

12

Projections of Solids

12.1 INTRODUCTION

A solid is a three dimensional object having length, breadth and thickness. In engineering practice, one often comes across solids bounded by simple or complex geometric surfaces. To represent a solid in orthographic projections, the number and types of views necessary will depend upon the type of solid and its orientation with respect to the principal planes of projections. Sometimes, additional views projected on auxiliary planes become necessary to make the description of a solid complete.

12.2 TYPES OF SOLIDS

Solids may be divided into two main groups:

(i) Polyhedra

(a) Regular Polyhedra

(b) Prisms

(c) Pyramids

(ii) Solids of Revolution

(a) Cylinders

(b) Cones

(c) Sphere

(i) Polyhedra : A polyhedron is defined as a solid bounded by planes called faces.

(a) Regular Polyhedra : A polyhedra is said to be regular if all its faces are similar, equal and regular. There are five regular polyhedra which may be defined as stated below:

- Tetrahedron : It has four equal faces, each are equilateral triangle as shown in Fig. 12.1

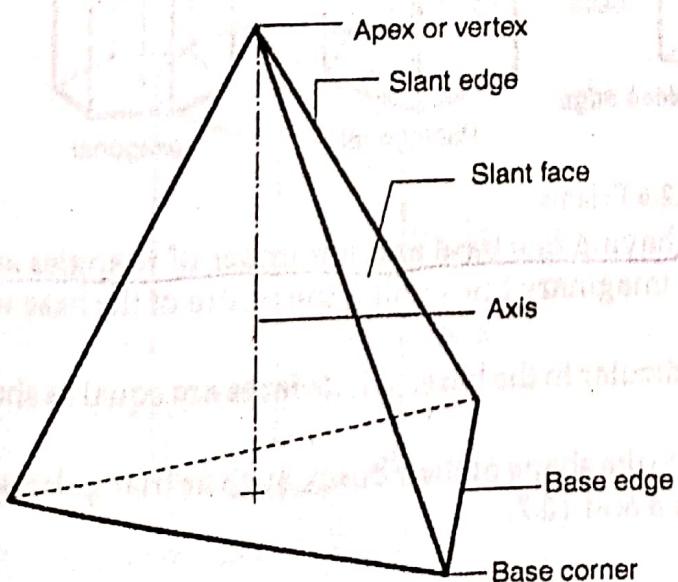


Fig. 12.1 Tetrahedron

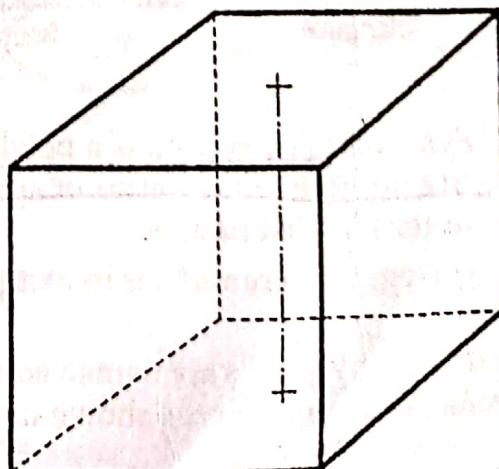


Fig. 12.2 Cube

- Cube or hexahedron : It has six equal faces, each a square as shown in Fig. 12.2.
- Octahedron : It has eight equal faces, each an equilateral triangle as shown in Fig. 12.3.
- Dodecahedron : It has twelve equal faces, each a regular pentagon as shown in Fig. 12.4.

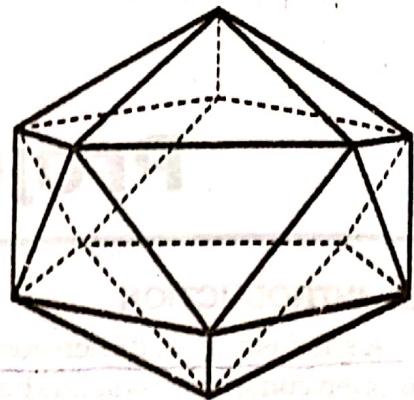
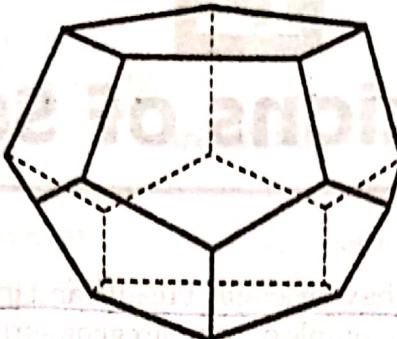
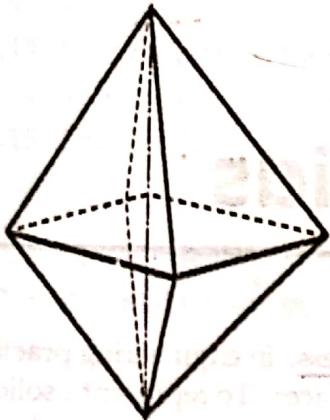


Fig. 12.3 Octahedron

Fig. 12.4 Dodecahedron

Fig. 12.5 Icosahedron

- Icosahedron : It has twenty equal faces, each an equilateral triangles as shown in Fig. 12.5.
- (b) Prisms : A prism is a polyhedron having two equal ends or bases, parallel to each other. The two bases are joined by faces which are rectangles or parallelograms. The imaginary line joining the centres of the bases is called the axis.

A right regular prism has its axis perpendicular to the base. All its faces are equal rectangles as shown in Fig. 12.6.

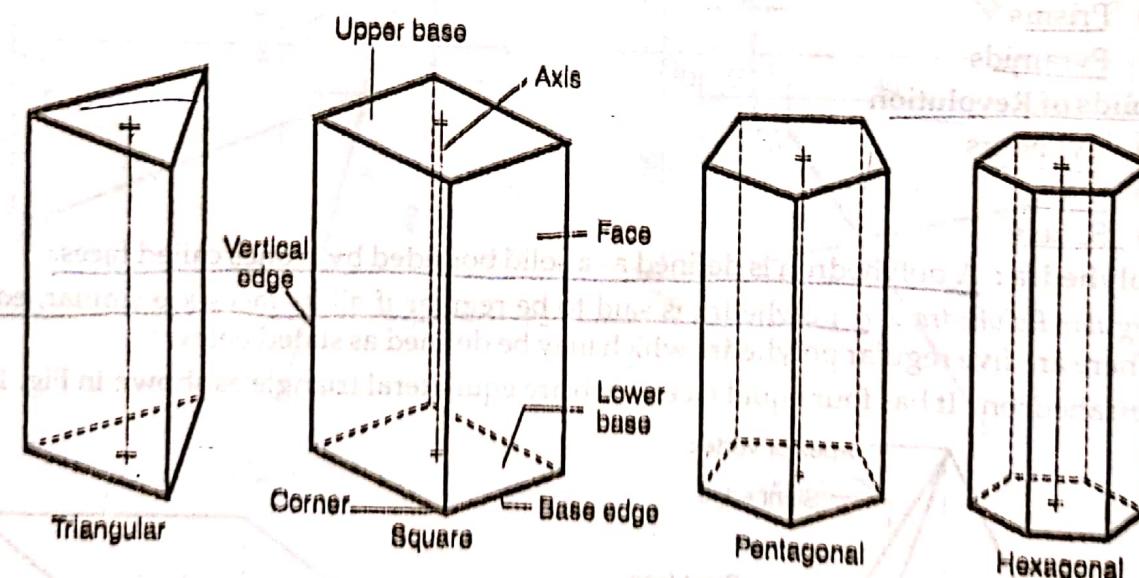


Fig. 12.6 Prism

- (c) Pyramids : A pyramid is a polyhedron having one base and a number of triangles as faces meeting at a point called as vertex or apex. The Imaginary line joining the centre of the base with its apex or vertex is called its axis.

A right regular pyramid has its axis perpendicular to the base. All its faces are equal as shown in Fig. 12.7.

Prisms and pyramids are named according to the shape of their bases, such as triangular, square, pentagonal, hexagonal etc. as shown in Figs. 12.6 and 12.7.

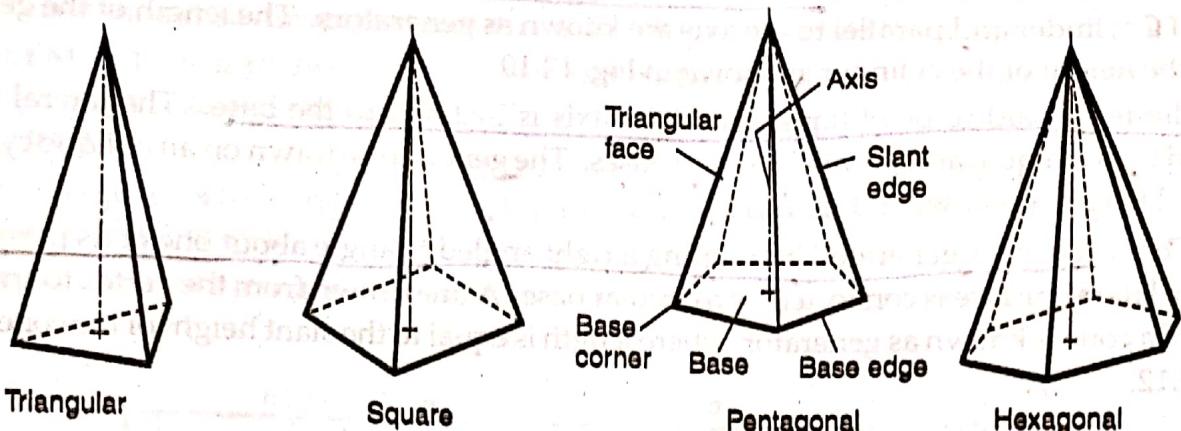


Fig. 12.7 Pyramids

In case of oblique solids, the axis is inclined to the base. The faces of an oblique prism are parallelograms and that of pyramids are triangles, which are not similar as shown in Figs. 12.8 and 12.9. The bases of oblique prisms are parallel, equal and similar.

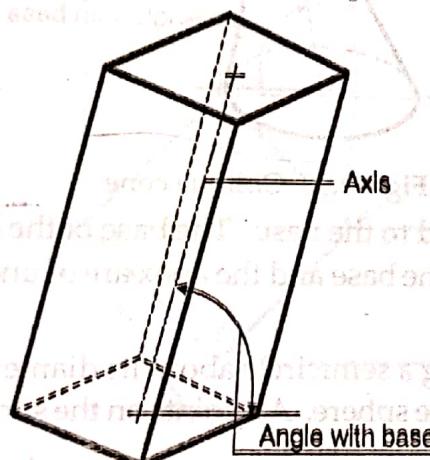


Fig. 12.8 Oblique prism

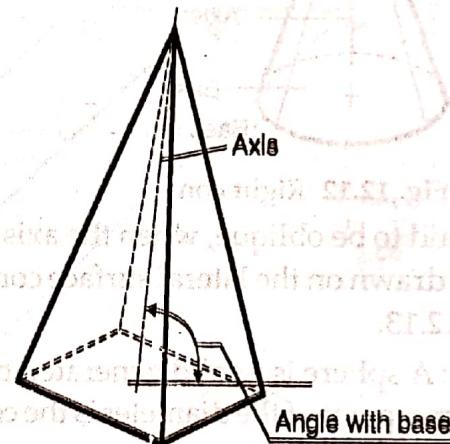


Fig. 12.9 Oblique pyramid

(ii) Solids of revolution: Solids of revolution are obtained or generated by rotating a plane figure about one of its edges.

(a) Cylinder: A cylinder is generated by rotating a rectangle about one of its edges, which remains fixed. It has two equal circular bases. The imaginary line joining the centres of the bases is called its axis.

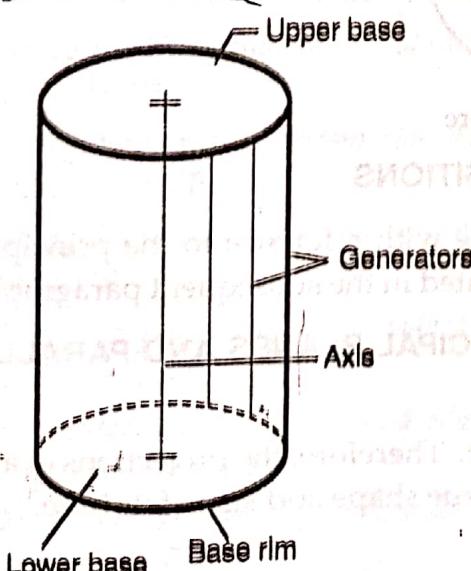


Fig. 12.10 Right cylinder

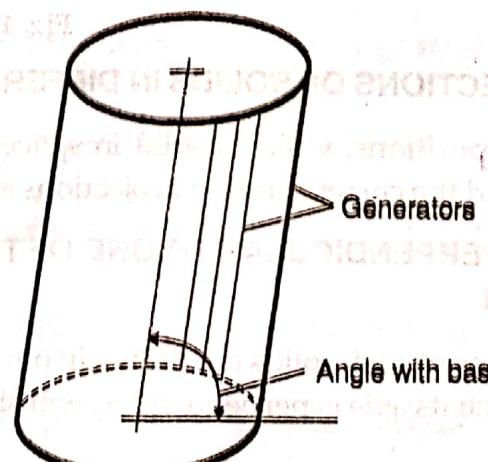


Fig. 12.11 Oblique cylinder

A cylinder is said to be right, when its axis is perpendicular to the base. The lines drawn on the surface of a cylinder and parallel to the axis are known as generators. The length of the generator is equal to the height of the cylinder as shown in Fig. 12.10.

A cylinder is said to be oblique, when the axis is inclined to the base. The lateral surface is connected by two equal and parallel circular bases. The generators drawn on an oblique cylinder are of unequal length as shown in Fig. 12.11.

(b) Cone : A cone is generated by rotating a right angled triangle about one of its perpendicular sides. The lateral surface is connected by a circular base. A line drawn from the vertex to any point on the base of a cone is known as generator, where length is equal to the slant height of the cone as shown in Fig. 12.12.

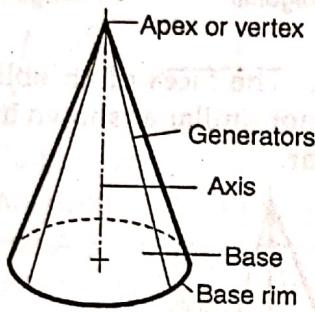


Fig. 12.12 Right cone

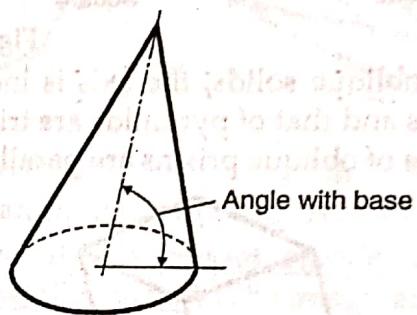


Fig. 12.13 Oblique cone

A cone is said to be oblique, when the axis is inclined to the base. The base of the cone is a circle. The generators drawn on the lateral surface connecting the base and the apex are of unequal length as shown in Fig. 12.13.

(c) Sphere : A sphere is a solid generated by rotating a semicircle about its diameter as shown in Fig. 12.14. The midpoint of the diameter is the centre of the sphere. All points on the surface of a sphere are equidistant from its centre.

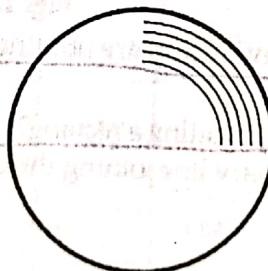


Fig. 12.14 Sphere

12.3 PROJECTIONS OF SOLIDS IN DIFFERENT POSITIONS

Various positions, which a solid in space can make with reference to the principal planes of projection and the corresponding projections are illustrated in the subsequent paragraphs.

12.4 AXIS PERPENDICULAR TO ONE OF THE PRINCIPAL PLANES AND PARALLEL TO THE OTHER

Projections of such solids are drawn in one step only. Therefore, the projections of a solid on the plane to which its axis is perpendicular, will show the true shape and size of its base.

(a) Axis perpendicular to the HP and parallel to the VP : When the axis is perpendicular to the HP, the top view should be drawn first and then front view is projected from it.

~~PROBLEM 12.1. Draw the three views of a cube 30 mm side when it is resting on its base on HP with one of the base edges making an angle of 45° to the VP.~~ (PTU, Jalandhar December 2007)

~~SOLUTION.~~

- Draw a square of base edge 30 mm in the top view, keeping its base edge say d-c or 4-3 inclined at angle of 45° to xy line.
- Project the front view and end view as shown in Fig. 12.15. Name all the corner points.

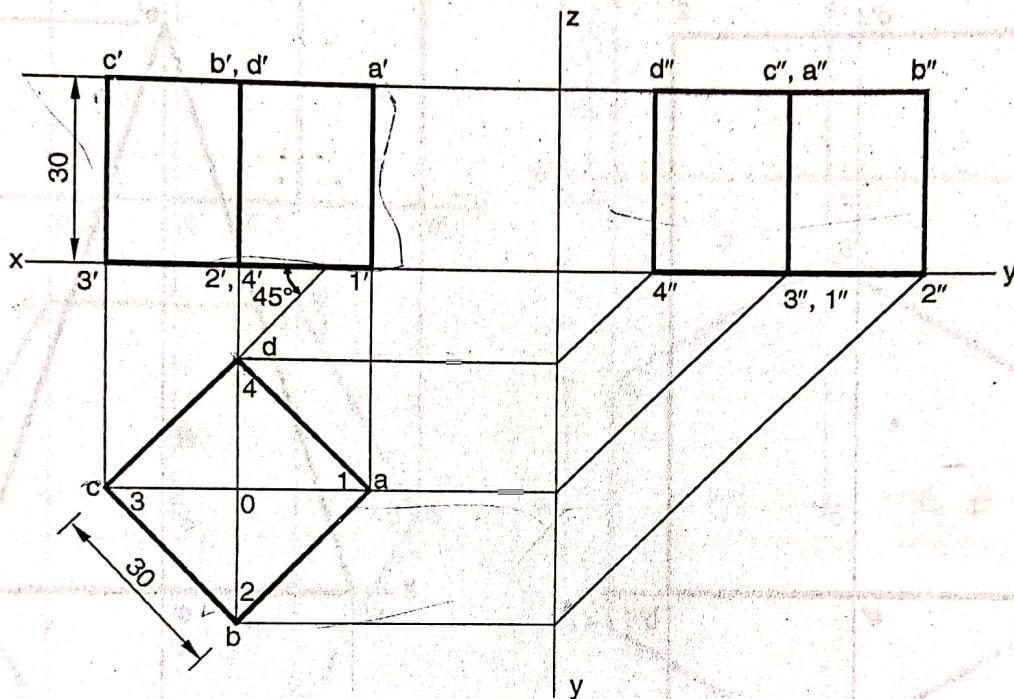


Fig. 12.15 Solution to problem 12.1

~~PROBLEM 12.2 Draw the projections of the following solids, resting in HP on their bases as given, using reference line xy.~~

- A cylinder, 40 mm base diameter and 60 mm height.
- A cone, 40 mm base diameter and 60 mm height.
- A right regular hexagonal prism, side of base 25 mm and axis 50 mm long, having one of its base edges parallel to the VP.
- A right regular hexagonal prism, side of base 25 mm and axis 55 mm long, having one of its base edges perpendicular to VP.
- A right regular pentagonal prism, side of base 25 mm and axis 55 mm long, having one of its base edges parallel to VP.
- A right regular pentagonal prism, side of base 25 mm and axis 55 mm long, having one of its base edges perpendicular to VP.
- A right regular pentagonal pyramid, edge of base 25 mm and height 60 mm long, having an edge of its base perpendicular to VP.
- A right regular pentagonal pyramid, edge of base 25 mm and height 60 mm long, having an edge of its base parallel to VP.

- (i) A right regular hexagonal pyramid, edge of base 25 mm and height 60 mm long, having one of its base edges perpendicular to VP.
- ~~Ques~~ A right regular pentagonal pyramid, edge of base 25 mm and axis 45 mm long, having one of its base edge inclined at an angle 30° to the VP. (PTU, Jalandhar December 2004, May 2005)

SOLUTION.

- (a) Draw a circle of 40 mm diameter in the top view and project the front view which will be a rectangle as shown in Fig. 12.16.

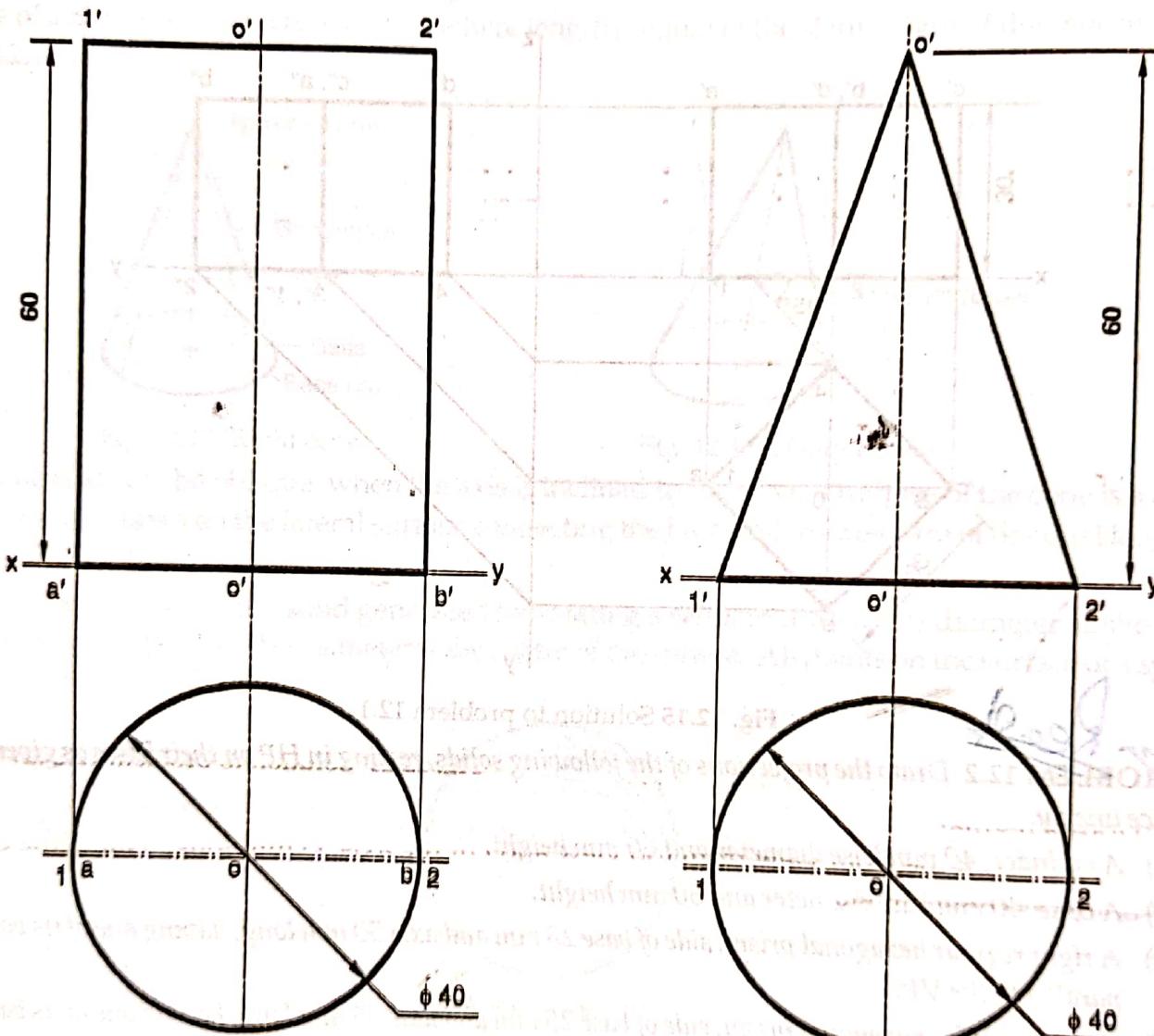


Fig. 12.16 Solution to problem 12.2 (a)

Fig. 12.17 Solution to problem 12.2 (b)

- (b) Draw a circle of 40 mm diameter in the top view. Through the centre O, project the apex or vertex O', 60 mm from xy. Complete the triangle in the front view as shown in Fig. 12.17.
- (c) Draw a regular hexagon of 25 mm side in the top view, keeping one of its sides parallel to the xy. Project the front view from top view and cut the axis 50 mm long. Name the corner points as shown in Fig. 12.18 and complete the views.
- (d) Draw a regular hexagon of 25 mm side in the top view, keeping one of its sides perpendicular to the xy. Project the front view from top view and cut the axis 55 mm long. Name the corner points as shown in Fig. 12.19 and complete the views.

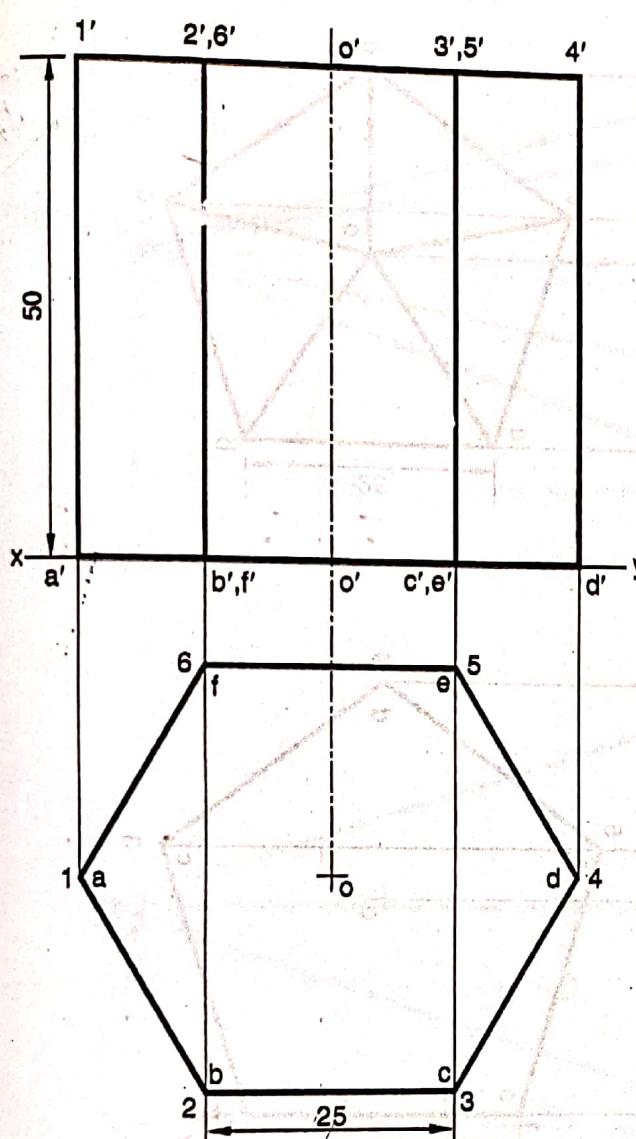


Fig. 12.18 Solution to problem 12.2 (c)

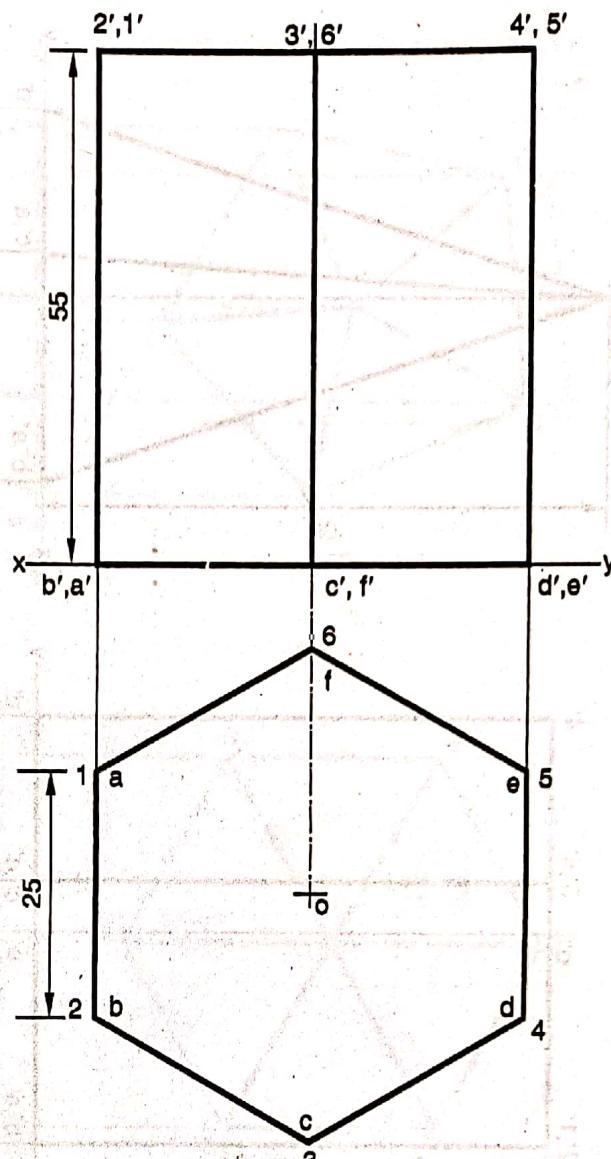


Fig. 12.19 Solution to problem 12.2 (d)

- (e) Draw a regular pentagon of 25 mm side in the top view, keeping one of its base edges parallel to xy . Project the front view from top view and cut the axis 55 mm long. Name the corner points as shown in Fig. 12.20 and complete the views.
- (f) Draw a regular pentagon of 25 mm side in the top view, keeping one of its base edges perpendicular to xy . Project the front view from top view and cut the axis 55 mm long. Name the corner points as shown in Fig. 12.21 and complete the views.
- (g) Draw a regular pentagon of 25 mm side in the top view, keeping one of its base edges perpendicular to xy . Through the centre O, project the apex or vertex O' 60 mm from xy . Complete the views as shown in Fig. 12.22. Name all the corner points.
- (h) Draw a regular pentagon of 25 mm side in the top view, keeping one of its base edges parallel to xy . Through the centre O, project the apex O' 60 mm from xy . Complete the views as shown in Fig. 12.23. Name all the corner points.
- (i) Draw a regular hexagon of 25 mm side in the top view, keeping one of its base edges perpendicular to xy . Through the centre O, project the apex or vertex O', 50 mm from xy . Complete the views as shown in Fig. 12.24.
- (j) Draw a pentagonal pyramid of 25 mm base edge in the top view, with a base edge 5-1 inclined at 30° to xy . Project the front view correspondingly as shown in Fig. 12.25.

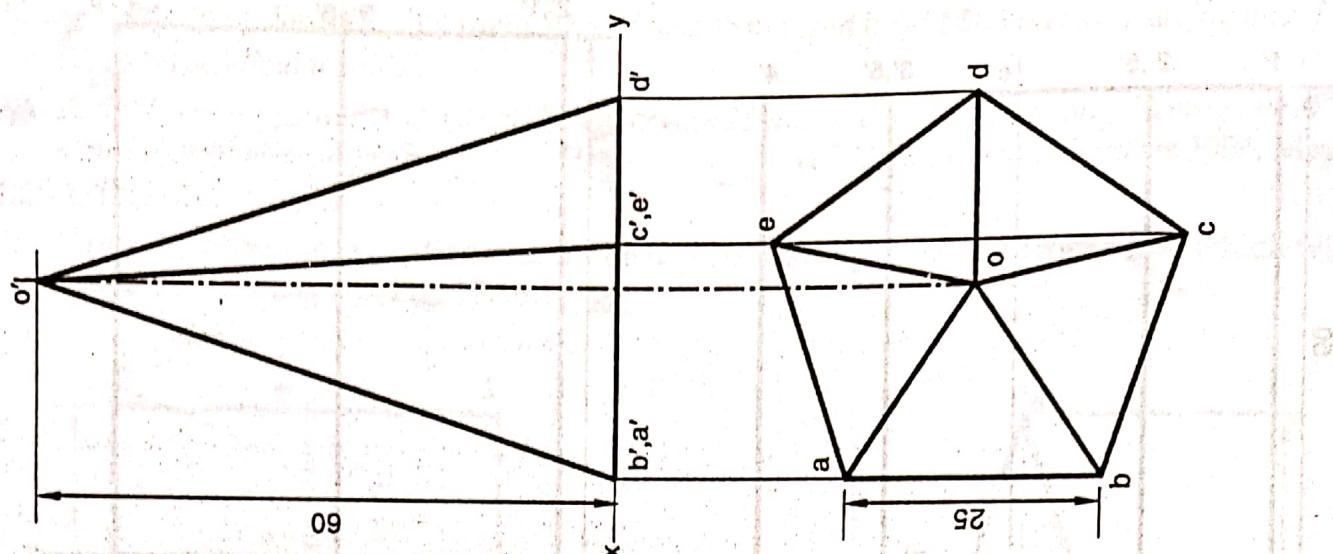


Fig. 12.22 Solution to problem 12.2 (g)

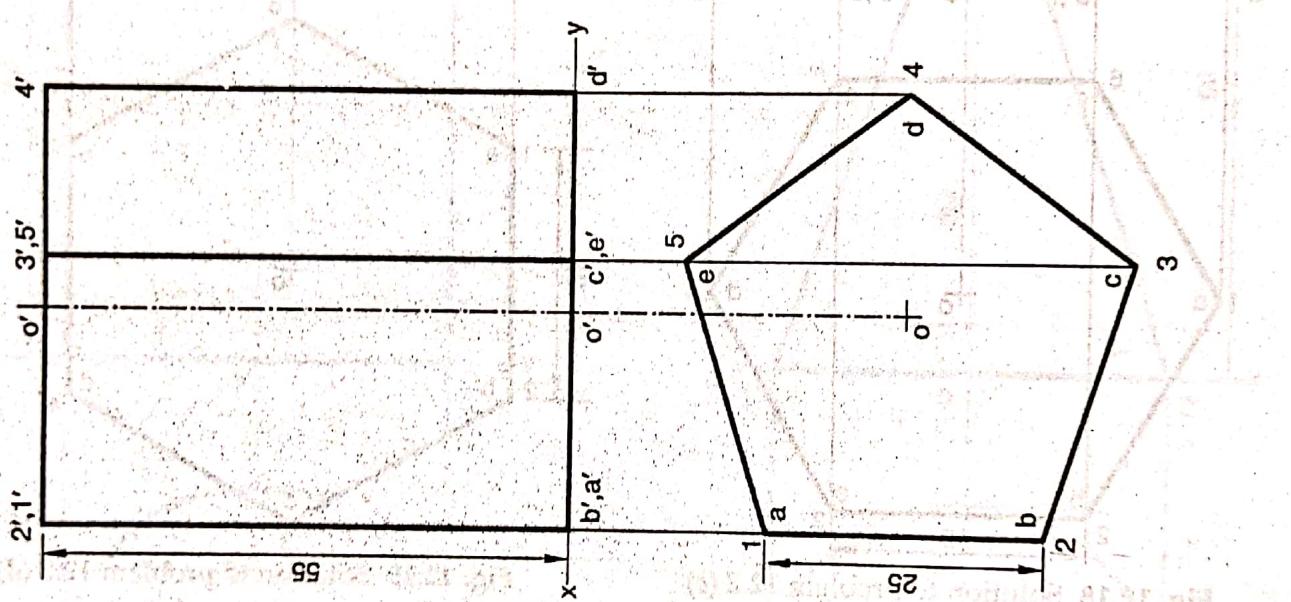


Fig. 12.21 Solution to problem 12.2 (f)

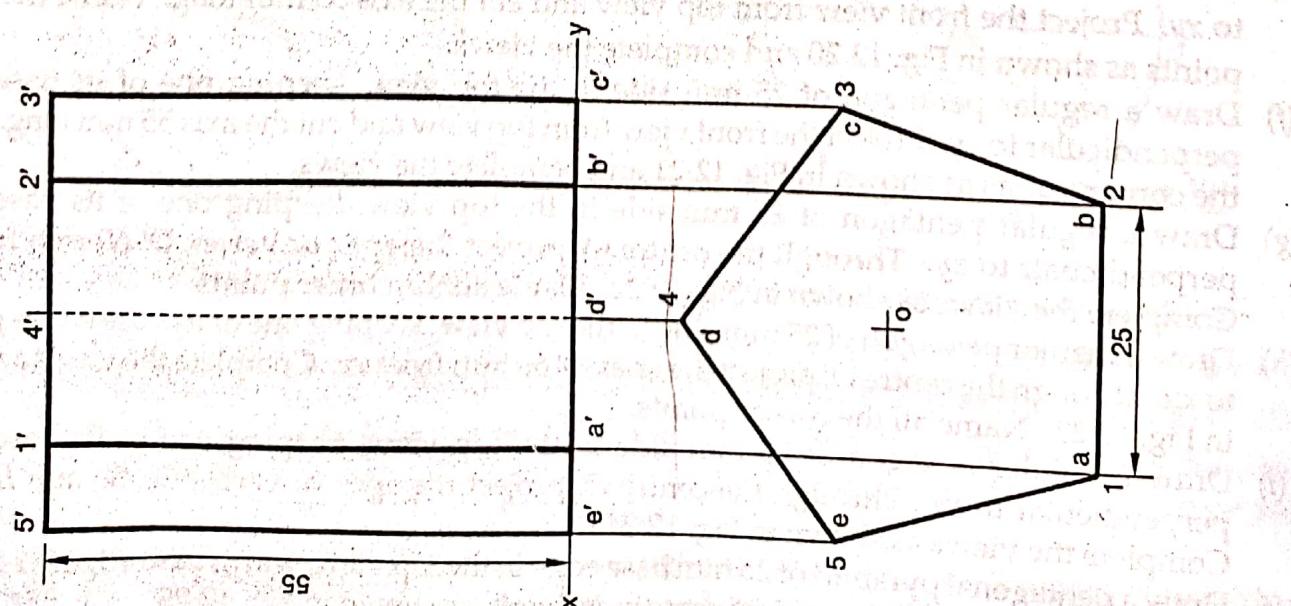


Fig. 12.20 Solution to problem 12.2 (e)

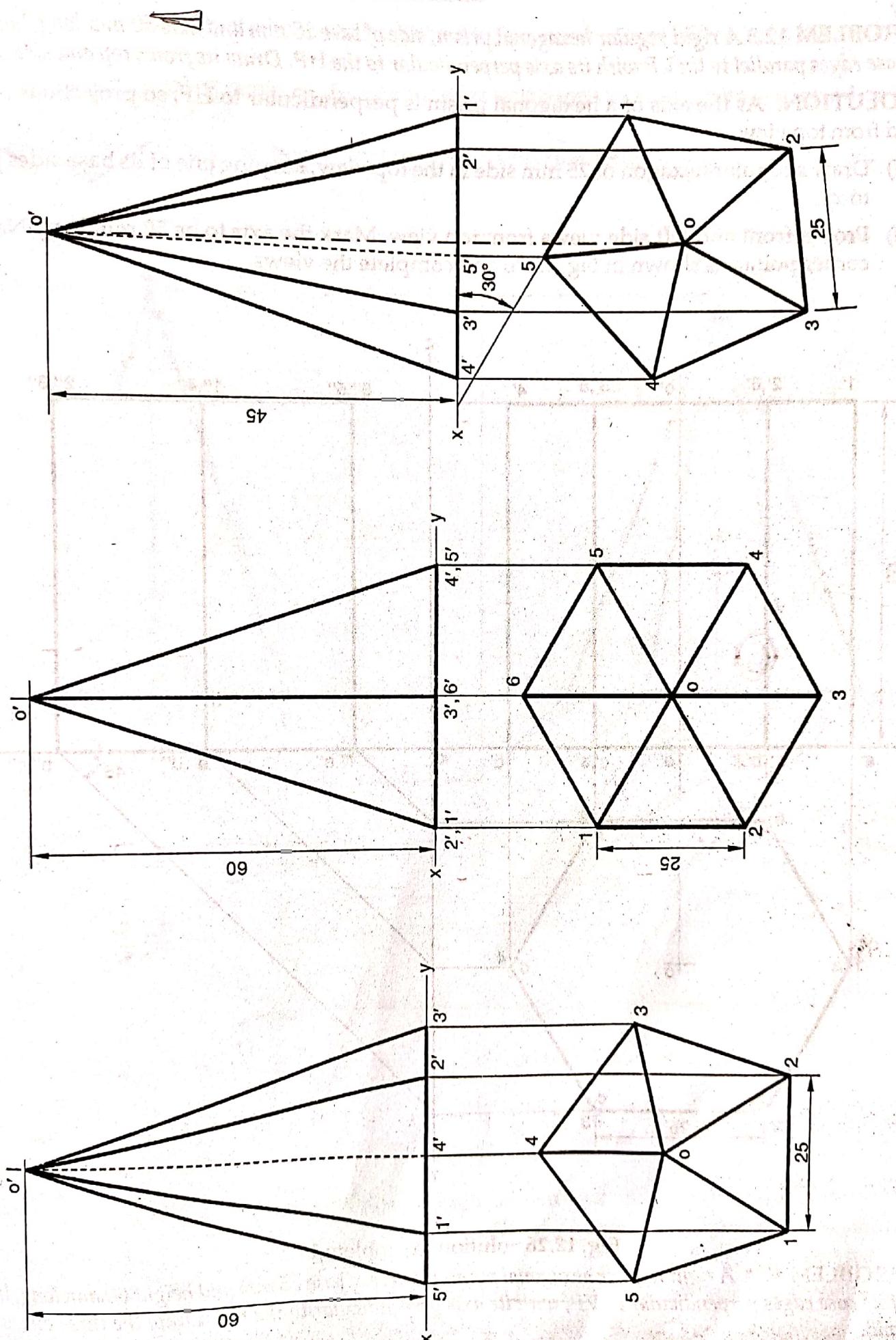


Fig. 12.23 Solution to problem 12.2 (h)

Fig. 12.24 Solution to problem 12.2 (i)

Fig. 12.25 Solution to problem 12.2 (j)

PROBLEM 12.3 A right regular hexagonal prism, side of base 25 mm and axis 50 mm long, having one of its base edges parallel to the VP with its axis perpendicular to the HP. Draw its front, top and side views.

SOLUTION. As the axis of a hexagonal prism is perpendicular to HP, so projections are to be started from top view.

- Draw a regular hexagon of 25 mm side in the top view, keeping one of its base sides parallel to xy .
- Project front and left side views from top view. Mark the axis to be 50 mm long. Name the corner points as shown in Fig. 12.26 and complete the views.

