

## 8

# Projections of Points

## 8.1 INTRODUCTION

A point may be situated in space, in any one of the four quadrants formed by two principal planes of projection or may lie in any one or both of them. Its projections are obtained by extending projectors perpendicular to the planes.

The two principal planes are made to lie in one plane by holding VP and rotating HP through  $90^\circ$  clockwise, so that the first and third quadrants are opened out. Then the projections are shown on a flat surface in their respective positions either above or below or in  $xy$ .

## 8.2 A POINT IS SITUATED IN THE FIRST QUADRANT

The pictorial view Fig. 8.1(a) shows a point A is situated above the HP and in front of the VP i.e. in the first quadrant. When the point is viewed in the direction of  $l$ , the view from front  $a'$  is obtained as the intersection point between the ray of sight through A and the VP. When the point is viewed in the direction  $m$ , the view from above or top  $a$  is obtained as the intersection point between the ray of sight and the HP.  $a'$  its front view and  $a$  the top view. Hold VP and rotate HP  $90^\circ$  in the clockwise, these projections are seen in the Fig. 8.1(b). The front view  $a'$  is above  $xy$  and top view  $a$  below it.

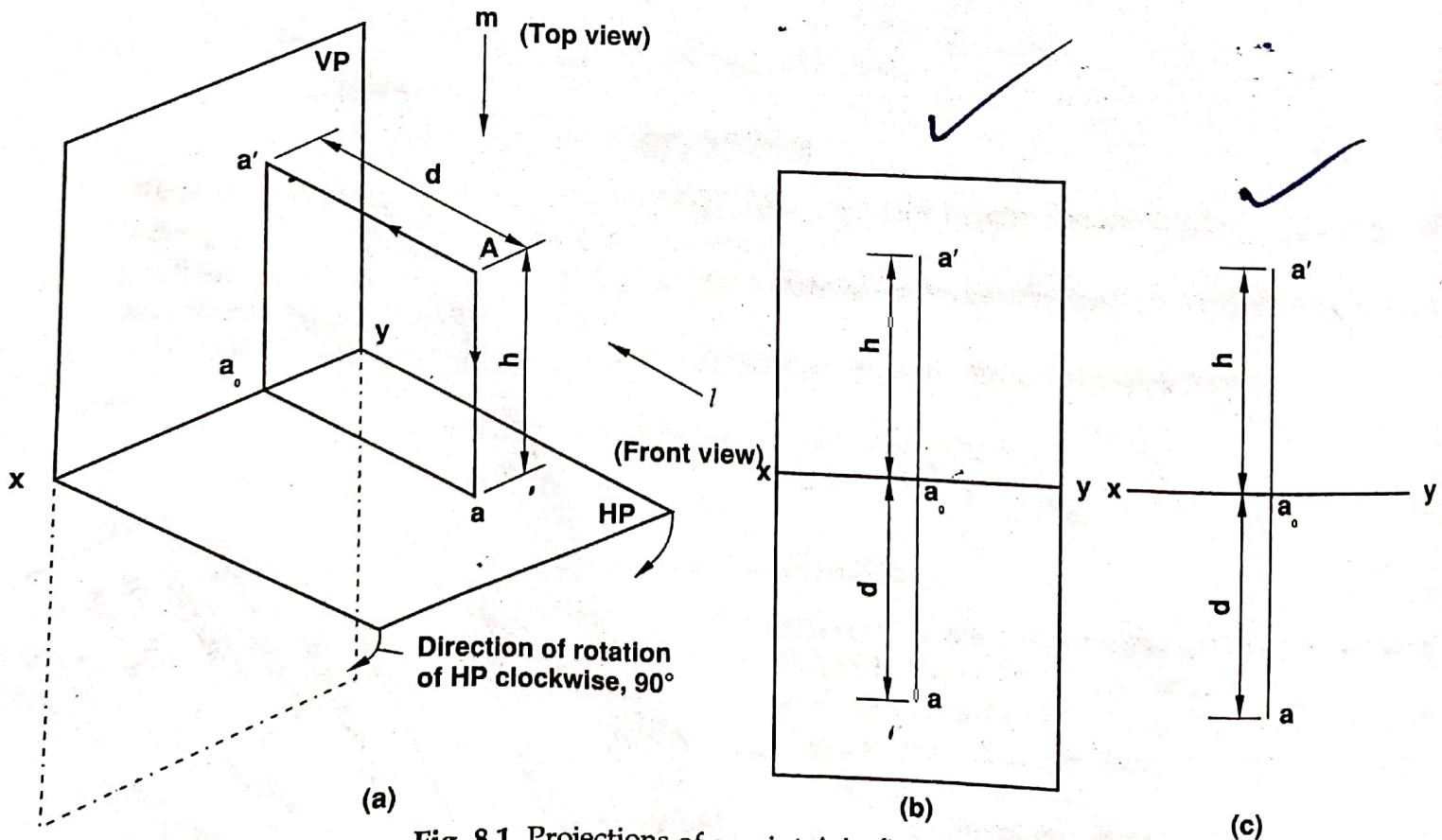


Fig. 8.1 Projections of a point A in first quadrant

The line joining  $a'$  and  $a$  (which is called as projector), intersect  $xy$  at right angles at a point  $a_0$ . It is quite evident from pictorial view that  $a'a_0 = Aa$ ; i.e., distance of front view from  $xy$  = distance of  $A$  from the HP viz.  $h$ . Similarly,  $aa_0 = Aa'$  i.e., distance of the top view from  $xy$  = distance of  $A$  from the VP viz.  $d$ . Fig. 8.1 (c), shows only the relative positions of the views, as it is customary not to show the planes of projection.

The following points may be noted from the study of Fig. 8.1.

- The line  $xy$  is the intersection line between HP and VP, as shown in Fig. 8.1(a). In Figs 8.1(b) and 8.1(c), the line is represented by  $xy$ , which is known as the reference line or common axis. Actually,  $xy$  is the line about which the rotation of the plane is made.
- It is customary to use capital letters to specify the position of the points in space and small letters for their projections. As an example, for the point  $A$  in space, the views from front, top and side are represented by  $a'$ ,  $a$  and  $a''$ .

### 8.3 A POINT IS SITUATED IN THE SECOND QUADRANT

As shown in Fig. 8.2 (a), a point  $B$  is above the HP and behind the VP i.e., in the second quadrant.  $b'$  and  $b$  are the front and top views obtained on VP and HP by viewing the points in the direction  $l$  and  $m$  respectively. Fig. 8.2 (b) shows the relative positions of the views. These are obtained by rotating the HP clockwise till it coincides with the VP. It may be noted that both the views are seen above reference line  $xy$ . Also  $b'b_0 = Bb$  and  $bb_0 = Bb'$ .

