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Magnetic Materials

(1) INTRODUCTION

Magnetic materials are generally divided into three categories. Depending upon the alignment of magnetic dipole moments of molecules/atoms, when substance is placed in external magnetic field, these are further divided into five groups as Paramagnetic, Ferromagnetic, Diamagnetic, Ferrimagnetic and Antiferromagnetic materials. Diamagnetism, paramagnetism and antiferromagnetism are very feeble effects. While Ferromagnetism and Ferrimagnetism are very strong effects. Magnetic materials are also classified as hard and soft magnetic materials. The coercivity of hard magnetic materials is very large and these find use in making permanent magnets while coercivity of soft magnetic materials is very small and these are used in a.c. circuits e.g. core of transformer. Magnetic materials are used widely in computer industry, storage devices and in electrical devices. Their magnetic behaviour is generally the result of interaction of magnetic dipole moment of each atom with external magnetic field.

(2) TERMINOLOGY

Before discussing behaviour of magnetic materials, we must introduce ourselves with certain physical quantities, which will appear in the expressions to be derived at later stage.

(1) **Magnetic flux density or magnetic induction (B)**. Consider a positively charged particle moving with velocity V at an angle θ to the direction of magnetic field, then magnetic force acting on charged particle is given by

$$F = qVB \sin \theta \quad \dots(1)$$

where q = charge on particle

B = magnetic flux density at a point.

If $q = 1$ coulomb, $V = 1 \text{ ms}^{-1}$ and $\theta = 90^\circ$ then above equation reduces to

$$F = B$$

"Thus magnetic flux density at a point is numerically equal to the force experienced by a particle of charge 1C moving with velocity 1 ms^{-1} in a direction perpendicular to the direction of magnetic field."

It is also defined as force experienced by a unit North placed at that point.

The SI unit of B is tesla and C.G.S. unit is Gauss.

$$1T = 10^4 G$$

The magnetic flux through an area \vec{ds} is given by

$$d\phi_B = \vec{B} \cdot \vec{ds}$$

$$= B ds \cos \theta$$

(where θ = angle between \vec{B} and \vec{ds})

or

$$B = \frac{d\phi_B}{ds \cos \theta} \quad \dots(2)$$

Thus B is also equal to normal flux per unit area. Hence its name.

(2) Magnetic dipole Moment (μ). A system of two equal and opposite magnetic poles separated by some distance is said to constitute a magnetic dipole.

The dipole moment of magnetic dipole is equal to product of pole strength of any pole and the distance between the poles and it is directed from south pole to north pole.

i.e. $\vec{\mu} = (m) \vec{l}$...(3)

where $\vec{\mu}$ = dipole moment

m = pole strength of each pole

l = distance between two poles

Unit of pole strength in $A m$ and of $\vec{\mu}$ is $A m^2$.

Every current carrying loop also behaves as magnetic dipole and its dipole moment is given by

$$\vec{\mu} = NIA \hat{n} \quad \dots(4)$$

where N = number of turns in the loop,

I = current through each turn

A = area of each turn

\hat{n} = a unit vector normal to plane of loop and its direction is found by applying right hand thumb rule to the sense of current through the coil.

Note that if current in the loop appears to be in anticlockwise sense, then that face possesses North polarity & if sense of current appears clockwise, then that face possesses South polarity.

(3) Magnetisation vector. If a magnetic material is placed in external magnetic field then the elementary magnetic dipoles experience torque and have tendency to align in the direction of external magnetic field. As a result the net magnetic dipole moment of the sample becomes non zero. This process is called magnetisation.

"The magnetic dipole moment per unit volume of the sample is called magnetisation vector."

It is denoted by \vec{M} .

Thus
$$\vec{M} = \lim_{V \rightarrow 0} \frac{\vec{\mu}}{V}$$

...(5)

where V = volume of the sample around a given point.

S.I. unit of \vec{M} is $A m^{-1}$

(4) Magnetic Intensity or magnetising force (\vec{H}). Fig. (1 (a)) shows a Rowland ring having toroidal winding of N turns. Let I_0 is real current passed by us through each turn of the

winding. We know that a current carrying conductor is a source of magnetic field (Oersted's experiment). Thus due to current i_0 magnetic field is produced inside the Rowland ring whose direction is along its circumferential length. Due to the magnetic field the internal dipoles in the core get aligned in the direction of field. A cross section of the Rowland ring at point A is shown separately in fig. 1 (b). It shows that all current loops (*i.e.* internal dipoles) are oriented identically because of torque applied by magnetic field. It is clear that the currents of current loops in the vicinity of material cancel each other due to effect of adjacent loops. As such there is no current inside the core. However the current in the outer portion of the outer loops remain uncancelled. Due to this a surface current i_s is produced along the circumference of the considered cross section. This current i_s is called "Surface Amperian Current."

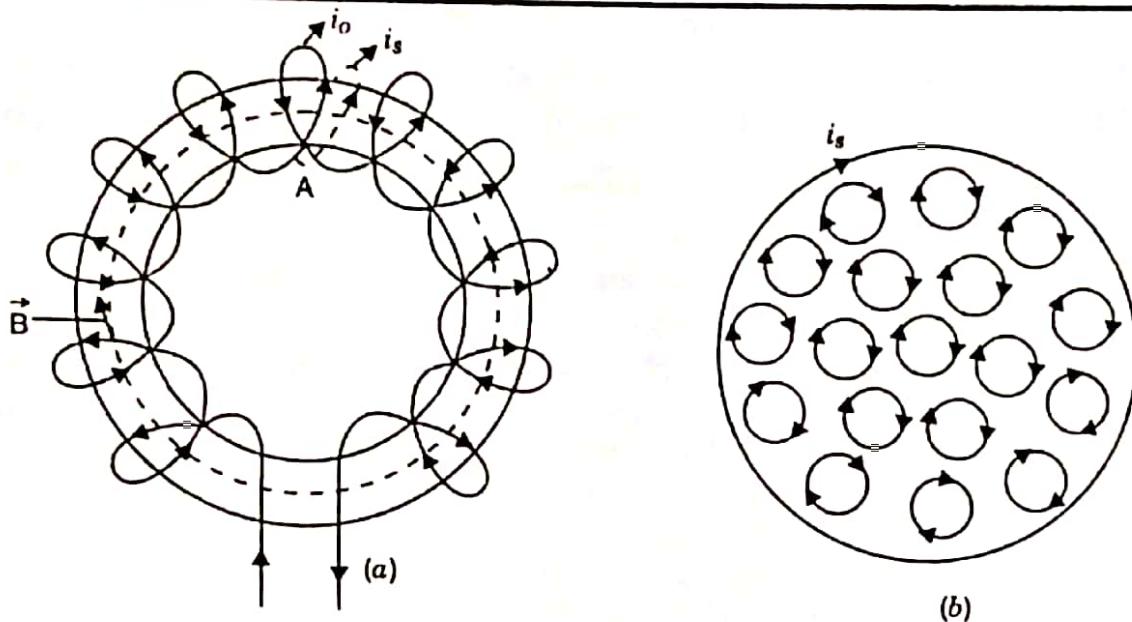


Fig. 1.

Let A is area of cross section and l is the length of the ring, then volume of material of ring is Al .

Also induced dipole moment is given by $i_s A$. Thus magnetisation vector is given by

$$\mathbf{M} = \frac{\Lambda i_s}{\Lambda l} = \frac{i_s}{l} \quad \dots(6)$$

Thus magnetisation vector is also equal to Amperian surface current per unit length.

The surface current i_s at every point behaves itself as a current loop and it produces its own magnetic flux density. The magnetic flux density at the centre of loop (shown at point A in fig. (1 (a)) due to surface current will be

$$B_M = \frac{\mu_0 i_s}{l} \quad \dots(7)$$

Also the magnetic flux density at same point due to real current i_0 (also called free current) is given as

$$B_0 = \frac{\mu_0 N i_0}{l} \quad \dots(8)$$

Thus net magnetic flux density at any point is given by

$$\begin{aligned} B &= B_0 + B_M \quad (\text{as both } B_0 \text{ and } B_M \text{ are in same direction}) \\ &= \mu_0 \left(\frac{N i_0}{l} + \frac{i_s}{l} \right) \end{aligned} \quad \dots(9)$$

or $B = \mu_0 \left(\frac{Ni_0}{l} + M \right)$... (10) (Using)

or $\frac{B}{\mu_0} - M = \frac{Ni_0}{l}$

or $H = \frac{Ni_0}{l}$... (11)

Here we have defined a new vector $\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$ called magnetic field intensity or simply as magnetic field.

From (11) it is clear that \vec{H} depends only upon distribution of free current and it is independent of the presence of medium. \vec{H} actually represents capability of the magnetic field to magnetise a material. It is actually the free current over which we have control and which we can measure easily. However \vec{B} is not under our direct control as it gets modified by the presence of a medium.

From equation (11) we can define magnetising force as the number of ampere turns flowing round the unit length of the Rowland ring.

(4) **Relative Permeability (μ_r)**. Relative permeability of a medium is defined as the ratio of number of magnetic field lines per unit area (i.e. magnetic flux density) in that material to the number of magnetic flux lines per unit area that would be present if the medium were replaced by vacuum.

i.e. $\mu_r = \frac{B}{B_0}$... (12)

where B is magnetic flux density in the presence of medium and B_0 is magnetic flux density in the absence of medium.

It is a dimensionless number.

(5) **Magnetic susceptibility (χ_m)**. It is observed that magnetisation vector of a medium is directly proportional to the magnetising force i.e. $\vec{M} \propto \vec{H}$

or $\vec{M} = \chi_m \vec{H}$... (13)

The constant of proportionality χ_m is called magnetic susceptibility. For given value of H , if χ_m is large then magnetisation produced is also large. Hence its name.

From (13) we have $\chi_m = \frac{|\vec{M}|}{|\vec{H}|}$

Thus magnetic susceptibility can be defined as the ratio of magnitude of magnetisation vector to magnetising force.

It is a dimensionless number. If χ_m is +ve, then \vec{M} and \vec{H} are in same direction and material will be attracted in the direction of magnetic field and vice versa.

Now we can rewrite equation (10) as

$$\begin{aligned} B &= \mu_0 (H + \chi_m H) \\ B &= \mu_0 (1 + \chi_m) H \\ B &= \mu H \end{aligned} \quad (\text{using (11) and (13)}) \quad \dots (14)$$

or
or

In vector form

$$\vec{B} = \mu \vec{H}$$

where

$$\mu = \mu_0 (1 + \chi_m) \quad \dots(15)$$

μ is called Absolute permeability of medium. It has same unit as that of μ_0 .

In case of vacuum $\chi_m = 0$ (i.e. there is no magnetisation) Hence (14) gives us $B_0 = \mu_0 H$ (where B_0 is external magnetic flux density).

Thus

$$\mu_r = \frac{\vec{B}}{\vec{B}_0} = \frac{\mu \vec{H}}{\mu_0 \vec{H}} = \frac{\mu}{\mu_0} \quad \dots(16)$$

Thus relative permeability is also equal to the ratio of permeability of medium to the permeability of free space.

Note. We shall use symbol $\vec{\mu}$ or $|\vec{\mu}|$ for magnetic dipole moment and μ for permeability.

(3) MAGNETIC MATERIALS

Magnetic materials are the substances, which when placed in external magnetic field, are attracted or repelled by magnetic field. These can be broadly categorised as Diamagnetic, Paramagnetic and Ferromagnetic materials.

(1) Diamagnetic Materials. The materials which when placed in external magnetic field are weakly magnetised in a direction opposite to applied field are called diamagnetic materials. e.g. Hydrogen, air, water, gold, silver, bismuth etc. The important properties of these materials are listed below.

