14 Development of Surfaces

14.1 INTRODUCTION

The complete surface of an object laid out on a plane is called the development of the surface or flat pattern of the object. Development is a term frequently used in sheet metal work where it means the unfolding or unrolling of a detailed into a flat sheet called as pattern (see Fig. 14.1). The development of geometrical surfaces is important in the fabrication of not only small, simple shapes made of thin sheet metal, but also sophisticated pieces of hardware also.

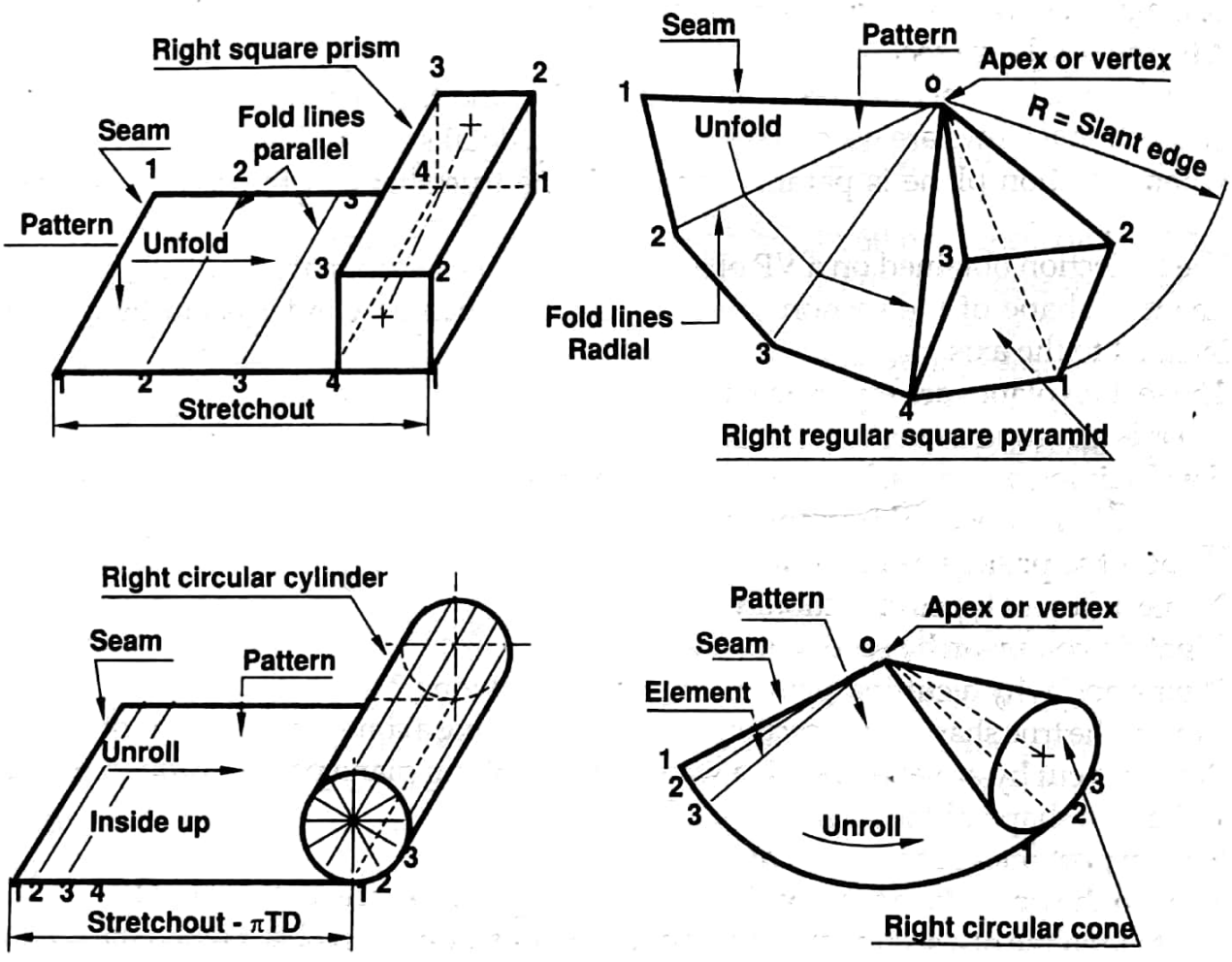


Fig. 14.1 Development of surfaces

In making the development of a geometric surface, the opening should be determined first. Every line used in making the development must represent the true length of that line on an actual surface.

The knowledge of development of surfaces is essential in many industries such as automobile, ship building, packaging and sheet metal work etc.

14.2 METHODS OF DEVELOPMENT

There are four methods of development. These are as follows :

(*i*) **Parallel line Method**

It is employed in case of prisms and cylinders in which stretch out line principle is used. Stretch out is given by the perimeter of the object measured in a plane at right angles to the axis.

(*ii*) **Radial line Method**

It is employed in case of pyramids and cones in which the true length of the slant edge or the generator is used as radius.

(*iii*) **Triangulation Method**

It is used to develop transition pieces. It is a method of dividing a surface into a number of triangles and transforming them into the development.

(*iv*) **Approximate Method**

It is used to develop objects of double-curved or warped surfaces as sphere, paraboloid, hyperboloid.

Only the lateral surfaces of the solids have been developed and shown as presented here, omitting the bases or ends of solids.

14.3 PARALLEL LINE METHOD

The surface of right prisms, oblique prisms, right cylinders and oblique prisms are developed by this method. In this method right section and stretchout line principle is used. Parallel lines, parallel to the axis of the detail, are shown on a view which show them as their true lengths.

(*a*) **Development of right prisms :** Development of the lateral surface of a right prism consists of the same number of rectangles in contact as the number of the sides of the base of the prism. One side of the rectangle is equal to the length of the axis and the other side equal to the length of the side of the base.

PROBLEM 14.1. *A right regular pentagonal prism, side of base 30 mm and height 50 mm, is truncated at the top and cut away from below as shown in Fig. 14.2 (a). Develop the lateral surface of the remaining prism.*

(PTU, Jalandhar May 2009)

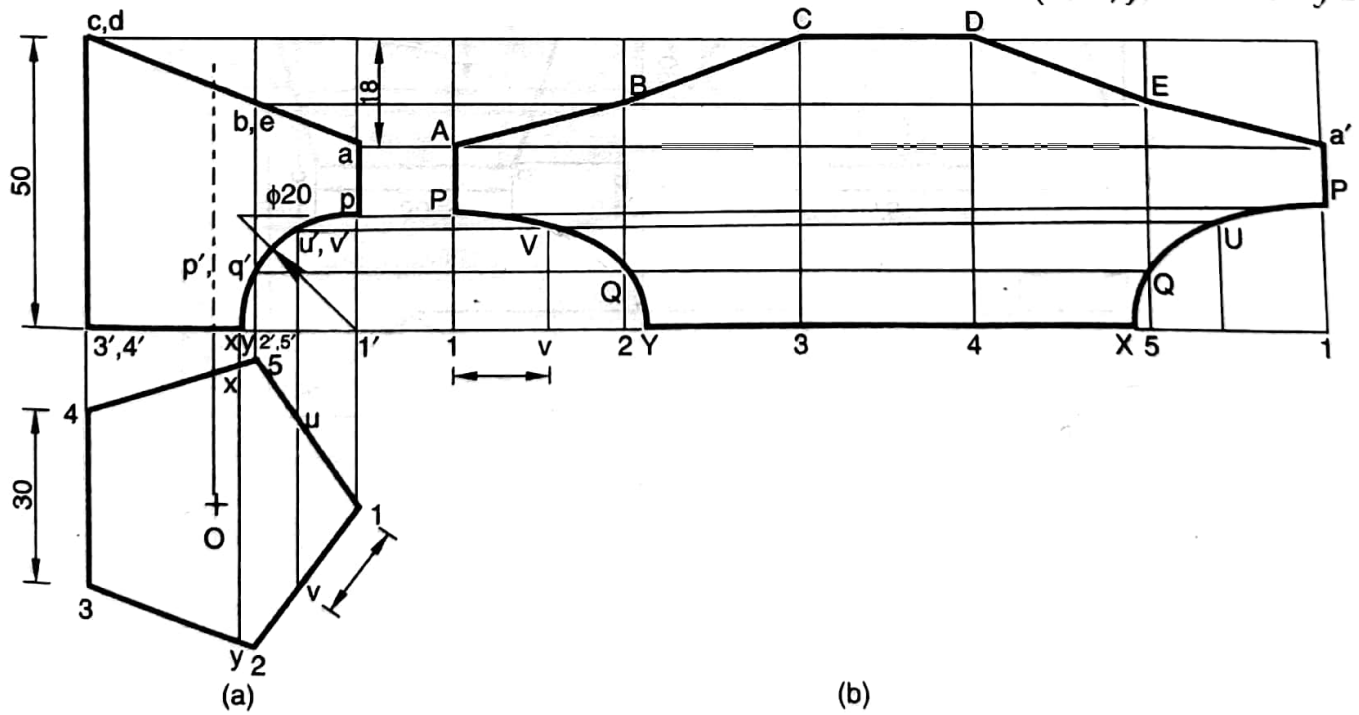


Fig. 14.2. Solution to problem 14.1

SOLUTION. For its solution, see Fig. 14.2 (*b*), which is itself a self explanatory.

PROBLEM 14.2 *Develop the part pentagonal sheet metal detail as shown in Fig. 14.3 (a).*

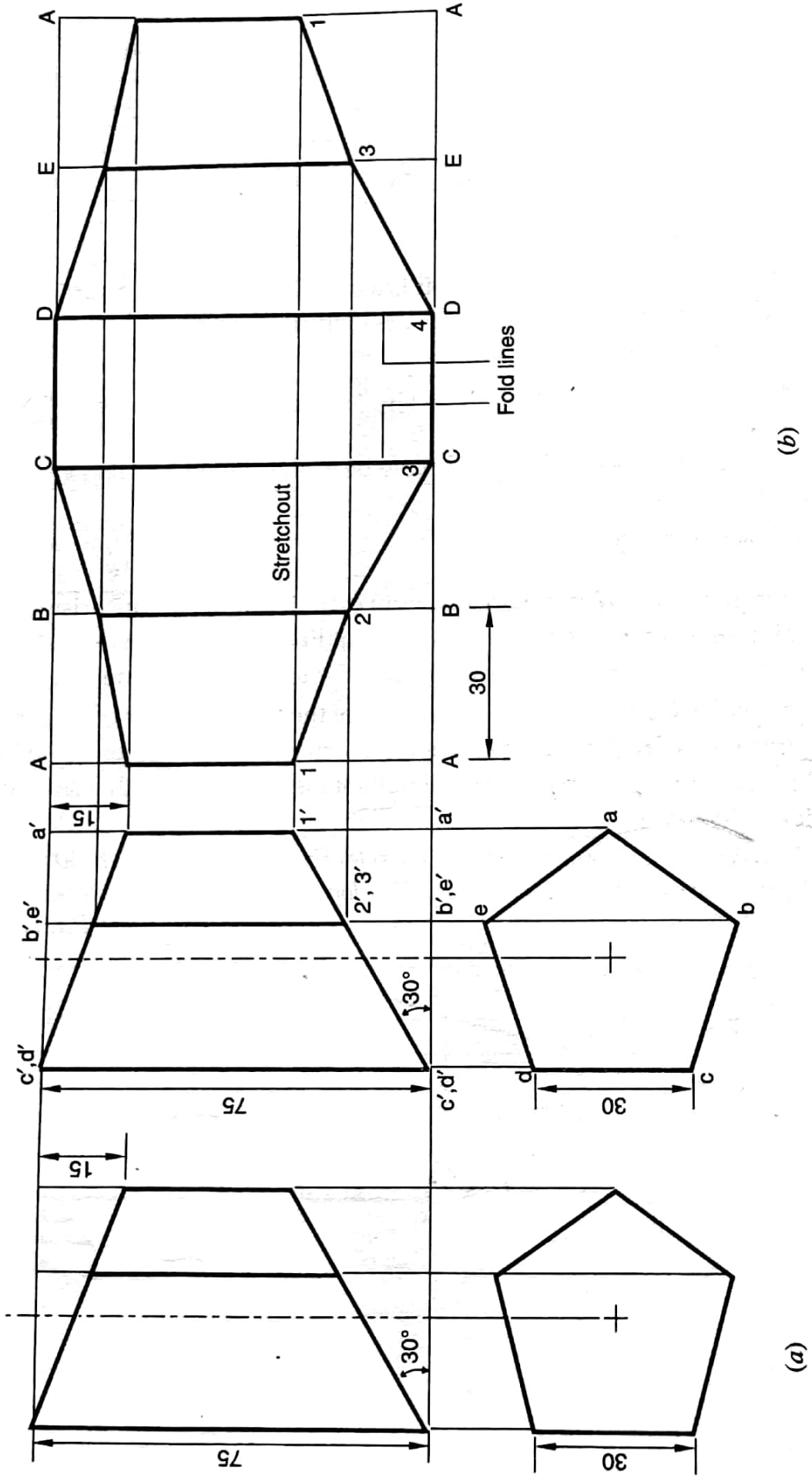


Fig. 14.3 Solution to problem 14.2

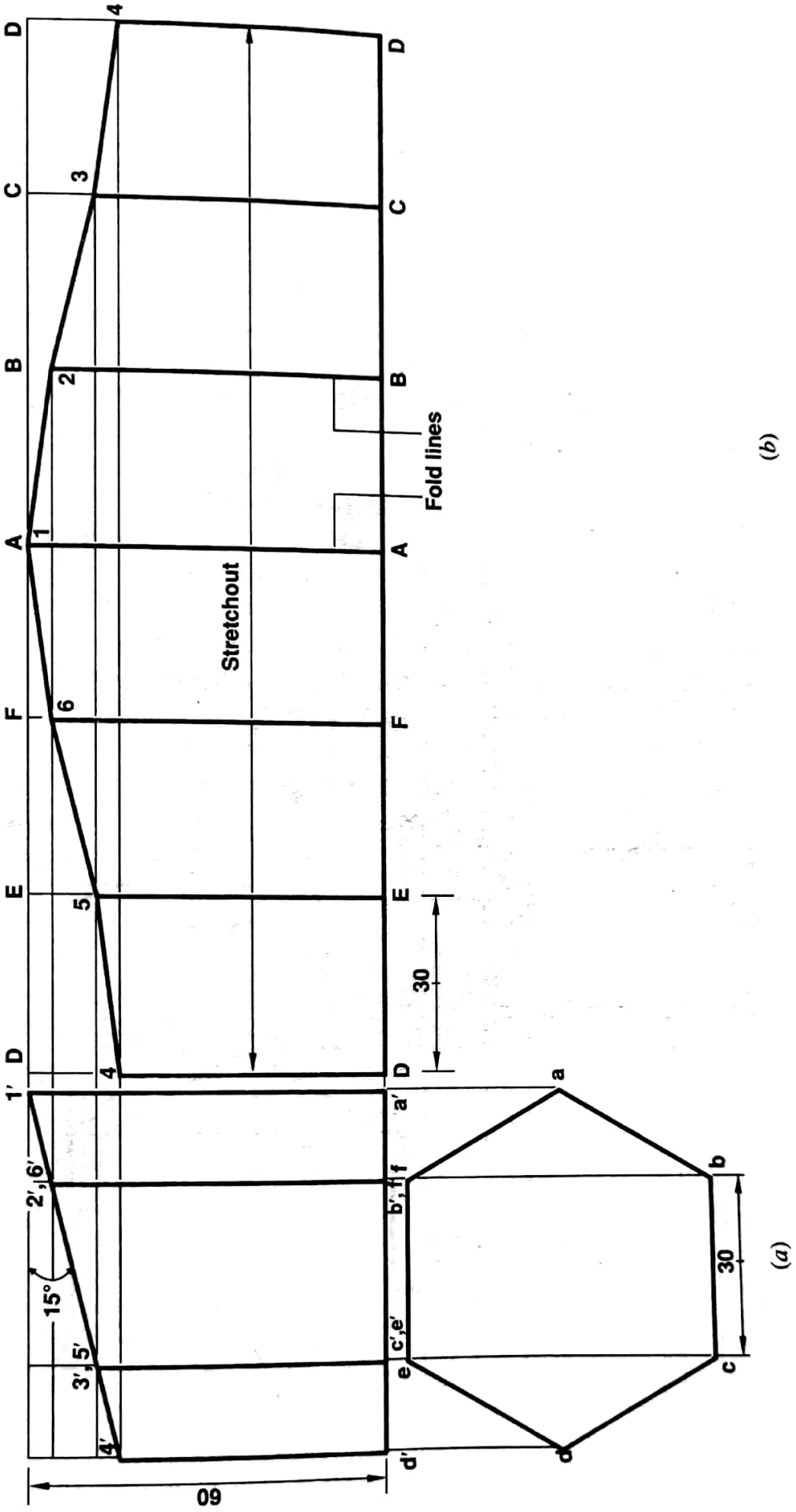


Fig. 14.4 Solution to problem 14.3

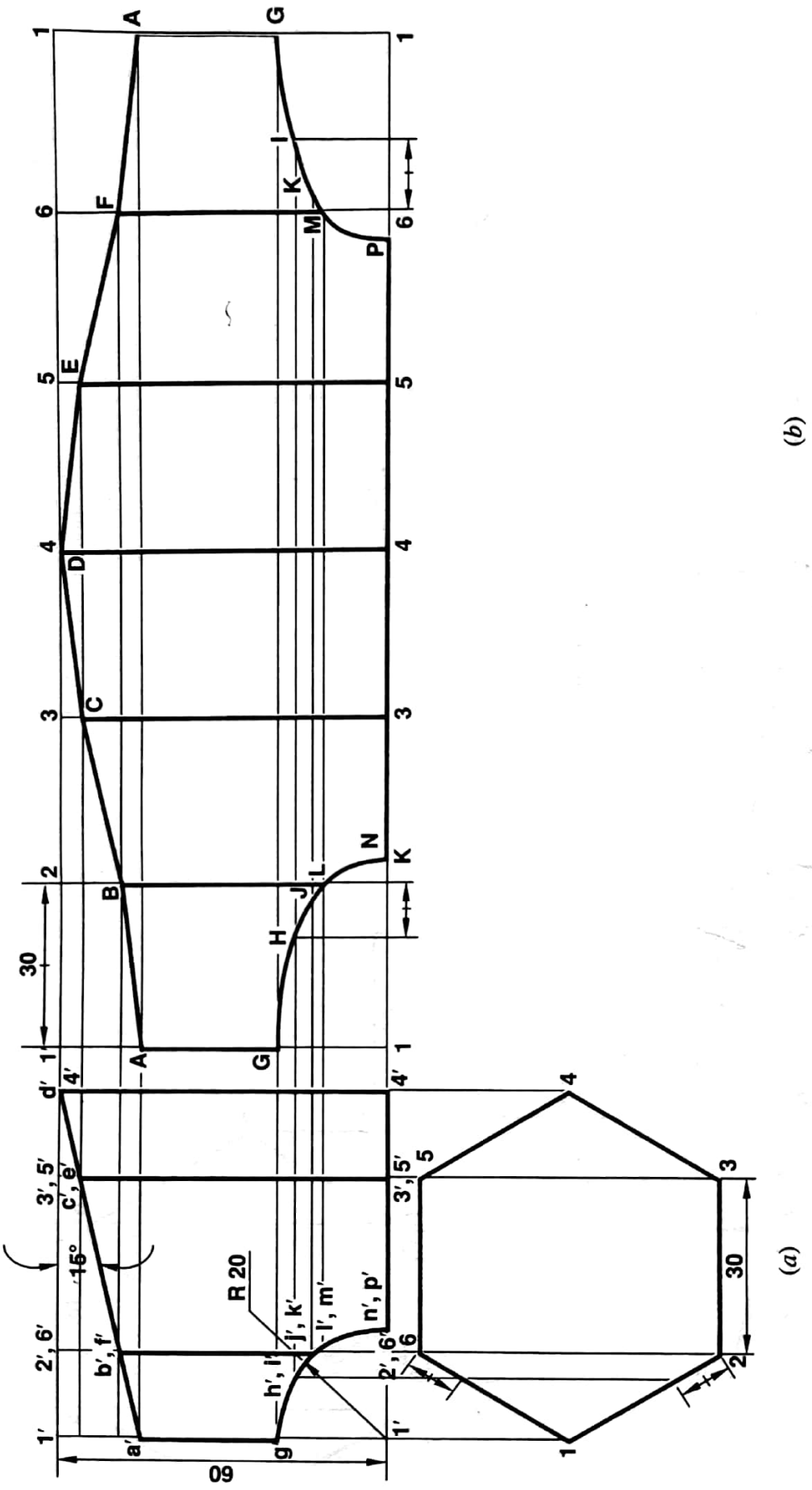


Fig. 14.5 Solution to problem 14.4

SOLUTION. (*i*) Draw the given views of the pentagonal prism.

(*ii*) Complete the development of the full pentagonal prism along the stretch out line.

(*iii*) Locate the intersection points on the edges of the prism.

(*iv*) Transfer the above points to the development by projecting them.

(*v*) Draw the lines joining these points and complete the development as shown in Fig. 14.4(*b*).

PROBLEM 14.3 *A right regular hexagonal prism, side of base 30 mm and height 60 mm, is truncated at the top as shown in Fig. 14.4 (a). Develop the lateral surface of the remaining prism.*

(PTU, Jalandhar May 2011)

SOLUTION. For its solution, see Fig. 14.4 (*b*), which is itself a self explanatory.

PROBLEM 14.4 *A right regular hexagonal prism, side of base 30 mm and height 60 mm, is truncated at the top and cut away from below as shown in Fig. 14.5 (a). Develop the lateral surface of the remaining prism.*

SOLUTION. (*i*) Draw the given views of the hexagonal prism.

(*ii*) Complete the development of the full hexagonal prism along its stretchout line.

(*iii*) Draw the stretchout line parallel to the edge view of the right section. Transfer along it the distance 1-2, 2-3, 3-4, 4-5, 5-6 and 6-1, measuring from the top view.

(*iv*) On the quarter circle in front view, take some points say g', h', i', j' etc. Mark their corresponding top views g, h, i, j etc. too. Distances of each of these points is measured and transfered from the top view to the development of the hexagonal prism by projecting them.

(*v*) Draw the lines joining these points and complete the development as shown in Fig. 14.5 (*b*).

PROBLEM 14.5 *A pentagonal prism of 25 mm base edge and 50 mm long is resting on its base with an edge of base at 45° to VP. The prism is cut by a sectional plane inclined at 30° to HP and passes through a point 25 mm from the base along its axis. Develop the truncated prism.* *(PTU, Jalandhar December 2003)*

SOLUTION. All the construction lines are retained to make the solution self-explanatory. See Fig. 14.6.

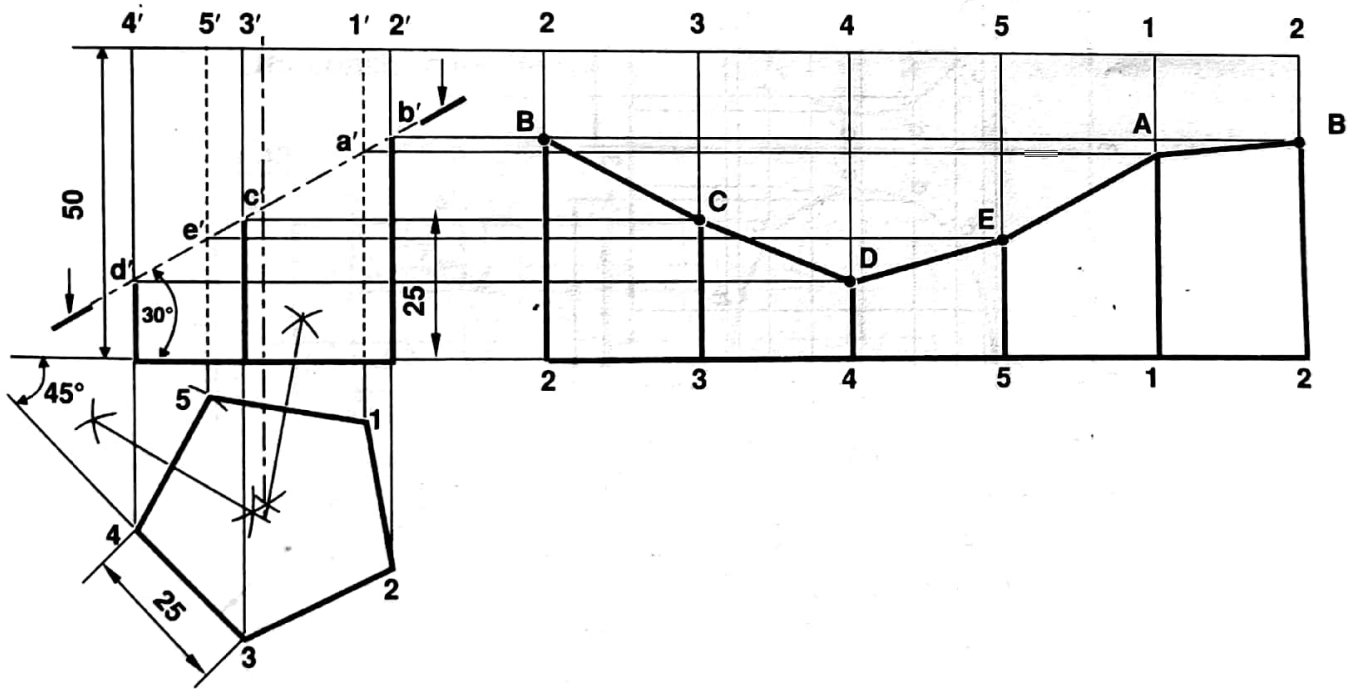


Fig. 14.6 Solution to problem 14.5

PROBLEM 14.6 *A right regular square prism, side of base 25 mm and height 50 mm, rests on its base on HP such that its vertical faces are equally inclined to the VP. A horizontal circular hole of diameter 30 mm drilled centrally through it such that the axis of the hole cuts the diagonally opposite vertical edges. Develop its lateral surfaces.* *(PTU, Jalandhar December 2009, May 2009)*

SOLUTION.

(*i*) Draw the front and top views of the prism satisfying all the condition.

(*ii*) Develop the lateral surface of the complete square prism.

(*iii*) Divide the circle in the front view into sixteen equal parts and name these points as a', b', c' and so on and then project these points into top view as a, b, c etc.

(*iv*) Transfer the distances of these points in top view from lateral corner points. For example, from point 2 towards 3, to the stretchout line of the development, such that $2g = 2g_1$, etc. Erect perpendiculars to the stretchout line through these points, to cut the corresponding horizontal projections, taken into the development, from the front views a', b', c', etc. of these points a, b, c, etc.

(*v*) Join these points by a smooth curve in the front view as shown in Fig. 14.7.

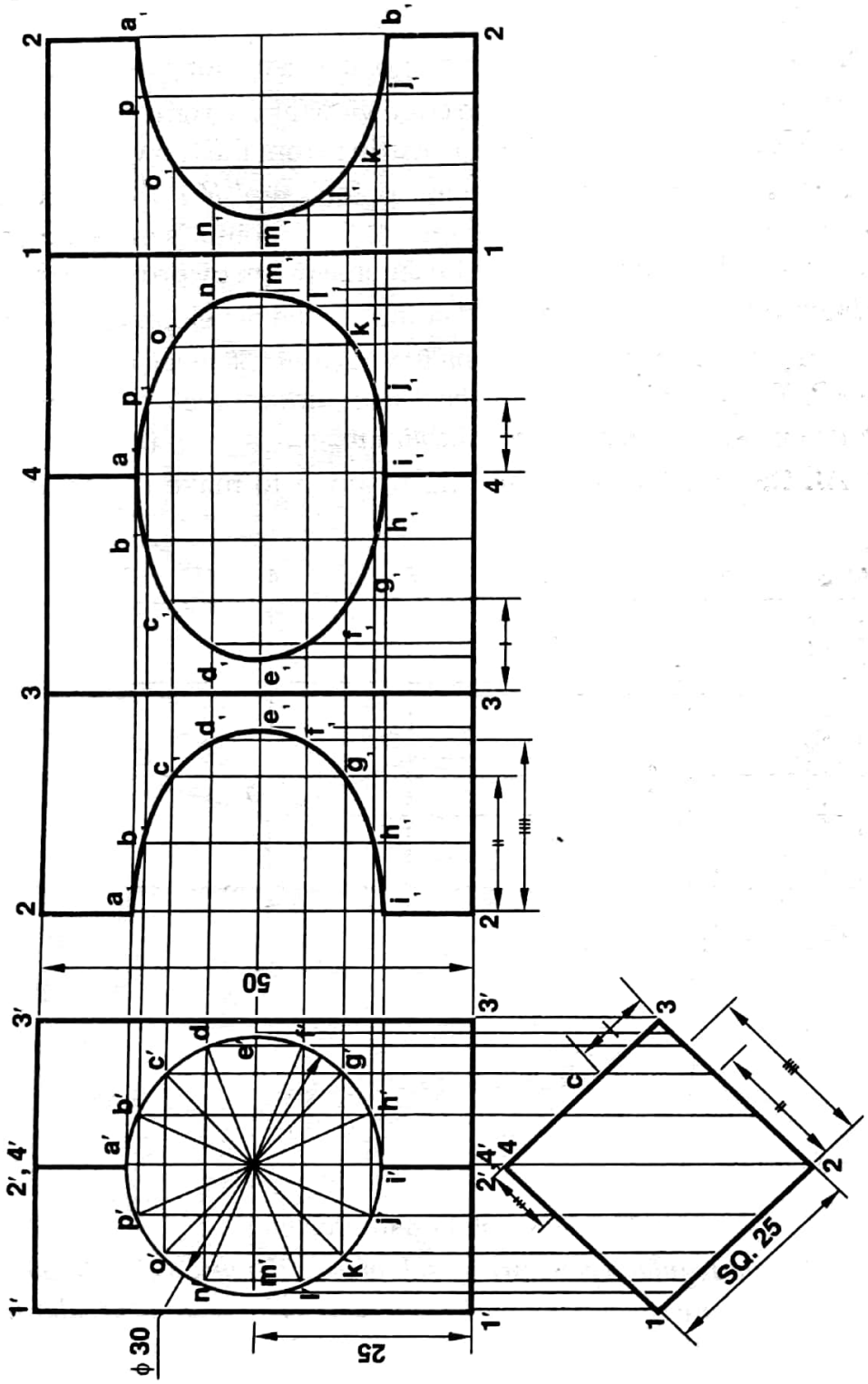


Fig. 14.7 Solution to problem 14.6

PROBLEM 14.7 *Front and top views of a right regular square prism are shown in Fig. 14.8 (a). Develop its lateral surface.*

SOLUTION. For its solution, see Fig. 14.8 (*b*), which is itself a self-explanatory.

