6 X-Rays

(1) INTRODUCTION

X-rays were discovered by chance by Roentgen in the year 1895. He was working in a dark room with a cathode ray tube. By chance, a few crystals of barium platinocyanide were lying in the same room. The crystals showed fluorescence, when cathode ray tube was working. Roentgen wrapped the cathode ray tube with thick black paper to prevent the escape of any visible light. But the flourescence of the crystals continued. Roentgen attributed this to some invisible rays coming out from cathode ray tube. He named these invisible rays as X-rays. It is now known that X-rays are electromagnetic waves of very short wavelength. Typical X-ray wavelengths lie between 1Å to 100Å. Using the expression $E = \frac{hc}{\lambda}$ for $\lambda = 100$ Å and then $\lambda = 1$Å, we can note that X-ray energies lie between 0.1 KeV to about 10 KeV.

(2) PRODUCTION OF X-RAYS

Principle

The production of X-rays is based on the principle that, whenever a fast moving electron gets strongly accelerated or de-accelerated, it radiates energy mostly in the form of X-rays.

A device used to produce X-rays is generally called an X-ray tube. Figure 1 shows a schematic diagram of such a device. This was originally designed by Coolidge and is known as coolidge tube to produce X-rays.

A filament F and a metallic target T are fixed in an evacuated glass chamber C. The filament is heated electrically and emits electrons by thermionic emission. A potential difference of several kilovolts is applied between filament and target using a DC power supply so that target is at a higher potential than the filament.

Thus the emitted electrons are accelerated by the electric field set up between the filament and hit the target with a very high speed. These electrons are stopped by the target and in this process X-rays are emitted. These X-rays are brought out of the tube through a window W made of thin mica or mylar or some other material, which does not absorb X-rays appreciably.

In this process, large amount of heat is generated and thus an arrangement is provided to cool down the tube continuously by running water. The exact design of tube depends on the type of use for which these X-rays are required.

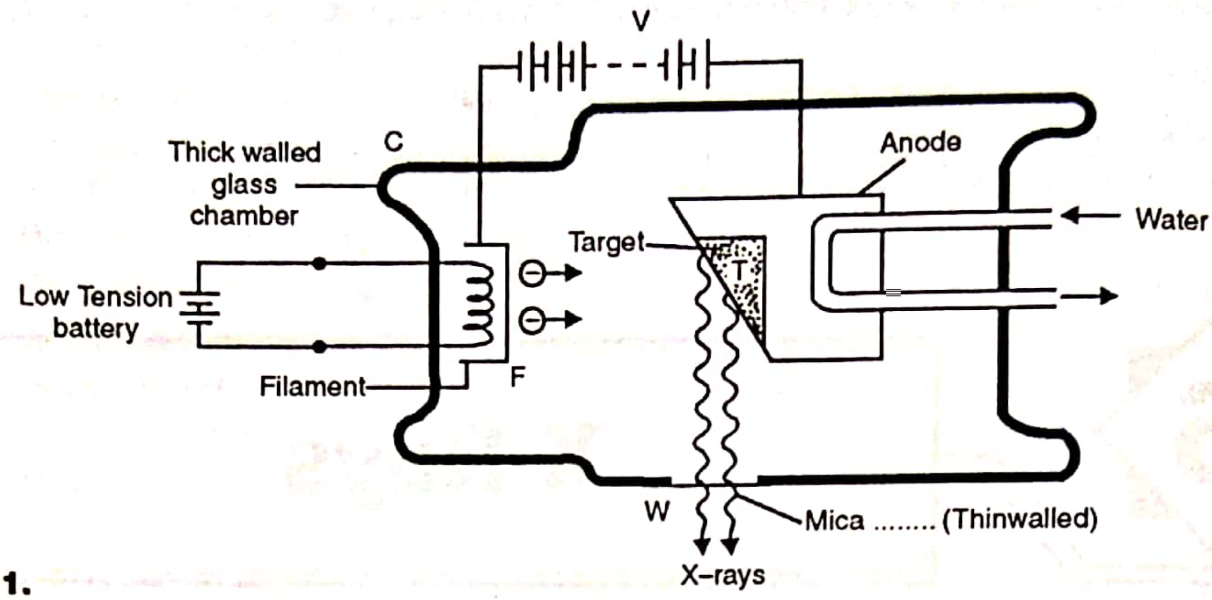

Fig. 1.

A high vacuum of the order of 10^{-6} mm of Hg is maintained in the glass tube for two reasons. (*i*) to prevent collision of high speed electrons emitted by cathode with gas molecules. (*ii*) To prevent oxidation of the filament.

The target in X-ray tube should have (*i*) high atomic number to stop bombarding electrons in a short distance of about 1 mm. (*ii*) high melting point, so that it is able to withstand a lot of heat produced in the generation of X-rays without melting. (*iii*) large coefficient of thermal conductivity because it would help the target in passing the heat quickly to the coolant.

(3) CONTINUOUS AND CHARACTERISTIC X-RAYS (X-RAY SPECTRUM)

If the X-rays coming from a coolidge tube are examined for wavelengths present, and the intensity of wavelength components are measured, we obtained a plot as shown in the fig. (2).

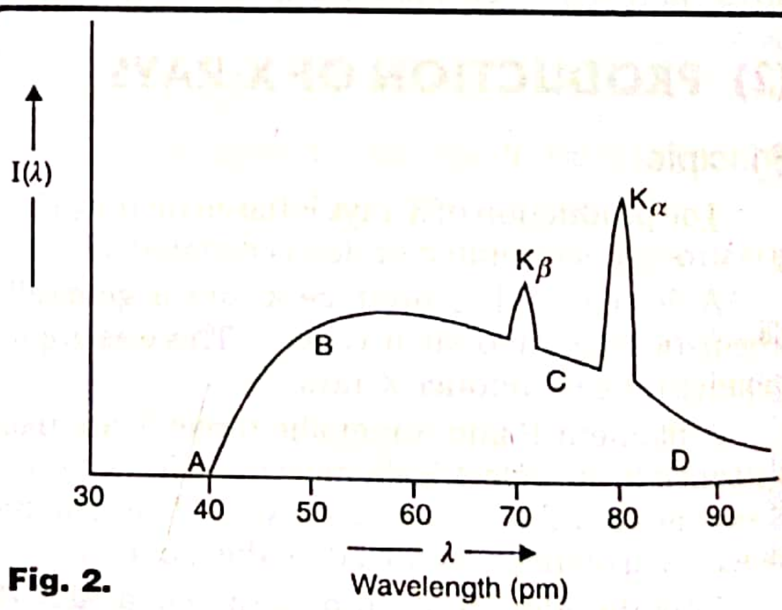

Fig. 2.

We see that there is a minimum wavelength λ_{min} below which no X-ray is emitted. This wavelength is called cut off wavelength or threshold wavelength. The above curve is called X-ray spectrum.

X-ray spectrum consists of two parts, the curve ABCD is called continuous spectrum. While peaks shown by K_α, K_β represent line or characteristic X-ray spectrum.

At the sharply defined peaks K_α and K_β, the intensity of X-rays is very large. These X-rays are called characteristic X-rays & produce characteristic or line spectrum. At other wavelengths the intensity varies gradually and these X-rays are called continuous X-rays giving rise to continuous spectrum.

(4) ORIGIN OF CONTINUOUS X-RAYS (SPECTRUM)

Suppose the potential difference applied between target and filament is V and electrons are emitted by the filament with negligible speed. The electrons are accelerated in their journey from the filament to the target. The kinetic energy of an electron when it reaches the target sample is

$$K = eV \qquad ...(1)$$

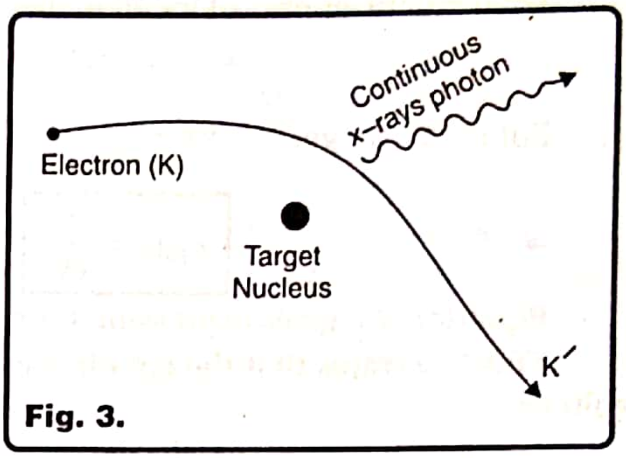

Fig. 3.