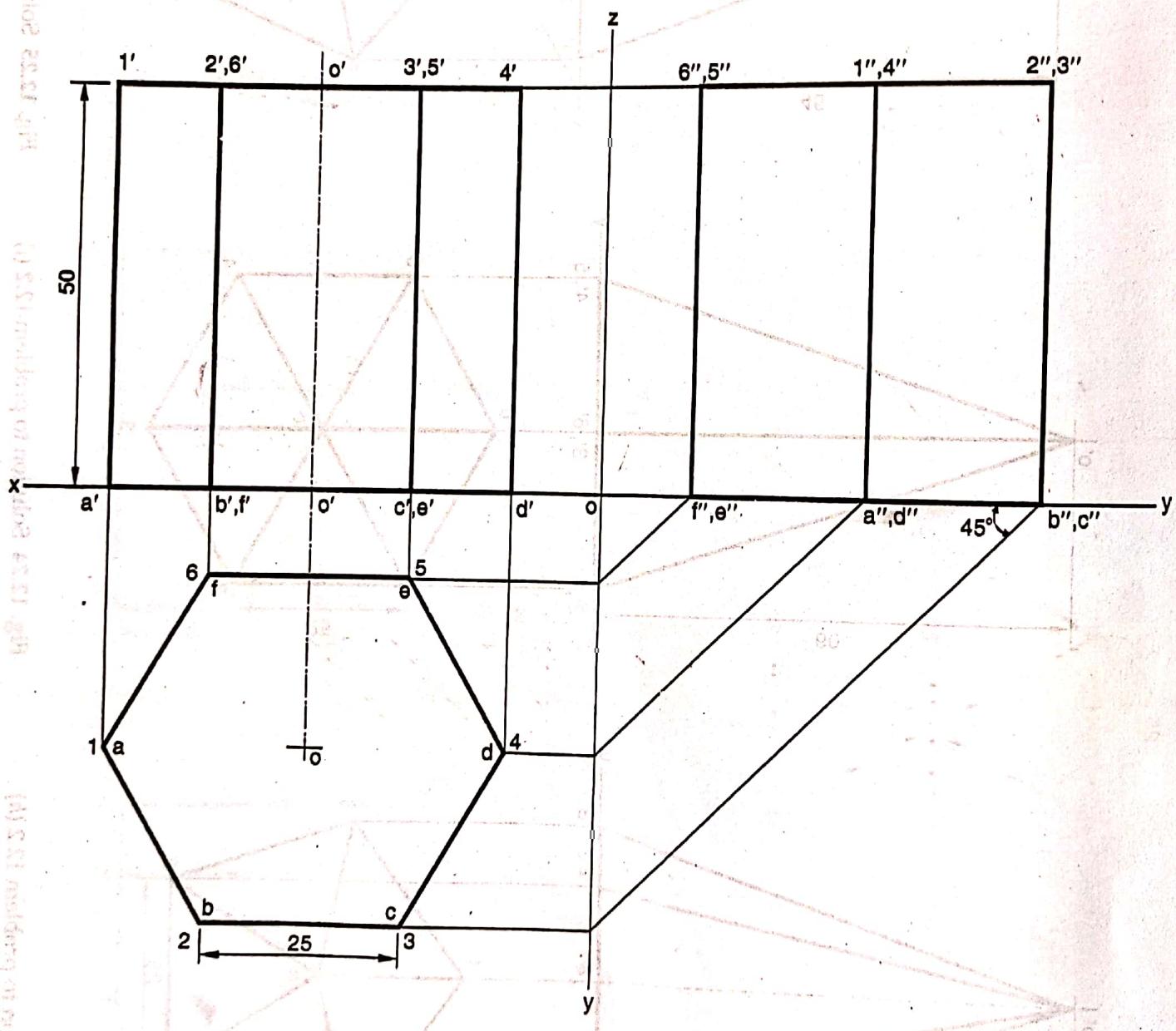


Fig. 12.26 Solution to problem 12.3

PROBLEM 12.4 A right regular hexagonal pyramid, edge of base 25 mm and height 60 mm long, having one of its base edges perpendicular to VP, with its axis perpendicular to the HP. Draw the three views of the pyramid.

SOLUTION. As the axis of a hexagonal pyramid is perpendicular to VP, so projections are to be started from front view.

- (i) Draw a hexagonal pyramid of 25 mm base edge in the top view, with a base edge 2-1 or 5-4 to be perpendicular to xy .
- (ii) Project front and left side views from top view. Name the corner points as shown in Fig. 12.27 and complete the views.

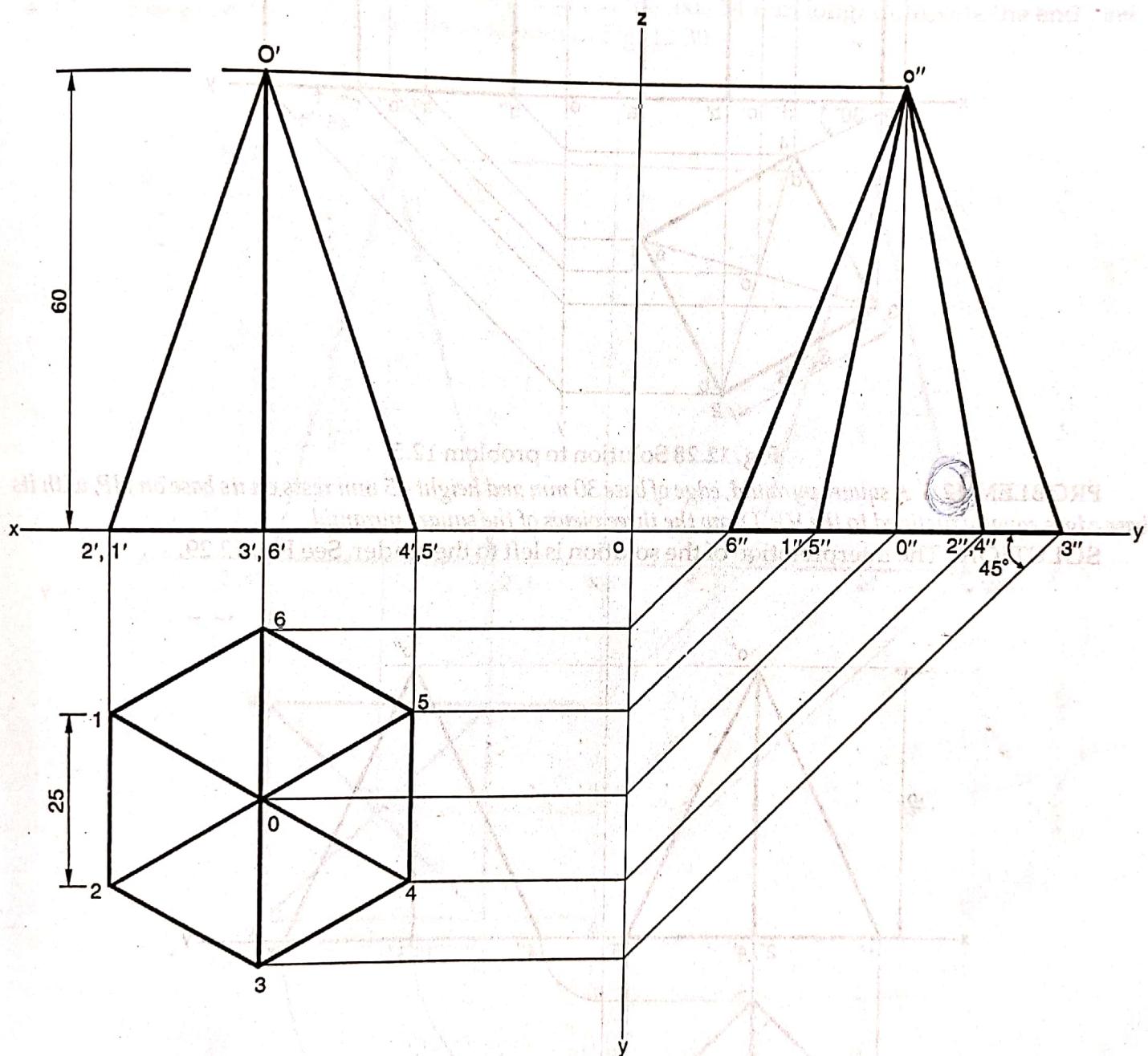


Fig. 12.27 Solution to problem 12.4

PROBLEM 12.5 A cube of 35 mm edge is resting on the HP on one of its faces, with a vertical face inclined at 30° to the VP. Draw the three views of the cube.
(PTU, Jalandhar May 2009)

SOLUTION. All the construction lines are retained to make the solution self-explanatory. See Fig. 12.28.

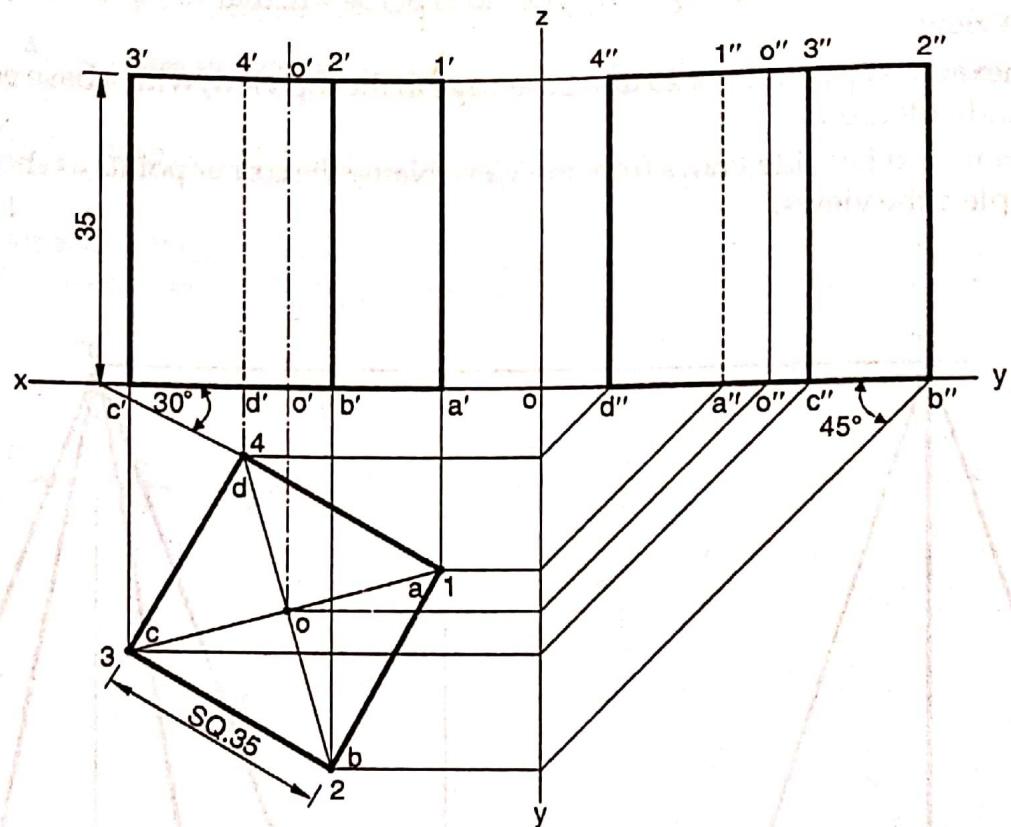


Fig. 12.28 Solution to problem 12.5

PROBLEM 12.5 A square pyramid, edge of base 30 mm and height 45 mm rests on its base on HP, with its base edges equally inclined to the VP. Draw the three views of the square pyramid.

SOLUTION. The interpretation of the solution is left to the reader. See Fig. 12.29.

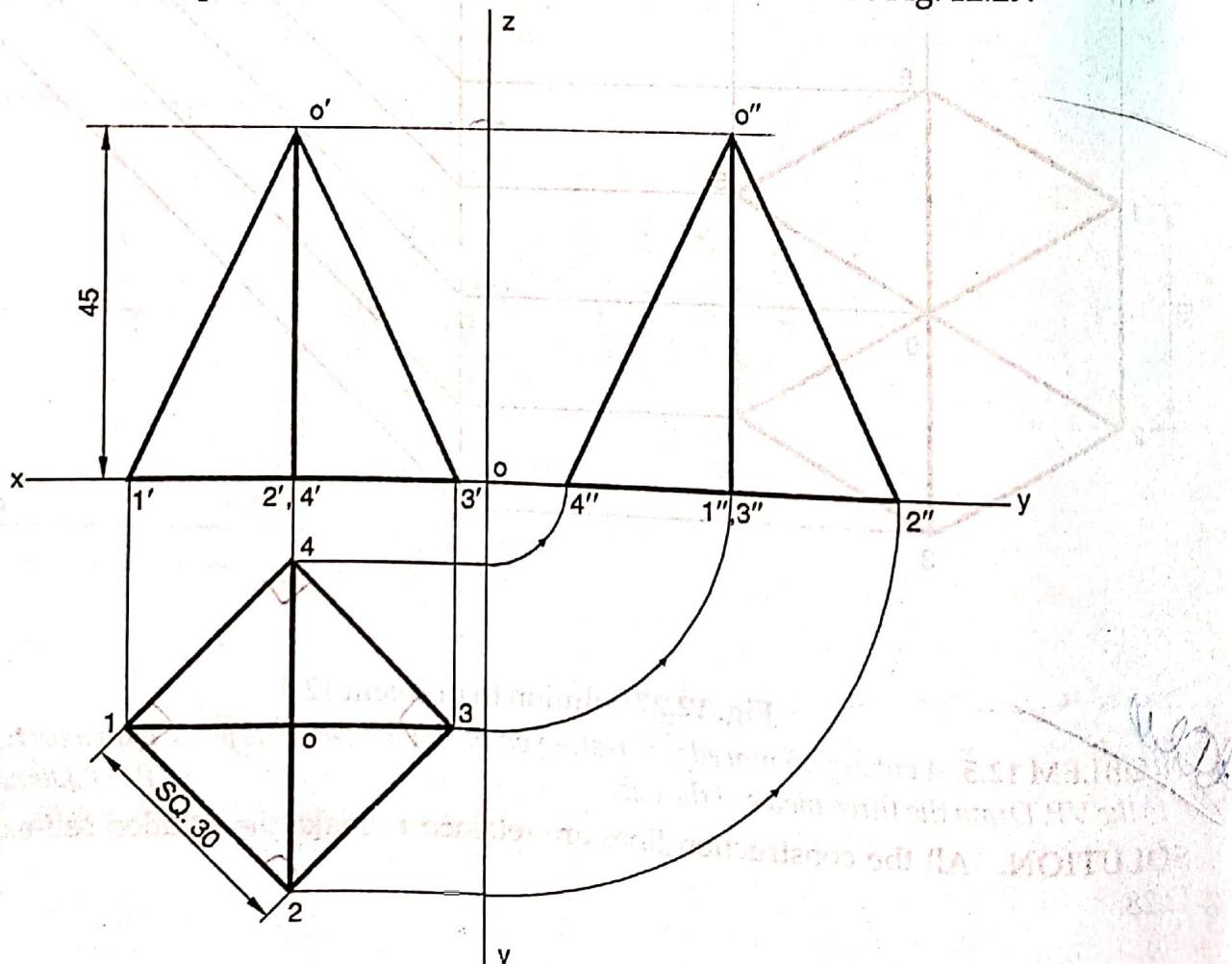


Fig. 12.29 Solution to problem 12.6

~~PROBLEM 12.7~~ Draw the projections of a square pyramid of base edges 30mm and axis 54 mm, resting on its base on HP with one of the base edge parallel to VP and axis perpendicular to the HP.

SOLUTION.

(PTU, Jalandhar December 2005).

- Draw a square of base edge 30 mm in the top view, keeping one of its base edges parallel to VP.
- Project front view from top view and cut the axis 54 mm long. Complete the end view too. Name all the corner points as shown in Fig. 12.30.

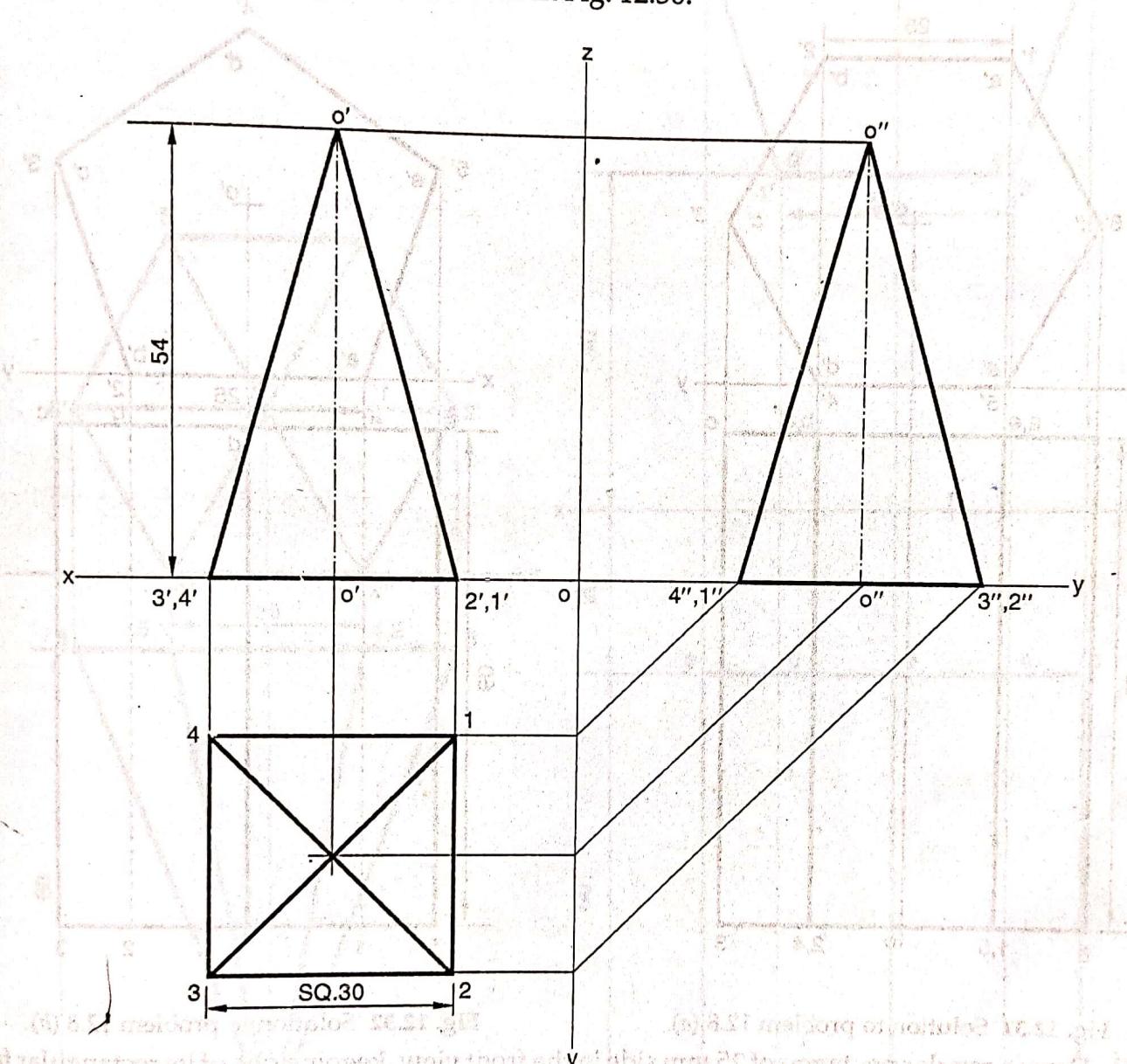


Fig. 12.30 Solution to problem 12.7

(b) Axis perpendicular to the VP and parallel to the HP : When the axis is perpendicular to the VP, the front view should be drawn first and then top view is projected from it.

PROBLEM 12.8 Draw the projections of the following solids :-

- A right regular hexagonal prism, side of base 25 mm and axis 60 mm long, lies on one of its rectangular faces on HP with its axis perpendicular to VP.
- A right regular pentagonal prism, side of base 25 mm and axis 60 mm long, lies on one of its rectangular faces on HP with its axis perpendicular to VP.

- (c) A right regular hexagonal pyramid, edge of base 25 mm and height 60 mm long, has its base parallel to VP with one of its base edges in HP.

SOLUTION.

- (a) Draw a regular hexagon of 25 mm side in the front view, keeping one of its rectangular faces on xy . Project the top view from front view and cut the axis 60 mm long. Complete the views as shown in Fig. 12.31. Name all the corner points.

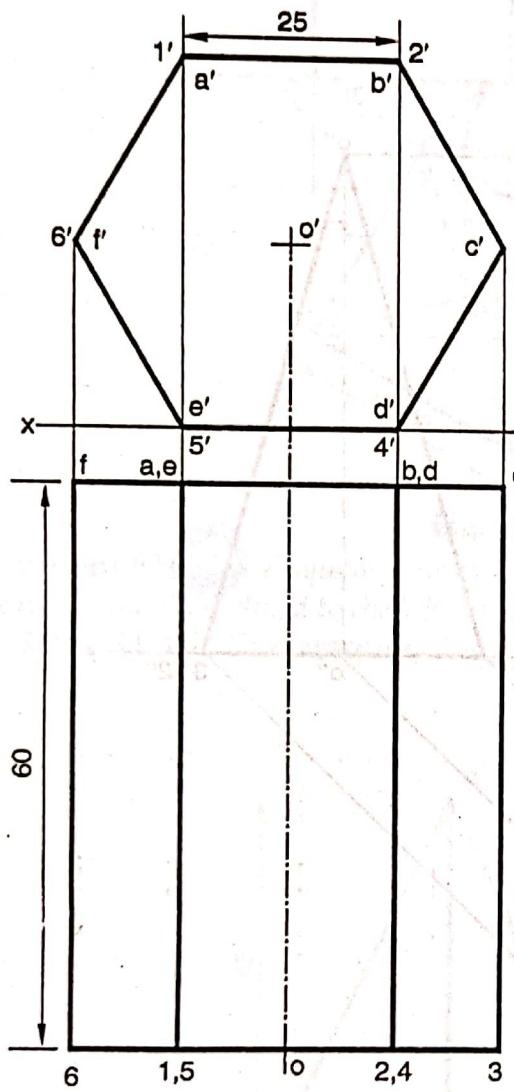


Fig. 12.31 Solution to problem 12.8 (a)

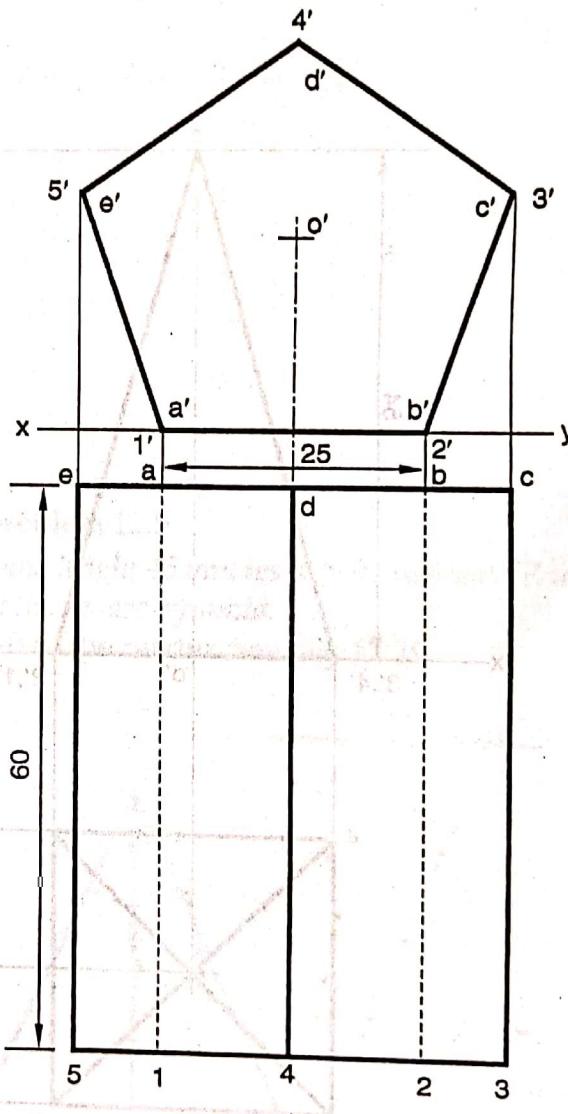


Fig. 12.32 Solution to problem 12.8 (b)

- (b) Draw a regular pentagon of 25 mm side in the front view, keeping one of its rectangular faces on xy . Project the top view from front view and cut the axis 60 mm long. Complete the views as shown in Fig. 12.32. Name all the corner points.
- (c) Draw a regular hexagon of 25 mm side in the front view, keeping one of its base edges parallel to xy . Through the centre O', project the vertex O, 60 mm long. Complete the views as shown in Fig. 12.33.

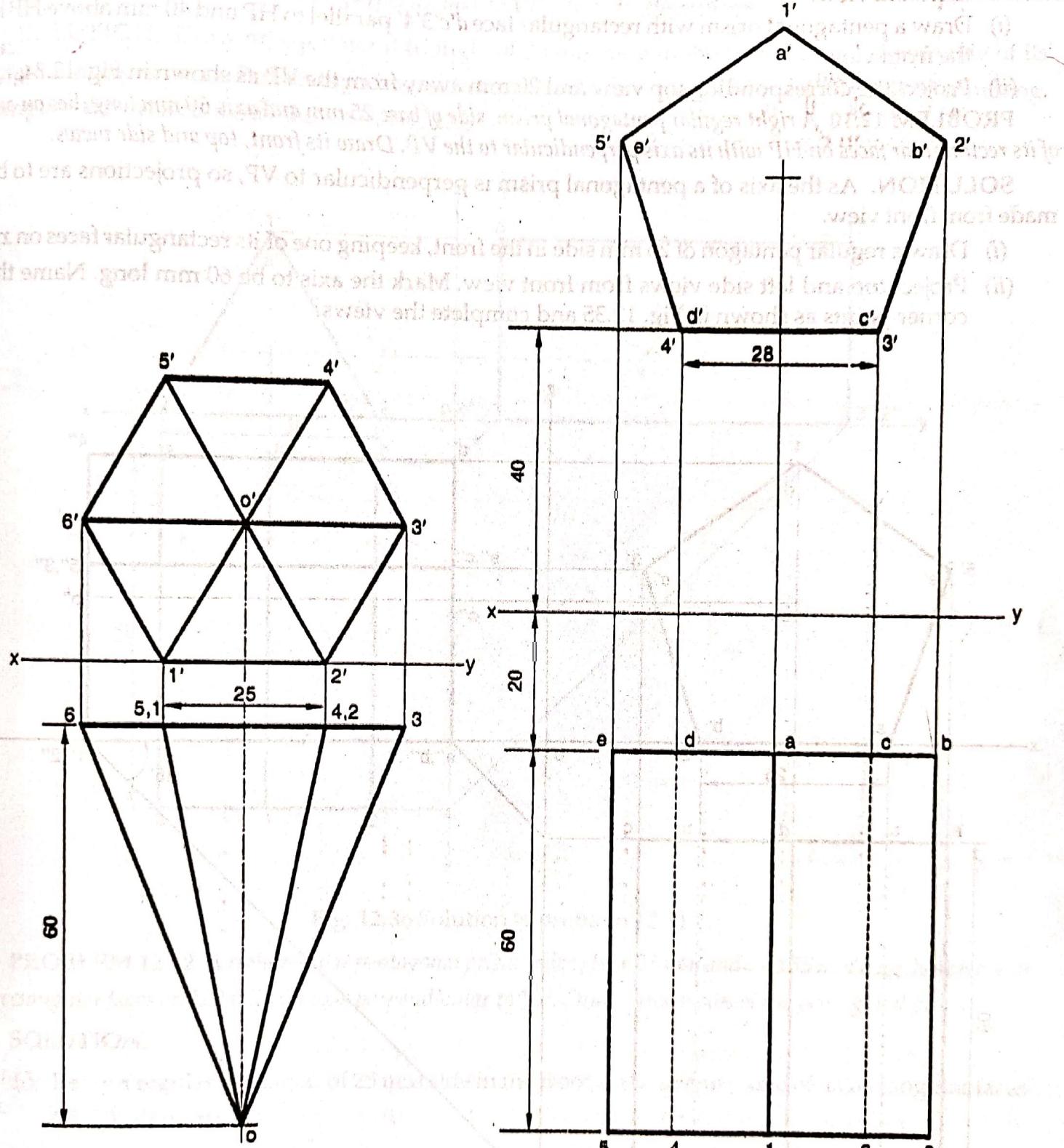


Fig. 12.33 Solution to problem 12.8 (c)

Fig. 12.34 Solution to problem 12.9

PROBLEM 12.9 A right regular pentagonal prism side of base 28 mm and axis 60 mm long has one of its rectangular faces parallel to HP with its axis perpendicular to VP and 40 mm above HP. Draw the elevation and plan of the pentagonal prism when the nearest end is 20 mm away from the VP.

(PTU, Jalandhar December 2004)

SOLUTION. As the axis of a pentagonal prism is perpendicular to VP, so projections are to be started from front view.

- Draw a pentagonal prism with rectangular face $d'c'3'4'$ parallel to HP and 40 mm above HP in the front view.

- Project the corresponding top view and 20 mm away from the VP as shown in Fig. 12.34.

PROBLEM 12.10 A right regular pentagonal prism, side of base 25 mm and axis 60 mm long, lies on one of its rectangular faces on HP with its axis perpendicular to the VP. Draw its front, top and side views.

SOLUTION. As the axis of a pentagonal prism is perpendicular to VP, so projections are to be made from front view.

- Draw a regular pentagon of 25 mm side in the front, keeping one of its rectangular faces on xy .
- Project top and left side views from front view. Mark the axis to be 60 mm long. Name the corner points as shown in Fig. 12.35 and complete the views.

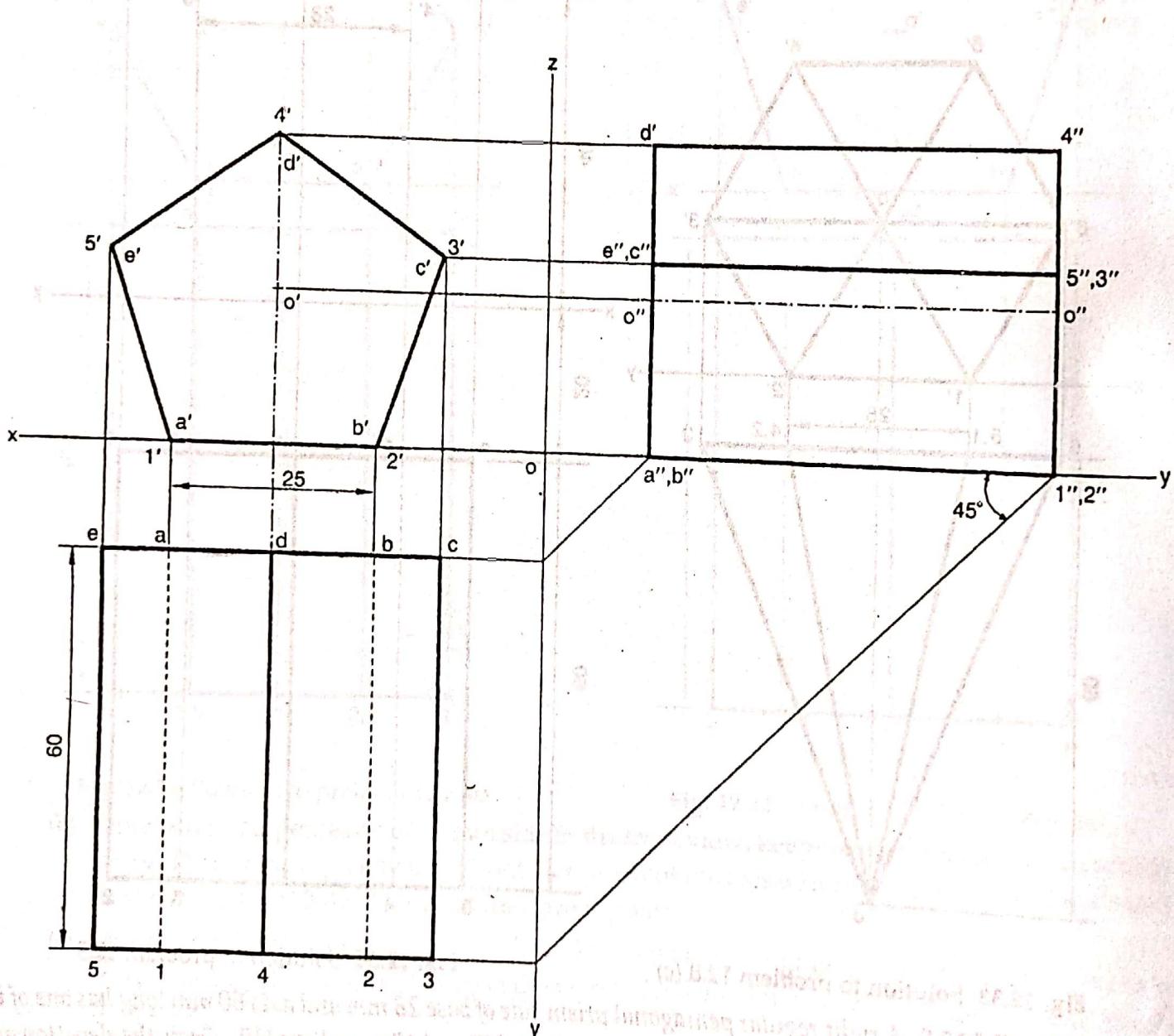


Fig. 12.35 Solution to problem 12.10

PROBLEM 12.11 A triangular prism side of base 30 mm and axis 55 mm long, lies on one of its rectangular faces in HP, with its axis perpendicular to VP. Draw its three views.

SOLUTION. Draw an equilateral triangle of 30 mm side in the front view, keeping one of its rectangular faces on xy . Project top view, side view from front view and mark the axis 55 mm long. Complete the projections as shown in Fig. 12.36.

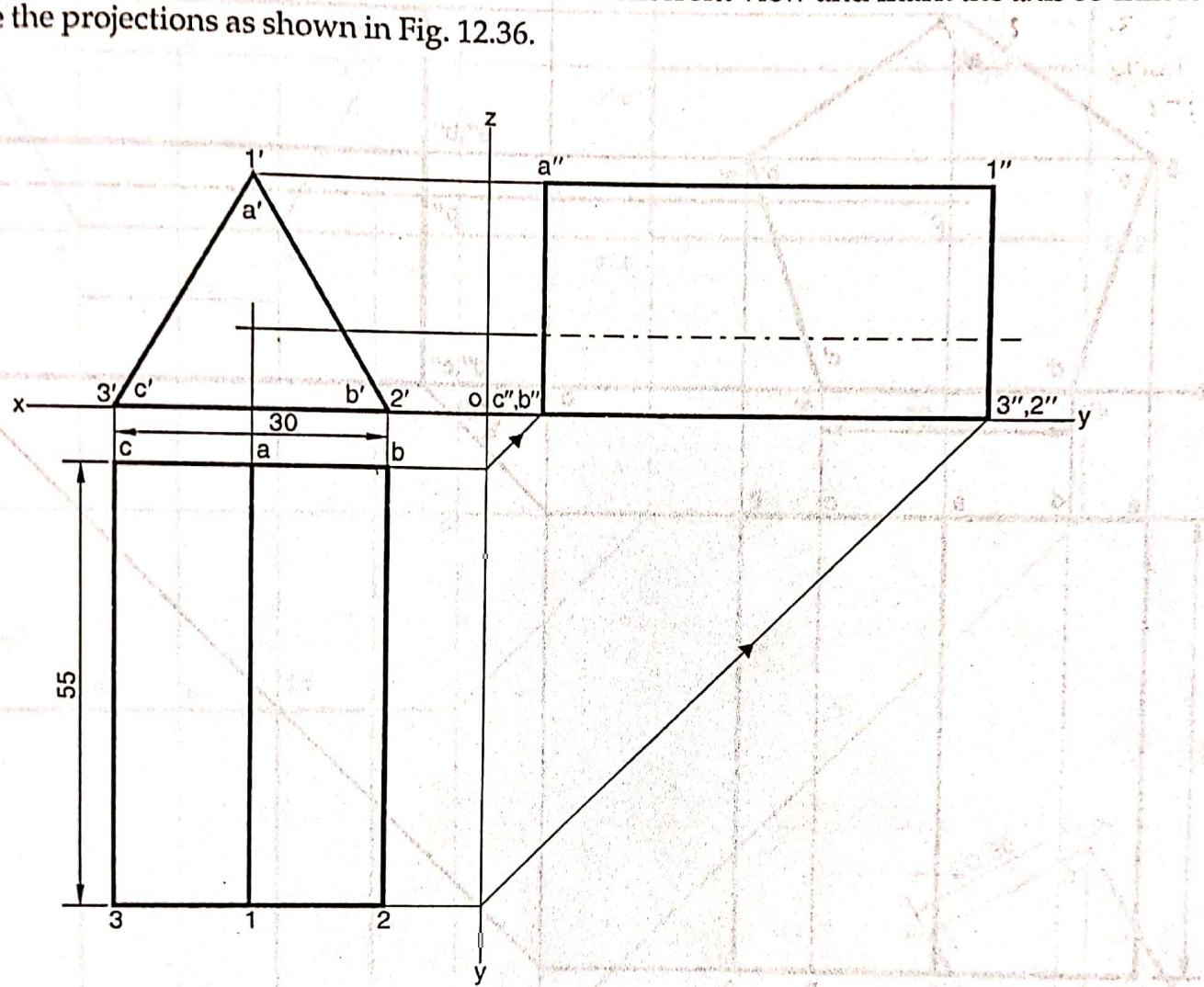


Fig. 12.36 Solution to problem 12.11

PROBLEM 12.12 A right regular pentagonal prism, side of base 25 mm and axis 55 mm long, lies on one of its rectangular faces on HP with its axis perpendicular to VP. Draw three views of the pentagonal prism.

SOLUTION.

- Draw a regular pentagon of 25 mm side in the front view, keeping one of its rectangular faces ($d' c' 3' 4'$) on xy .
- Project top view from front view and cut the axis 55 mm long. Complete the end view too, as shown in Fig. 12.37. Name all the corner points.

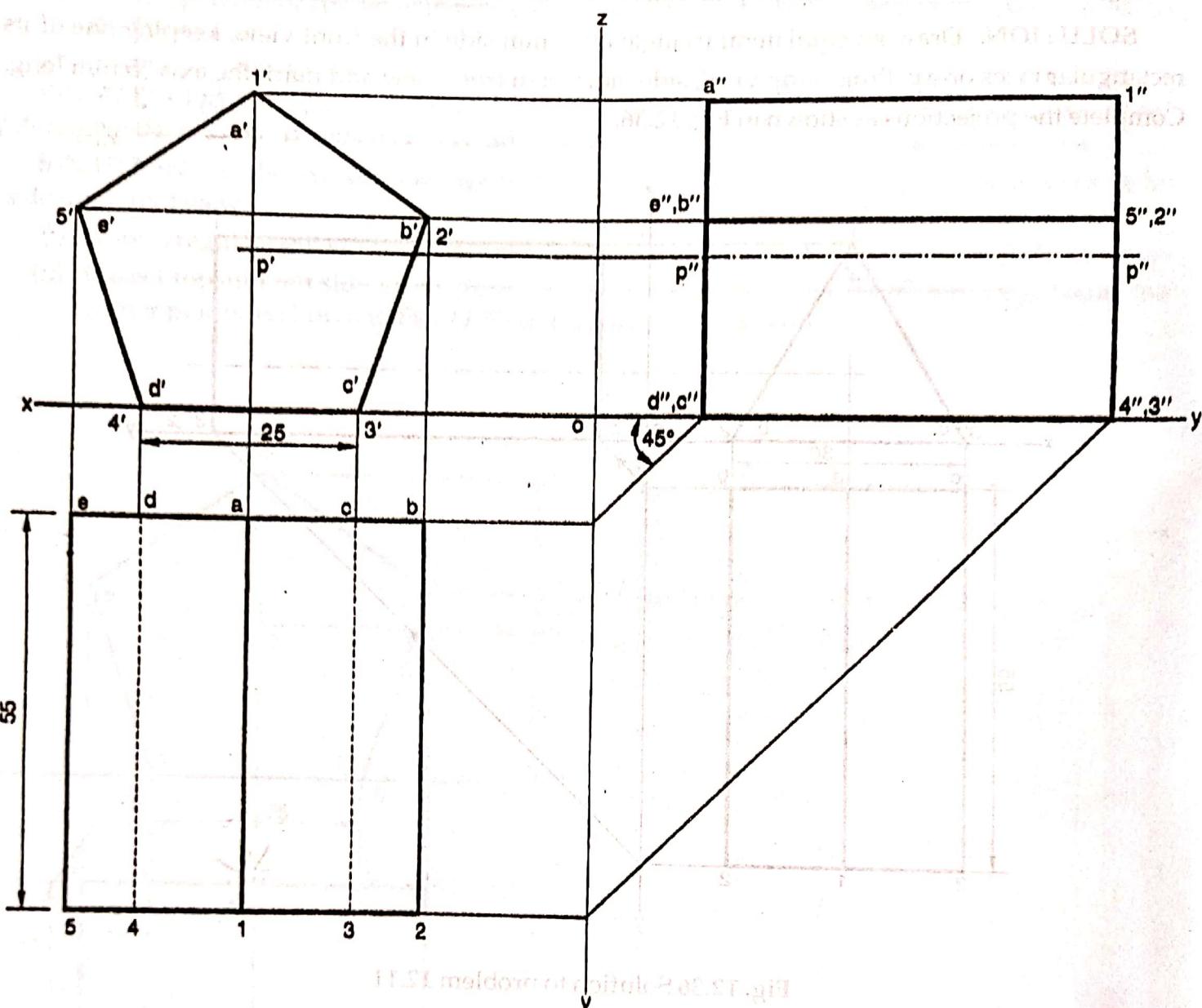


Fig. 12.37 Solution to problem 12.12

PROBLEM 12.13 A hexagonal prism, side of base 25 mm and axis 55 mm long, has one of rectangular faces parallel to HP with its axis perpendicular to VP and 35 mm above HP. Draw the elevation, plan and side view of the hexagonal prism when the nearer end is 25 mm away from the VP.

(PTU, Jalandhar December 2005)

SOLUTION. The solution to this problem has already been explained in previous problems. See Fig. 12.38.