- (a) These materials are repelled by magnetic field. If a diamagnetic material is placed in a region where some external magnetic field exists, then the magnetic field lines do not prefer to pass through the material. i.e. magnetic field lines in the region will be lesser than these would be in the absence of the material.
- (b) If a U-tube is filled with a diamagnetic liquid and external magnetic field is applied across one of the limb, then level of liquid in that limb shows depression.
- (c) Their magnetic susceptibility is slightly negative having magnitude $\approx 10^{-6}$ and relative permeability is slightly less than unity ($\mu_r < 1$)
- (d) The magnetic susceptibility is almost independent of the temperature.

(2) Paramagnetic Materials. The materials, which when placed in external magnetic field, get weakly magnetised in its direction are called Paramagnetic substances e.g. Platinum, Aluminium, manganese, copper sulphate, liquid, oxygen etc. Their important properties are given below :

- (a) When a bar of paramagnetic material is placed in external magnetic field, then field lines prefer to pass through the bar i.e. number of field lines in the material are more closer to each other than outside it.
- (b) If a U-tube is filled with a paramagnetic liquid and magnetic field is applied across one of limb, then the level of liquid in that limb rises.
- (c) Their magnetic susceptibility is slightly positive and has magnitude $\approx 10^{-6}$. The relative permeability is slightly greater than unity ($\mu_r > 1$).
- (d) The magnetic susceptibility is strongly dependent upon temperature. In fact at high temperature $\chi_m \propto \frac{1}{T}$ where T is absolute temperature of substance.

or

$$\chi_m = \frac{C}{T} \quad \dots(17)$$

where C is a constant called curie's constant. Equation (17) is called curie's law of paramagnetism. According this law graph between χ_m and $\frac{1}{T}$ is a straight line.

(3) Ferromagnetic Materials. These are the materials, which when placed in external magnetic field, get strongly magnetised in the direction of the field. e.g. Iron, Nickel, Cobalt, Gadolinium and their alloys. A few important properties of these materials are listed below

- (a) These materials are strongly attracted by magnetic field. Thus when a ferromagnetic material is placed in external magnetic field then field lines prefer to pass through the ferromagnetic sample. This property is infact used to protect a substance from external magnetic field (Magnetostatic shielding) i.e. the substance to be protected is placed in a shell made from a ferromagnetic material.
- (b) They possess very high value of magnetic susceptibility and relative permeability Susceptibility as large as 10^6 and relative permeability of the order of a few thousand are common.
- (c) The relation between magnetic flux density B and magnetising force H is not linear.
- (d) With increase in temperature their permeability μ decreases and at a certain temperature T_c , called Curie temperature it becomes almost equal to μ_0 . For temperature more than T_c the material becomes paramagnetic. For Iron T_c is $\approx 770^\circ\text{C}$.

(4) ATOM AS A MAGNETIC DIPOLE

We know that every current carrying coil behaves as a magnetic dipole and possesses a definite magnetic dipole moment. In an atom electron revolves around the nucleus in a closed orbit. Since electron is a charged particle, so its orbit around the nucleus is equivalent to a current loop. Hence it behaves as a magnetic dipole.

Let us assume that orbit of electron is circular.

If r = radius of orbit

e = charge on electron, T = time period of orbital motion then equivalent current in the orbit is given by

$$I = \frac{e}{T} = \frac{eV}{2\pi r} \quad \dots(18)$$

where V = velocity of the electron and $V = \frac{2\pi r}{T}$. The magnitude of magnetic dipole moment is given by

$$\begin{aligned} |\vec{\mu}_I| &= IA \\ &= \frac{eV}{2\pi r} \cdot \pi r^2 = \frac{eVr}{2} \\ &= \left(\frac{e}{2m}\right)mVr \quad (\text{where } m \text{ is mass of electron}) \\ |\vec{\mu}_I| &= \left(\frac{e}{2m}\right)L \end{aligned}$$

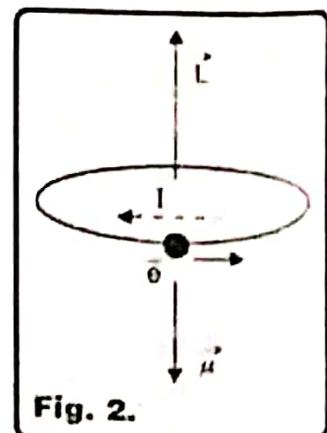


Fig. 2.

However for finding direction of \vec{L} we apply right hand thumb rule to motion of electron but direction of $\vec{\mu}$ is found by applying right hand thumb rule to the sense of conventional

current. Since electron is negatively charged thus $\vec{\mu}_l$ and \vec{L} are oppositely directed. Hence in vector form, equation (18) becomes

$$\vec{\mu}_l = \frac{-e}{2m} \vec{L} \quad \dots(19)$$

If we accept Bohr's model of atom, then the angular momentum of orbitting electron must be equal to an integral multiple of

$$\hbar = \frac{h}{2\pi}$$

i.e.

$$L = n\hbar \text{ where } n = 1, 2, 3, 4, \dots$$

Hence equation (18) gives

$$|\vec{\mu}_l| = n \left(\frac{e\hbar}{2m} \right)$$

or

$$|\vec{\mu}_l| = n |\vec{\mu}_B| \quad \dots(20)$$

where

$$|\vec{\mu}_B| = \frac{e\hbar}{2m} = 9.274 \times 10^{-24} \text{ Am}^2 \quad \dots(21)$$

$|\vec{\mu}_B|$ is called Bohr magneton. Thus no electron can have magnetic moment less than $|\vec{\mu}_B|$. In fact orbital magnetic moment of an electron is always quantised, the minimum quantum being Bohr's magneton.

Just as electron revolves around nucleus, in the same manner, it is simultaneously revolving around its own axis. Thus it possesses spin angular momentum and hence spin magnetic moment μ_s . It is given by

$$\vec{\mu}_s = -\frac{e}{m} \vec{S}_z \quad \dots(22)$$

where \vec{S}_z is spin angular momentum. Its magnitude is given by

$$|\vec{S}_z| = \frac{\pm 1\hbar}{2}$$

Thus

$$\vec{\mu}_s = \pm \frac{e\hbar}{2m} = \pm \vec{\mu}_B$$

Thus net magnetic dipole moment of the atom is given by vector sum of $\vec{\mu}_l$ and $\vec{\mu}_s$

or

$$\vec{\mu} = \vec{\mu}_l + \vec{\mu}_s$$

We know that each orbital can accomodate two electrons having opposite spin and orbital angular momentum. Thus spin and orbital magnetic moments of this electron in same orbit cancel each other. As a result, an atom having completely filled electron subshells does not posses permanent magnetic dipole moment. Thus magnetic moment of an atom is determined only by the unpaired outer electrons.

(5) ATOMIC THEORY OF MAGNETISM (QUALITATIVE)

We know that an atom behaves as a magnetic dipole and its dipole moment is equal to the vector sum of orbital and spin magnetic moment of the unpaired electrons in the outermost shell,

(i) **Paramagnetism.** In case of materials showing paramagnetism, their each molecule contains one or more unpaired electrons. Thus dipole moment of all electrons except the last unpaired electron cancel each other. Due to last unpaired electron, the magnetic moment of each atom or molecule is non zero in the absence of external magnetic field. However due to thermal agitations at room temperature, various dipole in the sample are randomly oriented. Thus overall magnetic dipole moment of the paramagnetic sample is zero when there is no external magnetic field.

However when some external magnetic field is applied across the sample then each dipole experiences torque which tends to align the dipoles in the direction of external magnetic field. Thus net dipole moment of sample becomes non zero as more dipoles are now ordered in the direction of magnetic field. This explains paramagnetism. Further with increase in magnitude of external magnetic field, more dipoles are aligned in the direction of field. On the other hand with increase in temperature, dipoles get thermal excitation, which provides randomness to the already aligned dipoles. Thus magnetic susceptibility of these materials decreases with increase in the temperature.

(ii) **Diamagnetism.** This effect is observable predominantly in those substances, whose atoms/molecules contain even number of electrons. Thus net magnetic dipole moment of each molecule is zero under natural conditions. When external magnetic field is applied across such a material then no torque is experienced by atomic dipoles, but the external magnetic field modifies the electronic motion within each atom. The orbit of each electron behaves as current loop and produces its own magnetic field. Those electrons, whose current loop is so oriented that its magnetic field is in a direction similar to external field get slowed down, while electrons whose loops are so oriented that their own magnetic field is opposite to external magnetic field, get accelerated because of Lenz's law. Thus every electron pair within same orbit acquires a net magnetic dipole moment in a direction opposite to external magnetic field. This explains diamagnetism. Thermal agitation does not affect this mechanism. Hence their magnetic susceptibility is independent of the temperature. Diamagnetism occurs in all materials, but in paramagnetic materials, it is over powered by paramagnetic effect.

(iii) **Ferromagnetism.** In ferromagnetic substances the atomic dipoles of sample exist in groups, called domains. In one domain the direction of all dipoles is same but is different from the direction of dipoles in other domain. The linear dimensions of each domain can be upto 1000 \AA .

In the absence of external magnetic field, the dipole moments of the domains are randomly oriented, so as to form closed loops so that net dipole moment of the sample is zero.

When material is placed in external field then the domains which are favourably magnetised in the direction of magnetic field, grow in size while other domains reduce in size. As a result material is rapidly magnetised in the direction of external magnetic field. If the external magnetic field is strong enough, then the domain may rotate as a single entity so as to align its magnetic dipole moment in the direction of external magnetic field. When external magnetic field is removed, then the domains do not move back to their original position (This is called Hysteresis) thus material becomes a permanent magnet. This explains ferromagnetism. If temperature is increased, it may rupture the magnetic domains and material behaves like a paramagnetic substance.

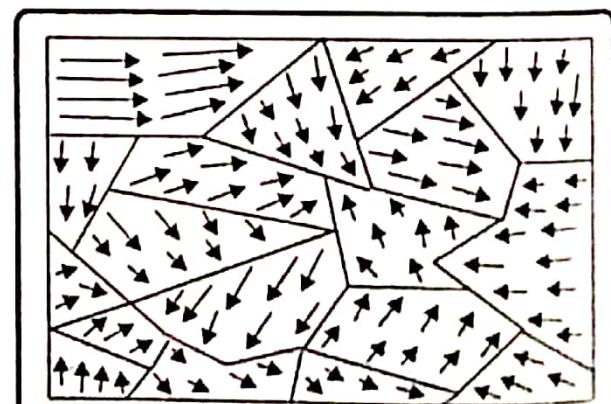


Fig. 3.

(6) LANGEVIN'S THEORY OF DIAMAGNETISM (QUANTITATIVE)

Each molecule of diamagnetic simple contains pairs of electrons. In fig. (4) we have shown two electrons in same orbit. For convenience, we have shown them separately. Both electrons revolve in the orbit in opposite sense. Let i is electronic current and r is radius of orbit (assumed circular) then magnetic dipole moment of each electron has magnitude.

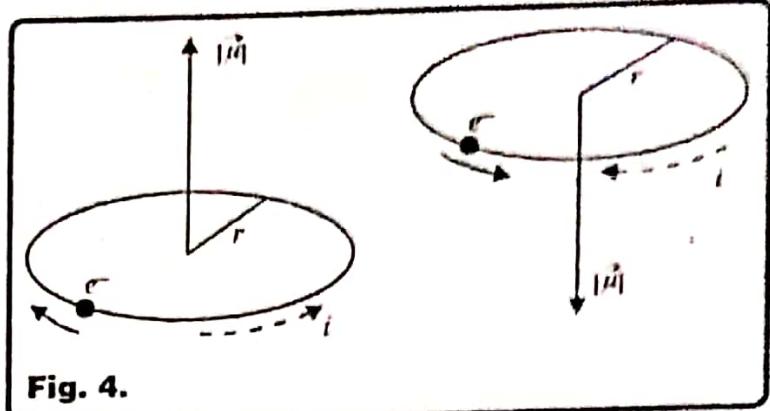


Fig. 4.

$$\begin{aligned} |\vec{\mu}| &= i(\pi r^2) \\ &= \left(\frac{e}{T} \right) (\pi r^2) = \left(\frac{e}{2\pi} \right) (\pi r^2) = \frac{1}{2} e \omega_0 r^2 \end{aligned} \quad \dots(23)$$

where ω_0 is angular frequency of each electron.

