density is zero.

(10) AMPERE'S CIRCUITAL LAW FOR VARYING CURRENTS

The differential form of Ampere's circuital law is given as

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \qquad ...(41)$$

Taking divergence on both sides, we get

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = \mu_0 \vec{\nabla} \cdot \vec{J} \qquad ...(42)$$

But we know that divergence of curl of any vector is always zero (see article 7(7)). Thus (42) gives us

$$0 = \mu_0 (\vec{\nabla} \cdot \vec{J}) \qquad \text{or} \qquad \vec{\nabla} \cdot \vec{J} = 0$$

i.e. current is steady.

Thus Ampere's circuital law derived by us earlier is valid only if current is steady.

On the other hand current need not be always constant. In that case $\vec{\nabla} \cdot \vec{J} \neq 0$. Hence equation (42) cannot be satisfied as its L.H.S. should be zero but right is non zero. So we must modify Ampere's law, which is valid for steady as well as varying currents.

For varying currents the equation of continuity gives

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0 \qquad ...(43)$$

But we know that $\vec{\nabla} \cdot \vec{D} = \rho$

(Here ρ means free change density)

Put in (43), we get

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial}{\partial t} (\vec{\nabla} \cdot \vec{D}) = 0 \qquad \text{or} \qquad \vec{\nabla} \cdot \vec{J} + \vec{\nabla} \cdot \frac{\partial \vec{D}}{\partial t} = 0$$

(Inter changing order of differentiation)

or
$$\vec{\nabla} \cdot \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) = 0 \qquad \text{or} \qquad \vec{\nabla} \cdot (\vec{J}_N) = 0 \qquad ...(44)$$

where $\vec{J}_N = \vec{J} + \dfrac{\partial \vec{D}}{\partial t}$ = Net current density.

Note that equation (42) implies that although $\vec{J}$ is varying but $\vec{J}_N$ represents a steady current density as its divergence is zero.

Thus equation (42) can be satisfied if we replace $\vec{J}$ by $\vec{J}_N$ in it.

Thus Ampere's law for varying currents is given by

$$\vec{\nabla} \times \vec{B} = \mu_0 (\vec{J}_N)$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \qquad ...(45)$$

The quantity $\dfrac{\partial \vec{D}}{\partial t}$ has units of current density and is called displacement current density.

(11) AMPERE'S CIRCUITAL LAW IN THE PRESENCE OF MAGNETIC MATERIALS

We have discussed polarisation of dielectrics in electrostatics. A similar mechanism takes place, when a magnetic material is placed in external magnetic field.

The sample may acquire net non zero magnetic dipole moment as it is magnetic in nature. This process of production of non zero magnetic dipole moment of a sample in the presence of external magnetic fields is called Magnetisation.

Just as polarisation of dielectric produces bound charges, whose volume charge density is given as

$$\rho_p = -\vec{\nabla} . \vec{P}$$

In the same way magnetisation of magnetic material produces bound currents, whose current density is given by

$$\vec{J}_M = \vec{\nabla} \times \vec{M} \quad \text{...(46)}$$

Here $\vec{M}$ is called magnetisation vector and it is equal to net magnetic dipole moment per unit volume of the sample. Its SI unit is A/m.

Thus net current density in the presence of magnetic materials becomes free current density + magnetisation current density + displacement current density.

Thus Ampere's law in the presence of magnetic materials is given by

$$\vec{\nabla} \times \vec{B} = \mu_0 (\vec{J}_N) = \mu_0 (\vec{J} + \vec{J}_M + \vec{J}_D) = \mu_0 \left(\vec{J} + \vec{\nabla} \times \vec{M} + \frac{\partial \vec{D}}{\partial t} \right)$$

or

$$\vec{\nabla} \times \frac{\vec{B}}{\mu_0} = \vec{J} + \vec{\nabla} \times \vec{M} + \frac{\partial \vec{D}}{\partial t} \quad \text{or} \quad \vec{\nabla} \times \left(\frac{\vec{B}}{\mu_0} - \vec{M} \right) = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \text{...(47)}$$

We define a new vector

$$\frac{\vec{B}}{\mu_0} - \vec{M} = \vec{H} \quad \text{...(48)}$$

$\vec{H}$ is called magnetic field or magnetising force, or magnetic field intensity. It has same unit as $\vec{M}$ *i.e.* A/m.

Thus Ampere's law in the presence of magnetic materials can be written as

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \text{...(49)}$$

Importance of $\vec{H}$. $\vec{H}$ plays same role in magnetism, as was played by $\vec{D}$ in electrodynamics. *i.e.* $\vec{H}$ is a quantity, independent of presence of medium. It only depends on the distribution of current in the region.

If $\vec{B}_a$ is applied magnetic flux density

$\vec{B}_M$ = magnetic flux density produced beacuse of magnetisation.

and $\quad \vec{B}$ = net flux density

then
$$\vec{B} = \vec{B}_a + \vec{B}_M \qquad ...(50)$$

However if μ is permeability of medium, then the expressions for $\vec{B}$, $\vec{B}_a$, $\vec{B}_M$ are given as

$$\vec{B} = \mu \vec{H},\ \vec{B}_a = \mu_0 \vec{H} \text{ and } \vec{B}_M = \mu_0 \vec{M}$$

$\vec{M}$ is found to be proportional to $\vec{H}$ *i.e.* $\vec{M} \propto \vec{H}$ or $\vec{M} = \chi_m \vec{H}$, χ_m is called magnetic ssusceptibility it has no unit and can be +ve, –ve or zero.

Put in (50), we get

$$\mu \vec{H} = \mu_0 (\vec{H} + \vec{M}) = \mu_0 (\vec{H} + \chi_m \vec{H})$$

or
$$\mu \vec{M} = \mu_0 (1 + \chi_m) \vec{H}$$

or
$$\frac{\mu}{\mu_0} = 1 + \chi_m$$

or
$$\mu_r = 1 + \chi_m \qquad ...(51)$$

where $\mu_r = \dfrac{\mu}{\mu_0}$ is called relative permeability of the medium.

Equation (51) gives relation between relative permeability and magnetic susceptibility.

(12) FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

Faraday performed various experiments on variation of magnétic flux linked with a coil and electric effects produced in it. He observed that a changing magnetic flux causes the production of induced emf. On the basis of his experiments, he formulated following two laws.

(*i*) Whenever magnetic flux linked with a coil changes, an induced emf is produced in the coil which will last as long as the flux linked with it is changing.

(*ii*) The magnitude of the induced emf is proportional to the time rate of change of magnetic flux linked with the coil.

i.e. If e = induced emf, then

$$|e| \propto \frac{\partial \phi_B}{\partial t} \quad \text{or} \quad |e| = \frac{\partial \phi_B}{\partial t} \qquad ...(52)$$

The constant of proportionality in SI system is found to be unity.

According to Lenz's law the emf is produced in such away that it opposes the cause producing it. Thus complete form of equation (52) can be written as

$$e = \frac{-\partial \phi_B}{\partial t} \qquad ...(53)$$

where negative sign comes from Lenz's law.

Also the emf in a circuit is defined as the total amount of work done by the electric field to move a unit +ve charge once along the closed circuit.

Thus if $\vec{E}$ is electric field existing along the coil then emf can be expressed as

$$e = \frac{\text{Work done}}{\text{Charge}} = \frac{q_0 \oint \vec{E} \cdot \vec{dl}}{q_0}$$

or

$$e = \oint \vec{E} \cdot \vec{dl} \quad ...(54)$$

We know from lower classes that $W_{AB} = -q_0 \int_{rA}^{rB} \vec{E} \cdot \vec{dl}$; where W_{AB} is work done against electric field to move a charge q_0 from A to B. Also note that work done along closed path is zero if electric field is conservative. However here electric field is non conservative so the work done along closed path in this case will be non zero.)

Equate (53) and (54), we get

$$\oint \vec{E} \cdot \vec{dl} = -\frac{\partial}{\partial t}(\phi_B)$$

or

$$\iint_S \vec{\nabla} \times \vec{E} \cdot \vec{ds} = -\frac{\partial}{\partial t} \iint_S \vec{B} \cdot \vec{ds} \quad ...(55)$$

(By applying stokes theorem on left side)

Here S is the open area of the coil.

Above equation can also be written as

$$\iint \left(\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} \right) \cdot \vec{ds} = 0$$

This integration is zero, independent of the area of coil *i.e.* limit of integration. Hence integrand should be zero itself.

Thus $$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \quad \text{or} \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad ...(56)$$

Equation (56) gives Faradays Laws of electromagnetic induction in differential form.

(13) GAUSS'S LAW OF MAGNETISM

Fig. (11) shows a conductor carrying a current I. Let $\vec{J}$ is current density.

According to Biot savart law the net magnetic flux density at point P due to the conductor is given as

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I\vec{dl} \times (\vec{r} - \vec{r'})}{|\vec{r} - \vec{r'}|^3} \quad ...(57)$$

Here $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

and $\vec{r'} = x'\hat{i} + y'\hat{j} + z'\hat{k}$

Now $I\vec{dl} = (J\, ds)\vec{dl} = \vec{J}\,(ds\, dl)$

($\because$ $\vec{J}$ and $\vec{dl}$ are in same direction)

$= \vec{J}\; dV'$

Where $dV' = ds\, dl$ = Volume of the current element it is written as dV' instead of dV just to indicate that volume of element is located by $\vec{r'}$ rather than $\vec{r}$.

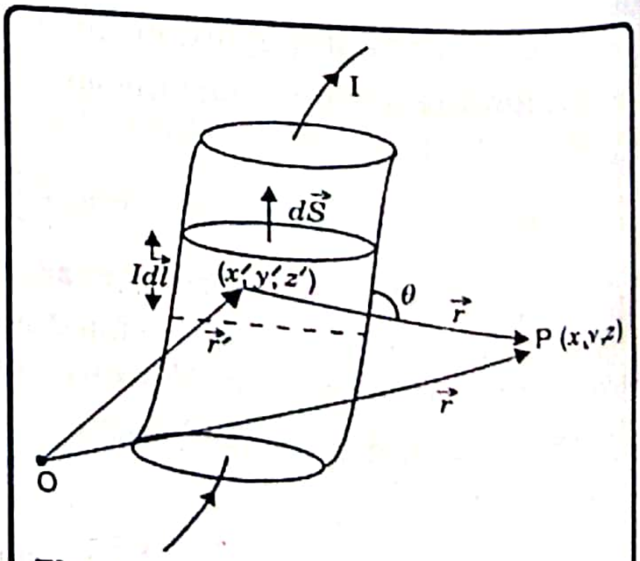

Fig. 11.