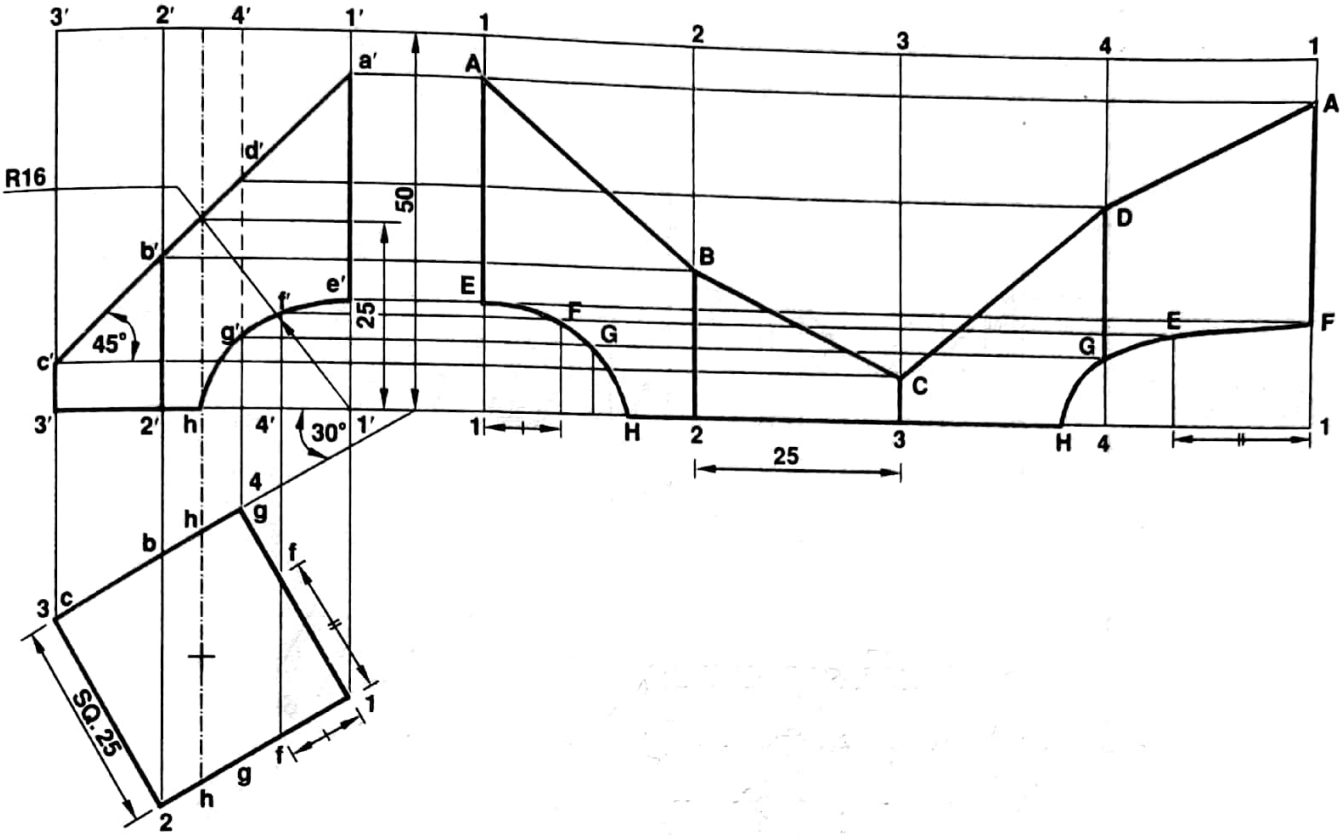


Fig. 14.8 Solution to problem 14.7

PROBLEM 14.8 *A right regular hexagonal prism, side of base 30 mm and height 60 mm, rests on its base on HP with one of its base side parallel to VP. A horizontal circular hole of diameter 40 mm drilled centrally through it, such that the axis of the hole in perpendicular to it. Develop its lateral surface.*

SOLUTION. All the construction lines are retained to make the solution self-explanatory. See Fig. 14.9.

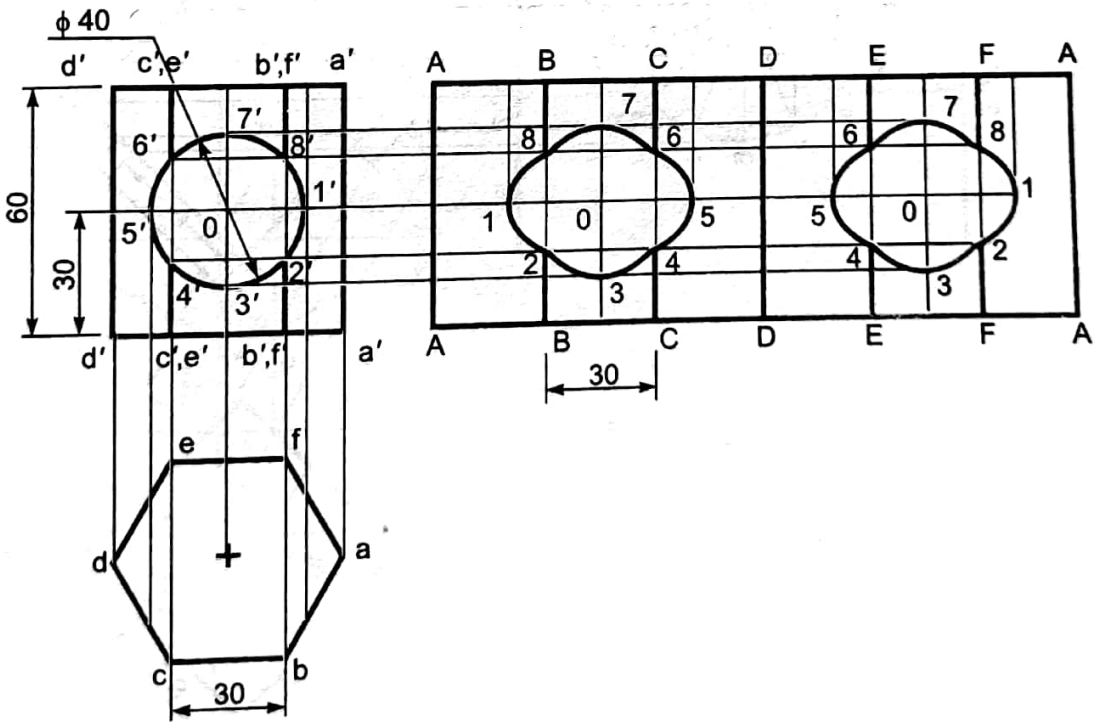


Fig. 14.9 Solution to problem 14.8

PROBLEM 14.9 *Front and top views of a right regular square prism are shown in Fig. 14.10 (a). Develop its lateral surface.*

SOLUTION. All the construction lines are retained to make the solution self– explanatory. See Fig. 14.10(*b*).

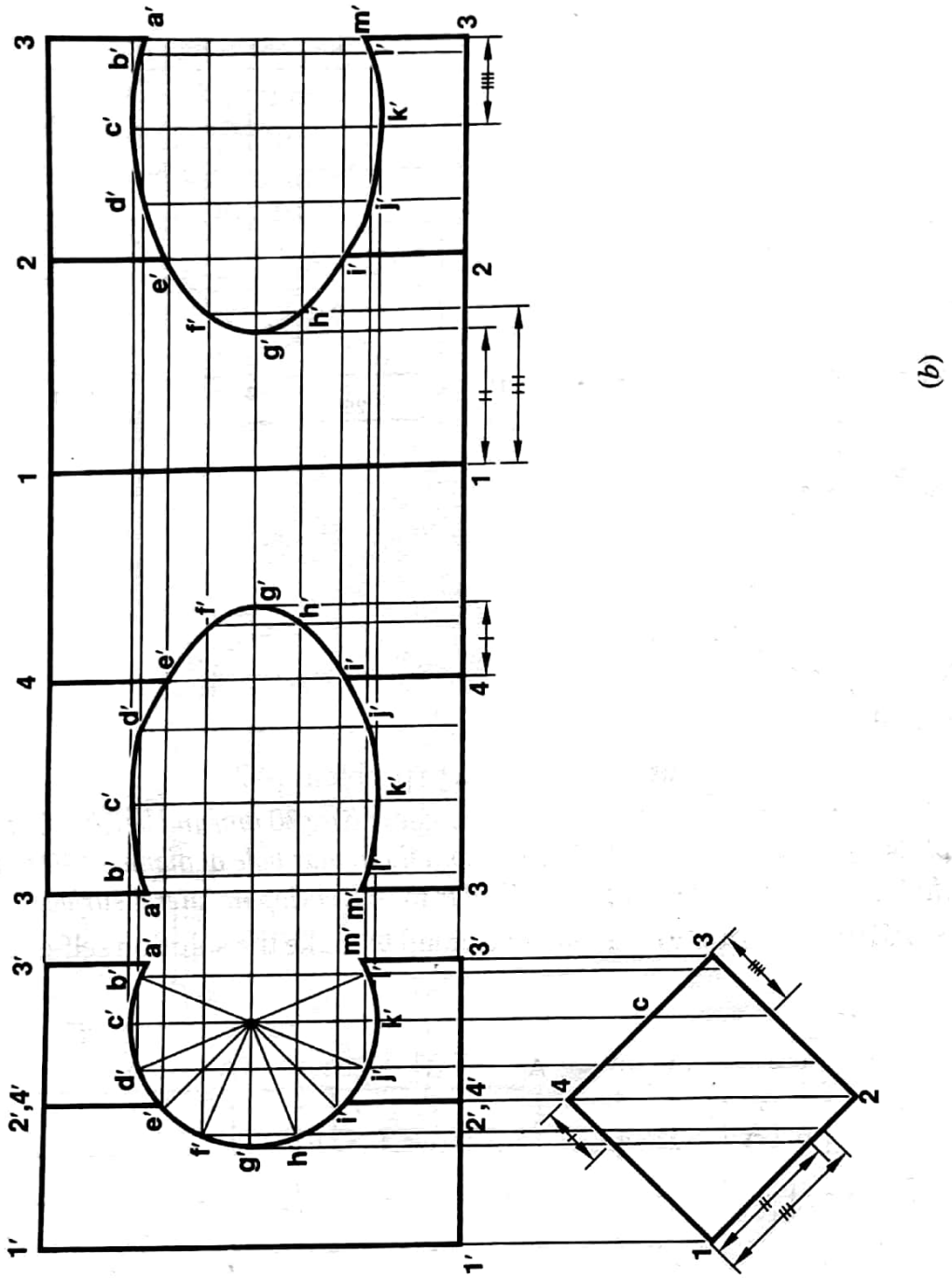


Fig. 14.10 Solution to problem 14.9

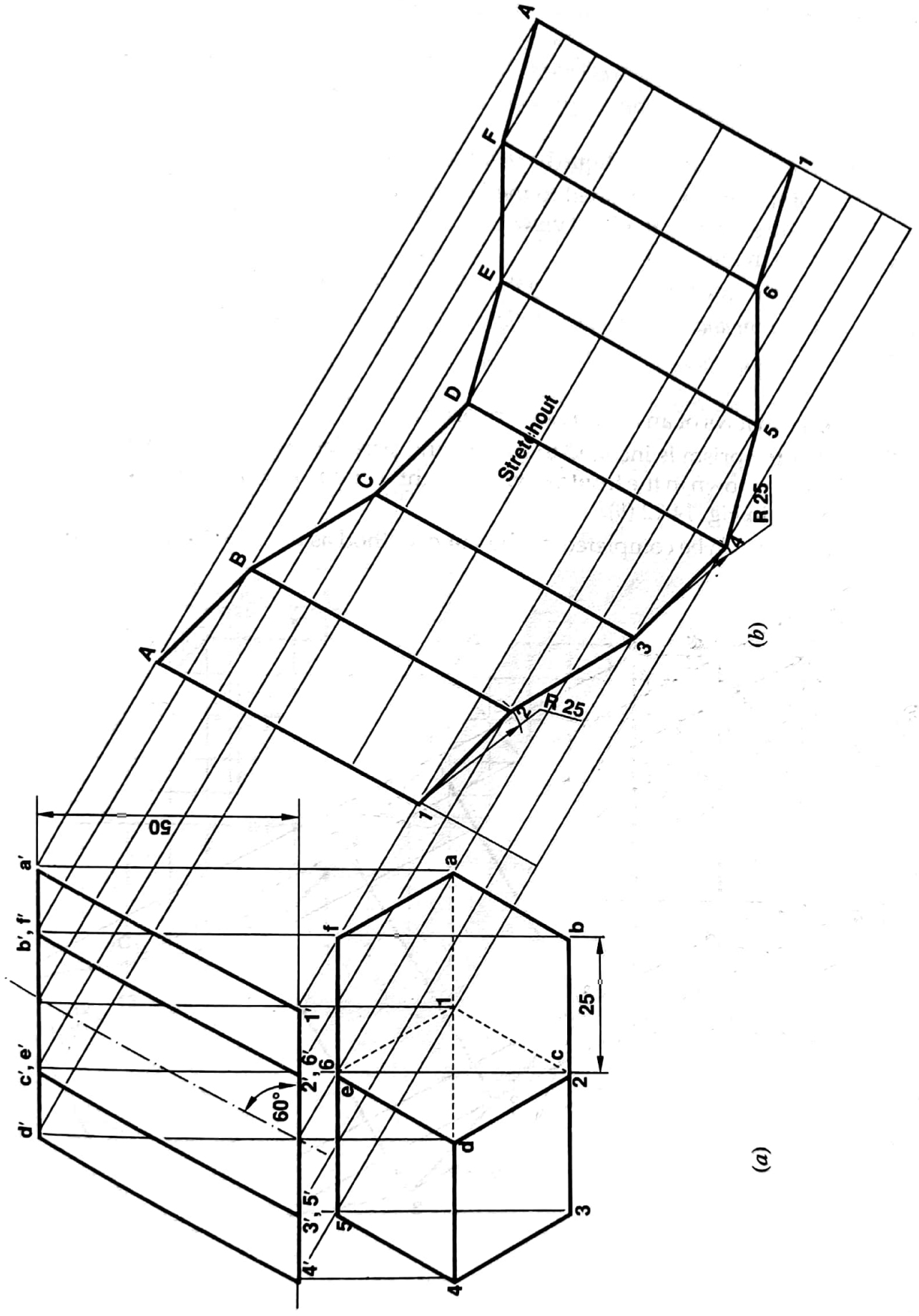


Fig. 14.11 Solution to problem 14.10

(b) Development of oblique prisms : When the axis of a prism is not at right angle to its base, the solid is known as oblique prism. The lateral surface of an oblique prism is developed by the same parallel line method as used for right prisms.

PROBLEM 14.10 *Front and top views of an oblique hexagonal prism are shown in Fig. 14.11 (a). Develop its lateral surface.*

SOLUTION.

(*i*) Draw the given views of an oblique hexagonal prism.

(*ii*) As the axis of the prism is inclined to the HP and parallel to the VP. The true length of the lateral edges are shown in the front view. The right section view can be drawn as an auxiliary view, as shown in Fig. 14.11 (*b*).

(*iii*) The development can be completed by the same method as used for the right prisms.

PROBLEM 14.11 *Front and top views of an oblique square prism are shown in Fig. 14.12 (a). Develop its lateral surface.*

SOLUTION.

(*i*) Draw the given views of an oblique square prism.

(*ii*) As the axis of the prism is inclined to the HP and parallel to the VP. The true length of the lateral edges are shown in the front view. The right section view can be drawn as an auxiliary view, as shown in Fig. 14.12 (*b*).

(*iii*) The development can be completed by the same method as used for the right prism.

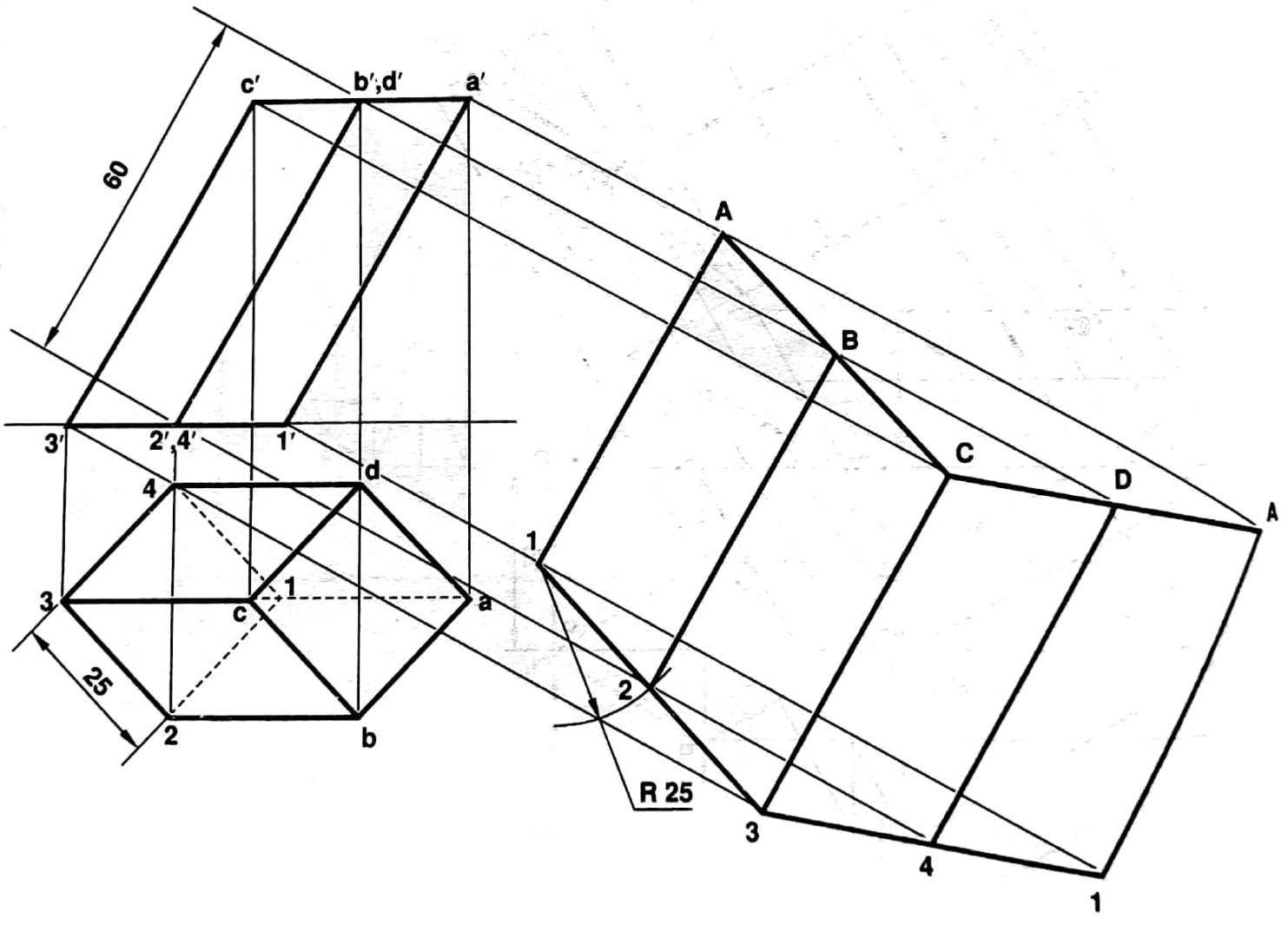


Fig. 14.12 Solution to problem 14.11

(c) Development of right cylinders : The development of the surface of a right circular cylinder is a rectangle, having one side equal to its circumference of its base circle and the other side equal to the height of the cylinder.

Since the surface of a cylinder is smooth and curved, it means that a cylindrical surface has no fold lines. However, a series of elements may be obtained by dividing its base circle into sixteen equal parts, is drawn as the surface of a cylinder.

PROBLEM 14.12 *A truncated cylinder is shown in Fig. 14.13 (a). Develop its lateral surface.*

Or

A cylinder of base 40 mm and height 60 is cut by a section plane which makes 45° with the HP at a distance of 40 mm from lower base. Draw the development for the lower part of the cylinder.

(PTU, Jalandhar May 2006, December 2006, May 2012)

SOLUTION. (*i*) Draw the given views of the truncated cylinder.

(*ii*) Complete the development of the full cylinder along its stretchout line.

(*iii*) Mark the points of intersection *a'*, *b'*, *c'* etc. between the generators and truncated zone of the cylinder.

(*iv*) Locate the points *a'*, *b'*, *c'* etc. on the generator 1, 2, 3 etc. by projecting from the front view.

(*v*) Join these points by a smooth curve as shown in Fig. 14. 13 (*b*).

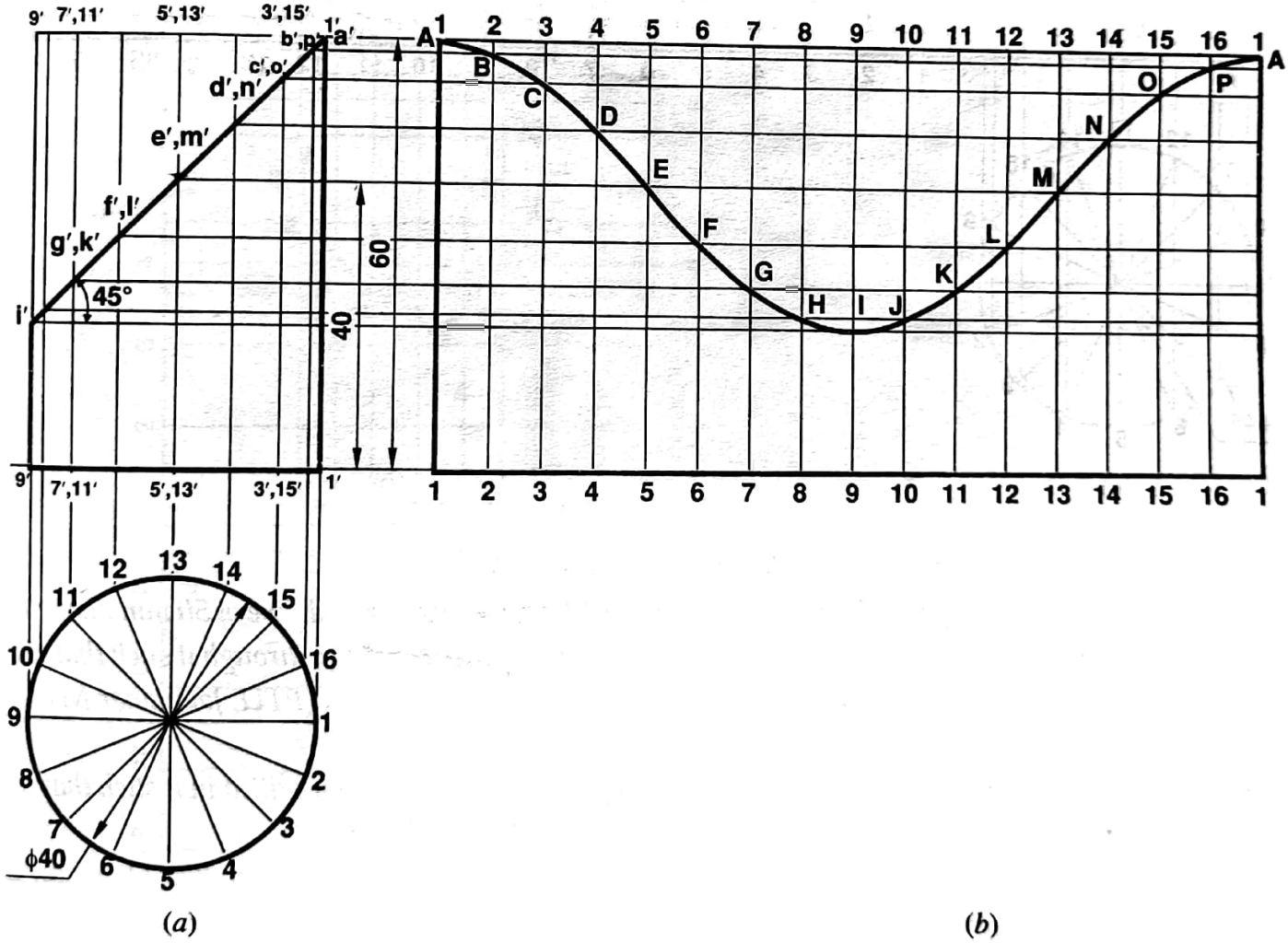
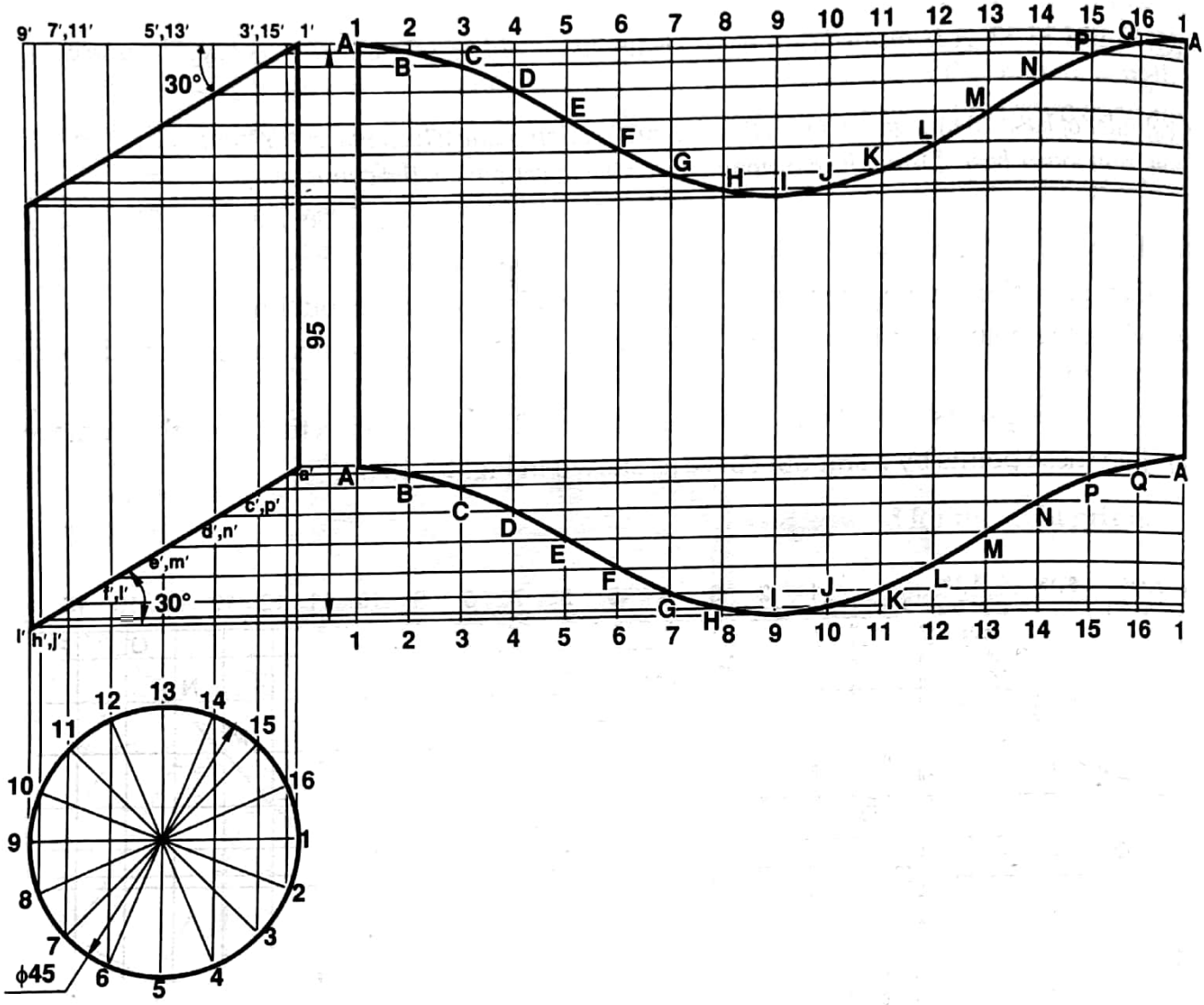


Fig. 14.13 Solution to problem 14.12

PROBLEM 14.13 *Develop the lateral surface of the right circular cylinder cut at top and bottom, as shown in Fig. 14.14 (a).*

SOLUTION. For its solution, see Fig. 14.14 (*b*), which is itself a self-explanatory.