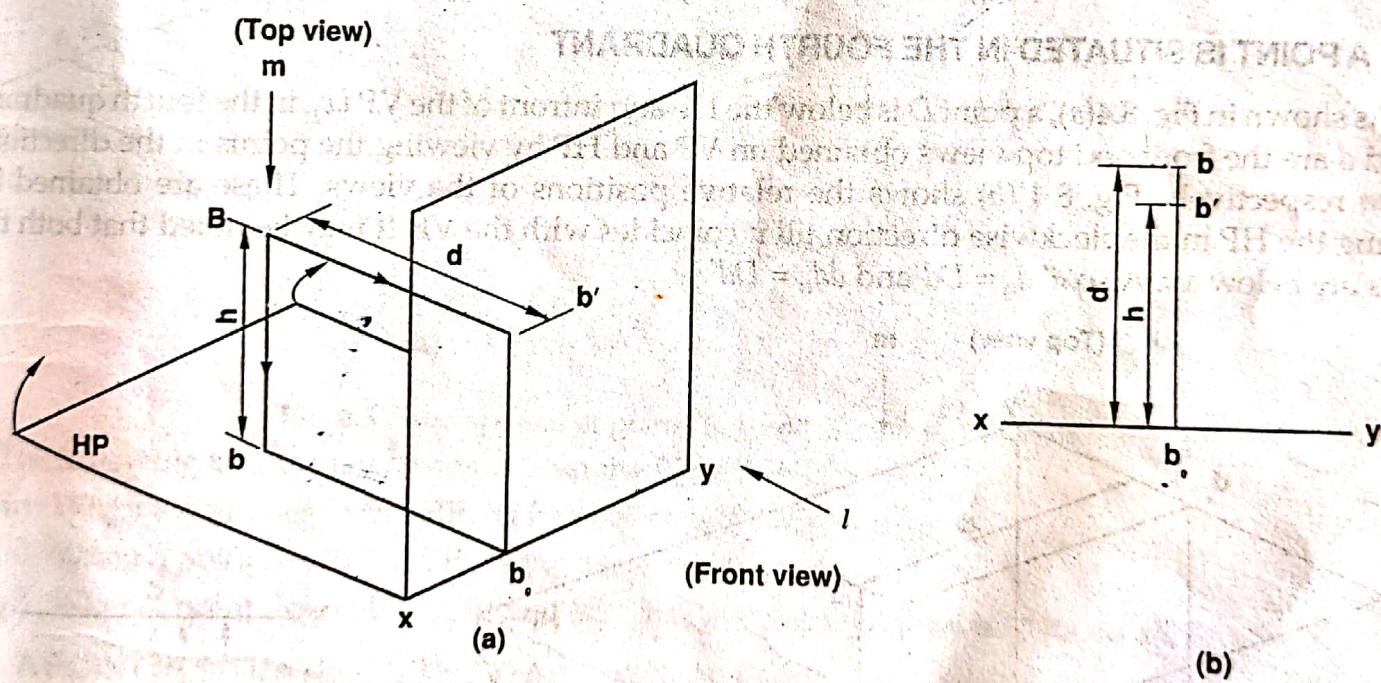


Fig. 8.2 Projections of a point  $B$  in second quadrant

### 8.4 A POINT IS SITUATED IN THE THIRD QUADRANT

As shown in Fig. 8.3(a), a point  $C$  is below the HP and behind the VP i.e., in the third quadrant.  $c'$  and  $c$  are the front and top views obtained on VP and HP by viewing the points in the direction  $l$  and  $m$  respectively. Here, it is assumed that both HP and VP are transparent. Fig. 8.3 (b) shows the relative positions of the views. These are obtained by rotating the HP clockwise till it coincides with the VP. It may be noted that front view  $c'$  is below the  $xy$  and top view  $c$  above the  $xy$ . Also  $c'c_0 = Cc$  and  $cc_0 = Cc'$ .

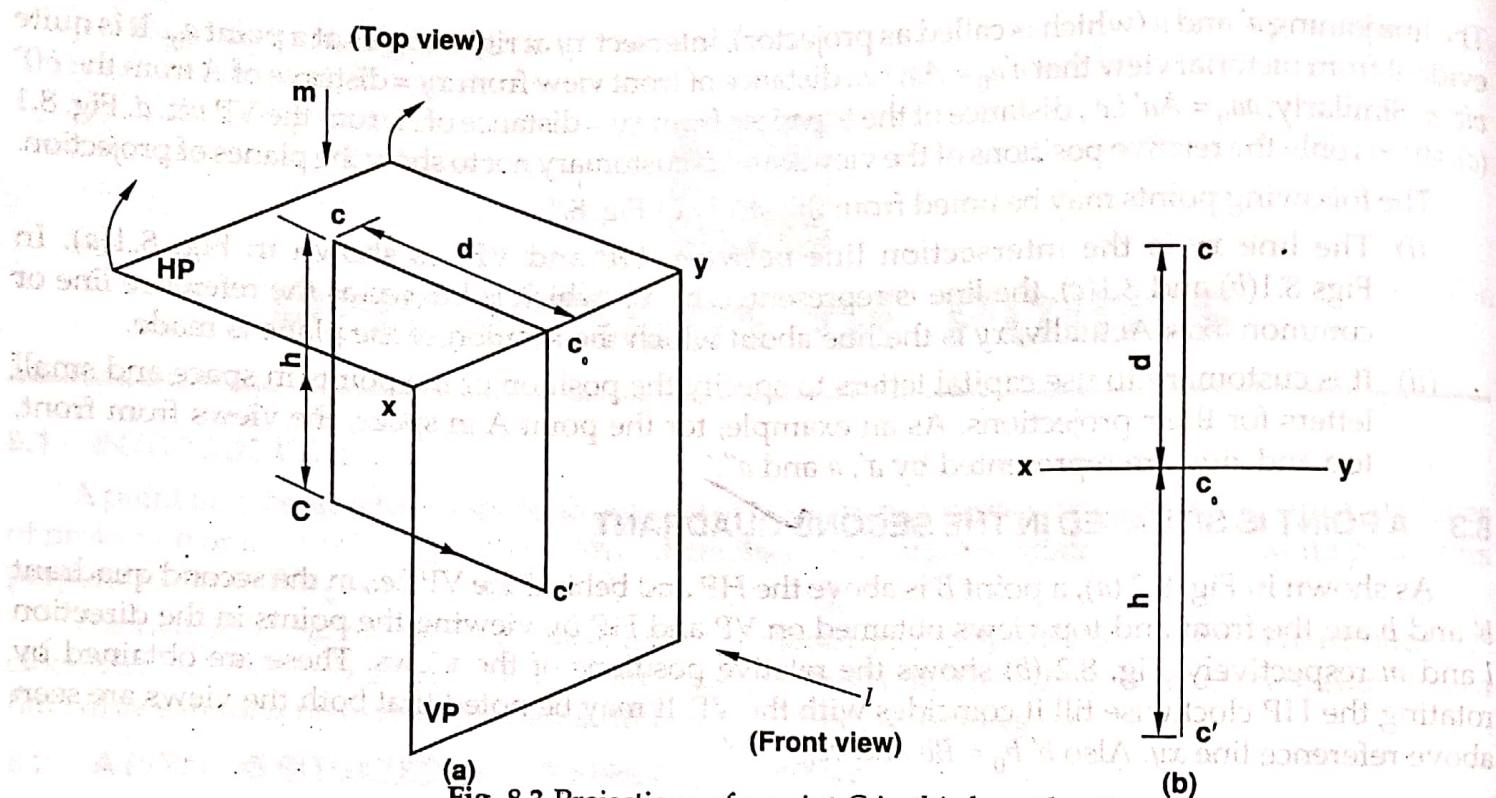


Fig. 8.3 Projections of a point  $C$  in third quadrant

## 8.5 A POINT IS SITUATED IN THE FOURTH QUADRANT

As shown in Fig. 8.4(a), a point  $D$  is below the HP and in front of the VP i.e., in the fourth quadrant.  $d'$  and  $d$  are the front and top views obtained on VP and HP by viewing the points in the direction  $l$  and  $m$  respectively. Fig. 8.4 (b) shows the relative positions of the views. These are obtained by rotating the HP in the clockwise direction till it coincides with the VP. It may be noted that both the views are below  $xy$ . Also  $d' d_0 = Dd$  and  $dd_0 = Dd'$ .