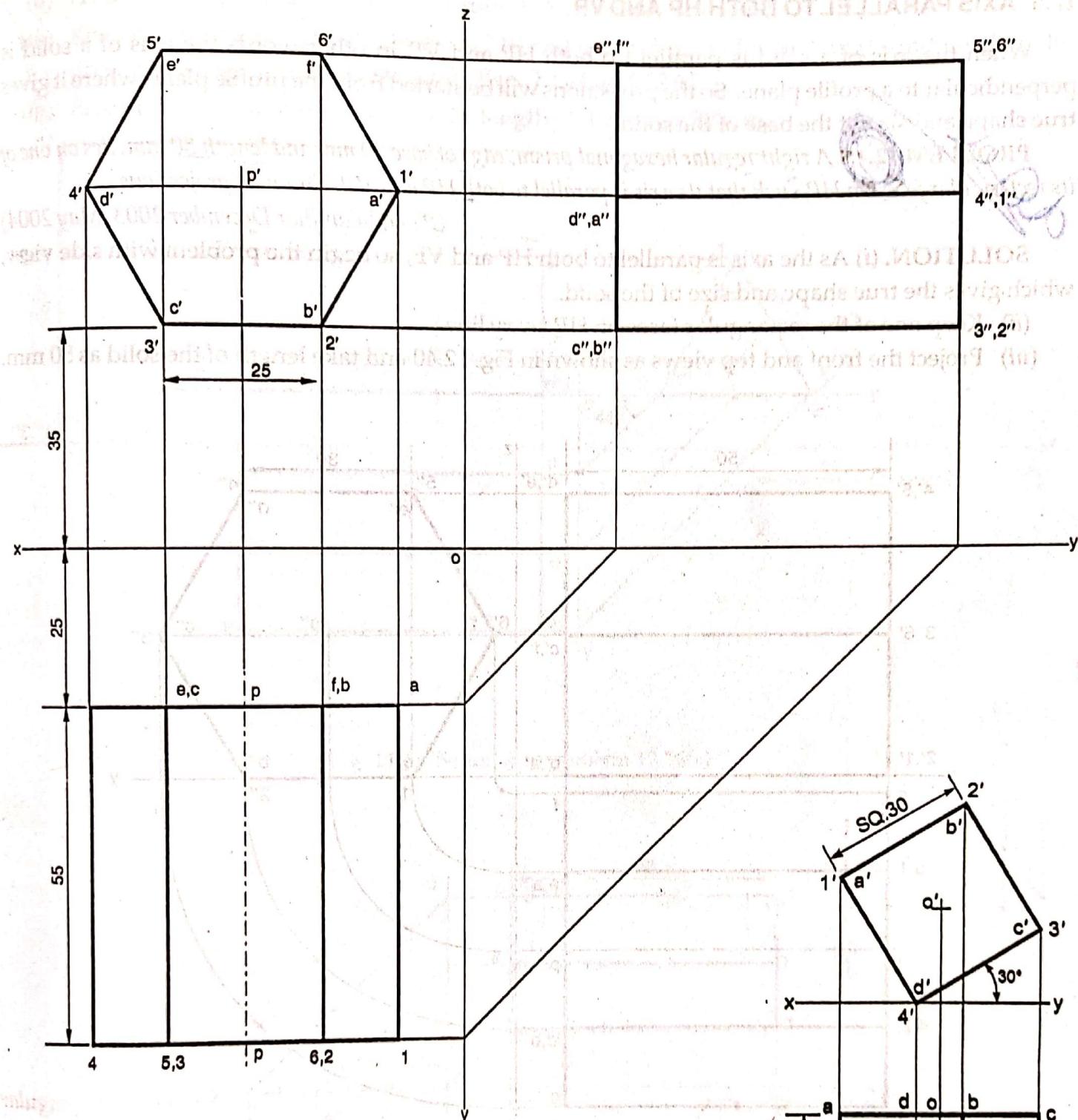


Fig. 12.38 Solution to problem 12.13

PROBLEM 12.14 A square prism, side of base 30 mm and axis 55 mm long is resting on one of its longer edge with a face containing the longer edge is inclined at 30° to the HP with its axis perpendicular to the VP. Draw its projections.

SOLUTION. All the construction lines are retained to make the solution self-explanatory. See Fig. 12.39.

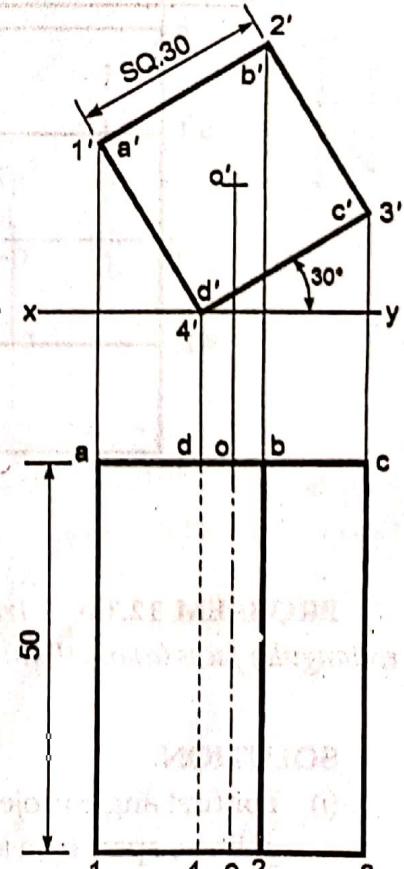


Fig. 12.39 Solution to problem 12.14

12.5 AXIS PARALLEL TO BOTH HP AND VP

When the axis of a solid is parallel to both HP and VP, in other words the axis of a solid is perpendicular to a profile plane. So the projections will be started from the profile plane, where it gives true shape and size of the base of the solid.

PROBLEM 12.15 A right regular hexagonal prism, edge of base 30 mm and length 50 mm, lies on one of its rectangular faces on HP, such that its axis is parallel to both HP and VP. Draw its projections.

(PTU, Jalandhar December 2003, May 2004)

SOLUTION. (i) As the axis is parallel to both HP and VP, so begin the problem with side view, which gives the true shape and size of the solid.

(ii) Keep one of the rectangular faces on HP i.e. xy line.

(iii) Project the front and top views as shown in Fig. 12.40 and take length of the solid as 50 mm.

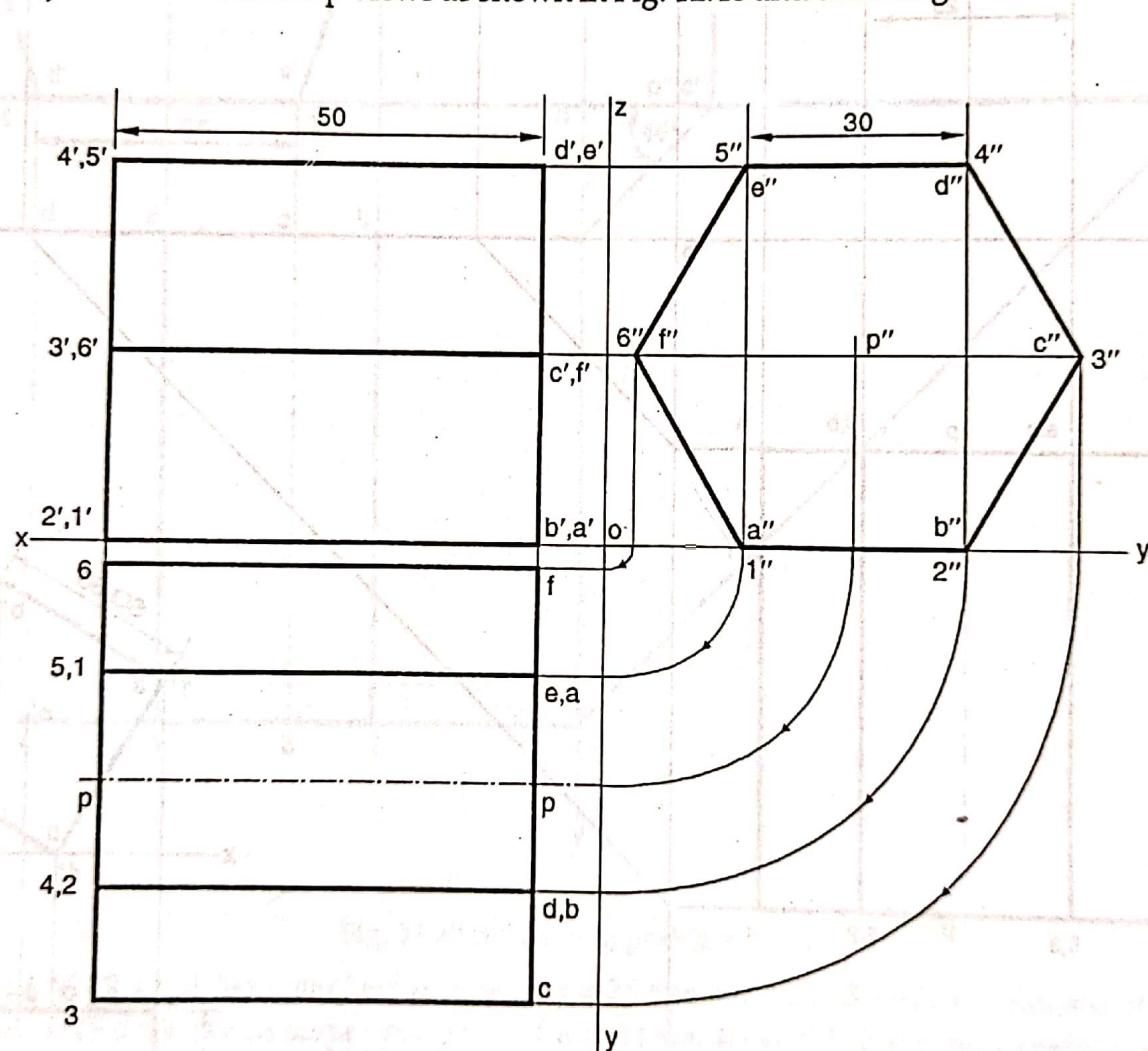


Fig. 12.40 Solution to problem 12.15

PROBLEM 12.16 A triangular prism side of base 25 mm and axis 50 mm long, lies on one of its rectangular faces (a) in HP (b) on ground plane, with its axis parallel to VP. Draw its projections

(PTU, Jalandhar June 2003)

SOLUTION.

(i) For first angle projection draw xy line only. Whereas for third angle projection draw xy and gl lines, apart by a suitable distance.

- (ii) As the axis is parallel to both the planes, so begin with side view.
- (iii) Keep one of the rectangular faces on HP and ground plane for first angle and third angle projections respectively as shown in Figs. 12.41 and 12.42.
- (iv) Project the front and top views. Take length of the axis is 50 mm.

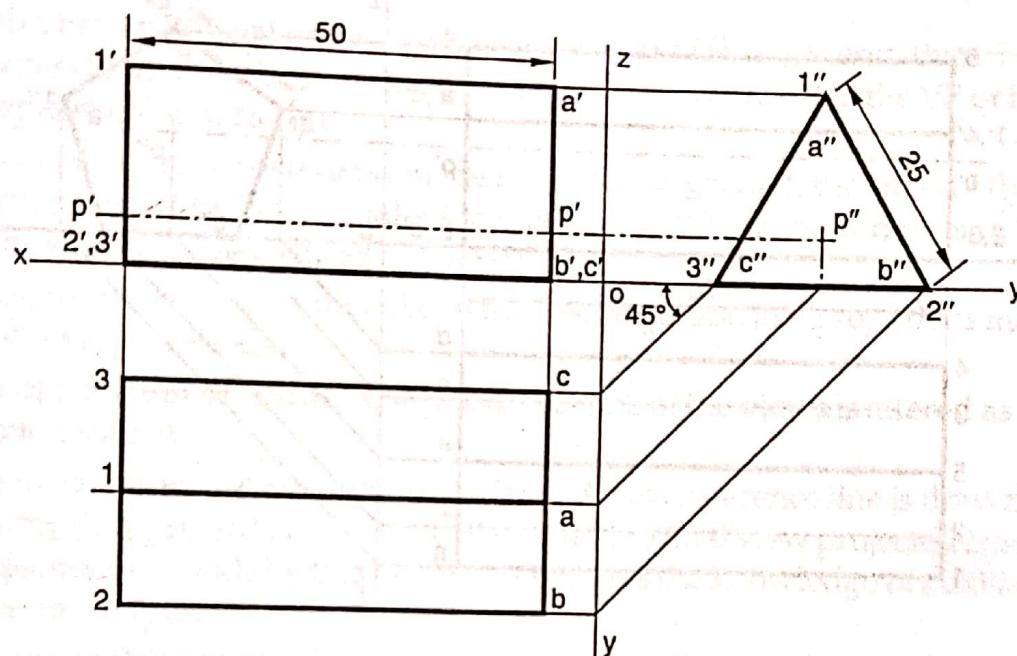


Fig. 12.41 Solution to problem 12.16(a)

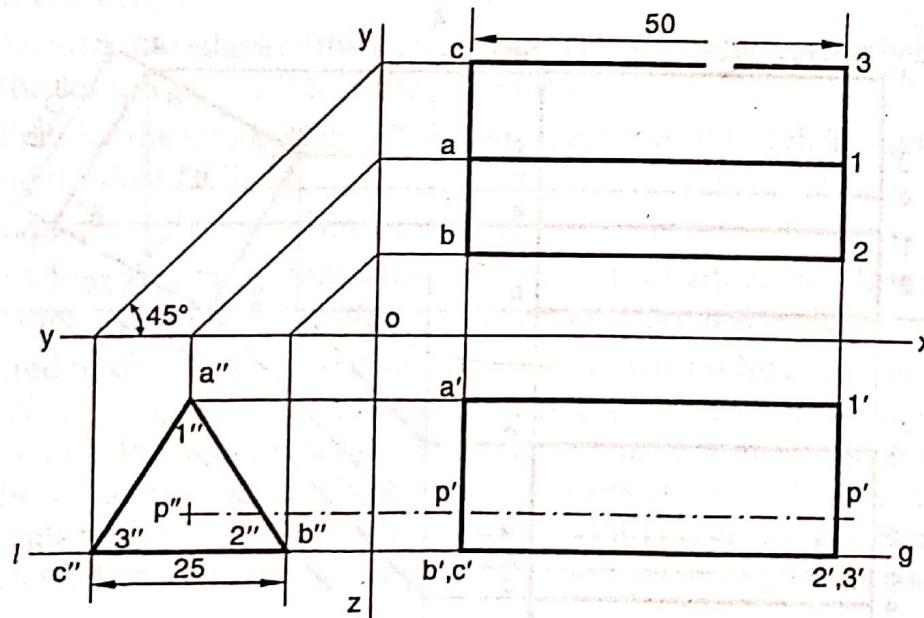


Fig. 12.42 Solution to problem 12.16(b)

PROBLEM 12.17 A pentagonal prism base 20 mm side and axis 75 mm long lies on one of its rectangular faces on HP with axis parallel to VP. Draw the three views of the prism.

(PTU, Jalandhar December 2002)

SOLUTION. The solution to this problem is self-explanatory. See Fig. 12.43.

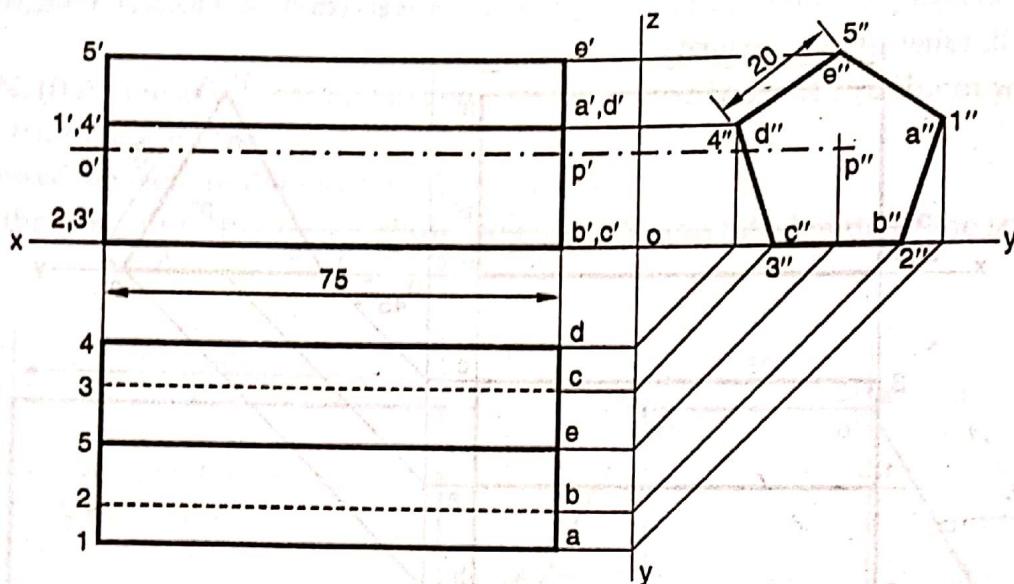


Fig. 12.43 Solution to problem 12.17

PROBLEM 12.18. A square prism, side of base 30 mm and axis 55 mm long is resting on HP on one of its longer edge with a face containing the longer edge is inclined at 30° to the HP. Draw its projections, when the axis is parallel to both HP and VP.

SOLUTION. All the construction lines are retained to make the solution self-explanatory. See Fig. 12.44.

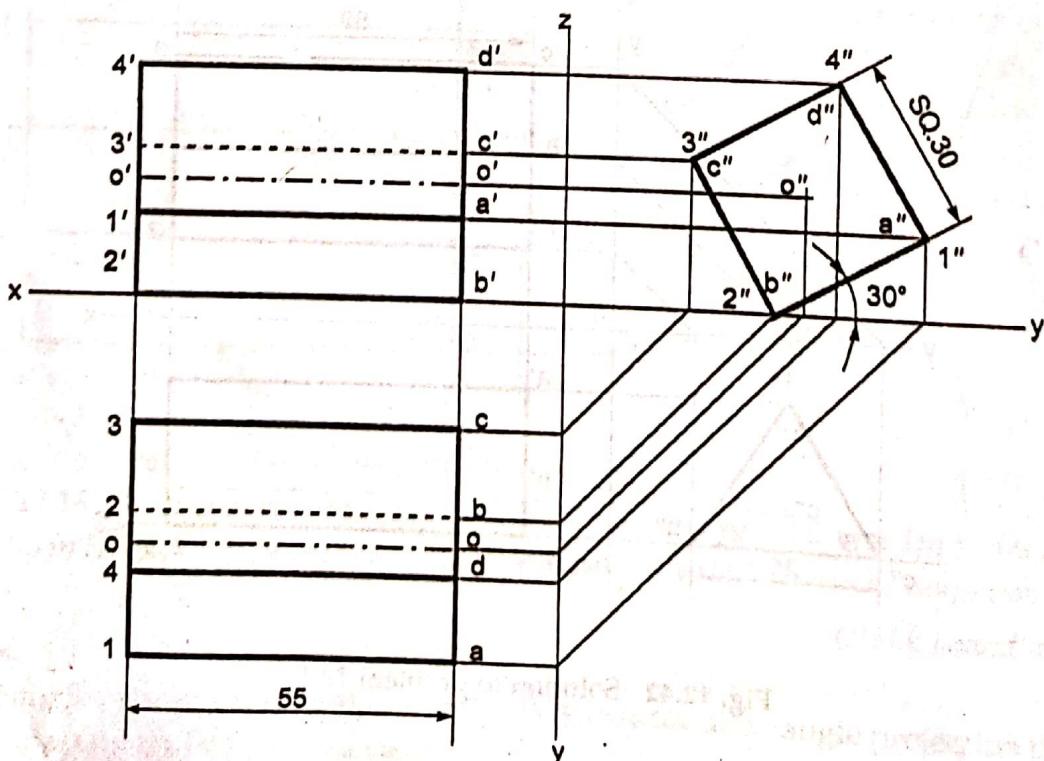


Fig. 12.44 Solution to problem 12.18

12.6 AXIS INCLINED TO ONE OF THE PRINCIPAL PLANES AND PARALLEL TO THE OTHER

When a solid has its axis inclined to one of the principal planes and parallel to the other, its projections are drawn into two stages. In the initial stage, its axis is kept perpendicular to that plane to which it is inclined. If the axis is to be inclined to the ground or HP, it is assumed to be perpendicular to the HP in the initial stage. Similarly, if the axis is to be inclined to the VP, it is assumed to be perpendicular to the VP in the initial stage. Moreover, the following two points need to be kept in mind :

- (i) If the solid has an edge of its base parallel to HP or in HP or ground, then that edge should be kept perpendicular to the VP. If the edge of the base is parallel to the VP or in VP, it should be kept perpendicular to the HP.
- (ii) If the solid has a corner of its base in the HP or on the ground, the sides of the base containing that corner should be kept equally inclined to the VP. If the corner is in the VP, the sides should be kept equally inclined to the HP.

After drawing the projections of the solid in the first stage, the final projections may be obtained by any one of the following methods :

(a) **Change of position of solids.** The position of one of the views is altered as required and the other view projected from it.

(b) **Change of reference line or auxiliary plane.** A new reference line is drawn according to the required conditions, to represent an auxiliary plane and the final view projected from it. To solve the problems on projections of solids by this method, it requires the knowledge of auxiliary projections as already described in Chapter 11.

The comparison of these methods reveal that the first method is laborious, as it take considerable time to reproduce the view, especially when the solid has curved surfaces or too many edges or corners. In such cases, it is easier and more convenient to adopt the second method. Sufficient care should be taken in transferring the distances of various points from their respective reference line.

After locating the positions of all the points representing the various corners in the final view, these are joined correctly to have the finished projection. The following sequence may be adopted for joining these points correctly :

- (i) Draw the lines for the edges of the visible base. The base which is further away from xy in one view, will be completely visible in the other view.
- (ii) Draw the lines for the longer edges. The lines which pass through the figure of the visible base should be short dashed lines.
- (iii) Draw the lines for the edges of the other base.

It should always be remembered that when two lines representing the edges cross each other, one of them must be hidden and should therefore be drawn as short dashed line.

(a) **Axis inclined to the HP and parallel to the VP.** When a solid has its axis inclined to the HP and parallel to the VP, its projections are drawn into two stages. In the initial stage, it is assumed to be perpendicular to the HP. In such problems, the top view will be drawn first, as it will show the true shape and size of the solid and then the front view is projected from it. The front view is reproduced making the given angle with HP. Project all the points vertically from this front view and horizontally from the first top view. Join all points in the final top view, observing the rules for establishing the visibility of lines.

PROBLEM 12.19. A right circular cone, diameter of base 50 mm, height 65 mm is resting on HP on its base with such that its axis is parallel to VP and inclined to HP at an angle of 60° . Draw its front, top and profile view.

SOLUTION. All the construction lines are retained to make the solution self-explanatory. See Fig. 12.45.

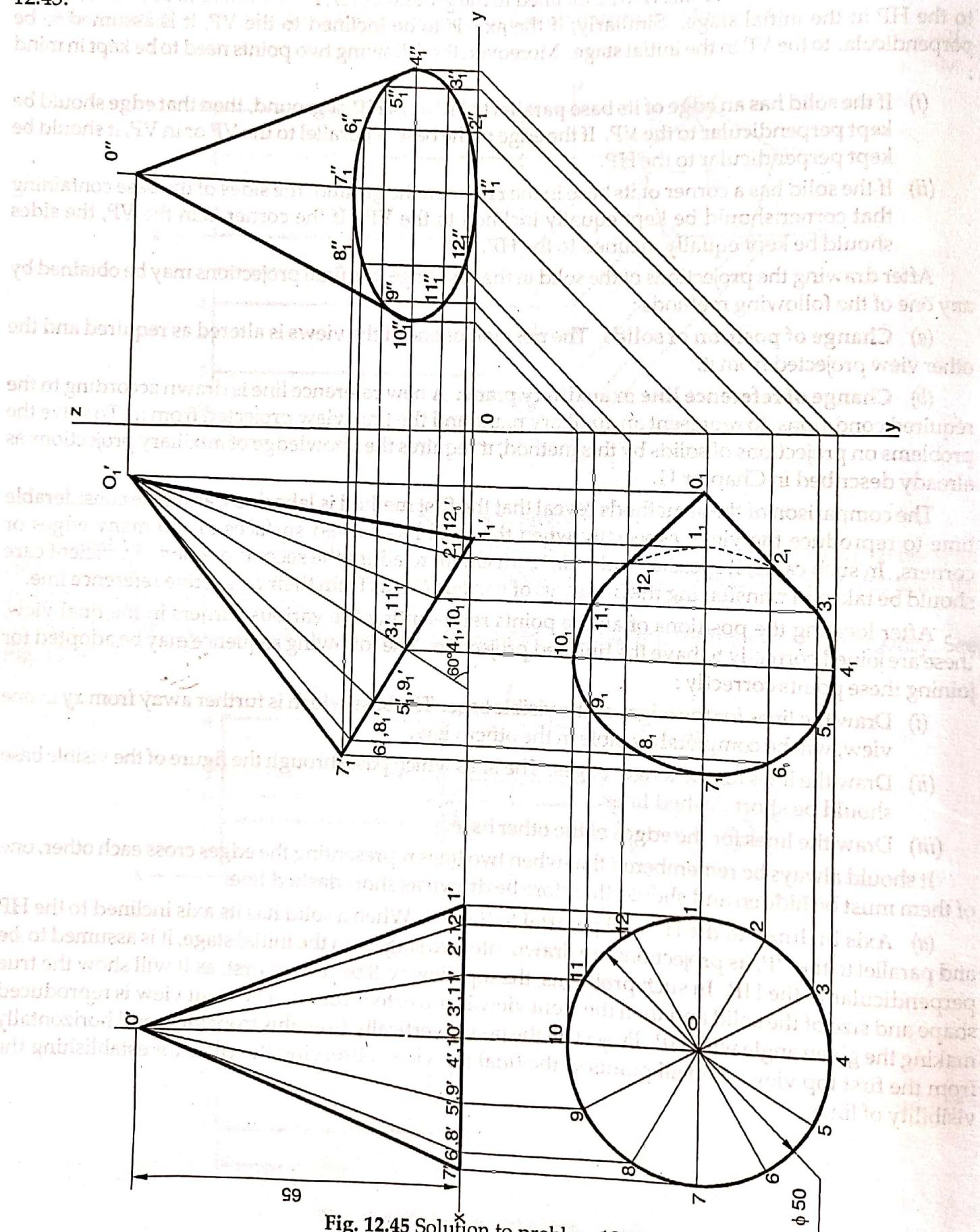


Fig. 12.45 Solution to problem 12.19

PROBLEM 12.20 A right regular hexagonal pyramid, edge of base 30 mm and axis 60 mm long, has an edge of its base in HP, such that its axis is inclined at 30° to the HP and parallel to the VP. Draw its projections by using both the methods.

SOLUTION. In the initial position or stage, assume the axis to be perpendicular to the HP. Draw the projections with the base in xy and its one edge perpendicular to the VP and label it.

Method I (Change of position of solid)

- Reproduce the front view so that the axis makes an angle of 30° with xy and the edge $2'1'$ remain in xy and name the points on it by adding suffix 1 to them.

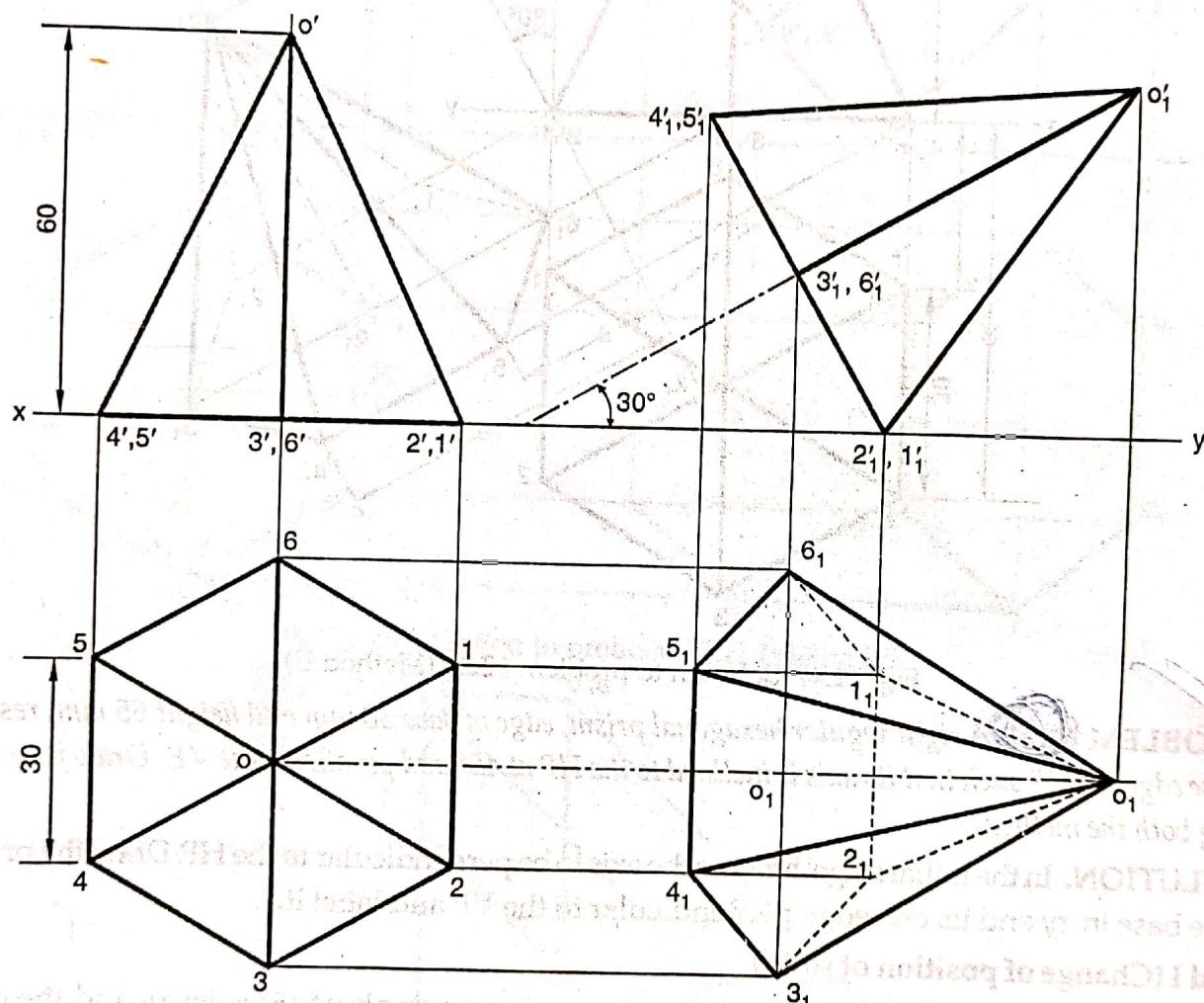


Fig. 12.46 Solution to problem 12.20 (Method I)

- Project all the points vertically from this front view and horizontally from the first top view. Complete the new top view as shown in Fig. 12.46. Join all the points in the final top view, observing the rules for establishing the visibility of lines.

Method II (Change of reference line)

- Draw a new reference line x_1y_1 inclined at 30° to the axis, to represent an auxiliary inclined plane (AIP) through edge $2'1'$.

- (ii) From the front view, project the required top view on x_1y_1 , keeping the distance of each point from x_1y_1 equal to the distance of its first top view from xy as shown in Fig. 12.47. Join all the points in the final top view, observing the rules establishing the visibility of lines.

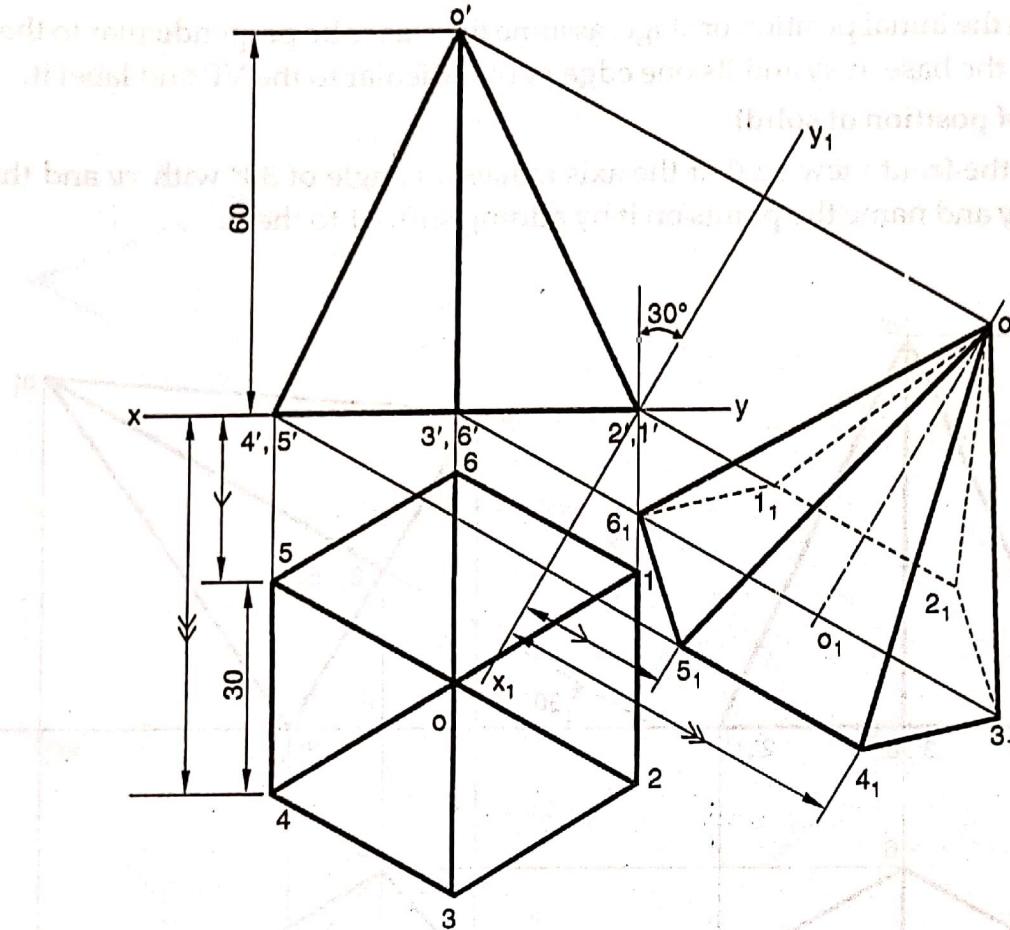


Fig. 12.47 Solution to problem 12.20 (Method II)

~~PROBLEM 12.21~~ A right regular hexagonal prism, edge of base 30 mm and height 65 mm, rests on one of its base edges in HP such that its axis is inclined to the HP at 45° and parallel to the VP. Draw its projections by using both the methods.

SOLUTION. In the initial stage, assume the axis to be perpendicular to the HP. Draw the projections with the base in xy and its one edge perpendicular to the VP and label it.

Method I (Change of position of solid)

- Reproduce the front view so that the axis makes an angle of 45° with xy and the edge $2'1'$ remains in xy and name the points on it by adding suffix 1 to them.
- Project from it, the corresponding top view and join all the points by observing the rules establishing the visibility of lines as shown in Fig. 12.48.

Method II (Change of reference line)

- Draw a new reference line x_1y_1 inclined at 30° to the axis, to represent an auxiliary inclined plane (AIP) through edge $2'1'$.
- From the front view, project the required top view on x_1y_1 as shown in Fig. 12.49.

PROJECTIONS OF SOLIDS

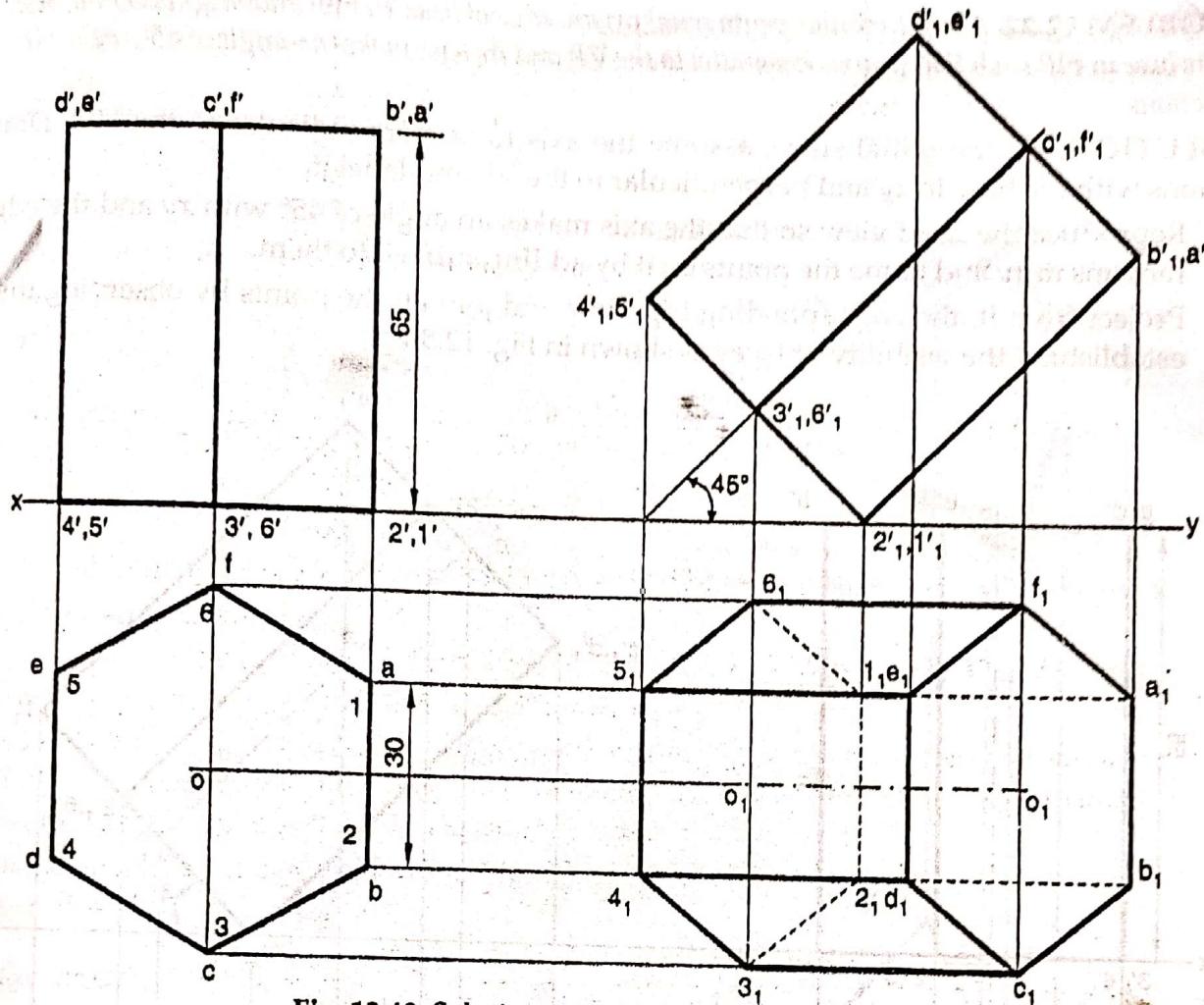


Fig. 12.48 Solution to problem 12.21 (Method I)

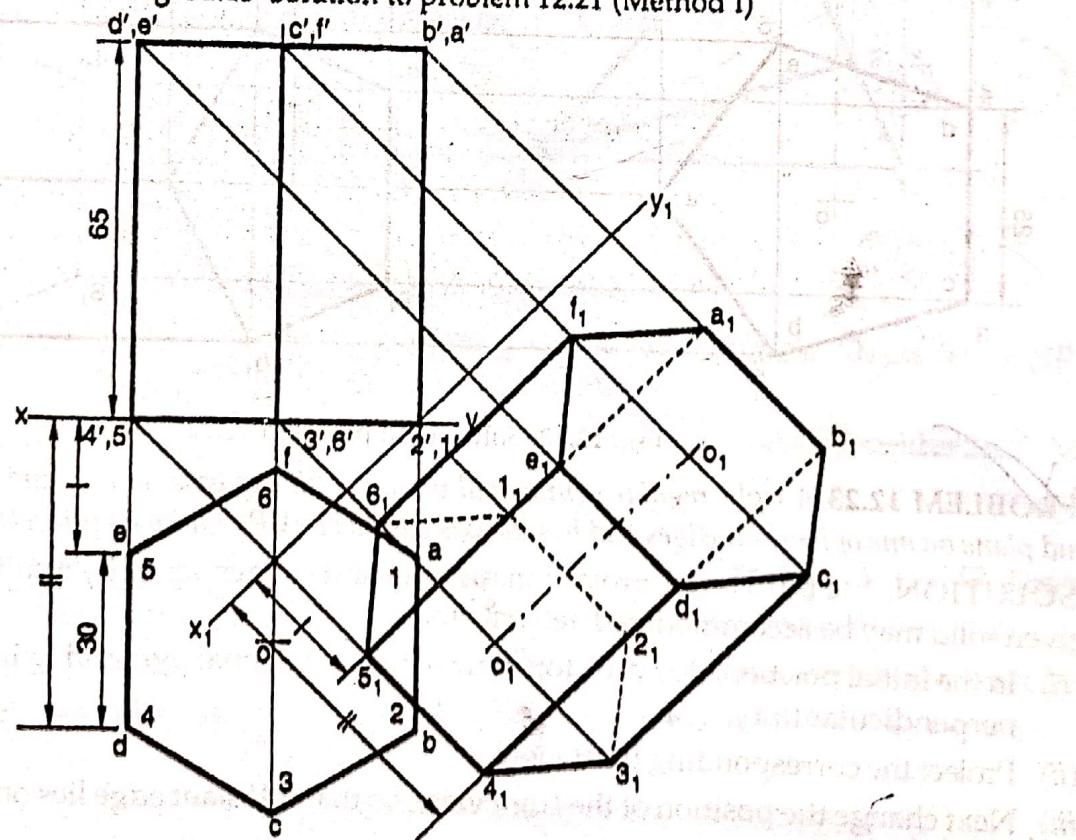


Fig. 12.49 Solution to problem 12.21 (Method II)

PROBLEM 12.22 A right regular pentagonal prism, edge of base 25 mm and height 55 mm, rests on an edge of its base in HP such that its axis is parallel to the VP and its base makes an angle of 45° to the HP. Draw its projections.

SOLUTION. In the initial stage, assume the axis to be perpendicular to the HP. Draw the projections with the base in xy and perpendicular to the VP and label it.

- Reproduce the front view so that the axis makes an angle of 45° with xy and the edge 3'4' remains in xy and name the points on it by adding suffix 1 to them.
- Project from it, the corresponding top view and join all the points by observing the rules establishing the visibility of faces as shown in Fig. 12.50.