(*a*) (*b*)

Fig. 14.14 Solution to problem 14.13

PROBLEM 14.14 *Develop the circular pattern of a right circular cylindrical pipe of 50 mm diameter and 60 mm height. It has a horizontal circular hole of diameter 30 mm drilled centrally through it such that the axes of the hole and the cylinder are mutually perpendicular to each other.* (*PTU, Jalandhar May 2000*)

Or

A cylinder of diameter 50 mm and height 60 mm has a hole of diameter 30 mm drilled in it such that its axis intersects that of the cylinder at the middle at right angle. Draw the development.

(*PTU, Jalandhar December 2002*)

SOLUTION.

(*i*) Draw the top and front views of the given cylindrical pipe.

(*ii*) Develop the complete cylinder by taking a stretchout line.

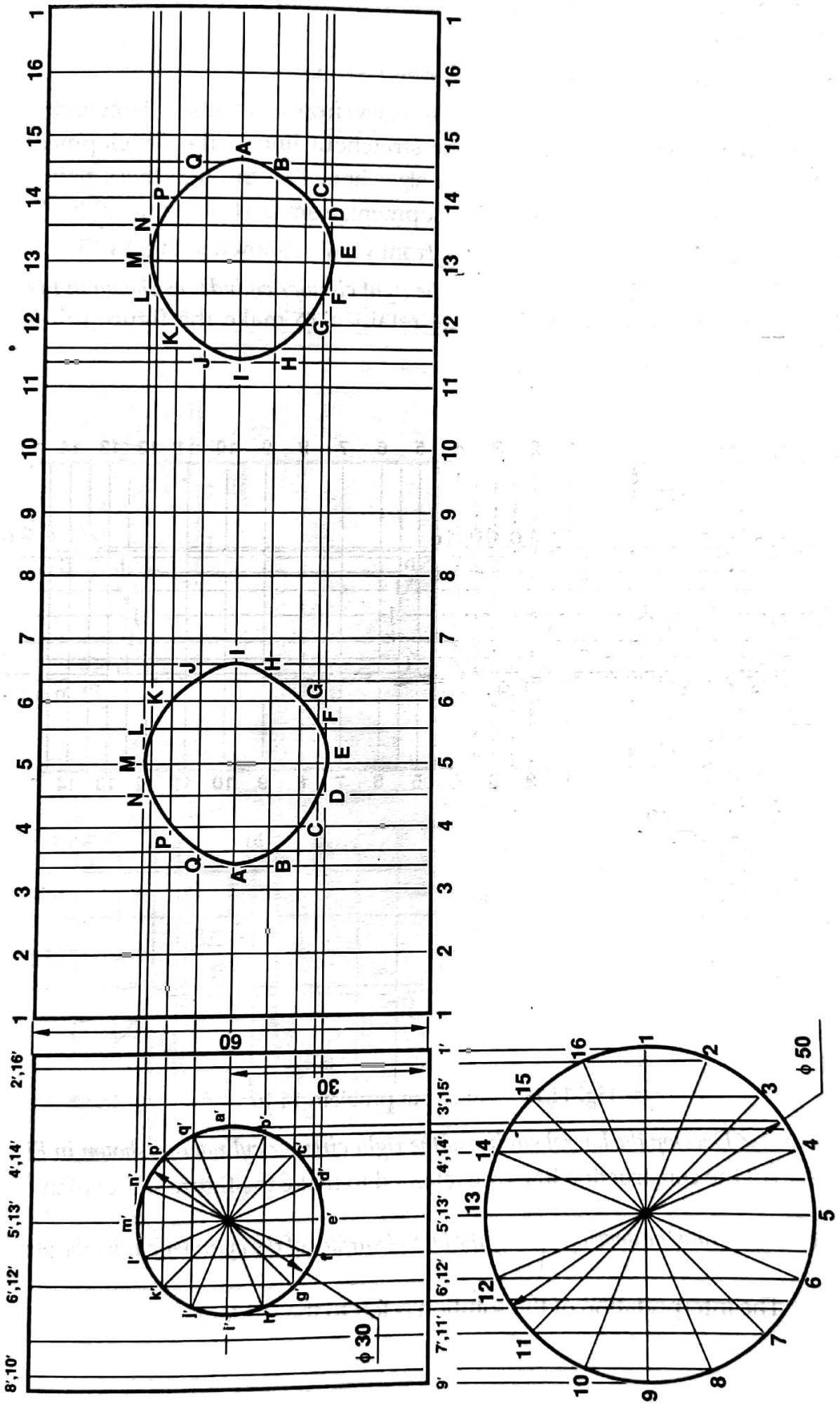


Fig. 14.15 Solution to problem 14.14

(*iii*) Name the points where the circle for the hole in the front view intersect the elements of the cylindrical pipe.

(*iv*) Project these points in the top view as *a*, *b*, *c* and so on.

(*v*) Transfer the distances of these points in the top view from elements of the cylindrical pipe. For example, from point 3 towards 4, to the stretchout line of the development, such that 3*a* = 3A etc. Erect perpendiculars to the stretchout line through these points, to cut the horizontal projections, taken into the development, from the front views *a'*, *b'*, *c'* etc.

(*vi*) Join these points by a smooth curve in the front view as shown in Fig. 14.15.

PROBLEM 14.15 *Develop the lateral surface of the right circular cylinder as shown in Fig. 14.15 (a).*

SOLUTION. All the construction lines are retained to make the figure self-explanatory. See Fig. 14.161 (*b*).

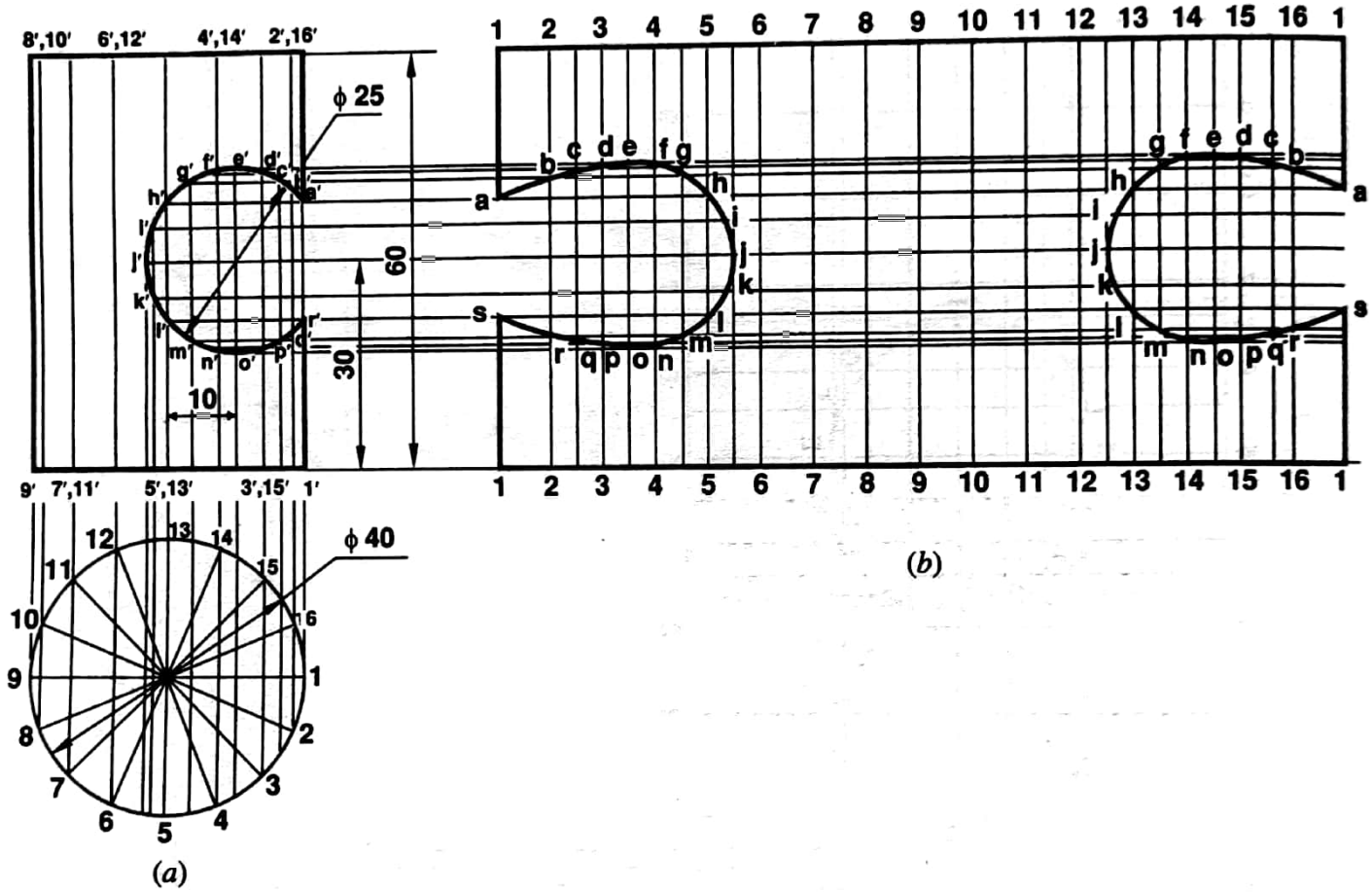


Fig. 14.16 Solution to problem 14.15

PROBLEM 14.16 *Develop the lateral surface of the right circular cylinder as shown in Fig. 14.17 (a).*

SOLUTION. All the construction lines are retained to make the figure self-explanatory. See Fig. 14.17 (*b*).

PROBLEM 14.17 *Draw the development of the lateral surface of the cylinder cut by the planes as shown in Fig. 14.18 (a).*

SOLUTION. The interpretation of the solution is left to the student. See Fig. 14.18 (*b*).

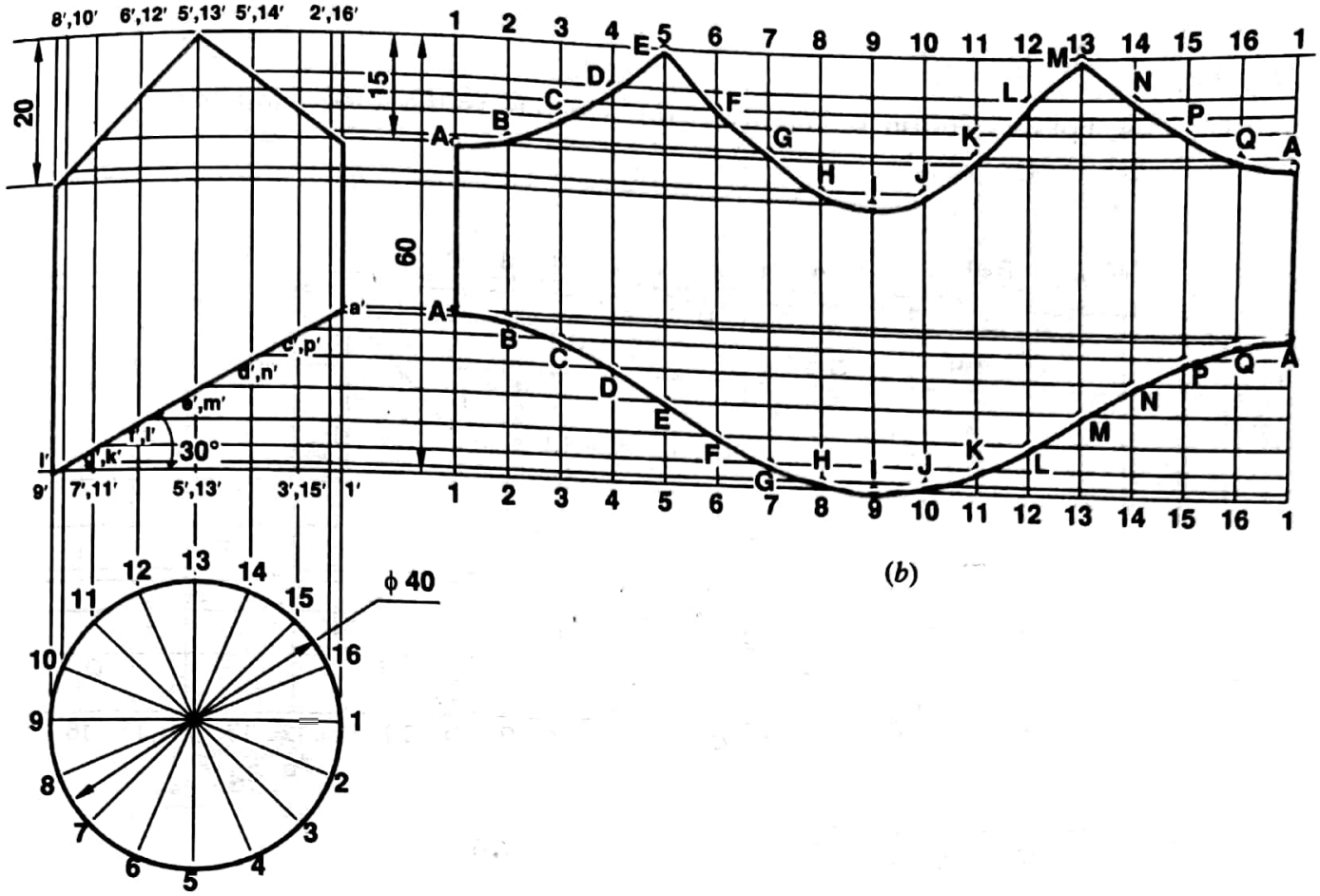


Fig. 14.17 Solution to problem 14.16

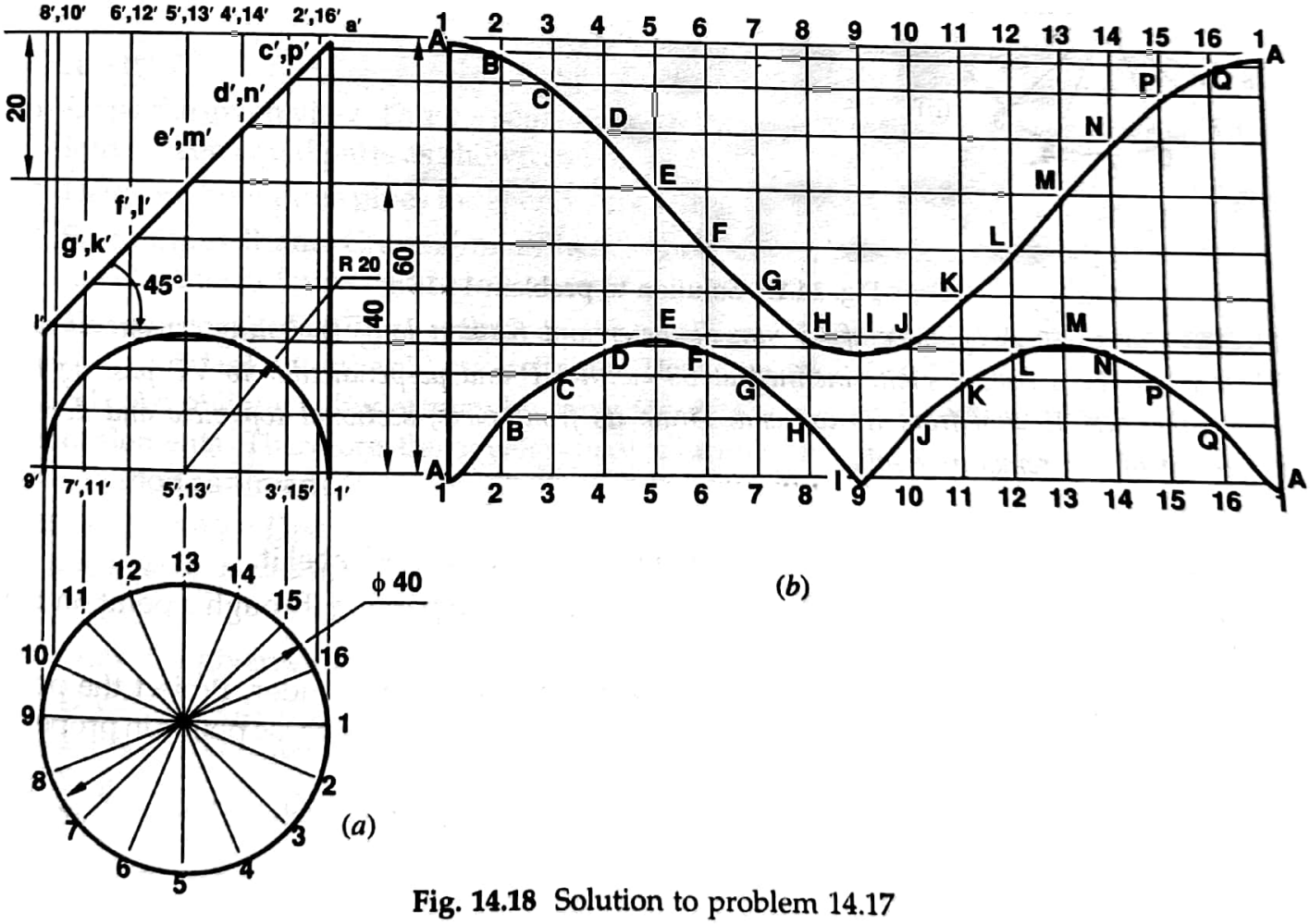


Fig. 14.18 Solution to problem 14.17

PROBLEM 14.18 *Draw the development of the lateral surface of the right circular cylinder as shown in Fig. 14.19 (a).*

SOLUTION. For its solution, see Fig. 14.19 (*b*), which is itself a self-explanatory.

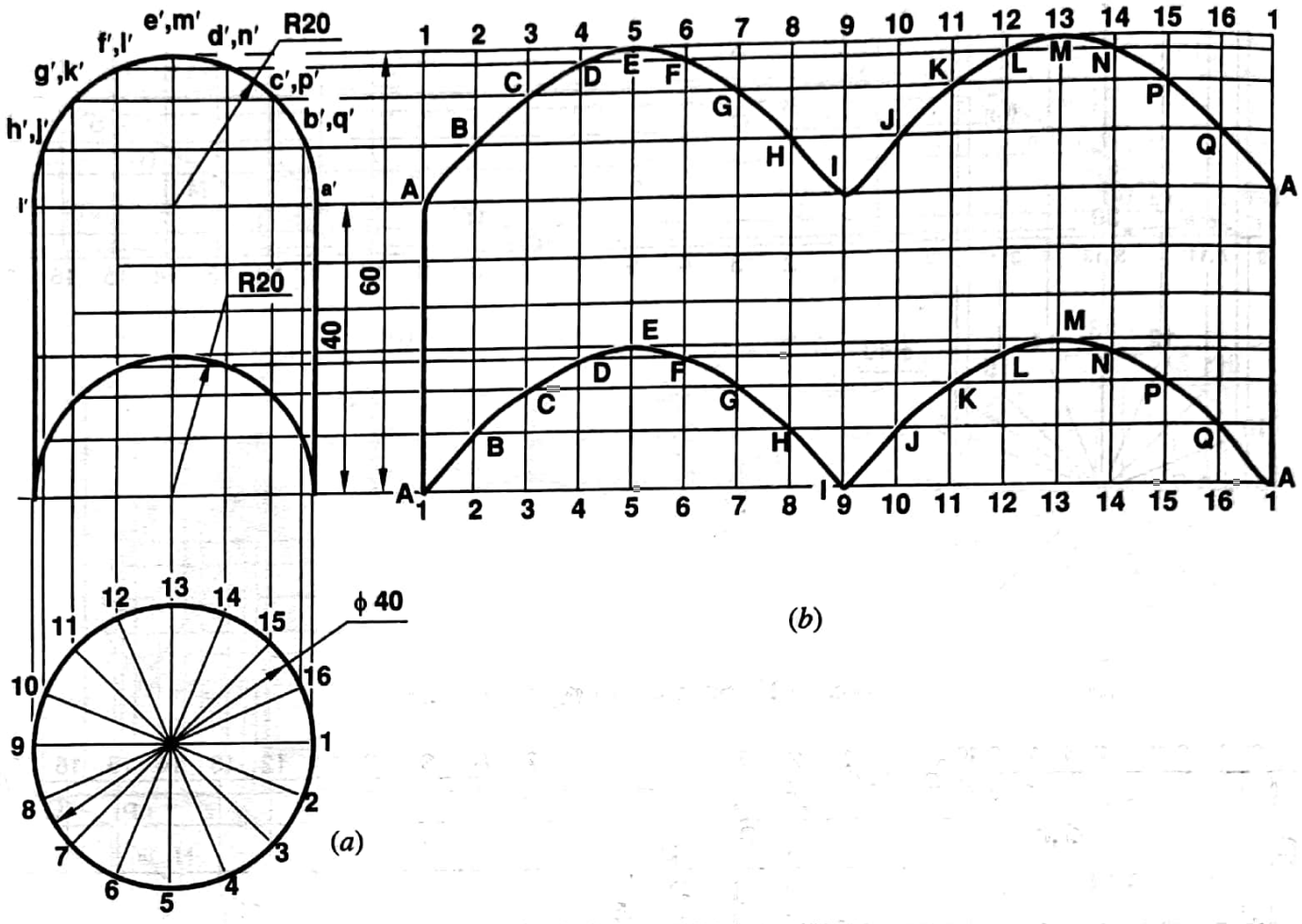


Fig. 14.19 Solution to problem 14.18

PROBLEM 14.19 *A cylinder of 45 mm diameter and 60 mm long is resting on one of its bases on HP. It is cut by a section plane inclined at 60° with HP and perpendicular to VP passing through a point on the axis 15 mm from its top end. Draw its front view, sectional top view and develop the lateral surface of the remaining solid.*

SOLUTION.

(*i*) Draw the projections of the cylinder in the given position and label it.

(*ii*) Draw the cutting plane line VT inclined at 60° to *xy* and passing through a point on the axis 15 mm from the top end of the cylinder.

(*iii*) The cutting plane line VT cuts the various elements of the cylinder. Project the points of intersection in the top view to their corresponding elements. Join these points in proper order and draw section lines in it.

(*iv*) Complete the development of the remaining cylinder along its stretchout line as shown in Fig. 14.20.

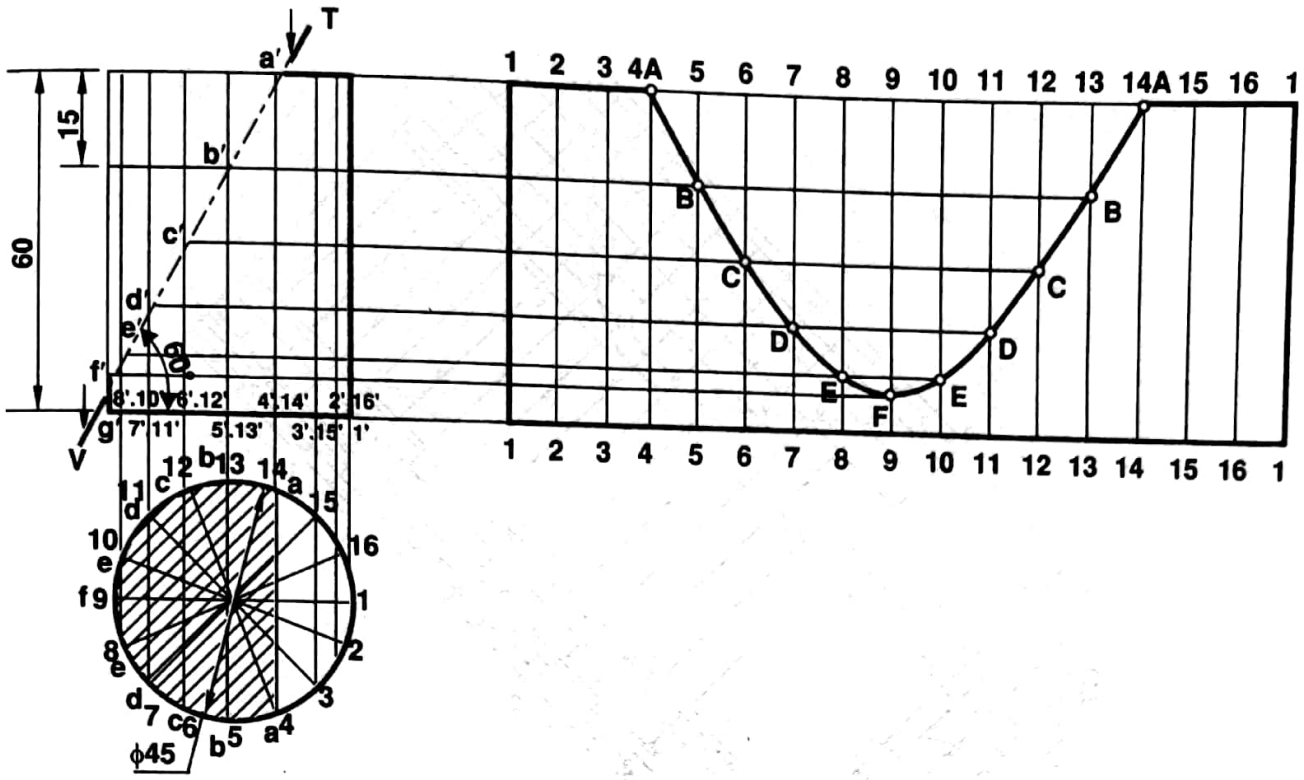


Fig. 14.20 Solution to problem 14.19

PROBLEM 14.20 *Three cylindrical pipes of diameter 30 mm form a Y-piece as shown in Fig. 14.21 (a). Draw the development of the lateral surface of each pipe.*

SOLUTION.

(*i*) Draw the given view. Draw a semi-circle on the base of the pipe A as diameter and divide it into sixteen equal parts as shown in Fig. 14.21 (*b*).

(*ii*) Develop these three pipes independently, using the principle of parallel line method.

(*iii*) Since pipes B and C are similar, hence their developments are also similar. Therefore, development of pipe B is done here.

(*d*) Development of oblique cylinders. When the axis of an oblique cylinder is not at right angle to its base, the solid is known as oblique cylinder. Therefore, its cross-section at right angles to the axis is elliptical. An oblique cylinder may be considered of as a regular oblique prism having infinite number of elements. Therefore, the development of an oblique cylinder may be constructed by using the same method as already been described for oblique prisms.

PROBLEM 14.21 *Draw the inside pattern of an oblique cylinder of 40 mm base diameter and 60 mm vertical height, with its axis inclined to its base at 60° as shown in Fig. 14.22 (a).*

SOLUTION.

(*i*) Draw the given views of an oblique cylinder.

(*ii*) Divide the circle into sixteen equal parts in the top view and draw surface lines for a right cylinder in the front view.

(*iii*) In this case, the stretchout line of the pattern will not be equal to the circumference of the circular top view.

(*iv*) Project the end points of the surface to the pattern as shown in Fig. 14.22 (*b*).