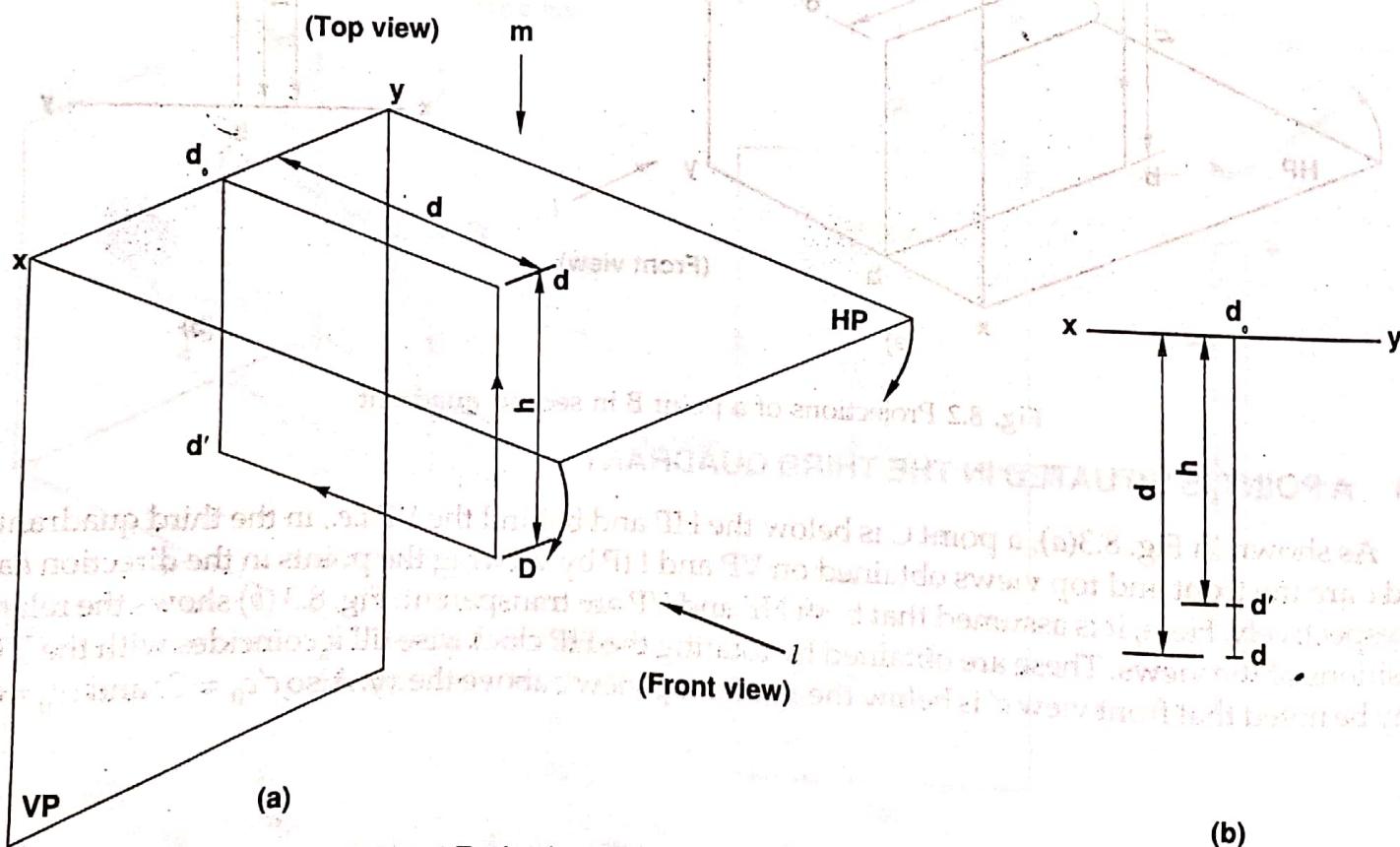


Fig. 8.4 Projections of a point  $D$  in fourth quadrant

## 8.6 SPECIAL CASES

- (i) A point E, is situated in the HP and in front of the VP.
- (ii) A point F, is situated in the VP and above the HP.
- (iii) A point G, is situated both in HP and VP.

All the three cases are shown in Fig. 8.5 (a). Fig. 8.5 (b) shows the relative positions of the views for each case.

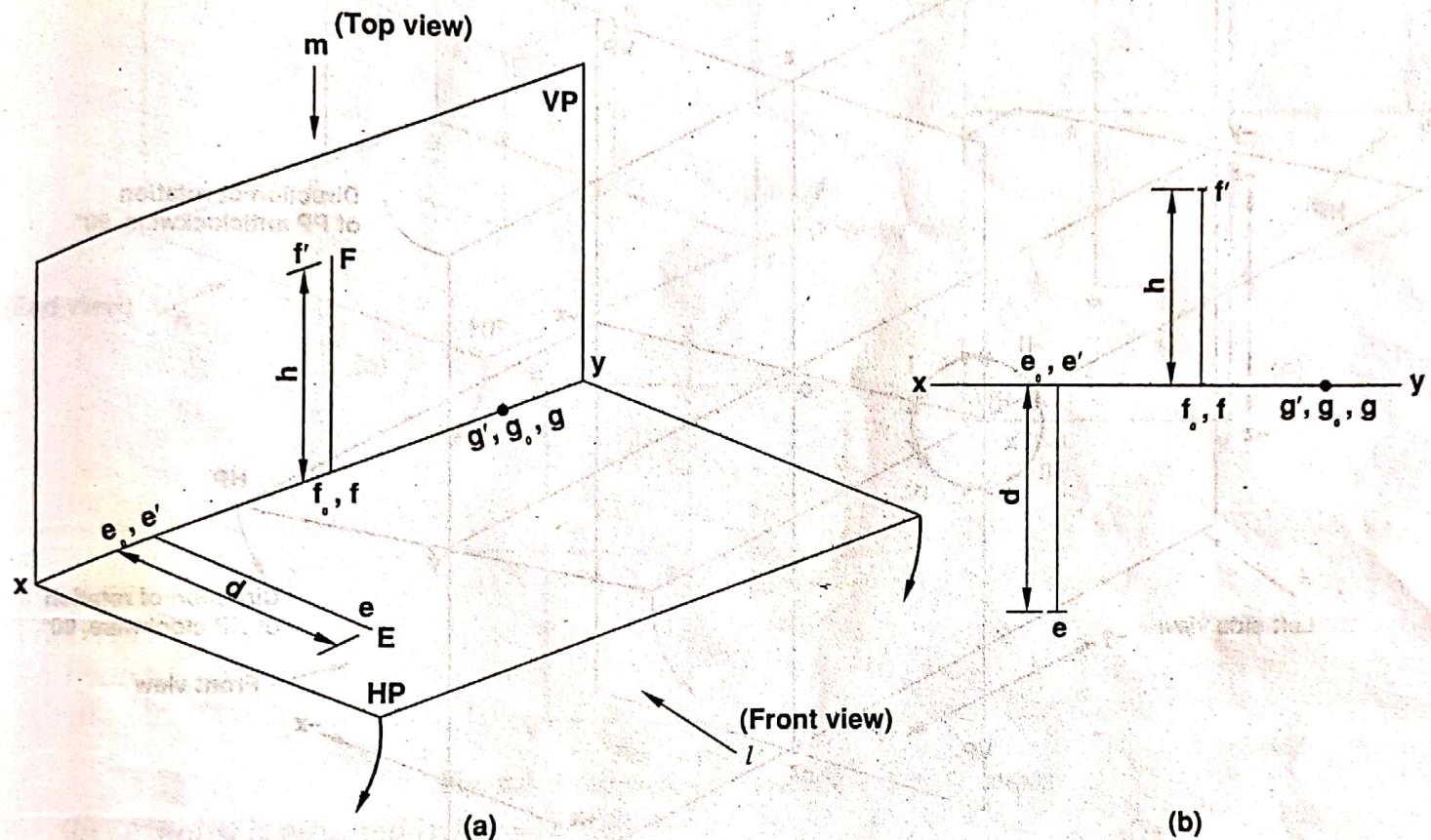


Fig. 8.5 Special cases of points E, F and G in first quadrant

The following points may be noted from the Fig. 8.5 (b).