Further we shall note that $\vec{J}$ is current density through current element, which is a function of $\vec{r'}$ and not of $\vec{r}$. Hence we shall write $\vec{J}$ as $\vec{J}(\vec{r'})$

We can rewrite equation (57) as

$$\vec{B} = \iiint \frac{\mu_0}{4\pi} \frac{\vec{J}(\vec{r'})\, dV' \times (\vec{r} - \vec{r'})}{|\vec{r} - \vec{r'}|^3}$$

$$= \iiint \frac{\mu_0}{4\pi} \vec{J}(\vec{r'})\, dV' \times \left[-\vec{\nabla}\left(\frac{1}{|\vec{r} - \vec{r'}|}\right)\right] \quad ...(58) \quad \left(\because \vec{\nabla}\left(\frac{1}{|\vec{r} - \vec{r'}|}\right) = -\frac{(\vec{r} - \vec{r'})}{|\vec{r} - \vec{r'}|^3}\right)$$

Now $$\vec{\nabla} \times \left(\frac{1}{|\vec{r} - \vec{r'}|} \vec{J}(\vec{r'})\right) = \vec{\nabla}\left(\frac{1}{|\vec{r} - \vec{r'}|}\right) \times \vec{J}(\vec{r'}) + \frac{1}{|\vec{r} - \vec{r'}|} \vec{\nabla} \times \vec{J}(\vec{r'}) \quad ...(59)$$

$$(\because \vec{\nabla} \times (\phi \vec{A}) = (\vec{\nabla}\phi) \times \vec{A} + \phi\, \vec{\nabla} \times \vec{A})$$

We know that $\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$ is a differential operator with respect to cordinates x, y, z. But $\vec{J}$ depends on $\vec{r'}$ *i.e.* on coordinate x', y', z' only. Hence if we change $\vec{r}$, then there is no variation in $\vec{J}$. Thus $\vec{\nabla} \times \vec{J}(\vec{r'}) = 0$.

Thus equation (59) becomes

$$\vec{\nabla} \times \left(\frac{1}{|\vec{r} - \vec{r'}|} . \vec{J}(\vec{r'})\right) = \vec{\nabla}\left(\frac{1}{|\vec{r} - \vec{r'}|}\right) \times \vec{J}(\vec{r'}) = -\vec{J}(\vec{r'}) \times \vec{\nabla}\left(\frac{1}{|\vec{r} - \vec{r'}|}\right) \quad (\because \vec{A} \times \vec{B} = -\vec{B} \times \vec{A})$$

Put this value in equation (58), we get

$$\vec{B} = \iiint \frac{\mu_0}{4\pi} \vec{\nabla} \times \left(\frac{\vec{J}(\vec{r'})}{|\vec{r} - \vec{r'}|}\right) dV' = \vec{\nabla} \times \iiint \frac{\mu_0}{4\pi} \frac{\vec{J}(\vec{r'})}{|\vec{r} - \vec{r'}|} dV' \quad ...(60)$$

(Interchanging order of differentation and integration as different with is w.r.t x, y, z while integration is w.r.t x', y', z' respectively)

or $$\vec{B} = \vec{\nabla} \times \vec{A} \quad ...(61)$$

where $\vec{A}$ is called magnetic vector potential.

It is given by $\vec{A} = \frac{\mu_0}{4\pi} \iiint \frac{\vec{J}(\vec{r'})\, dV'}{|\vec{r} - \vec{r'}|}$

Taking divergence on both sides of equation (61), we get

$$\vec{\nabla} . \vec{B} = \vec{\nabla} . (\vec{\nabla} \times \vec{A}) \quad \text{or} \quad \vec{\nabla} . \vec{B} = 0 \quad ...(62)$$

$$(\because \text{div (curl } \vec{A}) = 0 \text{ for any vector } \vec{A})$$

Equation (62) is Gauss law of magnetosatics in differential form.

Integrating this equation w.r.t volume we get

$$\iiint \vec{\nabla}.\vec{B}\,dV = 0$$

Apply Gauss Divergence theorem, we get

$$\oiint \vec{B}.\,\vec{ds} = 0 \qquad \text{...(64)}$$

This equation gives Gauss law of magnetism in integral form.

(14) MAXWELL'S EQUATIONS

The frame work of electromagnetism is based on four base equations called Maxwell's equations. There four equations are given below.

(i) $\vec{\nabla}\times\vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$ (ii) $\vec{\nabla}\times\vec{H} = \vec{J} + \dfrac{\partial \vec{D}}{\partial t}$

(iii) $\vec{\nabla}.\vec{D} = \rho$ (iv) $\vec{\nabla}.\vec{B} = 0$

For derivation of first equation see article (12)

For derivation of second equation, see article (11)

For derivation of 3rd equation see article (7)

For derivation of 4th equation, see article (13)

Any process of electromagnetism can be explained by the use of these four equations. Thus Maxwells equations play same role in electromagnetism, as is played by Newton's laws in mechanics. Thus these equations are actually foundation of electromagnetism.

Importance of Maxwell's first equation $\vec{\nabla}\times\vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

(i) It relates electric field vector and magnetic flux density vector.

(ii) It is a time dependent equation.

(iii) It describes Faraday's laws of electromagnetic induction along with Lenz's law.

(iv) It states that time varying magnetic field is a source of electric field in space.

Importance of Maxwell's second equation $\vec{\nabla}\times\vec{H} = \vec{J} + \dfrac{\partial \vec{D}}{\partial t}$

(i) It relates magnetic field vector with displacement vector.

(ii) It is a time dependent equation

(iii) It summarizes Ampere's law for varying currents (also called Maxwell Ampere's law or modified Ampere's law).

(iv) It tells that electric current $(\vec{J})$ is a source of magnetic field, which agrees well will Oersted's experiment.

(v) It tells that time varying electric field $(\because \vec{D} = \varepsilon\vec{E})$ is a source of magnetic field in space.

Importance of Maxwell's 3rd equation.

$$\vec{\nabla}.\vec{D} = \rho_f \quad \text{or} \quad \vec{\nabla}.\vec{E} = \frac{\rho_f}{\varepsilon_0}$$

(*i*) It summarizes Gauss law.

(*ii*) It is a time independent or steady state equation.

(*iii*) $\vec{\nabla}.\vec{E}$ can be visualised as derivative of E. If ρ_f is +ve then $\vec{\nabla}.\vec{E}$ is also +ve. It means that +ve charge is a source of electric field lines. Similarly –ve charge is a sink for electric field lines.

(*iv*) Since $\vec{\nabla}.\vec{D}$ is always a scalar quantity. So this relation shows that charge density and hence charge is a scalar quantity.

Importance of Maxwell's fourth equation. $\vec{\nabla}.\vec{B}=0$

(*i*) It tells that isolated magnetic monopoles do not exist.

(*ii*) It is a time independent or steady state equation.

(*iii*) It tells that there are no sources or sinks for magnetic field lines *i.e.* magnetic field lines form closed continuous loops.

(15) MAXWELL'S WAVE EQUATION FOR LINEAR ISOTROPIC HOMOGENEOUS MEDIUM

Let us now use Maxwell's equations to discuss propagation of electromagnetic waves through an isotropic, homogeneous medium.

We assume that the medium is neutral. Hence its volume charge density will be zero. If ε and μ denote permittivity and permeability of the medium, then Maxwell's equations for such a medium can be written as

$$\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} \qquad ...(65)$$

$$\vec{\nabla}\times\vec{B}=\mu\left(\sigma\vec{E}+\varepsilon\frac{\partial\vec{E}}{\partial t}\right) \qquad ...(66)$$

$$\vec{\nabla}.\vec{E}=0 \qquad ...(67)$$

$$\vec{\nabla}.\vec{B}=0 \qquad ...(68) \quad \left(\because \rho=0,\ \vec{H}=\frac{\vec{B}}{\mu},\ \vec{D}=\varepsilon\vec{E},\ \vec{J}=\sigma\vec{E}\right)$$

Taking curl of both sides of equation (65), we get

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E})=-\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

or

$$\vec{\nabla}(\vec{\nabla}.\vec{E})-\nabla^2\vec{E}=-\frac{\partial}{\partial t}\left(\mu\sigma\vec{E}+\mu\varepsilon\frac{\partial\vec{E}}{\partial t}\right) \qquad \text{(using 66)}$$

or

$$-\nabla^2\vec{E}=-\frac{\partial}{\partial t}\left(\mu\sigma\vec{E}+\mu\varepsilon\frac{\partial\vec{E}}{\partial t}\right) \qquad \text{(using 67)}$$

or

$$\nabla^2\vec{E}-\mu\sigma\frac{\partial\vec{E}}{\partial t}-\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}=0 \qquad ...(69)$$

Taking curl of equation (66), we get

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{B})=\mu\left(\sigma\vec{\nabla}\times\vec{E}+\varepsilon\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E})\right)$$

$$\vec{\nabla}(\vec{\nabla}.\vec{B})-\nabla^2\vec{B}=\mu\sigma\left(-\frac{\partial\vec{B}}{\partial t}\right)+\mu\varepsilon\frac{\partial}{\partial t}\left(\frac{-\partial\vec{B}}{\partial t}\right) \quad \text{(using (65))}$$

or $$-\nabla^2\vec{B}=-\mu\sigma\frac{-\partial\vec{B}}{\partial t}-\mu\varepsilon\frac{\partial^2\vec{B}}{\partial t^2} \quad \text{(using (68))}$$

or $$\nabla^2\vec{B}-\mu\sigma\frac{\partial\vec{B}}{\partial t}-\mu\varepsilon\frac{\partial^2\vec{B}}{\partial t^2}=0 \quad ...(70)$$

Equations (69) and (70) give maxwell's wave equations for propagation of electric and magnetic component of electromagnetic wave through a medium.

If wave is travelling along X-axis, having electric field variations along Y axis and magnetic field variations along z-axis then above equations can be written as

$$\frac{d^2E_y}{dx^2}-\mu\sigma\frac{dE_y}{dt}-\mu\varepsilon\frac{d^2E_y}{dt^2}=0 \quad ...(71)$$

and $$\frac{d^2B_z}{dx^2}-\mu\sigma\frac{dB_z}{dt}-\mu\varepsilon\frac{d^2B_z}{dt^2}=0 \quad ...(72)$$

Case I. Solution for a non conducting medium

For a non conducting medium $\sigma=0$

Thus equation (71) and (72) become

$$\frac{d^2E_y}{dx^2}-\mu\in\frac{d^2E_y}{dt^2}=0 \quad ...(73)$$

and $$\frac{d^2B_z}{dx^2}-\mu\in\frac{d^2B_z}{dt^2}=0 \quad ...(74)$$

Suppose trial solution of equation (73) is of the form

$$E_y=E_0\sin\left[\frac{kt}{\sqrt{\mu\in}}-kx\right] \quad ...(75)$$

Where E_0 and k are constants of integration.

Thus from (75), we have

$$\frac{d^2E_y}{dt^2}=\frac{-k^2}{\mu\in}E_y \quad \text{and} \quad \frac{d^2E_y}{dx^2}=-k^2E_y$$

Put these values in (73), we get

$$-k^2E_y-\mu\in\left(\frac{-k^2}{\mu\in}\right)E_y=0 \quad \Rightarrow \quad 0=0$$

This means our trial solution is a true solution of equation (73). Note that if E_0 and k are not constants then equation (73) will not be satisfied by equation (75).

But equation (75) is a standard equation of a plane progressive wave travelling along X-axis with constant amplitude (peak value of electric field intensity) E_0 and propagation vector k.

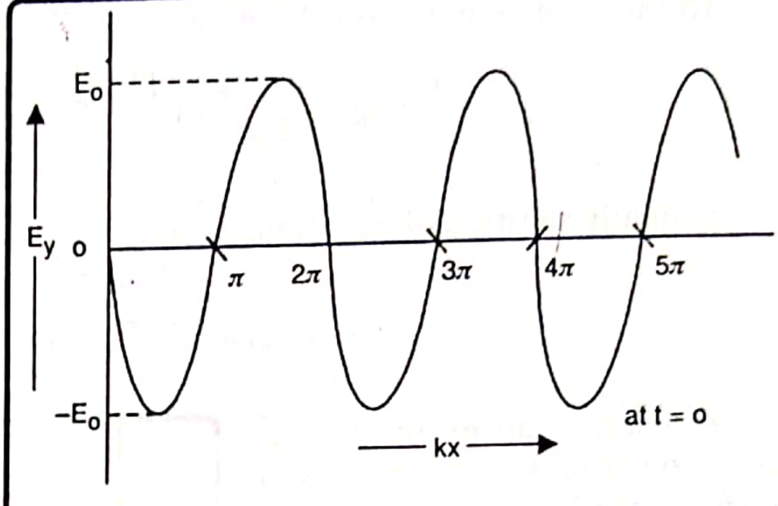

Fig. 12.