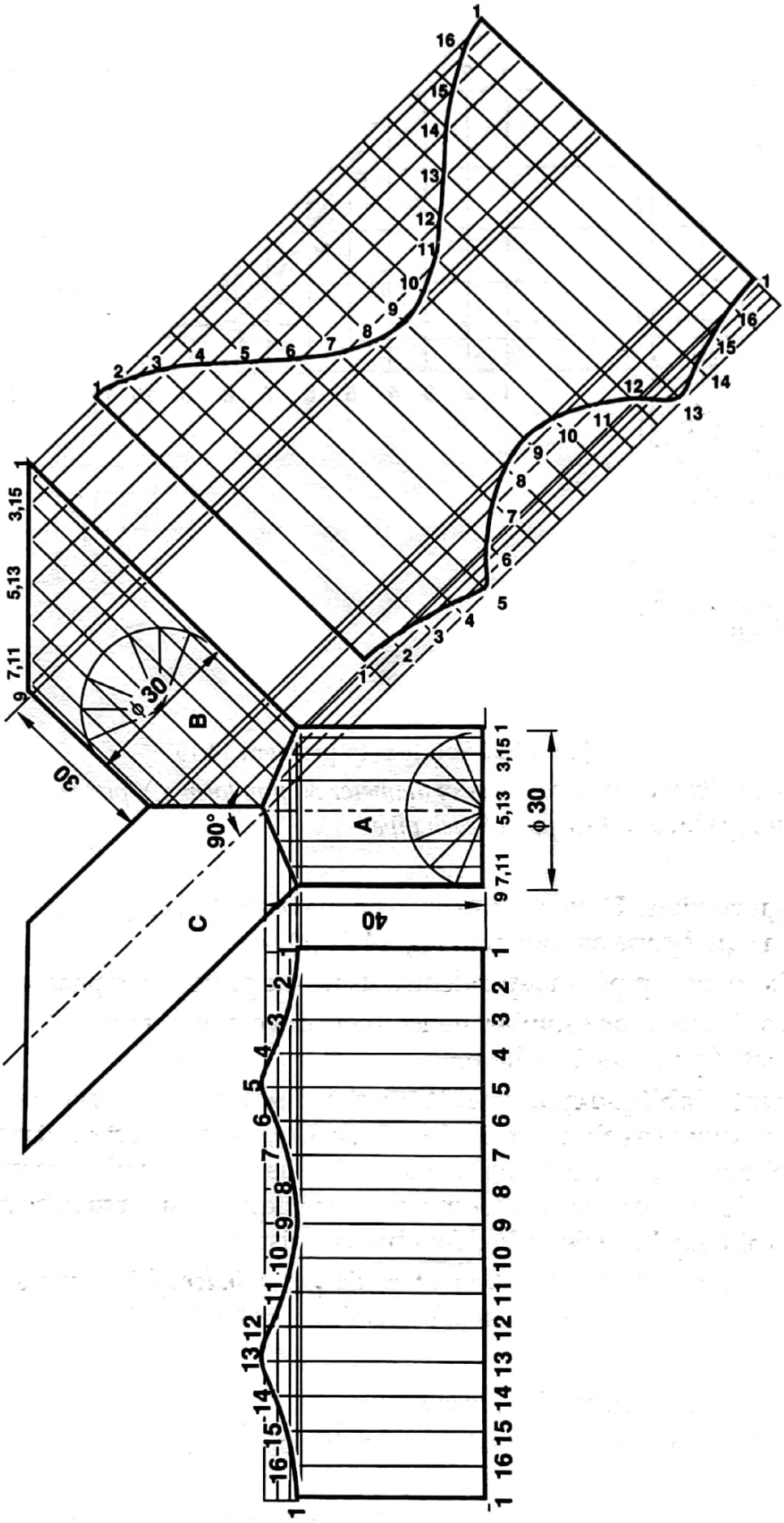


Fig. 14.21 Solution to problem 14.20

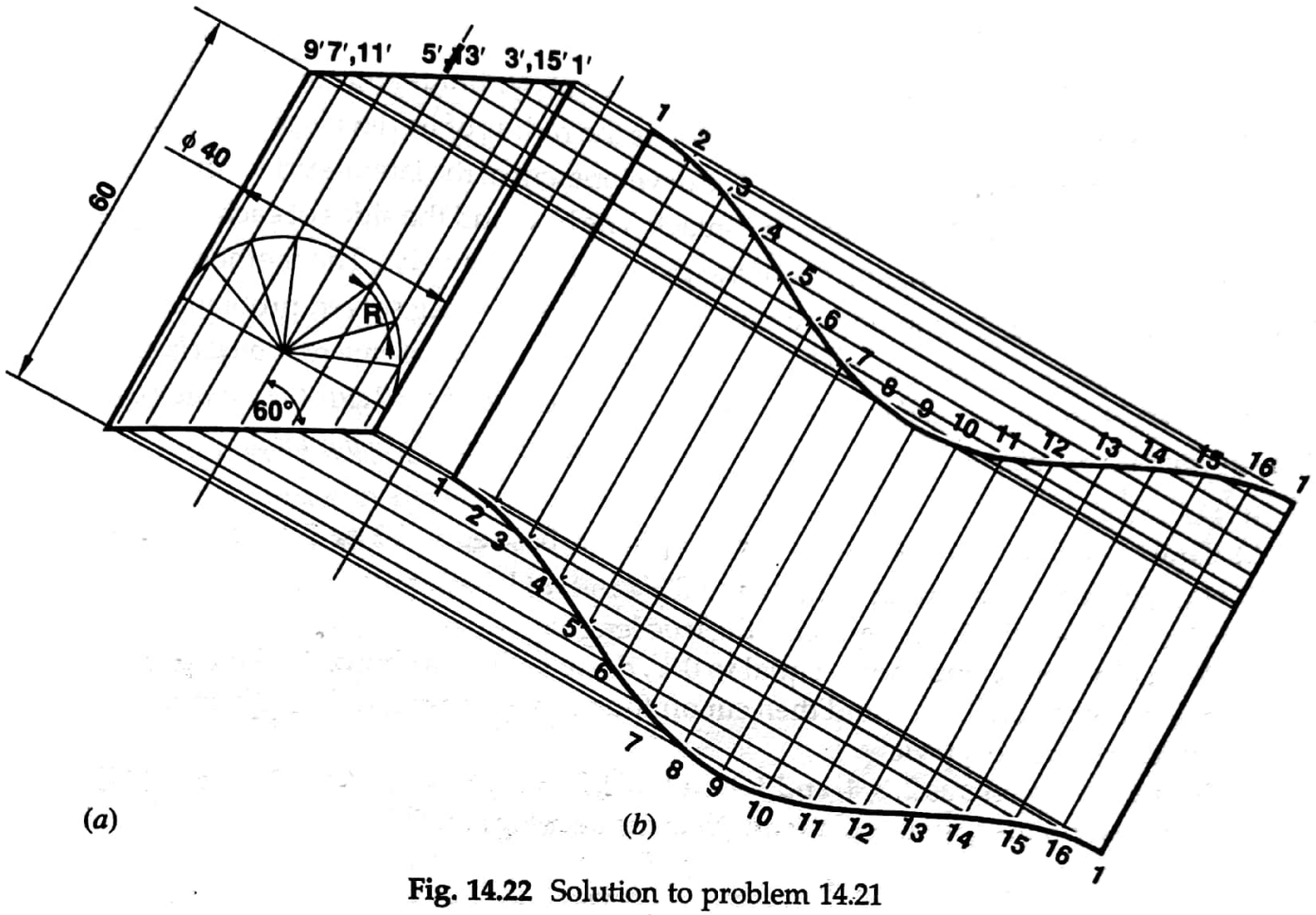


Fig. 14.22 Solution to problem 14.21

PROBLEM 14.22 *Develop the full size inside pattern of an oblique cylindrical piece made of sheet metal, shown in Fig. 14.23 (b).*

SOLUTION. All the construction lines are retained to make the figure self-explanatory. See Fig. 14.23 (*b*).

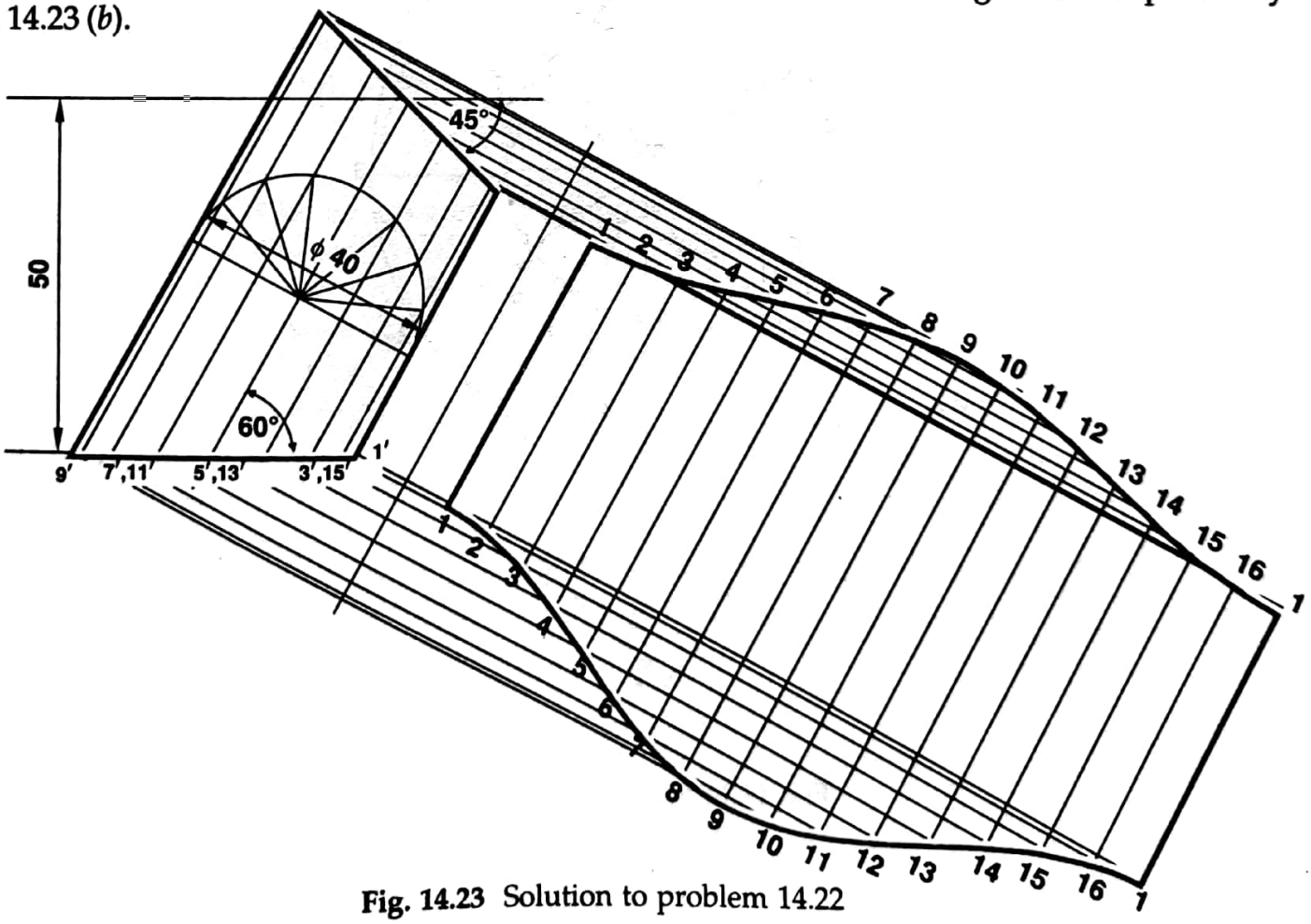


Fig. 14.23 Solution to problem 14.22

14.4 RADIAL LINE METHOD

The radial line method is used for the development of pyramids and cones in which apex or vertex is taken as centre and its slant edge or generator as the radius of its development.

(a) Development of right pyramids : The development of the lateral surface of a pyramid consists of a number of equal isosceles triangles in contact. The base and the sides of each triangle are equal to the edge of the base and the slant edge of the pyramid respectively. The method of developing the inside pattern of the right pyramids are explained in the following solved problems.

PROBLEM 14.23 *A square pyramid edge of base 30 mm and height 50 mm rests on its base in HP such that all of its base edges are equally inclined to VP. A section plane parallel the HP cuts the pyramid bisecting its axis. Draw its front view, sectional top view and develop the lateral surface of the pyramid.*

SOLUTION.

(*i*) Draw a square 1234 in the top view, keeping its base edge 4-1 or 4-3 inclined to the VP at 45°. Project its corresponding front view 1′2′3′4′.

(*ii*) Here 0′ 1′ or 0′ 3′ slant edges give the true length.

(*iii*) With 0′ as centre and radius equal to 0′1′, draw an arc as shown in the Fig. 14.24.

(*iv*) Select point 1 on this arc and then cut off four equal divisions of length equal to the length of an edge from the top view.

(*v*) Name these points as 2, 3, 4, and 1. Join 1 to 2, 2 to 3, 3 to 4 and 4 to 1 by straight lines and also join these points to points 0′ to complete the development.

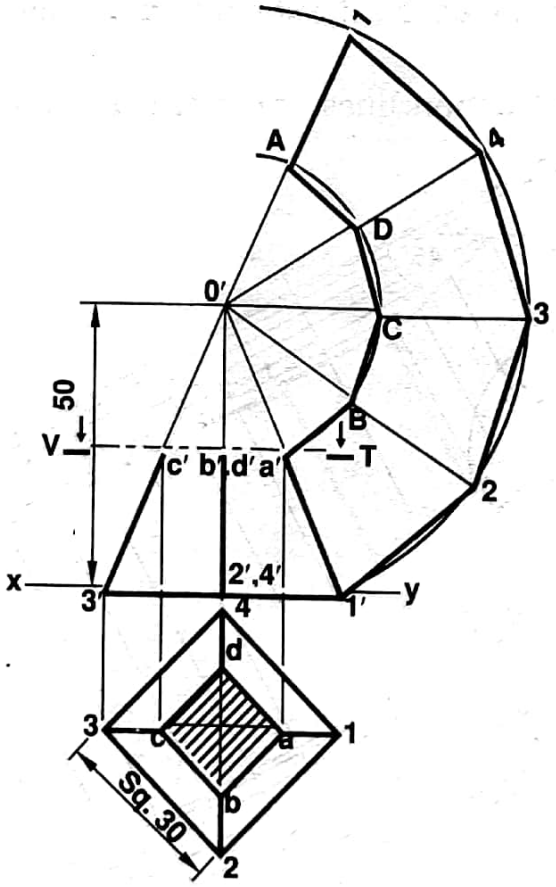


Fig. 14.24 Solution to problem 14.23

PROBLEM 14.24 *A right regular hexagonal pyramid, edge of 20 mm and height 40 mm, rests on its base in HP such that one of its base edge parallel to the VP. Draw its projections and develop its lateral surface.*

(PTU, Jalandhar May 2007)

SOLUTION.

(*i*) Draw a regular hexagon, 123456 in the top view, keeping one of its base edge say 5-6 or 3-2 parallel to the VP. Project its corresponding front view 1′ 2′ 3′ 4′ 5′ 6′.

(*ii*) The true lengths of the slant edges of 0′ 1′ or 0′ 4′ of a regular hexagon pyramid are to be measured from the front view, as the top view of these edges 01 or 04 are parallel to the xy.

(*iii*) So with 0′ as centre and radius equal to 0′ 1′, draw an arc as shown in Fig. 14.25.

(*iv*) Select point 1 on this arc and then cut off six equal divisions of length equal to the length of an edge from the top view.

(*v*) Name these points as 2, 3, 4, 5, 6 and 1. Join 1 to 2, 2 to 3, 3 to 4, 4 to 5, 5 to 6 and 6 to 1 by straight lines and also join these points to point 0′ to complete the development.

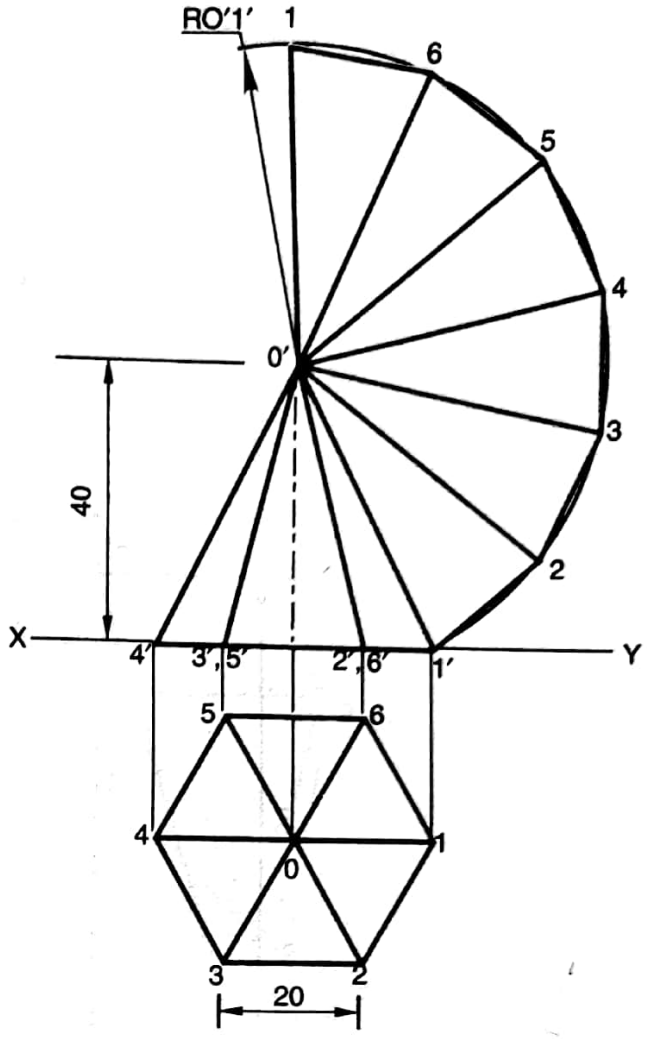


Fig. 14.25 Solution to problem 14.24

PROBLEM 14.25 *A square pyramid, edge of base 30 mm and height 50 mm, rests on its base on HP such that one of its base edges is parallel to the VP. Draw its projections and develop its lateral surface.*

(PTU, Jalandhar December 2004, May 2014)

SOLUTION.

(*i*) Draw a square 1234 in the top view, keeping its base edge 1-4 parallel to the VP. Project its corresponding front view 1′2′3′4′.

(*ii*) Here, none of the slant edges gives the true length. The true length of a slant edge $o'1_1'$ of a square pyramid is to be measured from the front view, as the top view of that edge is parallel to *xy*.

(*iii*) As all the slant edges are of the same length for a square pyramid, so with $o'1_1'$ as radius and *o*′ as centre, draw an arc as shown in Fig. 14.26.

(*iv*) Select point 1 on this arc and then cut off four equal divisions of length equal to the length of an edge from the top view.

(*v*) Name these points as 2, 3, 4 and 1. Join 1 to 2, 2 to 3, 3 to 4 and 4 to 1 by straight lines and also join these points to point *o*′ to complete the development.

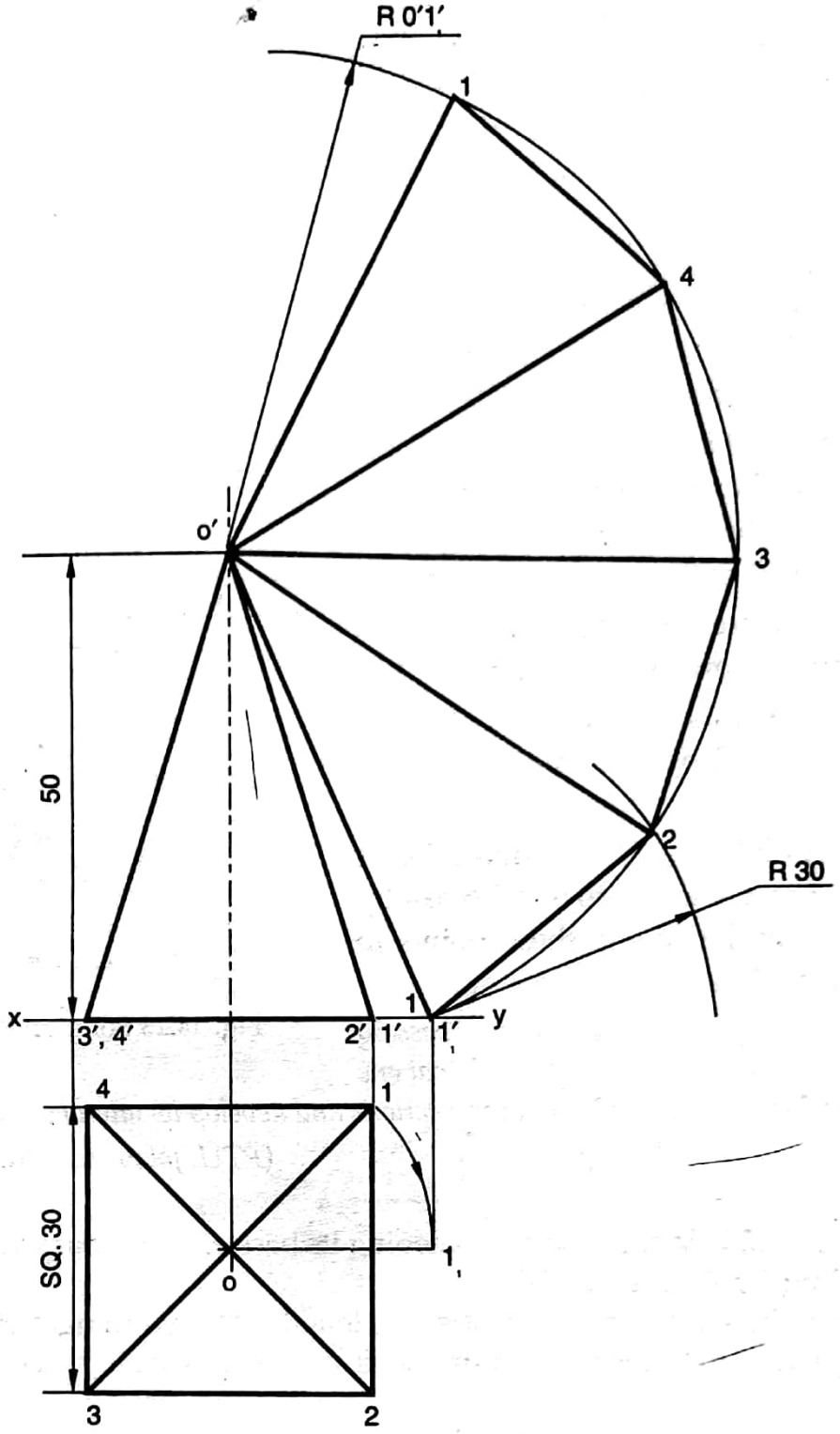


Fig. 14.26 Solution to problem 14.25

PROBLEM 14.26 *A square pyramid, edge of base 30 mm and height 50 mm, rests on its base in HP such that one of its base edges is parallel to the VP. A section plane perpendicular to the VP and inclined to the HP at 30° cuts the pyramid bisecting its axis. Draw its front view, sectional top view and develop the lateral surface of the truncated pyramid.*

SOLUTION.

(*i*) The development is first drawn as a complete pyramid from the apex or vertex to the base.

(*ii*) The true lengths of cut away parts of the slant edges, from the apex 0 to the points, *a*, *b*, *c* and *d* are found by projecting horizontally the point *a'*, *b'*, *c'* and *d'* in the front view to new positions on $0'1_1'$ slant edge. (true length).

(*iii*) These true length distances are measured along their respective radial lines from *o'* in the development at points *A*, *B*, *C* and *D*. These points are then joined by straight lines as shown in Fig. 14.27 to complete the development of the truncated pyramid.

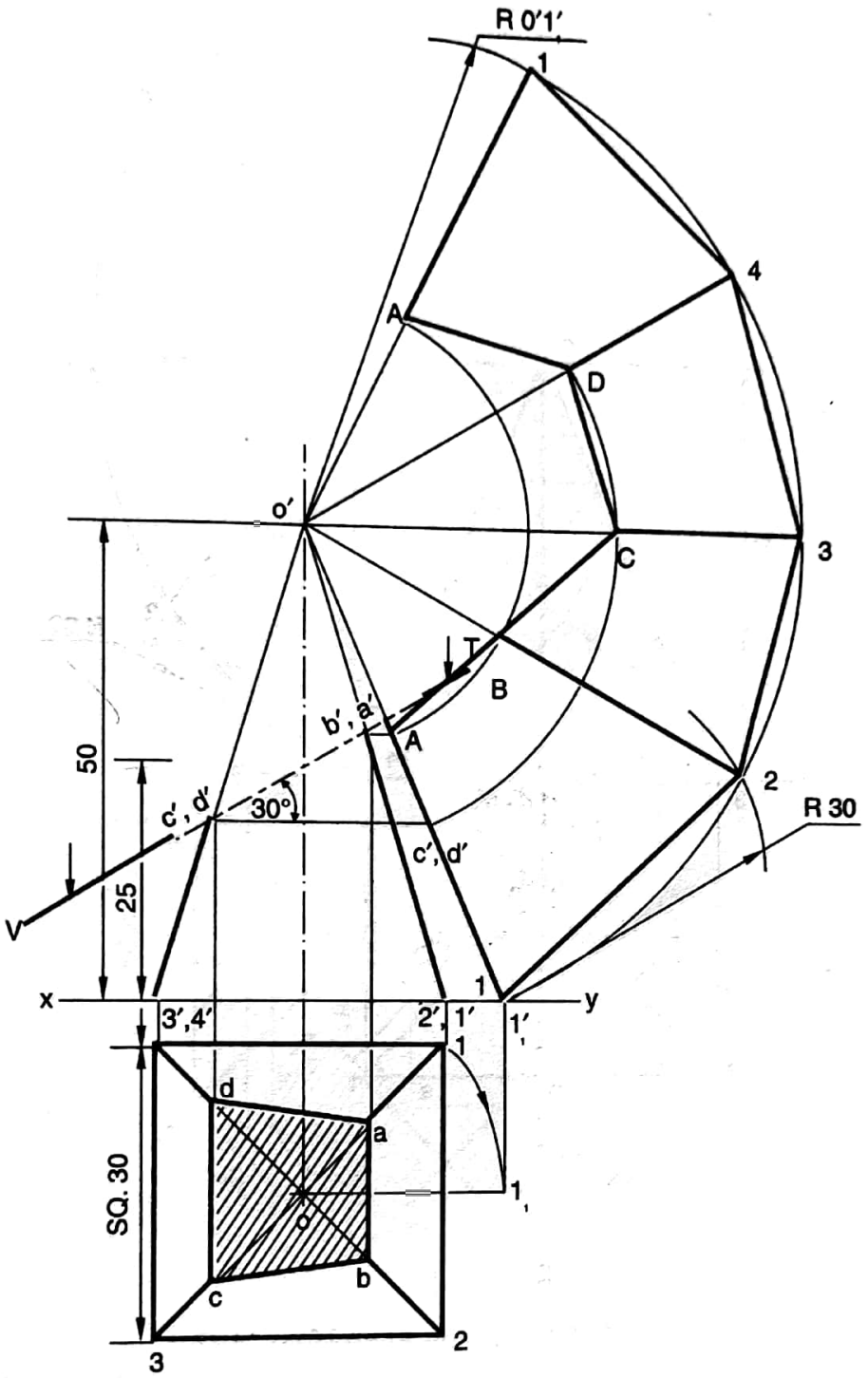


Fig. 14.27 Solution to problem 14.26

PROBLEM 14.27 *A square pyramid, edge of base 30 mm and height 50 mm, rests on its base, in HP such that all its base edges are equally inclined to the VP. Draw its projections and develop its lateral surface.*

(PTU, Jalandhar December 2004)

SOLUTION.

(*i*) Draw a square, 1234 in the top view, keeping its base edges equally inclined to the VP. Project its corresponding front view, 1'2'3'4'.

(*ii*) The true lengths of the slant edges of 0'1' and 0'3' of a square pyramid are to be measured from the front view, as the top view of these edges 01 and 03 respectively are parallel to the *xy*.

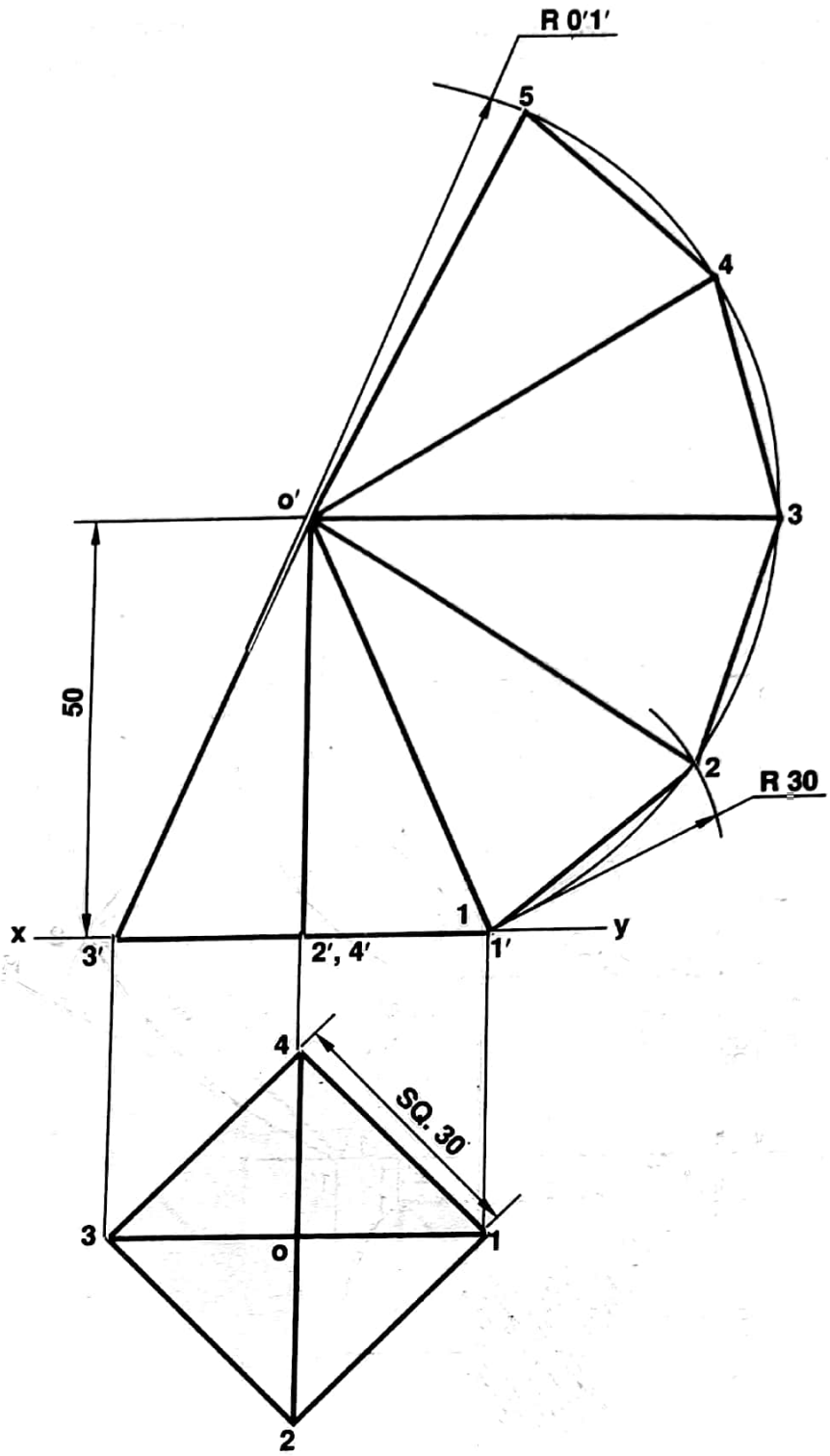


Fig. 14.28 Solution to problem 14.27

(*iii*) As all the slant edges are of the same length for a square pyramid, so with 0'1' as radius and *o'* as centre, draw an arc as shown in Fig. 14.28.