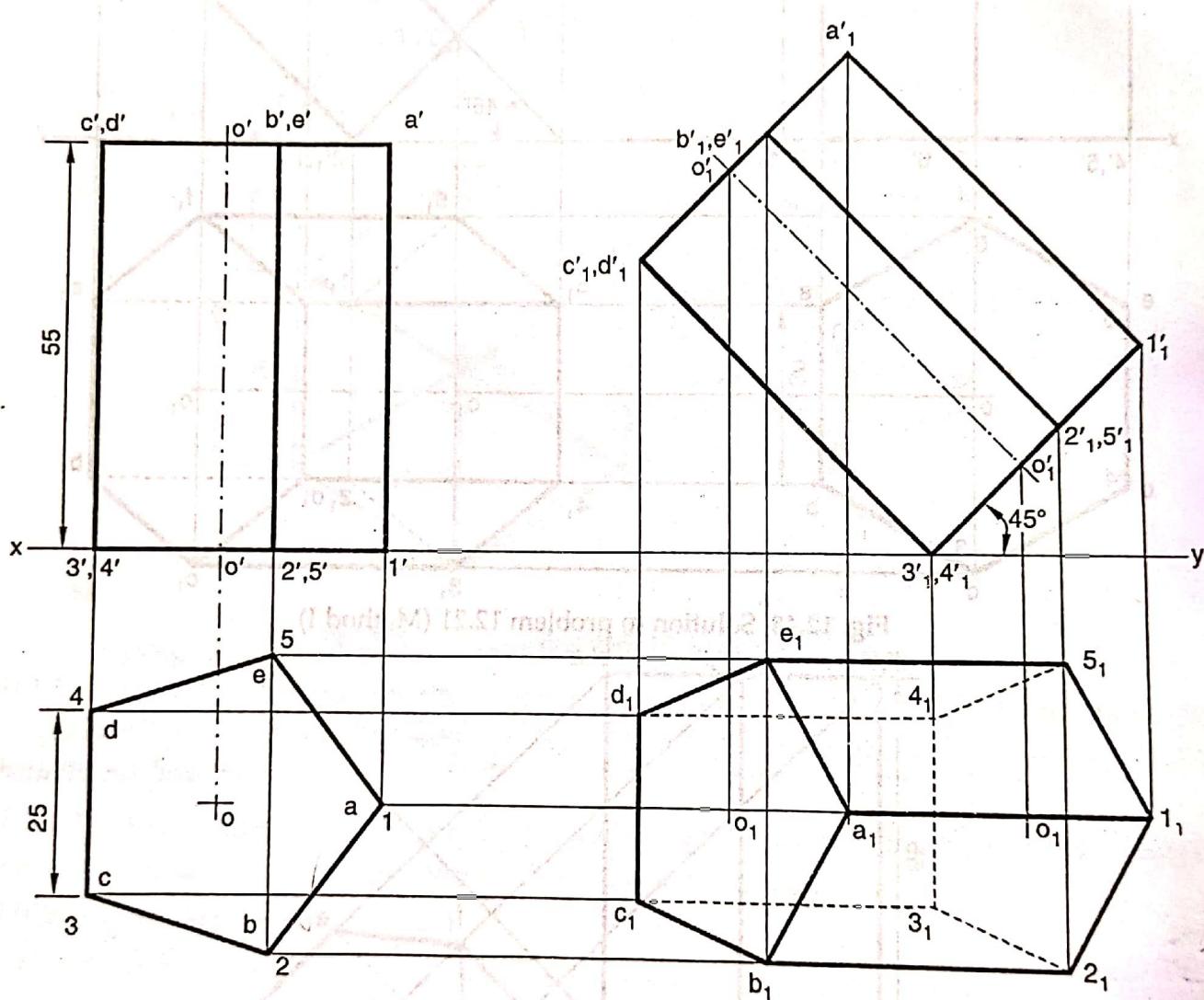


Fig. 12.50 Solution to problem 12.22

PROBLEM 12.23 A right regular pentagonal pyramid, side of base 30 mm and height 50 mm, lies on ground plane on one of its slant edges and has its axis parallel to VP. Draw its projections in third angle.

SOLUTION. For third angle projection draw xy and gl lines, apart by a suitable distance so that the given solid may be accommodated between them.

- In the initial position, draw the top view of the pentagonal pyramid, with one of its base edge perpendicular to xy .
- Project the corresponding front view.
- Next change the position of the front view, so that 0'1' slant edge lies on ground and add the suffix 1 to them.

- (iv) Project all the points vertically upwards from this front view and horizontally from the first top view. Complete the new top view as shown in Fig. 12.51, observing the rules of visibility of faces.

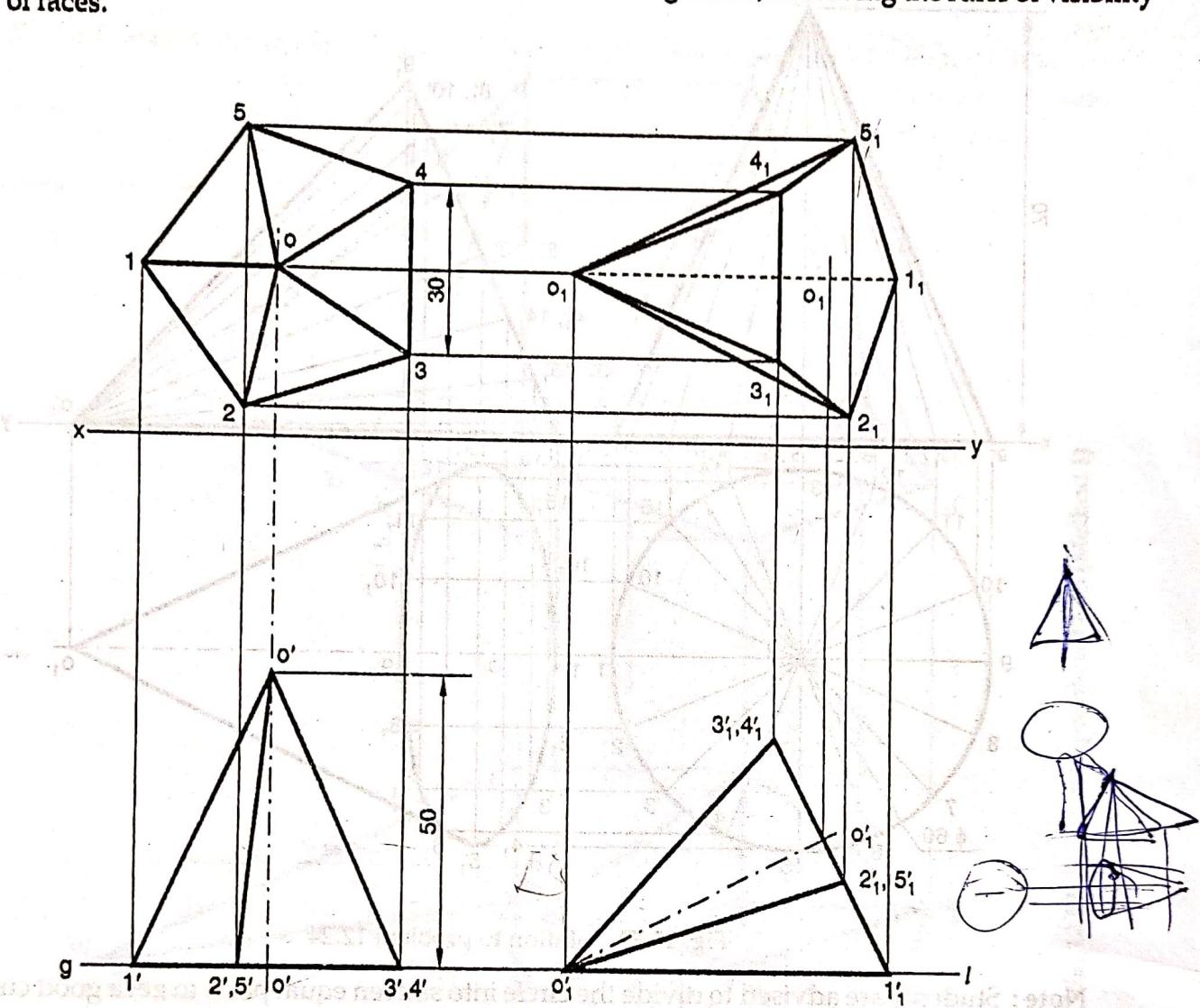


Fig. 12.51 Solution to problem 12.23

PROBLEM 12.24 A right circular cone, diameter of base 60 mm and height 70 mm, lies on HP on one of its elements, such that its axis is parallel to VP. Draw its projections.

(PTU, Jalandhar December 2003, May 2011)

SOLUTION.

- In the initial stage, assume the axis to be perpendicular to HP and parallel to VP. Draw the top view first. Divide the base circle into sixteen equal parts and name all the points.
- Join these points to the centre of the circle O, by continuous thin lines as shown in Fig. 12.52.
- Project the corresponding front view. Here only two elements i.e. O'1' and O'9' provides the true lengths.
- Reproduce the front view by placing O'1' element on the xy and add suffix 1 to all of them.
- Project all the points vertically from this front view and horizontally from the first top view. Complete the new top view by observing the rules of visibility of faces.

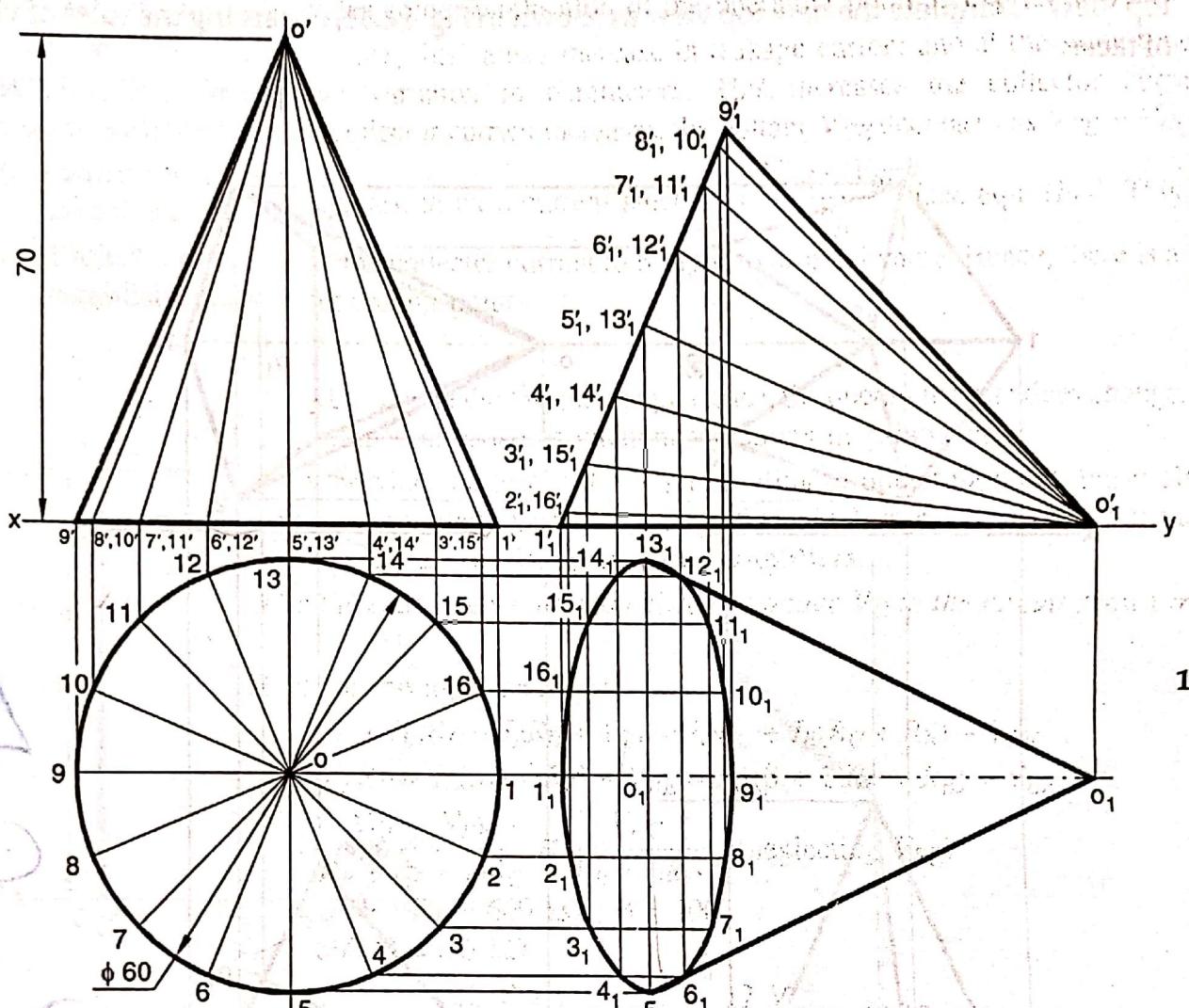


Fig. 12.52 Solution to problem 12.24

Note : Students are advised to divide the circle into sixteen equal parts to get a good curve.

PROBLEM 12.25 A right regular pentagonal pyramid, edge of base 30 mm and height 55 mm lies on HP on one of its slant edges and has its axis parallel to VP. Draw its projections by using both the methods.

SOLUTION. In the initial stage, assume the axis to be perpendicular to the HP and parallel to the VP. Draw the projections with the base in xy .

Method I (Change of position of solid)

- Reproduce the front view so that the slant edge lies on HP and name the points on it by adding suffix 1 to them.
- Project from it, the corresponding top view and join all the points by observing the rules establishing the visibility of lines as shown in Fig. 12.53.

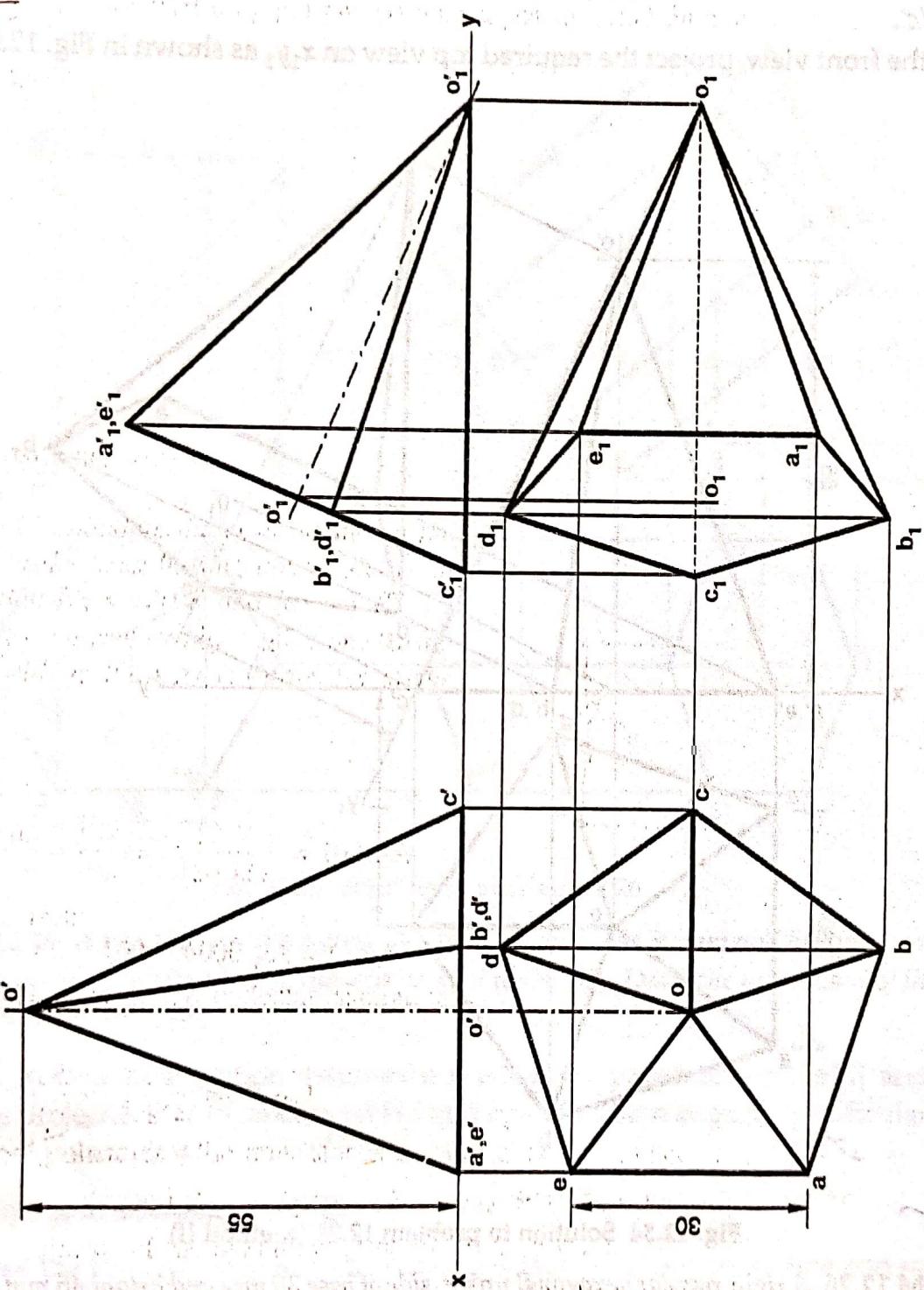


Fig. 12.53 Solution to problem 12.25 (Method I)

Method II (Change of reference line)

- Draw a new reference line x_1y_1 to represent an auxiliary inclined plane (AIP) through slant edge $o'c'$.
- From the front view, project the required top view on x_1y_1 as shown in Fig. 12.54.

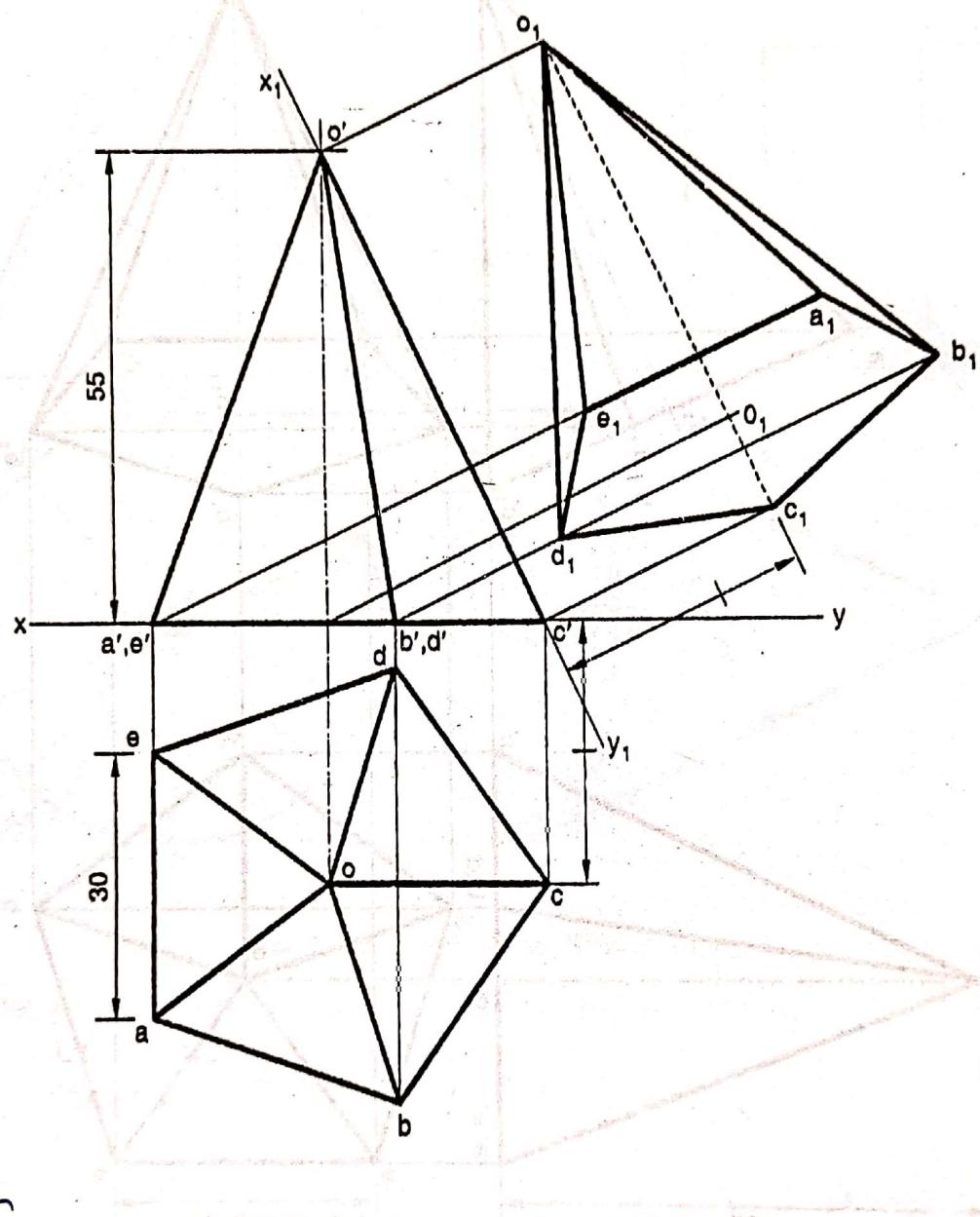


Fig. 12.54 Solution to problem 12.25 (Method II)

PROBLEM 12.26 A right regular hexagonal prism, side of base 20 mm and height 45 mm rests on a corner of its base on the HP with the longer edge passing through this corner making an angle of 30° to the HP. Draw the projections of the hexagonal prism.
 (PTU, Jalandhar December 2002)

SOLUTION. In the initial stage, assume the axis to be perpendicular to the HP. Draw the front and top views in this position and label it.

- Reproduce the front view such that the corner a' lies on HP and the edge $a'1'$ makes an angle 30° to the xy and name all points by adding suffix 1 to them.

(ii) Project from it, the corresponding top view and join all the points by observing the rules of establishing the visibility of the faces as shown in Fig. 12.55.

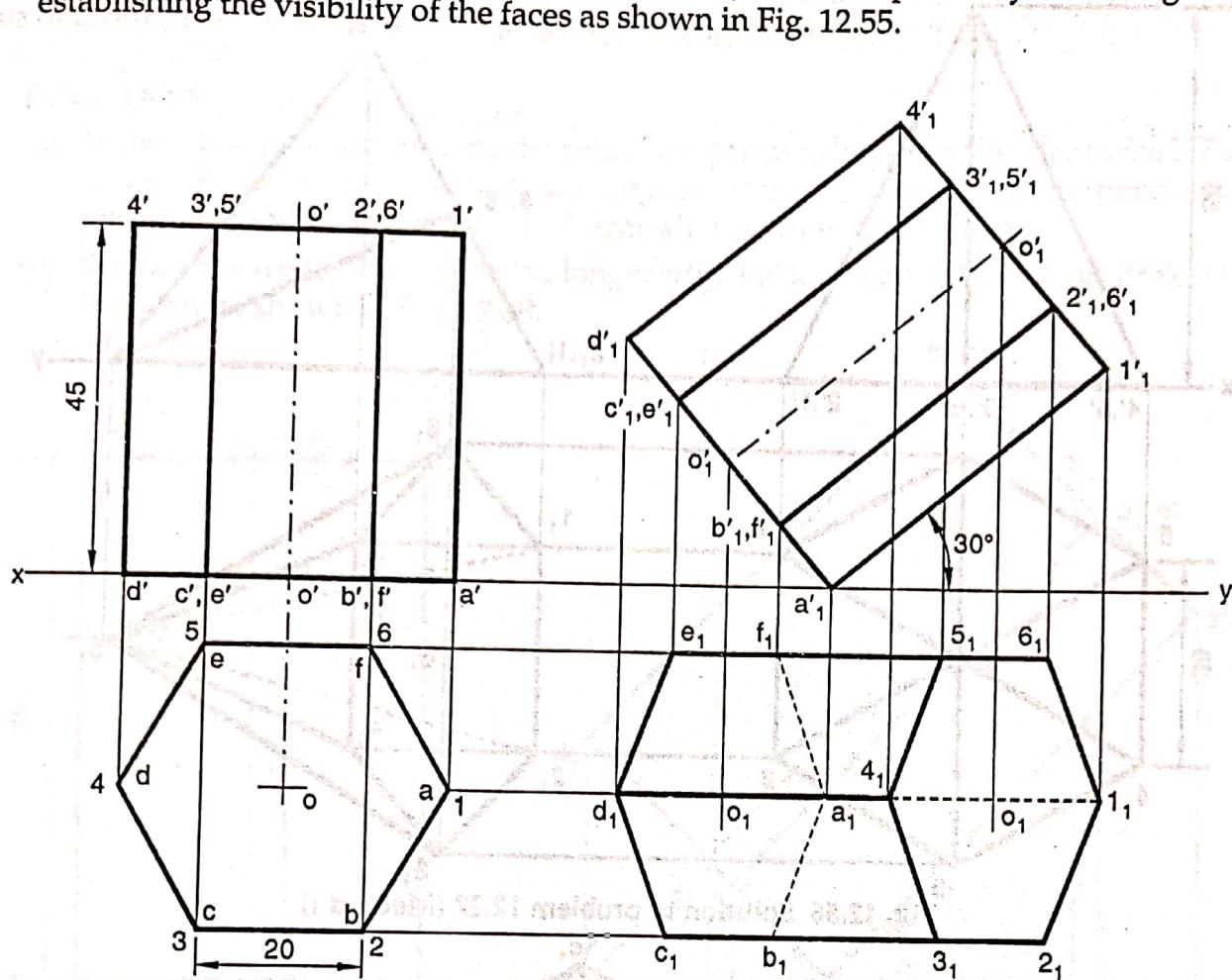


Fig. 12.55 Solution to problem 12.26

PROBLEM 12.27 A right regular hexagonal pyramid, edge of base 30 mm and height 50 mm is resting on one of its triangular faces on the HP with its axis parallel to the VP. Draw the projections of the hexagonal pyramid by using both the methods.

SOLUTION. In the initial position, assume the axis to be perpendicular to the HP and parallel to the VP. Draw the projections with its base on HP and one of its base edge perpendicular to the VP. Name all the corner points on it.

Method I (Change of position of solid)

- Reproduce the front view so that the triangular face $0'1'2'$ lies on xy line and name all the points on it by adding suffix 1 to them.
- Project all the points vertically from this front view and horizontally from the first top view. Complete the new top view as shown in Fig. 12.56.

Method II (Change of reference line)

- Draw a new reference line x_1y_1 coinciding with $0'1'2'$ in the front view. From the front view project the required top view on x_1y_1 (keeping the distance of each point from x_1y_1 equal to the distance of its first top view from xy) as shown in Fig. 12.57.

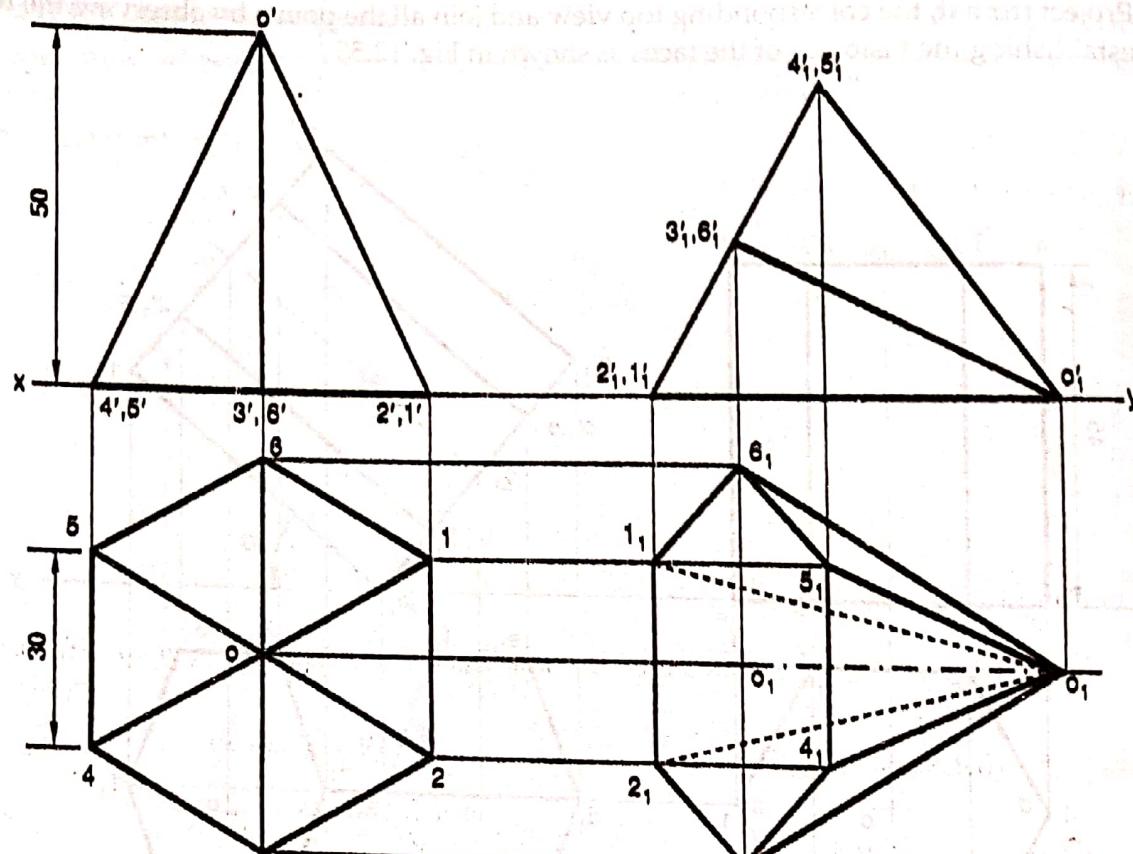


Fig. 12.56 Solution to problem 12.27 (Method I)

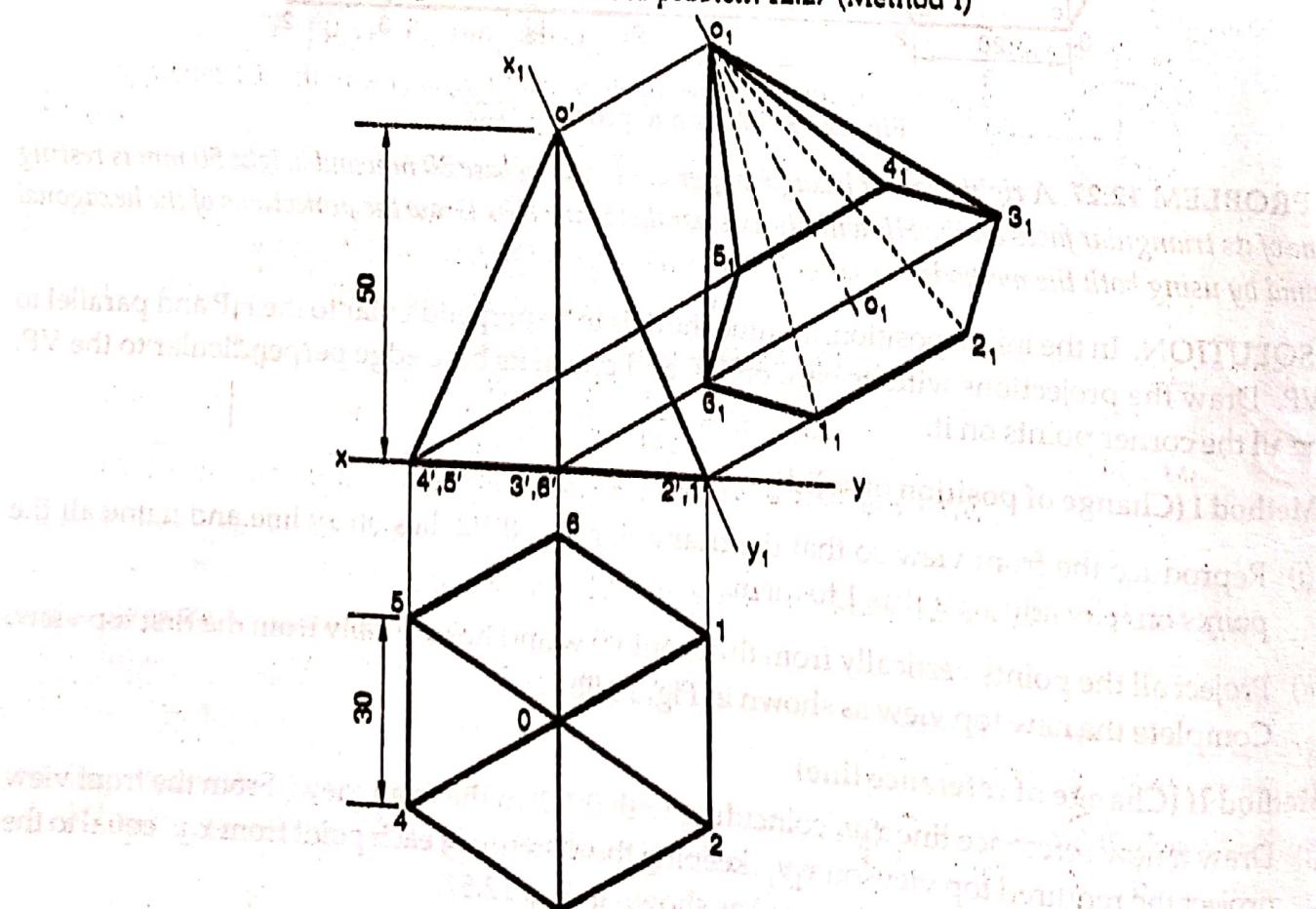


Fig. 12.57 Solution to problem 12.27 (Method II)

PROBLEM 12.28 A right regular pentagonal prism, side of base 30 mm and height 70 mm, rests on one of its base corners on HP such that its long edge containing that corner is inclined to the HP at 45° with its axis parallel to VP. Draw its projections.

SOLUTION.

- In the initial position, assume the axis of the prism to be perpendicular to the HP and parallel to the VP. Draw its projections keeping one of the sides of its base perpendicular to xy and whole of the base lies on the HP. Name all the corner points on it.
- Redraw the front view so that the longer edge $1'a'$ is inclined at 45° to xy . Project the required top view as shown in Fig. 12.58.

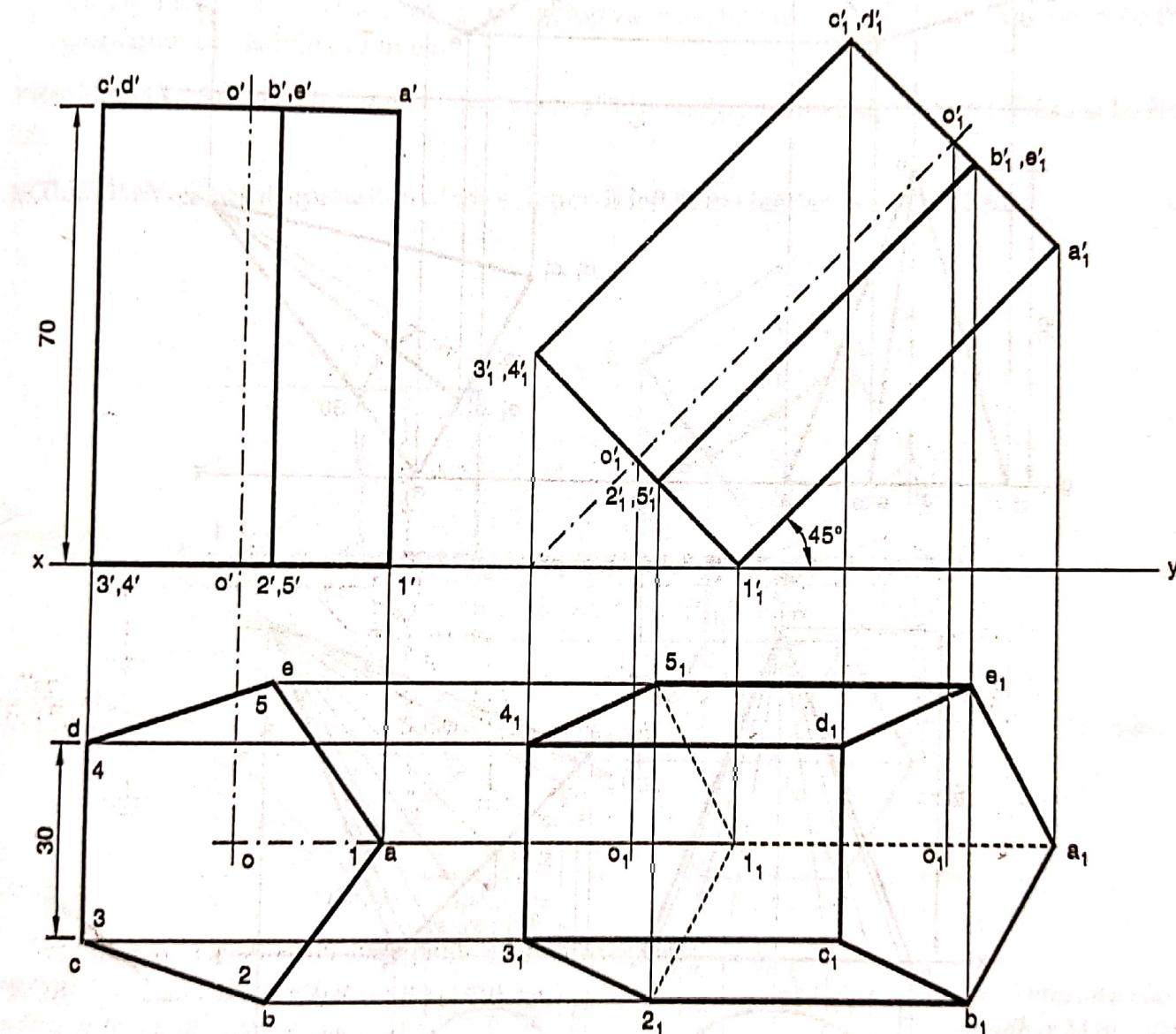


Fig. 12.58 Solution to problem 12.28

PROBLEM 12.29 A right regular pentagonal pyramid, edge of base 25 mm and height 55 mm, is held on ground plane on one of its base corners, such that its axis is inclined at 30° to the ground plane and is parallel to the VP. Draw its projections. (PTU, Jalandhar December 2009)

SOLUTION.

- As the pyramid is given to be resting on its base corner in ground plane, therefore the projections are to be made in third angle. Draw xy and gl lines, a suitable distance apart.
- Draw top and front views assuming the axis of the pyramid to be perpendicular to the ground plane.

(iii) Redraw the front view such that its axis is inclined at 30° to gl line. Project the required top view as shown in Fig. 12.59.

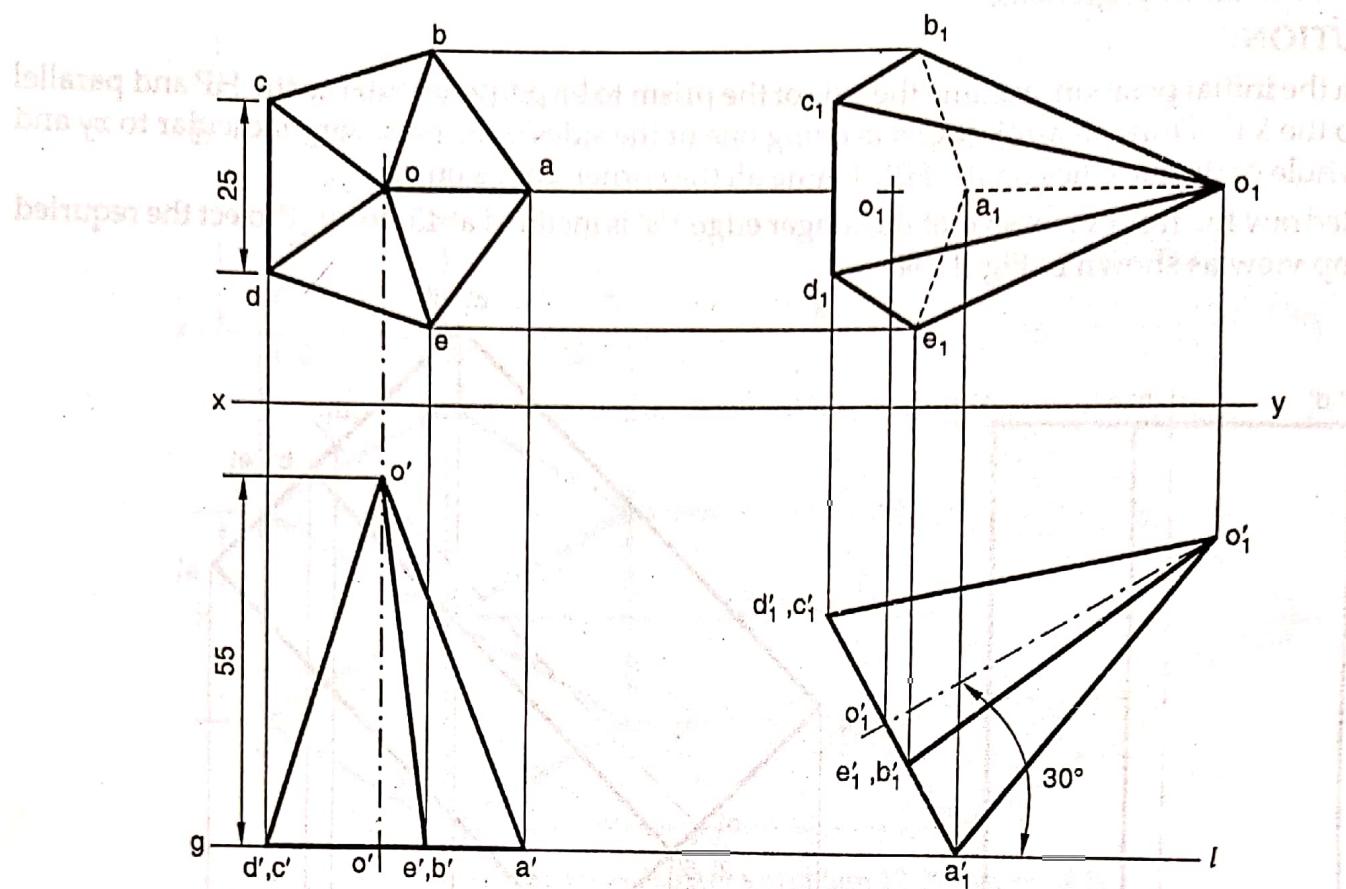


Fig. 12.59 Solution to problem 12.29

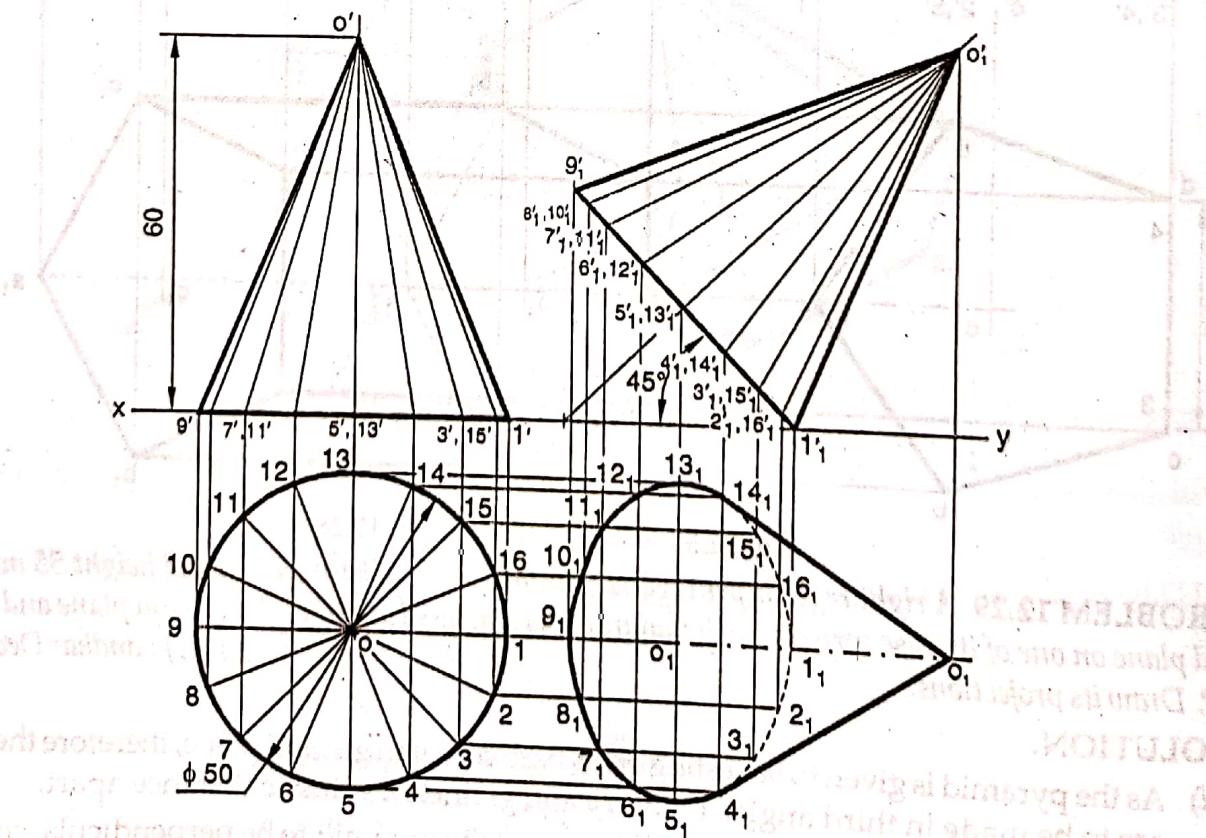


Fig. 12.60 Solution to problem 12.30

PROBLEM 12.30 A right circular cone diameter of base 50 mm and height 60 mm, rests on its base rim on HP such that its axis is parallel to the VP and its base inclined to the HP at 45° . Draw its projections.