The variation of E_y as a function of x at a fixed value of t (at $t = 0$ in this example) is shown in figure (12). We see that as electromagnetic wave covers distance in the medium, the amplitude of wave remains constant at E_0 (for electric component) throughout. Thus the electromagnetic waves are not attenuated through a non conducting medium. Due to this reason optical fibers are better wave guides for electromagnetic waves as compared to metallic wires because optical fibers are made from transparent glass or plastic which is non conducting.

Similarly we can prove that solution of equation (74) is of the form

$$B_z = B_0 \sin\left[\frac{kt}{\sqrt{\mu \in}} - kx\right] \quad ...(76)$$

If we compare equation (75) with standard equation of a plane progressive wave *i.e.*

$$E_y = E_0 \sin(wt - kx)$$

Then, we get $$w = \frac{k}{\sqrt{\mu \in}}$$

and the velocity of wave propagation is given as V_p = phase velocity

$$= \frac{\text{Coefficient of } t}{\text{Coefficient of } x}$$

$$= \frac{\frac{k}{\sqrt{\mu \in}}}{k} = \frac{1}{\sqrt{\mu \in}}$$

For vaccum, the velocity of electro magnetic waves is given as

$$V_p = \frac{1}{\sqrt{\mu_0 \in_0}}$$

Putting $\mu_0 = 4 \times 3.14 \times 10^{-7}\ \text{T}\, m\, \text{A}^{-1}$

and $\in_0 = 8.854 \times 10^{-12}\, c^2\, \text{N}^{-1}\, m^{-2}$

We get $V_p = 3 \times 10^8\ \text{ms}^{-1} = c = \text{Velocity of light}$

Since V_p in vacuum is not zero (in fact it is maximum in vacuum) so electromagnetic waves can travel in vacuum. Thus electromagnetic waves donot require any medium for propagation.

Case II. Solution for a conducting medium.

In this case $\sigma \neq 0$

The solution of equation (71) is given as

$$E_y = E_0 e^{-\left(\sqrt{\frac{w\mu\sigma}{2}}\right)x} \cos\left[wt - \left(\sqrt{\frac{w\mu\sigma}{2}}\right)x\right] \qquad ...(77)$$

Similarly solution of equation (72) is given as

$$B_z = B_0 e^{-\left(\sqrt{\frac{\mu\sigma w}{2}}\right)x} \cos\left[wt - \left(\sqrt{\frac{w\mu\sigma}{2}}\right)x\right] \qquad ...(78)$$

It is left as an exercise to the reader to verify that these solutions satisfy respective equations.

We would like to discuss consequencies of these solutions. If we carefully analyze equation (77), we see that amplitude of electric field decreases exponentially with distance due to presence of term $e^{-\left(\sqrt{\frac{\mu\sigma w}{2}}\right)x}$ The larger is conductivity, faster is the decrease in amplitude as shown in fig. 13. Thus conducting medium attenuates electromagnetic waves. Hence these are not suitable waveguides for electromagnetic waves.

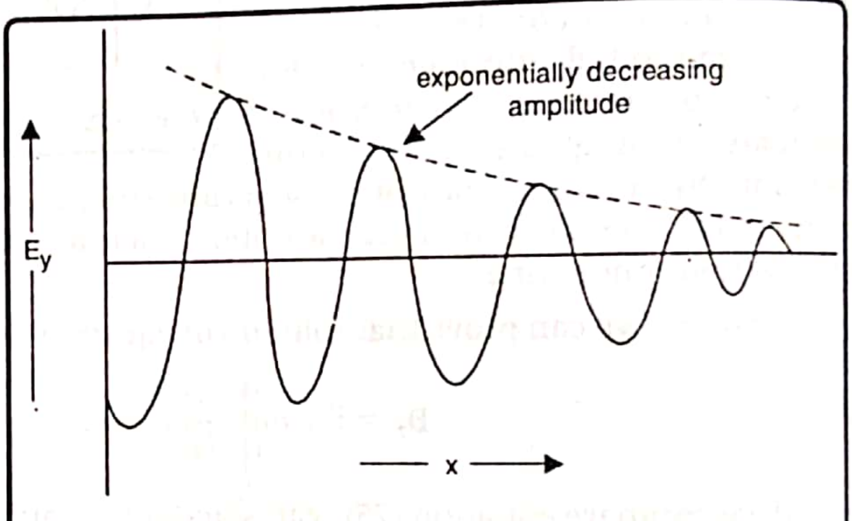

Fig. 13.

(16) RELATION BETWEEN AMPLITUDES OF ELECTRIC AND MAGNETIC FIELD FOR A PLANE ELECTROMAGNETIC WAVE

Consider a plane electromagnetic wave travelling along + X-axis is vacuum. Let electric field occillates along Y-axis and magnetic field along Z-axis. Then expression for instantaneous values of electric and magnetic field are given as

$$E = E_y = E_0 \sin\left[\frac{kt}{\sqrt{\mu_0 \epsilon_0}} - kx\right] \qquad ...(79)$$

$$B = B_z = B_0 \sin\left[\frac{kt}{\sqrt{\mu_0 \epsilon_0}} - kx\right] \qquad ...(80)$$

Let us consider the rectangular path JKLM in the XY plane as shown in figure 14*a*. Let us calculate line integration of electric field along the path JKLM.

$$\oint_{JKLM} \vec{E}.\vec{dl} = \int_J^K \vec{E}.\vec{dl} + \int_K^L \vec{E}.\vec{dl} + \int_L^M \vec{E}.\vec{dl} + \int_M^J \vec{E}.\vec{dl}$$

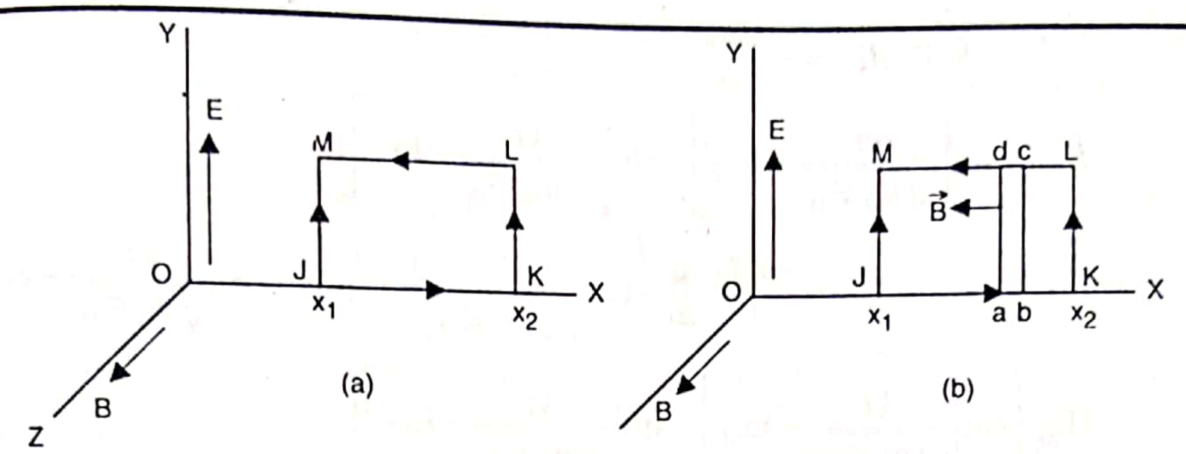

Fig. 14.

$$= \int_J^K E\,dl\cos 90° + \int_K^L E_2\,dl\cos 0° + \int_L^M E\,dl\cos 90° + \int_M^J E_1\,dl\cos(180°)$$

(where E_1, E_2 are magnitudes of electric fields along pathsMJ and KL respectively)

$$= E_2\int_K^L dl - E_1\int_M^J dl = E_2 l - E_1 l \qquad \text{(where KL = MJ} = l \text{ (say))}$$

$$= lE_0\sin\left[\frac{kt}{\sqrt{\mu_0 \in_0}} - kx_2\right] - lE_0\sin\left[\frac{kt}{\sqrt{\mu_0 \in_0}} - kx_1\right] \qquad \text{[Using (79)]}$$

$$= lE_0\left\{\sin\left[\frac{kt}{\sqrt{\mu_0 \in_0}} - kx_2\right] - \sin\left[\frac{kt}{\sqrt{\mu_0 \in_0}} - kx_1\right]\right\} \qquad \ldots(81)$$

Let us now calculate magnetic flux through the open area enclosed by closed path JKLM (note that area vector for all area elements of rectangle JKLM is directed along +Z axis)

$$\phi_B = \iint_{JKLM} \vec{B}\cdot\vec{ds}$$

$$= \int_{y=0}^{l}\int_{x=x_1}^{x_2} B\,dx\,dy \qquad (\because \vec{B} = B\hat{k} \text{ and } \vec{ds} = dx\,dy\,\hat{k})$$

$$= \int_{y=0}^{l}\int_{x=x_1}^{x_2} B_0\sin\left[\frac{kt}{\sqrt{\mu_0 \in_0}} - kx\right]dx\,dy$$

$$= l\,B_0\int_{x_1}^{x_2}\sin\left[\frac{kt}{\sqrt{\mu_0 \in_0}} - kx\right]dx$$

$$= \frac{lB_0}{k}\left\{\cos\left[\frac{kt}{\sqrt{\mu_0 \in_0}} - kx_2\right] - \cos\left[\frac{kt}{\sqrt{\mu_0 \in_0}} - kx_1\right]\right\} \qquad \ldots(82)$$

But according to Faraday's law of electromagnetic induction, we have

$$\oint \vec{E}.\vec{dl} = -\frac{d\phi_B}{dt}$$

or

$$l E_0 \left\{\sin\left[\frac{kt}{\sqrt{\mu_0 \epsilon_0}} - kx_2\right] - \sin\left[\frac{kt}{\sqrt{\mu_0 \epsilon_0}} - kx_1\right]\right\}$$

$$= \frac{-lB_0}{k}\frac{d}{dt}\left\{\cos\left[\frac{kt}{\sqrt{\mu_0 \epsilon_0}} - kx_2\right] - \cos\left[\frac{kt}{\sqrt{\mu_0 \epsilon_0}} - kx_1\right]\right\}$$

⇒

$$l E_0 \left\{\sin\left[\frac{kt}{\sqrt{\mu_0 \epsilon_0}} - kx_2\right] - \sin\left[\frac{kt}{\sqrt{\mu_0 \epsilon_0}} - kx_1\right]\right\}$$

$$= \frac{lB_0}{\sqrt{\mu_0 \epsilon_0}}\left\{\sin\left[\frac{kt}{\sqrt{\mu_0 \epsilon_0}} - kx_2\right] - \sin\left[\frac{kt}{\sqrt{\mu_0 \epsilon_0}} - kx_1\right]\right\}$$

or

$$E_0 = \frac{B_0}{\sqrt{\mu_0 \epsilon_0}}$$

or

$$E_0 = cB_0 \qquad \left(\because \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c\right)$$

Which is the required relation between amplitudes of electric and magnetic field.