(*iv*) Select point 1 on this arc and then cut off four equal divisions of length equal to the length of an edge from the top view.

(*v*) Name these points as 2, 3, 4 and 1. Join 1to 2, 2 to 3, 3 to 4 and 4 to 1 by straight lines and also join these points to point *o'* to complete the development.

PROBLEM 14.28 *A square pyramid, edge of base 30 mm and height 50 mm, resting on its base in HP such that all of its base edges are equally inclined to the VP. A section plane perpendicular to the VP and inclined to the HP at 30° cuts the pyramid, bisecting its axis. Draw its front view, sectional top view and develop the lateral surface of the truncated pyramid.*

(PTU, Jalandhar December 2014)

SOLUTION. The solution to this problem is self-explanatory. See Fig. 14.29.

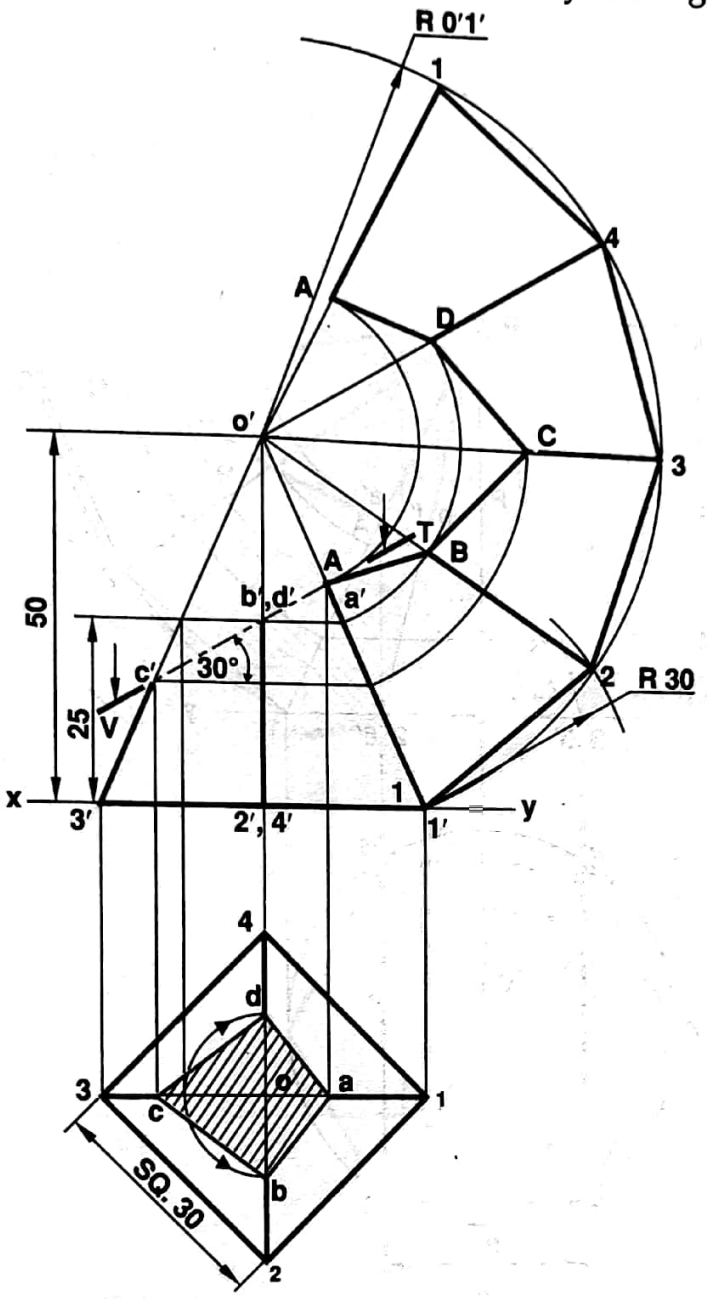


Fig. 14.29 Solution to problem 14.28

PROBLEM 14.29 *Draw the development of the lateral surface of the part P of the pyramid, the front view of which is shown in Fig. 14.30 (a). The pyramid is hexagonal, two sides of the base parallel to the VP.*

(PTU, Jalandhar December 2002)

SOLUTION. All the construction lines are retained to make the solution self-explanatory. See Fig. 14.30 (*b*).

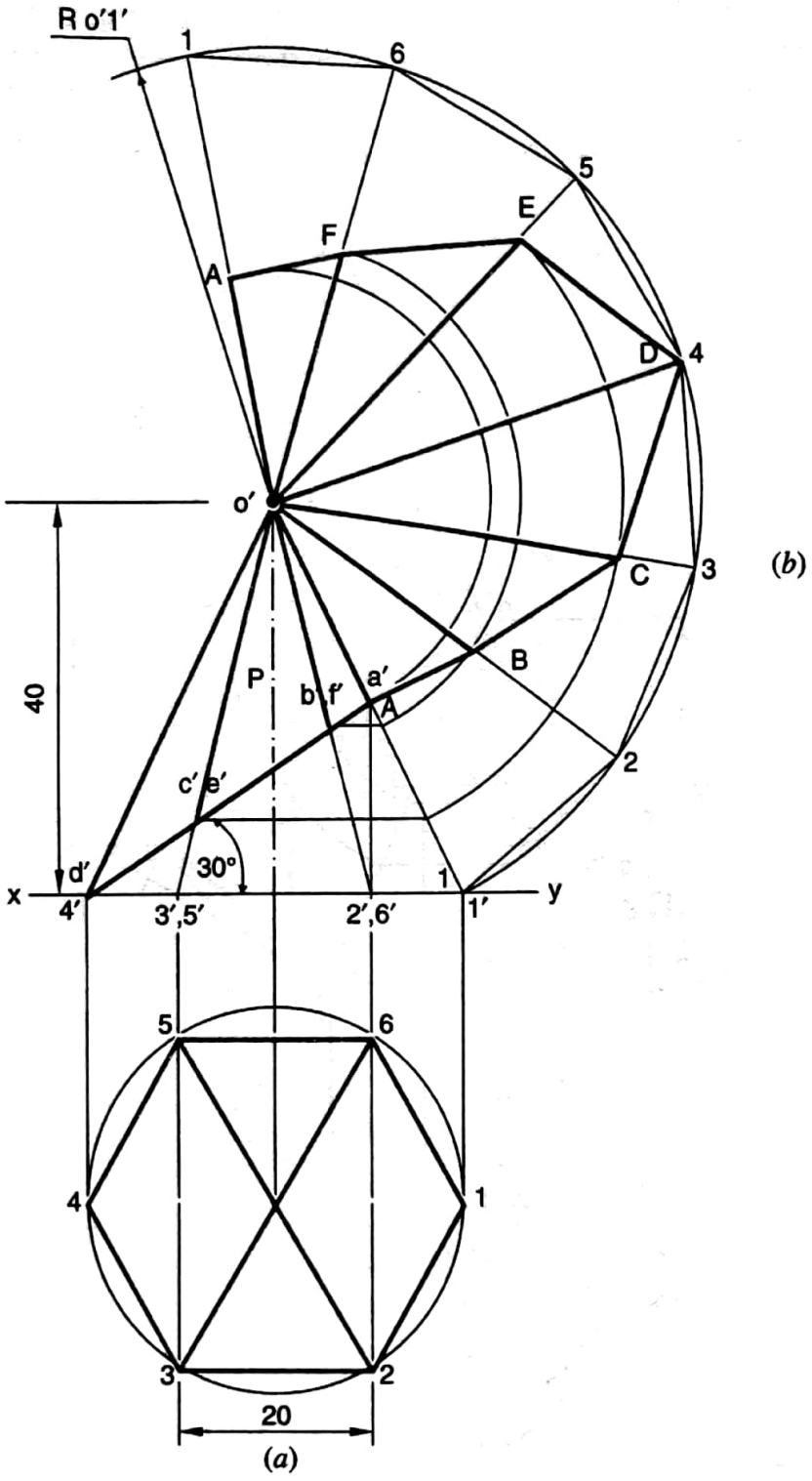


Fig. 14.30 Solution to problem 14.29

PROBLEM 14.30 *A right regular pentagonal pyramid, edge of base 25 mm and height 60 mm, is lying on one of its triangular faces in HP, with its axis parallel to the VP. A section plane perpendicular to the HP and inclined to the VP at 30° cuts pyramid, bisecting its axis. Draw its top view, sectional front view and develop the lateral surface of the remaining pyramid.*

SOLUTION. (*i*) Draw the projections of the pentagonal pyramid in the given conditions.

(*ii*) Then cut the solid in the top view by a section plane perpendicular to the HP and inclined to the VP at 30°, fulfilling all the given conditions and then draw the sectional front view from it.

(*iii*) Develop the remaining part of the pyramid as discussed in the previous problems using radial line method. See Fig. 14.31.

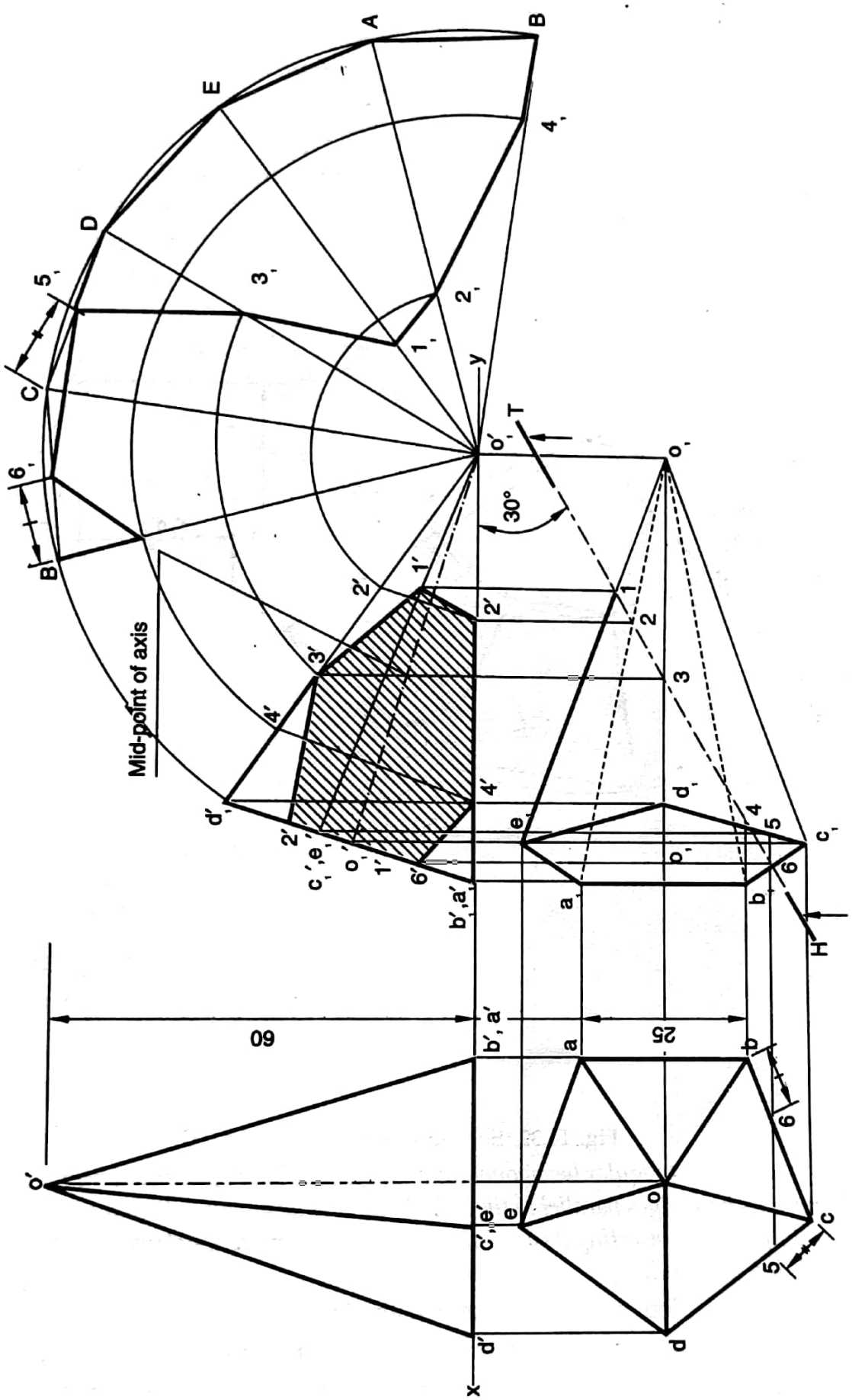


Fig. 14.31 Solution to problem 14.30

PROBLEM 14.31 *A right regular pentagonal pyramid, edge of base 30 mm and height 50 mm, rests on its base in HP with one of its base edges perpendicular to the VP. A section plane perpendicular to the VP and parallel to the HP cuts the pyramid, bisecting its axis. Draw its front view, sectional top view and develop the lateral surface of the remaining pyramid.*

SOLUTION. (*i*) Draw the projections of the pentagonal pyramid in the given conditions.

(*ii*) Cut the pyramid in the front view by a section plane perpendicular to the VP and parallel to the HP, fulfilling all the given conditions and then draw the sectional top view.

(*iii*) Develop the remaining part of the pyramid as discussed in the earlier problems using radial line method. See Fig. 14.32.

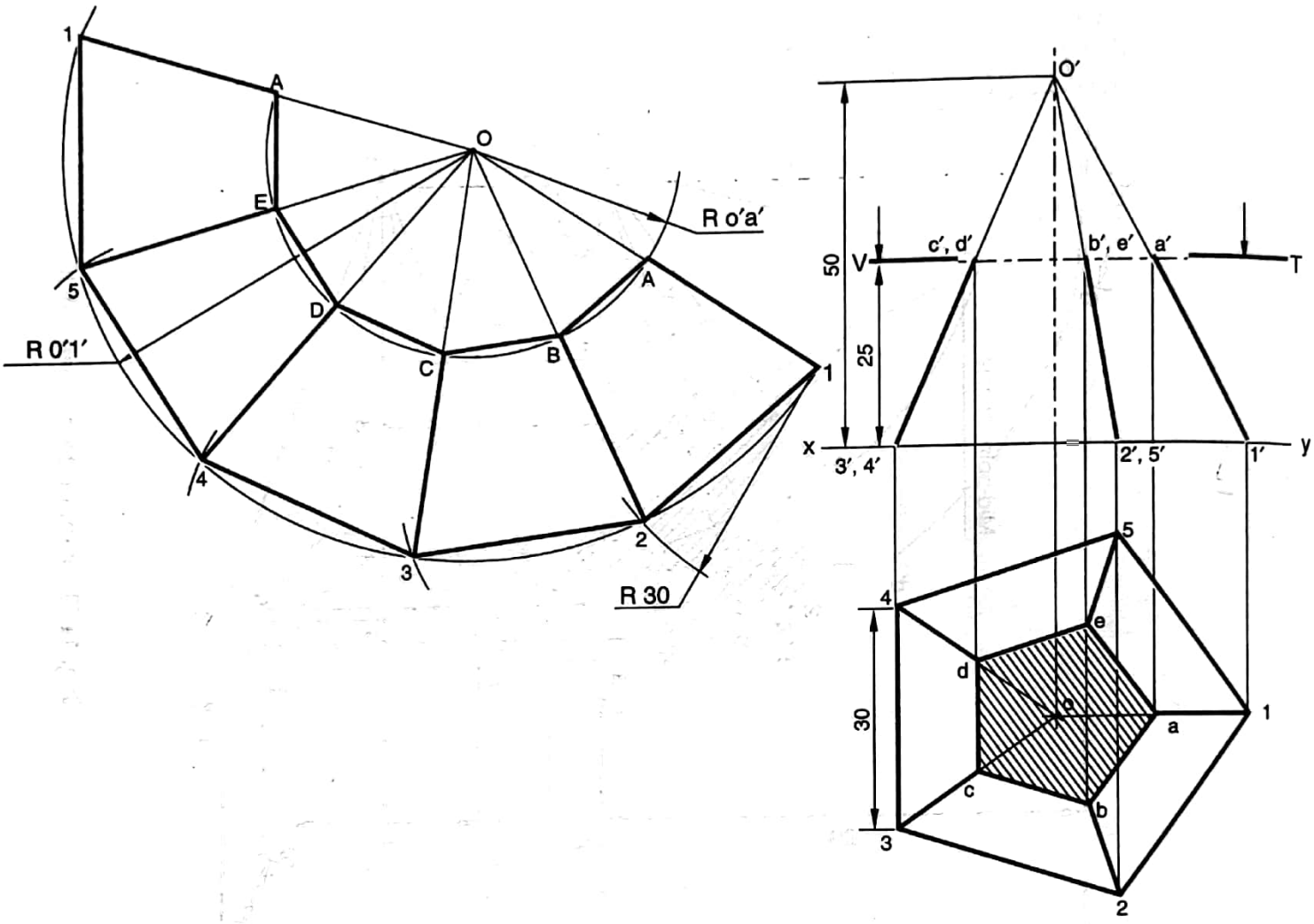


Fig. 14.32 Solution to problem 14.31

PROBLEM 14.32 *A right regular hexagonal pyramid, edge of base 25 mm and height 55 mm, rests on its base in HP with one of its base edges parallel to the VP. A section plane perpendicular to the VP and inclined to the HP at 30°, cuts the pyramid, bisecting its axis. Draw its front view, sectional top view and develop the lateral surface of the remaining pyramid.*

SOLUTION. This problem consists of three distinct parts :

(*i*) Firstly, draw the projections of the hexagonal pyramid in the given conditions.

(*ii*) Secondly, add the section plane to the front view to satisfy the given conditions and draw the sectional top view applying principles of section of the solids.

(*iii*) Finally, develop the remaining part of the pyramid as discussed in the previous problems using radial line method. See Fig. 14.33.

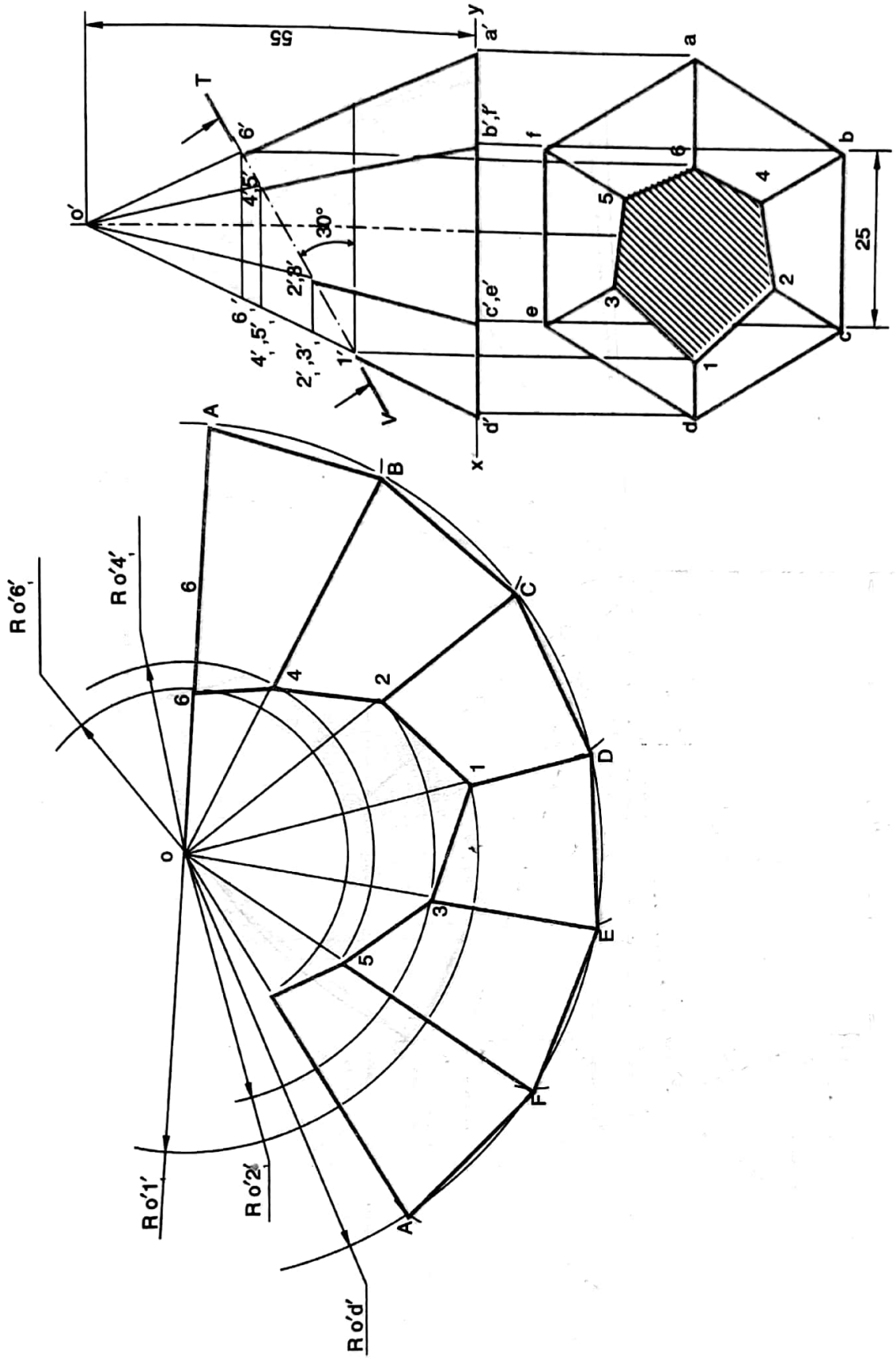


Fig. 14.33 Solution to problem 14.32

PROBLEM 14.33 *Develop the surface of a hexagonal pyramid side of base 28 mm and height 60 mm. The pyramid is resting on its base on the ground and the edge of the base is inclined at 20° to VP.*

(*PTU, Jalandhar May 2005*)

SOLUTION. All construction lines are shown to make the figure self-explanatory. See Fig. 14.34.

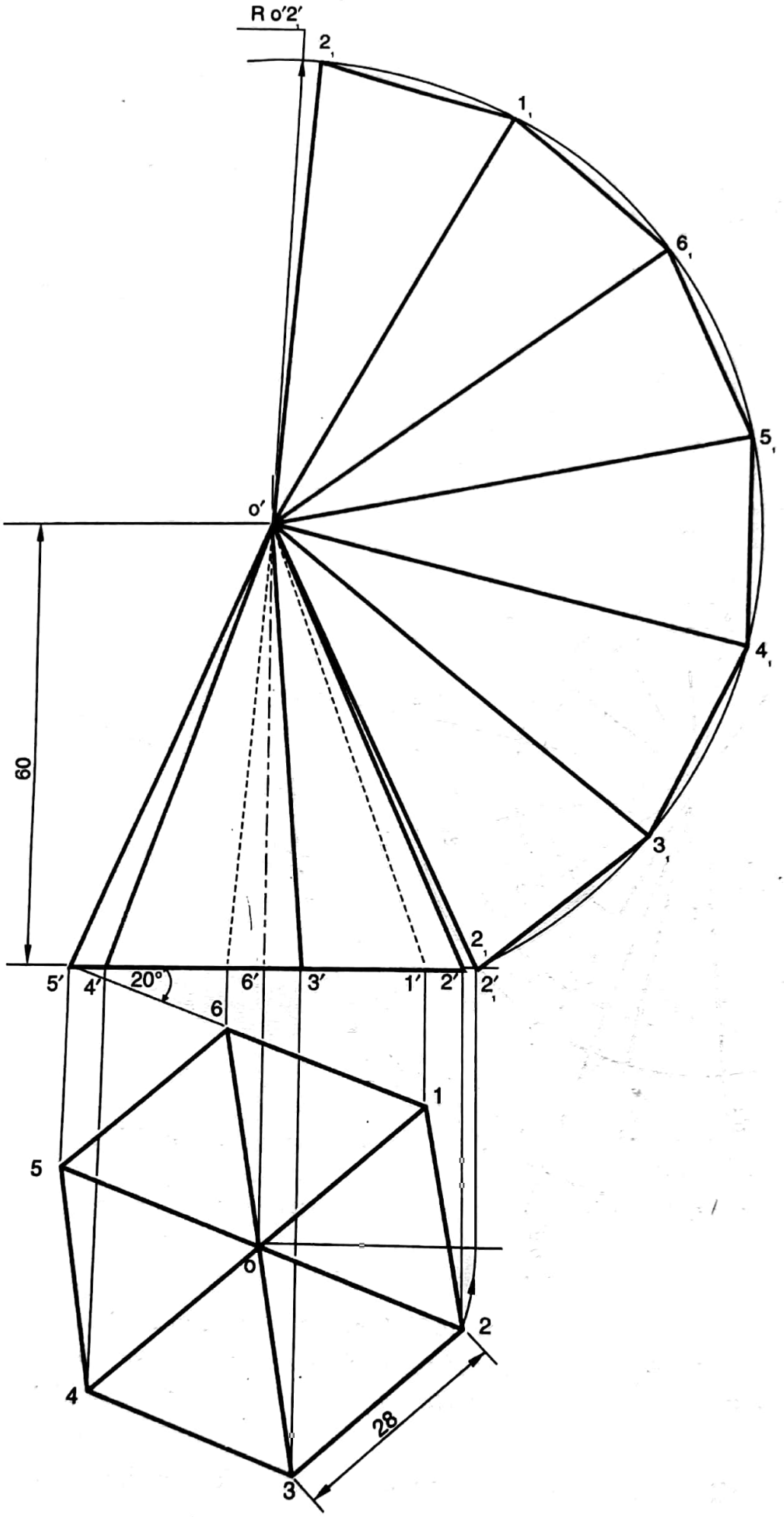


Fig. 14.34 Solution to problem 14.33

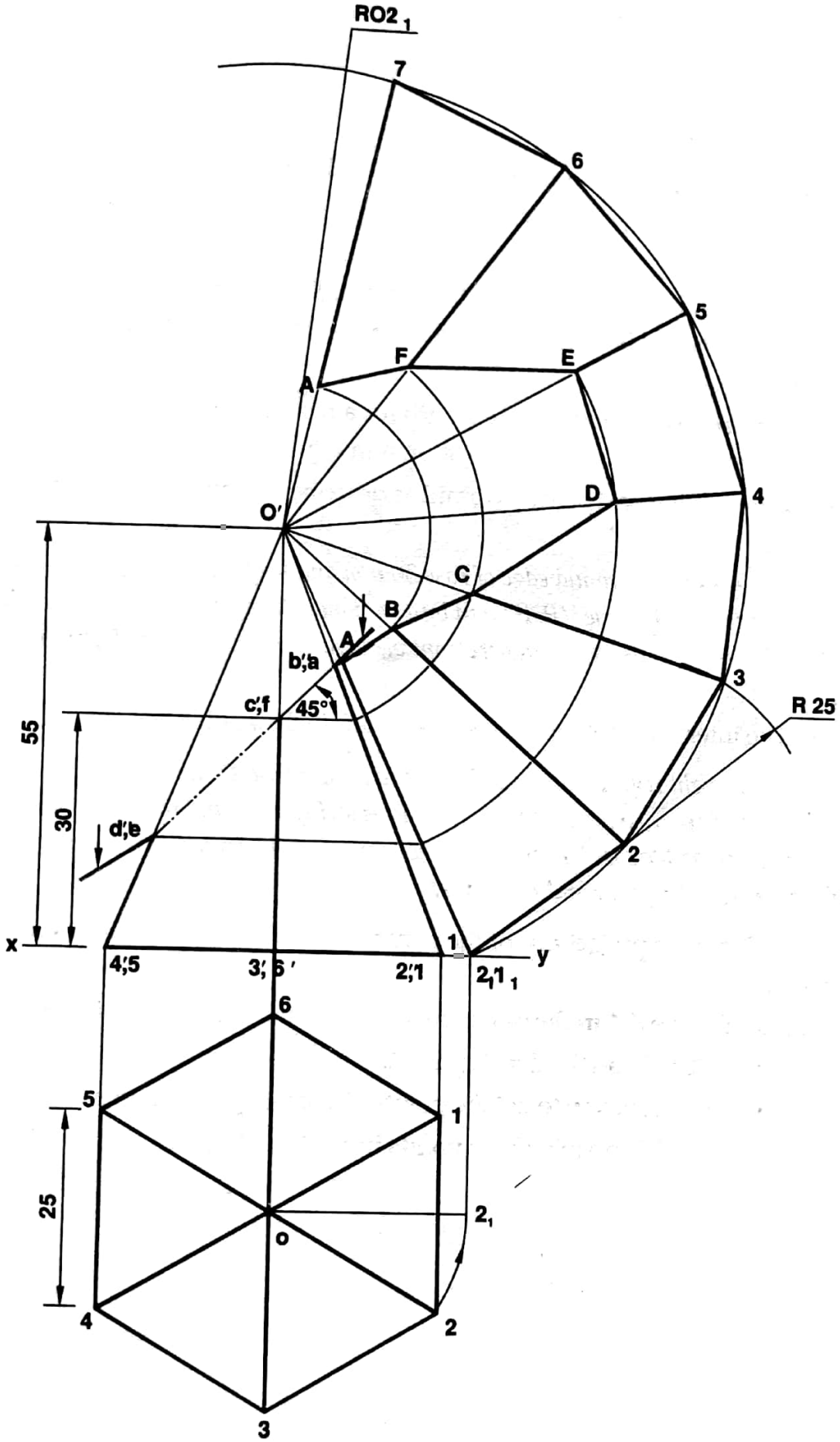


Fig. 14.35 Solution to problem 14.34

PROBLEM 14.34 *A right regular hexagonal pyramid edge of base 25 mm and height 55 mm is resting on its base in HP with one of its base edges perpendicular to the VP. It is cut by a plane, which is inclined at 45° to the HP and perpendicular to the VP, at a distance of 30 mm from its base. Draw its projections and develop its lateral surface.*

SOLUTION.