As the electron enters the target material, it experiences an attractive force due to nucleus of target atom. This force changes the path of electron as shown in figure (3). Thus electron suffers an acceleration and hence it will emit a photon of electromagnetic radiation (because whenever a charged particle undergoes accelerated motion, it emits a photon). In the process, the kinetic energy of electron is lost. This process is equivalent to collision between electron & nucleus. The extent of kinetic energy lost by electron depends upon how quickly electron is deviated from path & by how much angle. This infact depends on impact parameter of collision. "Impact parameter is the perpendicular distance between direction of motion of incident electron and a line drawn through centre of nucleus and parallel to incident direction of electron." Smaller is impact parameter, more is angle of scattering hence more is energy lost by electron (or of emitted X-ray photon).

It is obvious that when a number of electrons enter target material, the impact parameter of different electrons will be different. Hence photons of emitted X-rays will have different wavelengths. Furthermore, it is also possible an electron suffers multiple collisions with different nuclei and loses a fraction of its kinetic energy each time. This will also lead to production of X-ray photons having different wavelengths. Hence these interactions are responsible for continuous X-ray spectrum. From this discussion it is clear that wavelength of continuous X-rays does not depend on nature of target.

(5) DUANE HUNT LAW (EXPRESSION FOR CUT OFF WAVE-LENGTH)

We have just studied the cause of continuous X-ray spectrum. It is because of multiple collisions suffered by an electron with different target nuclei. Since energy of emitted continuous X-ray photon is equal to kinetic energy lost by an electron. Hence the emitted X-ray photon will have maximum energy or minimum wavelength if it loses all its kinetic energy in just one collision.

Let K is kinetic energy of incident electron

K' = kinetic energy of scattered electron

λ = wavelength of emitted X-ray photon.

Thus by law of conservation of energy

KE lost by electron = energy of emitted X-ray photon

$$\Rightarrow \qquad K - K' = \frac{hc}{\lambda} \qquad ...(2)$$

λ is minimum (energy of photon is maximum) when $K' = 0$

Hence equation (2) gives

$$K = \frac{hc}{\lambda_{min}} \quad ...(3)$$

Let V = accelerating voltage

Then kinetic energy of incident electron is

$$K = eV \quad ...(4)$$

Put in (3), we get $$eV = \frac{hc}{\lambda_{min}}$$

$$\Rightarrow \quad \boxed{\lambda_{min} = \frac{hc}{eV}} \quad ...(5)$$

Equation (5) gives expression for cut off wavelength and it is called Duane Hunt law.

This law states that the cut off wavelength is inversely proportional to the accelerating voltage.

It should be noted that the cut off wavelength is independent of nature of target material.

(6) ORIGIN OF CHARACTERISTIC X-RAY SPECTRUM

When the bombarding electrons in the X-ray tube strike the target, they have sufficient energy to eject an orbital electron from the target atom. Suppose an incident electron knocks out an orbital electron from K-shell.

This will create a vacancy in the K-shell in the sense that now there is only one electron with $n = 1$, whereas two could be accommodated by Pauli's Exclusion Principle. An electron from a higher energy state may make a transition to this vacant state. When such a transition takes place, the difference in energy of the levels involved in transition is emitted in the form of X-ray photon.

Let ΔE = difference of energies between levels participating in transition

λ = Wavelength of emitted photon

Then by law of conservation of energy

$$\frac{hc}{\lambda} = \Delta E$$

If electron from L shell makes transition to vacant state in K-shell, the emitted X-ray is called K_α X-ray. In that case above equation becomes

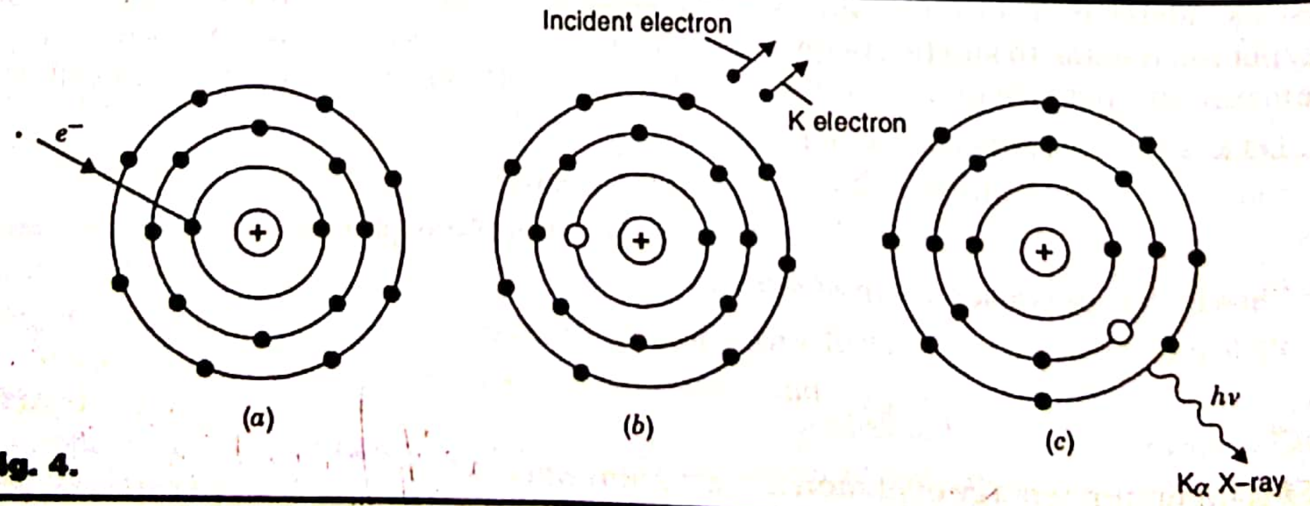

Fig. 4.

$$\frac{hc}{\lambda_\alpha} = E_L - E_k \qquad ...(6)$$

If an electron from the M shell makes transition to K shell, a K_β X-ray is emitted. Similarly we can define other members of K-series.

If vacancy in K shell is filled by an electron from L shell, then a vacancy is created in L shell. This vacancy can be filled by an electron in higher shell. If electron from M shell fills this vacancy, the emitted X-ray photon is called L_α and its wavelength is given as

$$\frac{hc}{\lambda_{\alpha'}} = E_M - E_L \qquad ...(7)$$

If this vacancy is filled by electron in N shell, we get L_β X-ray and so on. Thus in general a series of characteristic X-rays is emitted. Various photons emitted in this process can have only certain specific energies defined by the energy levels involved in transition. This gives rise to line or characteristic spectrum. The frequency or wavelength of various photons (lines) is found to be a function of nature of target. Hence line spectrum is characteristic of the target.

Due to this reason, line spectrum is also called characteristic spectrum. The line spectrum corresponding to various possible transitions of electron in the target atom is shown in figure (5). The energies E_k, E_L,.....etc. are characteristic properties of the material. For different materials, the values of these energies will be different. The wavelengths of the X-rays emitted corresponding to these transitions are

$$\lambda = \frac{hc}{E_L - E_K} \; (for\ K_\alpha)$$

$$\lambda = \frac{hc}{E_M - E_K} \; (for\ K_\beta)$$

$$\lambda = \frac{hc}{E_M - E_L} \; (for\ L_\alpha)$$

etc. Line spectrum is very helpful in identifying the target atom (*i.e.* atomic number Z can be found) by observing the wave lengths of various lines of X-ray spectrum of that target.