(17) RELATION BETWEEN ENERGY DENSITY AND INTENSITY OF A PLANE ELECTROMAGNETIC WAVE

When electromagnetic waves are passing through a region of space then that region is filled by extra energy due to oscillating electric and magnetic fields in there waves.

Consider a parallel plate capacitor such that area of each plate is A and distance between its plates in vacuum is l. Let V is potential difference between plates and E is electric field existing betweeen plates. The capacitor stores energy purely in the form of electric flux in the region between its plates. This electric energy is given as

$$U_e = \frac{1}{2}CV^2$$

$$= \frac{1}{2}\left(\frac{A\epsilon_0}{l}\right)(Ed)^2 \qquad \left(\because C = \frac{A\epsilon_0}{d} \text{ and } E = \frac{V}{d}\right)$$

$$= \frac{1}{2}(Al)\epsilon_0 E^2 \qquad ...(83)$$

Further volume of the region between plates of capacitor is Al. Thus electric energy stored per unit volume of the region filled by electro flux is

$$\frac{U_e}{\text{Volume}} = \frac{\frac{1}{2}(Al)\epsilon_0 E^2}{Al}$$

⇒

$$U_{ed} = \frac{1}{2}\epsilon_0 E^2 \qquad ...(84)$$

Now consider a solenoid of inductance L and area of cross section A and having length l. If I is current through each turn of solenoid, then magnetic energy stored inside the region occupied by solenoid is

$$U_m = \frac{1}{2}LI^2 = \frac{1}{2}\left(\frac{\mu_0 N^2 A}{l}\right)I^2 \qquad \left(\because L = \frac{\mu_0 N^2 A}{l}\right)$$

(where N = total number of turns in solenoid)

$$= \frac{1}{2}\left(\mu_0 \frac{NI}{l}\right)^2 \times \left(\frac{Al}{\mu_0}\right) = \frac{1}{2}(Al)\frac{B^2}{\mu_0}$$

Thus magnetic energy per unit volume of region occupied by solenoid is

$$U_{md} = \frac{U_m}{\text{Volume}} = \frac{\frac{1}{2}(Al)\frac{B^2}{\mu_0}}{Al}$$

$$= \frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2}\frac{(\mu_0 H)^2}{\mu_0} \qquad (\because B = \mu_0 H)$$

$$\Rightarrow \quad U_{md} = \frac{1}{2}\mu_0 H^2 \qquad \text{...(85)}$$

Note that solenoid stores energy purely in the form of magnetic flux.

Now if electromagnetic waves are propagating through a given region of space then that region will be filled by electric flux as well as magnetic flux. Thus total energy density in that region will be equal to sum of electric energy density and magnetic energy density (because energy is a scalar).

Thus U_d = total energy density

$$= U_{ed} + U_{md}$$

$$U_d = \frac{1}{2}(\epsilon_0 E^2 + \mu_0 H^2) \qquad \text{...(86)}$$

Note that E, H are instantaneous values of electric and magnetic field so energy density charges from time to time.

To find average value of energy density for long time, we can integrate equation (86) over large number of cycles of E & B and dividing by total time for which integration is done.

i.e.

$$\langle U_d \rangle = \frac{1}{nT}\int_0^{nT} U_d\, dt$$

(where T = period of osillation and n = very large integer)

$$= \frac{1}{nT}\left[\frac{1}{2}\epsilon_0 \int_0^{nT} E^2\, dt + \frac{1}{2}\mu_0 \int_0^{nT} H^2\, dt\right] \qquad \text{...(87)}$$

But

$$E = E_0 \sin\left[\frac{kt}{\sqrt{\mu_0 \epsilon_0}} - kx\right]$$

$$\therefore \quad \int_0^{nT} E^2\, dt = E_0^2 \int_0^{nT} \sin^2\left[\frac{kt}{\sqrt{\mu_0 \epsilon_0}} - kx\right] dt$$

$$= \frac{E_0^2}{2}\left[\int_0^{nT}\left(1-\cos\left[\frac{2kt}{\sqrt{\mu_0 \in_0}} - 2kx\right]\right)dt\right.$$

$$= \frac{E_0^2}{2}[nT - 0] = \frac{E_0^2 nT}{2}$$

($\because$ Cosine is a cyclic function so its average value over any number of complete cycles is zero)

Similarly we can prove that

$$\int_0^{nT} H^2\, dt = \frac{H_0^2 nT}{2}$$

Put these values in (v), we get

$$\langle U_d \rangle = \frac{1}{nT}\left[\frac{1}{2}\in_0\left(\frac{E_0^2 nT}{2}\right) + \frac{1}{2}\mu_0\left(\frac{H_0^2 nT}{2}\right)\right]$$

$$= \frac{1}{4}(\in_0 E_0^2 + \mu_0 H_0^2) \qquad \text{...(88)}$$

Intensity. It is defined as energy crossing per unit area per unit time such that area is perpendicular to the direction of propagation of wave.

Consider plane electromagnetic waves moving alog +X-axis. Consider a cylinderical region of area of cross section A and length $c\, dt = dx$ which is the distance covered by electromagnetic wave in time dt. Here c is the speed of electromagnetic wave in vacuum.

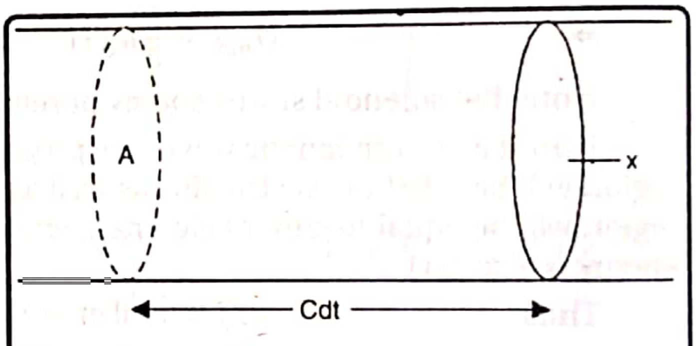

Fig. 15.

The total energy contained in the region is given by

$$U = \langle U_d \rangle \times \text{Volume of region}$$

$$= \frac{1}{4}(\in_0 E_0^2 + \mu_0 H_0^2)(c\,dt)\,A$$

$$= \frac{1}{4}\left(\in_0 E_0^2 + \mu_0\left(\frac{B_0}{\mu_0}\right)^2\right)(c\,dt)(A) \qquad (\because B_0 = \mu_0 H)$$

$$= \frac{1}{4}\left[\in_0 E_0^2 + \frac{B_0^2}{\mu_0}\right](c\,dt)(A) = \frac{1}{4}\left[\in_0 E_0^2 + \frac{E_0^2}{c^2\mu_0}\right](c\,dt)(A) \qquad (\because E_0 = cB_0)$$

$$= \frac{1}{4}\left[\in_0 E_0^2 + \frac{E_0^2}{\left(\frac{1}{\mu_0 \in_0}\right)\mu_0}\right](c\,dt)(A) \qquad \left(c^2 = \frac{1}{\mu_0 \in_0}\right)$$

$$= \frac{1}{4}\left[\in_0 E_0^2 + \in_0 E_0^2\right](c\,dt)(A) = \frac{1}{2}\in_0 E_0^2\,(c\,dt)(A)$$

Thus intensity of electromagnetic wave is given as

I = Intensity

$$= \frac{\text{energy passing normally to area}}{\text{area of cross section} \times \text{time}} = \frac{U}{A\,dt} = \frac{\frac{1}{2}\epsilon_0 E_0^2 (c\,dt)(A)}{A\,dt}$$

$$\therefore \quad I = \frac{1}{2}\epsilon_0 E_0^2 c = \frac{1}{2}\frac{B_0^2}{\mu_0}c \qquad \left(\because E_0 = \frac{B_0}{c} \text{ and } c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}\right)$$

This is very important result most often used in interference and diffraction problems where we say that intensity is directly proportional to square of amplitude of wave.

(18) POYNTING VECTOR ($\vec{P}$)

It is defined as energy flow per second per unit area held perpendicular the direction of flow or propagation.

Consider a plane electromagnetic wave travelling in vacuum along +X-axis. Let electric field is oscillating along Y-axis and magnetic field along Z-axis. Then vector form of $\vec{E}$ and $\vec{B}$ are given as

$$\vec{E} = \hat{j} E_0 \sin\left[\frac{kt}{\sqrt{\mu_0 \epsilon_0}} - kx\right] \quad \text{...(90)}$$

$$\vec{B} = \hat{k} B_0 \sin\left[\frac{kt}{\sqrt{\mu_0 \epsilon_0}} - kx\right] \quad \text{...(91)}$$

$$\therefore \quad \vec{E} \times \vec{B} = \hat{i} E_0 B_0 \sin^2\left[\frac{kt}{\sqrt{\mu_0 \epsilon_0}} - kx\right]$$

$$= \hat{i}\frac{E_0 B_0}{2}\left\{1 - \cos\left(\frac{2t}{\sqrt{\mu_0 \epsilon_0}} - 2kx\right)\right\}$$

The average value of $<\vec{E} \times \vec{B}>$ over very large number of cycles of oscillation of $\vec{E}$ or $\vec{B}$ is given as

$$<\vec{E} \times \vec{B}> = \frac{1}{nT}\int_0^{nT} \vec{E} \times \vec{B}\, dt$$

(where n is very large integer)

$$= \frac{1}{nT}\frac{\hat{i} E_0 B_0}{2}[nT - 0]$$

[$\because$ Cosine function is cyclic so its average value over any number of complete cycles is zero]

$$= \frac{\hat{i} E_0 B_0}{2} \qquad (\because E_0 = c B_0)$$

$$= \frac{\hat{i}\, c B_0^2}{2} \quad \text{...(92)}$$

We know that $\vec{B} = \mu_0 \vec{H}$. Thus (92) can be written as

$$<\vec{E} \times \mu_0 \vec{H}> = \hat{i}\frac{cB_0^2}{2}$$

or
$$<\vec{E} \times \vec{H}> = \hat{i}\, c\left(\frac{B_0^2}{2\mu_0}\right) \quad ...(93)$$

But $c\left(\frac{B_0^2}{2\mu_0}\right)$ is the intensity of plane wave *i.e.* energy flowing per unit area per second along the direction of motion of electromagnetic wave. Hence by definition it is equal to Poynting vector. Thus equation (93) can be written as

$$\vec{P} = <\vec{E} \times \vec{H}> \text{ Hence proved.}$$

(19) RELATION BETWEEN ELECTRIC FIELD AND ELECTRIC POTENTIAL AT A POINT

Figure 16 shows a volume charge distribution. Let A (x', y', z') is any point in the distribution having position vector $\vec{r'}$. Thus

$$\vec{r'} = x\hat{i} + y\hat{j} + z\hat{k} \quad ...(94)$$

Draw a small volume $dv' = dx'\, dy'\, dz'$ around point A. Let charge enclosed in elementary volume dv' is dq.

Suppose $\rho\,(x', y', z') \equiv \rho\,(\vec{r'})$ is volume charge density at point A. Note that ρ need not be uniform everywhere in the distribution, so it is a function of position of point A in the distribution.