- (i) When a point lies in the HP, its front view will lie on  $xy$ .
- (ii) When a point lies in the VP, its top view will lie on  $xy$ .
- (iii) When a point lies on both HP and VP, its front and top views will lie on  $xy$ .

## 8.7 A POINT IS SITUATED IN THE THREE PLANES OF PROJECTION

Sometimes two views of an object are not sufficient to describe the shape and size completely, then it is necessary to draw a third view preferably a side view or an end view on a profile plane. As already been described HP and VP divide the space into four quadrants. Introduction of PP will divide each quadrant, further into two parts i.e., four to the left and four to the right of the PP. Thus there become eight spaces called octants as shown in Fig. 8.6. Only anticlockwise or left hand system will be used throughout the book.

A point may be situated in either of the eight octants. As discussed in the Chapter 7, rotate the HP through  $90^\circ$  clockwise and PP through  $90^\circ$  anticlockwise, so as to be coplanar with the VP, such that the first octant opens out. On the same pattern, it can be noted that the planes after rabatment do not

overlap each other in the seventh octant. Except first and seventh octants, in other octants, there is overlapping of planes, which will result into overlapping of views too. So only first and seventh octants will be discussed here.

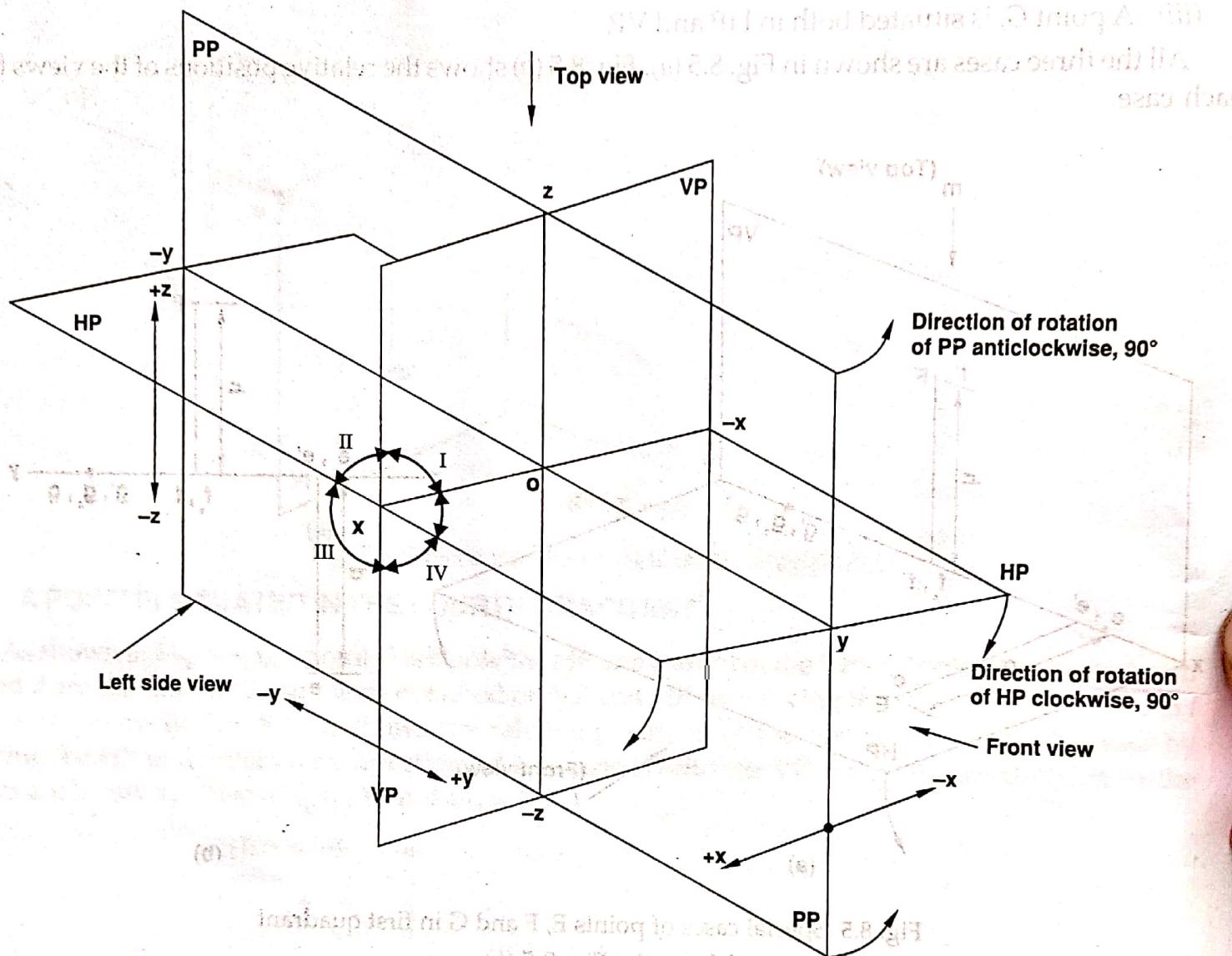


Fig. 8.6 Three planes of projection with front, top and side views

#### (a) A point is situated in the first octant

As shown in Fig. 8.7 (a), a point  $E$  is placed in the first quadrant along with the profile plane (PP). The projection of a point  $E$  on the PP is obtained by viewing the point in the direction  $n$ . The view obtained on the PP i.e.,  $e''$  is known as left side view. Fig. 8.7 (b) shows the relative positions of the views along with the three planes of projections. Fig. 8.7 (c) shows the relative positions of the