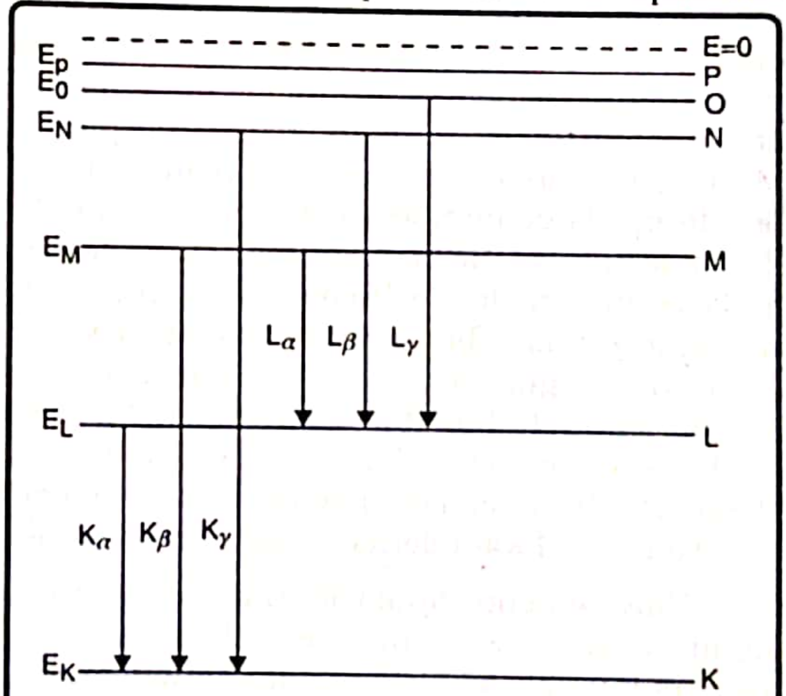

Fig. 5.

(7) SOFT AND HARD X-RAYS

The quality of X-rays is defined as their penetrating power i.e. their ability to pass through a given part of the body. It is determined by the energy of X-rays. Thus quality of X-rays depend upon their energy.

The energy of X-rays infact depends on the accelerating voltage used for accelerating electrons in the X-ray tube. If V is accelerating voltage then cut of wavelength is given by

$$\lambda_{min} = \frac{hc}{eV}.$$

Clearly with increase in accelerating voltage, the value of cut off wavelength λ_{min} will decrease and hence the energy of emitted X-rays will increase $\left(\because E \propto \frac{1}{\lambda_{min}}\right)$, thereby increasing penetrating power or quality of X-rays.

On the basis of quality, X-rays may be classified as Hard or Soft :

(*i*) If penetrating power or quality of X-rays is high then these are called Hard X-rays. These contain high energy or low wavelength photons. Generally their wavelength is much less than 1Å. Hard X-rays can be produced by using high value of accelerating voltage and by using target of high atomic number.

(*ii*) X-rays of low penetrating power (low quality) are called soft X-rays. Their wavelength is generally much greater than 1Å and these can be produced when relatively low accelerating voltage is applied between cathode and anti cathode in X-ray tube or by using target of low atomic number.

(8) MOSELEY'S LAW

Moseley's experiments on characteristic X-rays played a very important role in developing the concept of atomic number. In those days, the elements were arranged in the periodic table in increasing order of atomic weight. The periodicity in chemical properties of elements was brought out from such arrangement though some anomalies were present. Moseley measured the frequency of characteristic X-rays from a large number of elements and plotted the square root of the frequency against its position in the periodic table. He discovered that plot is very similar to a straight line. A portion of Moseley's plot is shown in figure (6). From this linear relation, Moseley concluded that there must be some fundamental property of the atom, which increases by regular steps as one moves from one element to the other. This quantity was later identified as number of protons in the nucleus of target atom and was referred to as Atomic Number.

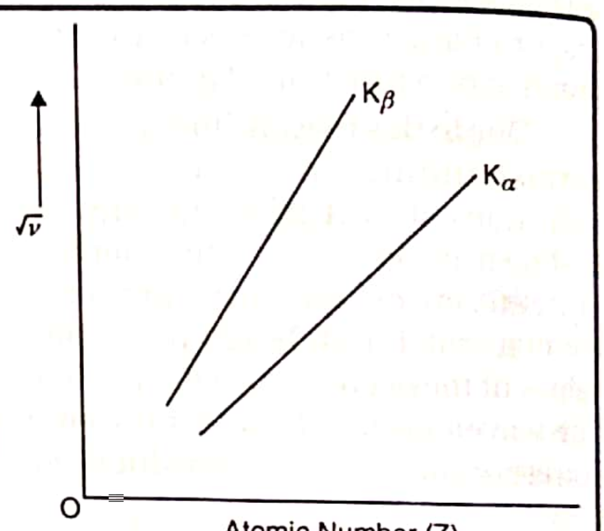

Fig. 6.

Thus elements should be arranged in the ascending order of atomic number and not the atomic weight. This removed several discrepancies in the periodic table of that time e.g. Nickel has atomic weight 58.7 whereas atomic weight of cobalt is 58.9. However, the frequency of K_α X-ray from cobalt is less than the frequency of K_α X-ray from Nickel. Thus Moseley rearranged the sequence as Cobalt, Nickel instead of Nickel, Cobalt. Similarly several other rearrangements were made.

From graph between $\sqrt{\nu}$ and z the empirical relation between these were written as (equation of straight lines) :

$$\sqrt{\nu} \propto (z - \sigma)$$

or

$$\nu = A\,(z - \sigma)^2 \qquad \text{...(8)}$$

Where A and σ are constants. $(z - \sigma)\,e$ is called effective charge on the nucleus as seen by the electron making a transition from higher to lower level (e = charge on proton).

Equation (8) is mathematical form of Moseley's law. We can state it as follows, "The frequency of a characteristic X-ray photon emitted from a substance is directly proportional to the square of effective charge on the nucleus".

For K_α line the value of A was found to be $\frac{3}{4}$RC where $R = 1.097 \times 10^7\ m^{-1}$ is Rydberg constant and C is speed of light. Also $\sigma = 1$ for K_α line. Thus for K_α line equation (8) becomes $\nu = \frac{3}{4} RC\,(z-1)^2$

For L_α line $A = \frac{5}{36} RC$ and $\sigma = 7.4$

Hence relation becomes $\nu = \frac{5}{36} RC\,(z-7.4)^2$

(9) MOSELEY'S LAW FROM BOHR'S THEORY

Let z = atomic number of atom

$-e$ = charge on electron

m = mass of electron

According to Bohr's theory, the total energy of an electron in n^{th} orbit is given as

$$E_n = \frac{-2\pi^2 mk^2 z^2 e^4}{n^2 h^2} \quad \text{...(9)}$$

where $k = \frac{1}{4\pi\varepsilon_0}$ and h is Planck's Constant.

In this result, it was assumed that atom contains only one electron. However if atom contains more than one electron, then inner electrons will screen the charge of nucleus. Thus although net charge on a nucleus is +ze, but due to screening effect of inner electrons, the outermost electron observes the charge on nucleus to be $+(z-\sigma)e$, which is less than actual charge + ze. Here σ is screening constant. Hence for a multi electron atom, we must replace z by effective atomic number $z-\sigma$. As a consequence equation (9) becomes

$$E_n = \frac{-2\pi^2 mk^2 (z-\sigma)^2 e^4}{n^2 h^2}$$

Hence energy of electron corresponding to two different energy levels n_1 & n_2 is given as

$$E_{n1} = \frac{-2\pi^2 mk^2 (z-\sigma_1)^2 e^4}{n_1^2 h^2}$$

&

$$E_{n2} = \frac{-2\pi^2 mk^2 (z-\sigma_2)^2 e^4}{n_2^2 h^2}$$

where σ_1, σ_2 are screening constants of level n_1 and n_2 respectively.

According to 3rd postulate of Bohr's atomic theory, when an atom jumps from higher energy level (n_1) to a lower energy level n_2, then it emits a photon of energy $h\nu$, which is equal to the difference in energy of levels involved.

i.e.

$$h\nu = E_{n_1} - E_{n_2} = \frac{2\pi^2 mk^2 e^4}{h^2}\left[\frac{(z-\sigma_2)^2}{n_2^2} - \frac{(z-\sigma_1)^2}{n_1^2}\right] \quad \text{(using (9))}$$

Since Z is very large compared to screening constants so we assume $\sigma_1 \approx \sigma_2 = \sigma$ (say)

Hence

$$h\nu \approx \frac{2\pi^2 mk^2 (z-\sigma)^2 e^4}{h^2}\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right]$$

or $$\nu = \frac{2\pi^2 mk^2 (z-\sigma)^2 e^4}{h^3}\left(\frac{1}{n_2^2}-\frac{1}{n_1^2}\right)$$

$$= C\,(z-\sigma)^2\left(\frac{2\pi^2 mk^2 e^4}{Ch^3}\right)\left(\frac{1}{n_2^2}-\frac{1}{n_1^2}\right) \quad \text{(where C = speed of light)}$$

or $$\nu = RC\,(z-\sigma)^2\left(\frac{1}{n_2^2}-\frac{1}{n_1^2}\right) \qquad ...(10)$$

where $R = \frac{2\pi^2 mk^2 e^4}{Ch^3} = 1.097 \times 10^7\ m^{-1}$

R is called Rydberg Constant.