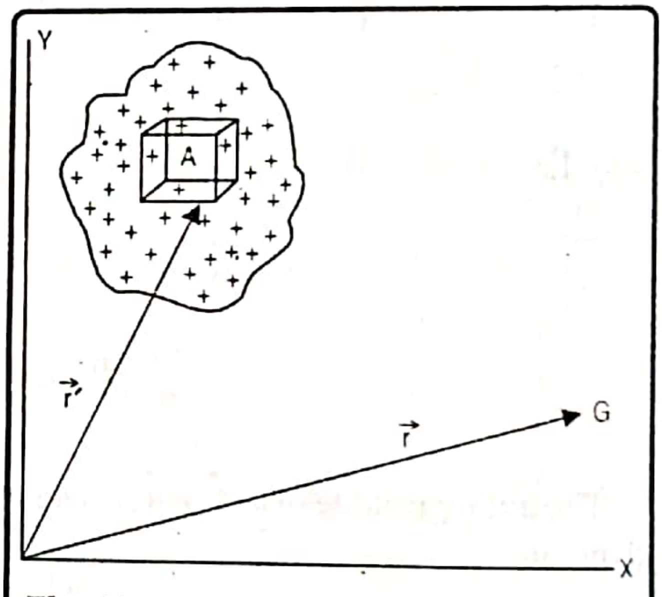

Fig. 16.

Consider an observation point G (x, y, z) in vacuum having position vector $\vec{r}$. Thus

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \quad ...(95)$$

Electric potential at point G due to elementary volume dv' is given as

$$dV = \frac{1}{4\pi \epsilon_0}\frac{dq}{|\vec{r} - \vec{r'}|}$$

$$= \frac{1.}{4\pi \epsilon_0}\frac{\rho(\vec{r'})\,dV'}{|\vec{r} - \vec{r'}|} \qquad \left(\because \rho(\vec{r'}) = \frac{dq}{dv'}\right)$$

Electric potential at point G due to complete distribution is given as

$$V = \iiint .dV = \frac{1}{4\pi \epsilon_0}\iiint \frac{\rho(\vec{r'})\,dv'}{|\vec{r} - \vec{r'}|} \quad ...(96)$$

Now, the electric field at point G due to volume element dv' is given as

$$\overrightarrow{dE} = \frac{1}{4\pi \epsilon_0} \frac{dq\,(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} = \frac{1}{4\pi \epsilon_0} \frac{\rho(\vec{r}')\,(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} dv'$$

Total electric field at G due to complete distribution is given as

$$\vec{E} = \iiint d\vec{E} = \frac{1}{4\pi \epsilon_0} \iiint \rho(\vec{r}') \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} dv'$$

$$= -\frac{1}{4\pi \epsilon_0} \iiint \rho(\vec{r}') \left\{ \vec{\nabla} \left(\frac{1}{|\vec{r} - \vec{r}'|} \right) \right\} dv' \qquad \left(\because \vec{\nabla} \left(\frac{1}{|\vec{r} - \vec{r}'|} \right) = \frac{-(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \right)$$

$$= -\vec{\nabla} \left\{ \frac{1}{4\pi \epsilon_0} \iiint \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} \right\} dv'$$

$$\Rightarrow \quad \vec{E} = -\vec{\nabla} V \qquad \text{Using equation } (iii)$$

Thus electric field at a point is equal to negative of gradient of potential.

(20) CONSERVATIVE NATURE OF ELECTRIC FIELD

A field is said to be conservative, if work done to move a particle along any closed path by or against that field is zero. Consider a charge q_0 which is moved along a closed path in a region where electric field exists due to some distribution of stationary charges. The electric field and electric potential at any point along the closed path are related as

$$\vec{E} = -\vec{\nabla} V$$

Taking curl on both sides, we get

$$\vec{\nabla} \times \vec{E} = \vec{\nabla} \times (-\vec{\nabla} V)$$

$$= -\vec{\nabla} \times \vec{\nabla} V = 0 \qquad [\because \text{curl (grad (V))} = 0]$$

$$\Rightarrow \vec{\nabla} \times \vec{E} = 0 \qquad \ldots(97)$$

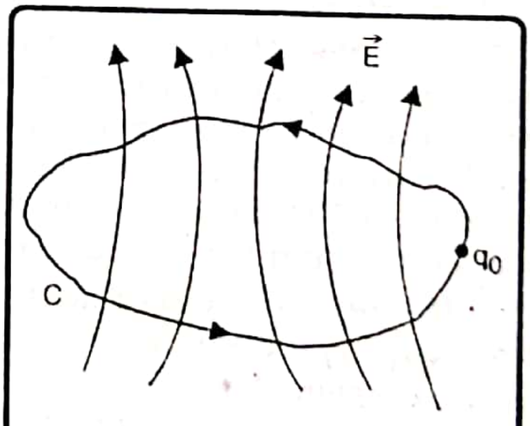

Fig. 17.

Integrating both sides over the open surface bounded by closed imaginary path C along which charge q_0 is moved, we get

$$\iint (\vec{\nabla} \times \vec{E}) . \vec{ds} = 0$$

or

$$\oint \vec{E} . \vec{dl} = 0 \qquad \ldots(98) \text{ (By applying Stoke's Theorem)}$$

The work done to move q_0 along closed path against the electric field $\vec{E}$ is given by

$$W = -q_0 \oint \vec{E} . \vec{dl}$$

$$= 0 \qquad [\text{Using } (98)]$$

Thus electrostatic field is conservative.

(21) ELECTROMAGNETIC SPECTRUM

"The orderly arrangement of electromagnetic waves (according to their wavelength or frequency) in the form of distinct groups having widely different properties is called electromagnetic spectrum. This is shown in table-1.

Table-1 (Electromagnetic Spectrum)

Electromagnetic wave group	Wavelength Range (m)	Frequency Range (Hz)
Gamma Rays	$6 \times 10^{-4} - 1 \times 10^{-11}$	$5 \times 10^{12} - 3 \times 10^{19}$
X-rays	$1 \times 10^{-11} - 3 \times 10^{-8}$	$3 \times 10^{19} - 1 \times 10^{16}$
Ultra Violet Rays	$6 \times 10^{-10} - 4 \times 10^{-7}$	$5 \times 10^{17} - 8 \times 10^{14}$
Visible Light	$4 \times 10^{-7} - 8 \times 10^{-7}$	$8 \times 10^{14} - 4 \times 10^{14}$
Infra Red Rays	$8 \times 10^{-7} - 3 \times 10^{-5}$	$4 \times 10^{14} - 1 \times 10^{13}$
Heat Radiations	$10^{-5} - 10^{-1}$	$3 \times 10^{13} - 3 \times 10^{9}$
Micro Waves	$10^{-3} - 0.3$	$3 \times 10^{11} - 1 \times 10^{9}$
Ultra High Frequency (UHF)	$1 \times 10^{-1} - 1$	$3 \times 10^{9} - 3 \times 10^{8}$
Very High Frequency (VHF)	$1 - 10$	$3 \times 10^{8} - 3 \times 10^{7}$
Radio Frequencies	$10 - 10^{4}$	$3 \times 10^{7} - 3 \times 10^{4}$
Audio Frequencies	$1.5 \times 10^{4} - 1.5 \times 10^{7}$	$20 \times 10^{3} - 20$
Power Frequencies	$5 \times 10^{6} - 6 \times 10^{6}$	$60 - 50$

It should be noted that there is over lap in some of the groups. Further the visible part of electro magnetic spectrum extends from 4×10^{-7} m (400 nm) to 8×10^{-7} m (800 nm). The wavelength range of different colours in visible spectrum is given in table 2.

Table-2 (Visible Spectrum)

Colour	Wavelength (nm)
Violet	400 – 450
Blue	450 – 500
Green	500 – 570
Yellow	570 – 590
Orange	590 – 620
Red	620 – 800

Some of the important constituents of electromagnetic spectrum are explained below :

(*a*) **γ-rays.** These waves are of nuclear origin and are mainly emitted by radioactive elements. They have maximum penetrating power and small ionising power. These are used in radiotherapy. In hospitals, these are used to treat cancer and tumors. In food industry, these are used to kill micro-organisms so as to preserve the food stuff for prolonged time. These are also used to study various nuclear reactions.

(*b*) **X-rays.** These have atomic origin. These are produced when a very fast moving electron strikes a heavy metal target. These rays find applications in surgery for detection of fractures, diseased organs. These are also used in radiotherapy to cure untractable skin diseases and malignant growths.In engineering, these are used in detecting flaws, cracks and gas pockets in metals. These are also used to detect explosives, opium and other contraband goods by custom officials. These are also used for studying crystal structure.

(*c*) **Ultraviolet Rays.** These rays can be produced by arcs of mercury and iron. These are used for checking mineral samples. These rays destroy bacteria, hence these are used for sterilising surgical instruments and in purifying water. These rays are also used in burglar alams based on photoelectric effect.

(*d*) **Infra-Red rays.** These rays are produced from hot bodies. Other commercial sources of IR are Nernst camp, Globar and Laser. These rays play very important role to keep earth warm at night due to Green House Effect.These rays are responsible for conversion of wood into coal in the interior of earth. These rays are used in solar water heaters and cookers. These rays are also used in weather forescasting. These rays are used to take pictures in case of fog, smoke and night. These rays are also used for producing dehydrated fruits. These rays are also used to treat muscular strains.

(*e*) **Radiowaves and Microwaves.** Radio waves and Microwaves are produced by using LC oscillators. These are used for wircless communication and TV transmission signals. These are also used in astronomy and radar system. Microwaves are used in cooking. Micro waves are also used in study of atomic and molecular structure.

Example 1. *An infinite charge sheet has a surface charge density $\sigma = 1 \times 10^{-7}\ Cm^{-2}$. How far apart are the equipotential surfaces whose potential differ by 5.0 volts ?*

Solution. Given $\sigma = 1 \times 10^{-7}\ \mathrm{Cm^{-2}}$

$$\therefore \qquad E = \frac{\sigma}{2\varepsilon_0} = \frac{1\times 10^{-7}}{2\times 8.854\times 10^{-12}} = 5.65 \times 10^{3}\ \mathrm{NC^{-1}}$$

Thus equipotential surfaces differing in potential by 5V are separated by a distance of

$$d = \frac{V}{E} \qquad \left(\because E = \frac{V}{d}\right)$$

$$= \frac{5}{5.65\times 10^{3}} = 8.854 \times 10^{-4}\ \mathrm{m}\ \text{Ans.}$$

Example 2. *The atomic weight and density of sulphur are 32 and 2.08 g cm^{-3} respectively. The electronic polarisability of the atom is 3.28×10^{-40} Fm^2. If solid sulphur has cubical symmetry, what will be its relative permitivity ?*

Solution. Given molar mass = 32g = 32×10^{-3} kg

ρ = density = 2.08g $\mathrm{cm^{-3}}$ = $2.08 \times 10^{3}\ \mathrm{kg\ m^{-3}}$

$\therefore$ n = number of sulphur atoms per unit volume

$$= \left(\frac{N_0}{M}\right)\times \rho = \frac{6.023\times 10^{23}}{32\times 10^{-3}}\times 2.08\times 10^{3}$$

$= 3.91 \times 10^{28}\ \mathrm{m^{-3}}$

[Note we have defined polarisability as $p = \varepsilon_0 \propto E$ so that $\propto$ has unit of volume.