(*i*) Draw a hexagon 123456 in the top view, keeping its base edge (12 or 45) perpendicular to the *VP*. Project its corresponding front view 1′2′3′4′5′6′.

(*ii*) Then cut the solid in the front view by a section plane perpendicular to the *VP* and inclined to the *HP* at 45°, fulfilling all the given conditions.

(*iii*) Here, none of the slant edges gives the true length. The true length of a slant edge $0'1_1'$ ($0'2_1'$) of a hexagonal pyramid is to be measured from the front view, as the top view of that edge is parallel to *xy*.

(*iv*) As all the slant edges are of the same length for a hexagonal pyramid, so with $0'1_1'$ ($0'2_1'$) as radius and 0′ as centre, draw an *arc* as shown in Fig. 14.35.

(*v*) Develop the remaining part of the pyramid as described is the previous problems using radial line method.

PROBLEM 14.35 *A square pyramid edge of base 30 mm and height 50 mm is resting on HP on its base with an edge of base inclined at 30° to the VP. It is cut by a sectional plane, which is inclined at 45° to the HP and perpendicular to the VP, bisecting its axis. Draw its front view, sectional top view and develop the lateral surface of the remaining pyramid.*

SOLUTION. The interpretation of the solution is left to the reader. See Fig. 14.36.

PROBLEM 14.36 *A right regular hexagonal pyramid edge of base 25 mm and height 50 mm is resting on its base is HP, with one of its base edges parallel to the VP. A string is wound round the surface of the pyramid from the right extreme point on the base and ending at the same point. Determine the shortest path required. Also show the path of the string in the front and top views.*

SOLUTION. (*i*) Draw the projections of the pyramid and develop its lateral surface as described in the earlier problems.

(*ii*) Draw the straight line *AA* in the development to get the shorter path of the string required.

(*iii*) Mark the points *BCDEF* on the development.

(*iv*) Proceed the reverse direction to get the points *b′c′d′e′f′* in the front view.

(*v*) Project the points *bcdef* and join these points in the top view. See Fig. 14.37.

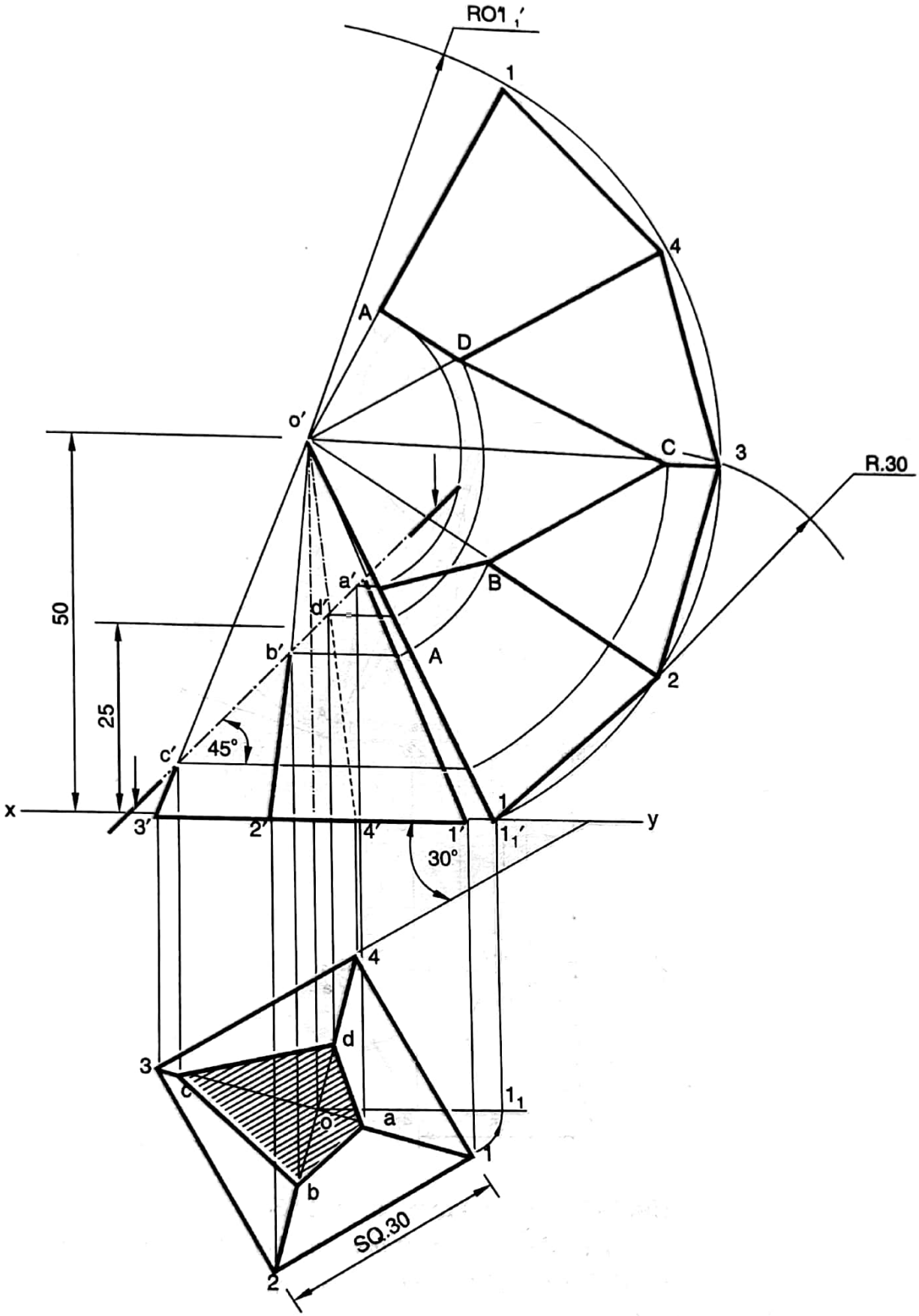


Fig. 14.36 Solution to problem 14.35

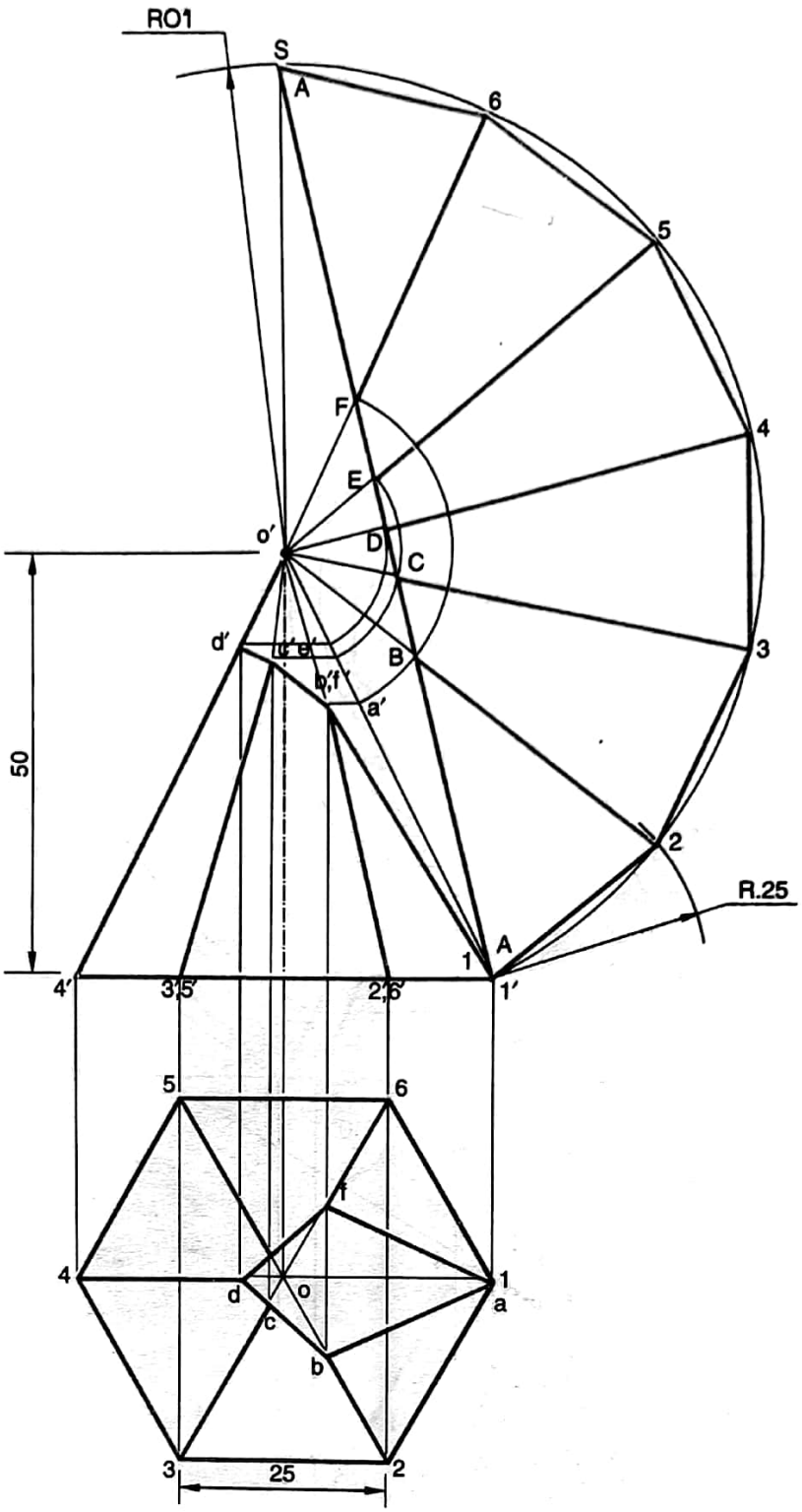


Fig. 14.37 Solution to problem 14.36

PROBLEM 14.37 *A square pyramid edge of base 40 mm and height 60 mm is lying on one of its triangular faces on HP, with its axis parallel to the VP. It is cut by a sectional plane which is perpendicular to the VP and parallel to the HP, bisecting its axis. Draw its front view, sectional top view and develop the lateral surface of the remaining pyramid.*

SOLUTION. All the construction lines are retained to make the solution self – explanatory. See Fig. 14.38.

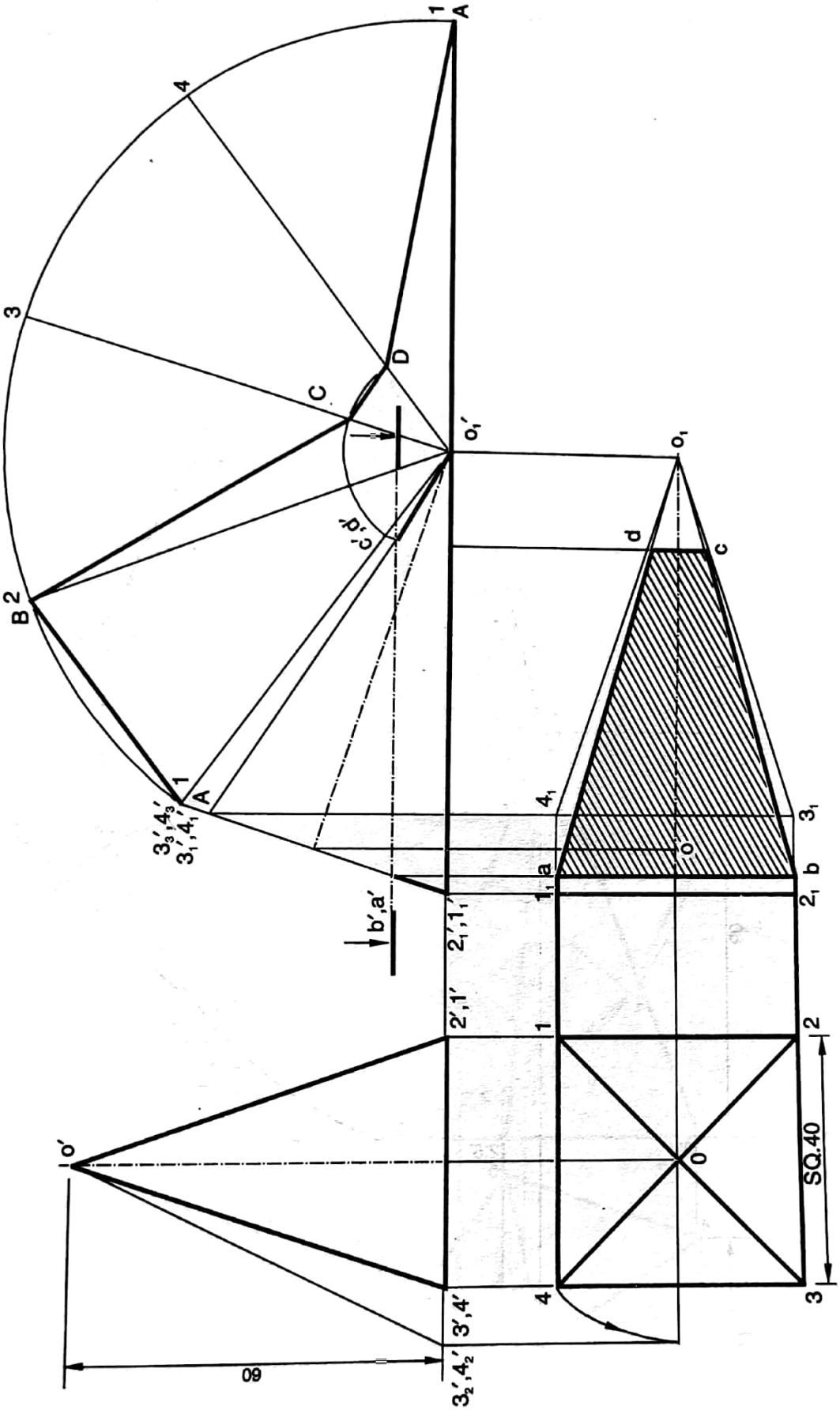


Fig. 14.38 Solution to problem 14.37

PROBLEM 14.38 *Develop the surface of a pentagonal pyramid having its base edge 30 mm and axis 60 mm long.* *(PTU, Jalandhar December 2005)*

SOLUTION. (*i*) Draw a pentagon 12345 in the top view. Project its corresponding front view 1′ 2′ 3′ 4′ 5′.

(*ii*) As the top view of slant edge 01 is parallel to *xy*, so 0′ 1′ slant edge of a pentagenal pyramid gives the true length.

(*iii*) With 0′ as centre, radius equal to 0′ 1′, draw an *arc* as shown in Fig. 14.39.

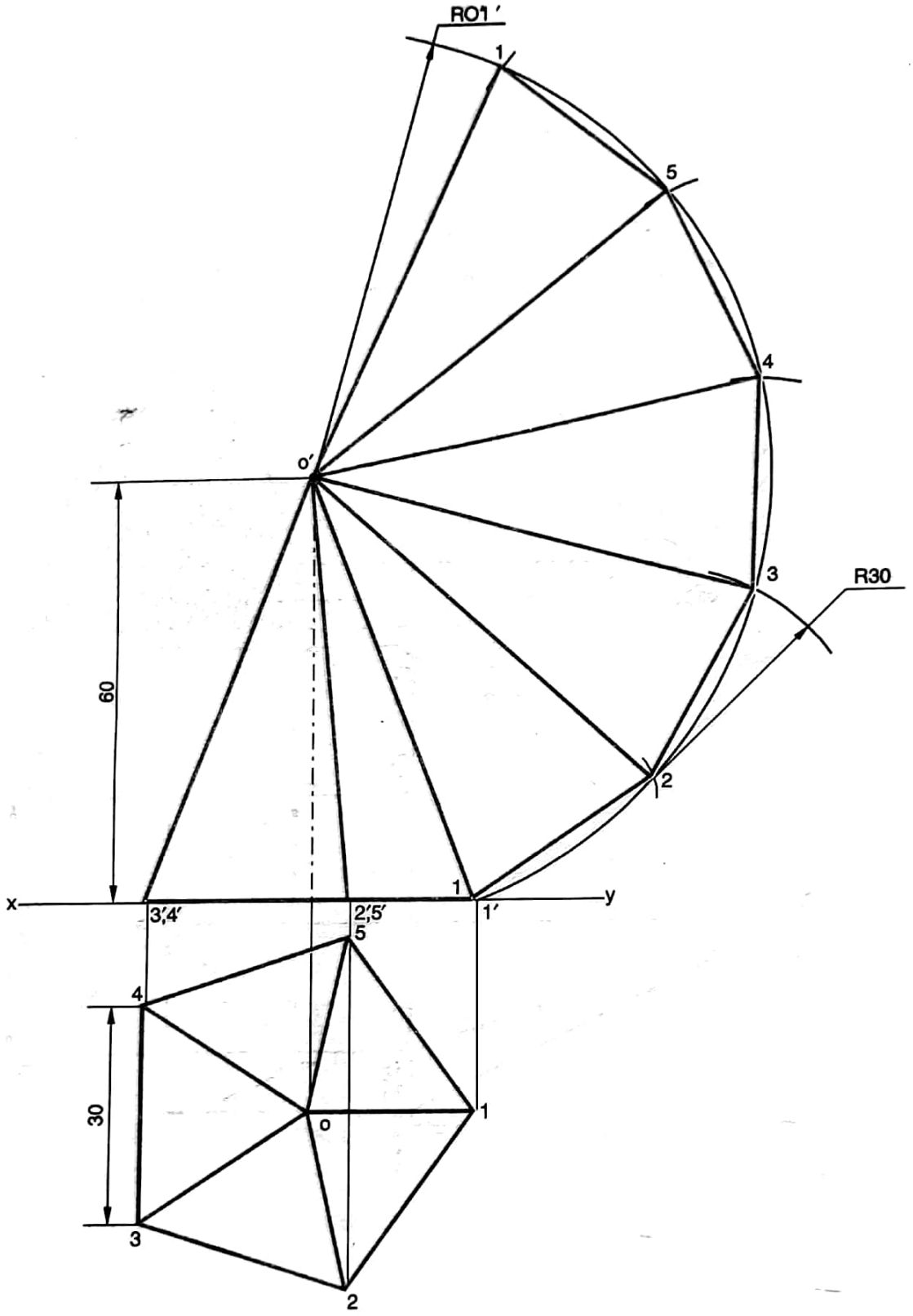


Fig. 14.39 Solution to problem 14.38

(*iv*) Select point 1 on this *arc* and then cut off five equal divisions of length equal to the length of an edge from top view.

(*v*) Name these points as 2, 3, 4 and 5. Join 1 to 2, 2 to 3, 3 to 4, 4 to 5 and 5 to 1 by straight lines and also join these points to point 0′ to complete the development.

PROBLEM 14.39 *A right regular hexagonal pyramid, edge of base 20 mm and height 40 mm, rests on its base in HP, with one of its base edges perpendicular to VP. It is cut by a section plane, which is perpendicular to VP and inclined to HP at 30°, cuts the pyramid, bisecting its axis. Draw its front view, sectional top view and develop the lateral surface of the truncated pyramid.*

SOLUTION. All the construction lines are retained to make the solution self-explanatory. See Fig. 14.40.

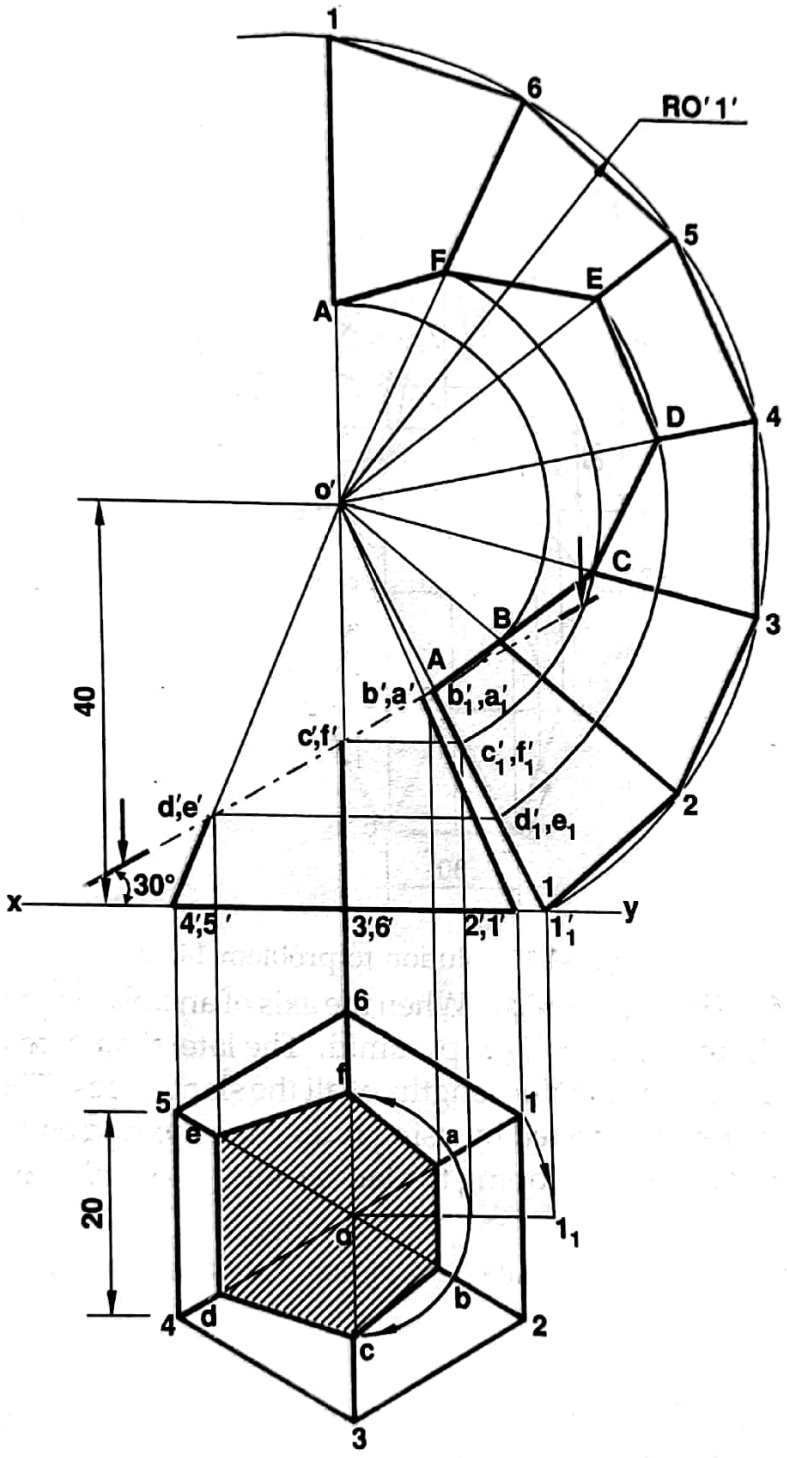


Fig. 14.40 Solution to problem 14.39

PROBLEM 14.40 *A right regular hexagonal pyramid, edge of base 30 mm and axis 65 mm long rests on its base on HP with one of its base edges is parallel to the VP. A horizontal circular hole of diameter 30 mm drilled at a distance of 20 mm from the base of the pyramid and to the axis perpendicular to the VP. Develop its lateral surfaces.*

SOLUTION. All the construction lines are retained to make the solution self-explanatory. See Fig. 14.41.

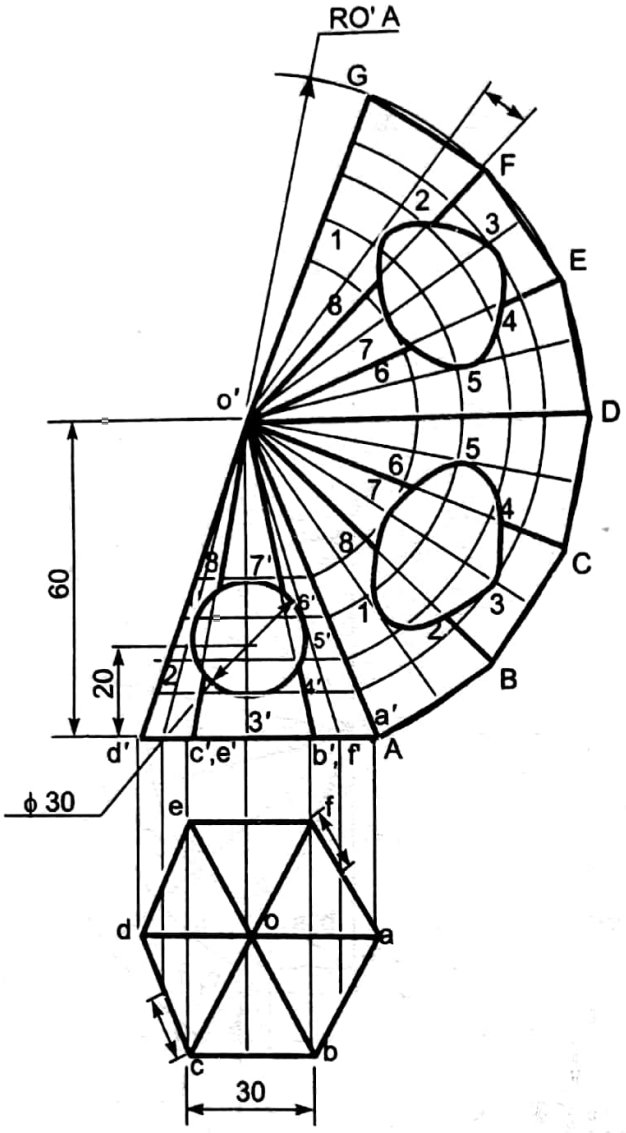


Fig. 14.41 Solution to problem 14.40

(*b*) Development of oblique pyramids. When the axis of an oblique pyramid is not at right angle to its base, then the solid is known as oblique pyramid. The lateral surfaces of these solids consist of triangles. It is needed to determine the true lengths of all the slant edges. Therefore, the development of an oblique pyramid may be constructed by using radial line method as already been described for right pyramids. The procedure of developing the lateral surface of oblique pyramids is explained in the following problems.

PROBLEM 14.41 *Two views of an oblique square pyramid are shown in Fig. 14.42 (a). Develop its lateral surface.*

SOLUTION.

(*i*) Draw the given views. Here the true lengths are different for the slant edges, whereas the base edges are of true length in the top view.

(*ii*) Find out the true lengths for all the slant edges as shown in Fig. 14.42 (*b*).

(*iii*) Complete the problem as discussed in the right pyramids.

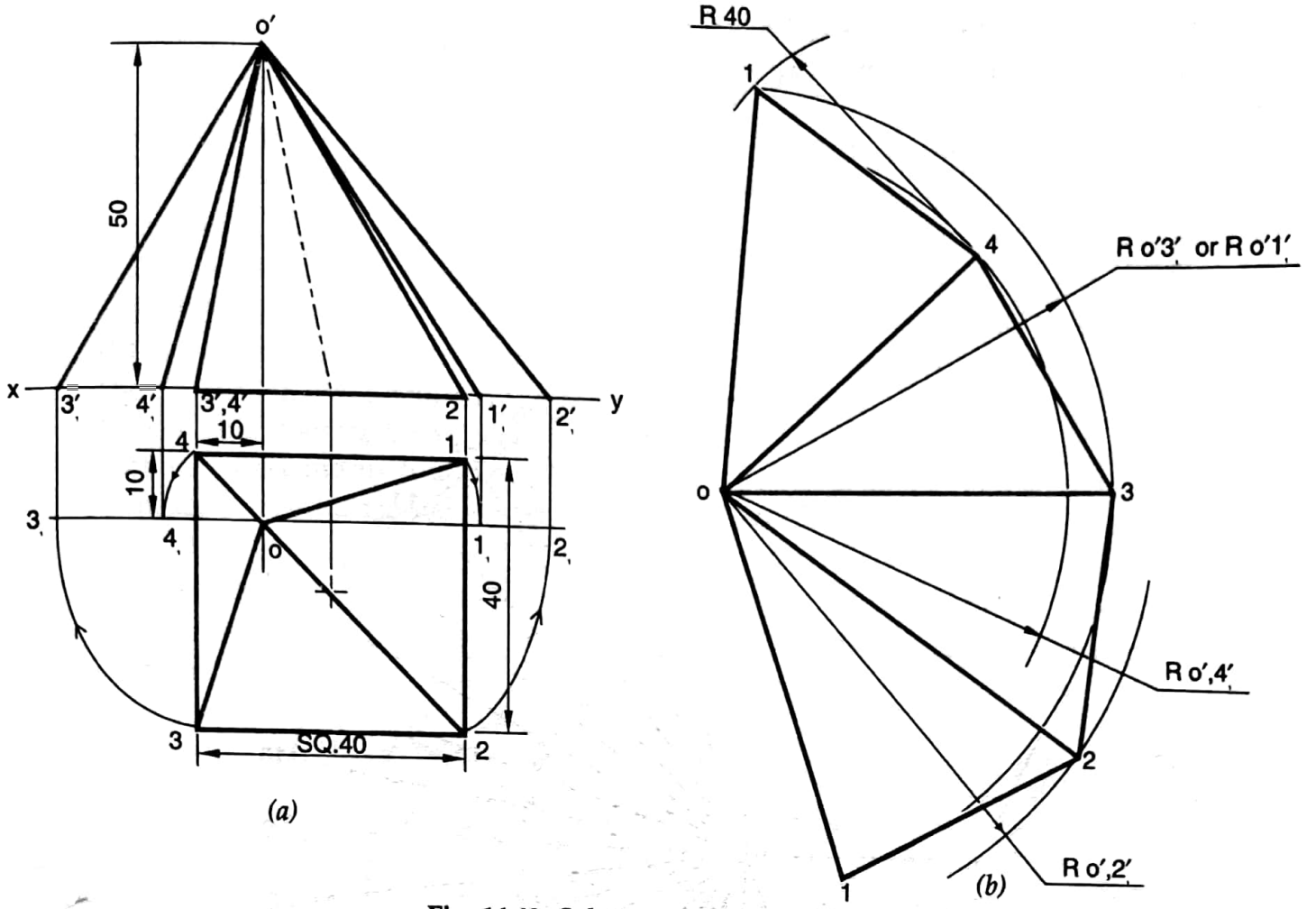


Fig. 14.42 Solution to problem 14.41

PROBLEM 14.42 *Two views of an oblique square pyramid are shown in Fig. 14.43 (a). Develop its lateral surface.*

SOLUTION. The interpretation of the solution is left to the reader. See Fig. 14.43 (*b*).