It is clear from above equation that frequency of emitted photon is proportional to $(z-\sigma)^2$, proving Moseley's law.

For K_α line electron jumps from L shell ($n_1 = 2$) to K shell ($n_2 = 1$). Thus equation (10) gives

$$\nu = RC\,(Z-1)^2\left(\frac{1}{1}-\frac{1}{4}\right) \qquad (\because \sigma = 1 \text{ for } K_\alpha \text{ line})$$

$$= \frac{3}{4} RC\,(z-1)^2$$

which agrees well with observed results.

For L_α line $n_1 = 3$, $n_2 = 2$ and $\sigma = 7.4$. Thus equation (10) gives

$$\nu = RC\,(Z-7.4)^2\left(\frac{1}{4}-\frac{1}{9}\right)$$

or $$\nu = \frac{5}{36} RC\,(Z-7.4)^2$$

This result is same as observed experimentally for L_α line.

Thus Moseley's law follows from Bohr's law.

(10) PROPERTIES OF X-RAYS

(*i*) X-rays travel in straight line in vacuum with speed of light.

(*ii*) These carry no charge and hence are not deflected by electric and magnetic field.

(*iii*) These are diffracted from certain crystals according to Bragg's law.

(*iv*) These affect photographic plate more strongly than light.

(*v*) When incident on certain materials such as barium platinocyanide, X-rays cause fluorescence.

(*vi*) When passed through a gas, X-rays ionize the molecules of the gas.

(*vii*) X-rays can penetrate into several metals and other materials. Thus they can penetrate through small thickness of aluminium, wood, plastics, human flesh etc. They are stopped by materials of high density and high atomic number.

(*viii*) They cannot be focussed by a lens.

(ix) They eject electrons on striking surface of certain metals. This phenomenon is called photoelectric effect.

(x) They show interference and diffraction phenomenon under suitable conditions.

(xi) They produce chemical and biological changes by the ionisation and excitation.

(xii) They produce slight amount of heat in passing through matter.

(xiii) They produce secondary X-rays on striking metals.

(11) PHOTOELECTRIC EFFECT WITH X-RAYS : AUGER EFFECT

When a metal is exposed to an X-ray beam, then X-ray photons will knock out the inner electrons from K shell of the atom. This will create a vacancy in the K shell. So the electron from the higher shells of the same atom will go to K shell accompanied by the emission of an X-ray photon. If all the K-shell vacancies were filled by emission of X-ray photon, then each photoelectron will be followed by an X-ray photon. So the number of X-ray photons and number of photoelectrons must be equal to each other. But experimentally it was found that the number of X-ray photons is less than number of photoelectrons.

Auger made a study of photoelectrons emitted due to X-ray bombardment of a metal in a cloud chamber. He made the following observations :

(i) Most of the K photoelectron's tracks were accompanied by another smaller electron track. This smaller electron track is due to an electron whose energy was much less than that of K-photoelectron.

(ii) The two tracks had a common origin.

(iii) The direction of shorter track electron was random independent of the direction of photoelectron.

(iv) The length of shorter track is independent of the frequency of incident X-ray photon.

From these observations, following conclusions can be drawn :

(i) Most of K-photoelectrons are accompanied by another electron. This extra electron is called Auger electron. Both Auger electron and photoelectron come from same atom.

(ii) The emission of photoelectrons and Auger electron are independent processes. Thus these processes donot occur simultaneously.

Explanation for emission of Auger Electron

When the incident X-ray photon ejects a K-electron from an atom, it leaves a vacancy in K-shell. This vacancy is filled by an electron from L-shell followed by emission of photon of energy $h\nu = E_L - E_K$. This photon is also of high energy or small wavelength and thus it is an X-ray photon which can eject another electron from L-shell. Hence two vacancies appear in L-shell. Thus an extra electron is ejected instead of an X-ray photon. This extra electron is called Auger electron and this effect is called Auger Effect. It should be noted that the two vacancies created in L-shell will be filled by electron from higher shell resulting in emission of a photon. But this photon will be of very less energy and thus it will not be an X-ray photon but an infrared ray photon.

(12) DIFFRACTION OF X-RAYS

We know that when a wave is obstructed by an object whose size is comparable to the wavelength of wave, then wave on bends around the sharp edges of object. This phenomenon is called Diffraction.

For diffraction of light we use a device called Plane transmission grating. It is nothing but a plane sheet consisting of parallel strips which are alternatively transparent and opaque. Light can pass through transparent strip of width 'a' and cannot pass through opaque strip (shown shaded) of width 'b'. Thus a collection of one transparent & one opaque strip is equivalent to a slit whose width is given by $d = a + b$. This slit width is called grating element.

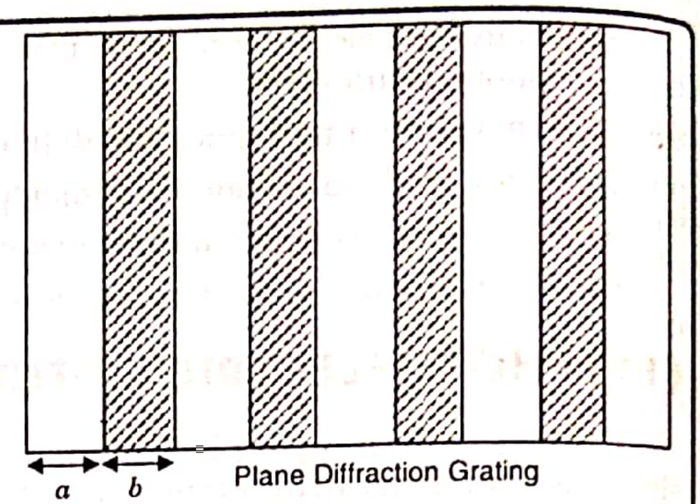

Fig. 7.

Normally used diffraction grating can have 6000 lines per cm (or 1500 lines per inch). Thus slit width or grating element is given by

$$d = a + b = \text{Grating Element} = \frac{1}{6000\ (\text{cm})^{-1}}$$

or $$d = 1667 \times 10^{-7}\ \text{cm}$$

This size is comparable to average wavelength of visible light. Hence plane diffraction grating can cause diffraction of light.

But X-rays have very small wavelength (0.1Å to 100Å). Thus for diffraction of X-rays, we need a diffraction grating whose slit width is comparable to this range. For this we have to increase number of lines for cm atleast 100 times than used for visible light. Such a fine grating has not been manufactured till date. Thus diffraction of X-rays seemed almost impossible in the absence of such fine gratings.

However Laue gave a suggestion that for diffraction of X-rays, a crystal can act as a three dimensional diffraction grating. It is because of the fact that in a crystal, atoms/molecules/ions are separated by a distance $\approx$ 1Å, which is comparable with wavelength of X-rays. Later on he performed an experiment with ZnS crystal and was successful to diffract X-rays from it.

(13) BRAGG PLANES IN A CRYSTAL

A crystal is a 3 dimensional arrangement of atoms/molecules/ions. So it is different from ordinary grating in respect that diffraction centres donot lie in a plane. Thus a crystal acts as a space grating rather than a plane grating. Within a crystal, there are many planes of atoms. Some planes are rich in atoms than others. This can be seen by two dimensional arrangement of atoms as shown in Fig. (8).