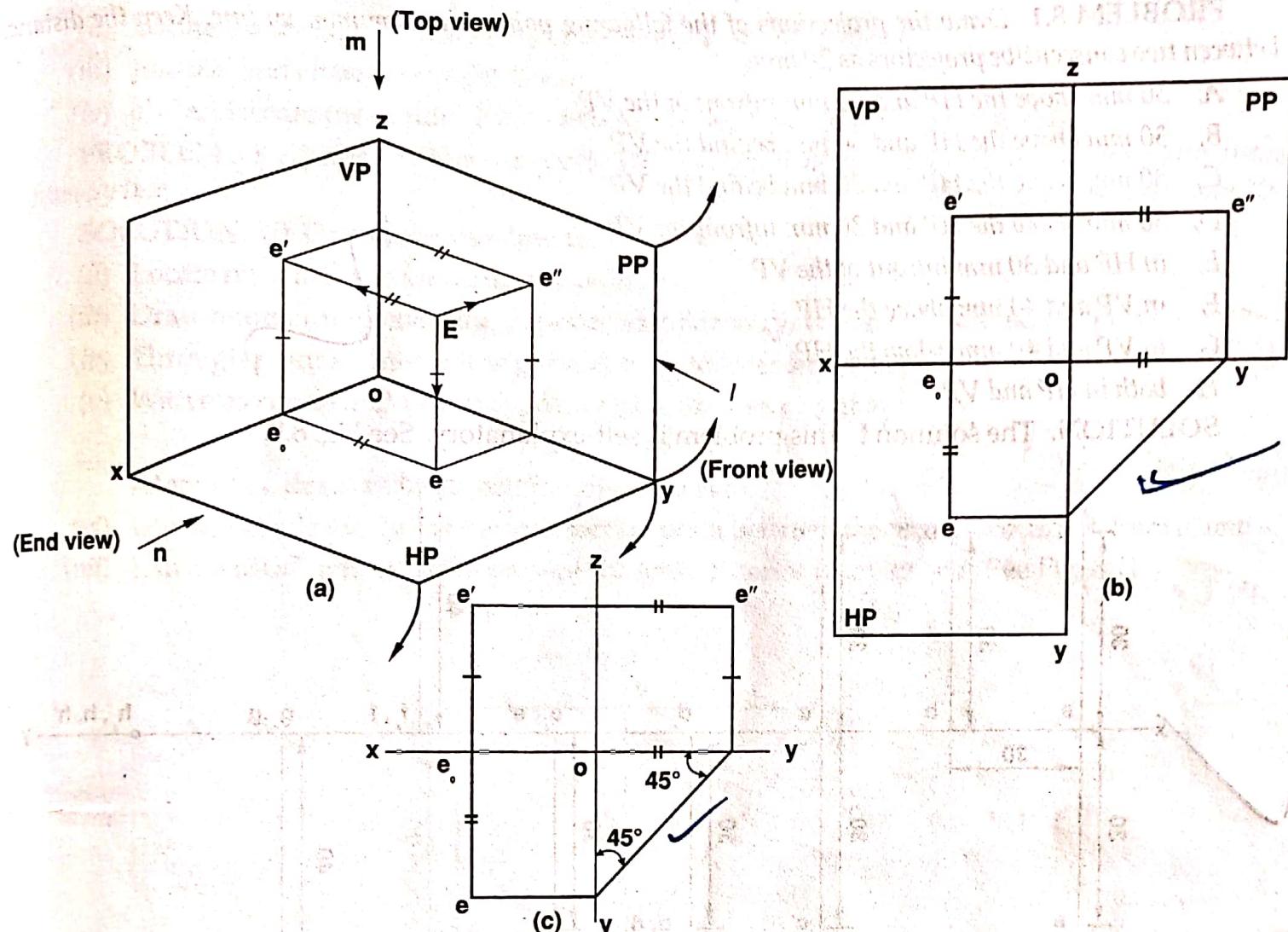


Fig. 8.7 Projections of a point E in first octant

### (b) A point is situated in the seventh octant

As shown in Fig. 8.8 (a), a point F is placed in the third quadrant along with the profile plane (PP). The projection of a point F on the PP is obtained by viewing the point in the direction  $n$ . Here it is assumed that all the three planes are transparent. Fig. 8.8(b) shows the relative positions of the views.

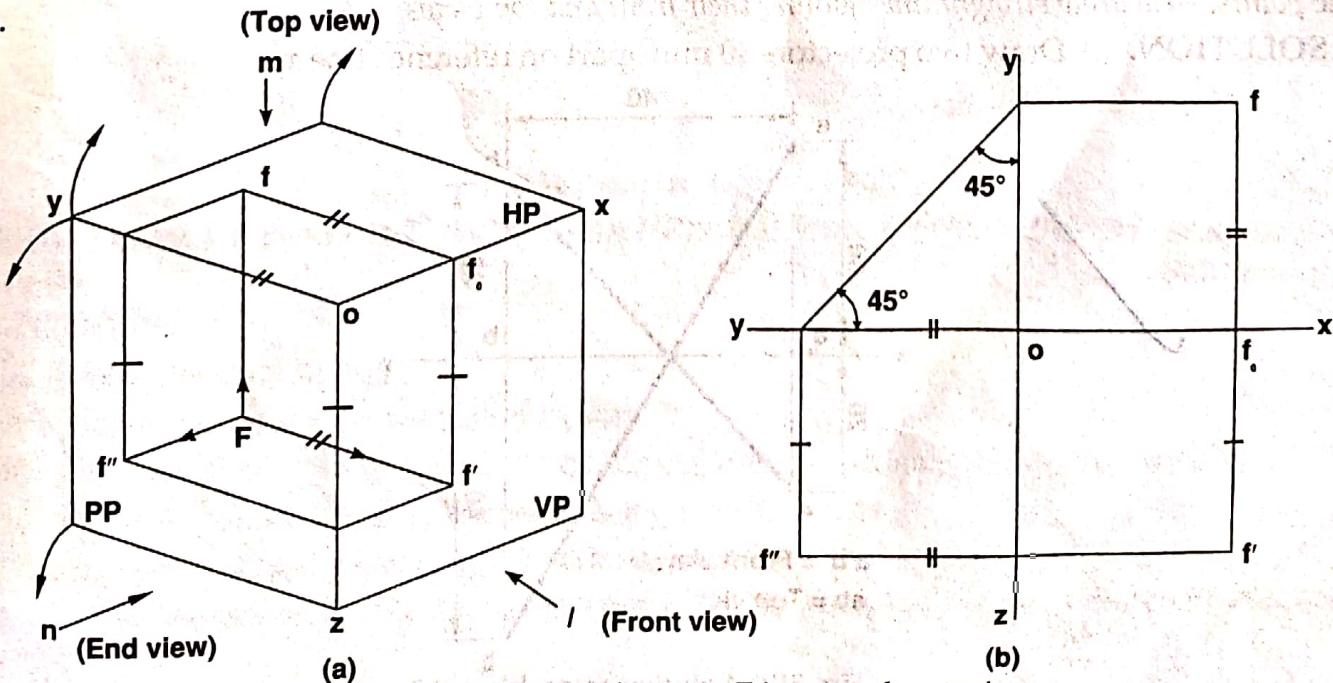


Fig. 8.8 Projections of a point F in seventh octant

**PROBLEM 8.1** Draw the projections of the following points on a common  $xy$  line. Keep the distance between two consecutive projectors as 20 mm.

- A, 30 mm above the HP and 30 mm in front of the VP
- B, 30 mm above the HP and 30 mm behind the VP
- C, 30 mm below the HP and 30 mm behind the VP
- D, 30 mm below the HP and 30 mm in front of the VP
- E, in HP and 30 mm in front of the VP
- F, in VP and 40 mm above the HP
- G, in VP and 40 mm below the HP
- H, both in HP and VP.