SOLUTION.

- In the initial stage, assume the axis to be perpendicular to the HP and parallel to the VP. Draw the top view of the cone and project the corresponding front view. Divide the base circle in the top view into sixteen equal parts and project these points into the front view too.
- Reproduce the front view such that its base rim 1 lies on xy and base makes an angle of 45° to the xy . Project from it the corresponding top view as shown in Fig. 12.60, keeping in mind the principles of visibility of lines.

PROBLEM 12.31 Draw the projections of a cube of 30 mm edge, when a body diagonal of the solid is kept vertical.

SOLUTION. The interpretation of the solution is left to the reader. See Fig. 12.61.

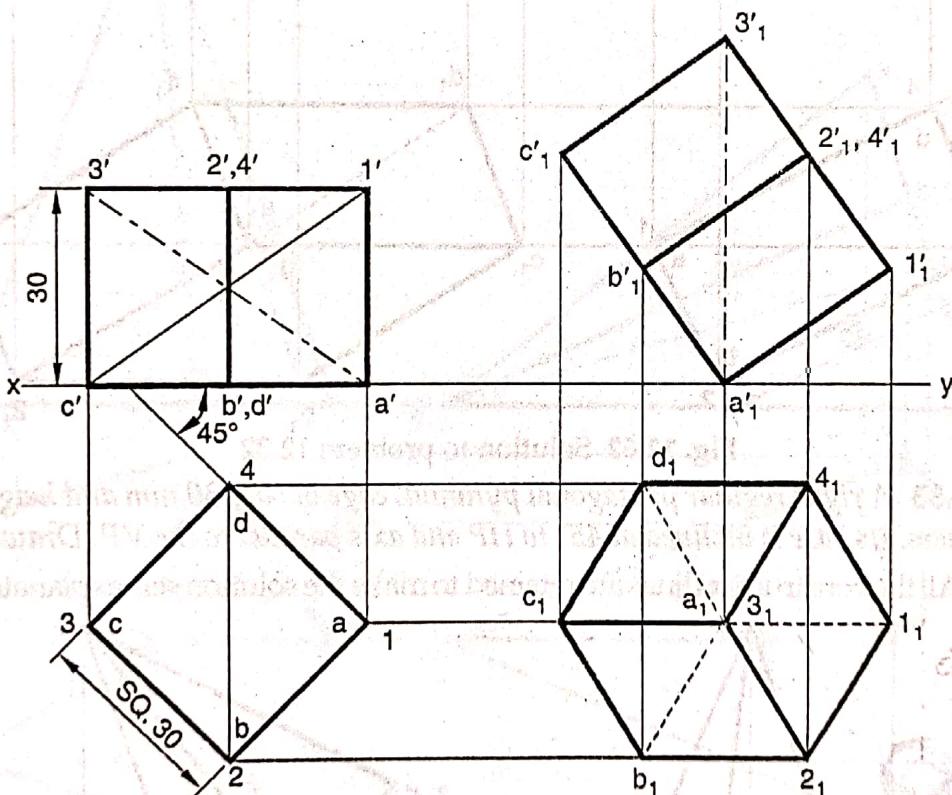


Fig. 12.61 Solution to problem 12.31

PROBLEM 12.32 Draw the projections of a solid rectangular prism 25 mm \times 50 mm \times 75 mm on a plane perpendicular to one of the diagonal of the solid.
(PTU, Jalandhar May 2004)

SOLUTION. All the construction lines are retained to make the solution self-explanatory. See Fig. 12.62.

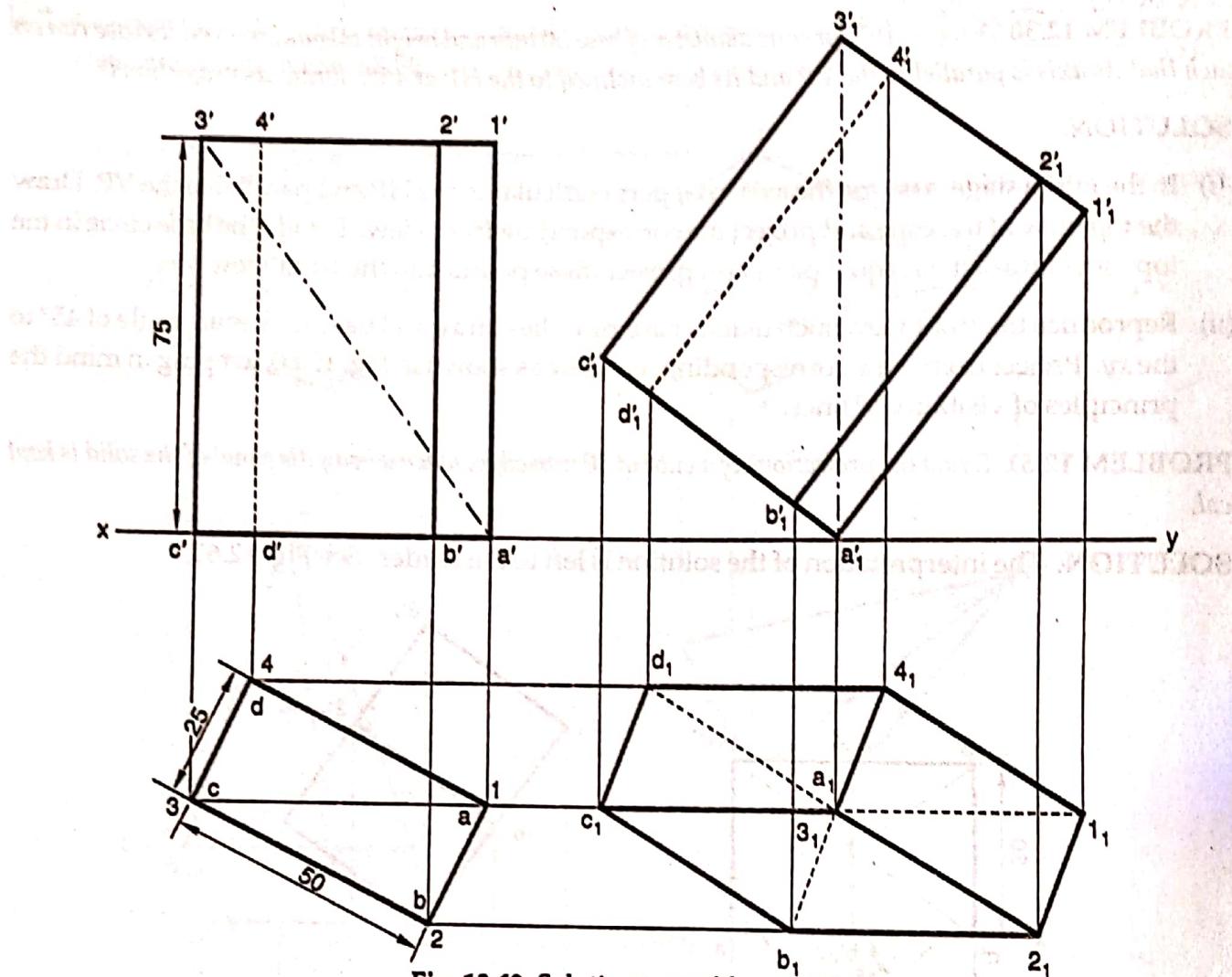


Fig. 12.62 Solution to problem 12.32

PROBLEM 12.33 A right regular pentagonal pyramid, edge of base 30 mm and height 60 mm, rests on HP on one of its corners. Its base is inclined at 45° to HP and axis parallel to the VP. Draw its projections.

SOLUTION. All the construction lines are retained to make the solution self-explanatory. See Fig. 12.63.

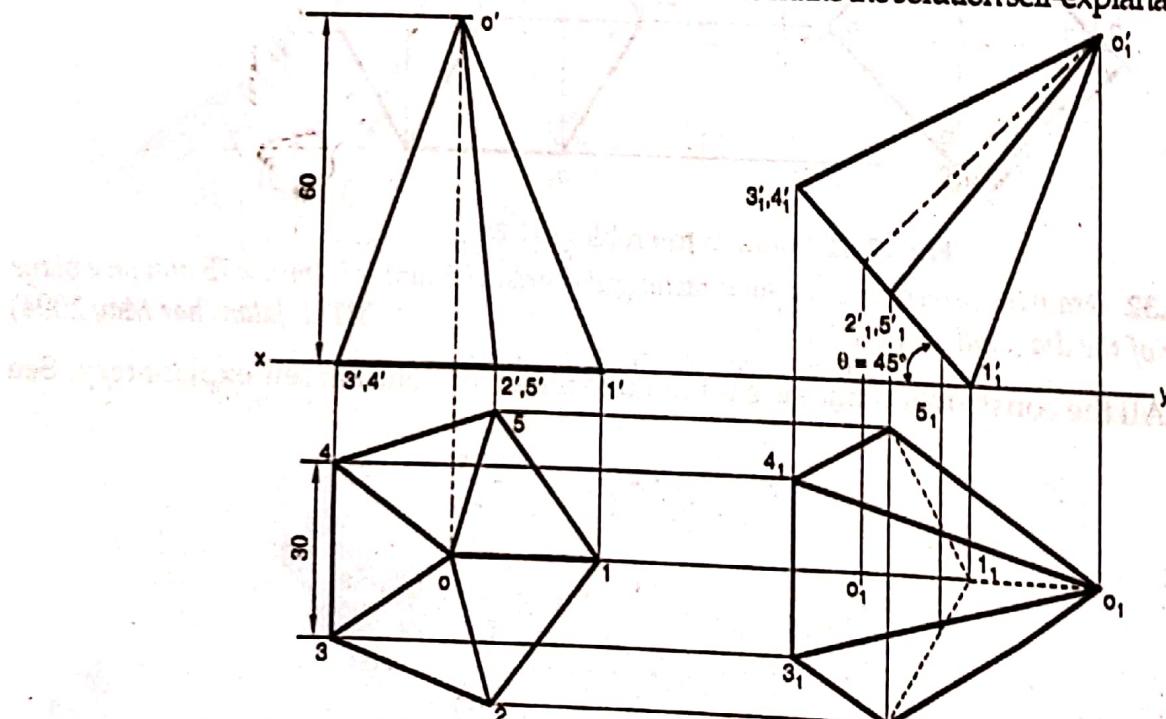


Fig. 12.63 Solution to problem 12.33

PROBLEM 12.34 A right regular pentagonal pyramid, edge of base 30mm and height 65 mm, lies on one of its triangular faces on HP with its axis parallel to the VP. Draw its projections by using both the methods.

SOLUTION. In the initial stage, assume the axis to be perpendicular to the HP and parallel to the VP. Draw the projections with the base in xy .

Method I (Change of position of solid)

- Reproduce the front view so that the triangular face ($o'c'd'$) lies on HP and name the points on it by adding suffix 1 to them.
- Project corresponding top view from it and join all the points by observing the rules establishing the visibility of lines as shown in Fig. 12.64.

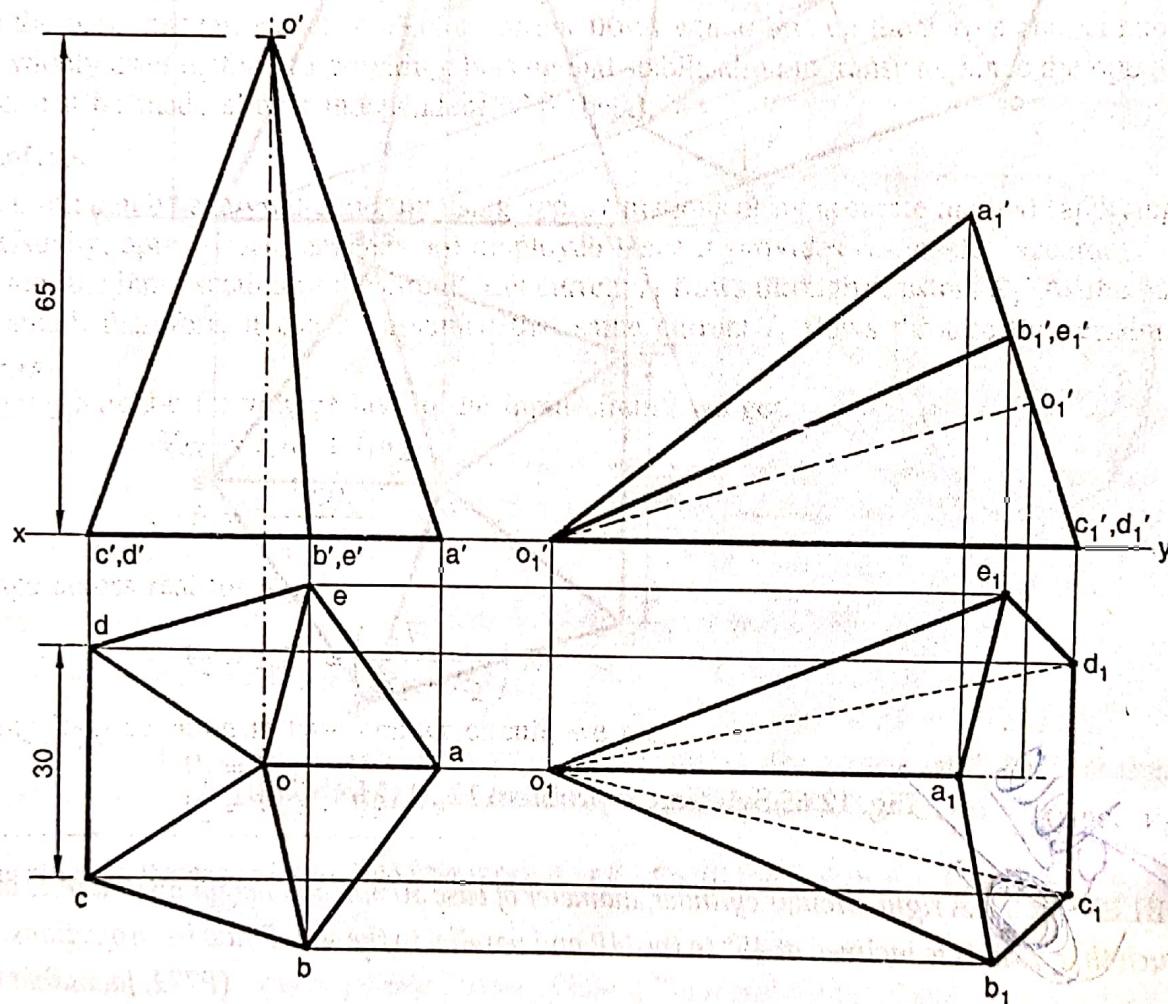


Fig. 12.64 Solution to problem 12.34 (Method I)

Method II (Change of reference line)

- Draw a new reference line x_1, y_1 , to represent an auxiliary inclined plane (AIP) through the triangular face ($o'c'd'$).
- From front view, project the required top view on $x_1 y_1$ as shown in Fig. 12.65.

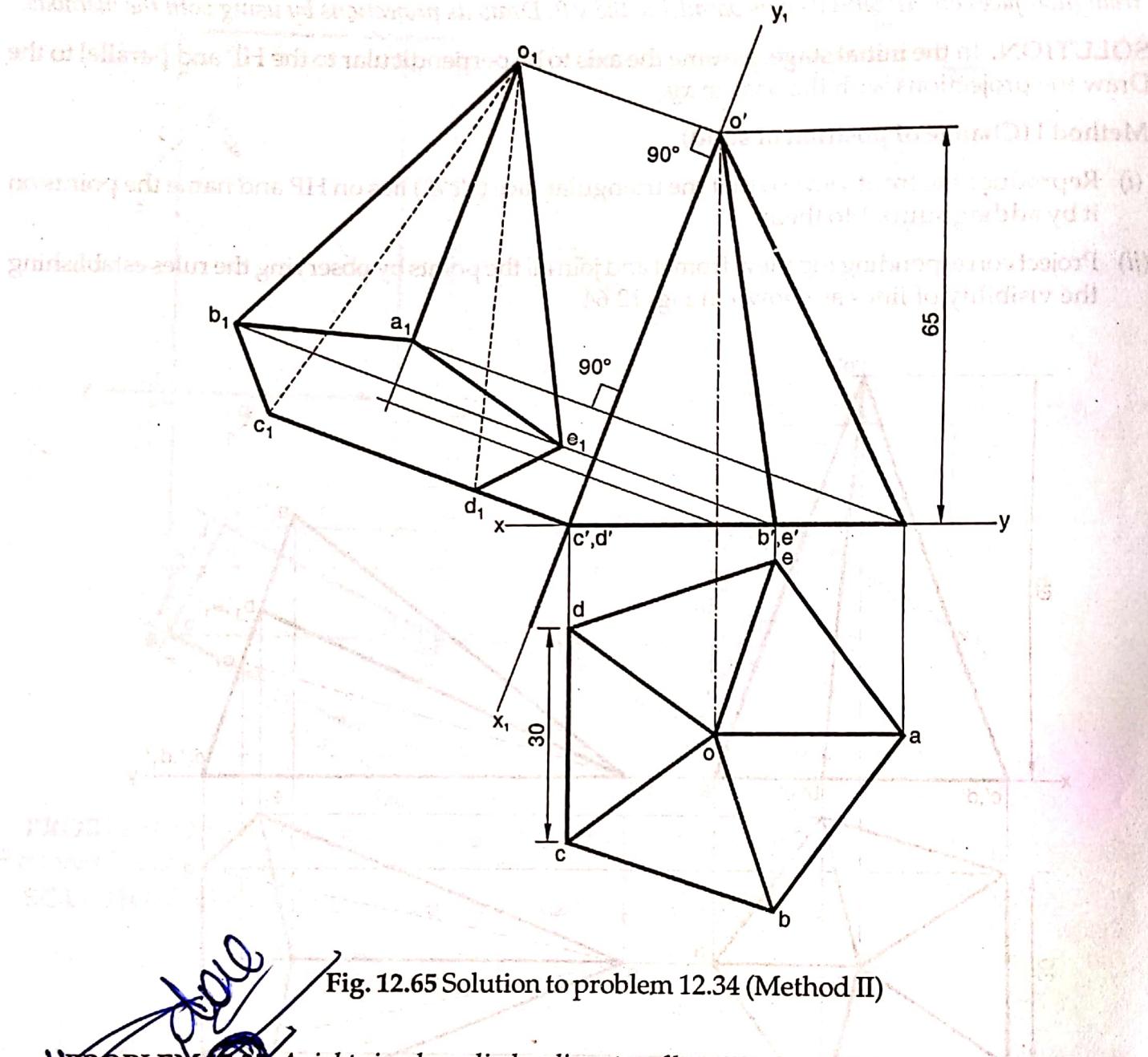


Fig. 12.65 Solution to problem 12.34 (Method II)

PROBLEM 12.34 A right circular cylinder, diameter of base 50 mm and height 65 mm rests on HP on its base rim such that its axis is inclined at 45° to the HP and parallel to the VP. Draw its projections.

(PTU, Jalandhar May 2011)

SOLUTION.

- In the initial stage, assume the axis to be perpendicular to the HP and parallel to the VP. Draw the top view of the cylinder and divide the base circle into sixteen equal parts and name all the points.
- Join these points to the centre of the circle o , by continuous thin lines as shown in Fig. 12.66.

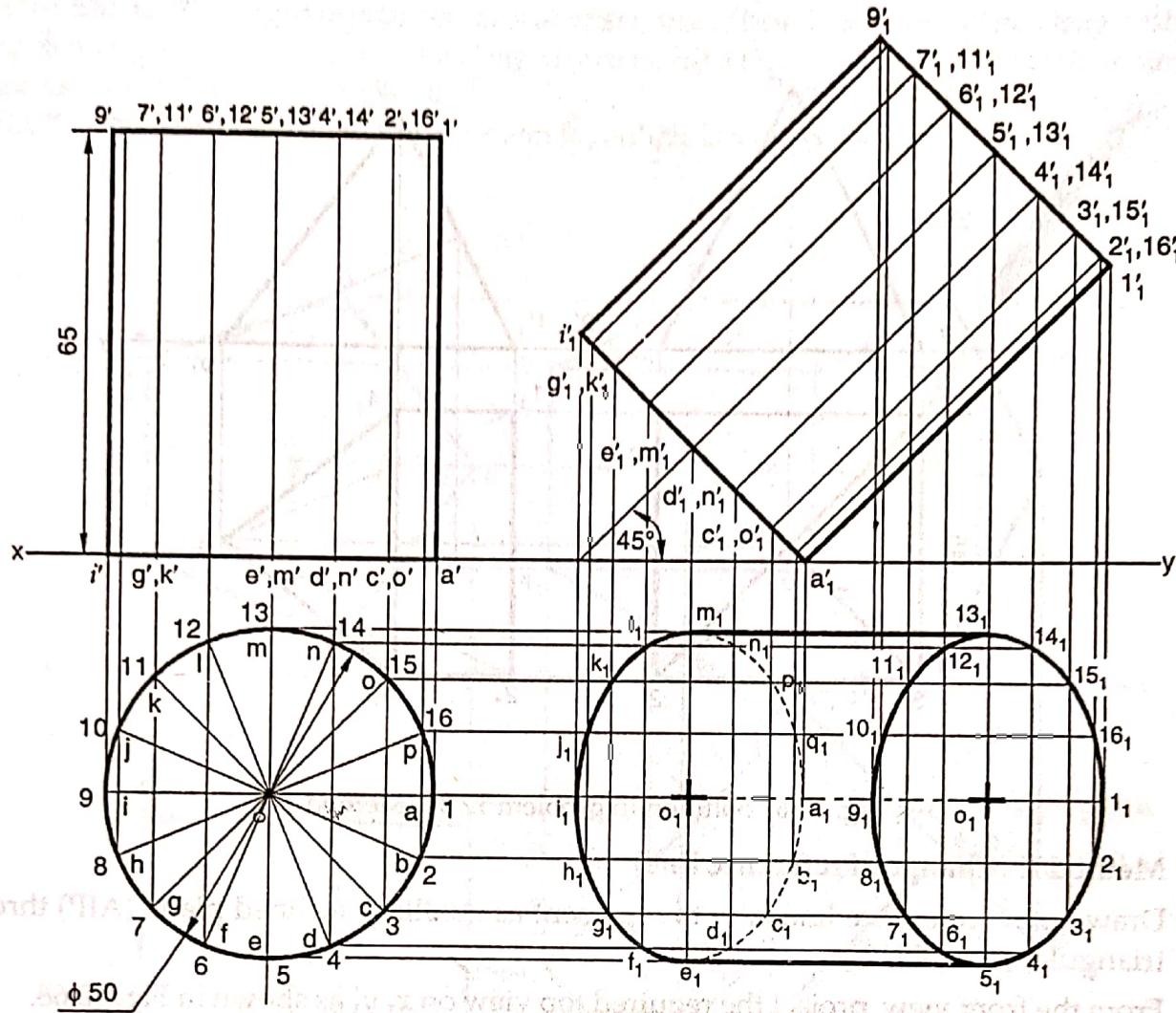


Fig. 12.66 Solution to problem 12.35

- (iii) Project the corresponding front view.
- (iv) Reproduce the front view by placing a' (base rim) on the xy so that the axis makes an angle of 45° with xy and name all the points on it by adding suffix 1 to them.
- (v) Project corresponding top view from it and join all the points by observing rules establishing the visibility of elements.

PROBLEM 12.36 A square pyramid, edge of base 45 mm and length of axis 45 mm is resting on one its triangular faces on HP with its axis parallel to the VP. Draw its projections by using both the methods.

SOLUTION. In the initial stage, assume the axis to be perpendicular to the HP and parallel to the VP. Draw the projections with the base in xy .

Method I (Change of position of solid)

- (i) Reproduce front view so that triangular face ($0'1'2'$) lies on HP and name the points on it by adding suffix 1 to them.
- (ii) Project the corresponding top view, observing the rules of visibility of surfaces.

See Fig. 12.67.

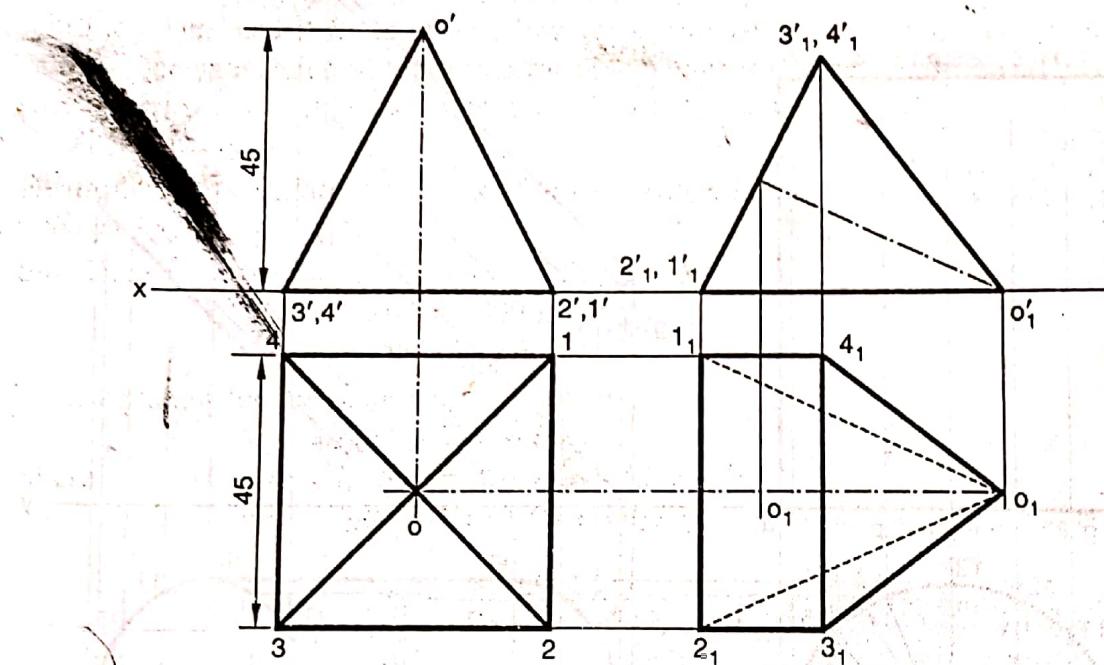


Fig. 12.67 Solution to problem 12.36 (Method I)

Method II (Change of reference line)

- Draw a new reference line $x_1 y_1$ to represent as auxiliary inclined plane (AIP) through the triangular face $(0' 1' 2')$.
- From the front view, project the required top view on $x_1 y_1$ as shown in Fig. 12.68.

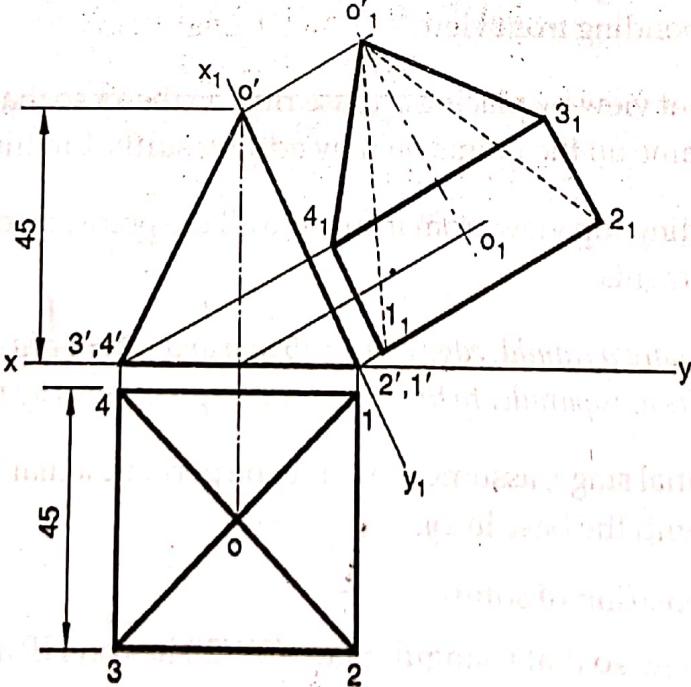


Fig. 12.68 Solution to problem 12.36 (Method II)

PROBLEM 12.37 A right regular pentagonal prism, side of base 30 mm and 55 mm long, rests on one of its base corners on ground plane such that its long edge containing that corner is inclined to the ground plane at 45° . Draw its projections in third angle.

SOLUTION. The solution to this problem is self-explanatory. See Fig. 12.69.

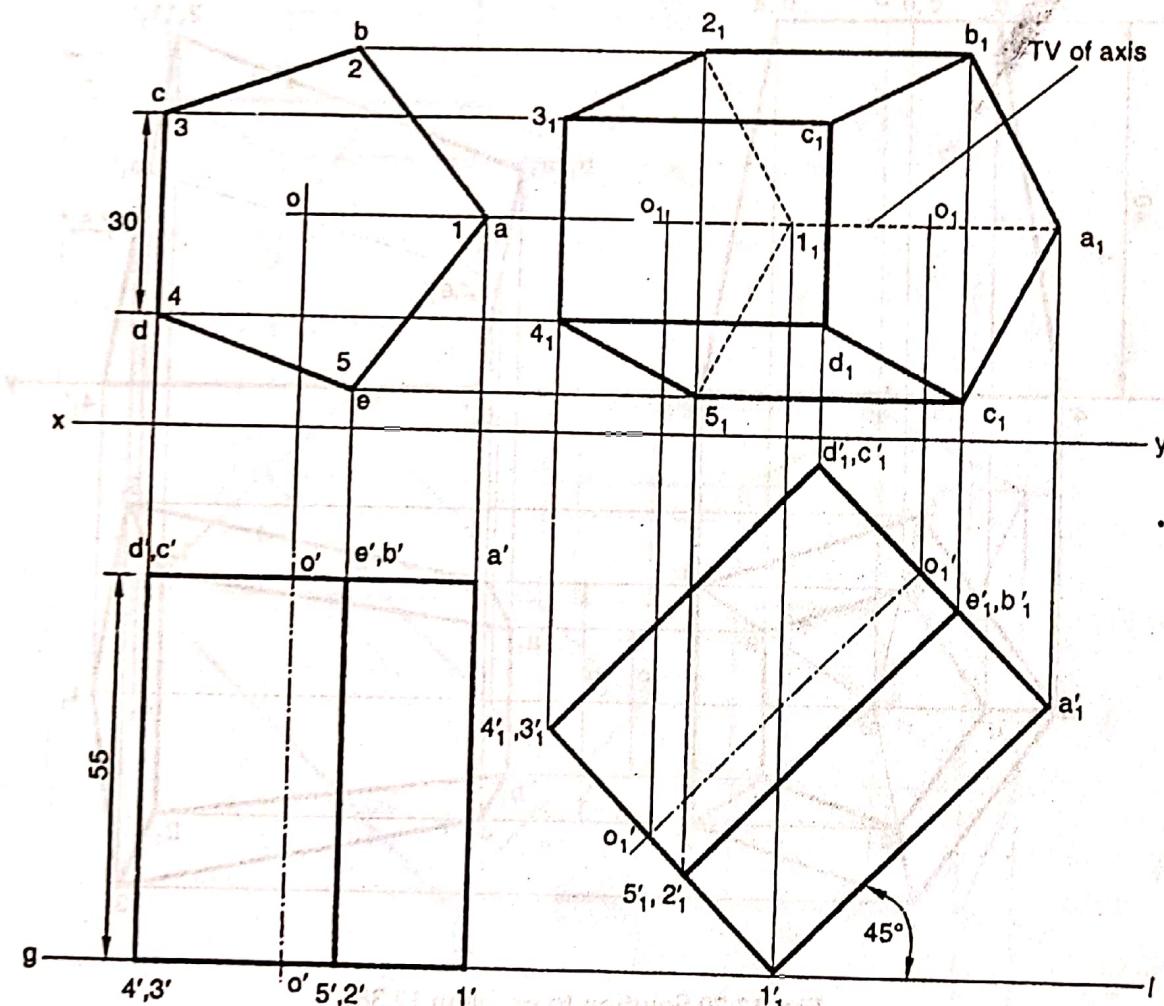


Fig. 12.69 Solution to problem 12.37

PROBLEM 12.38 A frustum of a right regular pentagonal pyramid, edge of lower base 25 mm, edge of upper base 15 mm and axis 40 mm long, is lying on one of its slant edges on HP with its axis parallel to VP. Draw its projections.

SOLUTION. All the construction lines are retained to make the solution self-explanatory. See Fig. 12.70.

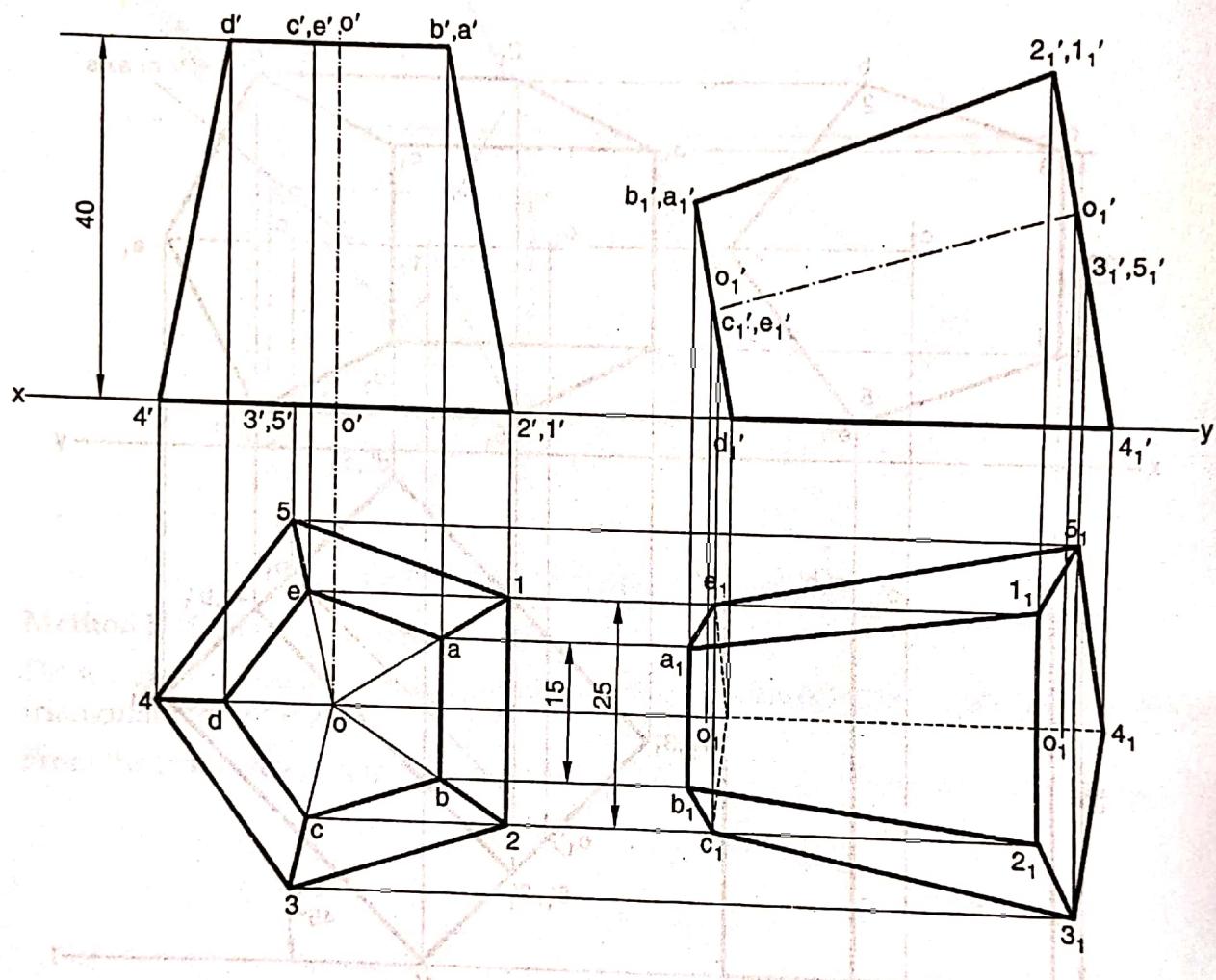


Fig. 12.70 Solution to problem 12.38

PROBLEM 12.39 A right regular pentagonal prism, edge of base 25 mm and height 60 mm, is resting on one of its base edges in HP, such that its axis is inclined at 45° to the HP and parallel to the VP. Draw three views of the pentagonal prism.

SOLUTION. All the construction lines are retained to make the solution self-explanatory. See Fig. 12.71.

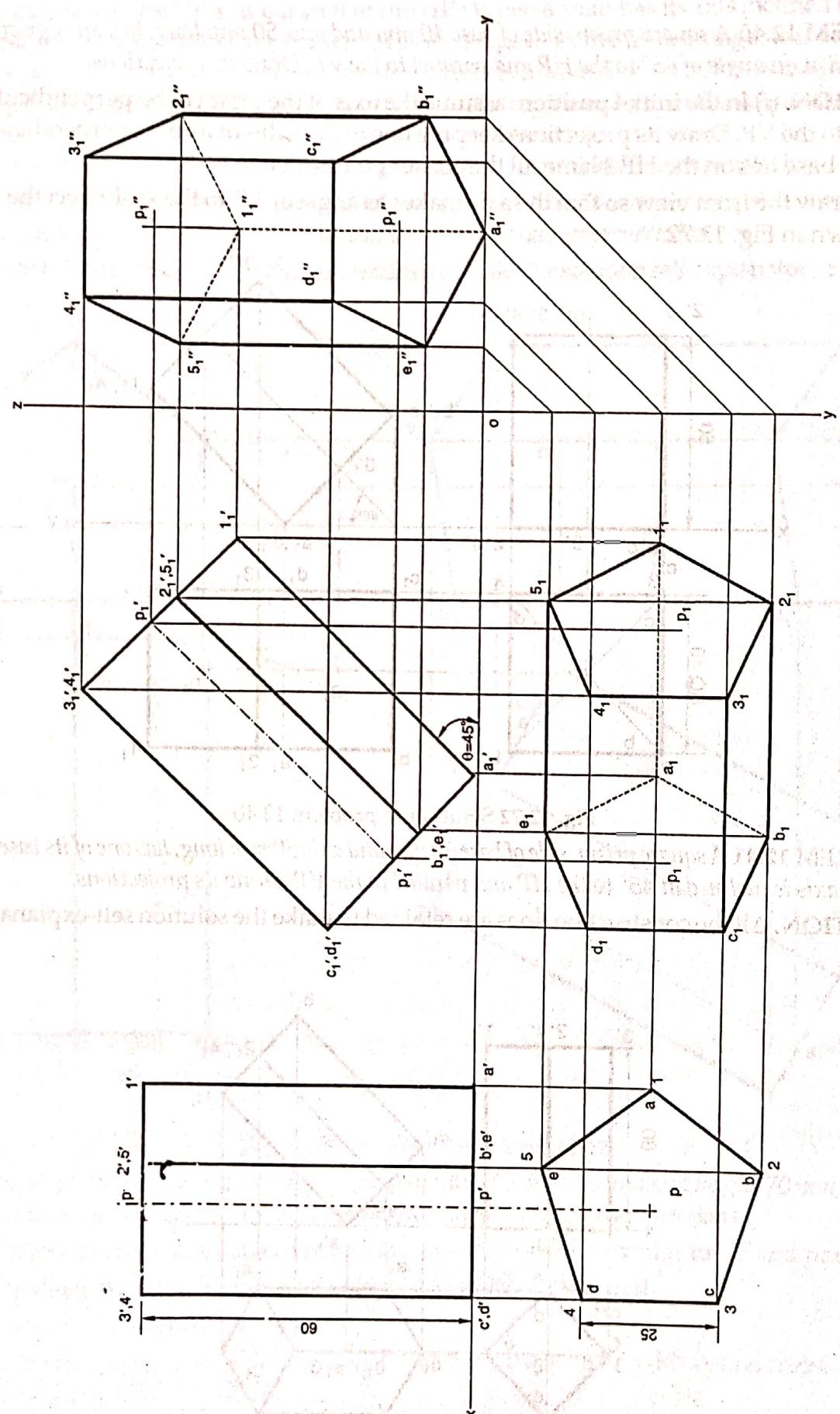


Fig. 12.71 Solution to problem 12.39

PROBLEM 12.40 A square prism, side of base 40 mm and axis 50 mm long, has an edge of its in HP. Its axis is inclined at an angle of 45° to the HP and parallel to the VP. Draw its projections.

