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classmate
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Superconductivity Revision

1. Define - Superconductors & Superconductivity critical temp / transition temp, critical mag. field, critical current, cooper pair.

2. When a material becomes superconductor $T < T_c$ (i) $R = 0, \rho = 0$ (ii) $B = 0$

Here B will include the sum of the applied and developed flux. Since it comes out to zero inside the superconducting sample &

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) = 0 \Rightarrow \chi = \frac{\vec{M}}{\vec{H}} = -1$$

(as $\mu_0 \neq 0$)

∴ Superconductors will have to behave like perfectly diamagnetic substance.

Now the salient feature of diamagnetism is that electrons are paired with equal & opp spin. ∴ In superconductors e^-s are paired, known as Cooper pairs. In a Cooper pair, spin and linear momenta of the e^-s will be equal & opp..

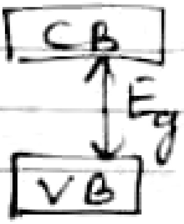
In a normal conductor, e^-s are not paired, rather they move individually.

Ideal Conductor or Perfect Conductor

$R, \rho = 0$ but $B_{ini} = B_{final}$

i.e. mag. nature of the perfect conductor will not change on changing the temp. Whereas in superconductors when $T < T_c$, B should change to zero, whatever may be its initial value, final value should be zero when temp. is lowered.

3. In insulators there is energy b/w VB & CB



independent of temp

In superconductors, energy gap is not b/w VB & CB, rather it the gap b/w two types of e^-s in CB — some are paired & settle at the bottom of CB and some are unpaired & are close to top of CB ~~or~~ [illegible] are above the Fermi level.

E_g in this case depends on temp. and is max. at 0K, when all the e^-s are paired.

$$E_g(0K) = 3.54\, K_B T_c$$

$K_B = 1.38 \times 10^{-23}$ J/K
$T_c \rightarrow$ critical temp.

(3) Since electrons are paired in Superconductors & have opp. Spin, when we apply mag. field to the sample, mag. force will act on the electrons but in opp. dir. Keep on increasing the field at one stage the pair will break.

Max. Value of mag field that can be applied safely to a superconductor without destroying superconductivity is known as Critical mag. field, H_c.

H_c also depends on temperature.

$$H_c(T) = H_c(0)\left[1 - \frac{T^2}{T_c^2}\right]$$

||arly $B_c(T) = B_c(0)\left[1 - \frac{T^2}{T_c^2}\right]$

$H_c(0)$, $B_c(0)$ are values at 0K.

Mag. field is also generated when we will pass current thro. a superconductor So mag. of current should be such that mag. field developed should be less than H_c.

$\therefore$ Critical current is the max. value of current that can be passed safely thro. a superconductor without destroying superconductivity $I_c(T)$. If there is a long current carrying superconducting wire then $I_c(T) = 2\pi r H_c(T)$

(Silsbee's Rule)

$$J_c(T) = \frac{I_c(T)}{\text{Area}}$$, Critical current density

Also note that in case of various isotopes of a given element,

$$T_c \propto \frac{1}{\sqrt{M}}, \quad M \text{ is isotopic mass}$$

or $T_c\sqrt{M}$ = constant

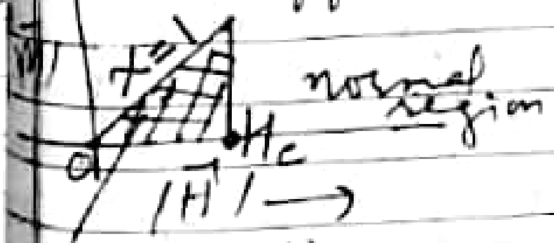
or $T_{c_1}\sqrt{M_1} = T_{c_2}\sqrt{M_2} = T_{c_3}\sqrt{M_3}$

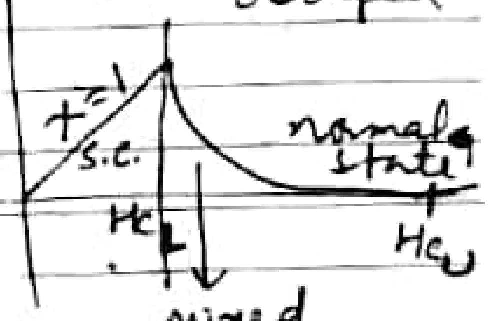
This is isotope effect. Different isotopes will diff. value of critical temp. at which they will become superconductor

Superconductors are of two types

Type I / Soft & Type II / Hard

Obey Meissner effect completely → partially obeyed





superconducting region

Usually are pure elements

no practical use

change from S.C. state to normal state is abrupt like step for.