**SOLUTION.** The solution to this problem is self-explanatory. See Fig. 8.9.

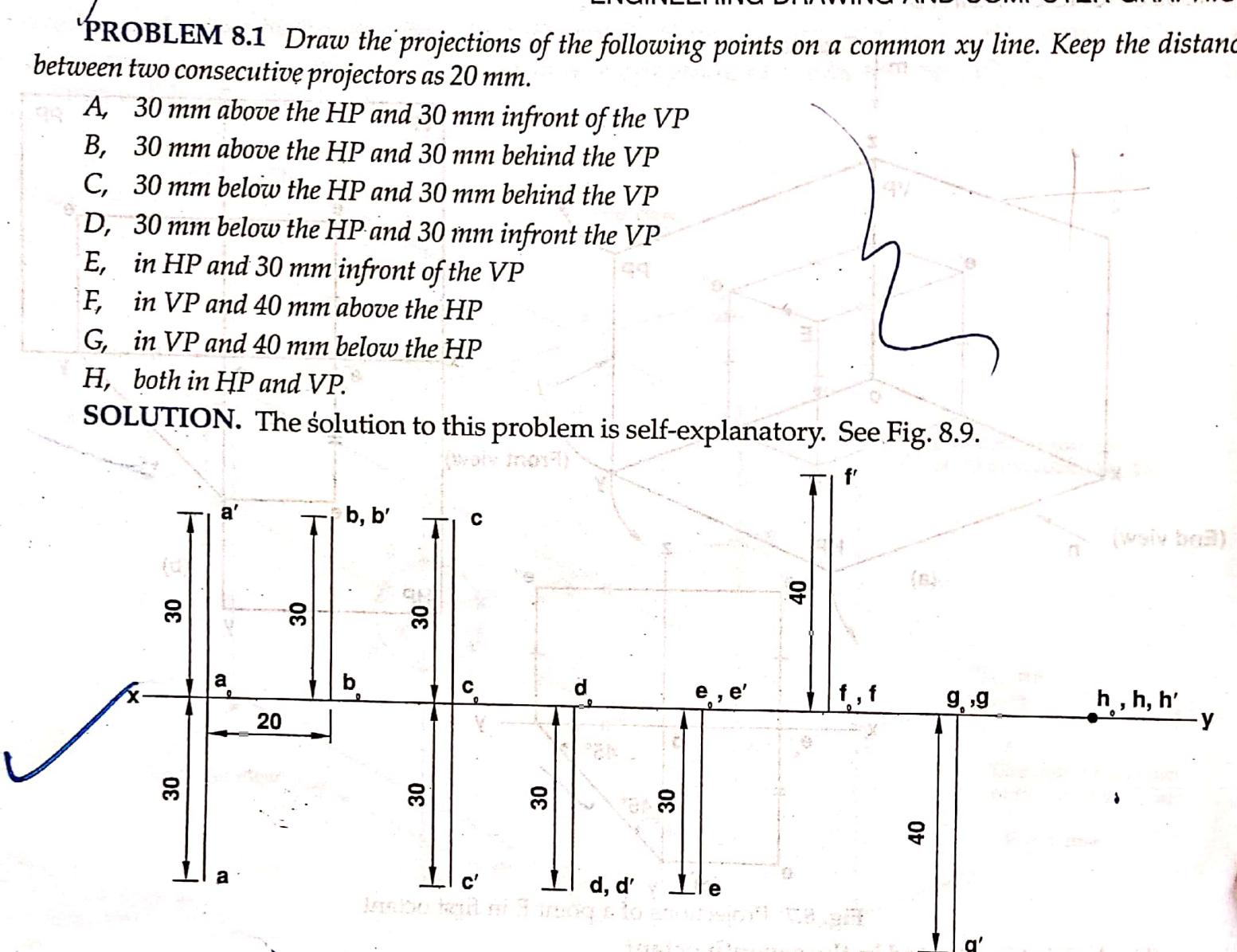


Fig. 8.9 Solution to problem 8.1

**PROBLEM 8.2** A point A is 30 mm above HP and 20 mm in front of the VP. Another point B is 40 mm below HP and 25 mm behind the VP. The distance between the end projectors is 40 mm. Draw the projections of the points. Also draw straight lines joining their front and top views.

**SOLUTION.** (i) Draw two projectors 40 mm apart on reference line  $xy$ .

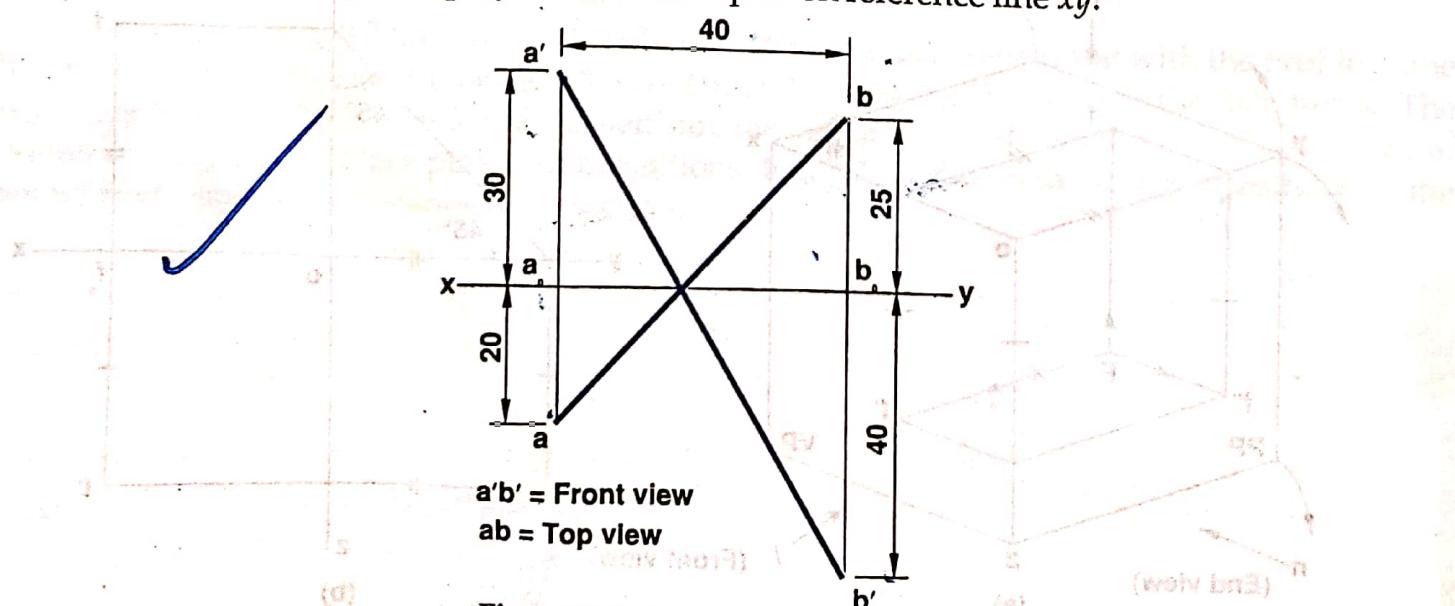


Fig. 8.10 Solution to problem 8.2

- (ii) Locate front and top views for points A and B on the projectors.
- (iii) Join  $a'b'$  and  $ab$  with straight lines.
- (iv)  $a'b'$  and  $ab$  are the required front and top views respectively. See Fig. 8.10.

**PROBLEM 8.3** A point A is 30 mm above the HP and 25 mm in front of the VP. Determine its least distance from  $xy$  line. (PTU, Jalandhar December 2008)

**SOLUTION.** (i) Draw reference line  $xy$ .

- (ii) Locate front and top views for the point A.
- (iii) Draw another reference line  $yz$  perpendicular to  $xy$ , which will intersect at point o.
- (iv) Through point a, draw a line parallel to  $xy$  to intersect  $yz$  at  $o_2$ .
- (v) With o as centre and radius  $oo_2$  draw an arc to meet  $xy$  at  $o_3$ .

or

At point  $o_2$ , draw an angle of  $45^\circ$  to meet  $xy$  at  $o_3$ .