If we define polarisability as $p = \propto' E$

then units of $\propto'$ will be $\mathrm{Fm^2}$ clearly $\propto' = \varepsilon_0 \propto$]

Given $\varepsilon_0\alpha = 3.28 \times 10^{-40}\ \mathrm{Fm^2}$

we know that $\chi_e = n\alpha$

Thus relative permittivity is given by

$$K = 1 + \chi_e = 1 + n\alpha = 1 + (3.91 \times 10^{28}) \times \left(\frac{3.28\times 10^{-40}}{8.854\times 10^{-12}}\right) = 1 + 1.45 = 2.45\ \text{Ans.}$$

Example 3. *A point charge of 11C is placed at the centre of a cube of side 6 cm. Calculate the electric flux through each of the surfaces and through cube.*

Solution. Since charge q is at the centre of cube. Thus by Gauss law net flux through the cube will be

$$\phi_E = \frac{q}{\varepsilon_0} = \frac{11}{8.854\times 10^{-12}} = 1.24 \times 10^{12}\ \mathrm{Nm^2}\,c^{-1}$$

Since the charge is at the centre of cube thus each face will subtend an equal solid angle $\left(\text{equal to } \frac{4\pi}{6} = \frac{2\pi}{3} \text{ steradian}\right)$ on the point charge *i.e.* centre of cube. Thus total flux through each face of cube will be

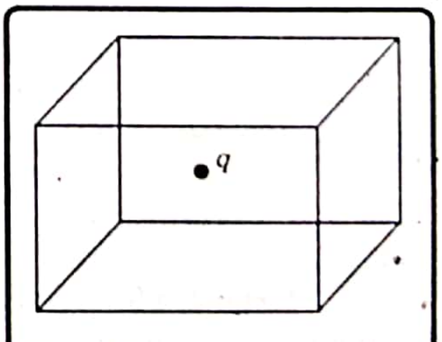

Fig. 18.

$$\phi_E' = \frac{\phi_E}{6} = \frac{q}{6\varepsilon_0} = 2.07 \times 10^{11}\ \mathrm{N\,m^2\,c^{-1}}$$

Example 4. *A square plane sheet of side 10 cm is placed in an electric field of 100 NC⁻¹ making angle 60° with the field. Calculate the electric flux through the sheet.*

Solution. $\phi = \mathrm{EA}\cos\theta = 100 \times (0.1 \times 0.1) \times \cos(90° - 60°)$

($\because$ If angle between sheet and $\vec{E}$ is 60° then angle between $\vec{E}$ and area vector is 30°)

$= 0.867\ \mathrm{N\,m^2\,c^{-1}}$

Example 5. *A solenoid, 4m long has mean diameter of 10cm and contains 1000 turns. If a current of 5A is flowing through it, calculate magentic field at its centre.*

Solution.

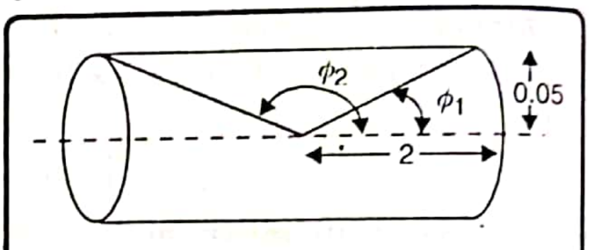

Fig. 19.

$$\mathrm{B} = \frac{\mu_0 n\mathrm{I}}{2}[\cos\phi_2 - \cos\phi_1] \qquad ...(i)$$

Here $\cos\phi_1 = \dfrac{2}{\sqrt{2^2 + (0.05)^2}} = 1$

Similarly $\cos\phi_2 = -1$

Hence $\mathrm{B} = \mu_0 n \mathrm{I}$

$$= \frac{\mu_0 \mathrm{NI}}{l} = \frac{4\pi \times 10^{-7} \times 1000 \times 5}{4} = 1.57 \times 10^{-3}\ \mathrm{T}$$

Example 6. *A charge q is uniformly distributed over a rod of length l. Consider an imaginary cube of edge l with centre of cube at one end of rod. Find the minimum and maximum possible electric flux through the entire surface of the cube.*

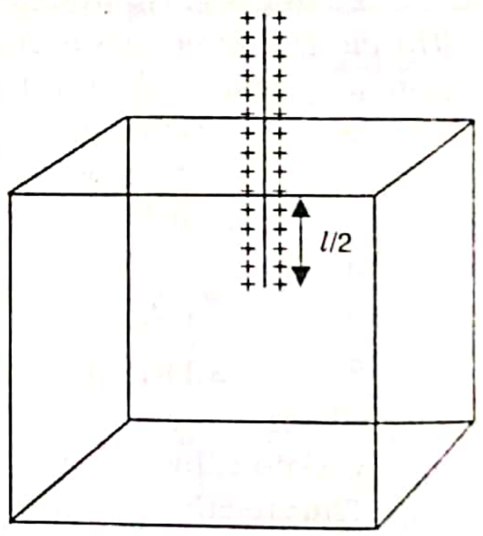

Fig. 20.

Solution. (*a*) For minimum flux through the cube charge enclosed by it should be minimum. This is possible when length of the rod inside cube is minimum. Since one end of rod is at centre of cube at all the times, thus rod must be parallel to at least one edge of cube and it will pass through centre of one of its face as shown in fig. 20.

Thus q' = charge enclosed $= \left(\dfrac{q}{l}\right) \times \dfrac{l}{2} = \dfrac{q}{2}$

$\therefore$ Minimum flux through cube $= \dfrac{q'}{\epsilon_0} = \dfrac{q}{2\epsilon_0}$

(*b*) For maximum flux the rod should be aligned as shown in figure (21). In this case length of rod inside the cube is half of length of its body diagonal *i.e.* $\dfrac{l}{\sqrt{2}}$.

Thus q' = charge enclosed $= \left(\dfrac{q}{l}\right) \times \dfrac{l}{\sqrt{2}} = \dfrac{q}{\sqrt{2}}$

$\therefore$ Electric flux $= \dfrac{q'}{\epsilon_0} = \dfrac{q}{\sqrt{2}\,\epsilon_0}$

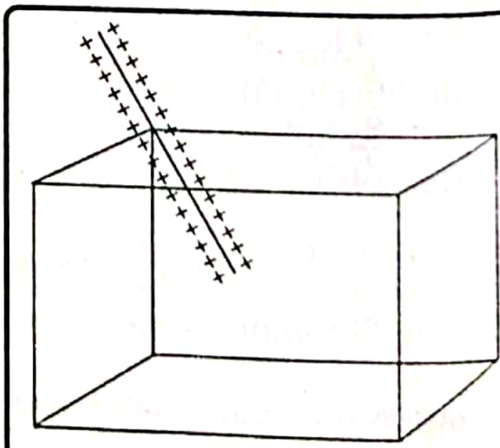

Fig. 21.

Example 8. *A charge q is distributed uniformly within the material of a hollow sphere is inner and outer radii 'a'* *and 'b' as shown in figure 22. Find electric field at any point P at a distance x from the centre of sphere such that* $a \le x \le b$.

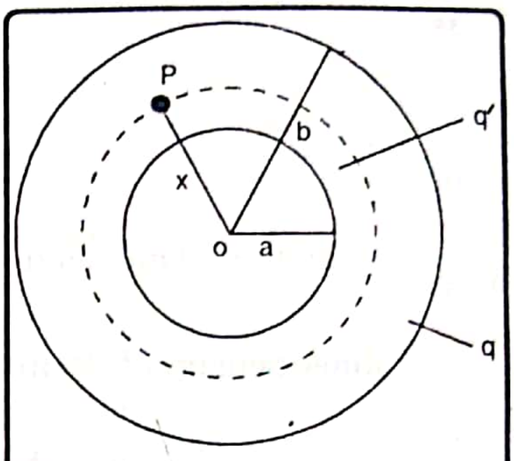

Fig. 22.

Solution. Since charge is uniformly distributed within the material of sphere (shown by shaded region), so its volume charge density is given by

$$\rho = \frac{q}{\frac{4\pi}{3}(b^3 - a^3)} \qquad ...(i)$$

Draw a spherical surface (shown dotted) around centre O having radius x so that point P lies onit. Let q' is charge enclosed by this surface. Since charge is uniformly distributed, so volume charge density is also given as

$$\rho = \frac{q'}{\frac{4\pi}{3}(x^3 - a^3)} \qquad ...(ii)$$

From (*i*) and (*ii*)

$$\frac{q'}{\frac{4\pi}{3}(x^3 - a^3)} = \frac{q}{\frac{4\pi}{3}(b^3 - a^3)} \Rightarrow q' = q\left(\frac{x^3 - a^3}{b^3 - a^3}\right) \qquad ...(iii)$$

The electric field at any point on the Gaussian surface is in the direction of area vector due to spherical symmetry of charge distribution. Thus by Gauss's law :

$$\oiint \vec{E}.\vec{ds} = \frac{q'}{\epsilon_0}$$

$$\Rightarrow \oiint E\,ds\cos 0° = \frac{q}{\epsilon_0}\left(\frac{x^3 - a^3}{b^3 - a^3}\right) \qquad \text{[Using (iii)]}$$

$$\Rightarrow E\oiint ds = \frac{q}{\epsilon_0}\left(\frac{x^3 - a^3}{b^3 - a^3}\right) \Rightarrow E\,(4\pi x^2) = \frac{q}{\epsilon_0}\left(\frac{x^3 - a^3}{b^3 - a^3}\right)$$

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0}\frac{q}{x^2}\left(\frac{x^3 - a^3}{b^3 - a^3}\right)$$

Example 8. *A long cylindrical wire carries a positive charge of linear density* 4×10^{-6} *Cm*$^{-}$. *An electron revolves around it in circular path under the influence of attractive electrostatic force. Calculate (i) speed of resolution of electron (ii) magnitude of linear momentum (iii) kinetic energy of electron.*

Solution. Let electron is resolving around conducting wire in circular orbit of radius r.

Given $\lambda = 4 \times 10^{-6}\ \text{Cm}^{-1}$

Magnitude of electric field of wire at distance r from it is given by

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \qquad ...(i)$$

The centripetal force on electron is provided by electrostatic force of attraction *i.e.*

$$\frac{mv^2}{r} = eE \quad \text{or} \quad \frac{mv^2}{r} = \frac{e\lambda}{2\pi\epsilon_0 r}$$

$$\Rightarrow \quad v = \sqrt{\frac{e\lambda}{2\pi \epsilon_0 m}} = \sqrt{\frac{1.6\times10^{-19}\times4\times10^{-6}}{2\times3.142\times8.854\times10^{-12}\times9.1\times10^{-31}}}$$

$$= 1.12 \times 10^{8} \text{ ms}^{-1}$$

Magnitude of momentum of electron is given by

$$p = mv = 9.1 \times 10^{-31} \times 1.12 \times 10^{8} = 1.02 \times 10^{-22} \text{ kg ms}^{-1}$$

Kinetic energy of electron is given by

$$\text{KE} = \frac{p^2}{2m} = \frac{(1.02\times10^{-22})^2}{2\times9.1\times10^{-31}} = 5.71\times10^{15} \text{ J}$$

Example 9. *A 20cm long thread is tied to a rigid support and a mass of 20g is hanged with it. The suspended body is given a charge of 2μc. A thick charged conducting plate is placed near the suspended body. In equilibrium, the thread makes an angle to 60° with the vertical. Find surface charge density of plate and tension in the thread.*

Solution. The situation is shown in figure (23). The electric field at any point due to thick plate is given by

$$\text{E} = \frac{\sigma}{\epsilon_0} \quad ...(i)$$

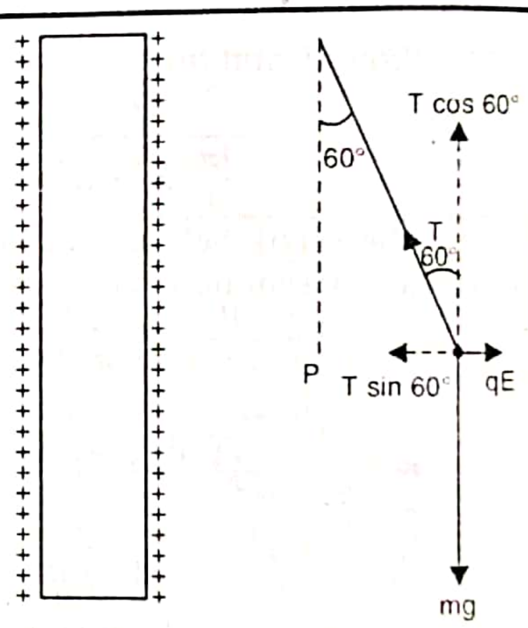

Fig. 23.