Equation (23) gives magnitude of dipole moment of each electron in same orbital. Since dipole moment of each electron is opposite to that of the other, hence net dipole moment of each atom is zero in the absence of any external magnetic field.

In the absence of external magnetic field, the centripetal force required by the electron is provided by the electrostatic force of attraction between electron and the nucleus

$$F_e = m \omega_0^2 r \quad \dots(24)$$

Direction of F_e is towards centre of orbit. Let now an external magnetic field is applied perpendicular to the plane of orbit as shown in fig. (5).

In fig. 5 (a), applied field and $\vec{\mu}$ are in same direction and by Fleming's left hand rule, the magnetic force F_m is produced, which is directed away from centre of orbit. While in case (b) it is directed towards centre of orbit. Force on electron becomes $F_e \pm F_m$.

The centripetal force changes by changing the angular frequency of orbitting electron, keeping radius of orbit constant (think why?). Thus if ω is new angular frequency of orbitting electron, in the presence of magnetic field, then new equation of motion of orbitting electron is

$$F_e \pm F_m = m \omega^2 r \quad \dots(25)$$

But

$$F_m = e V B = e(r\omega) B$$

Thus (25) gives

$$m \omega_0^2 r \pm e \omega r B = m \omega^2 r$$

$$\pm e \omega r B = m(\omega^2 - \omega_0^2)r = m(\omega - \omega_0)(\omega_0 + \omega_0)r = m \Delta \omega (2\omega)r$$

or

$$\omega = \omega_0 \pm \frac{\Delta \omega}{2}$$

where

$$\Delta\omega = \omega - \omega_0 \text{ and } \omega + \omega_0 \approx 2\omega$$

$$\Rightarrow \Delta\omega = \pm \frac{eB}{2m} \quad \dots(26)$$

From equation (23), we have

$$|\vec{\Delta\mu}| = \frac{1}{2} e r^2 \Delta\omega$$

(i.e. change in angular frequency produces change in $\vec{\mu}$)

or

$$|\vec{\Delta\mu}| = \frac{e^2 r^2 B}{4m} \quad \dots(27)$$

Since centripetal force decreases in fig. 5 (a). It means electron angular frequency decreases and hence dipole moment of electron decreases in this case by amount $|\vec{\Delta\mu}|$. Similarly dipole moment of 2nd electron increases by same amount $|\vec{\Delta\mu}|$ as it is accelerated. Thus net dipole moment of one sample in the presence of magnetic field becomes

$$|\vec{\mu}| + |\vec{\Delta\mu}| - (|\vec{\mu}| + |\vec{\Delta\mu}|) = 2|\vec{\Delta\mu}|$$

and this net dipole moment is directed opposite to the applied magnetic field. Thus each molecule will experience repulsion from electron. This explains diamagnetism.

If there are N electrons in one atom, then total induced dipole moment of one atom will be

$$|\vec{\mu}_t| = \sum_{i=1}^N |\vec{\Delta\mu}_i| = \frac{e^2 B}{4m} \sum_{i=1}^N r_i^2 \quad \dots(28)$$

Let n = number of atoms per unit volume of the sample,

Then magnetisation vector of sample is given by

$$\vec{M} = n |\vec{\mu}_t| = \frac{n e^2 B}{4m} \sum_{i=1}^N r_i^2 \quad \dots(29)$$

Now

$$B = \mu_0 (H + M)$$

For diamagnetic materials, M is very small. Hence we can put $B \approx \mu_0 H$ in (29), so that

$$M \approx \frac{n e^2 \mu_0 H}{4m} \sum_{i=1}^N r_i^2$$

In vector form

$$\vec{M} = \frac{-n e^2 \mu_0 H}{4m} \sum_{i=1}^N r_i^2 \quad \dots(30)$$

Negative sign appears because H and M are directed opposite to each other

But

$$\vec{M} = \chi_m \vec{H} \quad \dots(31)$$

Compare (30) and (31) we get magnetic susceptibility of diamagnetics as

$$\chi_m = \frac{-n e^2 \mu_0}{4m} \sum_{i=1}^N r_i^2 \quad \dots(32)$$

In this discussion, we have assumed magnetic field to be perpendicular to plane of each orbit. In actual practice, it may not be so. If random orientation of electronic orbits is taken into account, then an additional factor $\frac{2}{3}$ appears in right side of equation (32). Hence we get

$$\chi_m = \frac{-n e^2 \mu_0}{6m} \sum_{i=1}^N r_i^2 \quad \dots(33)$$

This result shows that diamagnetic susceptibility is negative and independent of the temperature.

(7) LANGEVIN'S THEORY OF PARAMAGNETISM (QUANTITATIVE)

According to this theory, every atom of a paramagnetic sample possesses permanent magnetic dipole moment. In the absence of any external magnetic field, all dipoles are randomly oriented so that net magnetisation of the sample is zero.

Let μ_m is dipole moment of one atom.

If this dipole is in a magnetic field of flux density \vec{B} and makes an angle θ with \vec{B} , then potential energy of such dipole is given by

$$U = -|\mu_m| B \cos \theta \quad \dots(34)$$

In the presence of external field all dipoles tend to align in the direction of external field, so as to occupy minimum energy. However the material is at room temperature so thermal kinetic energy $\left(\frac{3}{2}kT\right)$ of each molecule tends to provide randomness to them. Thus there is a competition between torque due to field and thermal agitation.

According to Maxwell Boltzmann Statistics the number of molecules in an energy level (called occupancy) corresponding to energy U are proportional to $e^{\frac{-U}{kT}}$, where k is Boltzmann constant. Consider the dipoles contained with in shaded region in figure (6). The semivertical angles of cones are θ and $\theta + d\theta$ respectively.

$d\Omega$ is solid angle subtended by the shaded region at O. Thus

$$\begin{aligned} d\Omega &= \frac{\text{area of ring between } \theta \text{ and } \theta + d\theta}{r^2} \\ &= \frac{2\pi(r \sin \theta)(r d\theta)}{r^2} = 2\pi \sin \theta d\theta \end{aligned} \quad \dots(35)$$

Let dn is number of dipoles per unit volume, which lie within the solid angle $d\Omega$ and whose energy is equal to U . Then dn is proportional to solid angle $d\Omega$ and occupancy $e^{\frac{-U}{kT}}$

i.e.

$$dn = C e^{\frac{-U}{kT}} d\Omega = 2\pi C \sin \theta e^{\frac{-U}{kT}} d\theta \quad (\text{where } C \text{ is some constant})$$

or

$$dn = A \sin \theta e^{\frac{-U}{kT}} d\theta \quad \dots(36)$$

where $A = 2\pi C$ is new constant.

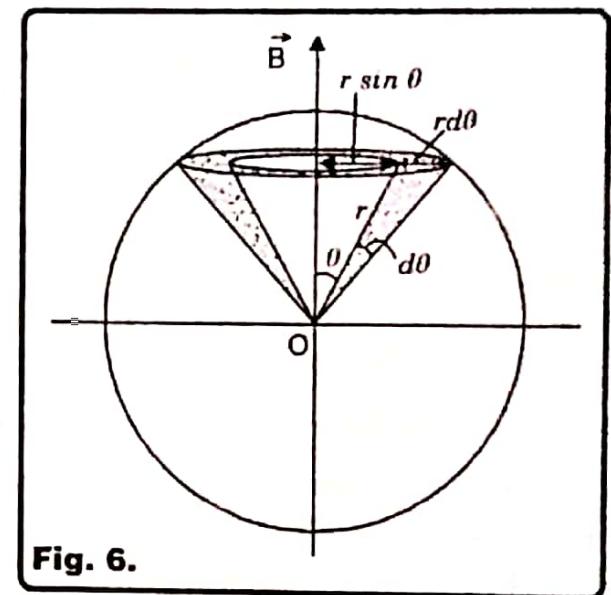


Fig. 6.

If magnetic field of flux density \mathbf{B} is applied then the potential energy U is given by equation (34).

Thus

$$dn = \Lambda e^{-\frac{|\mu_m| B \cos \theta}{kT}} \cdot \sin \theta d\theta = \Lambda e^{x \cos \theta} \sin \theta d\theta \quad \dots(37)$$

where

$$x = \frac{|\mu_m| B}{kT} \quad \dots(38)$$

Thus total number of dipoles per unit volume are given by

$$\begin{aligned} n &= \int_0^{\pi} dn = \Lambda \int_0^{\pi} e^{x \cos \theta} \cdot \sin \theta d\theta \\ &= -\Lambda \int_{-1}^{+1} e^{xu} du \quad (\text{where } \cos \theta = u) \\ &= -\Lambda \left[\frac{e^{xu}}{x} \right]_{-1}^{+1} = \frac{\Lambda}{x} (e^x - e^{-x}) \\ &= \frac{2\Lambda}{x} \sinhx \end{aligned} \quad \dots(39)$$

In our discussion we have considered dipoles which are oriented at angle θ with \mathbf{B} . Thus component of dipole moment of each dipole in the direction of field is $|\mu_m| \cos \theta$ while the component transverse to field is $|\mu_m| \sin \theta$. But by symmetry, the transverse components due to all dipoles will cancel each other while cosine components will add up. Thus magnetisation vector of the sample is given by

$$\begin{aligned} M &= \frac{\text{Net dipole moment}}{\text{Volume}} \\ &= \text{dipole moment of one molecule in the direction of } \mathbf{B} \times \text{number of molecules per unit volume} \\ &= \int_0^{\pi} |\mu_m| \cos \theta \cdot dn = |\mu_m| \Lambda \int_0^{\pi} e^{x \cos \theta} \cdot \cos \theta \cdot \sin \theta d\theta \\ &= |\mu_m| \Lambda \int_{-1}^{+1} u e^{xu} du \quad (\text{where } \cos \theta = u) \\ &= |\mu_m| \Lambda \left[\{e^x + e^{-x}\} - \frac{1}{x} \{e^x - e^{-x}\} \right] \\ &= \frac{2|\mu_m| \Lambda}{x} \left[\cosh x - \frac{\sinhx}{x} \right] \\ &= \frac{|\mu_m| n}{\sinhx} \left[\cosh x - \frac{\sinhx}{x} \right] \quad \left(\because \text{from (38)} \frac{2\Lambda}{x} = \frac{n}{\sinhx} \right) \\ &= |\mu_m| n \left[\cot hx - \frac{1}{x} \right] = n |\mu_m| L(x) \end{aligned} \quad \dots(40)$$

Here $L(x)$ is called Langevin's function.

But

$$x = \frac{|\vec{\mu}_m| B}{kT} \quad \text{or} \quad x \propto \frac{B}{T}$$

The value of x is very small even for strong fields at room temperature. (However at temperature very close to 0 Kelvin, x becomes very large) Hence $L(x)$ can be approximated as

$$\begin{aligned} L(x) &= \cosh x - \frac{1}{x} = \left(\frac{1}{x} + \frac{x}{3} - \frac{x^3}{45} + \frac{2x^5}{945} + \dots \right) - \frac{1}{x} \\ &\approx \frac{x}{3} = \frac{|\vec{\mu}_m| B}{3kT} \end{aligned}$$

Put in (39), we get

$$M = \frac{n |\vec{\mu}_m|^2 B}{3kT} \quad \dots(41)$$

Now

$$B = \mu_0 (H + M)$$

For paramagnetic materials M is very small, hence we can use $B \approx \mu_0 H$ in (41). Thus

$$M \approx \frac{n |\vec{\mu}_m|^2 \mu_0 H}{3kT} \quad \dots(42)$$

Thus magnetic susceptibility is given by

$$\chi_m = \frac{M}{H} = \frac{n |\vec{\mu}_m|^2 \mu_0}{3kT}$$

$$\text{or } \chi_m = \frac{C}{T} \quad \dots(43)$$

where

$$C = \frac{n |\vec{\mu}_m|^2 \mu_0}{3k} \quad \dots(44)$$

C is called Curie's constant.