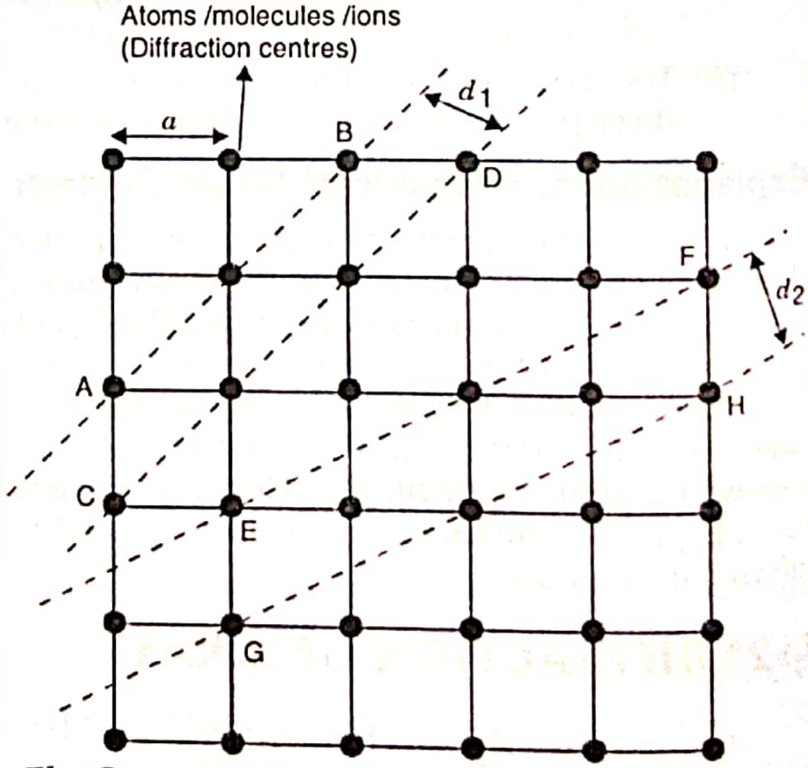

Fig. 8.

Suppose distance between two successive atoms is 'a'. Consider the lines such as AB and CD. The atoms of these lines are at a distance of $\sqrt{2}a$. Similarly in lines EF and GH the distance between adjacent atoms is $\sqrt{5}a$. These lines will become planes if we take into account 3-dimensional nature of crystal. Further distance between two adjacent planes in the set represented by AB, CD etc, is d_1 while distance between two adjacent plans in the set represented by EF and GH is d_2.

It is possible to choose planes in a crystal such that atoms in such planes are regularly spaced. These planes are called Bragg's planes. These planes are possible for Bragg Reflection. Thus Bragg's planes are those planes in the crystal, which are responsible for Bragg Reflection.

(14) BRAGG'S LAW OF X-RAY DIFFRACTION FROM A CRYSTAL

Consider a beam of X-rays of wavelength λ falling on a crystal at an angle of glance θ. Let d is distance between adjacent lattice planes. One X-ray is reflected from point A on Bragg's plane X_1X_1'. Another X-ray is reflected from point B on adjacent Bragg's plane. According to laws of reflection, angles of glance for incident and reflected ray are equal.

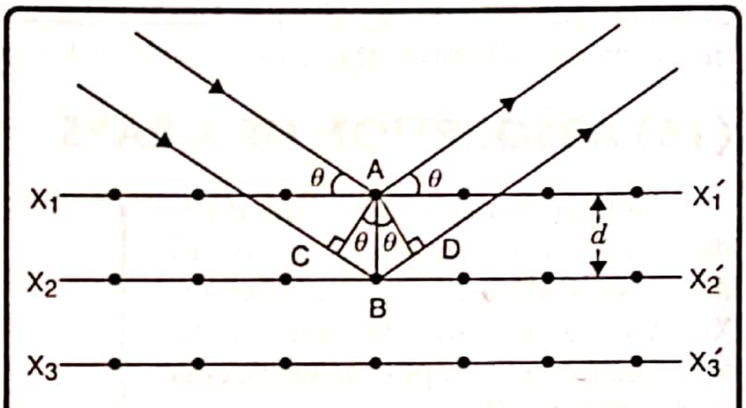

Fig. 9.

Let d = distance between Bragg's Planes

The path difference between the two reflected waves is given by

$$x = CB + BD \quad \text{...(11)}$$

In ΔABC $\qquad \frac{CB}{AB} = \sin\theta$

$\Rightarrow \qquad CB = d\sin\theta$

Similarly $\qquad BD = d\sin\theta$

Put in (11) $\Rightarrow \qquad x = 2d\sin\theta$

The two reflected X-rays will interfere constructively if the path difference is an integral multiple of λ.

i.e. $\qquad x = n\lambda$

$$\Rightarrow \qquad 2d\sin\theta = n\lambda \quad \text{...(12)}$$

Equation (12) is mathematical form of Bragg's Law and is called Bragg's equation. The value of n determines the order of reflection.

Thus Bragg is law can be defined as follows :

When X-rays fall on a crystal then diffraction maxima will be formed only when twice the lattice spacing multiplaced with sine of angle of glance is integral multiple of wavelength.

(15) X-RAY SPECTROMETER

The schematic design of an X-ray spectrometer is based on Bragg's analysis is shown in figure. A narrow beam of X-rays falls upon a crystal at an angle θ and a detector is placed so that it records those rays, whose angle of scattering is also θ. To achieve this crystal and detector are so adjusted that if crystal is rotated through angle θ, then detector rotates by double angle 2θ. Any X-ray reaching the detector therefore obeys Bragg's condition. As θ is varied, the detector will record intensity peaks corresponding to the orders predicted by Bragg's equation. If the spacing d between adjacent Bragg planes in the crystal is known, the X-ray wavelength λ may be calculated.

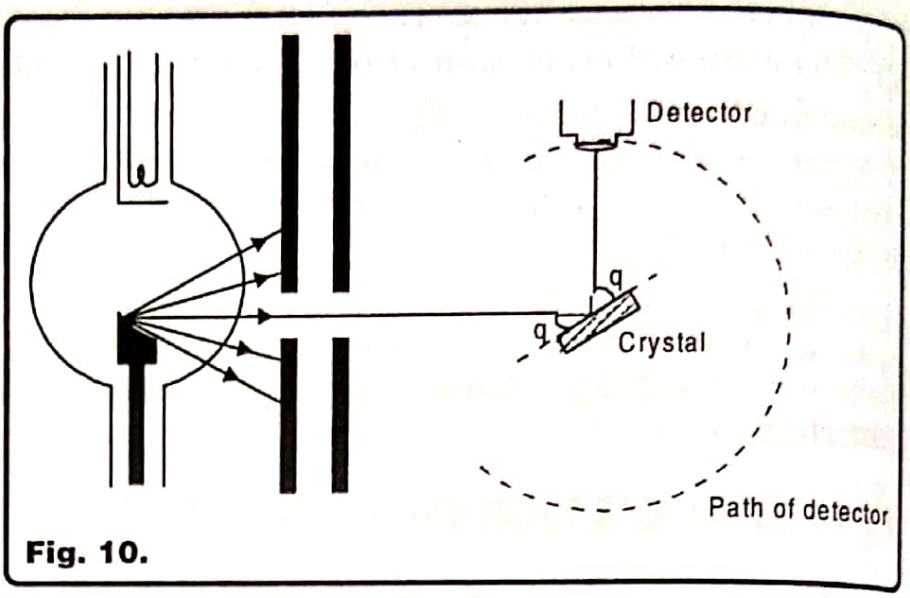

Fig. 10.

(16) ABSORPTION OF X-RAYS

When X-rays pass through a material, their intensity gradually decreases with distance. This is because X-rays suffer absorption in the substance chiefly by photoelectric effect and compton effect.

Let I is intensity of X-rays at a distance x inside the substance.

Let dI is decrease in intensity of X-rays when they further travel a distance dx in absorber.

The decrease in intensity is proportional to the intensity of X-rays at that point and the thickness of absorber.

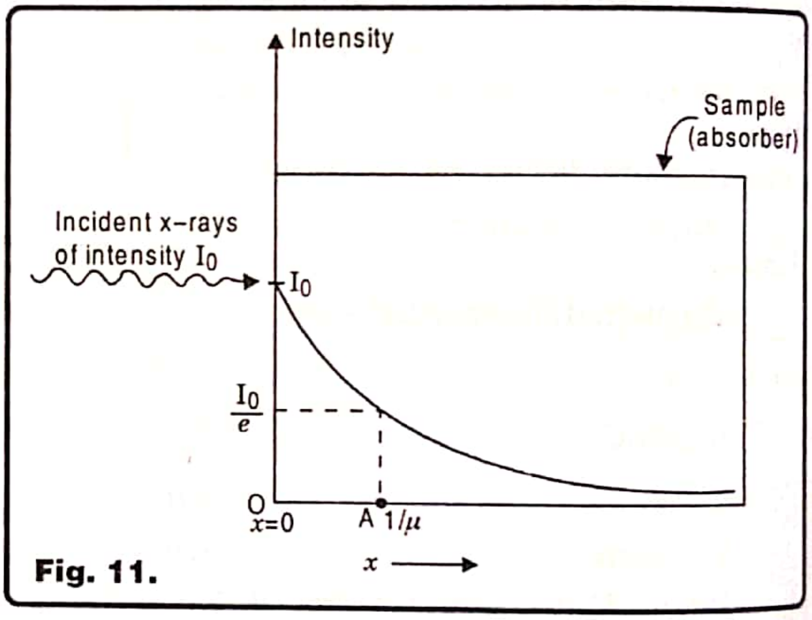

Fig. 11.

i.e. $$dI \propto I$$
$$\propto dx$$

or $$dI = -\mu I\, dx \qquad ...(13)$$

Here μ is a constant of proportionality called linear absorption coefficient. It depends upon the nature of substance. Larger value of μ means strong absorption.