SOLUTION. (i) In the initial position, assume the axis of the prism to be perpendicular to the HP and parallel to the VP. Draw its projections keeping one of the sides of its base perpendicular to xy and whole of the base lies on the HP. Name all the corner points on it.

(ii) Redraw the front view so that the axis makes an angle of 45° to the xy. Project the required top view as shown in Fig. 12.72.

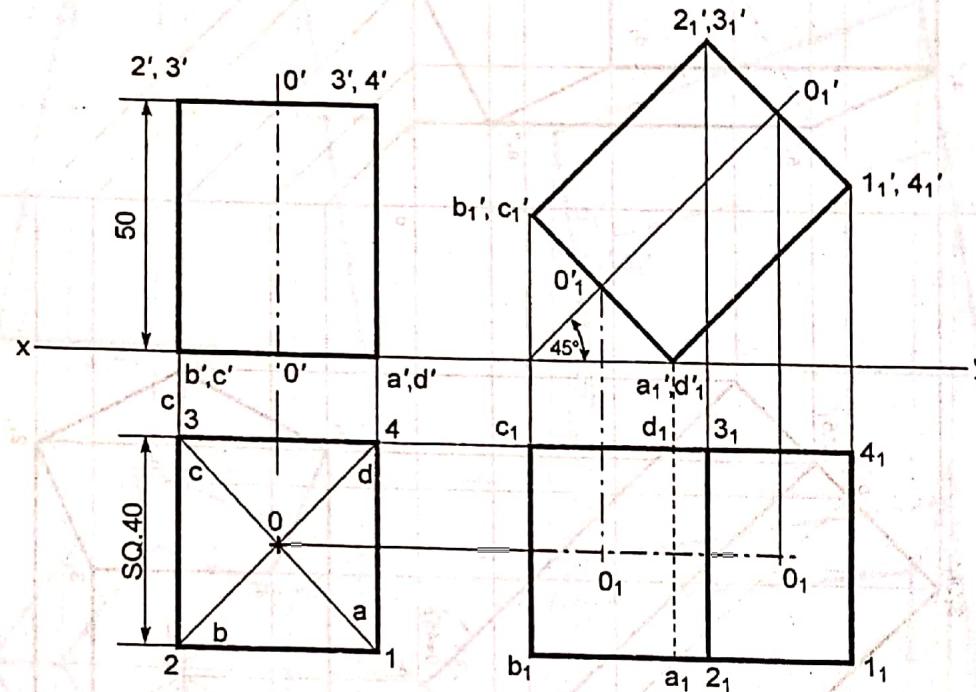


Fig. 12.72 Solution to problem 12.40

PROBLEM 12.41 A square prism, side of base 30 mm and axis 60 mm long, has one of its base corner in HP such that its axis is inclined at 45° to the HP and parallel to the VP. Draw its projections.

SOLUTION. All the construction lines are retained to make the solution self-explanatory. See Fig. 12.73.

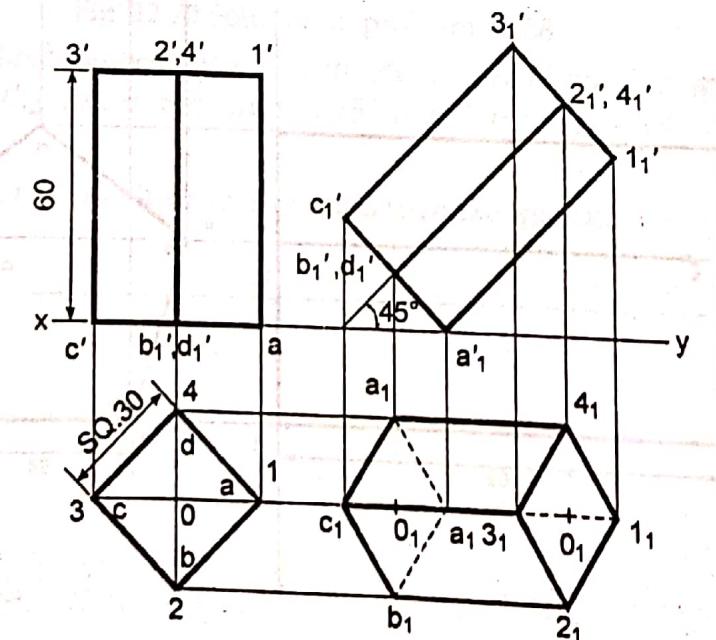


Fig. 12.73 Solution to problem 12.41

(b) Axis inclined to the VP and parallel to the HP : When a solid has its axis inclined to the VP and parallel to the HP, its projections are drawn into two stages. In the initial stage, it is assumed to be perpendicular to the VP. In such problems, the front view is drawn first, as it shows the true shape and size of the object and then top view is projected from it. The top view is reproduced making given angle with VP. Project all the points vertically from this top view and horizontally from the first front view. Join all points in the final front view, observing the rules for establishing the visibility of lines.

PROBLEM 12.42 A right regular pentagonal prism, side of base 30 mm and height 70 mm, lies on one of its rectangular faces on HP with its axis inclined at 30° to VP. Draw its projections.

SOLUTION. All the construction lines are retained to make the solution self-explanatory. See Fig. 12.74.

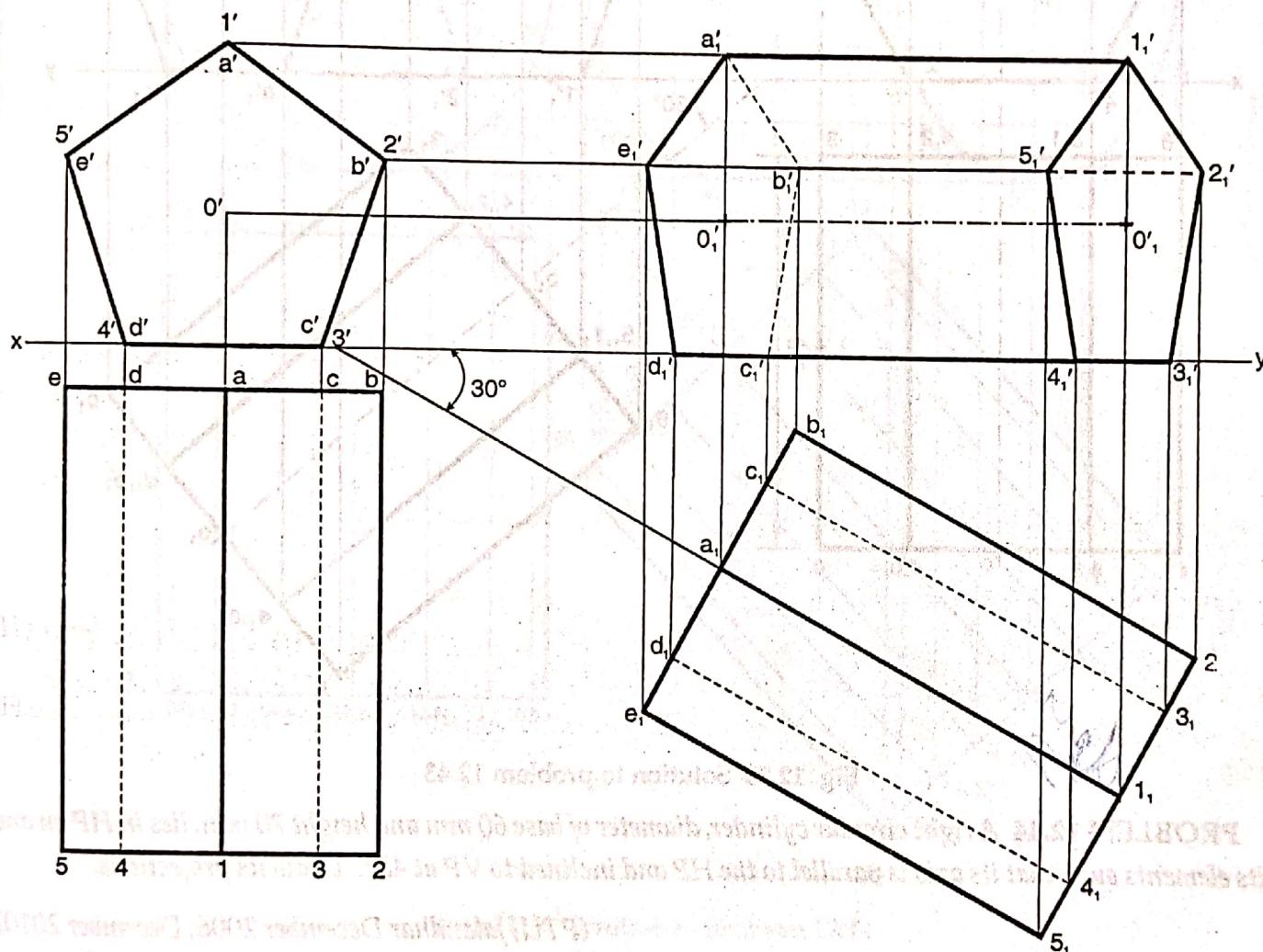


Fig. 12.74 Solution to problem 12.42

PROBLEM 12.43 A right regular hexagonal prism, side of base 30 mm and height 70 mm, lies on one of its rectangular faces on HP and its axis inclined at 30° to the VP. Draw its projections.

SOLUTION. In the initial stage, assume the axis to be perpendicular to VP and parallel to HP.

- Draw the front view, keeping one of its faces in HP and label it.
- Project the corresponding top view.
- Reproduce the top view so that the axis makes an angle of 30° with xy and name the points on it by adding suffix 1 to them.
- Project from it, the corresponding front view and join all the points by observing the rules establishing the visibility of faces as shown in Fig. 12.75.

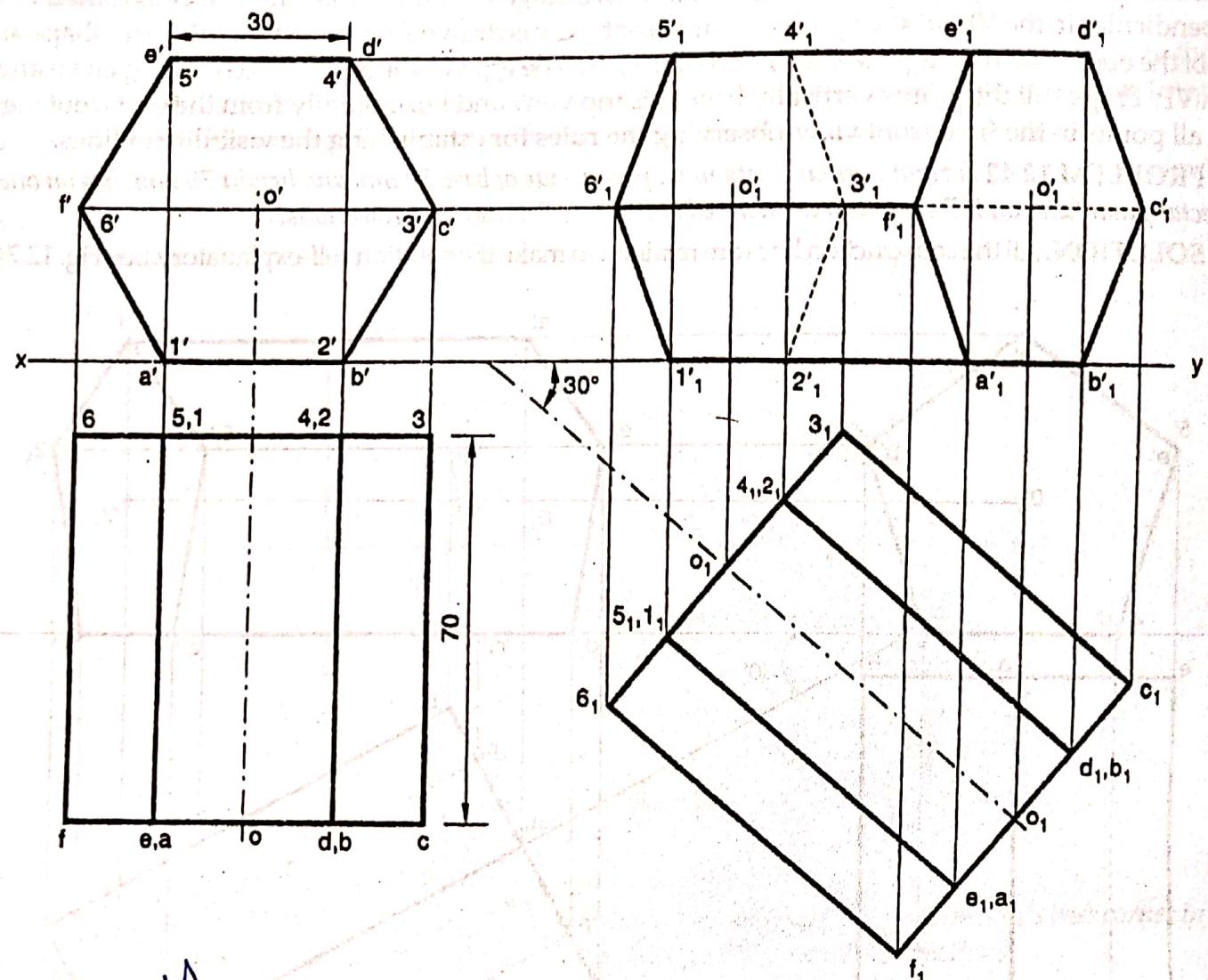


Fig. 12.75 Solution to problem 12.43

PROBLEM 12.44 A right circular cylinder, diameter of base 60 mm and height 70 mm, lies in HP on one of its elements such that its axis is parallel to the HP and inclined to VP at 45°. Draw its projections.

(PTU, Jalandhar December 2006, December 2010)

SOLUTION. In the initial stage, assume the axis to be perpendicular to VP and parallel to HP.

- Draw the front view, keeping one of its elements in HP. Divide the circle into sixteen equal parts and name all the points.
- Join these points to the centre of the circle O', by continuous thin lines as shown in Fig. 12.76.
- Project the corresponding top view.
- Reproduce the top view so that the axis makes an angle of 45° with xy and name all the points by adding suffix 1 to them.
- Project the corresponding front view and join all the points by observing the rules of visibility of lines.

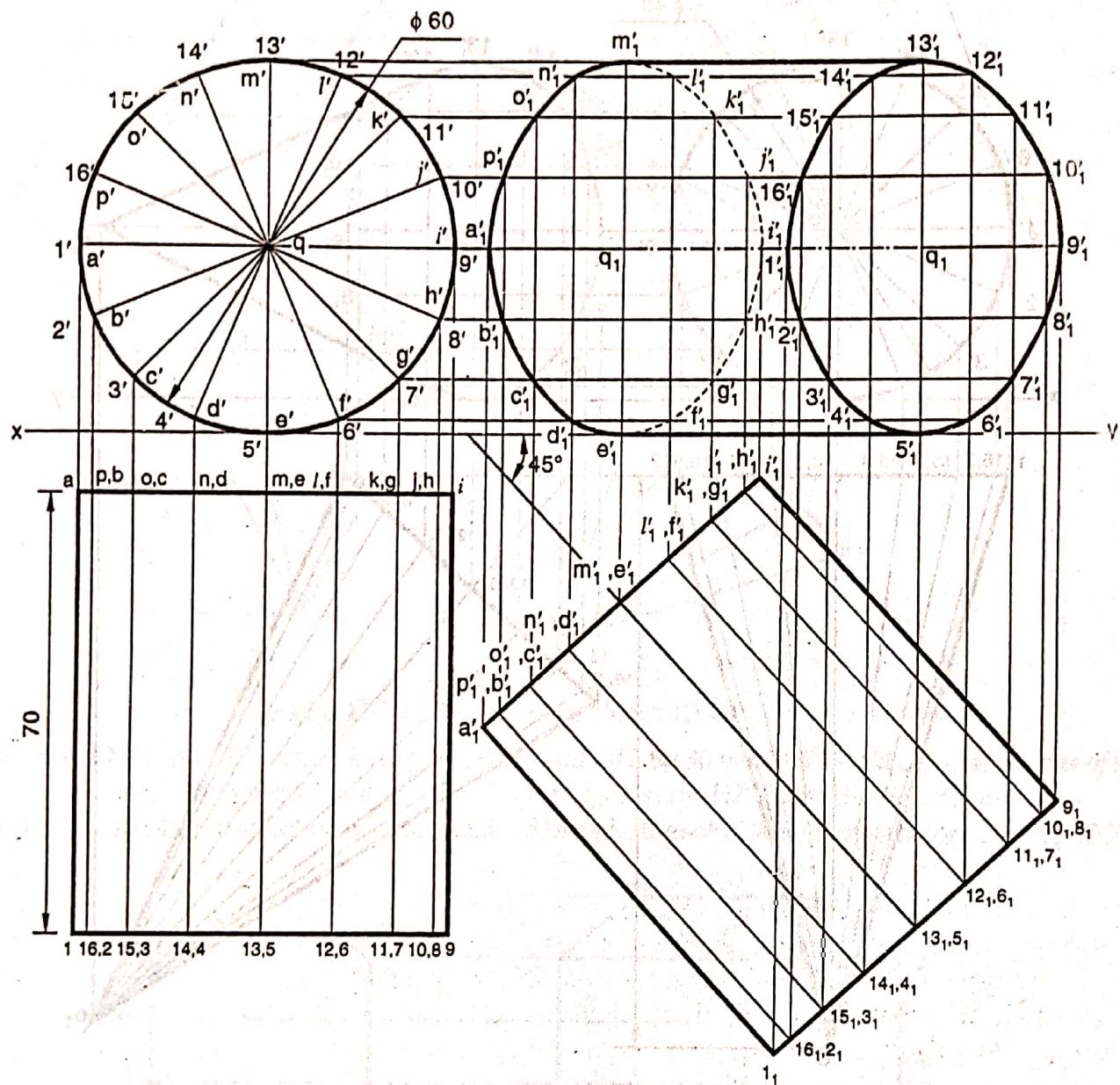


Fig. 12.76 Solution to problem 12.44

PROBLEM 12.45 A right circular cone, diameter of base 40 mm and height 70 mm is held on its base rim in HP such that its axis is inclined at 45° to VP and parallel to HP. Draw its projections, keeping vertex towards the observer.

SOLUTION.

- In the initial stage, assume the axis to be perpendicular to VP and parallel to HP. Draw the front view first, keeping its base rim on HP. Divide the base circle into sixteen equal parts and label it.
- Project the corresponding top view.
- Reproduce the top view as shown in Fig. 12.77 and add suffix 1 to them.
- Project all the points vertically from this top view and horizontally from the first front view. Complete the new front view by observing the rules of visibility of lines.

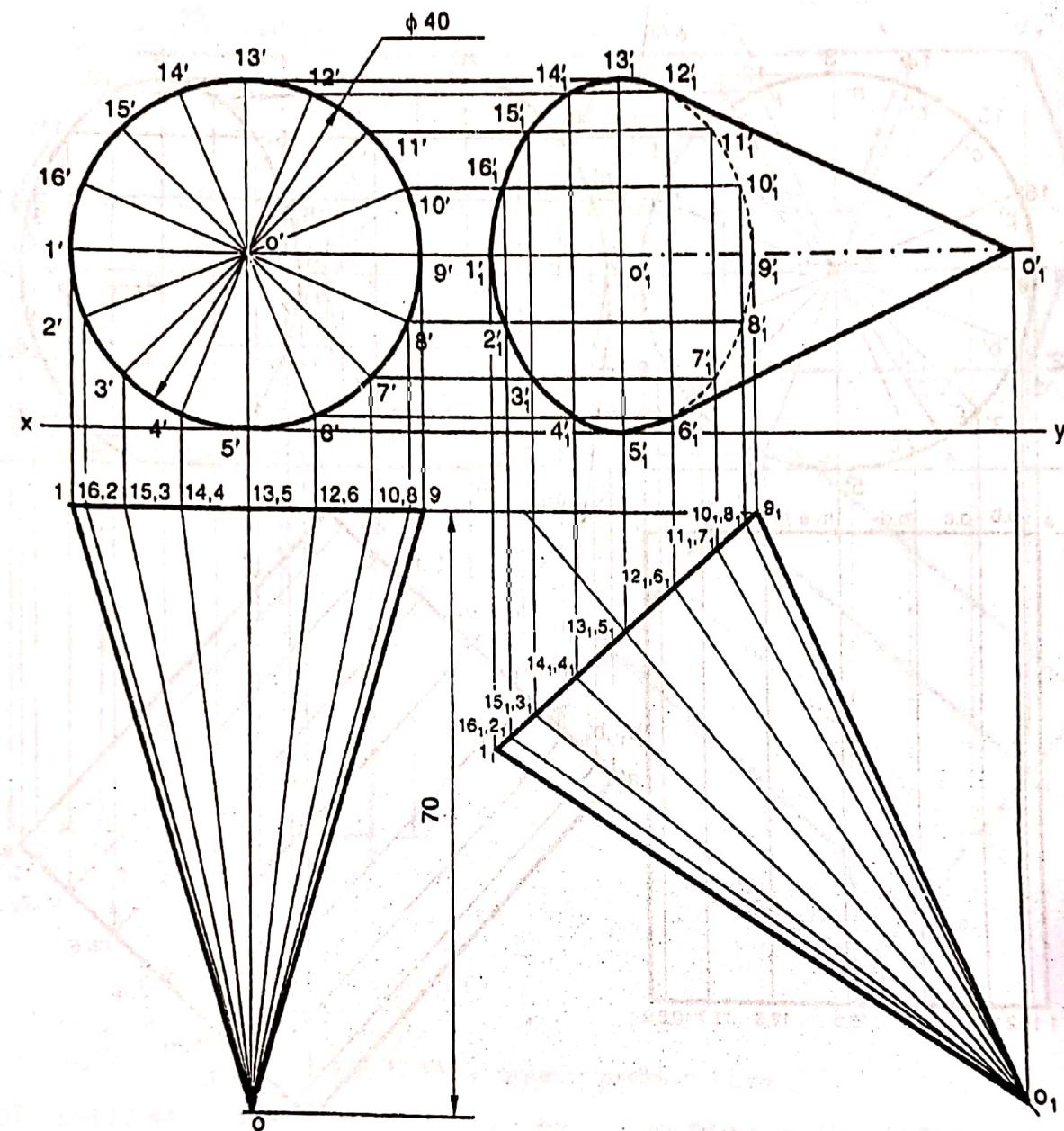


Fig. 12.77 Solution to problem 12.45

PROBLEM 12.46 A right regular hexagonal pyramid, edge of base 25 mm and length of axis 60 mm, has one of its corners in the VP such that, its axis inclined at 45° to the VP and parallel to the HP. Draw its projections.

SOLUTION.

- In the initial stage, assume the axis to be perpendicular to VP and parallel to HP. Draw the front view first, keeping its base in VP. Name all the corner points on it.
- Project the corresponding top view.
- Reproduce the top view such that the corner point 4 lies on VP and the axis makes an angle of 45° to VP. Name all the points on it by adding suffix 1 to them.
- From the top view, project the required front view as shown in Fig. 12.78.

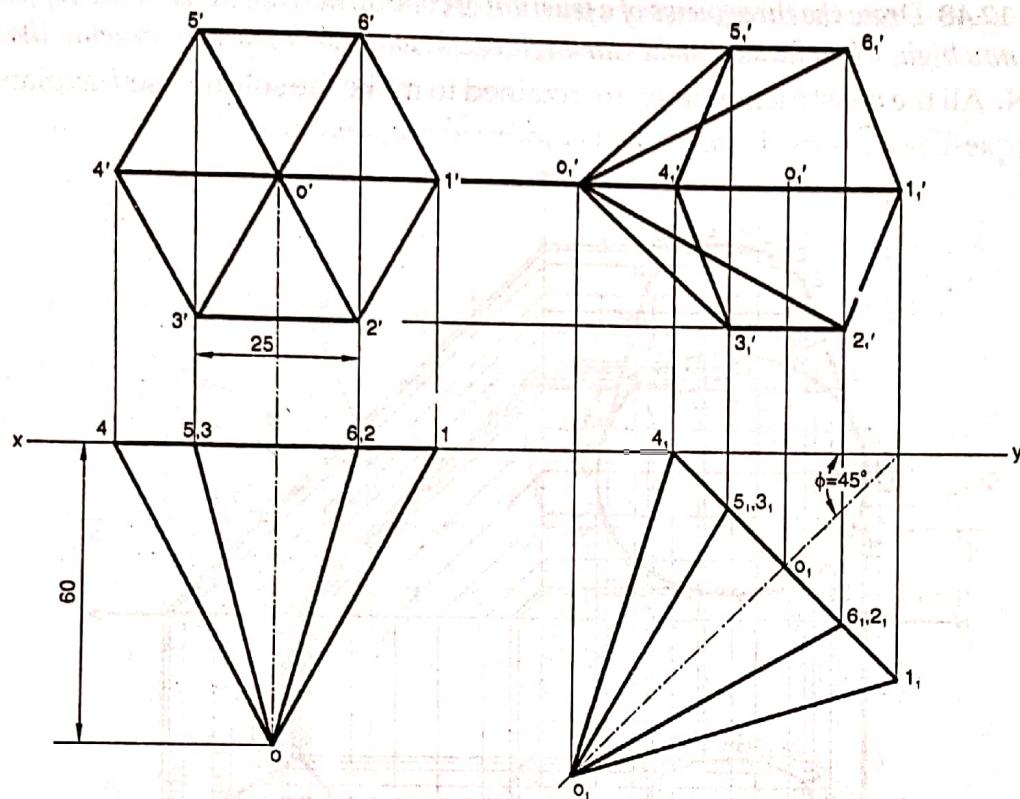


Fig. 12.78 Solution to problem 12.46

PROBLEM 12.47 A right regular hexagonal prism, side of base 30 mm and height 60 mm, has one of its side (or face) inclined at 45° to the VP and one of its longer edges on the HP. Draw its projections.

SOLUTION. All the construction lines are retained to make the solution self-explanatory. See Fig. 12.79.

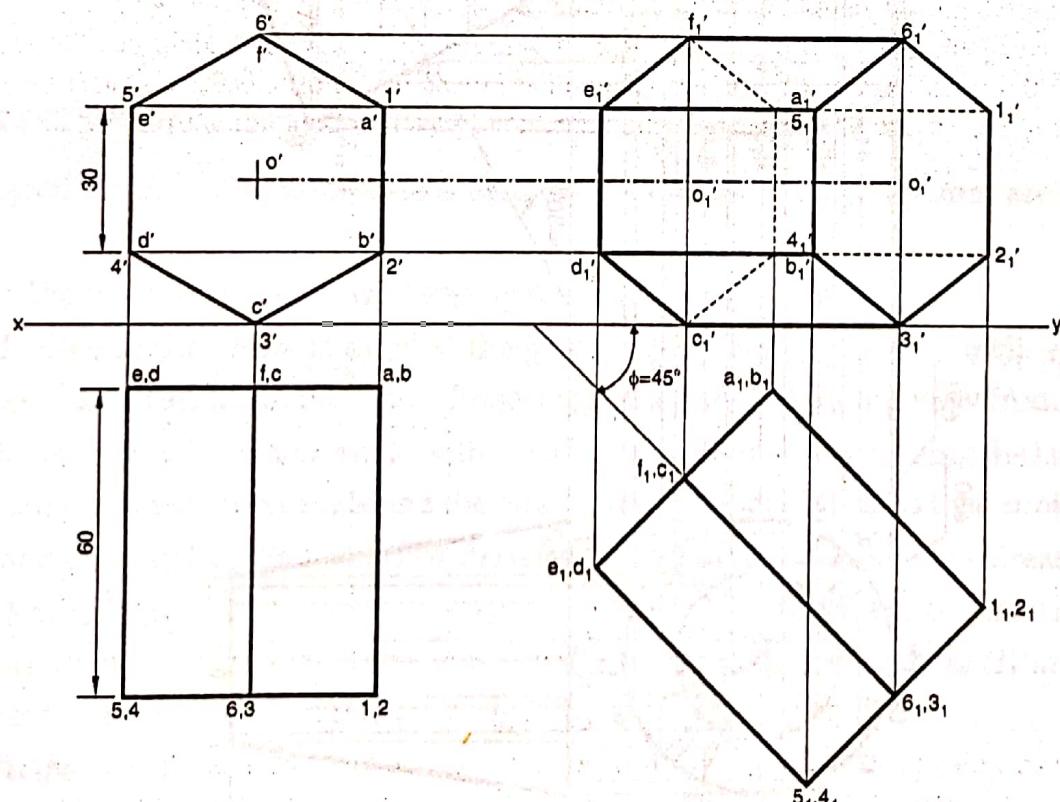


Fig. 12.79 Solution to problem 12.47

PROBLEM 12.48 Draw the three views of a frustum of cone 50 mm diameter at the top, 30 mm diameter at the bottom, 60 mm high, when its axis makes an angle of 30° with the VP and parallel to the HP.

SOLUTION. All the construction lines are retained to make the solution self-explanatory. See Fig. 12.80.

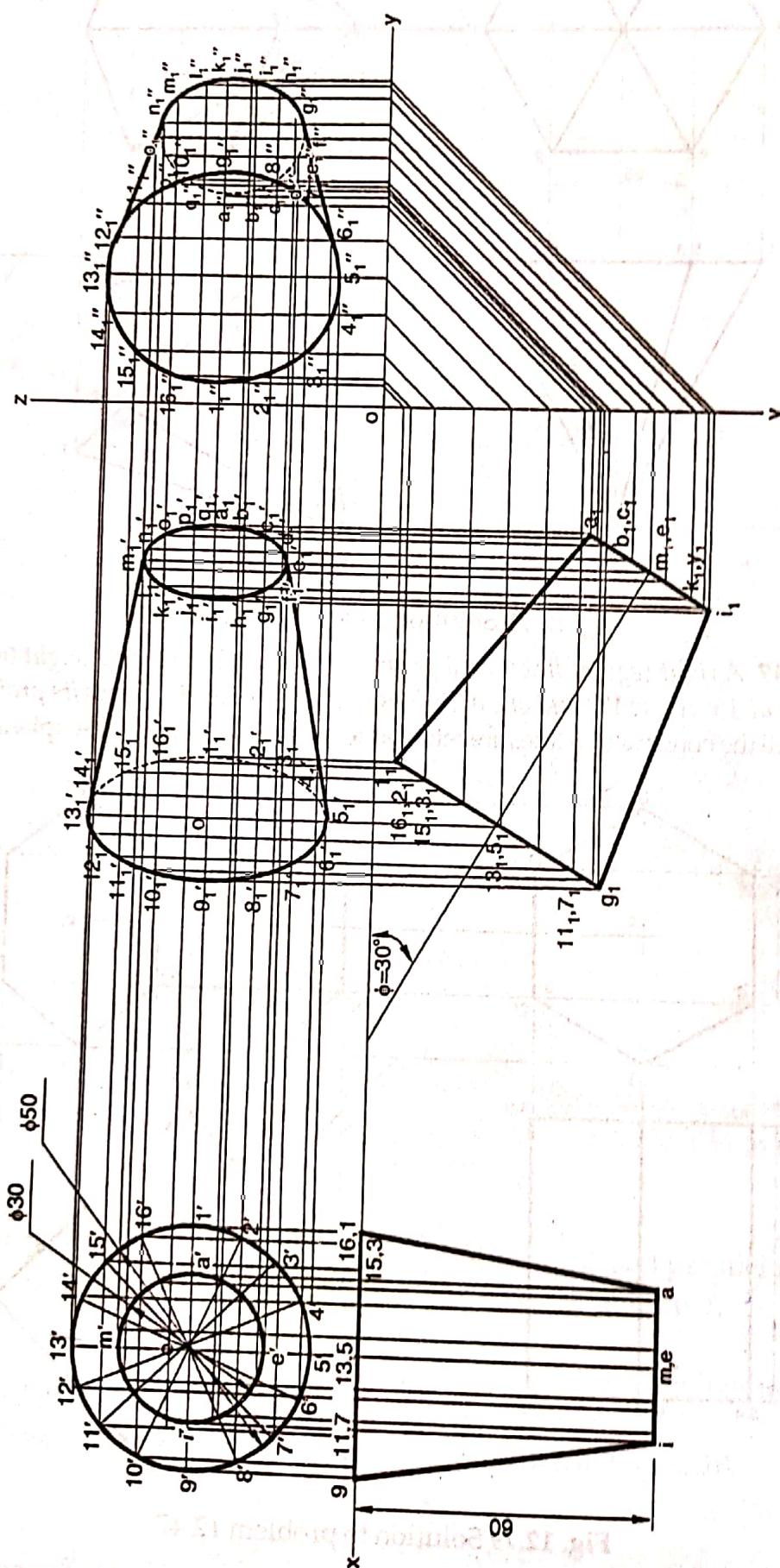


Fig. 12.80 Solution to problem 12.48

PROBLEM 12.49 A square prism, side of base 30 mm and axis 55 mm long is resting on HP on one of its longer edge with a face containing the larger edge is inclined at 30° to the HP with its axis inclined at 30° to the VP. Draw its projections.

SOLUTION. All the construction lines are retained to make the solution self-explanatory. See Fig. 12.81.

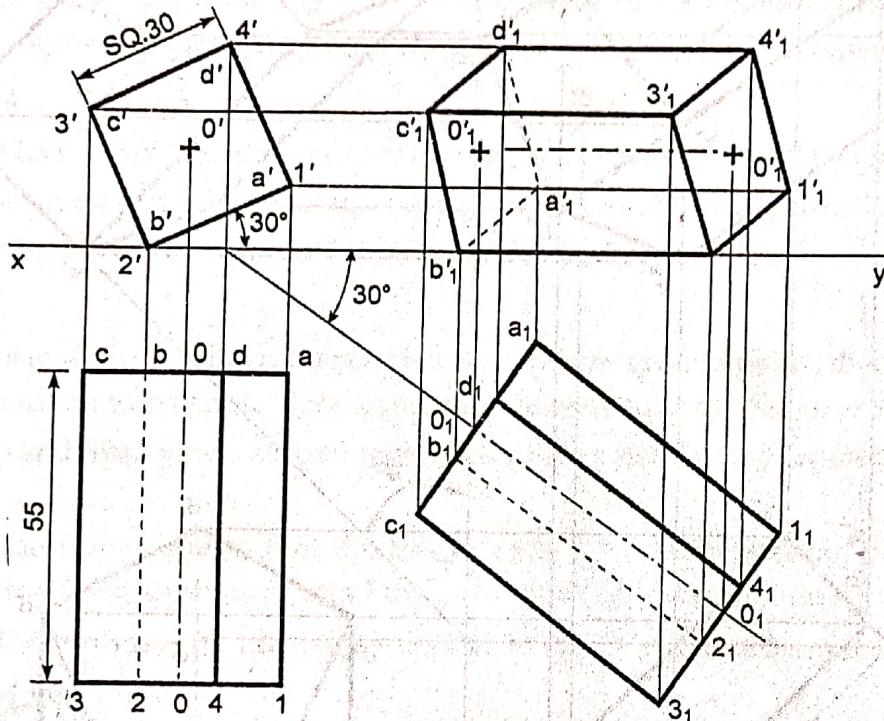


Fig. 12.81 Solution to problem 12.49

12.7 AXES INCLINED TO BOTH HP AND VP

The projections of a solid with its axis inclined to both the principal planes are drawn in three stages :

Stage I : The two views are drawn keeping the solid in its simple position.

Stage II : One of the views is titled at the given angle, keeping the axis inclined to one of the principal planes and parallel to the other. Project the other corresponding view from it.

Stage III : In the final position, angle with the other plane is made by turning the later drawn view of the second stage at the given angle and the other view of the final stage is then projected.

The second and final positions may be drawn by either of the two methods already discussed.

PROBLEM 12.50 A right regular pentagonal prism, side of base 30 mm and height 70 mm, rests on one of its base corners on HP such that its long edge containing the corner is inclined to the HP at 45° and the side of base opposite to the corner is inclined at 45° to the VP. Draw its projections.

SOLUTION.

- In the initial position, assume the prism to be resting on its base on the horizontal plane. Draw its projections, keeping one of the sides of its base perpendicular to xy .

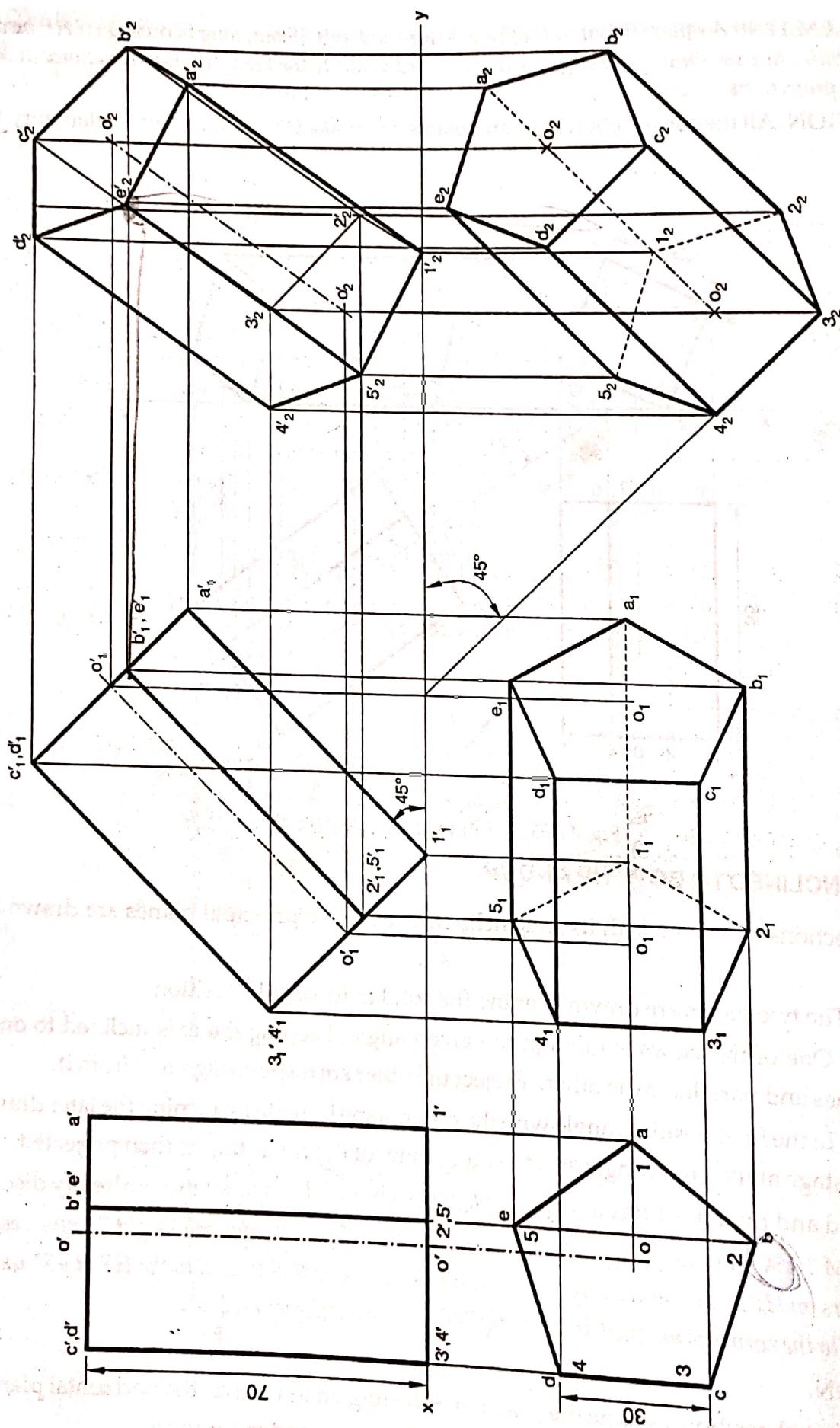


Fig. 12.82 Solution to problem 12.50

- (ii) Redraw the front view so that the longer edge $1'a'$ is inclined at 45° to xy . Project the required top view and add suffix 1 to all the points. Complete the top view, keeping in mind the principles of visibility of faces.
- (iii) Redraw the top view so that the side 4_13_1 (which is still true length) at an angle of 45° to the xy .
- (iv) Project all the points vertically from this top view and horizontally from the second front view. Complete the new front view by observing the rules of visibility of lines as shown in Fig. 12.82.

PROBLEM 12.51 A right regular pentagonal prism, side of base 30 mm and height 55 mm, rests on one of its base corners on ground plane such that its long edge containing that corner is inclined to the ground plane at 45° and the top view of the axis is inclined at 30° to VP. Draw its projections.

SOLUTION.

- (i) As the prism is given to be resting on its base corner in ground plane, therefore the projections will be made in third angle. Draw xy and gl lines, a suitable distance apart.
- (ii) Draw top and front views assuming the axis of the prism to be perpendicular to the ground plane.
- (iii) Redraw the front view so that the longer edge $1_1'a_1'$ is inclined at 45° to gl . Project the required top view, keeping in mind the principles of visibility of lines.
- (iv) Redraw the top view by turning its axis at 30° to xy . Project from it the final front view. See Fig. 12.83.

PROBLEM 12.52 A right circular cone diameter of base 50 mm and height 60 mm lies on one of its elements in HP such that the element is inclined to the VP at 30° . Draw its projections.

SOLUTION.

- (i) In the initial stage, assume the axis to be perpendicular to the HP and parallel to the VP. Draw the top and front views of the cone assuming the base on HP. Divide the base circle in top view into sixteen equal parts and project these points in the front view.
- (ii) Redraw the front view such that the element $0_1'1_1'$ (which is true length) lies on xy . Project from it the corresponding top view, keeping in mind the principles of visibility of lines.
- (iii) Redraw the top view, as the element 0_21_2 (in second stage, which is true length) is inclined at 30° to xy .
- (iv) Project all the points vertically from this top view and horizontally from the second front view. Complete the new front view by observing the rules of visibility of lines as shown in Fig. 12.84.