Equation (42) shows that paramagnetic susceptibility is positive and varies inversely as absolute temperature. This is nothing but curie's law.

(8) DOMAIN THEORY OF FERROMAGNETISM

Ferromagnetism arises when the magnetic moments of adjacent atoms are arranged in a regular order i.e. pointing in the same direction. Ferromagnetic substances possess a magnetic moment even in the absence of external magnetic field. The magnetic dipole moment per unit volume in the absence of external magnetic field is called spontaneous magnetisation. There is a certain temperature T_c , below which magnetisation is non zero. This temperature is called critical temperature and above this temperature spontaneous magnetisation disappears and material behaves as paramagnetic substance. Normally value of T_c is greater than room temperature. Hence these materials can be permanent magnets at room temperature. Even below T_c , the magnetisation of these materials decreases with increase in temperature. When these materials are placed in external magnetic field, then field lines tend to concentrate in the region occupied by Ferromagnetic substance.

To explain Ferromagnetism, Weiss put forward Domain theory. According to this theory, a single crystal of Ferromagnetic solid is divided into a large number of small regions called

Domains such that all magnetic dipoles within one domain are in same direction (*i.e.* each domain is magnetically self saturated). Even the thermal agitation at room temperature is unable to disturb the dipoles of one particular domain. However the magnetic domains within each sample are randomly oriented (see fig. (7)) so that net magnetisation of sample is zero. When external magnetic field is applied across the specimen then material acquires non zero magnetisation. This magnetisation is prominent even if applied field is weak.

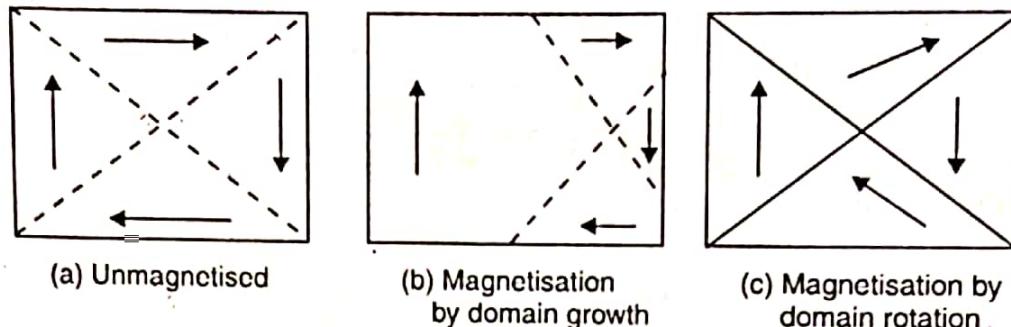


Fig. 7.

The material can show magnetisation by two different mechanisms.

(i) When small magnetic fields are applied then domains pointing approximately in the direction of magnetic field grow at the expense of the domains, which are not parallel to applied field. The magnetisation acquired by specimen is small in magnitude and is illustrated in fig. (7 (b)). If the applied magnetic field is made stronger, then the magnetisation increases due to further growth of domains in the direction nearly parallel to applied field. The process continues until all the favourably oriented domains attain maximum volume. At this stage, each domain is still magnetised in what is locally the easiest direction of magnetisation *i.e.* all domains are not in the direction of applied field and magnetisation of the sample is still less than the maximum value, it can attain (called Saturation Magnetisation).

(ii) If applied magnetic field is increased further then the magnetic domains which are not in the direction of applied field rotate as a single entity (see fig. 7(c)). Ultimately at some value of magnetic field, all domains are oriented in the direction of applied magnetic field. The magnetisation of the sample is maximum at this stage and its value is called Saturation Magnetisation. With further increase in H (magnetic field), magnetisation remains constant.

If the externally applied magnetic field is decreased then magnetisation of sample also decreases but not in same proportion as was during the magnetisation of the specimen. Thus domains do not return back to original positions on complete removal of magnetic field. Thus material becomes permanent magnet after the removal of magnetic field.

(9) EXPERIMENTAL EVIDENCE FOR EXISTENCE OF DOMAINS (HYSTERESIS)

The technique for observing magnetic domains was given by Fowler and Fryer. In this technique, the domains are illuminated by polarised light and the reflected light was viewed through an analyser. Because of Kerr magneto-optical effect, the plane of polarisation is rotated and the amount of rotation depends on the magnitude of magnetisation of a particular domain. Thus in this way magnetic domains can be seen as areas of different intensity.

The domain theory of Ferromagnetism can be employed to explain Hysteresis. To understand Hysteresis we consider magnetisation curve of a ferromagnetic material as shown in fig. (8). Initially in the absence of external field ($H = 0$) the domains in the sample are

randomly oriented so that magnetisation of the sample is zero ($M = 0$). When small magnetic field is applied, then domains which are favourably in the direction of field start growing in size slowly. Thus magnetisation M of specimen becomes non zero and increases with H slowly along curve 'OJ.' The process is reversible upto point J i.e. if field is made zero then magnetisation will also become zero by following path 'JO' back.

However if external field is increased further, then magnetisation of sample increases fastly along the path JK. The magnetisation acquired in this case also due to growth of favourably oriented domains and decay of others. If applied field is increased more then magnetisation increases further but slowly along the path KA. In this portion the domains which are not in the direction of applied field, rotate as a single unit. Corresponding to point A on the curve are domains are favourably oriented in the field direction and material is saturately magnetised. Further increase in H does not produce any change in magnetisation. The process KA is also irreversible. If we decrease field then magnetisation also decreases but along a different path 'KB'. When applied field is totally removed then the material still possesses some non zero value of magnetisation M_r . This value is called Retentivity or Residual magnetism. The material shows all aspects of permanent magnet at this stage.

If one wants to demagnetise this specimen then an external magnetic field $-H_c$ must be applied across the specimen in a direction opposite to the earlier. This reverse value of magnetic field, that must be applied across sample to demagnetise it, is called Coercive force or Coercivity. If we continue to increase field in reverse direction, then material is again magnetised in opposite direction. The magnetisation curve now follows path BC and at C material is saturately magnetised in opposite direction.

If field is again removed to zero then magnetisation curve follows another path CD and is again a permanent magnet. To demagnetise material again, we must now apply magnetic field across the specimen in a direction same as the start of the process at point O of the figure. The curve now follows path DE and with further increase in H , the magnetisation curve follows path EA and again saturation is achieved at point A again. The closed loop ABCDEA is called Hysteresis loop and is a consequence of presence of domains in the Ferromagnetic sample.

"The process of taking a Ferromagnetic specimen along one complete cycle of magnetisation and demagnetisation along two opposite directions is called Hysteresis.

It should be noted that in whole process of Hysteresis, we have to apply magnetic field of varying magnitude just to orient the domains in a certain direction. Thus the energy is wasted in this process as heat.

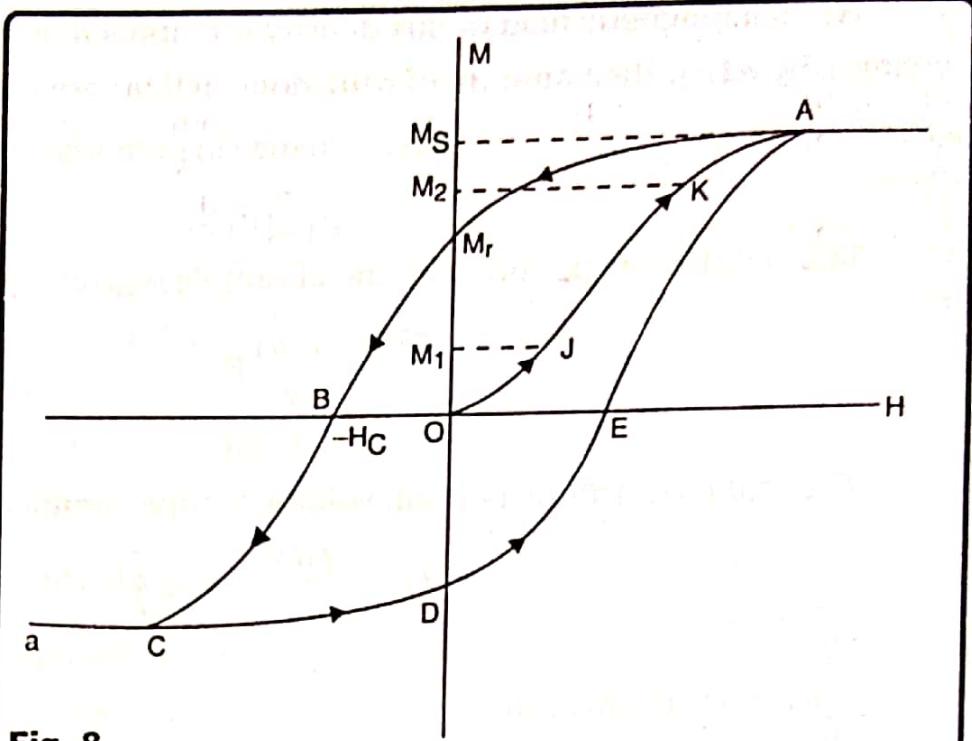


Fig. 8.

When a magnetic field of flux density B is applied, let the dipole moment of specimen changes by $d|\mu|$, then amount of work done in this case is given by

$$\begin{aligned} dW &= \text{change in potential energy} \\ &= d|\mu| B \cos 0^\circ \end{aligned}$$

Thus work done per unit volume of sample is given by

$$\begin{aligned} \frac{dW}{V} &= \frac{d|\mu|}{V} B && (\text{where } dM = \text{magnetisation}) \\ &= \mu_0 H dM && (\because B = \mu_0 H) \end{aligned}$$

Thus total work done per unit volume for one complete cycle is given by

$$\begin{aligned} W_d &= \oint \frac{dW}{V} = \mu_0 \oint H dM \\ \Rightarrow W_d &= \mu_0 \times (\text{area of Hysteresis loop}) && \dots(45) \end{aligned}$$

This equation gives the importance of Hysteresis loop. "The area of loop is proportional to the heat lost per cycle per unit volume." This is called warburg law.

(10) CURIE WEISS LAW OF FERROMAGNETISM

In 1907, P. Weiss proposed the molecular field theory to explain qualitatively salient features of ferromagnetism. According to Weiss every molecule in a ferromagnetic material is magnetic dipole and is a source of magnetic field itself. A molecule experiences the magnetic field of the other molecules. This magnetic field is called internal field. Thus it is the internal field, which is responsible for lining up all the dipoles of a local cluster in one direction and results in the formation of Ferromagnetic domains.

Let H is applied magnetic field.

M is magnetisation of sample

Then according to Weiss, the internal field H_i is given by

$$H_i = H + \gamma M \quad \dots(46)$$

γ is called Molecular field constant or Weiss constant.

It determines the intensity of interaction between the dipoles.

The internal field H_i of the Ferromagnetic material is found to obey Curie's law.

$$\text{i.e. } \chi_i = \frac{M}{H_i} = \frac{C}{T} \quad \dots(47)$$

where C is Curie's constant

Inserting (46) in (47), we get

$$\frac{M}{H + \gamma M} = \frac{C}{T}$$

or

$$MT = CH + C\gamma M$$

or

$$M = \frac{CH}{T - \gamma C} \quad \dots(48)$$

Thus Ferromagnetic susceptibility is given by

$$\chi_m = \frac{M}{H}$$

$$= \frac{C}{T - \gamma C}$$

(where H = applied field)

or

$$\chi_m = \frac{C}{T - T_c} \quad \dots(49)$$

Here $T_c = \gamma C$ is called Curie temperature.

Equation (49) is called Curie Weiss law. If $T = T_c$ then $\chi_m \rightarrow \infty$. It means that the interactions of the individual magnetic moments reinforce each other, causing them to align parallel at $T = T_c$.

From measurement of susceptibility as a function of temperature, value of γ can be calculated. It is found to be of the order of 10^3 . This value is enormously high than theoretically predicted value. The disagreement is because of the fact that interaction between various dipoles is not classical magnetic interaction but it is quantum mechanical magnetic interaction originating from exchange forces between various dipoles.