From (13) $$\frac{dI}{I} = -\mu\, dx$$

If I_0 is intensity of X-rays incident at absorber (i.e. at $x = 0$) then above equation can be integrated to get

$$\int_{I_0}^{I} \frac{dI}{I} = -\mu \int_0^x dx$$

$$\ln(I) - \ln(I_0) = -\mu x$$

$\Rightarrow$
$$\ln\left(\frac{I}{I_0}\right) = -\mu x \qquad ...(14)$$

or
$$\boxed{I = I_0\, e^{-\mu x}} \qquad ...(15)$$

This equation shows that intensity decreases exponentially which agrees well with experimental plot shown in fig. (11).

Experimental data shows that X-ray absorption depends only on the number of atoms in the given thickness of substance. Thus $\frac{\mu}{\rho}$ = constant for a substance. Here ρ is density of material.

We define $\mu_m = \frac{\mu}{\rho}$ as mass absorption coefficient. Then $\mu = \rho\mu_m$

Hence equation (15) can be written as

$$I = I_0\, e^{\left(\rho x \frac{\mu}{\rho}\right)}$$

or
$$I = I_0\, e^{-(m\mu_m)} \qquad ...(16)$$

Here m is mass per unit area of the sample (absorbing sheet).

If we put $\mu = \frac{1}{x}$ in (15), we get

$$I = \frac{I_0}{e}$$

Thus linear absorption coefficient represents reciprocal of the thickness of the sample such that intensity of X-rays reduce to $\frac{1}{e}$ of the intensity of incident X-ray beam.

From equation (14) we can also write

$$x = \frac{\ln\left(\frac{I_0}{I}\right)}{\mu}$$

This equation gives absorption thickness required such that intensity reduces to I from incident value I_0.

(17) X-RAY RADIOGRAPHY

This technique is used for detecting & estimating the location and size of defects present in the material. The defects, which are detected by this method are helpful to design components which can withstand high temperature and pressures employed in power plants, atomic reactors, chemical and pressure vessels and oil refining equipments.

In this method short wavelength X-rays or γ-rays are allowed to penetrate through the material under test. The intensity of penetrating radiations is modified by passage through the material and by defects in the material. The transmitted radiations are exposed on the photographic film. The contrast on the developed film between the image of an area containing a defect and the image of defect free area of material permits the observer to distinguish the cracks or flaws present in the material.

X-ray flourography is another technique similar to X-ray Radiography. In this technique, the transmitted X-rays are projected on a fluorescent screen instead of a photographic plate. This method is widely used in industry for ready inspection of manufactured goods.

(18) APPLICATIONS OF X-RAYS

(*i*) **In Industry.** X-rays are widely used for testing materials in industry. X-ray photography techniques are used to study the structure of cellulose, rubber fibres, plastics etc., to distinguish between real and artificial gems and to detect pearls in oysters.

(*ii*) **In Medicine.** X-rays are playing an important role in medical science. These are used in the examination and in the diagnosis of diseases or fracture in the body. X-rays are also used in treatment of deadly diseases like cancer. This technique of treating cancer with X-rays is called Radiotherapy. These rays are used to kill cancerous tissue.

Example 1. *A crystal when studied with monochromatic X-rays of wavelength 0.58Å shows diffraction maxima for glancing angles of 6.45°, 9.15° and 13°. Calculate the interplanar spacings in the crystal.*

Solution. According to Bragg's law

$$\frac{d}{n} = \frac{\lambda}{2 \sin \theta}$$

(*i*) When $\theta = 6.45°$

$$\Rightarrow \qquad \frac{d}{n} = \frac{0.58}{2 \sin (6.45°)} = 2.58 \text{ Å}$$

(*ii*) When $\theta = 9.15°$

$$\Rightarrow \qquad \frac{d}{n} = \frac{0.58}{2 \sin (9.15°)} = 1.82 \text{ Å}$$

(*iii*) When $\theta = 13°$

$$\Rightarrow \qquad \frac{d}{n} = \frac{0.58}{2 \sin (13°)} = 1.29 \text{ Å}$$

Careful observation reveals that $\frac{d}{n}$ of case (*i*) is double that of case (*iii*). It means the reflections at $\theta = 6.45°$ and $\theta = 13°$ are from same set of parallel planes of spacings $d = 2.58$Å. For these planes at $\theta = 6.45°$ we have first order & at $\theta = 13°$ we are obtaining 2nd order diffraction.

The reflection at $\theta = 9.15°$ must be first maximum of diffraction due to another set of planes of spacing 1.82 Å. Remember it cannot be higher order 2nd and 3rd etc. because first order is most intense. It means if a higher order is observed then all orders lower to that should also be observed for same set of parallel planes.

Example 2. *The wavelength of L_α X-ray line of platinum (atomic number 78) is 1.321Å. An unknown substance emits L_α, X-ray of wavelength 4.174Å. Calculate the atomic number of the unknown substance. Given screening constant for L_α line is 7.4.* (*PTU Exams*)

Solution.
$$\frac{1}{\lambda} = R(z-\sigma)^2 \left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

$$= R(z-7.4)^2 \left(\frac{1}{4} - \frac{1}{9}\right) \qquad (\because \text{for } L_\alpha \text{ line } \sigma = 7.4,\ n_1 = 3,\ n_2 = 2)$$

$$= R(z-7.4)^2 \times \frac{5}{36}$$

$$\therefore \qquad \frac{\lambda_2}{\lambda_1} = \frac{(z_1-7.4)^2}{(z_2-7.4)^2}$$

$$\Rightarrow \qquad \frac{z_2-7.4}{z_1-7.4} = \sqrt{\frac{\lambda_1}{\lambda_2}}$$

$$\Rightarrow \qquad \frac{z_2-7.4}{z_1-7.4} = \sqrt{\frac{1.321}{4.174}} = 0.563$$

$\Rightarrow$ $z_2 = 47$ Thus unknown substance is silver.

Example 3. *If the potential difference across an X-ray tube is 10kV and the current through it is 2mA, calculate the velocity of electrons, with which they strike the target ?*

Solution. By work energy theorem

Gain in kinetic energy = electrostatic work done

$$\Rightarrow \qquad \frac{1}{2}mv^2 = eV$$

$$\Rightarrow \qquad v = \sqrt{\frac{2eV}{m}}$$

$$\Rightarrow \qquad v = \sqrt{\frac{2\times 1.6\times 10^{-19}\times 10\times 10^3}{9.1\times 10^{-31}}} = 5.93\times 10^7\ m/s$$

Example 4. *X-rays of wavelength 0.71 Å are reflected from the (110) plane of a rock salt crystal (a = 2.82 Å). Calculate the glancing angle corresponding to second order reflection.*

Solution. $$d = \frac{a}{\sqrt{h^2+k^2+l^2}} = \frac{2.82\ \text{Å}}{\sqrt{1^2+1^2+0}} = 2.0\ \text{Å}$$

By Bragg's Law $$\sin\theta = \frac{n\lambda}{2d} = \frac{2\times 0.71\ \text{Å}}{2\times 2.0\ \text{Å}}$$

$$\Rightarrow \qquad \theta = 20°42'$$

Example 5. *Calculate the highest reflecting order that could be observed with a radiation of wavelength 1.2Å from a quartz crystal, which has plane spacing d = 4.225 Å.*

Solution. By Bragg's Law $2d\sin\theta = n\lambda$

or $$\sin\theta = \frac{n\lambda}{2d}$$

But $$\sin\theta \le 1$$

$$\Rightarrow \qquad \frac{n\lambda}{2d} \le 1$$

$$\Rightarrow \qquad n \le \frac{2d}{\lambda}$$

or $$n \le \frac{2\times 4.225}{1.2}$$

$$n \le 7.04$$

Thus highest order observable is 7.