Let T is tension in the thread.

For vertical equilibrium

$$\text{T} \cos 60° = mg \quad ...(ii)$$

For horizontal equilibrium

$$\text{T} \sin 60° = q\text{E}$$

or

$$\text{T} \sin 60° = \frac{q\sigma}{\epsilon_0} \quad ...(iii) \text{ [Using } (ii)]$$

Dividing (*iii*) by (*ii*)

$$\Rightarrow \quad \tan 60° = \frac{q\sigma}{\epsilon_0 mg}$$

$$\Rightarrow \quad \sigma = \frac{\epsilon_0 mg \tan 60°}{q} = \frac{8.854\times10^{-12}\times20\times10^{-3}\times9.8\times\sqrt{3}}{2\times10^{-6}}$$

$$= 1.5 \times 10^{-6} \text{ Cm}^{-2}$$

Example 10. *Two oppositely charged large conducting sheets are placed parallel to each other having a separation of 4cm between them. A proton starting from rest near the positive sheet reaches the negative sheet in 2μs. Assuming that both sheets have equal surface charge densities, find the value of charge density.*

Solution. Let σ is magnitude of surface charge density of each sheet. Since both seets are oppositely charged so magnitude of electric fixed between the sheets is given by

$$\text{E} = \frac{\sigma}{\epsilon_0} \quad ...(i)$$

Let a is acceleration of proton and S is distance between two sheets then

$$\text{F} = ma$$

or

$$q\text{E} = ma$$

$$\therefore \quad a = \frac{qE}{m} = \frac{q\sigma}{\in_0 m} \qquad \text{...(ii) [Using (i)]}$$

Now $\qquad S = ut + \frac{1}{2}at^2$

$$\Rightarrow \quad S = 0 + \frac{1}{2}\left[\frac{q\sigma}{\in_0 m}\right]t^2 \qquad (\because u = 0)$$

$$\therefore \quad \sigma = \frac{2S\in_0 m}{qt^2} = \frac{2\times 4\times 10^{-2}\times 8.854\times 10^{-12}\times 1.66\times 10^{-27}}{1.6\times 10^{-19}\times (2\times 10^{-6})^2}$$

$$= 1.84 \times 10^{-9}\,\text{Cm}^{-2}$$

Example 11. *The electric potential in a certain region is given by $V(x, y, z) = 10x^2 + 20y + 5z^3$. Calculate the electric field at the point P (2, 3, 1). Is the field uniform ?*

Solution. The electric field is a related to electric potential as

$$\vec{E} = -\vec{\nabla}V \qquad \text{(provided electric field is conservative)}$$

$$= -\left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right](10x^2 + 20y + 5z^3)$$

$$= -[\hat{i}(20x + 0 + 0) + \hat{j}(0 + 20 + 0) + \hat{k}(0 + 0 + 15z^2)]$$

$$= -20x\hat{i} - 20\hat{j} - 15z^2\hat{k} \qquad \text{...(i)}$$

Since $\vec{E}$ depends upon position co-ordinates. Hence $\vec{E}$ is non uniform electric field.

The electric field at point P (2, 3, 1) is given from relation as

$$\vec{E}(2, 3, 1) = -20(2)\hat{i} - 20\hat{j} - 15(1)^2\hat{k} = -40\hat{i} - 20\hat{j} - 15\hat{k} \quad \text{units}$$

Exercise 1. A point charge of $11\mu C$ is placed at the top of a cube of side 6 cm. Calculate electric flux through the cube. [Ans. $6.21 \times 10^3\,\text{Nm}^2 C^{-1}$]

Exercise 2. Calculate the total charge enclosed by a closed surface, if the number of electric field lines entering in it are 10,000 and leaving it are 30,000. [Ans. 1.77×10^{-7} C]

Exercise 3. A charge of 40nC is distributed uniformly on the surface of a sphere of radius 1cm. It is covered by a concentric hollow conducting shpere of radius 5cm. (*a*) Find the electric field at a point 2cm away from the centre. (*b*) A charge of 60nC is placed on the hollow sphere. Find the surface charge density onthe outer surface of the hollow sphere.

[Ans. (*a*) $9 \times 10^5\,\text{NC}^{-1}$, (*b*) 100 *n*C]

Exercise 4. An electric field of $(2\hat{i} + 3\hat{j})$ NC^{-1} exists in a region. Calculate electric flux through a small surface of area $10^{-3} m^2$ placed in (*i*) XY plane (*ii*) YZ plane (*iii*) XZ plane.

[Ans. (*i*) 0, (*ii*) $2 \times 10^{-3}\text{Nm}^2 C^{-1}$, (*iii*) $3 \times 10^{-3}\,\text{Nm}^2 C^{-1}$]

Example 5. Show that electrostatic pressure at the surface of a conducting body is proportional to square at surface charge density at that point.

Exercise 6. A point charge q is placed at (*i*) corner (*ii*) middle point of an edge of a cube of each side *l*. Find electric flux through the cube in each case. [Ans. (*i*) $\frac{q}{8\in_0}$, (*ii*) $\frac{q}{4\in_0}$]

Exercise 7. Find the magnitude of electric field at a point 2cm away from a line charge of density $4 \times 10^{-6}\,Cm^{-1}$. [Ans. $3.60 \times 10^6\,\text{NC}^{-1}$]

SHORT ANSWER TYPE QUESTIONS

Q. 1. Differential Equations of Electrodynamics are more useful than corresponding equations in integral form. Why ?

Ans. It is because of the fact that differential equations are point equations. The solution of a differential equation will provide us expression for the value of the variable at a point (x, y, z) in space. By putting various values of x, y, z we can find the value of variable at any point. On the other hand in the integral equations, we find average values of the physical quantity in the region. Moreover differential equations are easier to solve as compared to integral equations.

Q. 2. What is a Gaussian Surface ?

Ans. A hypothetical closed surface of any shape drawn in the electric field to solve the problems using Gauss law is called Gaussian Surface. In practical problems, the shape of Gaussian Surface is decided keeping in view the symmetry considerations of the problem.

Q. 3. Is the charge on a particle invariant ?

Ans. Yes, charge on a particle does not charge when it is at rest or in relative motion. Thus the charge does not depend on velocity of particle.

Q. 4. Which law of electrostatics can be used for measuring charges in motion ?

Ans. Coulomb's law is not valid for charges in motion, because magnitude as well as direction of force between moving charges varies continuously. However, Gauss law is valid even for moving charges. It is because of the fact that magnitude of charge does not vary with motion but shape of Guassian Surface may vary with motion. But shape of Gaussian surface is immaterial for applying Gauss law. Hence we can use Gauss law for measuring charges in motion.

Q. 5. The relation $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ is not sufficient to determine $\vec{B}$ at a point, even when $\vec{J}$ at that point is known. Why ?

Ans. It is because of the reason that circulation of $\vec{B}$ can be obtained around any of the infinitely many possible paths enclosing a current element in which current density is $\vec{J}$. That is many different values of $\vec{B}$ can satisfy above mentioned relation. Hence this equation alone is not sufficient to determine $\vec{B}$. To find $\vec{B}$ uniquely, we use $\vec{\nabla} \cdot \vec{B} = 0$ in addition to above equation.

Q. 6. What are Dielectrics ?

Ans. Dielectrics are insulating materials. When they are placed in electric field, then these allow field lines to pass through them to some extent.

Q. 7. What is atomic polarizability ?

Ans. The dipole moment (p) of an atom/molecule of a dielectric is proportional to Net Electric field (E). That is

$$p \propto E \qquad \text{or} \qquad p = \alpha \varepsilon_0 E$$

$$\Rightarrow \qquad \alpha = \frac{p}{\varepsilon_0 E}$$

α is called atomic/molecular polarizability. It is defined as the ratio of dipole moment of the atom to the product of electric field intensity and permittivity of free space.

Q.8. What is importance of dielectric constant ?

Ans. Dielectric constant of a substance gives its suitability to be used for increasing the capacity of a capacitor. Larger is the value of dielectric constant more is the increase in capacity of capacitor. Further liquids having high value of dielectric constant are better solvents as most of dielectric of ionic solids dissolve in them.

Q.9. What is the importance of polarizability ?

Ans. Atomic polarizability is a measure of the radius of molecule of dielectric substance. Hence by knowing atomic polarizability we can calculate molecular size.

Q.10. On what factors, dielectric strength of a material depends ?

Ans. It depends on thickness of material, temperature, dielectric constant, humidity and time for which electric field is applied.

Q.11. What is the relation between displacement current and conduction current ?

Ans.

$$I_d = \iint \vec{J}_D . \vec{ds} = \iint \frac{\partial \vec{D}}{\partial t} . \vec{ds}$$

$$= \frac{\partial}{\partial t} \iint \vec{D} . \vec{ds} = \frac{\partial}{\partial t} \iint \in_0 \vec{E} . \vec{ds}$$

$$= \frac{\partial}{\partial t} \in_0 \iint E ds \cos 0°$$

($\because$ between plates of capacitor, $\vec{E}$ and $\vec{ds}$ are parallel)

$$= \frac{\partial}{\partial t} (\in_0 EA) = \frac{\partial}{\partial t} \left(\frac{\in_0 A}{d} \right) V \qquad \left(\because E = \frac{V}{d} \right)$$

$$= \frac{\partial}{\partial t} (CV) = \frac{\partial q}{\partial t} \qquad (\because q = cV)$$

$$= I$$

Thus displacement and conduction current are equal.

Q.12. Write Maxwell's equations in differential form and obtain their integral form from these.

Solution. First equation

$$\vec{\nabla} \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$$

Integrating on both sides over the open area of coil, we get

$$\iint \vec{\nabla} \times \vec{E} . \vec{ds} = \frac{-\partial \vec{B}}{\partial t} \iint \vec{B} . \vec{ds}$$

$$\Rightarrow \quad \oint \vec{E} . \vec{dl} = \frac{-\partial \phi_B}{\partial t} \qquad ...(i) \text{ [Using Stoke's Theorem]}$$

This is integral form of first equation.

Second Equation.

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Integrating both sides over open area of closed path, we get

$$\iint \vec{\nabla} \times \vec{H} . \vec{ds} = \iint \vec{J} . \vec{ds} + \frac{\partial}{\partial t} \iint \vec{D} . \vec{ds}$$

$$\Rightarrow \oint \vec{H}.\vec{dl} = I + I_d \quad \text{...(ii) [Using Stoke's Theorem]}$$

Here
$$I = \iint \vec{J}.\vec{ds} = \text{conduction current}$$

$$I_d = \frac{\partial}{\partial t}\iint \vec{D}.\vec{ds} = \text{displacement current}$$

Equation (*ii*) is integral form of Maxwell's second equation.