Note that temperature dependence of susceptibility given by equation (49) is valid only for $T > T_c$ i.e. when Ferromagnetic material behaves as paramagnetic material. For $T < T_c$ (when sample behaves as Ferromagnet), χ_m varies in a complex manner with temperature and exact expression has not been derived.

(11) ANTIFERROMAGNETISM

Antiferromagnetism arises when the spin moments of the neighbouring atoms are oriented in antiparallel order. In other words Antiferromagnetic crystals are those, in which alternate atoms have their spin moments parallel to each other but adjacent atoms have not.

Such a crystal can be visualised as two interpenetrating sublattices A & B. All atoms of type A have spin up, which is in opposite directions to that of type B (spin down). One such model of a 'b.c.c.' unit cell is shown in figure (9). We know that an atom at the corner of unit cell can be shared by eight similar cubes. So contribution of a corner atom is only $\frac{1}{8}$ for one cube. Since there are eight corners of one cube thus net contribution of atoms at corners is $8 \times \frac{1}{8} = 1$. While atom B is totally inside the unit cell contains effectively two dipoles one from A type corner atoms having spin up and other from type B atom having spin down. Thus net dipole moment or magnetisation of each unit cell & hence of specimen is zero in the absence of any external magnetic field.

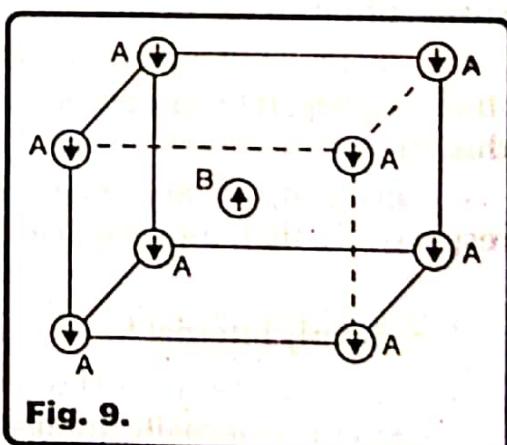


Fig. 9.

If external magnetic field is applied then a small magnetisation in the direction of applied field is produced. This magnetisation increases with increase in temperature. Finally a temperature T_N called Neel temperature is obtained, at which magnetisation is maximum (see fig. 10 (a) and (b)). With further rise in temperature magnetisation starts decreasing. Thus above Neel temperature specimen starts behaving like Paramagnetic Substance.

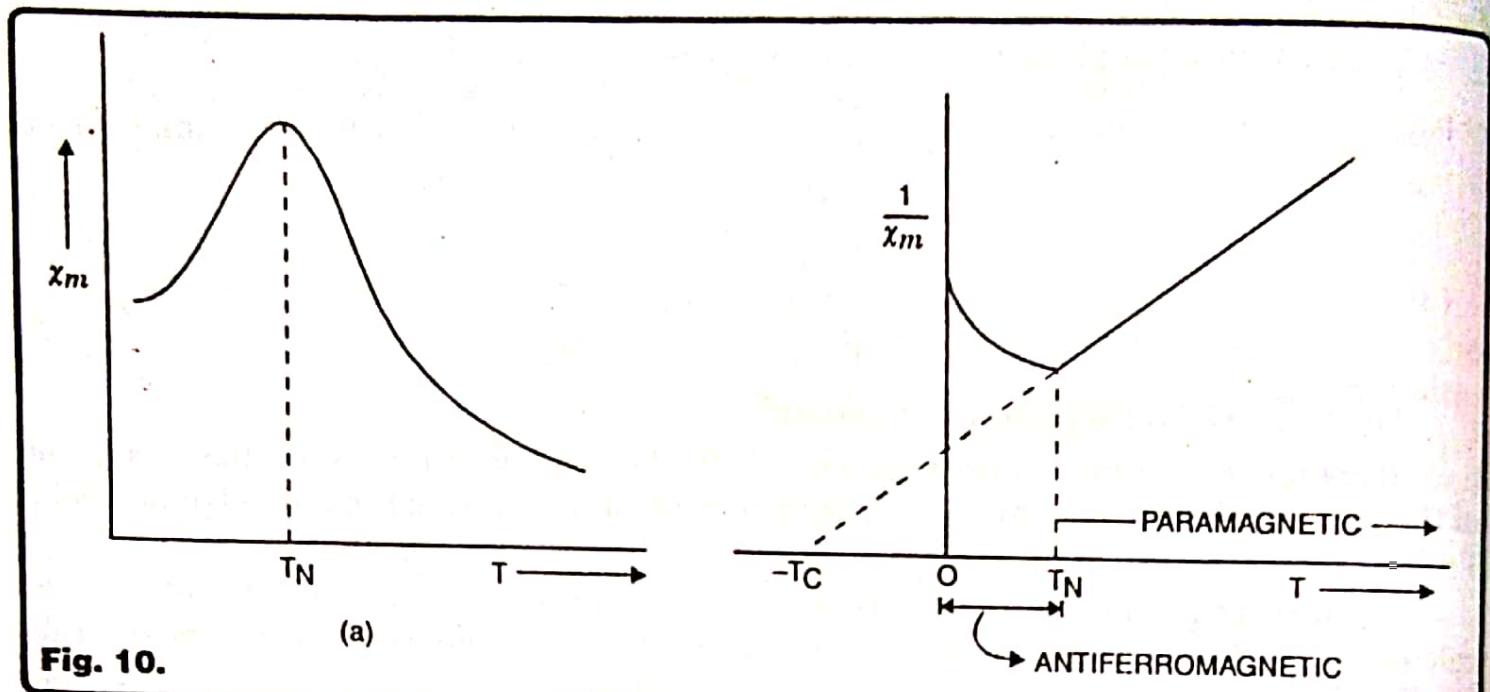


Fig. 10.

For $T > T_N$, the Antiferromagnetic material becomes paramagnetic and its susceptibility varies according to the relation.

$$\chi_m = \frac{C_0}{T + T_N} \quad \dots(50)$$

where C_0 is curie's constant

and $C_0 = \frac{2n|\vec{\mu}_m|^2 \mu_0}{3k} = 2C$ (see eq. (44))

Thus Curie's constant in this case is twice the Curie's constant for paramagnetic material.

The elements manganese and chromium exhibit antiferromagnetism at room temperature. Most of the antiferromagnetic materials are ionic compounds and their susceptibility is small and positive and lies in the range 10^{-3} to 10^{-5} .

The proof of equation (50) is given by molecular field theory. According to this theory there is a negative nearest neighbour interaction between atom A and B (see fig. (9)). Besides this, there is also next nearest neighbour interaction between atoms AA and BB.

Let α and β be molecular field constant for AA (or BB) interaction and AB (or BA) interaction respectively, then internal field at atom A is given by

$$H'_A = -\beta M_B - \alpha M_A \quad \dots(51)$$

Similarly internal field at atom B is given by

$$H'_B = -\beta M_A - \alpha M_B \quad \dots(52)$$

Let H is externally applied magnetic field, then, effective magnetic field at A and B is given as

$$H_A = H + H'_A = H - \beta M_B - \alpha M_A \quad \dots(53)$$

$$H_B = H + H'_B = H - \beta M_A - \alpha M_B \quad \dots(54)$$

If $T > T_N$ then dipoles are randomly oriented and both sublattices are unsaturated. Thus magnetisations M_A and M_B are given by equation (42) i.e.

$$M_A = \frac{n|\vec{\mu}_m|^2 \mu_0 H_A}{3kT} \quad \text{and} \quad M_B = \frac{n|\vec{\mu}_m|^2 \mu_0 H_B}{3kT}$$

Magnetic Materials

Thus total magnetisation M of the sample is equal to the sum of magnetisation of sublattices A and B.

$$\Rightarrow M = M_A + M_B \quad \dots(55)$$

or

$$M = \frac{n|\vec{\mu}_m|^2 \mu_0}{3kT} (H_A + H_B)$$

$$= \frac{n|\vec{\mu}_m|^2 \mu_0}{3kT} (2H - (\alpha + \beta) (M_A + M_B)) \quad (\text{Using (53) and (54)})$$

$$= \frac{n|\vec{\mu}_m|^2 \mu_0}{3kT} (2H - (\alpha + \beta)M)$$

$$\frac{n|\vec{\mu}_m|^2 \mu_0}{3kT} (2H)$$

or

$$M = \frac{\frac{n|\vec{\mu}_m|^2 \mu_0}{3kT} (2H)}{1 + \frac{n|\vec{\mu}_m|^2 \mu_0}{3kT} (\alpha + \beta)}$$

Thus

$$X_m = \frac{M}{H} = \frac{\frac{2n|\vec{\mu}_m|^2 \mu_0}{3kT}}{1 + \frac{n|\vec{\mu}_m|^2 \mu_0}{3kT} (\alpha + \beta)}$$

or

$$X_m = \frac{C_0}{T + T_N} \quad \dots(56)$$

where $C_0 = \frac{2n|\vec{\mu}_m|^2 \mu_0}{3k}$ is Curie's constant and $T_N = \frac{n|\vec{\mu}_m|^2 \mu_0}{3k} (\alpha + \beta)$

Thus relation has been proved. This relation describes fairly well the variation of Antiferromagnetic susceptibility with temperature above Neel Temperature.

(12) FERRIMAGNETISM

Ferrimagnetism is a special case of Anti ferromagnetism. It occurs when the magnetisation of two sublattices, in terms of which, we consider antiferromagnetic crystal, are unequal. Thus net magnetisation of the crystal, which is equal to sum of magnetisations of the two sublattices has a non zero value. The ferrimagnetic crystals are magnetic compounds consisting of two or more different kinds of atoms. The saturation magnetisation of Ferrimagnetic materials is not as high as Ferromagnetic materials, but their macroscopic behaviour is very much similar to Ferromagnetic substances. Thus Ferrimagnetism is a strong effect. With increase in temperature, the saturation Magnetisation decreases and vanishes at Neel temperature or even earlier than that. Above Neel temperature these materials exhibit paramagnetic behaviour. However in

paramagnetic behaviour, Curie Weiss law is not obeyed and relation between X_m and $\frac{1}{T}$ is not linear. These materials have Residual magnetism, which is characterised by small coercive force. They posses small domains in which electron spins are spontaneously aligned. However alignment is antiparallel.

(13) HARD AND SOFT MAGNETIC MATERIALS

On the basis of area of Hysteresis loop, magnetic materials can be classified as Hard and Soft. Magnetic materials, which are easily magnetised and demagnetised are called soft magnetic materials.

The Hysteresis loop of soft materials is thin and area of loop is very small. These are characterised by low coercivity and high retentivity. The material can thus be magnetised with low magnetic field and can be used to increase flux linked with a coil. Low coercivity corresponds to easy movements of domains with applied magnetic field. If the sample suffers from structural defects, then the movements of domains become difficult, which increases coercivity. Thus soft magnetic material must be free from structural defects.

If a.c. is passed through coil wound over a soft material, then material is magnetised and demagnetised regularly at a rate equal to the frequency of a.c. In this process material shows Hysteresis and energy is wasted as heat. Since area of Hysteresis loop is minimum in soft magnetic materials, so these are best suited for making transformer cores. Further when flux linked with a conducting material changes then an additional energy is wasted in the form of heat due to eddy currents. To reduce this loss, the resistivity of soft materials can be increased by doping the metal with some suitable non metal impurity e.g. iron is slightly doped with silica. To further reduce the Eddy losses, a number of thin laminated pieces are used instead of a single bulky piece of material.

Soft magnetic materials are further classified as : (i) Heavy duty flux multipliers (ii) Light duty flux multipliers (iii) Square loop materials (iv) Microwave system components. Type (i) materials find use in cores of transformers, motors & generators. Type (ii) in communication equipments. Type (iii) in magnetic amplifiers, computers etc. and in type (iv) soft ferrites and garnets are used.