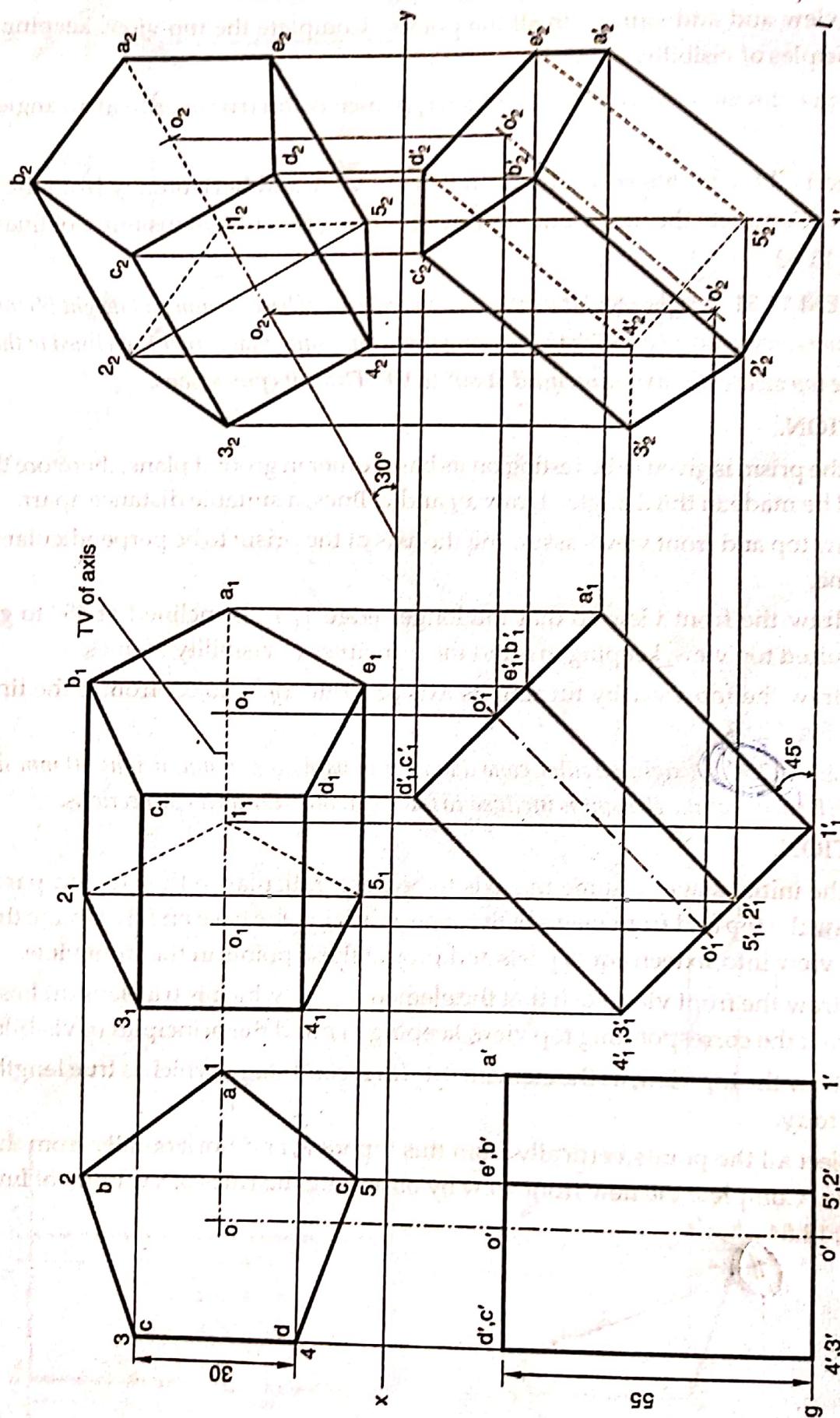


Fig. 12.83 Solution to problem 12.51

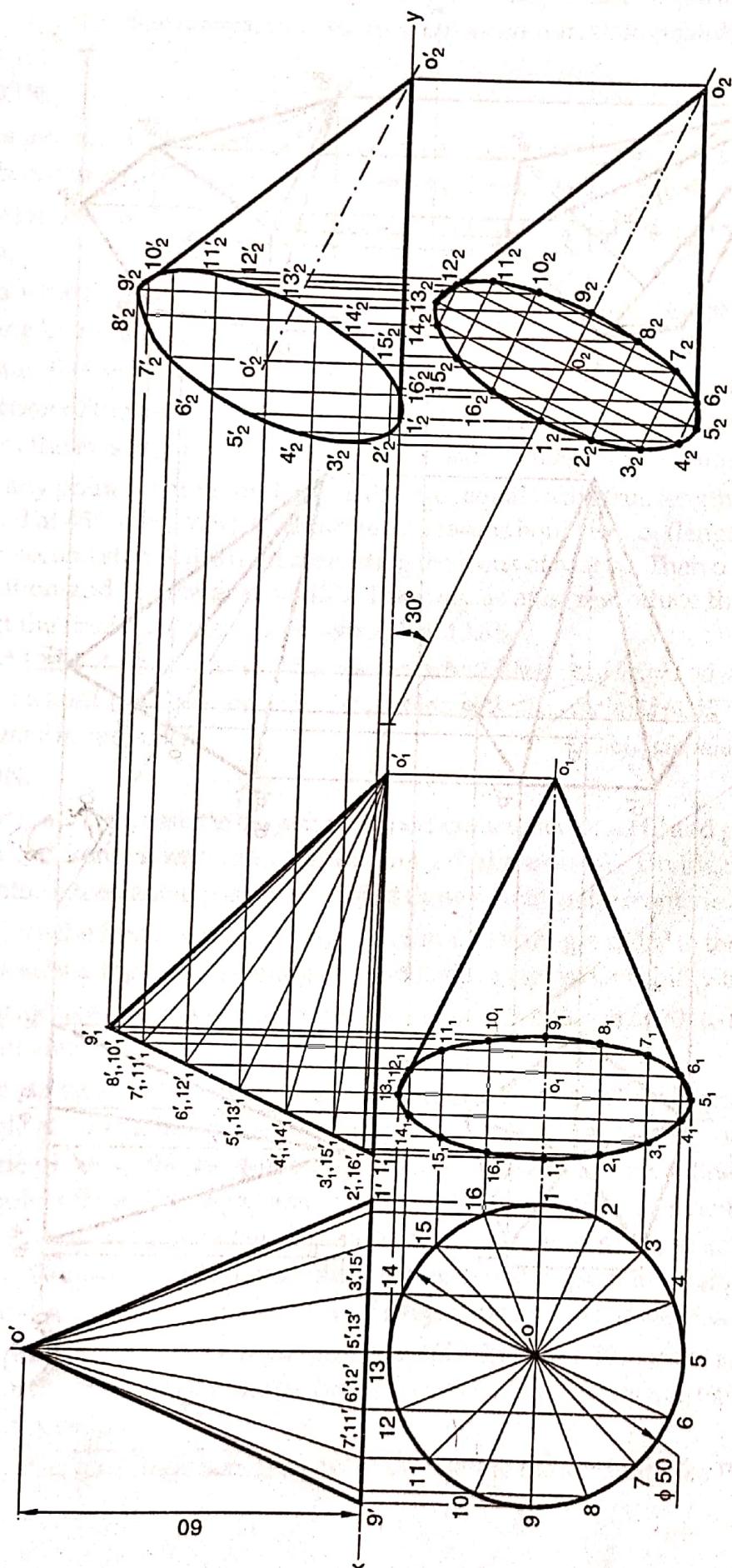


Fig. 12.84 Solution to problem 12.52

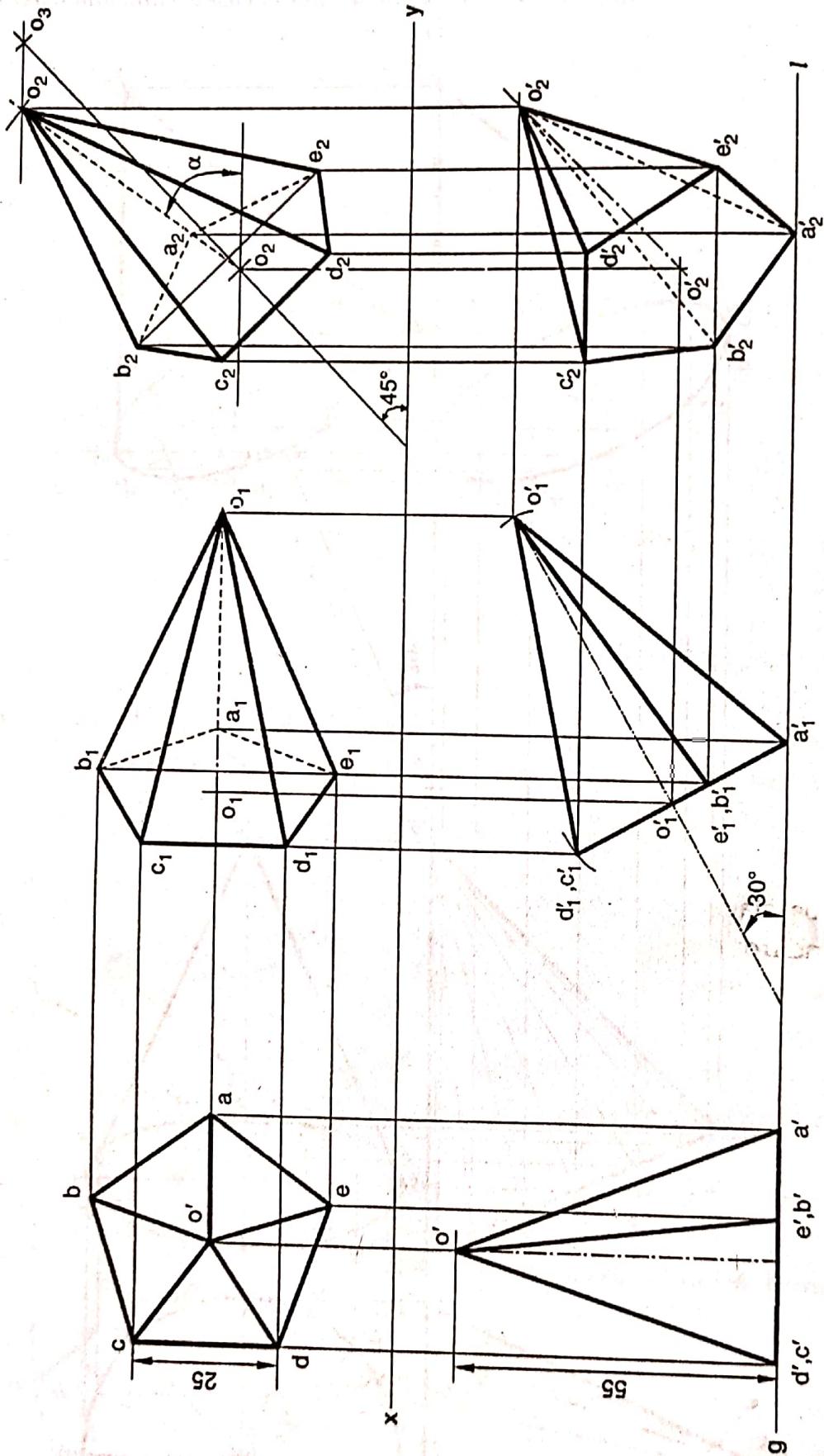


Fig. 12.85 Solution to problem 12.53

PROBLEM 12.53 A right regular pentagonal pyramid, edge of base 25 mm and height 55 mm is held on ground plane on one of its base corners, such that its axis is inclined at 30° to ground plane and 45° to VP. Draw its projections.

SOLUTION.

- (i) As the pyramid is given to be resting on its base corner in ground plane, therefore the projections will be made in third angle. Draw xy and gl lines, a suitable distance apart.
- (ii) Draw top and front views assuming the axis of the pyramid to be perpendicular to the ground plane.
- (iii) Redraw the front view such that its axis is inclined at 30° to gl . Project the required top view, keeping in mind the principles of visibility of lines.
- (iv) Redraw the top view by turning its axis at 45° to xy , but in the second stage top view, the projection of the axis is not true length. So first find out the apparent angle at which the top view of the axis is to be inclined so that the axis should make an angle of 45° to the xy .
- (v) Mark any point o_2 above xy . Draw a line o_2o_3 equal to the true length of the axis i.e. $o_1'o_1'$ and inclined at 45° to xy . With o_2 as centre and radius equal to o_1o_1 (length of the top view of the axis in second stage), draw an arc cutting the locus of o_3 at o_2 . Then α is the apparent angle of inclination and is greater than 45° . Take o_2o_2 as axis, reproduce the second top view and project the final front view as shown in Fig. 12.85.

PROBLEM 12.54 A right circular cone, diameter of base 50 mm and height 60 mm rests on its base rim on HP with its axis inclined at 45° to it such that (a) top view of the axis inclined at 30° to VP (b) axis inclined at 30° to the VP. Draw its projections. (PTU, Jalandhar May 2005, May 2010)

SOLUTION.

- (i) In the initial stage, assume the axis to be perpendicular to the HP and parallel to the VP. Draw the top and front views of the cone assuming the base on HP. Divide the base circle in the top view into sixteen equal parts and project these points in the front view.
- (ii) Reproduce the front view such that its axis makes an angle of 45° to the xy . Project from it the corresponding top view, keeping in mind the principles of visibility of lines.
- (iii) (a) Reproduce the top view of the axis such that it is inclined at 30° to the xy . Project the final front view.
 (b) The top view of the axis in the second stage does not give true length. So first the apparent angle at which the top view of the axis is to be inclined so that the axis should make an angle of 30° to the xy . Mark any point p_3 below xy . Draw a line o_3o_4 equal to the true length of the axis i.e. $o_1'o_1'$ and inclined at 30° to xy . With o_3 as centre and radius equal to o_1o_1 (length of the top view of the axis in second stage), draw an arc cutting the locus of o_4 at o_3 . Then is the apparent angle of inclination and is greater than 30° . Take o_3o_3 as axis, reproduce the second top view and project the final front view as shown in Fig. 12.86.

PROBLEM 12.55 A right regular pentagonal pyramid, edge of base 30 mm and height 50 mm rests on one of its base corners on HP and the base is inclined at 25° to the HP. The side opposite the corner is inclined at 30° to the VP. Draw its projections.

SOLUTION. The interpretation of the solution is left to the student. See Fig. 12.87.

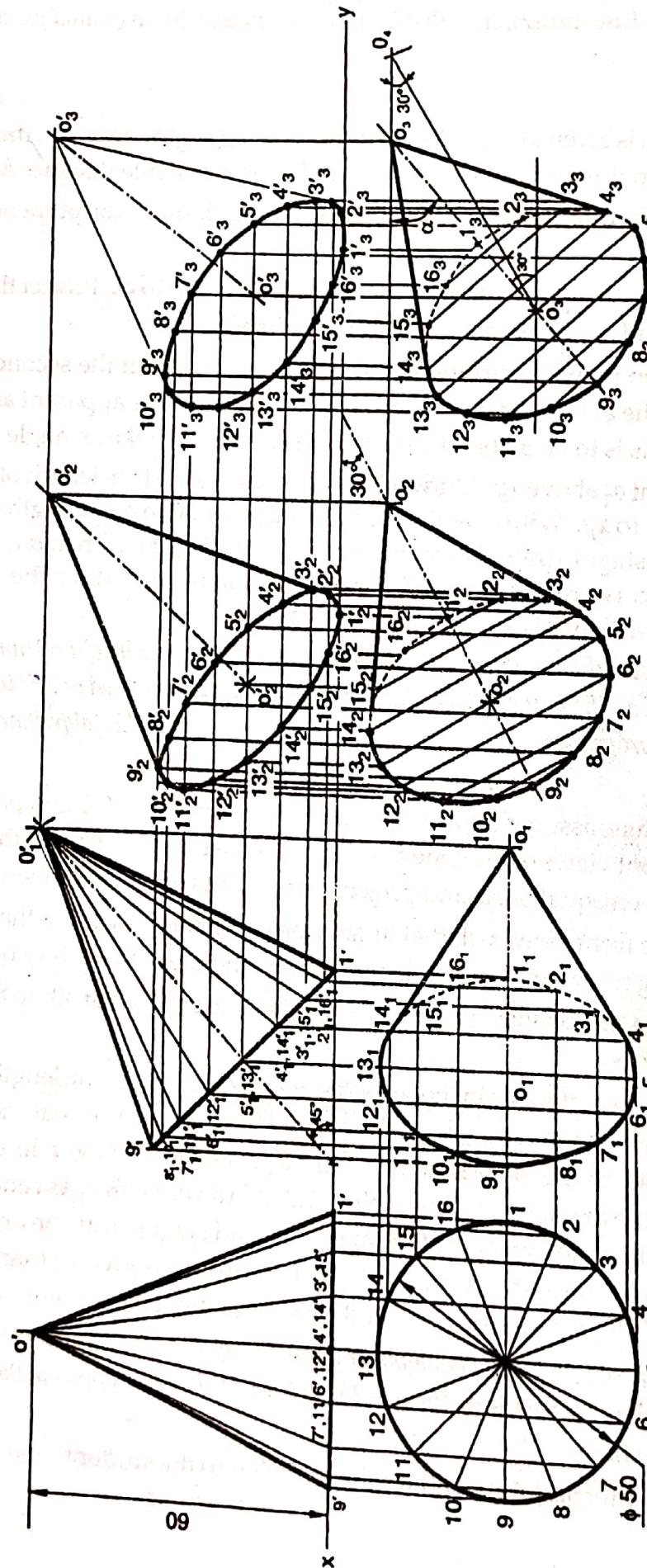


Fig. 12.86 Solution to problem 12.54

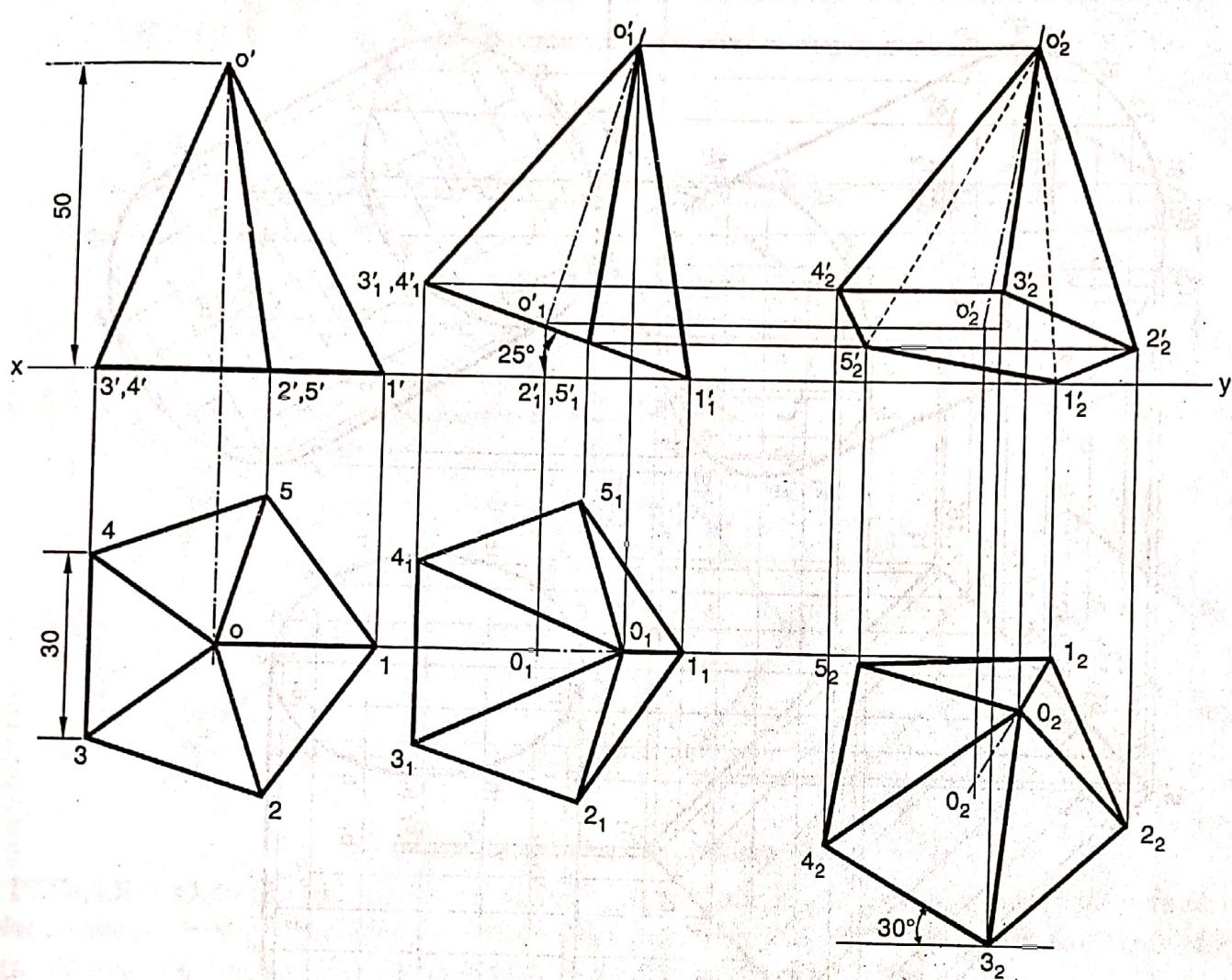


Fig. 12.87 Solution to problem 12.55

PROBLEM 12.55 A right circular cylinder, diameter of base 50 mm and height 65 mm rests on HP on its base rim such that its axis is inclined at 45° to the HP and the top view of the axis is inclined at 60° to the VP. Draw its projections.

SOLUTION. The interpretation of the solution is left to the student. See Fig. 12.88.

PROBLEM 12.57 A cube of 30 mm side, rests on HP on one of its corners with a body diagonal perpendicular to the VP. Draw its projections.

SOLUTION.

- In the initial position, assume the cube to be resting on one of its faces on the HP with a body diagonal parallel to the VP. Draw a square 1234 in the top view with its sides inclined at 45° to xy . Project the front view too.
- Tilt the front view about the corner a' so that the line $c'1'$ becomes parallel to xy . Project the corresponding top view. The body diagonal $c'1'$ is now parallel to both HP and VP. Name the points on it by adding suffix 1 to them.
- Reproduce the second top view such that the top view of the body diagonal i.e. c_{11_1} is perpendicular to xy and add suffix 2 to all the points.

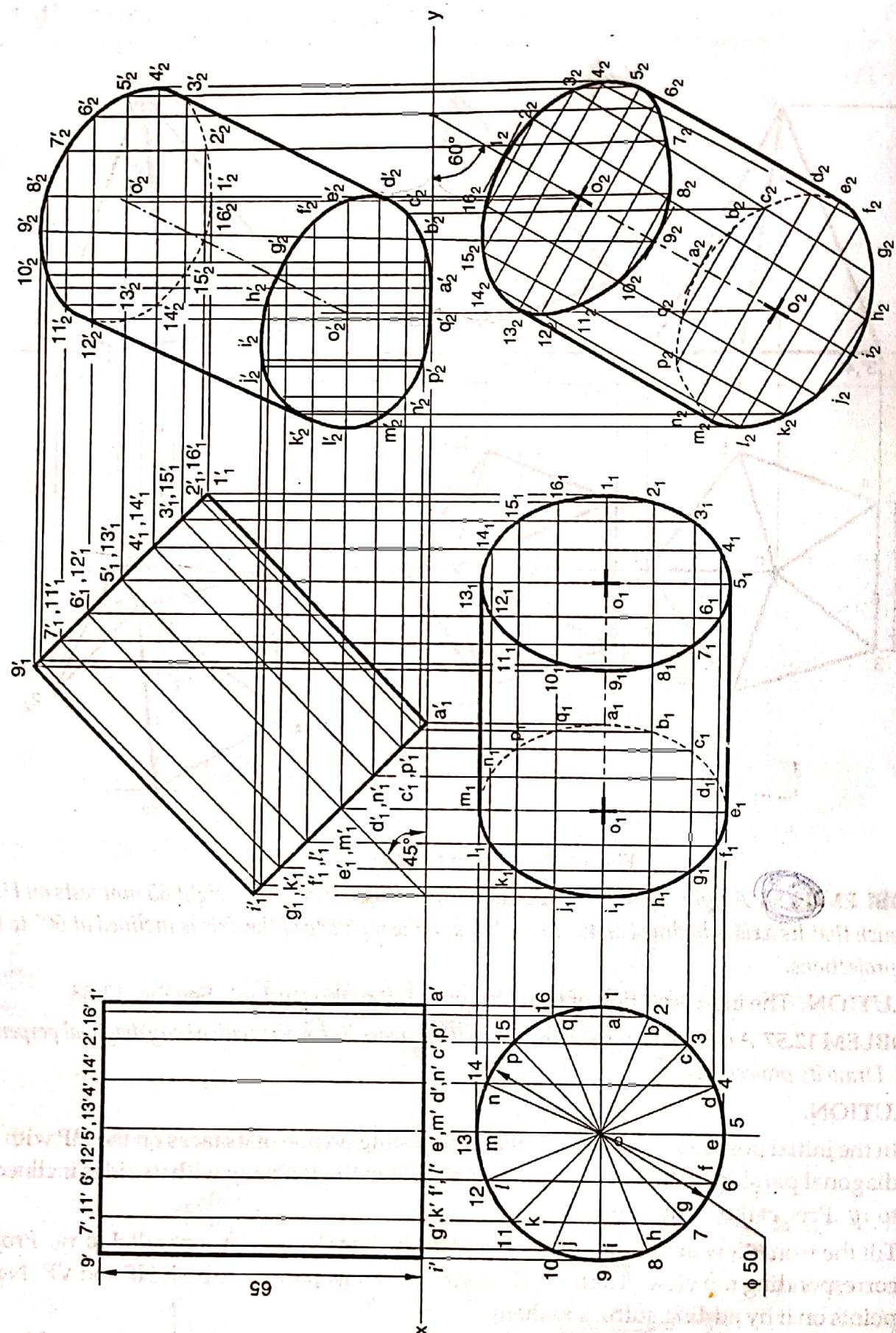


Fig. 12.88 Solution to problem 12.56

- (iv) Project all the points vertically from this top view and horizontally from the second front view. Complete the new front view by observing the rules of visibility of lines or surfaces as shown in Fig. 12.89.

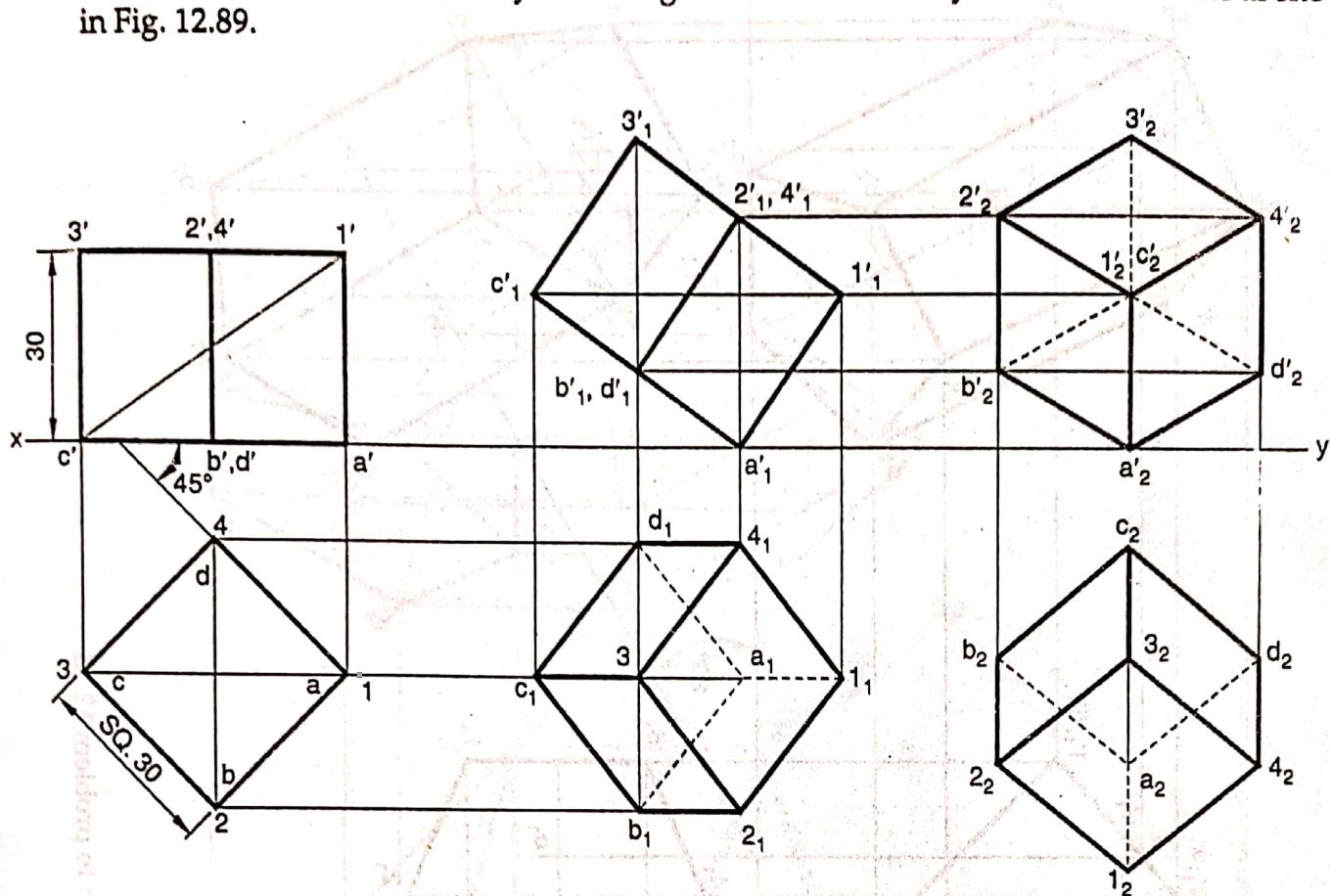


Fig. 12.89 Solution to problem 12.57

PROBLEM 12.58 A right regular pentagonal prism 70 mm height with each side of the base 30 mm is resting on one of the base edges on the horizontal plane and inclined at 30° to VP and the face containing that edge is inclined at 45° to the HP. Draw the projections of the pentagonal prism.

(PTU, Jalandhar December 2004)

SOLUTION.

- In the initial position, draw the pentagon in top view with one of its base sides (ae), perpendicular to xy line. Project the front view from it.
- Tilt the front view about the base side ae i.e. $a'e'1'5'$ (face) at an angle of 45° to the HP. Project the corresponding top view observing the rules of visibility of surfaces. Name all the points on it by adding suffix 1 to them.
- Reproduce the second top view about the side a_1e_1 (True length) at an angle of 30° to the VP i.e. xy line and add suffix 2 to all the points.
- Project all the points vertically from this top view and horizontally from the second front view. Complete the new front view as shown in Fig. 12.90.

PROBLEM 12.59 A right regular pentagonal pyramid of base 30 mm sides and height 60 mm rests on one of its slant edges on HP. The plan of the axis is inclined at 30° to VP. Its apex is nearer to VP. Draw the projections of the pyramid.

(PTU, Jalandhar May 2001)

SOLUTION. The solution to the problem is left to the reader. All construction lines are retained to easy to understand the solution. See Fig. 12.91.

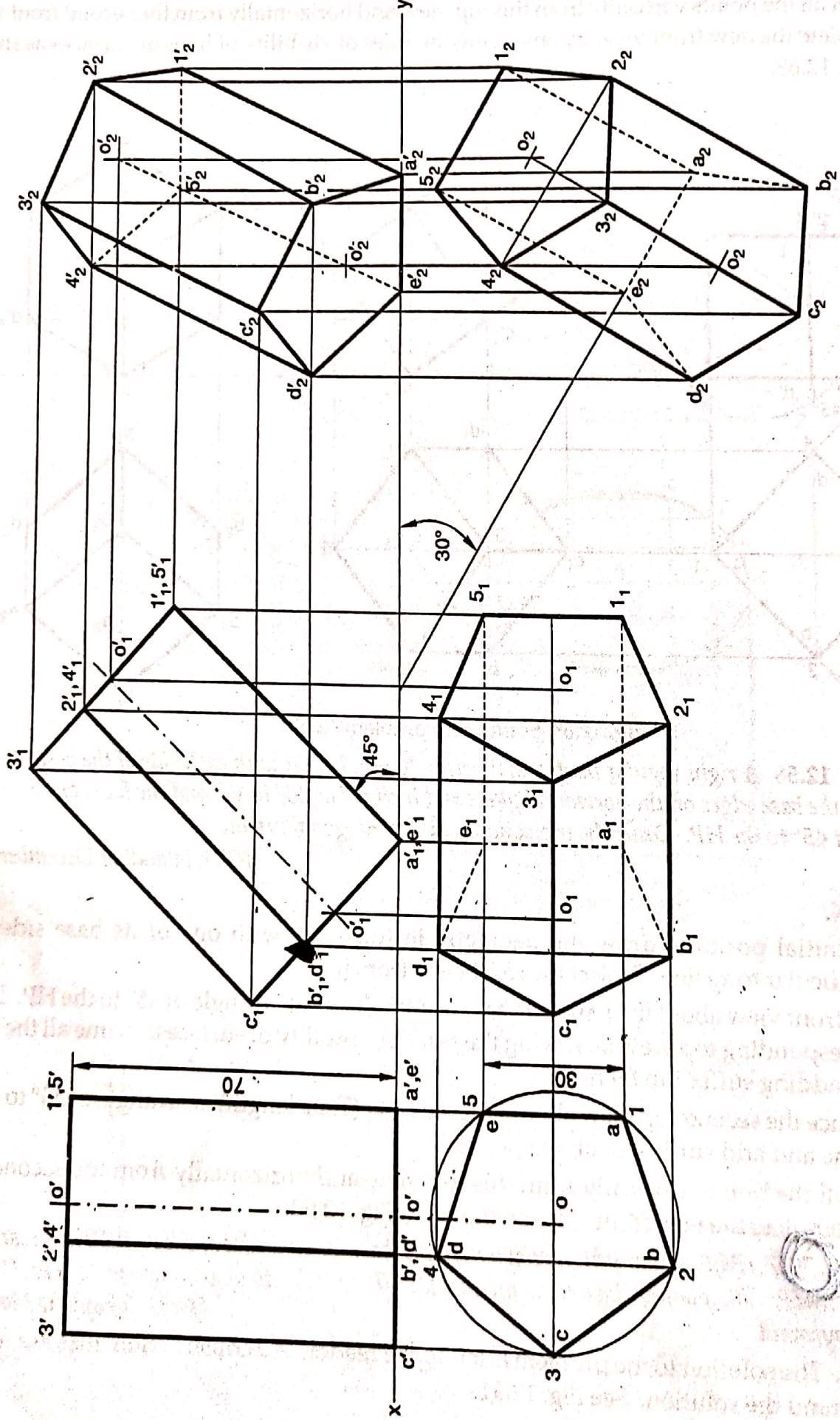


Fig. 12.90 Solution to problem 12.58

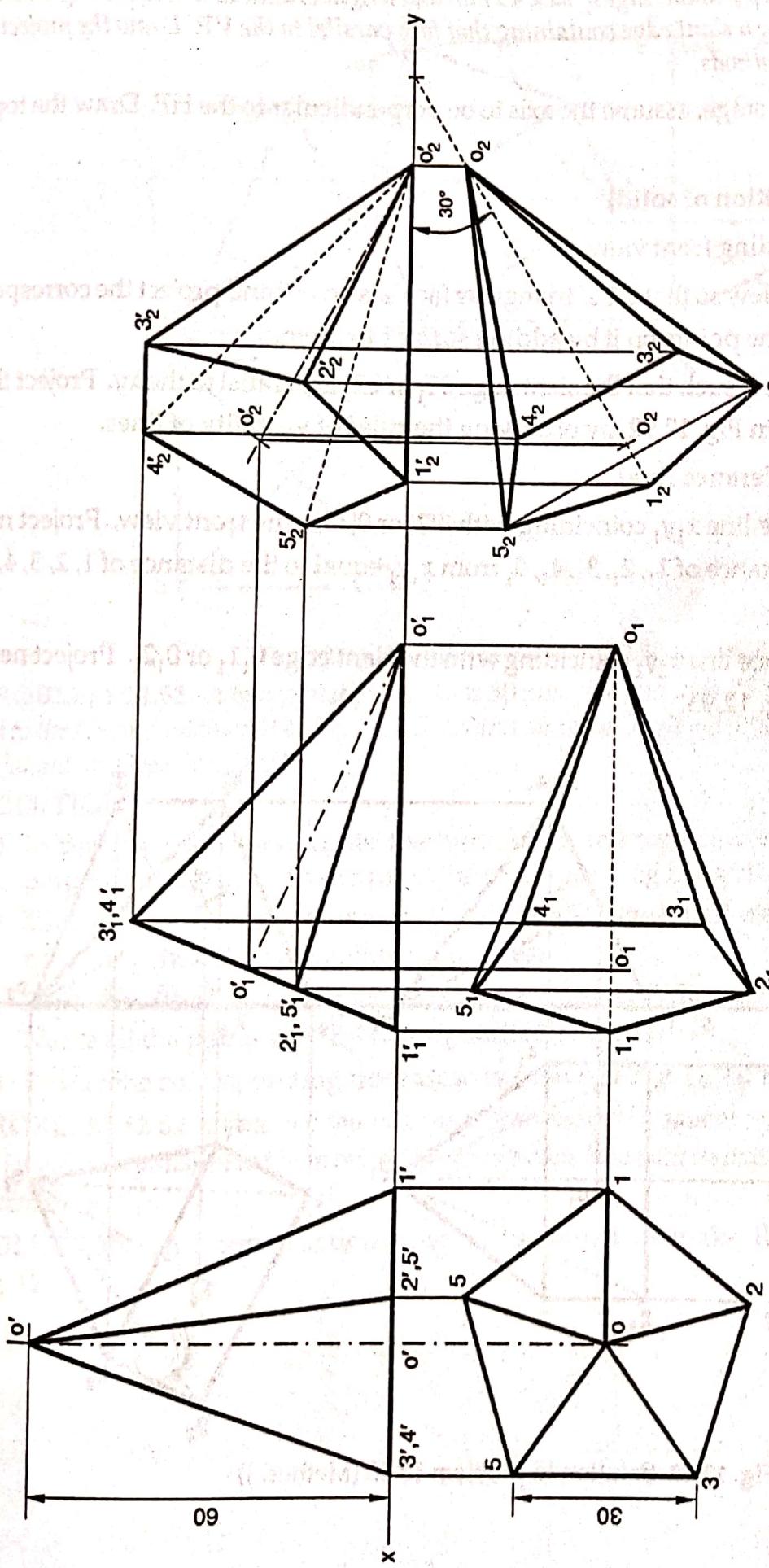


Fig. 12.91 Solution to problem 12.59

PROBLEM 12.60 A square pyramid, edge of base 45 mm and length of axis 45 mm is resting on one of its triangular faces on the HP having a slant edge containing that face parallel to the VP. Draw the projections of the pyramid by using both the methods.

SOLUTION. In the initial stage, assume the axis to be perpendicular to the HP. Draw the top view and label it.

Method I (Change of position of solid)

- Project the corresponding front view.
- Reproduce the front view so that $0'1'2'$ triangular face lies on HP and project the corresponding top view. Name all the points on it by adding suffix 1 to them.
- Reproduce the top view such that the slant edge 01_1 or 02_2 is parallel to the xy . Project the new front view as shown in Fig. 12.92, by observing the rules of visibility of lines.

Method II (Change of reference line)

- Draw a new reference line x_1y_1 coinciding with $0'2'$ or $0'1'$ in the front view. Project new top view, keeping the distance of $1_1, 2_1, 3_1, 4_1, 0_1$ from x_1y_1 equal to the distance of $1, 2, 3, 4, 0$ from xy .
- Draw another reference line x_2y_2 coinciding with the slant edge 0_11_1 or 0_12_1 . Project new front view as shown in Fig. 12.93.