Example 6. *Calculate the maximum frequency of continuous X-rays emited from an X-ray tube whose operating voltage is 50kV.*

Solution. $$\lambda_{min} = \frac{hc}{eV} \Rightarrow \nu_{max} = \frac{c}{\lambda_{min}} = \frac{eV}{h}$$

$$\Rightarrow \nu_{max} = \frac{1.6 \times 10^{-19} \times 50 \times 10^3}{6.625 \times 10^{-34}} = 12.06 \times 10^{18}\ \text{Hz}$$

Exercise 1. An X-ray tube operates at 18 *k*V. Find the maximum speed of electrons striking the target. [Ans. $8 \times 10^7\ ms^{-1}$]

Exercise 2. Which element has K_α line of wavelength 1.7825 Å. [Ans. Cobalt]

Exercise 3. If K_α radiation of M_0 (Z = 42) has wavelength 0.71Å, find the wavelength of corresponding radiation of copper (Z = 29). [Ans. 1.52Å]

Exercise 4. The potential difference between cathode and the target in the coolidge tube is 120 *k*V. Find the momentum of the photon corresponding to minimum wavelength of X-rays emitted by the tube. [Ans. $6.4 \times 10^{-23}\ kgms^{-1}$]

Example 7. *The wavelength of K_α line is 1.54 Å for copper. Calculate the ionisation potential of the K shell electron of copper. Given energy of L shell is –0.923 keV.*

Solution.
$$\lambda_\alpha = 1.54 \times 10^{-10}\ m$$
$$E_L = -0.923\ keV$$
$$\therefore \quad E_\alpha = \frac{hc}{\lambda_\alpha} = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{1.54 \times 10^{-10} \times 1.6 \times 10^{-19}} = 8.072\ keV$$

This is the energy of X-rays corresponding to K_α member.

Now K_α line is obtained, when electron jumps from L shell to K shell

$$\Rightarrow \quad E_L - E_K = E_\alpha$$
$$\Rightarrow \quad E_K = E_L - E_\alpha$$
$$= -0.923 - 8.072 = -8.995\ keV$$

Thus ionisation energy of K-shell

$$= E_\infty - E_K$$
$$= 0\ eV - (-8.995\ keV) = 8.995\ keV$$

Example 8. *Calculate the longest wavelength, that be analysed by a rock salt crystal of spacing d = 2.82 Å in the first order (ii) Second order.*

Solution. Given $d = 2.82$ Å

By Bragg's law $$\lambda = \frac{2d \sin\theta}{n}$$

For longest wave length $\sin\theta = 1$

$$= \lambda_{max} = \frac{2d}{n}$$

(*i*) For first order $n = 1$

$$\Rightarrow \quad \lambda_{max} = \frac{2 \times 2.82}{1} = 5.64\ \text{Å}$$

(*ii*) For second order

$$\lambda_{max} = \frac{2d}{2} = d = 2.82\ \text{Å}$$

Example 9. *An X-ray tube operates at 20 kV. A particular electron loses 5% of its kinetic energy to emit an X-ray photon at first collision. Find the wavelength corresponding to this photon.*

Solution. Kinetic enegy acquired by electron = $eV = 20 \times 10^3$ eV

The energy of photon = $0.05 \times 20 \times 10^3 = 10^3$ eV

Thus
$$\frac{hc}{\lambda} = 10^3 \text{eV}$$

$$\Rightarrow \qquad \lambda = \frac{hc}{10^3 \text{eV}} = \frac{(4.14 \times 10^{-5}\ \text{eVs}) \times 3 \times 10^8\ m/s}{10^3\ \text{eV}} = 1.24\ nm$$

Example 10. *An X-ray tube is operated at 20kV and the current through the tube is 0.5 mA. Find (a) the number of electrons hitting the target per second. (b) The energy falling on the target per second as kinetic energy of the electrons (c) The cut off wavelength of X-rays emitted.*

Solution. (*a*) I = ne where n = number of electrons hitting the target per second.

$$\Rightarrow \qquad n = \frac{I}{e} = \frac{0.5 \times 10^{-3}}{1.6 \times 10^{-19}} = 3.1 \times 10^{15}\ s^{-1}$$

(*b*) The kinetic energy of an electron reaching the target is

$$\text{KE} = eV$$

Thus energy falling on target in one second is

$$\text{E} = neV$$
$$= \text{IV} = 0.5 \times 10^{-3} \times 20 \times 10^3 = 10\text{J}/s$$

(*c*)
$$\frac{hc}{\lambda_{min}} = eV$$

$$\Rightarrow \qquad \lambda_{min} = \frac{hc}{eV}$$

$$= \frac{1242\ \text{eV}-nm}{e\,(20 \times 10^3)\ \text{V}} = 0.062\ nm$$

Example 11. *Find the constants a & b in the Moseley's equation $\sqrt{\nu} = a\,(z - b)$ from the following data:*

Element	*Z*	*Wave length of K_α, X-ray*
Mo	*42*	*71pm*
Co	*27*	*178.5 pm*

Solution. Moseley's equation is

$$\sqrt{\nu} = a\,(z - b)$$

$$\Rightarrow \qquad \sqrt{\frac{c}{\lambda_1}} = a\,(z_1 - b) \qquad \text{...(i)}$$

and
$$\sqrt{\frac{c}{\lambda_2}} = a\,(z_2 - b) \qquad \ldots(ii)$$

From (*i*) & (*ii*)

$$\sqrt{c}\left(\frac{1}{\sqrt{\lambda_1}} - \frac{1}{\sqrt{\lambda_2}}\right) = a\,(z_1 - z_2)$$

or
$$a = \left(\frac{\sqrt{c}}{z_1 - z_2}\right)\left(\frac{1}{\sqrt{\lambda_1}} - \frac{1}{\sqrt{\lambda_2}}\right)$$

$$= \left(\frac{\sqrt{3 \times 10^8}}{42 - 27}\right)\left(\frac{1}{\sqrt{71 \times 10^{-12}}} - \frac{1}{\sqrt{178.5 \times 10^{-12}}}\right)$$

$$= 5.0 \times 10^7\ (\text{Hz})^{\frac{1}{2}}$$

Dividing (*i*) by (*ii*)

$$\Rightarrow \qquad \sqrt{\frac{\lambda_2}{\lambda_1}} = \frac{z_1 - b}{z_2 - b}$$

$$\Rightarrow \qquad \sqrt{\frac{178.5}{71}} = \frac{42 - b}{27 - b}$$

$$\Rightarrow \qquad b = 1.37$$

Example 12. *Find the atomic spacing in a crystal of rock salt (NaCl) for which unit cell is simple cubic and there is one NaCl molecule for unit cell. The density of rock salt is 2.16 × 10³ kgm⁻³ and average masses of Na and Cl atoms are respectively 3.82 × 10⁻²⁶ kg and 5.89 × 10⁻²⁶ kg.*

Solution. Let M = molecular weight of crystal

ρ = density of crystal

Volume per mole of crystal $= \dfrac{M}{\rho}$

There are 2N ions in 1 mole ($\because$ each unit cell contains one Na^+ and one Cl^- ion). Here $N = 6.023 \times 10^{23}$.