- (vi) Locate the side view  $a''$  as the intersection point between the projectors drawn from  $o_3$  and  $a'$ .
- (vii) Join o with  $a''$ , which will represent the least distance from  $xy$  line. See Fig. 8.11.

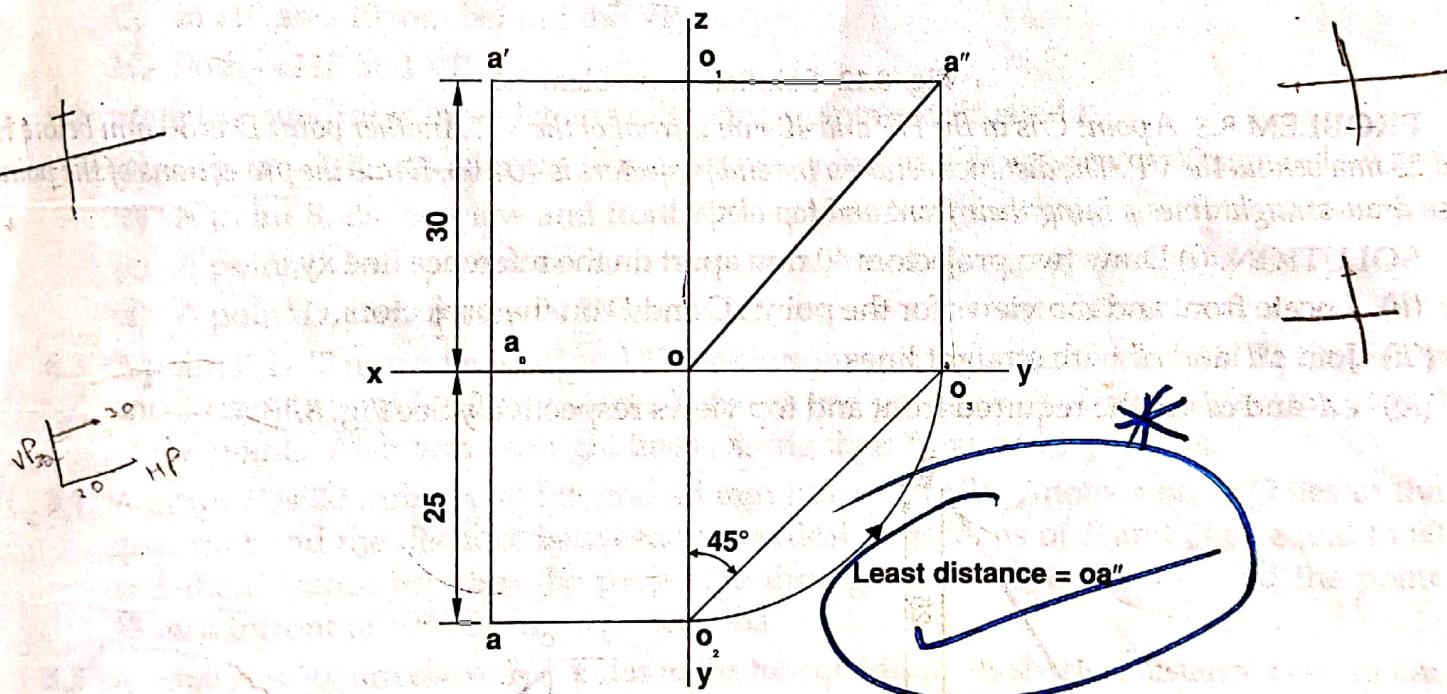


Fig. 8.11 Solution to problem 8.3

**PROBLEM 8.4** A point P is 25 mm below the HP and 30 mm behind the VP. Determine its least distance from  $xy$  line. (PTU, Jalandhar May 2009)

**SOLUTION.**

- (i) Draw reference line  $xy$ .
- (ii) Locate front and top views for the point P.
- (iii) Draw another reference line  $yz$  perpendicular to  $xy$ , which will intersect at point o.
- (iv) Through point p, draw a line parallel to  $xy$  to intersect  $yz$  at  $o_2$ .
- (v) At point  $o_2$ , draw an angle of  $45^\circ$  to meet  $xy$  at  $o_3$ .
- (vi) Locate the side view  $p''$  as the intersection point between the projectors drawn from  $o_3$  and  $p'$ .

(vii) Join  $o$  with  $p''$ , which will represent the least distance from  $xy$  line. See Fig. 8.12.

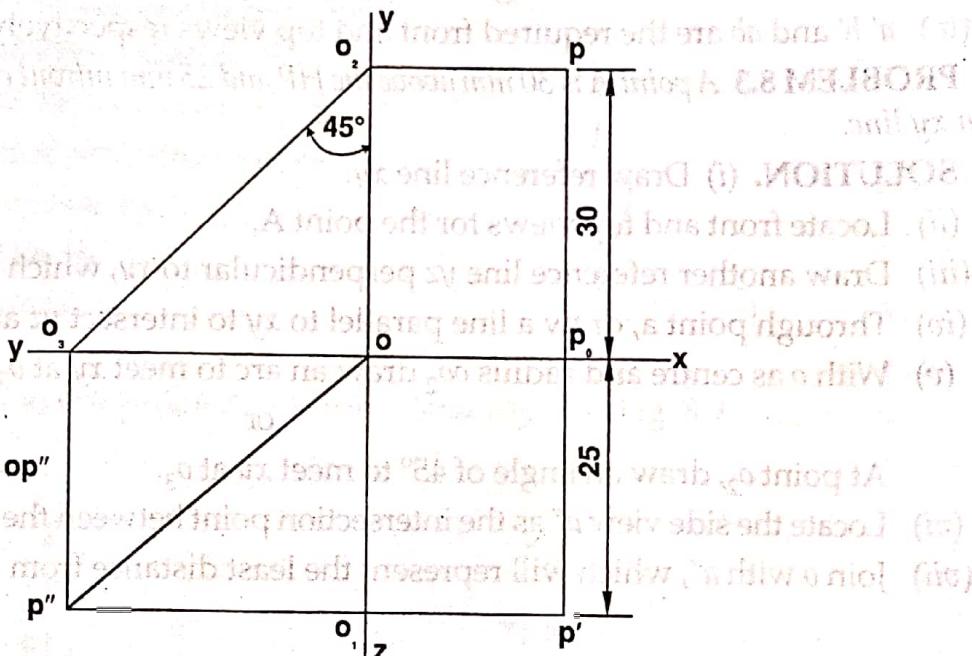


Fig. 8.12 Solution to problem 8.4

**PROBLEM 8.5** A point  $C$  is in the HP and  $40$  mm in front of the VP. Another point  $D$  is  $30$  mm below HP and  $25$  mm behind the VP. The distance between the end projectors is  $40$  mm. Draw the projections of the points. Also draw straight lines joining their front and top views.

**SOLUTION.** (i) Draw two projectors  $40$  mm apart on the reference line  $xy$ .

(ii) Locate front and top views for the points  $C$  and  $D$  on the projectors.

(iii) Join  $c'd'$  and  $cd$  with straight lines.

(iv)  $c'd'$  and  $cd$  are the required front and top views respectively See Fig. 8.13.