The magnetic materials are said to be hard if these are difficult to magnetise and demagnetise. These are characterised by low retentivity and high coercive force. The area of Hysteresis loop of such materials is large. Once magnetised by using high, magnetic field, these materials cannot be easily demagnetised by magnetic field of earth or field of other nearby magnets. This property makes them ideal materials for making permanent magnets.

We can draw fig. (12) in terms of B and H also. It should be noted that in first and 3rd quadrant, Hysteresis loop shows magnetisation

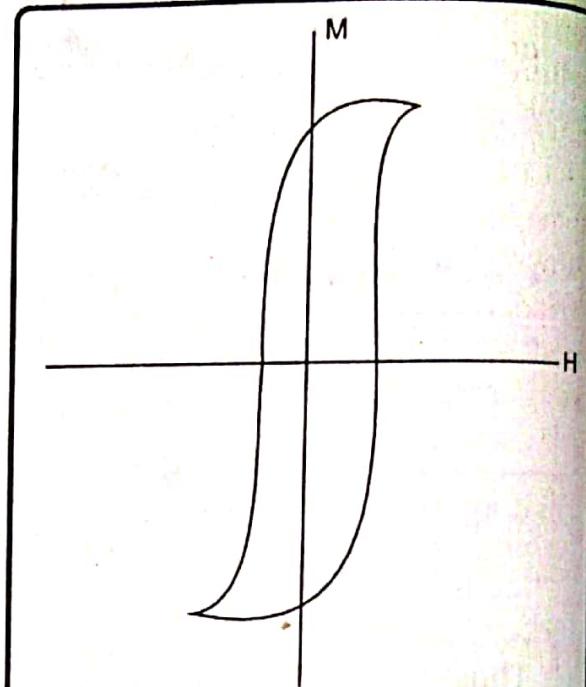


Fig. 11.

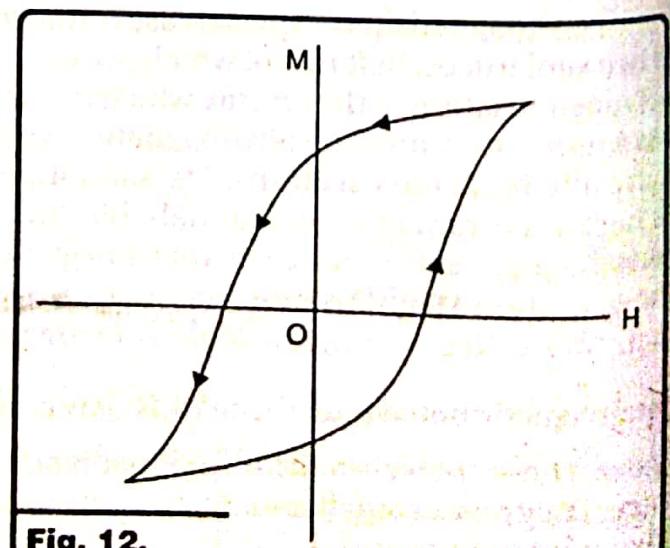


Fig. 12.

of sample and in 2nd and 4th quadrant, material is being demagnetised. The product of flux density B and magnetic field H is equal to energy per unit volume and is called energy product.

In fig. 13(a) the demagnetisation curve is shown and in fig. 13(b) variation of B with energy product is shown. The energy product is zero for $H=H_c$ and $B=B_r$ (Residual flux density). Energy product is maximum corresponding to flux density B_d . This maximum energy product is called power or external energy of hard magnet. It is directly related to the area of Hysteresis loop. i.e. large loop area means more power of magnet.

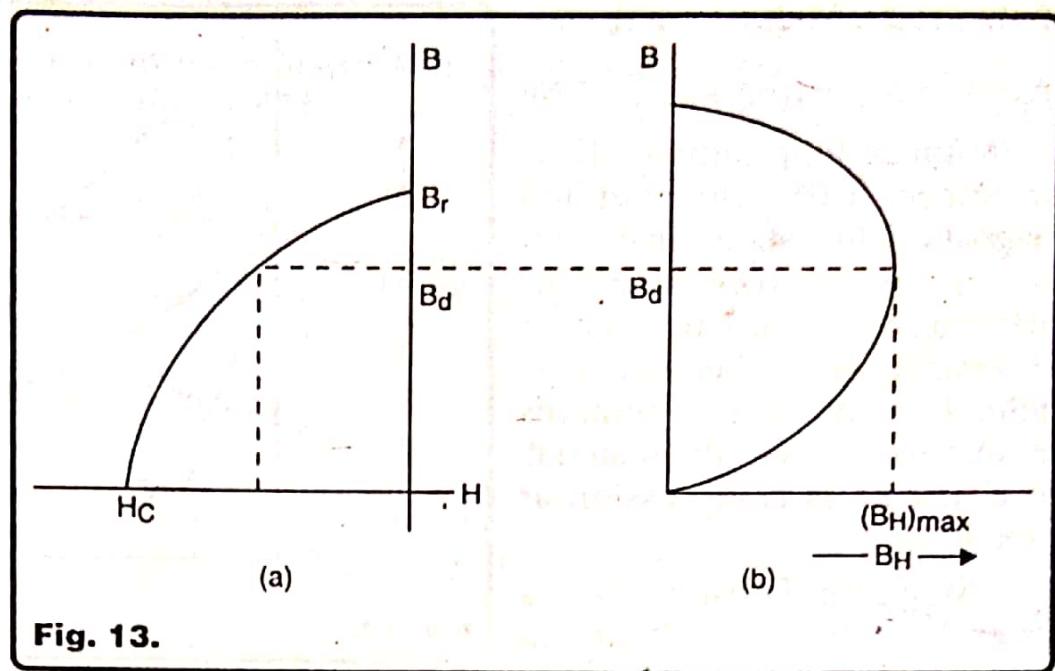


Fig. 13.

(14) FERRITES

Ferrites are a class of compounds of Ferrimagnetic substances. These are represented by general formula $\text{Me}^{++} \text{Fe}_2^{+++} \text{O}_4$, where Me^{++} stands for a suitable divalent ion such as Co^{++} , Mn^{++} , Ni^{++} , Fe^{++} , Mg^{++} etc. Fe_2^{+++} is trivalent ferric ion. If we insert Co^{++} for Me^{++} , then the Ferrite $\text{Co}^{++} \text{Fe}_2^{+++} \text{O}_4$ is called Cobalt Ferrite etc. If two different Ferrites are mixed to form a solid solution (alloy), then it is called composite ferrite. e.g. Ni-Zn Ferrite is an example of composite Ferrite. The electrical resistance of Ferrites is 10^5 to 10^{15} times the resistance of metallic Ferromagnets (these are ceramic materials) hence materials are good insulators. Thus Ferrites possess electric properties of dielectrics combined with magnetic properties of Ferromagnetic materials. Hence these materials can be used for applications at high frequencies without loss of energy due to Eddy currents.

Ferrites can also be classified as Soft and Hard Ferrites. The area of Hysteresis loop of soft Ferrites is small. These are used for low voltage signal, memory core, audio visual and recording applications. These are also used for low energy inductors and convergence coils for television receivers. Ferrites having nearly square shaped Hysteresis loop were earlier used as memory devices or logic operation devices in computers, as switching devices and in information storage.

The area of Hysteresis loop of Hard Ferrites is large. These are used to make permanent magnets. These are also widely used in generators, relays, loud speakers, telephone ringers, toys etc. where high energy product is not required. Hard ferrite powders mixed with plastic material to form flexible magnets are used as door closers of refrigerator doors.

(15) MAGNETOSTRICTION

When a ferromagnetic material is magnetised, its length either expands or contracts in the direction of magnetisation. This effect is called Magnetostriiction or Joule Effect. The longitudinal

strain produced due to magnetostriction is written as $\frac{\Delta l}{l}$. The variation of longitudinal stress produced with the applied magnetic field is shown in fig. (14) for certain materials. Magnetostriction occurs due to the rotation of domains on applied magnetic field. This rotation of domains produces a stress in the material, resulting in its compression or expansion.

Magnetostriction is a reversible effect i.e. change in magnetisation can be produced by applying stress on the material. This effect is called Villari effect.

Magnetostriction has been largely of theoretical interest and of limited practical applications. A few applications are given below : (i) This effect is used in high frequency oscillators and generators of super sound i.e. ultrasonic waves (see chapter 3) (ii) For under water sound projectors and sound detectors.

(16) MAGNETIC ANISOTROPY

In certain single crystals, such as iron, the magnetic properties depend upon the direction in which these are measured. This phenomenon is called magnetic anisotropy. e.g. when external magnetic field is absent, then spontaneous magnetisation takes up certain specific direction with respect to crystal axis instead of any arbitrary direction. In case of iron, there are six preferred directions of spontaneous magnetisation. In case of polycrystalline solids, the various crystals in a polycrystal are randomly oriented, so that properties are same in all directions. However if specific treatment such as cold rolling is given to certain polycrystalline substances, then magnetic properties become different in different directions. This anisotropy produced in the material is called Induced Anisotropy. This property is of considerable practical importance. e.g. thin films of Ni-Fe alloy deposited on to a substrate by evaporation in vacuum with magnetic field applied in the plane of the substrate show spontaneous magnetisation in the direction of applied field. These magnetic films are used as storage devices in computers.

Example 1. The area of Hysteresis curve for 15 kg iron piece is equivalent to 300 J/m^3 . What will be the energy loss per minute in magnetising this piece at a frequency of 50 Hz. Give density of iron is 7.5 g cm^{-3} .

Solution. $\rho = 7.5 \text{ g cm}^{-3} = 7.5 \times 10^3 \text{ kg m}^{-3}$

$$\text{Volume of specimen} = V = \frac{m}{\rho} = \frac{15}{7.5 \times 10^3} = 2 \times 10^{-3} \text{ m}^3$$

$$\text{Number of cycles completed in one minute} = 60 \times 50 = 3000$$

$$\text{Now energy lost per B-H cycle} = 300 \text{ J} \times 3000 = 9 \times 10^5 \text{ J m}^{-3}$$

$$\therefore \text{total energy lost in specimen} = 9 \times 10^5 \text{ J m}^{-3} \times 2 \times 10^{-3} \text{ m}^3 = 1800 \text{ J}$$

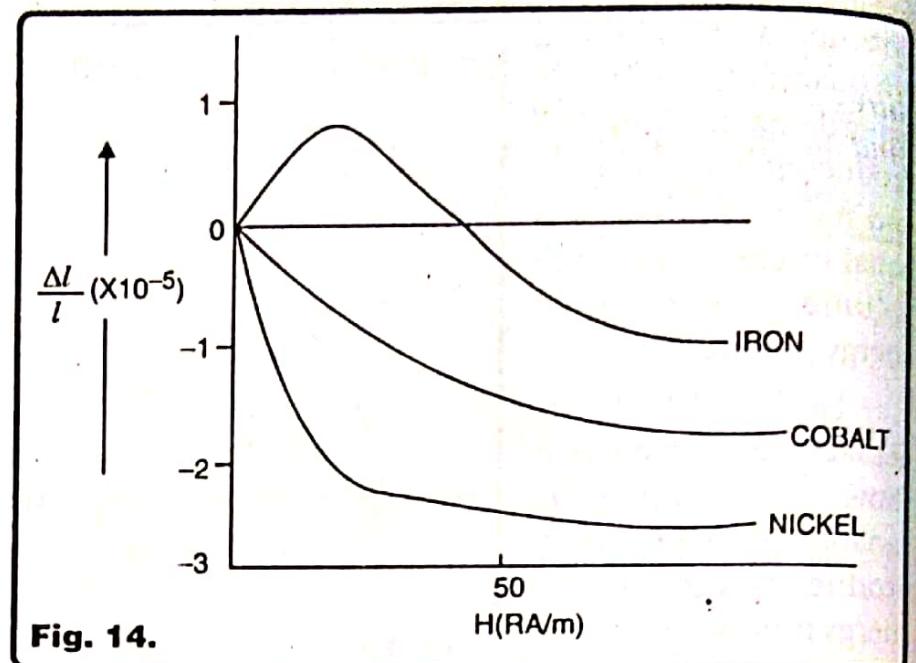


Fig. 14.