Change is gradual. mixed state

$H_{c_1} \rightarrow$ process begins

$H_{c_2} \rightarrow$ process completes

Upto H_{c_1} meissner effect is obeyed & beyond that it is not.

→ Alloys, used commercially

Meissner effect is also known as flux expulsion. Remember Meissner effect is for bulk superconductors whose one of the dimensions is greater than penetration depth (λ).

Penetration depth (λ) inside a superconductor is actually a distance inside the sample where we can feel the presence of mag. flux and is defined as the distance inside the sample at which flux becomes $\frac{1}{e}$ times the value of the flux at the surface of superconductor. λ is also a fn. of temp, T.

$$\lambda(T) = \frac{\lambda(0)}{\left[1-\frac{T^4}{T_c^4}\right]^{\frac{1}{2}}}$$

$\lambda(0)$ at 0K & $\lambda(0) = \sqrt{\frac{m}{\mu_0 n_s e^2}}$

$m = 9.1 \times 10^{-31}$ kg

$\mu_0 = 4\pi \times 10^{-7}$ A/m

$n_s \rightarrow$ no. of superconducting e^-s

$e = 1.6 \times 10^{-19}$ C

(6) In case of thin films, whole of the flux will penetrate inside the sample & $B \neq 0$

London eqns can explain both ~~Flux Expulsion (Meissner effect)~~ & Flux Penetration

London eqns also give us the substitute of Ohm's law for superconductors. Ohm's law is not valid for superconductors

derive these

$$\frac{d\vec{J}}{dt} = \frac{n_s e^2}{m}\vec{E} \quad (1)$$

$$\vec{\nabla}\times\vec{J} = -\frac{n_s e^2}{m}\vec{B} \quad (2)$$

As $\vec{\nabla}\cdot\vec{B} = 0$ & div(curl) = 0

$\therefore$ $\vec{B}$ can be written as curl of a vector fld

$\therefore$ $\vec{B} = \vec{\nabla}\times\vec{A}$, $\vec{A}$ is known as mag. vector potential

from (2), $\vec{J} = -\frac{n_s e^2}{m}\vec{A}$

just like $\vec{J} = \sigma\vec{E}$ (ohm's law) for conductors.

$\vec{A}$ plays the same role in mag field as is played by scalar pot. V in electric field.

Also note $B = B_0 e^{-x/\lambda}$ for superconductors

(derive this expression)

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(7) <u>BCS Theory</u> – Two e^-'s which usually repel each other, start attracting each other at $T<T_c$ due to the intermediate vibrating lattice ions and thus form a pair known as Cooper pair.

$$e^- \text{---} \underset{\text{lattice ion}}{\oplus} \text{---} e^- \quad \text{at } T<T_c$$

Paired e^- → Bosons (spin 0)
Single e^- → Fermion (spin $\pm\frac{1}{2}$)
No. of Cooper pairs is max at 0k and is min. at $T=T_c$

(8) <u>Persistent Currents</u> → mag. of the current remains same & is not diminished. $\therefore I(t)=I(0)$ as $R=0$ in superconductors & from $I(t)=I(0)e^{-\frac{R}{L}t}$ (conductor)

$$\Rightarrow I(t)=I(0) \rightarrow \text{persistent current}$$

(9) Since E_g is max at 0K and if we apply energy (maybe a photon of energy $h\nu$) equal to E_g, Cooper pair will break

$$h\nu = E_g = 3.54\, k_B T_c \quad (\text{at } 0K)$$

from here, we can find ν or λ as required.