Third Equation.

$$\vec{\nabla}.\vec{D} = \rho_f$$

Integrating both sides over volume occupied by closed surface, we get

$$\iiint \vec{\nabla}.\vec{D}\,dV = \iiint \rho_f\,dV$$

$$\Rightarrow \oiint \vec{D}.\vec{ds} = q_f \quad \text{...(iii) (Using Gauss's Divergence Theorem)}$$

This is integral form of Maxwell's 3rd equation.

Fourth Equation

$$\vec{\nabla}.\vec{B} = 0$$

Integrating both sides over the volume occupied by closed surface, we get

$$\iiint \vec{\nabla}.\vec{B}\,dV = 0$$

$$\Rightarrow \oiint \vec{B}.\vec{ds} = 0 \quad \text{...(iv) (Using Gauss's Divergence Theorem)}$$

This is integral form of Maxwell's Fourth Equation.

Q.13. What is the importance of current element in Biot Savart's law ?

Ans. The unit of current element is

$$I\,dl = Am$$

This unit is same as that of unit of magnetic pole strength. Thus although isolated magnetic monopoles do not exist, yet we can treat a current element like a monopole.

Q.14. What is the cause of producing displacement current ?

Ans. Changing electric flux is the cause of displacement current.

Q.15. Is displacement current like conduction current a source of magnetic field ?

Ans. Yes, because displacement current is produced whenever electric flux within the plates of capacitor is changing (*i.e.* capacitor is charging or discharging) and changing electric flux is a source of magnetic field according to Maxwell's equations. Thus displacement current is a source of magnetic field.

Q.16. What are various kinds of electromagnetic waves ?

Ans. Electromagnetic waves are mainly classified as (*i*) spherical waves (*ii*) cylindrical waves (*iii*) Plane waves.

The equation of spherical wave is $\psi = \frac{\psi_0}{r}\sin(\omega t - kx)$; where $\psi = E, B$

Thus in case of cylindrical waves intensity is inversity proportional to square of distance. There are called spherical waves because their wave front is spherical.

The equation of cylindrical wave is $\psi = \frac{\psi_0}{\sqrt{r}}\sin(\omega t - kx)$

Their wave front is cylindrical and intensity varies as $\frac{1}{r}$.

The equation of plane wave is $\psi = \psi_0 \sin(\omega t - kx)$

Their intensity is independent of distance and wave front is plane wave front.

Q.17. What are sources of various kinds of electromagnetic waves ?

Ans. A point source produces spherical electromagnetic waves. A line source produces cylindrical electromagnetic waves. While LASER produces plane electromagnetic waves.

Q.18. What are various kinds of Polarization ?

Ans. Polarisation is of four kinds :

(*i*) **Electronic Polarisation.** It arises due to displacement of centres of positive and negative charge clouds in external electric field.

(*ii*) **Ionic Polarisation.** It arises due to displacement of positive and negative ions in an ionic solids in the presence of external electric field.

(*iii*) **Orientation Polarisation.** It arises due to rotation of dipoles in a polar dielectric when external electric field is applied.

(*iv*) **Space charge Polarisation.** It arises at the interface where thus dielectrics having different resistivity are joined. It arises due to concentration gradient of charges across the interface.

Q.19. What is a Dielectric strength of a substance ?

Ans. It is the maximum value of electric field that can be applied across a dielectric without its breakdown.

Q.20. Will Gauss's law remain valid if electric field varies as $\frac{1}{r^3}$?

Ans. Let $E \propto \frac{1}{r^3}$ *i.e.* $E = \frac{1}{4\pi \epsilon_0}\frac{q}{r^3}$ (for a point charge)

Thus electric flux through a closed surface in vacuum is given as

$$\phi_E = \oiint \vec{E}.\vec{ds} = \oiint E ds\cos\theta \quad \text{(where } \theta = \text{angle between } \vec{E} \text{ and } \vec{ds})$$

$$= \frac{1}{4\pi\epsilon_0} q \oiint \frac{ds\cos\theta}{r^3} \quad \ldots(i)$$

But $\oiint \frac{ds\cos\theta}{r^3} \neq 4\pi$ $\left(\because \oiint \frac{ds\cos\theta}{r^2} = \text{solid angle} = 4\pi\right)$

Thus equation (*i*) will not give us $\phi_E = \frac{q}{\epsilon_0}$. Hence Gauss's law will not be valid in this case.

Q.21. What are limitations of Coulomb's law ?

Ans. (*i*) It is valid only for point charges.

(*ii*) It is valid only for charges at rest.

(*iii*) It gives force between only two point charges.

(*iv*) It is not valid if distance between point charges is less than 10^{-15}m.

Q.22. What are limitations of Gauss's law of electrostatics ?

Ans. (*i*) It does not give direction of electric field and it is found from symmetry properties of charge distribution.

(*ii*) It is valid only for closed surface and not for open surface.

Q.23. A particle of charge *q* moves undeviated through a region containing perpendicular electric and mangetic fields. What is the speed of particle ?

Ans. Small particle goes undeviated, so net force on it should be zero.

i.e. Electric force = magnetic force

$$\Rightarrow \quad qE = qVB \sin 90°$$

$$\Rightarrow \quad V = \frac{E}{B}$$

Q.24. Explain how a dielectric inserted between plates of capacitor increases its capacity.

Ans. On inserting dielectric between plates of capacitor, polarisation of dielectric takes place. Due to this an electric field opposite to applied electric field a produced. This decreases net electric field between plates of capacitor by a factor of K (where K = dielectric constant). Due to this potential difference between plates also decreases by a factor of K. But charge on plates of capacitor is unchanged *i.e.*

$$Q' = Q \quad \Rightarrow \quad C'V' = CV$$

$$\Rightarrow \quad C'\frac{V}{K} = CV \quad \Rightarrow \quad C' = KC$$

So capacity becomes K times the old capacity.

Since K is always more than one, so capacity increases.

Q.25. Is electric field always conservative ?

Ans. No. Only electrostatic field (field due to stationary charges) is conservative. Electric field produced by moving charges and electric field produced by changing magnetic field are not conservative.

Q.26. What do you mean be irrotational and solenoidal fields ?

Ans. If divergence of a vector field is zero, then such a field is called solenoidal.

If curl of a vector field is zero, then such a field is called irrotational.

Q.27. What do you mean by Dielectric loss ? Explain briefly.

Ans. When electric field is applied across a polar dielectric, then due to torque on electric dipoles, they start orienting themselves in the direction of electric field. With increase in strength of electric field the number of dipoles in the direction of electric field goes on increasing untill saturation polarisation is achieved when all dipoles are in the direction of electric field. If electric field is suddenly switched off then electric dipoles donot orient themselves so randomly as was before applying electric field *i.e.* a large number of dipoles are still inthe direction of external field although electric field in switched off. Hence the polarisation vector of sample will be non zero after switching off electric field. To reduce the polarisation to zero, one has to apply an external electric field in opposite direction. But again if we continue to increase strength of electric field in opposite direction, then dielectric specimen is polarized in opposite direction and soon saturation polarisation is achieved in that direction. On switching off electric field the polarisation vector of sample doesnot become zero as earlier. Hence we have to again apply electric field in the same direction as at the starting of our discussion. Further we can again polarise the sample by increasing electric field strength in the initially taken direction. We thus see that external electric field is always needed to polarise the dielectric in two opposite directions and the energy spent in this process is wasted in the form of heat. When alternating electric field is applied across a dielectric, then material is repeteadly polarized and depolarized and in this process energy is continuously wasted as heat. This loss of energy is called Dielectric Loss.

Q.28. Write some applications of Dielectric.

Ans. (*i*) Glass and plastics are dielectrics used in transmission of optical signals in optical fibres.

(*ii*) Dielectrics are used in providing electric insulation.

(*iii*) A dielectric slab is inserted between plates of a capacitor to increase its capacity. Dielectrics used for this purpose are cellulose, polyvinyl chloride (PVC), mica etc.

(*iv*) When ionic salts are dissolved in a dielectric liquid like water, then molecules of salt dissociate into ions due to decrease in coulomb attraction by factor of $\frac{1}{K}$ where K is dielectric constant. Hence the liquid solution becomes electrolyte and can be used to carry electrolysis.

Q.29. Explain various types of polarisation.

Ans. There are four different types of polarisation. These are explained briefly as below:

1. Electronic Polarisation. This type of polarisation arises due to shifting of centre of positive and negative chargs in the molecule in opposite directions when external electric field is applied. Non polar Dielectric molecules in gas phase show this kind of polarisation.

2. Ionic Polarisation. When external electric field is applied across an ionic solid, then cations and anions in the molecule move in opposite direction. This type of polarisation is called ionic polarisation.

3. Orientation Polarisation. When external electric field is applied across polar dielectric material, then dipoles of the material feel torque and start rotating so as to aligen themselves in the direction of external electric field. Due to this, the substance gets polarised in the direction of external electric field. This is called orientation polarisation.

4. Interfacial or Space Charge Polarisation. When external electric field is applied across a multiphase (heterogereous) dielectric, then bound charges are accumulated at interphases of the material. Such kind of polarisation is called space charge polarisation.

If $\overrightarrow{P_e}, \overrightarrow{P_i}, \overrightarrow{P_o}$ and $\overrightarrow{P_s}$ are polarisation vectors due to electronic, inonic, orientation and space charge polarisation respectively, then net polarisation vector is given by

$$\vec{P} = \overrightarrow{P_e} + \overrightarrow{P_i} + \overrightarrow{P_o} + \overrightarrow{P_s}$$

QUESTIONS

1. What is meant by polarisation in a dielectric material ?
2. Deduce Gauss law in differential form and further deduce coulomb's law from Gauss law.
3. By using Maxwell's equations, develop wave equation for transverse electric and magnetic field in free space.
4. State Faraday's laws of electromagnetic induction.
5. Define dielectric polarisation.
6. State and prove Gauss law in electrostatics and express it in differential form $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$, where $\vec{E}$ is the electric field, ρ is charge density and ε_0 is permittivity of vacuum.

7. What do you understand by curl of a vector ?
8. Explain electromotive force.
9. Define electric potential and electric field at a point. Explain how we may calculate the electric field from the electric potential and vice versa ?
10. What is dielectric polarisation and electric displacement ?
11. Give Maxwell's equations.
12. Explain electric field and electric lines of force.
13. Explain dielectric polarisation in brief.
14. Discuss the propagation of electromagnetic waves in a dielectric medium.
15. Give the physical significance of divergence and curl of a vector.
16. State Maxwell's equations and solve them to obtain velocity of the electromagnetic waves in a homogeneous istropic dielectric medium.
17. State Gauss law.
18. Write the conditions under which Maxwell's equations are applicable.
19. Describe the behaviour of a dielectric in a static electric field. Explain the meaning of polarisation $\vec{P}$ and electric displacement $\vec{D}$. Also find relation between these.
20. Write down Maxwell's equations and state the laws of electrodynamics, to which these correspond. Deduce the wave equation for electromagnetic waves in free space.
21. Discuss Faraday's law of electromagnetic induction and express it in integral form.
22. Why are electric field lines normal to an equipotential surface at all point ?
23. Write down Maxwell's equations.
24. Explain the meaning of the gradient of a scalar field.

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