Thus volume associated with each ion $= V' = \dfrac{V}{2N} = \dfrac{M}{2\rho N}$ ...(*i*)

Let d = distance between two ions = plane spacing

$\therefore$ Volume of elementary cube of NaCl = d^3 ...(*ii*)

Comparing (*i*) & (*ii*) we get

$$d^3 = \frac{M}{2\rho N}$$

$$\Rightarrow \qquad d = \left(\frac{M}{2\rho N}\right)^{1/3}$$

For NaCl crystal $$\frac{M}{N} = \frac{M_{Na}}{N} + \frac{M_{Cl}}{N}$$

$$= 3.82 \times 10^{-26} + 5.89 \times 10^{-26} = 9.71 \times 10^{-26}\ kg$$

and $$\rho = 2.16 \times 10^3\ kg/m^3$$

Thus $$d = \left(\frac{9.71 \times 10^{-26}}{2 \times 2.16 \times 10^3}\right)^{1/3} = 2.82\ \text{Å}$$

Exercise 5. The K_α X-ray of molybdenum has wavelength 71 *pm*. If the energy of a molybdenum atom with a K electron knocked out is 23.32 *keV*, what will be the energy of this atom when an L electron is knocked out ? [Ans. 5.82 *keV*]

Exercise 6. If the operating potential in an X-ray tube is increased by 1%, by what percentage does the cut off wavelength decrease ? [Ans. $\approx$ 1%]

Exercise 7. An X-ray tube operates at 40 *kV*. Suppose the electron converts 70% of its energy into a photon at each collision. Find the lowest three wavelengths emitted from the tube. Neglect the energy imparted to the atom, with which the electron strikes.

[Ans. 44.3 *pm*, 148 *pm*, 493 *pm*]

Exercise 8. The K_β X-ray of argon has wavelength of 0.36 *nm*. The minimum energy needed to ionize an argon atom is 16 *eV*. Find the energy needed to knock out an electron from the K-shell of an argon atom. [Ans. 3.47 *keV*]

Exercise 9. What thickness of lead will attenuate a beam of 0.4 MeV X-rays by a factor of 2 ? Give $\mu = 2.3\ cm^{-1}$. [Ans. 0.30 cm]

Exercise 10. Heat at the rate of 200 W is produced in an X-ray tube operating at 20 *kV*. Find the current in the circuit. Assume that only a small fraction of the kinetic energy of electrons is converted into X-rays. [Ans. 10 mA]

Exercise 11. A free atom of iron emits K_α X-rays of energy 6.4 *keV*. Calculate the recoil kinetic energy of the atom. Mass of an iron atom = 9.3×10^{-26} kg. [Ans. 3.9×10^{-4} *eV*]

SHORT ANSWER TYPE QUESTIONS

Q.1. What is the importance of Moseley's Law ?

Ans. According to Moseley's law, the frequency of characteristic X-ray photon is directly proportional to square of effective atomic number. Thus if we determine frequency of X-ray line experimentally from an unknown substance, then its atomic number can be calculated from Moseley's Law. Hence the unknown substance can be identified.

Q. 2. How will you explain the fact that diffracted X-rays from a crystal are observed all around the crystal ?

Ans. Fig. (12) shows arrangement of lattice points in a crystal.

Let a beam of X-rays is incident on the crystal. If the Bragg's condition is satisfied by the planes AA′, then the reflected X-rays will go in direction (1) making angle θ_1 with this plane.

If X-rays satisfy Bragg's condition for planes BB′, then reflected X-rays will go in direction (2), making an angle θ_2 with plane BB′.

Similarly it Bragg's condition is satisfied by planes CC′, then reflected X-rays will be observed in direction (3), making an angle θ_3 with CC′.

In general there will be more than one distinct set of parallel planes which will satisfy Bragg's condition for the incident X-ray beam. Hence diifracted X-rays will be observed all around the crystal.

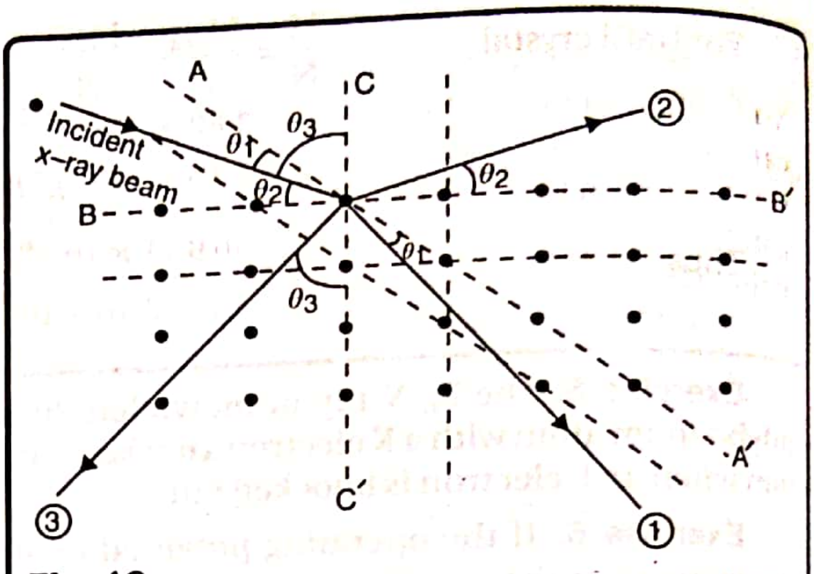

Fig. 12

Q. 3. Why the target used in X-ray tube should have high atomic number, high melting point and large value of thermal conductivity.

Ans. Atomic number should be high so that electrons can be stopped quickly by the target. The melting point should be high so that it can with stand large amount of heat produced in the generation of X-rays without meeting. The thermal conductivity should be large so that, it may pass on the heat produced to the coolant, as early as possible.

Q. 4. What is the limitation of Laue's method ?

Ans. It cannot be used to find lattice spacing. It can only tell the symmetry properties of crystal.

Q. 5. Why X-rays donot show diffraction from a plane transmission Grating ?

Ans. Because the wavelength of X-rays in very small and it is not feasible to make a plane transmission grating, with slit width comparable to their wavelength.

Q. 6. What is Bremsstrahlung ?

Ans. When a rapidly moving electron is suddenly brought to rest, then due to de-acceleration, it produces radiations. Radiations produced under these circumstances are called Bremsstrahlung.

Q. 7. Hydrogen cannot emit X-rays ? Why ?

Ans. The energy of an electron in nth orbit of hydrogen atom is

$$E_n = -\frac{13.6 \text{ eV}}{n^2}$$

Thus if an electron jumps down from highest excited state ($n = \infty$) to the lower most state ($n = 1$) then energy of emitted photon will be maximum or the wavelength of emitted photon will be minimum. This wavelength is given as

$$\frac{\lambda_C}{\lambda_{min}} = \frac{-13.6}{\infty} + \frac{13.6}{1}$$

$$= 13.6 \text{ eV}$$

$$= 13.6 \times 1.6 \times 10^{-19} \text{ J}$$

$$\Rightarrow \quad \lambda_{min} = \frac{\lambda_C}{13.6 \times 1.6 \times 10^{-19}}$$

$$\lambda_{min} = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{13.6 \times 1.6 \times 10^{-19}} = 913 \text{ Å}$$

This wavelength lies in ultraviolet region. Hence Hydrogen cannot emit X-rays.

Q. 8. What are the limitations of Roentgen tube ?

Ans. This tube was used by Roentgen to produce X-rays. However in this tube intensity of X-rays cannot be controlled. The quality of X-rays cannot be controlled. The quality of X-rays cannot be controlled. The accelerating voltage required is very high. Further the high voltage was produced in tube using induction coil, which produces high voltage pulses. These pulses can produce large amount of heat on cathode & may damage it. These defects were removed in collidge tube.

Q. 9. Write some important applications of X-rays.

Ans. (*i*) These rays are used to locate internal structure of a body. These are also used to locate imperfections in the mould forgings and castings.

(*ii*) These are used in metallurgy to analyse the structure various alloys.

(*iii*) These are used in studying the molecular grouping in rubber and plastic industry.

(*iv*) These are used to study the crystal structure of various solids.

(*v*) These are used to study atomic structure.

(*vi*) These are used to find atomic number of various elements and their identification.

(*vii*) In X-ray therapy to cure skin diseases, malignant tumors, etc.

Q. 10. X-rays cannot show pair production. Why ?

Ans. The minimum energy required for as photon to show pair production is equal to twice the rest mass of an electron, which is equal to 1.02 MeV. Since the energy of X-ray photon is in the keV range, hence X-rays donot show pair production.

QUESTIONS

1. Differentiate between continuous and characteristic X-ray.
2. What is Moseley's law ? Give its importance.
3. How do X-rays differ from γ-rays ? Explain the origin of continuous and characteristic X-rays.
4. State and explain Moseley's law.
5. What is Bragg's law of X-ray diffraction ? Derive it.
6. Describe Bragg's Spectrometer. Explain how it is used to determine the wavelength of X-rays ?
7. What are applications of X-rays in practical life ?
8. Discuss X-rays radiography.