Example 2. An iron rod 50 cm long and 1 cm in diameter is placed in a long solenoid of 5 turns per cm, carrying a current of 0.2 A. Assuming that relative permeability of iron is 1000, find the magnetic moment of the rod.

Solution. Given $n = 5 \text{ cm}^{-1} = 500 \text{ m}^{-1}$

$$I = 0.2 \text{ A}$$

$$H = nI = 0.2 \times 500 = 100 \text{ A m}^{-1}$$

$$V = \frac{\pi D^2 l}{4} = \frac{3.14 \times (10^{-2})^2 \times 10^{-2}}{4} = 7.85 \times 10^{-5} \text{ m}^3$$

$$\mu_r = 1000$$

Now

$$\frac{M}{H} = \chi_m = \mu_r - 1$$

\Rightarrow

$$M = (\mu_r - 1) H = (1000 - 1) \times 100 = 99900 \text{ A m}^{-1}$$

\Rightarrow

$$|\vec{\mu}| = MV = 99900 \times 7.85 \times 10^{-5} = 7.84 \text{ Am}^2$$

Example 3. The magnetic intensity H at the centre of a long solenoid carrying a current of 4.0 A is found to be 2000 Am^{-1} . Find the number of turns per centimeter of the solenoid.

Solution.

$$H = nI$$

$$\therefore n = \frac{H}{I} = \frac{2000}{4} = 500 \text{ m}^{-1} = 5 \text{ cm}^{-1}$$

Example 4. The dipole moment of each atom of iron is two times Bohr magneton. If concentration of iron atoms in a sample is $8.52 \times 10^{28} \text{ atoms/m}^3$, find (a) maximum magnetisation of a long cylindrical iron bar (b) maximum magnetic flux density along the axis of rod inside it.

Solution.

$$|\vec{\mu}| = 2|\vec{\mu}_B| = 2 \times 9.27 \times 10^{-24} = 1.854 \times 10^{-23} \text{ Am}^2$$

Here $|\vec{\mu}|$ = dipole moment of one atom.

Let l = length of cylinder, A = Area of cross section

$\therefore N$ = total no. of atoms = $8.52 \times 10^{28} \times Al$... (i)

$|\vec{\mu}_m|$ = maximum dipole moment for 100% magnetisation

$$= N|\vec{\mu}| = 8.52 \times 10^{28} \times Al \times 1.854 \times 10^{-23} = 1.58 \times 10^6 \times Al \text{ Am}^2$$

Thus maximum value of magnetisation is given by

$$M = \frac{|\vec{\mu}_m|}{Al} = 1.58 \times 10^6 \text{ Am}^{-1}$$

$$(b) B = \mu_0 M = 4\pi \times 10^{-7} \times 1.58 \times 10^6 = 1.98 \text{ T}$$

Example 5. A rod is inserted inside a current carrying solenoid. The current through each turn of solenoid is 4.0 A and number of turns per centimeter is 5.0. (i) Find magnetic intensity M at the centre, (ii) If magnetisation of the core is found to be 0.15 Am^{-1} , find magnetic susceptibility of the material. (iii) What is the nature of material?

Solution.

$$I = 4.0 \text{ A } n = 5 \text{ cm}^{-1} = 500 \text{ m}^{-1}$$

$$(i) H = nI = 500 \times 4 = 2000 \text{ Am}^{-1}$$

$$(ii) M = 0.15 \text{ Am}^{-1}$$

$$\therefore \chi_m = \frac{M}{H} = \frac{0.15}{2000} = 7.5 \times 10^{-5}$$

(iii) Since χ_m is slightly positive, so material is paramagnetic.

Example 6. A bar magnet of length 2cm and area of cross section 4cm² produces a magnetic field of 2.5×10^{-4} T at a point on the axial line at a distance of 20cm from its centre. (i) Find magnetic dipole moment of magnet (ii) Magnetisation of magnet (iii) Magnetic flux density at the centre of magnet.

Solution.

$$2l = 2\text{cm} = 2 \times 10^{-2}\text{m} = \text{length of magnet}$$

$$A = 4\text{cm}^2 = 4 \times 10^{-4}\text{m}^2$$

$$n = 20\text{ cm} = 20 \times 10^{-2}\text{m}$$

$$B_a = \text{Magnetic flux density on axial line} = 2.5 \times 10^{-4}\text{T}$$

We know

$$B_a = \frac{\mu_0}{4\pi} \frac{2|\vec{\mu}|r}{(r^2 - l^2)^2} = \frac{\mu_0}{4\pi} \frac{2|\vec{\mu}|r}{r^4} \quad (\because r \gg l)$$

$$= \frac{\mu_0}{4\pi} \frac{2|\vec{\mu}|}{r^3}$$

$$|\vec{\mu}| = \frac{4\pi r^3 B}{2\mu_0} = \frac{(20 \times 10^{-2})^3 \times 2.5 \times 10^{-4}}{2 \times 10^{-7}} = 10 \text{ Am}^2$$

$$(ii) M = \frac{|\vec{\mu}|}{A(2l)} = \frac{10}{4 \times 10^{-4} \times 2 \times 10^{-2}} = 1.25 \times 10^6 \text{ Am}^{-1}$$

(iii) B = magnetic flux density at the centre of magnet

$$= \frac{\mu_0}{4\pi} \frac{2m}{l^2} \quad (\text{where } m = \text{pole strength})$$

$$= \frac{\mu_0}{4\pi} \frac{|\vec{\mu}|}{l^3} \quad (|\vec{\mu}| = m(2l))$$

$$= \frac{10^{-7} \times 10}{(1 \times 10^{-2})^3} = 1.0 \text{ T}$$

Example 7. The susceptibility of magnesium is 1.2×10^{-5} at room temperature (300K). Find its susceptibility at 400K.

Solution. Since susceptibility at room temperature is small and positive so material is paramagnetic and obeys Curie's law.

$$\chi_1 \propto \frac{1}{T_1} \quad \text{and} \quad \chi_2 \propto \frac{1}{T_2}$$

$$\frac{\chi_2}{\chi_1} = \frac{T_1}{T_2}$$

$$\chi_2 = \chi_1 \left(\frac{T_1}{T_2} \right) = 1.2 \times 10^{-5} \left(\frac{300}{400} \right) = 9 \times 10^{-6}$$

Example 8. The coercivity for a ferromagnetic substance is $5.0 \times 10^4 \text{ Am}^{-1}$. It is placed inside a long solenoid of turns 50 cm^{-1} and a current is passed in the solenoid to demagnetise it completely. Find current through each turn.

Solution.

$$M = 5.0 \times 10^4 \text{ Am}^{-1}$$

$$n = 50 \text{ cm}^{-1} = 5000 \text{ m}^{-1}$$

$$H = nI$$

⇒

$$I = \frac{H}{n} = \frac{5.0 \times 10^4}{5000} = 10 \text{ A}$$

Exercise 1. A specimen of magnetised iron of volume 20 cm^3 is placed in a magnetic field of 100 Am^{-1} . If the magnetic dipole moment produced in it is 6 Am^2 , find magnetic flux density inside it at its centre. [Ans. 0.377 T]

Exercise 2. A toroid has a mean radius equal to $\frac{10}{\pi} \text{ cm}$ and total number of turns 200, such that current through each turn is 3 A . An aluminium ring at temperature 300 K inside the toroid provides the core. (i) If the magnetisation is $6.0 \times 10^{-2} \text{ Am}^{-1}$, find susceptibility of aluminium at 300 K . (ii) If temperature of ring is raised to 350 K , what will be new value of magnetisation? [Ans. (i) 6.0×10^{-5} , (ii) 5.14×10^{-5}]

Exercise 3. A long cylindrical iron core of cross sectional area 6.00 cm^2 is inserted into a long solenoid having 1000 times/m and current through each turn is 1.00 A . The magnetic flux density inside the core is found to be 2.0 T . Find the magnetisation of core and pole strength developed. [Ans. (i) $1.59 \times 10^6 \text{ Am}^{-1}$, (ii) 954.3 Am]

Exercise 4. Find the percentage change in magnetic flux density where spare within a current carrying solenoid is filled with a medium of susceptibility 2.5×10^{-5} . [Ans. $2.5 \times 10^{-5}\%$ (Increase)]

Exercise 5. The area of hysteresis loop of a ferromagnetic substance is 500 J m^{-3} . It is subjected to an alternating magnetic field of 60 Hz . If density of substance is 9 g cm^{-3} and its mass is 36 kg , find energy lost in process in 5 minutes. [Ans. $3.6 \times 10^4 \text{ J}$]

SHORT ANSWER TYPE QUESTIONS

Q. 1. Why magnetic lines of force prefer to pass through the ferromagnetic substance?

Ans. Due to very high value of permeability or susceptibility of iron, the concentration of magnetic field lines is more inside the ferromagnetic substance.

Q. 2. Can a paramagnetic material show diamagnetism ?

Ans. Yes, diamagnetism is a universal property of all the substances and all substances have a diamagnetic contribution to their susceptibility. However the effect is very feeble and it is overshadowed by paramagnetism in paramagnetic materials. Thus practically it cannot be observed though it does take place.

Q. 3. What is the susceptibility of a perfect diamagnetic substance ?

Ans. The susceptibility of a perfect diamagnetic material is -1 . Such materials are called Super Conductors.

Q. 4. What is the importance of Hysteresis loop ?

Ans. The area of loop is proportional to the energy lost per unit volume, per cycle of magnetisation. Thus we can select a material suitable for making permanent magnet or Transformer core depending upon the value of area of loop.

Q. 5. Why Ferrites are more useful than Ferromagnetic substances ?

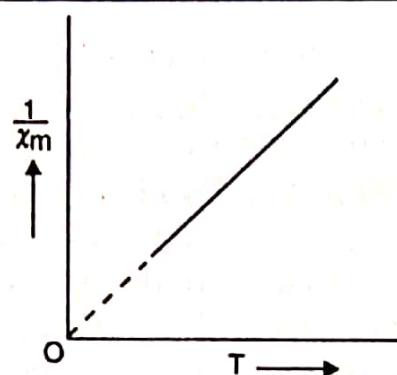
Ans. Ferromagnets are good conductors of electricity. Hence more energy is lost when flux is passed through them due to Hysteresis and Eddy currents. On the other hand Ferrites are dielectrics and do not conduct electricity. Such wastage of energy is minimum in Ferrites.

Q. 6. What is the origin of Magnetisation ?

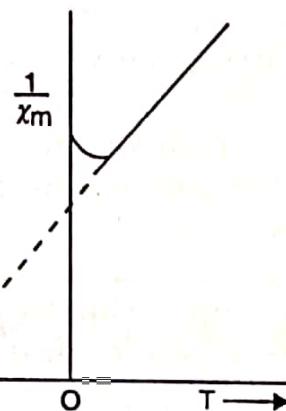
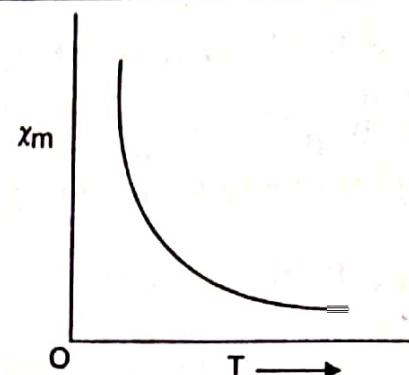
Ans. A current loop is a source of magnetic field. Inside an atom the motion of charged particles (electron & proton) in closed orbits is equivalent to current loops and hence possesses magnetic dipole moment. Thus every atom is a complete magnet in itself. This is the origin of magnetisation.