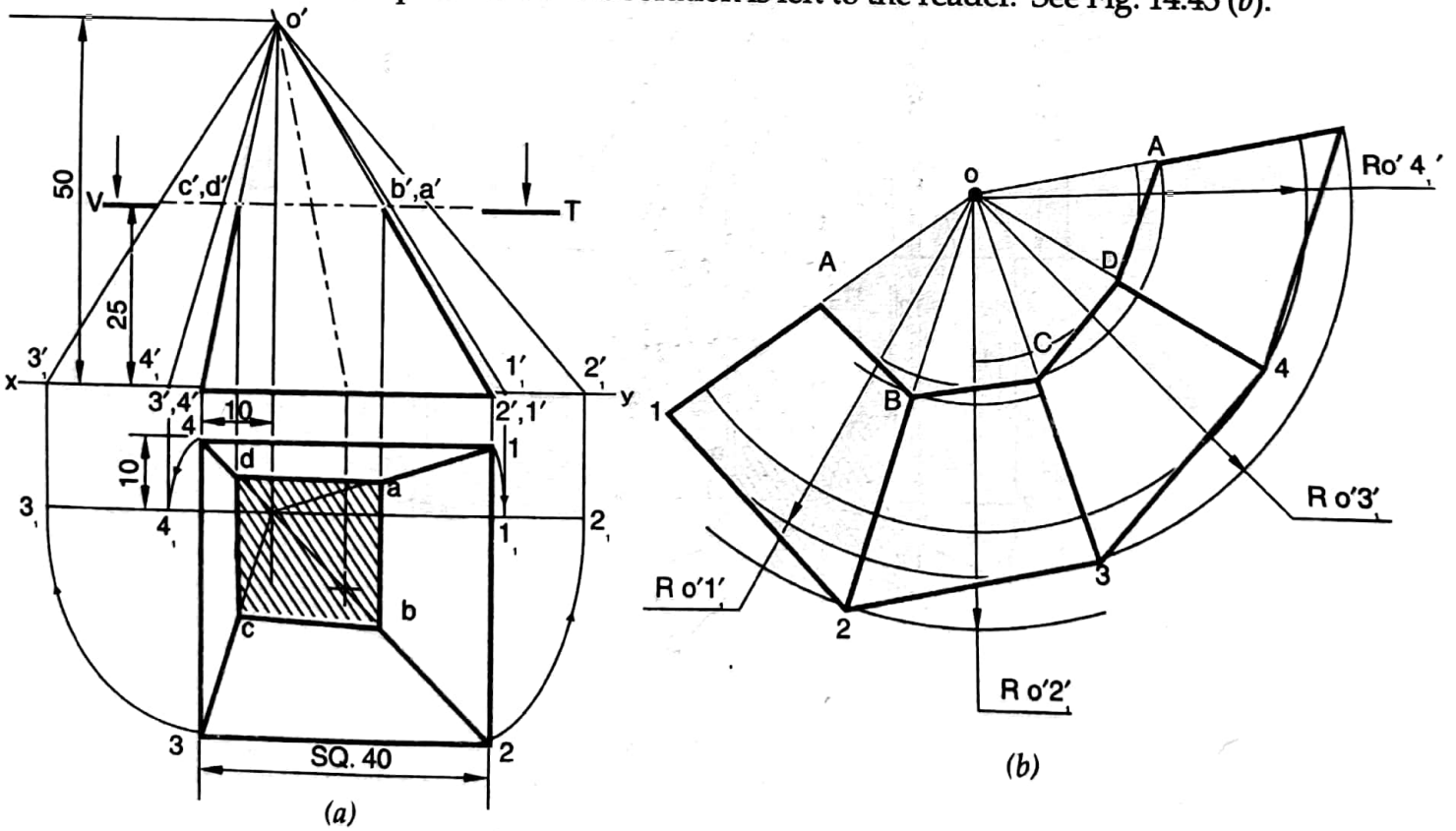


Fig. 14.43 Solution to problem 14.42

(c) Development of right cones : Development of the curved surface of a cone is a sector of a circle having its radius equal to the generator *i.e.*, slant height and length of the arc is equal to its circumference of the base circle.

PROBLEM 14.43 *A right circular cone, diameter of base 50 mm and height 60 mm, rests on its base in HP. A section plane perpendicular to the VP and parallel to the HP cuts the cone, bisecting its axis. Draw its front view, sectional top view and develop the lateral surface of the remaining part of the cone.*

(PTU, Jalandhar May 2010)

SOLUTION. (*i*) Draw the front and top views of the cone.

(*ii*) Divide the base circle into sixteen equal parts.

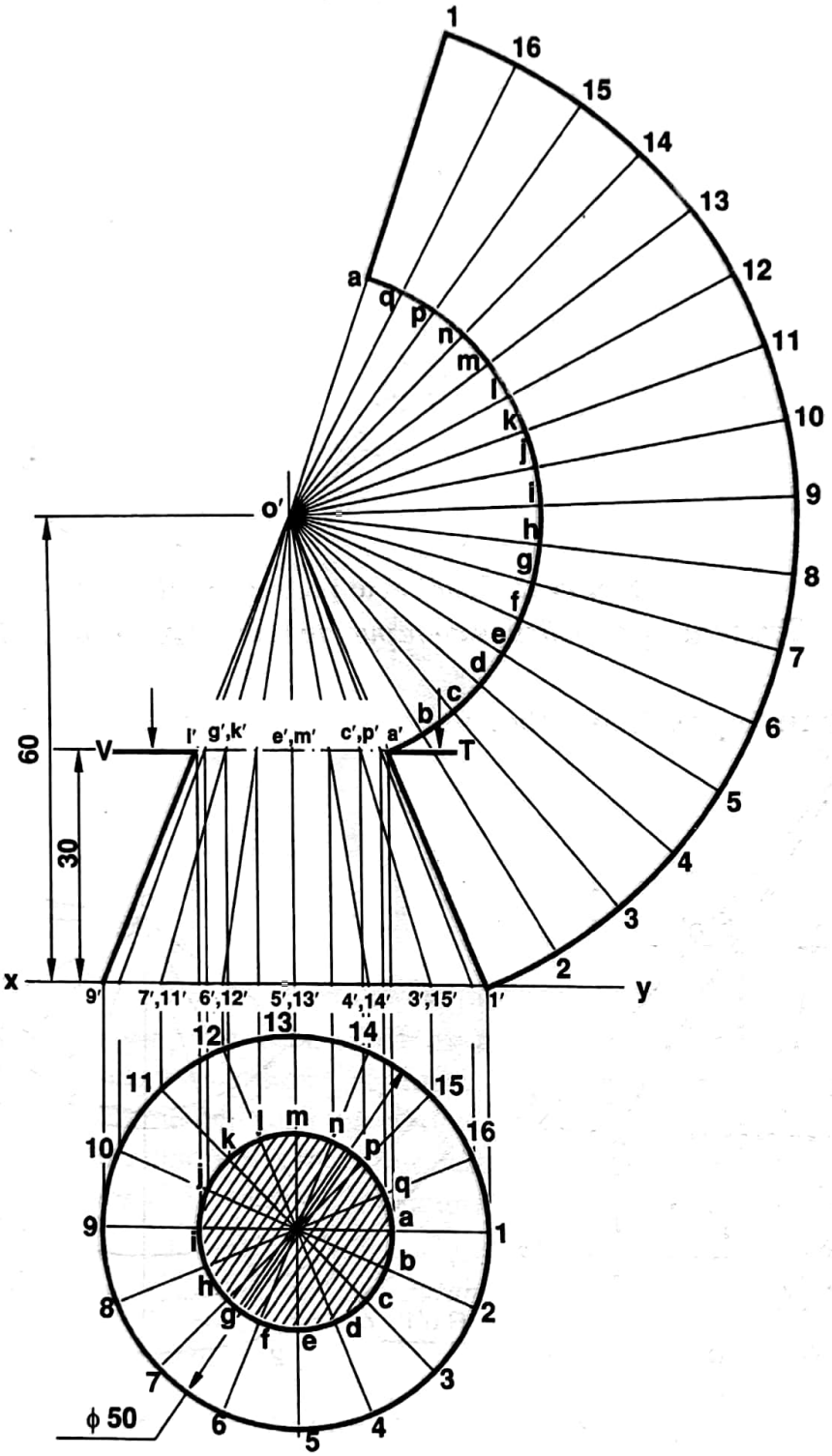


Fig. 14.44 Solution to problem 14.43

(*iii*) With centre 0′ and radius equal to 0′1′ or 0′9′, draw an arc of the circle. The length of this arc should be equal to the circumference of the base circle. This arc can be determined in two ways :

(*a*) Calculate the subtended angle θ by the formula

$$\theta = 360^\circ \times \frac{\text{Radius of the base circle}}{\text{Slant height of the cone}}$$

Cut-off the arc so that it makes the angle θ at the centre and then divide it into sixteen equal parts.

(*b*) By dividing the arc with a compass or a divider into sixteen equal parts or divisions, where each division must be equal to the one of the divisions of the base circle. This method will give an approximate length of the circumference.

(*iv*) Join the division points with *o*, thereby completing the development of the whole cone.

(*v*) The cut portion of the cone may be deducted from this development by making the positions of the points at which the generators are cut. The true lengths of these points on the cut away portion of the cone are found by projecting their points of intersection with the section plane in the front view horizontally on to true length *o*'1' or *o*'9'. There true length distances are then transferred to their respective elements as shown in Fig. 14.44.

(*vi*) Join all these points by a smooth curve to complete the development of the frustum.

(*vii*) Also draw it front view and sectional top view, as discussed in the sections of solids chapter 13.

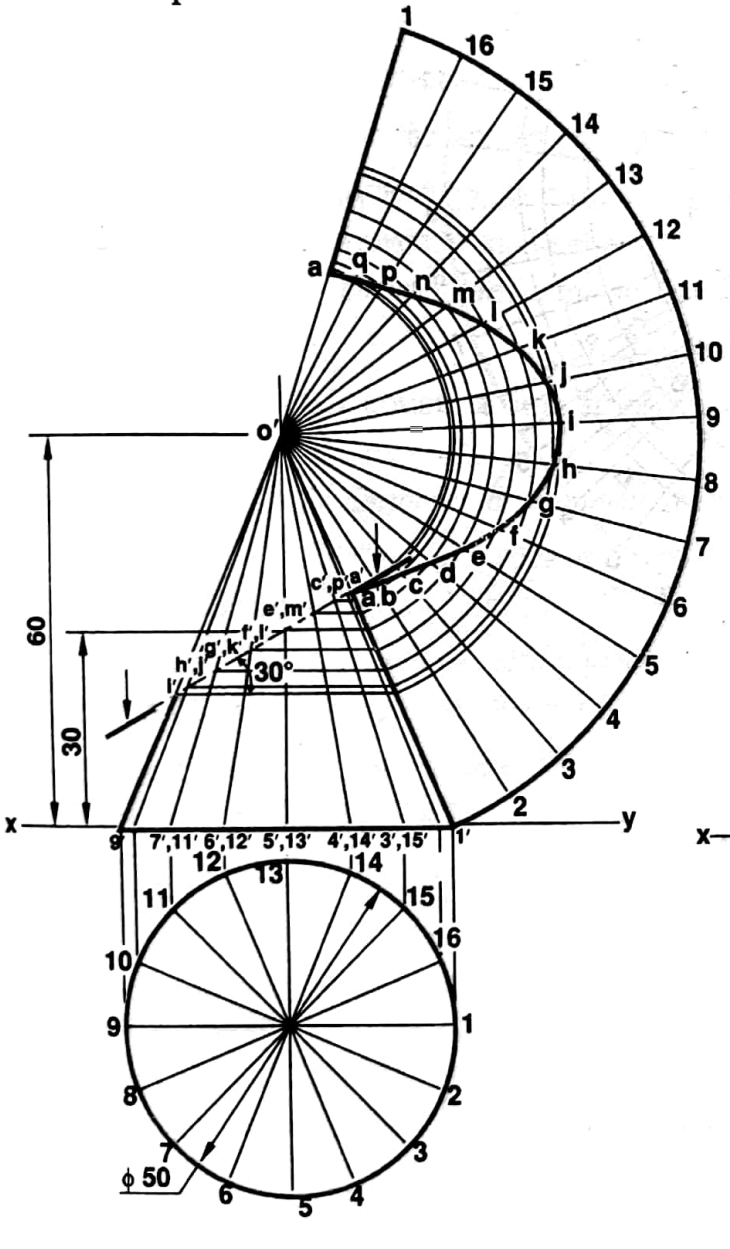


Fig. 14.45 Solution to problem 14.44

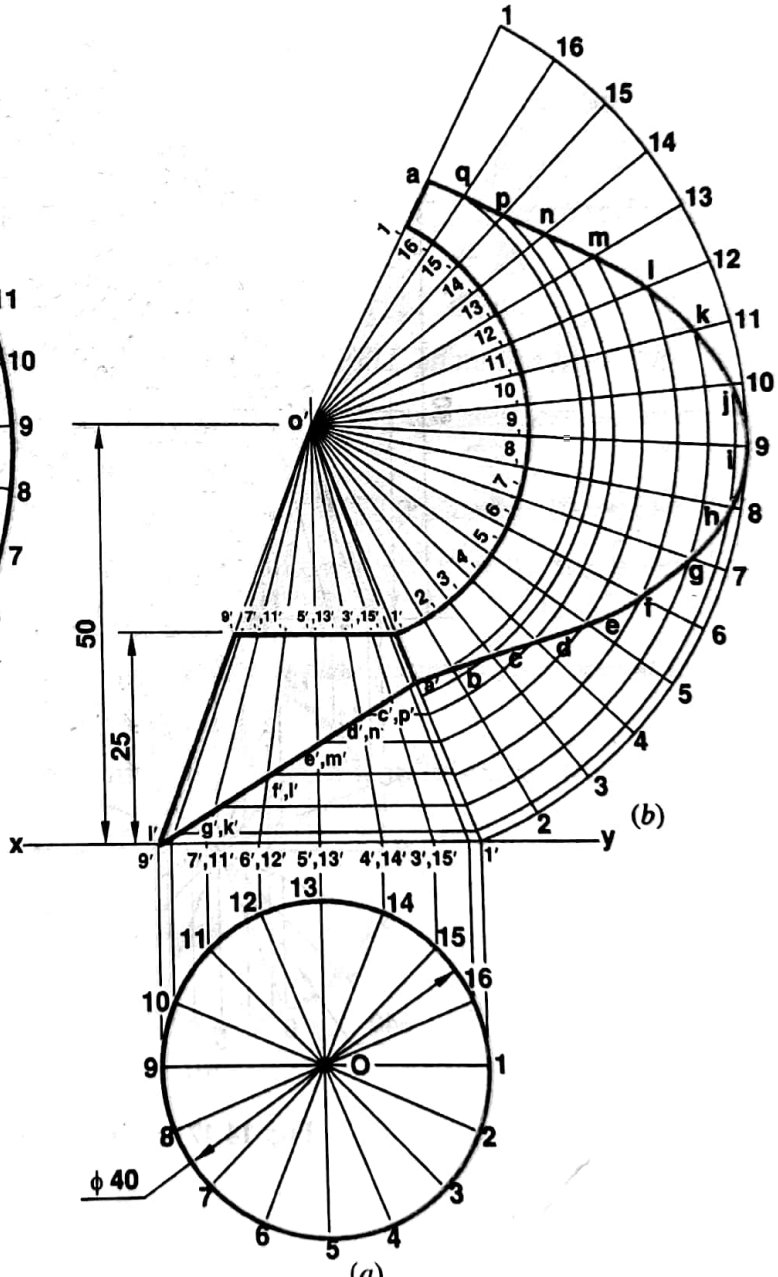


Fig. 14.46 Solution to problem 14.45

PROBLEM 14.44 *A right circular cone, diameter of base 50 mm and height 60 mm, rests on its base in HP. A section plane perpendicular to the VP and inclined to the HP at 30° cuts the cone, bisecting its axis. Draw projections of the truncated cone and develop its lateral surface.*

SOLUTION. All construction lines are shown to make the figure self-explanatory. See Fig. 14.45.

PROBLEM 14.45 *Develop the lateral surface of a right circular cone as shown in Fig. 14.46 (a).*

SOLUTION. All construction lines are retained to make the solution easy to understand. See Fig. 14.46 (*b*).

PROBLEM 14.46 *Develop the lateral surface of a right circular cone with an equilateral triangular hole as shown in Fig. 14.47 (a).*

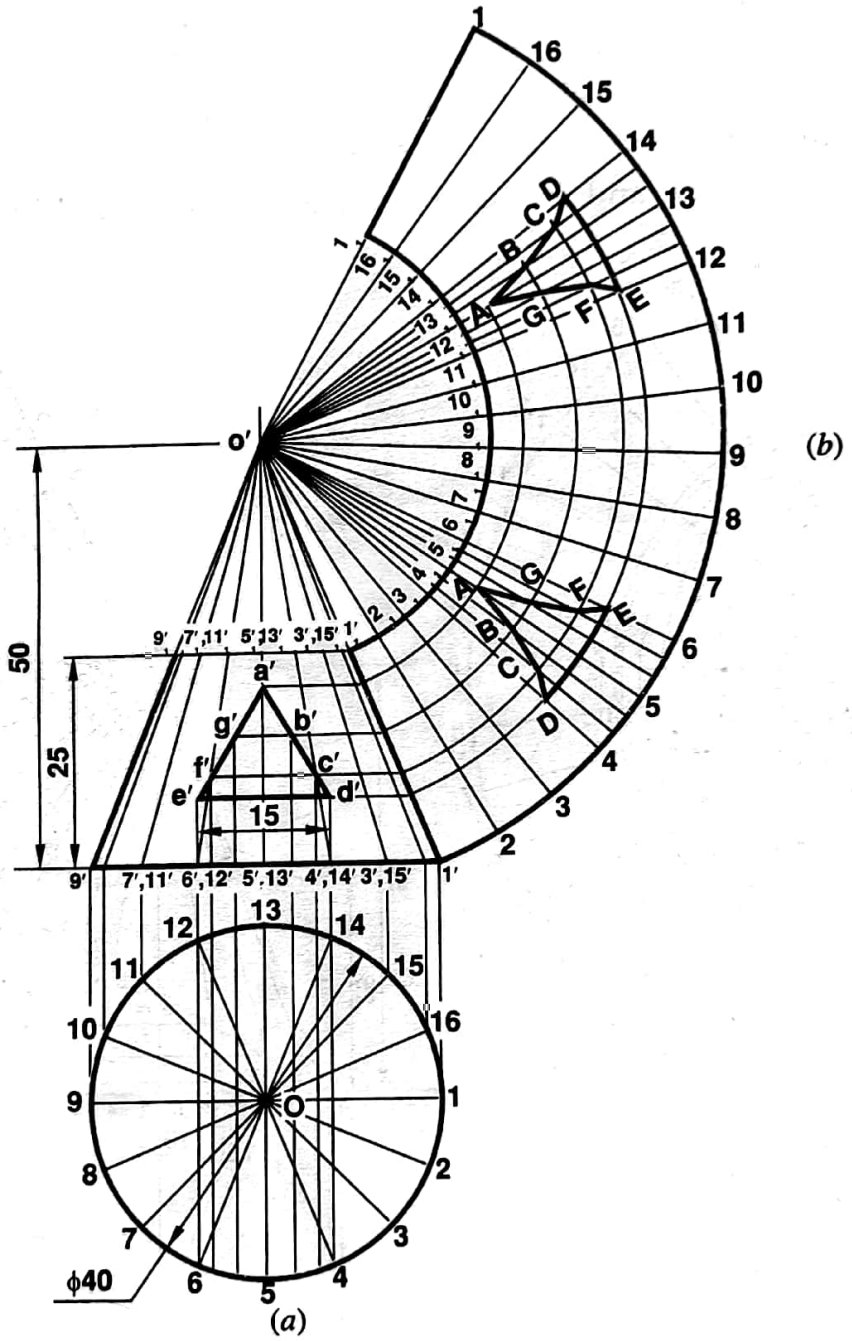


Fig. 14.47 Solution to problem 14.46

SOLUTION.

(*i*) Draw the given front view and project the corresponding top view.

(*ii*) Develop the lateral surface of the frustum of the cone by the radial line method.

(*iii*) Take a number of points *e.g.* $a', b', c', d', e', f', g'$ on the triangular hole in the front view and transfer these points to their respective positions in the development.

(*iv*) Join these points by a smooth curve as shown in Fig. 14.47 (*b*).

PROBLEM 14.47 *A right circular cone, diameter of base 50 mm and height 60 mm, is lying on one of its generators on HP with its axis parallel to the VP. A section plane parallel to the HP and perpendicular to the VP cuts the axis at a distance of 15 mm from its base. Draw front view, sectional top view and develop the lateral surface of the cut cone.*

SOLUTION. This problem consists of three different parts :

(*i*) First of all, draw the projections of the cone in the given conditions.

(*ii*) Next add the section plane in the front view to satisfy the given conditions and draw the sectional top view, applying principles of section of solids.

(*iii*) Finally develop the lateral surface of the cut cone using radial line method.

All construction lines are retained to make the solution more understandable. See Fig. 14.48.

PROBLEM 14.48 *Develop the lateral surface of a funnel as shown in Fig. 14.49 (a).*

SOLUTION. This object consists of two parts : (*i*) Frustum of a cone and (*ii*) A right cylinder. All construction lines are shown to make the figure self-explanatory. See Fig. 14.49 (*b*).

PROBLEM 14.49 *Develop the lateral surface of a funnel as shown in Fig. 14.50 (a).*

(PTU, Jalandhar May 2001, 2011)

SOLUTION. All construction lines are shown to make the figure self-explanatory. See Fig. 14.50 (*b*).

PROBLEM 14.50 *Draw the development of a given Fig. 14.51 (a).* *(PTU, Jalandhar December 2002)*

SOLUTION. All the construction lines are retained to make the solution to the given figure self-explanatory. See Fig. 14.51 (*b*).

PROBLEM 14.51 *Draw the development of a conical object with base diameter as 50 mm and the top diameter as 35 mm. Height of the object as 35 mm.*

SOLUTION. All the construction lines are retained to make the solution to the given figure self-explanatory. See Fig. 14.52.