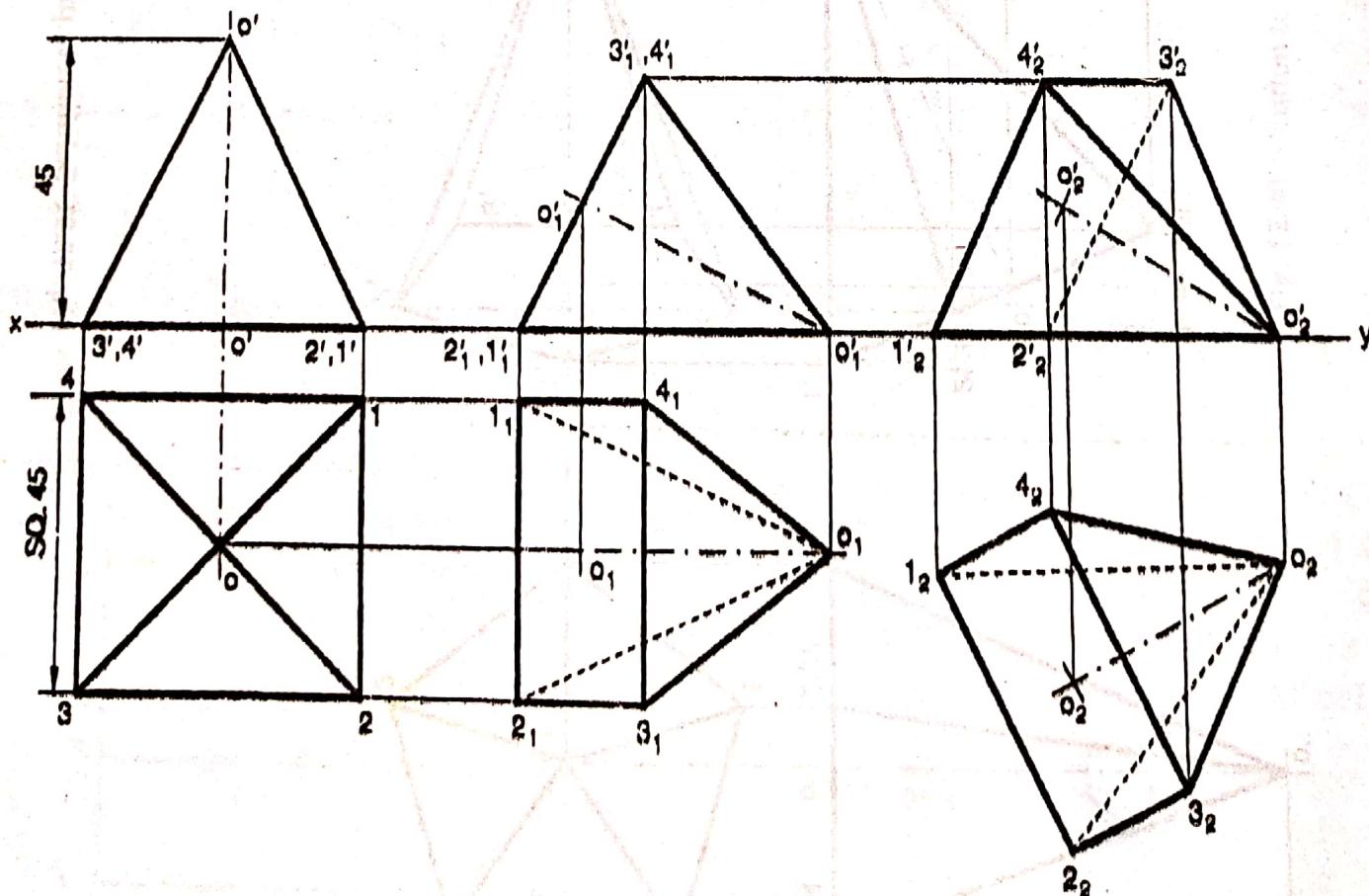


Fig. 12.92 Solution to problem 12.60 (Method I)

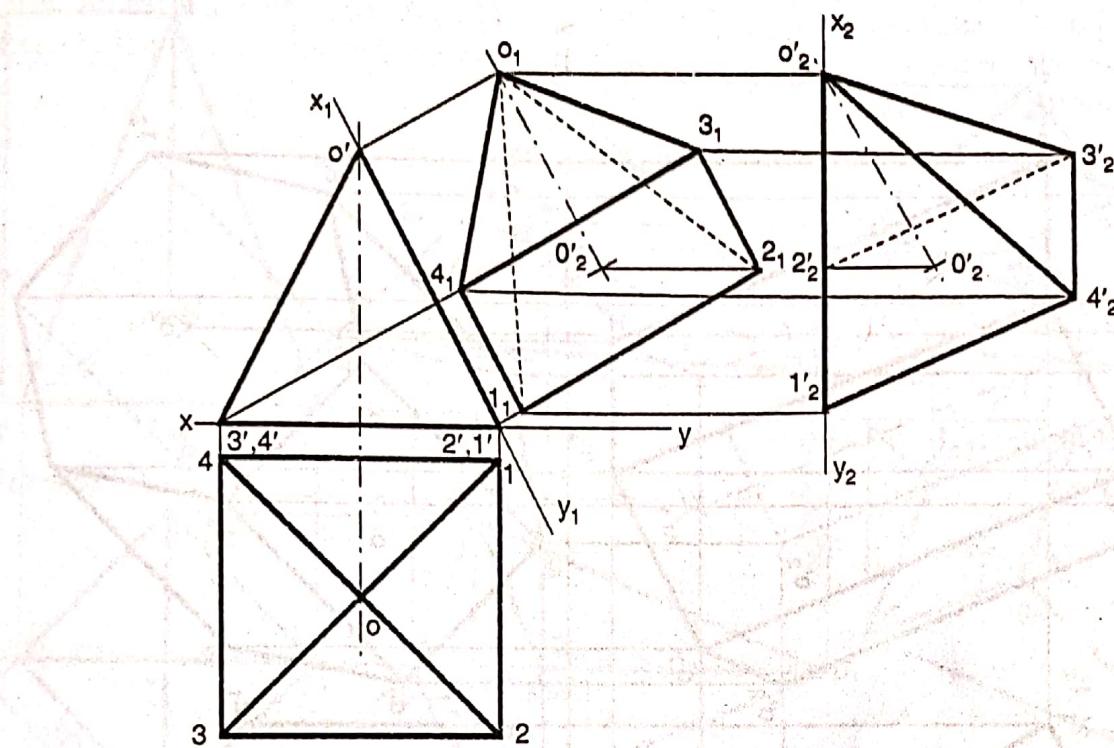


Fig. 12.93 Solution to problem 12.60 (Method II)

PROBLEM 12.61 A hexagonal prism, base 30 mm side and axis 75 mm long, has an edge of the base parallel to the HP and inclined at 45° to the VP. Its axis makes an angle of 60° with the HP. Draw the projections. (PTU, Jalandhar December 2002)

SOLUTION.

- In the initial position, draw the hexagon in the top view with one of its base sides (ab) perpendicular to xy . Then project the corresponding front view from it.
- Tilt the front view about the axis at angle of 60° to the HP. Project the corresponding top view, observing the rules of visibility of surfaces.
- Reproduce the second top view about the side a_1b_1 (True length) at an angle of 45° to the VP. Name all the points on it by adding suffix 2 to them.
- Project the corresponding front view as shown in Fig. 12.94.

PROBLEM 12.62 Draw the top view and front view of a square pyramid, side of base 30 mm, axis 55 mm long, is freely suspended from one of the corners of its base with its axis in a vertical plane makes an angle of 45° with the VP. (PTU, Jalandhar December 2004)

SOLUTION. All construction lines are retained to make the solution self-explanatory. See Fig. 12.95.

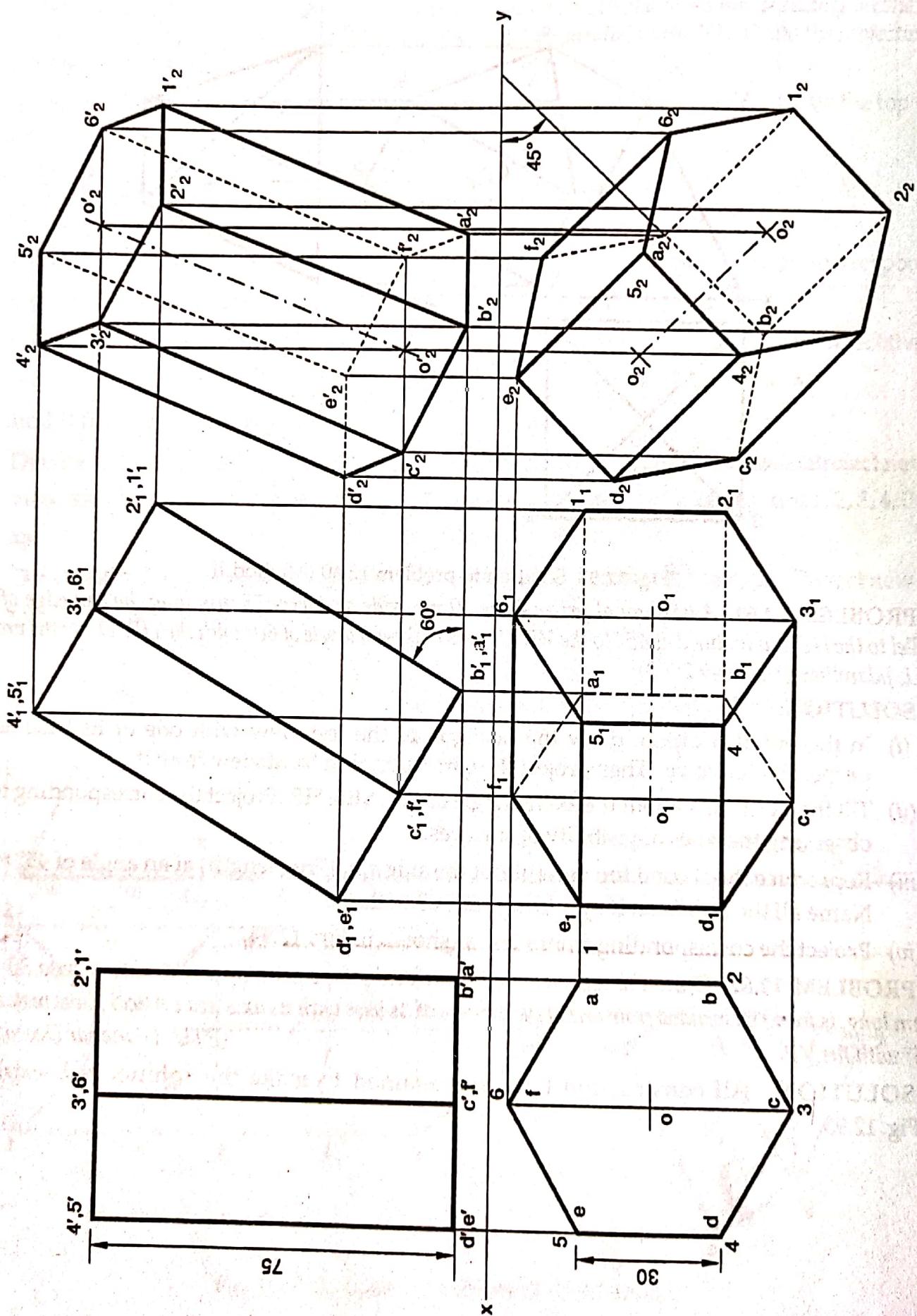


Fig. 12.94 Solution to problem 12.61

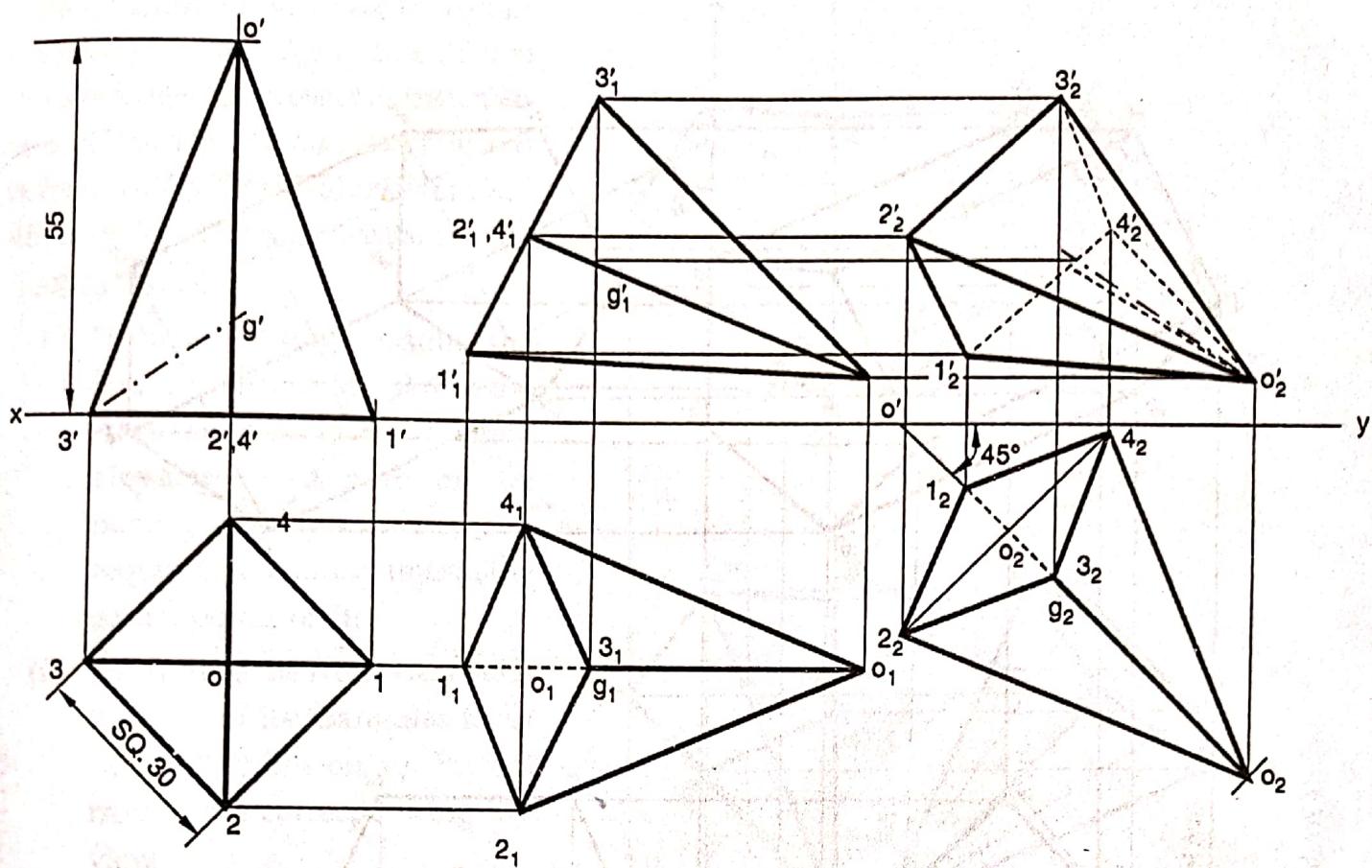


Fig. 12.95 Solution to problem 12.62

PROBLEM 12.63 A right regular pentagonal prism, side of base 25 mm and axis 60 mm long is resting on one of its base edges in the HP with its axis inclined at 45° to the HP and the top view of the axis is inclined at 60° to the VP. Draw its projections.

SOLUTION.

- In the initial position, draw a pentagon in the top view with one of its base edges perpendicular to xy . Then project the corresponding front view from it. Name all the corner points on it.
- Tilt the front view about the axis at angle of 45° to the HP. Project the corresponding top view, observing the rules of visibility of surfaces. Name all the points on it by adding suffix 1 to them.
- Reproduce the top view such that the top view of the axis is inclined at 60° to the VP. Project the corresponding front view. See Fig. 12.96.

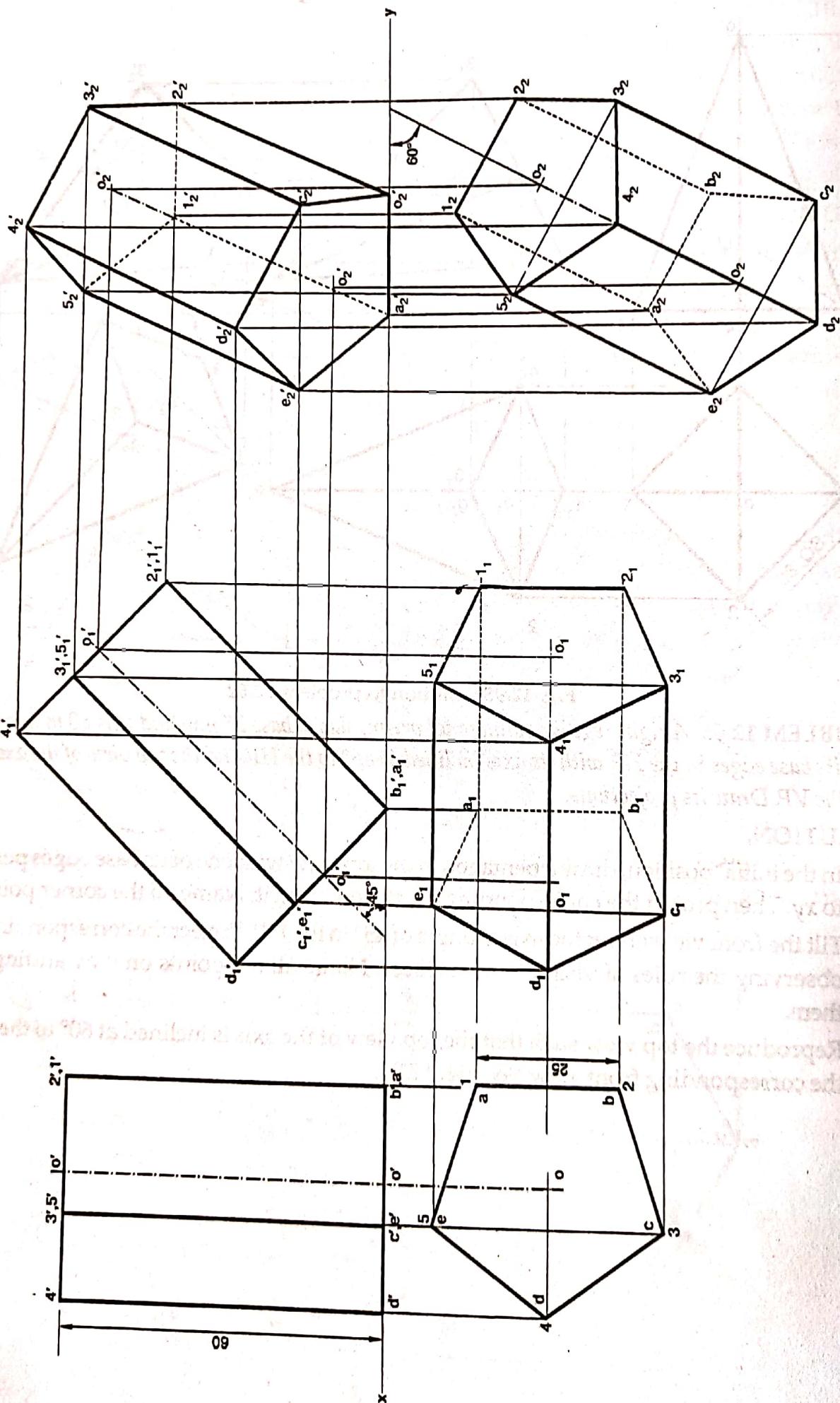


Fig. 12.96 Solution to problem 12.63

PROBLEM 12.64 A right regular pentagonal pyramid, edge of base 25 mm and height 50 mm lies on one of its triangular faces in HP such that (a) top view of the axis is inclined at 45° to the VP (b) axis is inclined at 45° to the VP. Draw its projections.

SOLUTION.

- In the initial stage, assume the axis to be perpendicular to the HP and parallel to the VP. Draw elevation and plan of the pentagonal pyramid in the required position. Name all the corner points on it.
- Reproduce the front view such that one of its triangular faces say (0'3'4') lies on xy . Project from it the corresponding top view.
- (a) Reproduce the top view, such that the top view of the axis is inclined at 45° to the VP. Project the final front view.
(b) The top view of the axis in the second stage does not give true length. So, firstly find out the true angle of inclination with xy . Complete the projections as shown in Fig. 12.97.

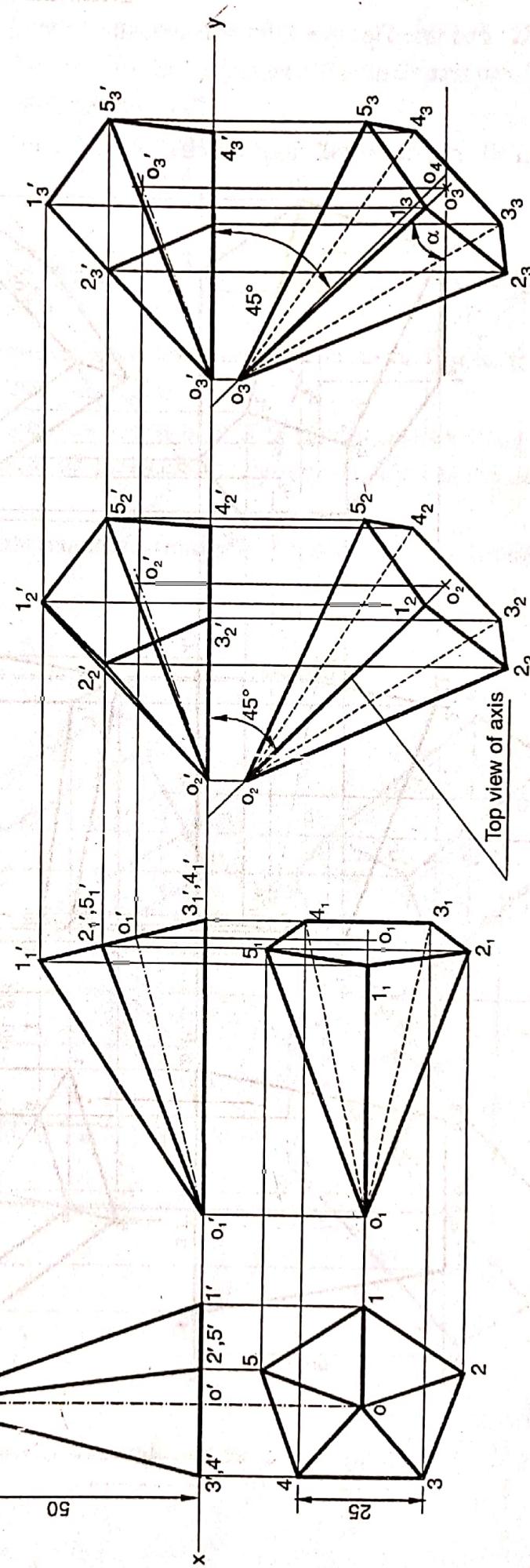


Fig. 12.97 Solution to problem 12.64

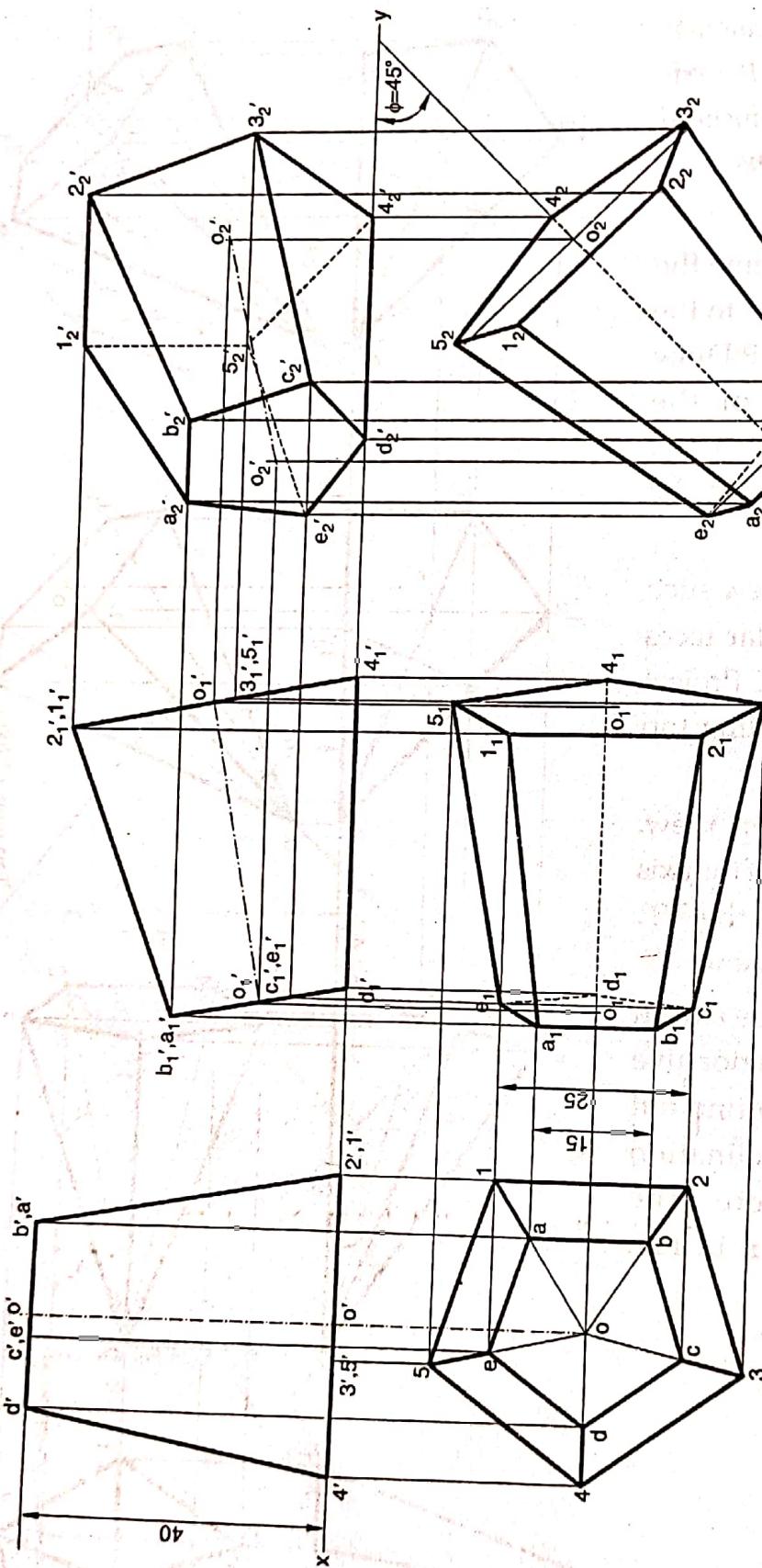


Fig. 12.98 Solution to problem 12.65

PROBLEM 12.65 A frustum of a right regular pentagonal pyramid, edge of lower base 25 mm, edge of upper base 15 mm and axis 40 mm long is lying on one of its slant edges on HP with its axis parallel to VP. The top view of axis is inclined at 45° to the VP. Draw its projections.

SOLUTION. (i) In the initial stage, assume the axis to be perpendicular to the HP. Draw the top view and label it.

(ii) Project the corresponding front view.

(iii) Reproduce the front view so that slant edge ($d' 4'$) lies on HP and project the corresponding top view. Name all the points on it by adding suffix 1 to them.

(iv) Reproduce the top view such that axis ($o_1 o_1$) makes an angle of 45° to xy . Project the final front view by observing the rules of visibility of lines. See Fig. 12.98.

PROBLEM 12.66 A right regular hexagonal prism, side of base 25 mm and axis 60 mm long is resting on one its base corner in the HP with its axis inclined at 30° to the HP and the top view of the axis is inclined at 45° to the VP. Draw its projections.

SOLUTION. All the construction lines are retained to make the solution self-explanatory. See Fig. 12.99.

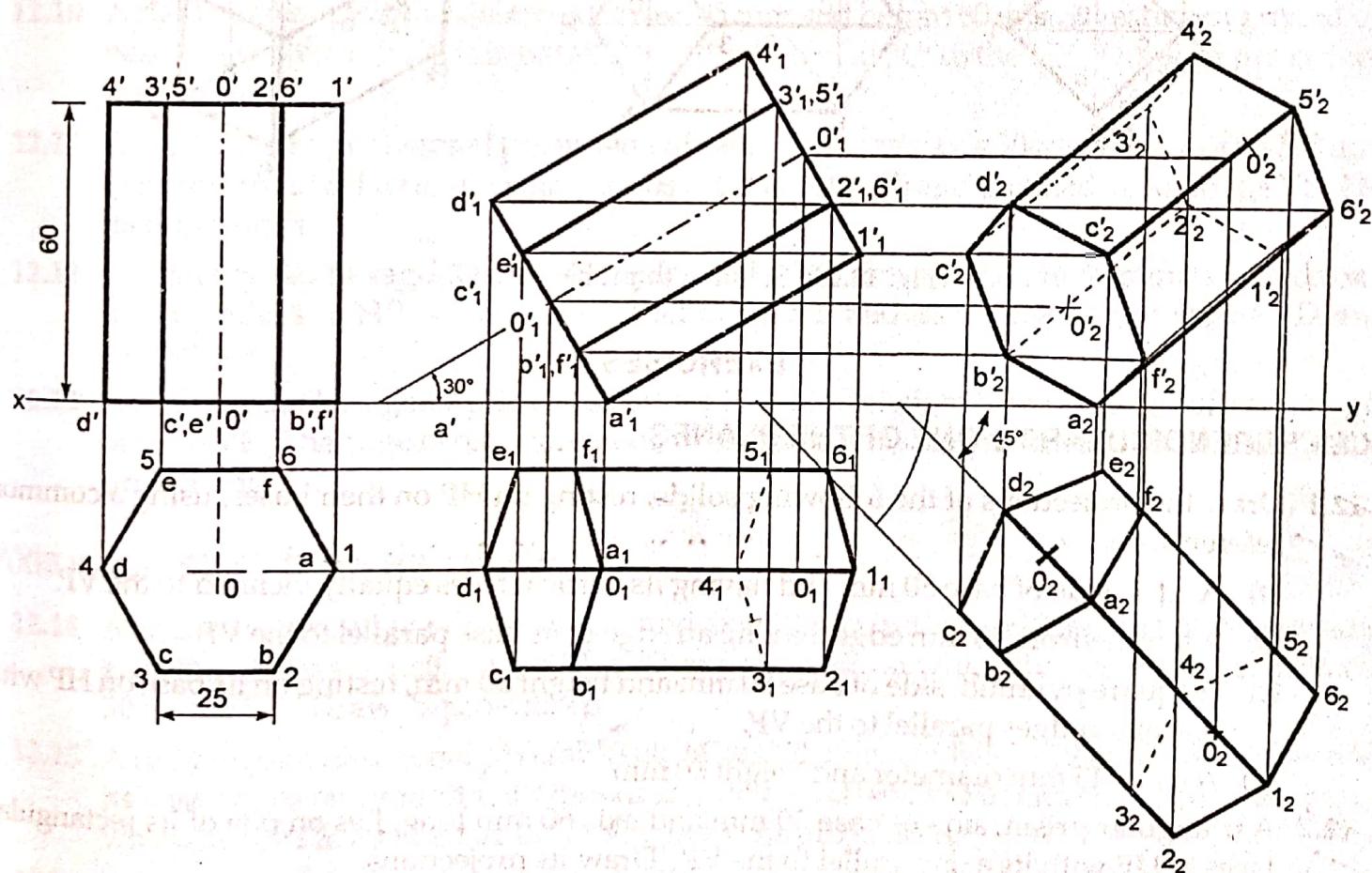


Fig. 12.66 Solution to problem 12.99

PROBLEM 12.67. A square prism, side of base 30 mm and axis 70 mm long is resting on a when of its base in HP with its axis inclined at 45° to the HP. The top view of the axis is inclined at 30° to the VP. Draw its projections.

SOLUTION. All the construction lines are retained to make the solution self-explanatory. See Fig. 12.100

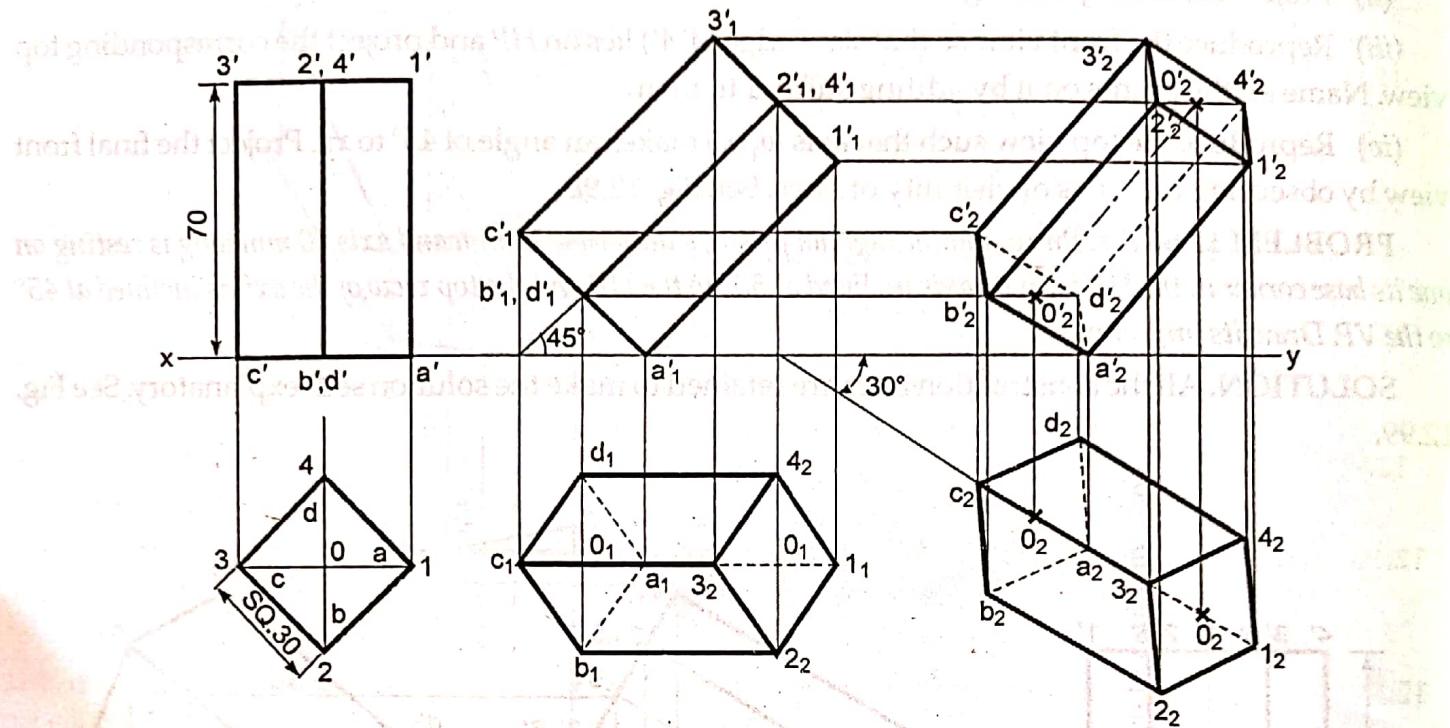


Fig. 12.100 Solution to problem 12.67

EXERCISES

AXIS PERPENDICULAR TO ONE OF THE PLANES

- 12.1 Draw the projections of the following solids, resting on HP on their bases, using a common reference line.
 - (a) A cube, side of base 30 mm and having its vertical faces equally inclined to the VP.
 - (b) A tetrahedron, 45 mm edge, having an edge of its base parallel to the VP.
 - (c) A square pyramid, side of base 30 mm and height 50 mm, resting on its base on HP with one of its edges parallel to the VP.
 - (d) A cone, 45 mm diameter and height 60 mm.
- 12.2 A triangular prism, side of base 30 mm and axis 60 mm long, lies on one of its rectangular faces in HP with its axis parallel to the VP. Draw its projections.
- 12.3 A right regular hexagonal prism, edge of base 30 mm and length 50 mm, lies on one of its rectangular faces on ground, such that its axis is parallel to both HP and VP. Draw its projections.

AXIS INCLINED TO ONE OF THE PLANES

- 12.4 A right regular pentagonal prism, edge of base 25 mm and height 50 mm, rests on an edge of its base in HP such that its axis is parallel to VP and one of its rectangular faces is perpendicular to VP and inclined to HP at 45° . Draw its projections.
- 12.5 Draw the projections of a cube of 40 mm edge, when a body diagonal of the solid is kept vertical.
- 12.6 A right regular pentagonal pyramid, side of base 20 mm and height 40 mm, rests on a corner of its base on HP, such that its axis is inclined at 45° to the HP and is parallel to the VP. Draw its projections by using both the methods.
- 12.7 A right regular hexagonal pyramid, edge of base 25 mm and height 50 mm, rests on one of its base edges in HP with its axis parallel to VP. Draw its projections when the base makes an angle of 45° to the HP.
- 12.8 A square pyramid, edge of base 30 mm and axis 50 mm long, rests on HP on one of its base corners, such that the slant edge containing that corner is perpendicular to the HP. Draw its projections.
- 12.9 A right circular cone diameter of base 45 mm and height 50 mm, rests on ground on its base rim such that its axis is parallel to the VP and inclined to the HP at 45° . Draw its projection.
- 12.10 A right circular cylinder diameter of base 45 mm and height 60 mm, is resting in ground on its base rim such that its axis is parallel to VP, inclined at 30° to the HP. Draw its projections in third angle.
- 12.11 A right regular pentagonal prism, side of base 25 mm and axis 50 mm long, lies on HP on one of its rectangular faces, such that its axis is parallel to HP and inclined at 30° to the VP. Draw its projections.
- 12.12 A right regular hexagonal pyramid, edge of base 25 mm and height 50 mm, is held on one of its base edges in HP. Its axis is parallel to the HP and inclined at 30° to the VP. Draw its projections.
- 12.13 A right regular hexagonal prism, side of base 30 mm and height 60 mm, has one of its rectangular faces in VP. The edge of base contained by that face is making an angle of 30° to HP. Draw its projections.

AXIS INCLINED TO BOTH HP AND VP

- 12.14 A square prism, edge of base 30 mm and axis 60 mm, rests on HP, on one of its base edges, such that its axis is inclined to HP at 45° and the edge on which the prism rests is inclined at 30° to the VP. Draw its projections.
- 12.15 A right regular hexagonal pyramid, side of base 25 mm and height 50 mm is resting on one of its base edges on ground with its axis inclined at 30° to the HP and the edge of the base on which its rests is inclined at 45° to VP. Draw its projections.
- 12.16 A tetrahedron of 40 mm edge, rests on HP on one of its edges such that the face containing that edge is inclined at 30° to HP and the edge is inclined at 45° to VP. Draw its projections.
- 12.17 A cube of 30 mm side, rests on HP on one of its corners with a body diagonal perpendicular to the VP. Draw its projections.
- 12.18 A right regular hexagonal prism, edge of base 30 mm and axis 70 mm long is held so that an edge of its base parallel to HP and inclined at 45° to VP, while its axis makes an angle of 60°

to the HP. Draw its projections.

- 12.19 A right regular hexagonal prism, edge of base 30 mm and axis 65 mm long rests on one of its base corners on HP with its axis inclined at 45° to HP and the top view of the axis inclined at 30° to the VP. Draw its projections.
- 12.20 A right regular hexagonal pyramid, edge of base 25 mm and axis 50 mm long, has one of its triangular faces in VP and the edge of base contained by that face is inclined at 30° to HP. Draw its projections.
- 12.21 A right regular pentagonal pyramid, edge of base 30 mm and height 60 mm, is held on ground plane on one of its base corners such that the slant edge containing the corner is inclined to HP at 45° and the base edge opposite the corner is inclined at 30° to VP. Draw its projections.
- 12.22 A square pyramid, whose faces are isosceles triangles of 45 mm base and 65 mm altitude, is lying on one of its triangular faces in HP. The projection of the axis of the pyramid on the HP makes an angle of 30° with the xy line. Draw its projections.
- 12.23 A tetrahedron of 30 mm edge and resting on one of its base corners on the HP has one of its edges perpendicular to the HP. The edge opposite to this in the view from above makes 45° with the VP. Draw the projections.
- 12.24 A right regular pentagonal prism, side of base 25 mm and axis 60 mm long is resting on one of its base edges in the HP with its axis inclined at 45° to the HP and the top view of the axis is inclined at 60° to the VP. Draw its three views.
- 12.25 A hexagonal prism, base 30 mm side and axis 75 mm long, has an edge of the base parallel to the HP and inclined at 60° to the VP. Its axis makes an angle of 45° with the HP. Draw the projections.

OBJECTIVE QUESTIONS

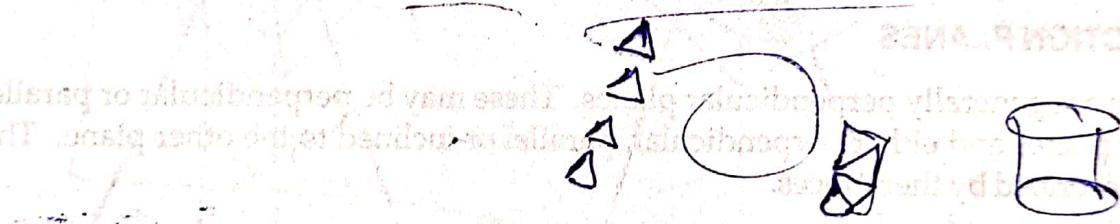
- 12.1 To represent a solid in orthographic projections, at least views are required.
- 12.2 A solid having four equal equilateral triangular faces is called Tetrahedron.
- 12.3 When the axis of a solid is parallel to both HP and VP, shows the true shape of the base.
- 12.4 If a solid rests on its base on HP, its axis must be kept to HP.
- 12.5 Differentiate between right and oblique solids ?
- 12.6 Differentiate between prism and pyramid ?
- 12.7 In the projections of a solid, when two lines representing the edges cross each other, one of them must be auxiliary.
- 12.8 An object having dimensions is called a solid.
- 12.9 An oblique solid is one, which has its axis to its base.
- 12.10 If a solid rests on an edge of its base on VP, it must be kept to VP.
- 12.11 What are the different types of solids ?
- 12.12 Illustrate solids of revolution with simple sketches ?
- 12.13 A cone is resting with its apex on the HP. Draw its projections.
- 12.14 What is the side view of a cylinder if its axis is parallel to both HP and VP ? rectangle

- ✓ 12.15 If a solid rests on an edge of its base on VP, it must be kept to HP.
- ✓ 12.16 A solid with two identical ends is called Prism.
- ✓ 12.17 A cone is generated by the revolution of a about its altitude.
- ✓ 12.18 What do you mean by right regular prism? Show it by a sketch.

ANSWERS

- | | | | |
|----------------------------|--------------------|-----------------------------------|---------------------|
| 12.1 Two | 12.2 Tetrahedron | 12.3 Side view | 12.4 Perpendicular |
| 12.7 Invisible | 12.8 Three | 12.9 Inclined | 12.10 Perpendicular |
| 12.15 Perpendicular | 12.16 Prism | 12.17 Right angle triangle | |

To sketch a prism with two parallel vertical edges, draw a horizontal line and a vertical line intersecting at one point. From the top of the vertical line, draw a horizontal line to the right. From the top of the horizontal line, draw a vertical line downwards. From the top of the vertical line, draw another horizontal line to the right. From the top of this second horizontal line, draw a vertical line downwards. This forms a rectangular prism. Now, from the top of the vertical line, draw a diagonal line upwards and to the left. From the top of this diagonal line, draw a vertical line downwards. This forms a triangular prism. Now, from the top of the vertical line, draw a diagonal line upwards and to the right. From the top of this diagonal line, draw a vertical line downwards. This forms a triangular prism.



ANSWER

For a right-angled triangular prism, draw a horizontal line and a vertical line intersecting at one point. From the top of the vertical line, draw a horizontal line to the right. From the top of this horizontal line, draw a vertical line downwards. This forms a rectangular prism. Now, from the top of the vertical line, draw a diagonal line upwards and to the left. From the top of this diagonal line, draw a vertical line downwards. This forms a triangular prism. Now, from the top of the vertical line, draw a diagonal line upwards and to the right. From the top of this diagonal line, draw a vertical line downwards. This forms a triangular prism.

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For a right-angled triangular prism, draw a horizontal line and a vertical line intersecting at one point. From the top of the vertical line, draw a horizontal line to the right. From the top of this horizontal line, draw a vertical line downwards. This forms a rectangular prism. Now, from the top of the vertical line, draw a diagonal line upwards and to the left. From the top of this diagonal line, draw a vertical line downwards. This forms a triangular prism. Now, from the top of the vertical line, draw a diagonal line upwards and to the right. From the top of this diagonal line, draw a vertical line downwards. This forms a triangular prism.