Q. 7. Draw susceptibility variation with respect to temperature for various magnetic materials.

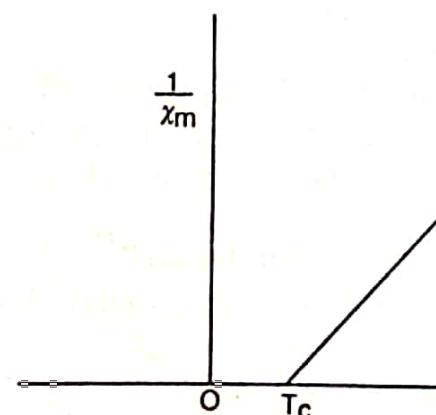
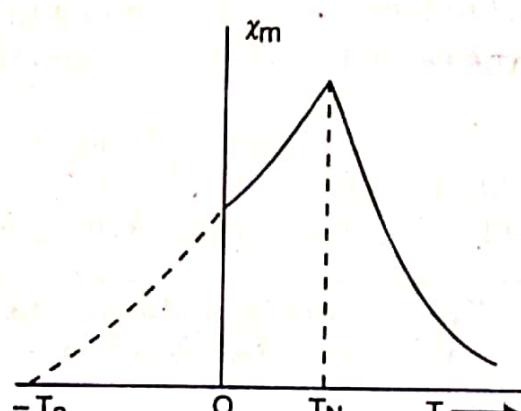
Ans. The variations are shown below :



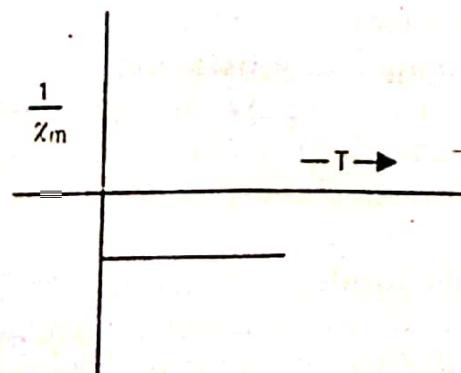
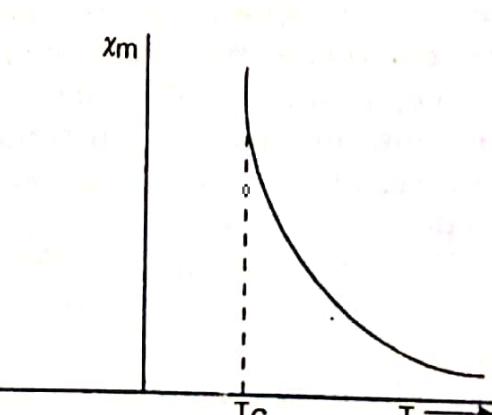
(a) Paramagnetic material



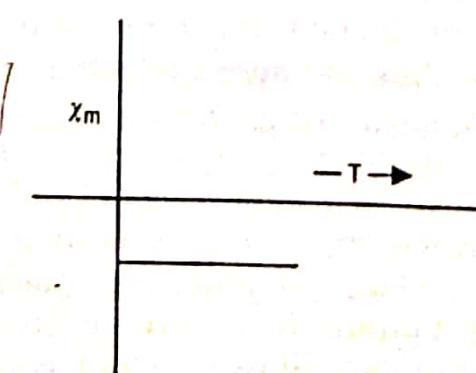
(b) Anti ferromagnetic material



(c) Ferromagnetic material



(d) Diamagnetic material



Q. 8. What kinds of dipole moments, an atom can possess ?

Ans. Inside an atom electron possesses two kinds of dipole moments dipole moment due to orbital motion, dipole moment due to spin motion. Further magnetic moment is also produced due to spinning of nucleus around its own axis. The nuclear magnetic moment is

$\frac{1}{1836}$ times the magnetic moment of a single electron.

Q. 9. What are Garnets ?

Ans. These are a class of Ferrimagnetic materials. Their general formula is $M_3 Fe_5 O_{12}$, where M represents a rare earth ion such as Samarium, Europium, Gadolinium, or Yttrium. The magnetic losses for these materials are substantially lower. This aspect is particularly important in applications where a very distinct magnetic resonance phenomena is required like in certain microwave power devices.

Q. 10. Can a free electron show diamagnetism ?

Ans. No, diamagnetism effect can be produced by a pair of two electrons orbiting in same orbit in opposite sense. While a free electron has only spin motion.

Q. 11. Why ferromagnetism is not observed in liquids or gases ?

Ans. Ferromagnetism arises due to exchange interaction, which exists only in those atoms, which are lying very close to one another. The distance between atoms/molecules of liquids is so large that exchange interactions does not occur in these. Hence liquids and gases do not show Ferromagnetism.

Q. 12. What is the difference between Antiferromagnetism and Ferrimagnetism ?

Ans. Ferrimagnetism is a special case of Antiferromagnetism. In antiferromagnetism, the spontaneous magnetisation of a unit cell is due to two interpenetrating sublattices. However the spontaneous magnetisation of one sublattice is equal and opposite to that of other, so that net spontaneous magnetisation is zero. In ferrimagnetic material the only difference is that spontaneous magnetisation of one sublattice is different from that of other. As a result net magnetisation of unit cell is non zero.

Q. 13. What is Curie temperature ?

Ans. It is that temperature above which, the ferromagnetic domains cease to exist and material behaves like a paramagnetic substance.

Q. 14. What would you suggest if we want to shield a specimen from external magnetic field ?

Ans. The specimen can be placed in a cabinet made from a ferromagnetic substance.

Q. 15. Name the material for which the relation $B = \mu H$ is not valid i.e. B is not a linear function of magnetic field.

Ans. For Ferromagnetic materials the variation of B (or M) with H is non linear. Hence $B = \mu H$ is not valid.

Q.16. Show that area of B-H curve is $\frac{1}{4\pi}$ times the energy dissipated per cm^3 of the metal during each magnetic cycle.

Ans. In CGS system

$$B = H + 4\pi M$$

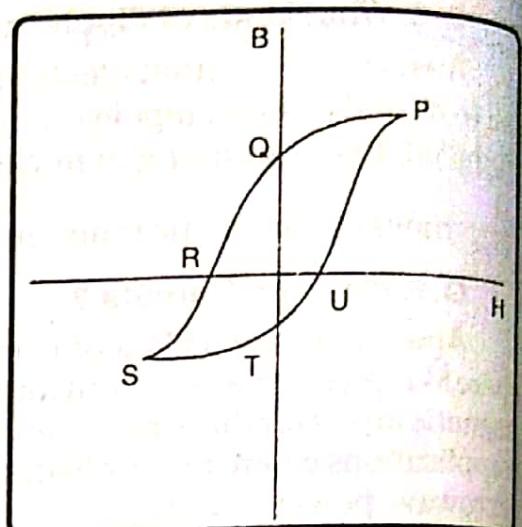
$$dB = dH + 4\pi dM$$

Multiply both sides by H, we get

$$HdB = HdH + 4\pi HdM$$

Integrating both sides over the area of Hysteresis loop PQRSTUP, we get

$$\begin{aligned}\oint \mathbf{H} d\mathbf{B} &= \oint \mathbf{H} d\mathbf{H} + 4\pi \oint \mathbf{H} d\mathbf{M} \\ \Rightarrow \oint \mathbf{H} d\mathbf{B} &= \left[\frac{\mathbf{H}^2}{2} \right]_P^P + 4\pi \oint \mathbf{H} d\mathbf{M} \\ &= \frac{[\mathbf{H}(P)]^2 - [\mathbf{H}(P)]^2}{2} + 4\pi \oint \mathbf{H} d\mathbf{M} \\ &= 0 + 4\pi \oint \mathbf{H} d\mathbf{M} \\ \Rightarrow \oint \mathbf{H} d\mathbf{M} &= \frac{1}{4\pi} \oint \mathbf{H} d\mathbf{B} \quad \dots(i)\end{aligned}$$



But $\oint \mathbf{H} d\mathbf{M}$ = energy lost per cm^3 in one Hysteresis cycle by the metal and

$$\oint \mathbf{H} d\mathbf{B} = \text{area PQRSTUP of Hysteresis loop}$$

Thus from (i) we conclude that

$$\text{Energy lost per } \text{cm}^3 = \frac{1}{4\pi} \times \text{area of Hysteresis loop.}$$

Q.17. Can we have hysteresis in paramagnetic and diamagnetic materials ?

Ans. No, because there are no domains in there materials.

Q.18. Earth's core is known to contain iron. Yet geologists do not regard this as a source of earth's magnetism. Why ?

Ans. Earth's core contain molten iron due to high temperature. This molten iron is not ferromagnet and hence it cannot be treated as source of earth's magnetism.

Q.19. Would the maximum possible magnetisation of a paramagnetic sample be of the same order of magnitude as the magnetisation of ferromagnet ?

Ans. Yes, the maximum magnetisation of paramagnetic and ferromagnetic substance will be of same order. However, value of magnetic field required to achieve saturation magnetisation will be very large for paramagnetic material.

Q.20. A system displaying hysteresis loop such as a ferromagnet is a device for strong memory. Explain meaning of this statement

Ans. Magnetisation of a ferromagnet depends on magnetising field as well as history of magnetisation (*i.e.* how many cycles of hysteresis, it has gone). Thus value of magnetisation of a sample is record of memory of cycles of magnetisation it has undergone. The system displaying such a hysteresis loop can act as a device for storing memory.

Q.21. What is a non magnetic material ?

Ans. A non magnetic substance is that which is not affected even by strong magnetic field. For example wood and plastic.

Q.22. When a dielectric is placed in an electric field, it gets polarised. The electric field in a polarised material is less than the applied field. When a paramagnetic substance is kept in a magnetic field, the field in the substance is more than the applied field. Explain.

Ans. In case of dielectric the electric field produced due to polarisation is opposite to applied electric field so that total electric field inside sample is decreased. While in case of paramagnetic materials, the induced magnetic field is in the direction of applied magnetic field so that net magnetic field in the material is more than applied magnetic field.

Q.23. The permanent magnetic dipole moment of atoms of material is not zero. What is the nature of material ?

Ans. Material can be paramagnetic or ferromagnetic.

Q.24. Can we classify magnetic materials on the basis of relative permeability ?

Ans. Yes, if $\mu_r < 1$ then material is diamagnetic if μ_r is slightly more than one then material is paramagnetic.

If μ_r is much greater than one, then material is ferromagnetic.

Q.25. Suppose that isolated magnetic monopoles exist. Then what will be the form of Gauss's law of magnetism ?

Ans. In magnetostatics pole strength m plays role of charge q permeability μ_0 plays role of $\frac{1}{\epsilon_0}$ and magnetic field \vec{B} plays role of electric field \vec{E} .

\therefore replacing \vec{E} by \vec{B} , $\frac{1}{\epsilon_0}$ by μ_0 and q by m , in Gauss's law of electrostatics, we get

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 m$$

i.e. Total magnetic flux through a closed surface in vacuum is equal to μ_0 times the pole strength enclosed by the surface.

QUESTIONS

1. Define magnetic susceptibility and permeability.
2. Describe properties of diamagnetic, paramagnetic and ferromagnetic materials.
3. Discuss various contributions to the permanent magnetic dipole moment of the atom.
4. Discuss Langevin's theory of paramagnetism and derive an expression for paramagnetic susceptibility.
5. Discuss Langevin's theory of Diamagnetism and obtain expression for diamagnetic susceptibility.
6. Define Curie's law of paramagnetism.
7. Discuss Domain theory of Ferromagnetism.
8. What is the origin of Ferromagnetic domains ?
9. Describe Antiferromagnetism and ferrimagnetism.
10. What are Ferrites ? Write their important use.
11. What do you mean by Hysteresis ? What is its importance ?
12. What is magnetic Anisotropy ? What is its importance ?
13. What do you mean by magnetostriction ? What is its use ?
14. Give Weiss theory of Ferromagnetism.
15. Define Neel Temperature.
16. What are Hard and Soft magnetic materials. Give their characteristic properties and applications.
17. Prove that the orbital magnetic moment of an electron must be an integral multiple of $\frac{e\hbar}{4\pi m}$, where symbols have their usual meanings.