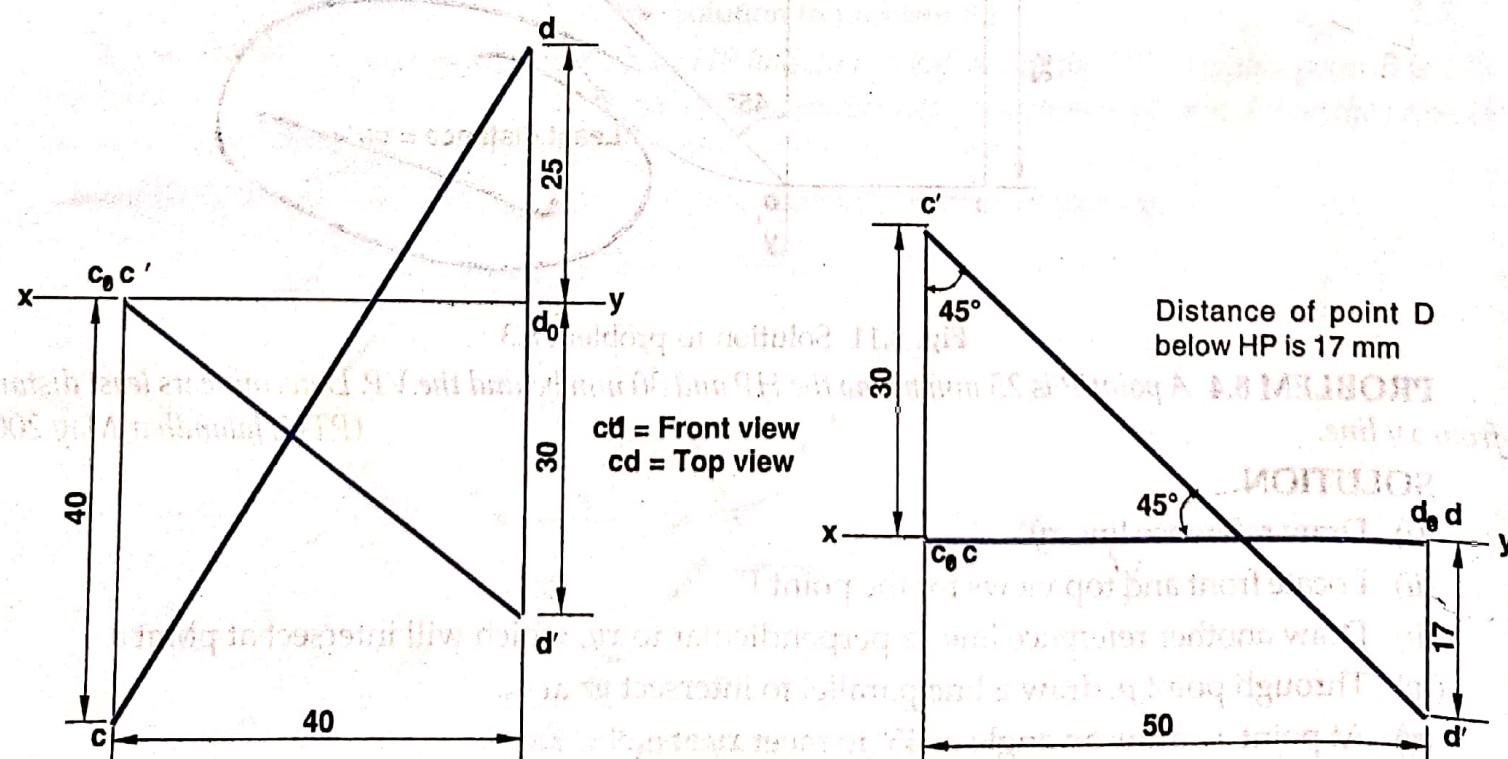


Fig. 8.13 Solution to problem 8.5

Fig. 8.14 Solution to problem 8.6

**PROBLEM 8.6** Two points C and D are in VP. Point C is 30 mm above HP while point D is below the HP. The distance between the end projectors is 50 mm. The line joining the front views of the two points makes angles of  $45^\circ$  with the reference line. Draw the projections of the line CD and find the distance of the point D below the HP.

**SOLUTION.** The solution to this problem is self-explanatory. See Fig. 8.14.

### EXERCISES

- 8.1 Draw the projections of the following points on a common  $xy$  line, keeping the distance between two consecutive projectors as 20 mm.
- 30 mm above HP and 40 mm in front of the VP
  - 30 mm above HP and 40 mm behind the VP
  - 30 mm below HP and 40 mm behind the VP
  - 30 mm below HP and 40 mm in front of the VP
  - in HP and 40 mm in front of VP
  - in VP and 30 mm above the HP
  - in VP and 30 mm below the HP
  - in HP and 40 mm behind the VP
  - both in HP and VP.
- 8.2 State the quadrants in which the following points are situated :
- A point A, its top view is 30 mm above  $xy$  line and front view is 40 mm below  $xy$  line.
  - A point B, its top view and front view coincide with each other 40 mm above  $xy$  line.
  - A point C, its top view on  $xy$  line and front view 30 mm above the  $xy$  line.
  - A point D, its top view is 35 mm below  $xy$  line and front view on  $xy$  line.
- 8.3 A point A is 40 mm above HP and 25 mm in front of VP. Another point B is 20 mm below HP and 30 mm behind VP. The distance between end projectors is 50 mm. Draw the projections of the points. Also draw straight lines joining their front and top views.
- 8.4 A point P is 20 mm above HP and 40 mm in front of VP. Another point Q lies in the first quadrant and the distance between the vertical projections of P and Q be equal to 60 mm and the distance between the projectors through P and Q be 45 mm and the point Q is 35 mm in front of VP. Draw its projections.
- 8.5 A point A is 30 mm above HP and is in the first quadrant. Its shortest distance from  $xy$  line is 55 mm. Draw its front and top views.
- 8.6 A point P is 40 mm below HP and 35 mm behind VP. Determine its least distance from  $xy$ .
- 8.7 A point P is 20 mm below HP and is in third quadrant. Its shortest distance from  $xy$  line is 50 mm. Draw its projections.

### OBJECTIVE QUESTIONS

- Draw the projection of a point 'P' when it is lying on the  $xy$  line i.e. where HP and VP meet each other.
- To represent the projections on a sheet, the planes must be rotated such that ..... or ..... quadrant always open out.
- When a point is below the HP, its front view is .....  $xy$  line.
- When a point lies on both HP and VP, its front view and top view .....  $xy$  line.
- How the side view of a point is obtained from its front and top views ?

- 8.6 The ..... is obtained as the intersection point between the ray of the sight and the VP.
- 8.7 The top view is obtained as the intersection point between the ray of the sight and the ..... .
- 8.8 When a point lies in the HP, its front view will lie .....
- 8.9 The line joining the projections of a point intersects at an angle of ..... to the  $xy$  line.
- 8.10 When a point lies in the VP, its top view will lie .....
- 8.11 If the front view is above  $xy$  line and the top view is below  $xy$  line, then the point is in ..... quadrant?
- 8.12 When a point is situated in the first quadrant it will be located above the HP and in front of VP. (True/False)
- 8.13 When a point lies on both HP and VP, its front and top view will lie \_\_\_\_\_.
- 8.14 The projector of a point is always \_\_\_\_\_ to the reference line.
- 8.15 What is a projector?

### ANSWERS

- |                  |                  |                   |                    |
|------------------|------------------|-------------------|--------------------|
| 8.2 First, third | 8.3 Below        | 8.4 Lie on        | 8.6 Front view     |
| 8.7 HP           | 8.8 On $xy$ line | 8.9 $90^\circ$    | 8.10 On $xy$ line  |
| 8.11 First       | 8.12 True        | 8.13 On $xy$ line | 8.14 Perpendicular |

### BONITA E VITÓRIO