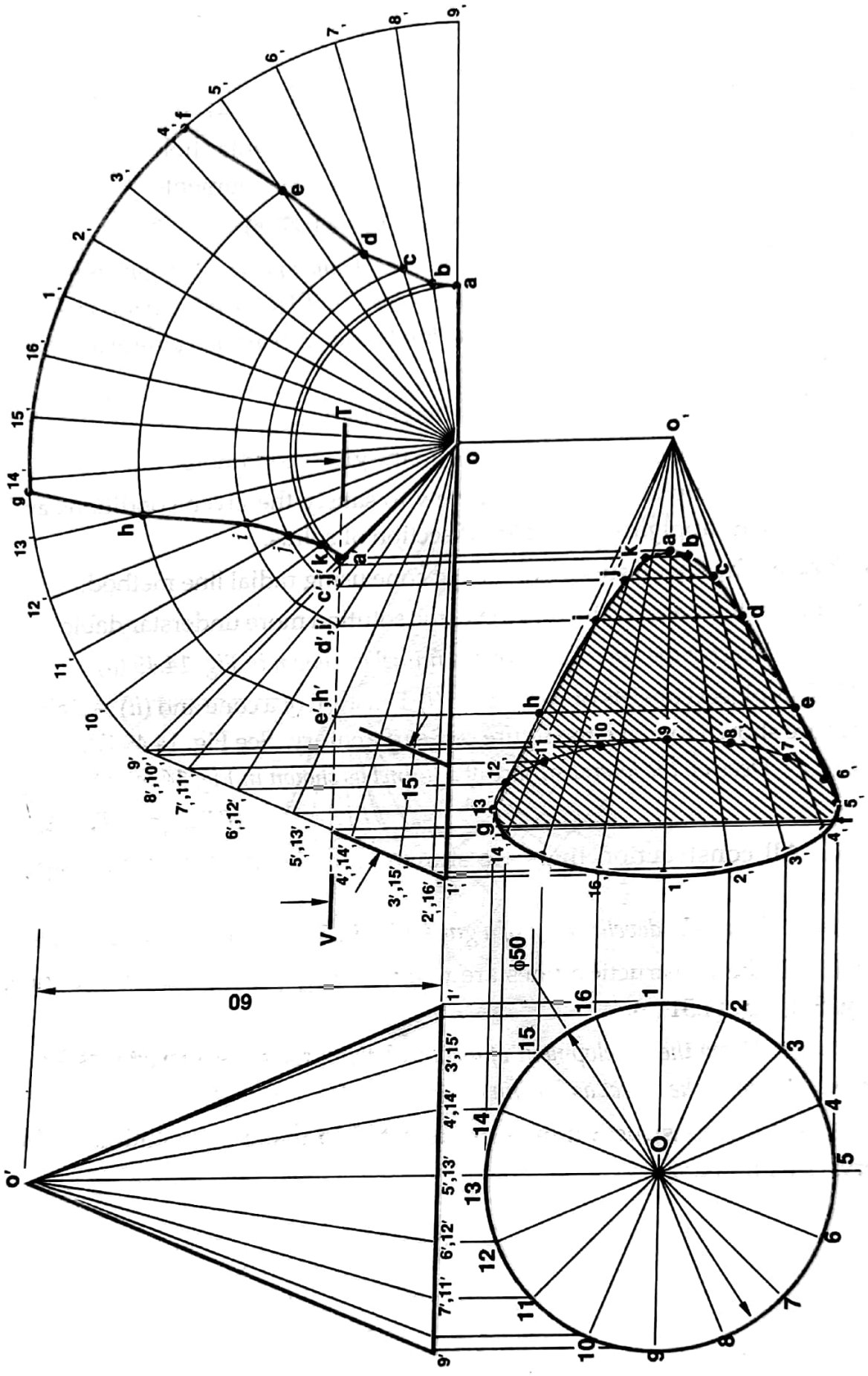


Fig. 14.48 Solution to problem 14.47

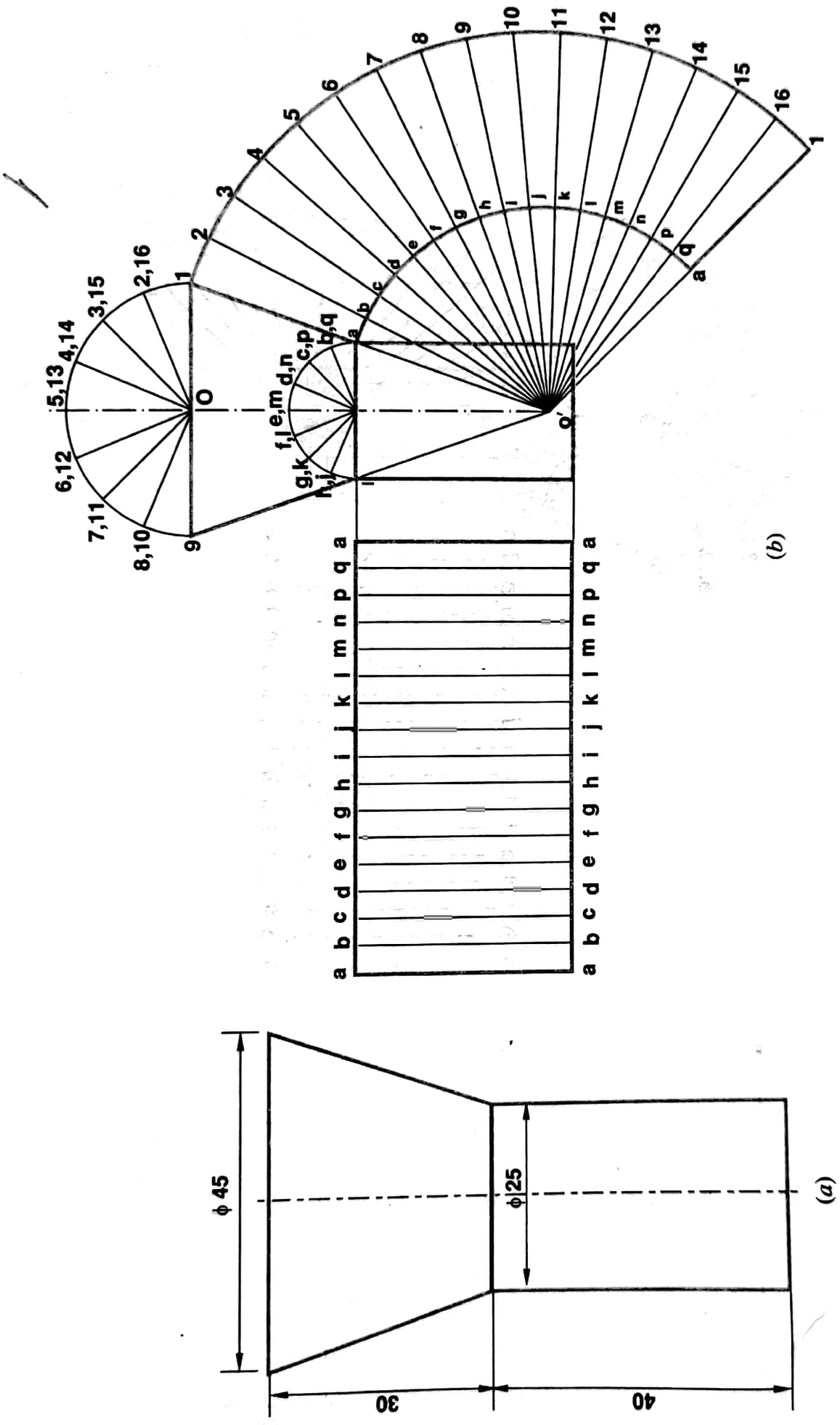


Fig. 14.49 Solution to problem 14.48

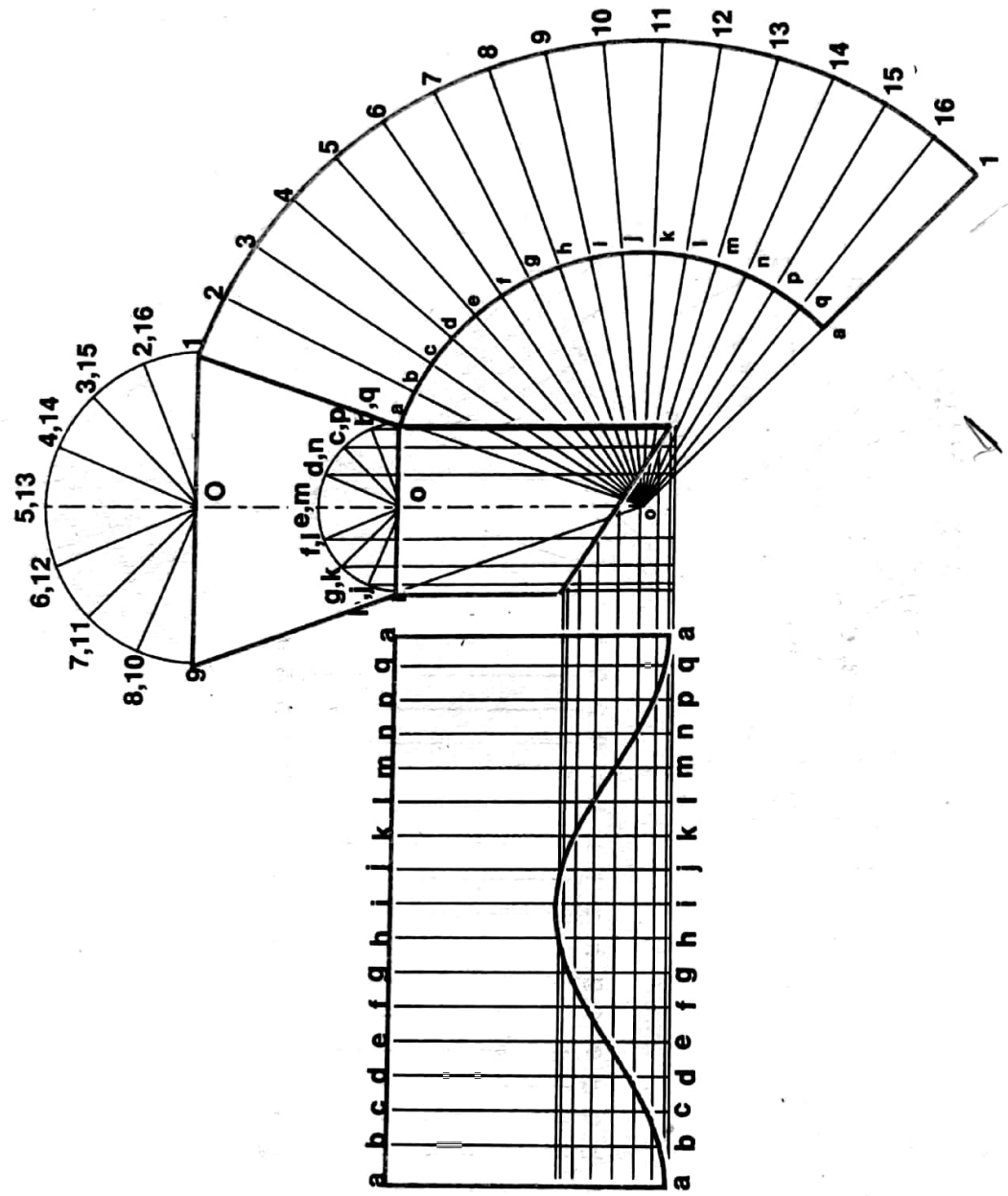
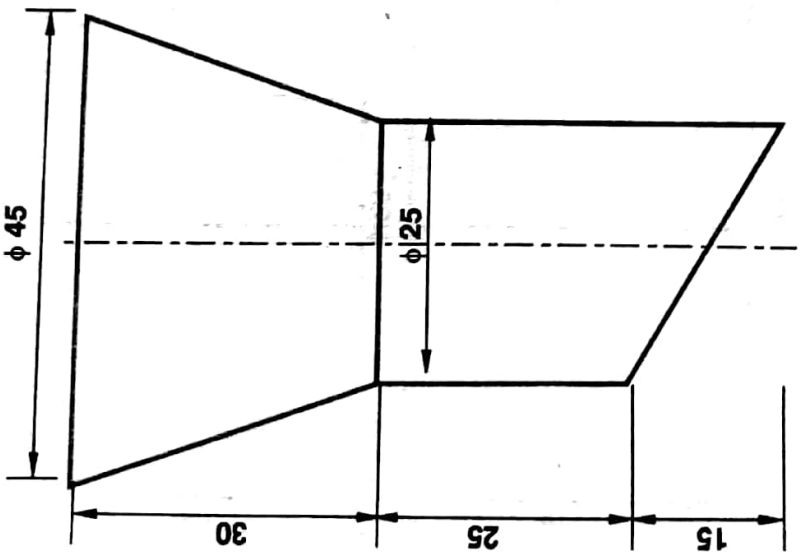


Fig. 14.50 Solution to problem 14.49



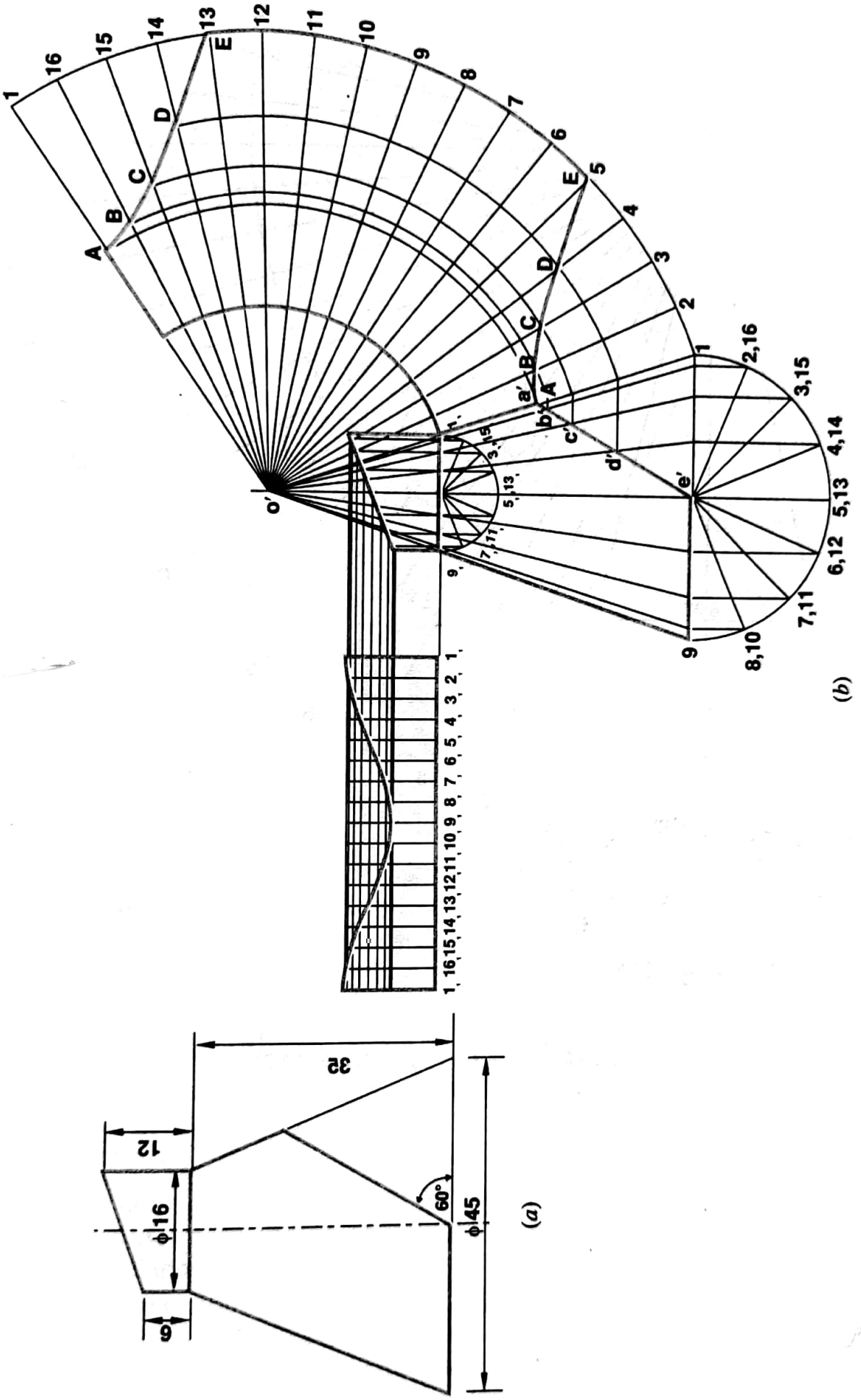


Fig. 14.51 Solution to problem 14.50

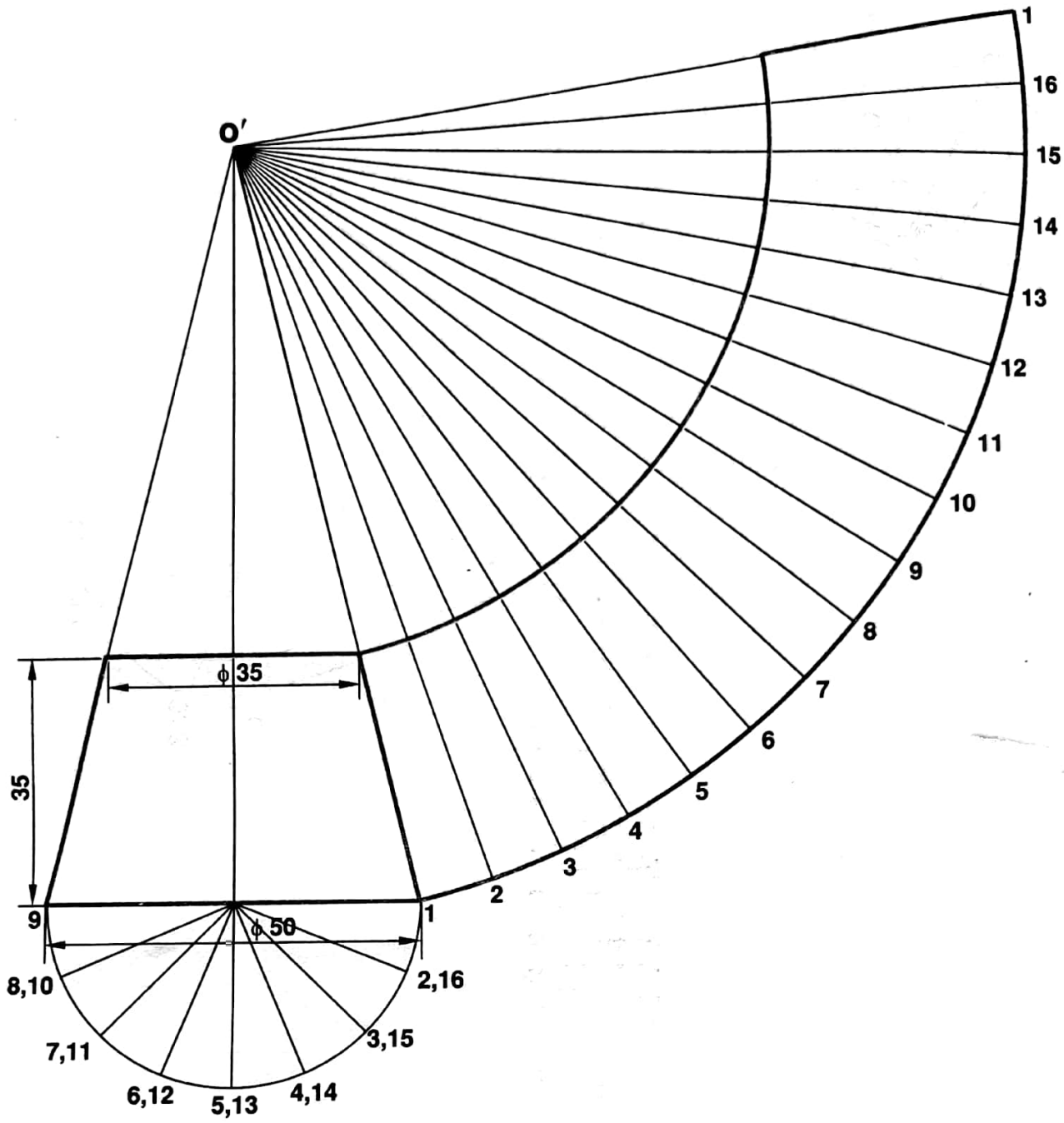


Fig. 14.52 Solution to problem 14.51

PROBLEM 14.52 *A right circular cylinder of 30 mm diameter and 35 mm height of axis is cut by sectional plane inclined at 30° to HP and passes 18 mm from the base along the axis. Draw the development of the truncated cylinder.* *(PTU, Jalandhar May 2004)*

SOLUTION. The solution to this problem is self-explantory. See Fig. 14.53.

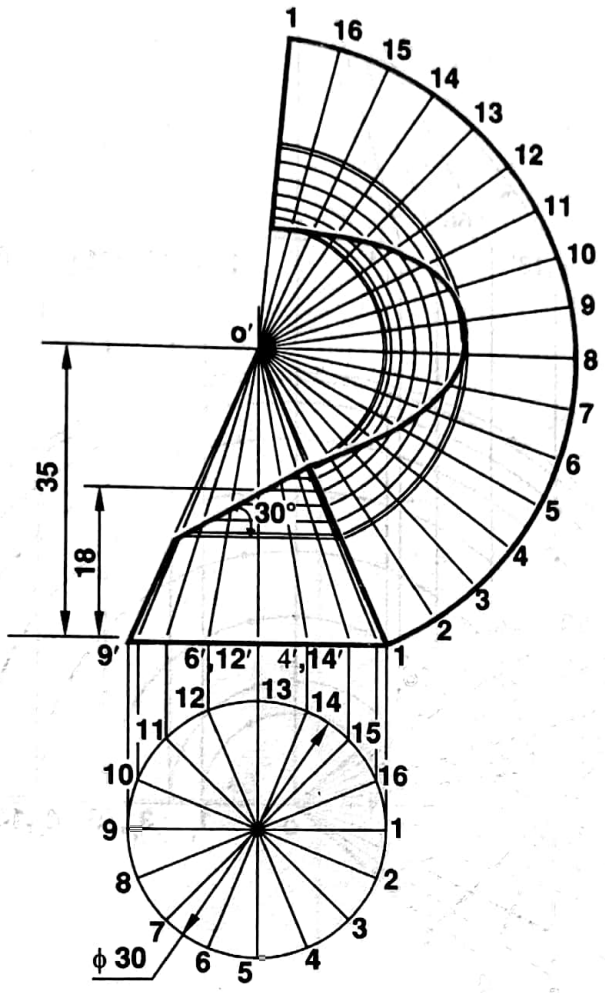


Fig. 14.53 Solution to problem 14.52

(*d*) Development of oblique cones. When the axis of an oblique cone is not at right angle to its base, then the solid is known as oblique cone. Therefore, its development is different from that of a right cone.

PROBLEM 14.53 *Develop the lateral surface of an oblique cone ; diameter of base 40 mm and height 40 mm, having its axis inclined at 60° at its base.*

SOLUTION. All the construction lines are retained to make the solution self-explanatory. See Fig. 14.54.

14.5 TRIANGULATION METHOD

This method is used for transition pieces. In most cases, transition pieces are composed of plane and conical surfaces. As transition pieces are made of different kinds of surfaces, so these are generally developed by triangulation method.

Development of transition pieces

The triangulation method is used for transition pieces. This method consists of dividing the surfaces into suitable triangles and placing these side by side after finding the true lengths of each side of the triangle. The procedure of development of a few transition pieces is illustrated in the following problems.

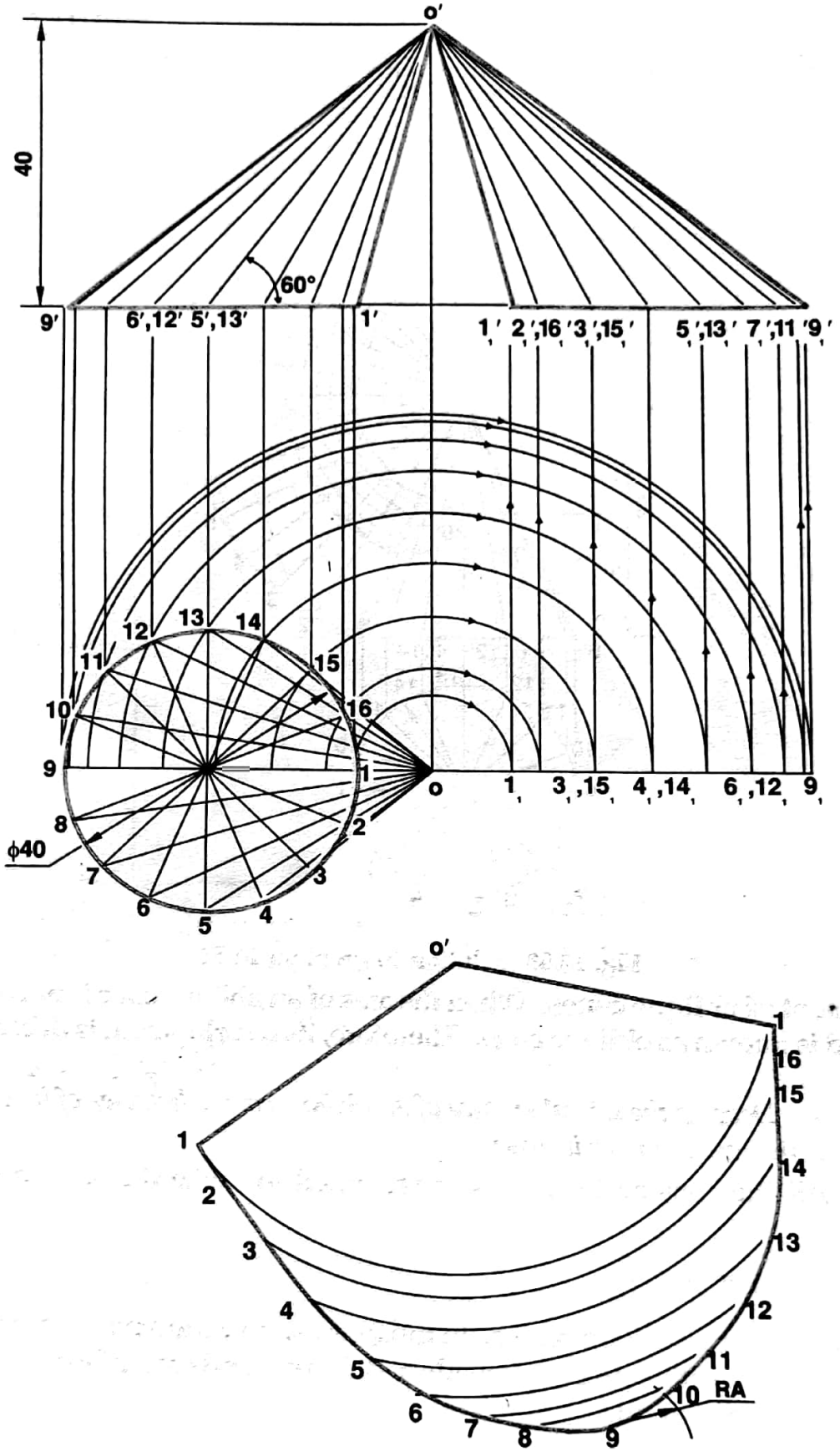
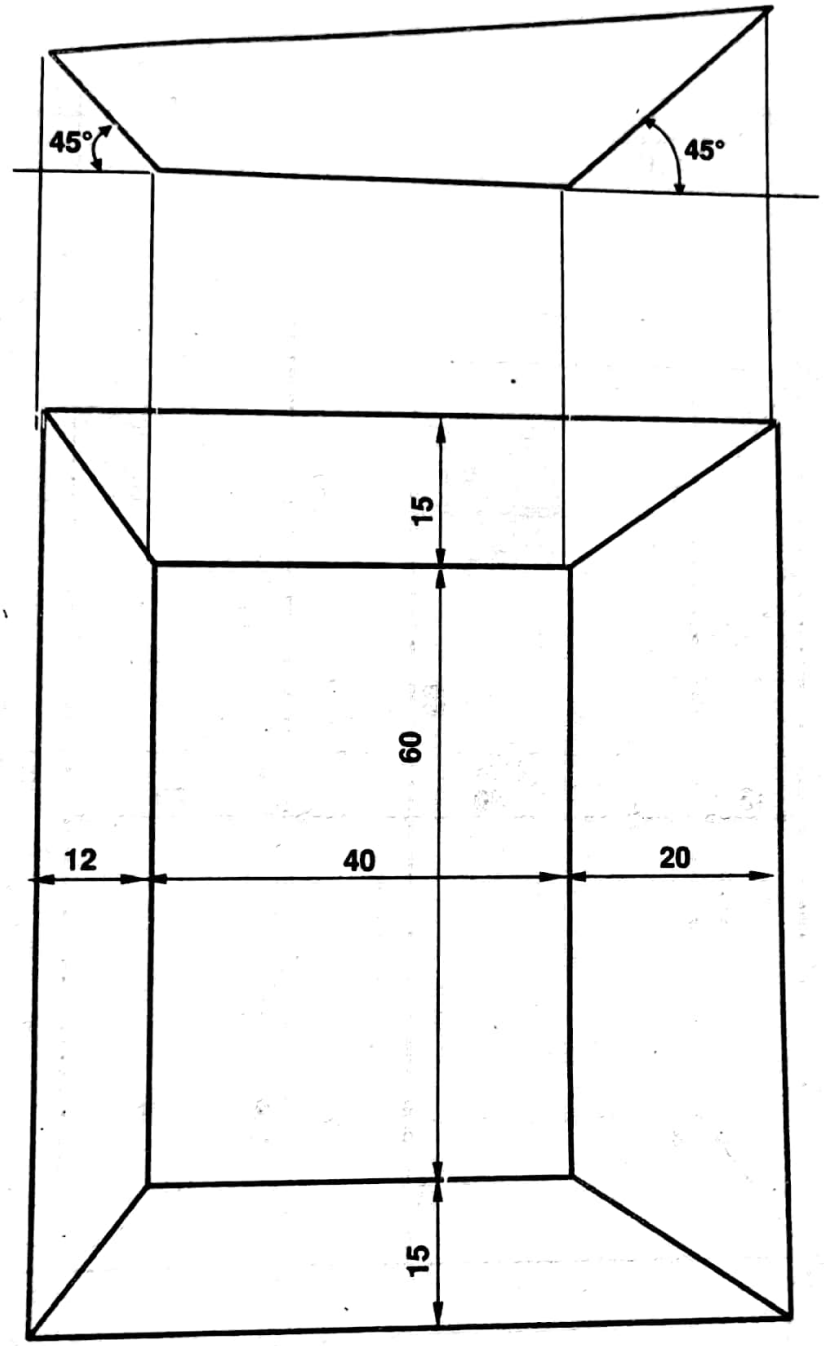


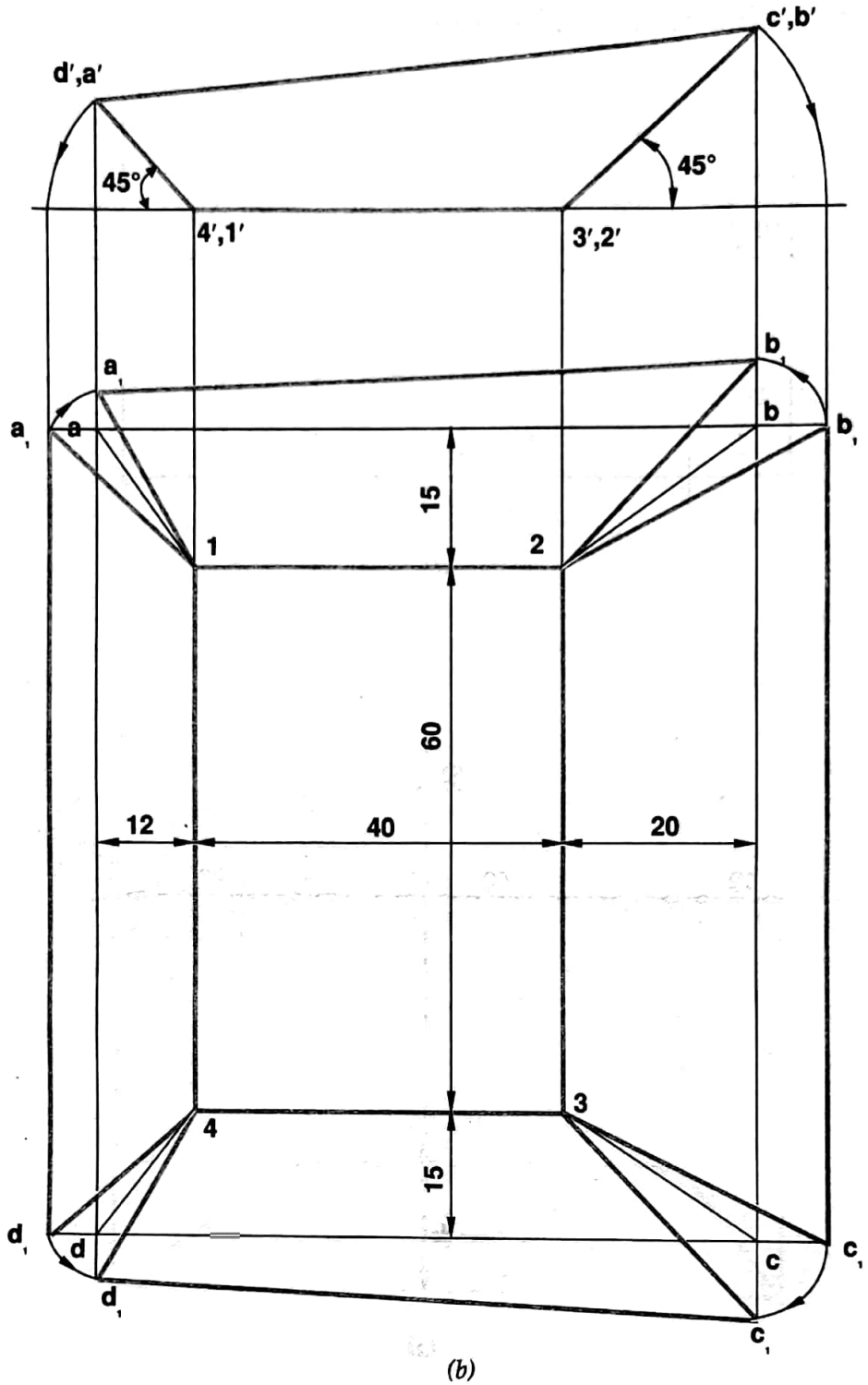
Fig. 14.54 Solution to problem 14.53

PROBLEM 14.54 *Fig. 14.55 (a) shows a paper tray model. Develop the surface of a paper tray model.*

SOLUTION. For its solution, see Fig. 14.55 (*b*).



(a)



(b)

Fig. 14.55 Solution to problem 14.54

PROBLEM 14.55 *An air conditioning duct of a square cross-section 50 mm × 50 mm connects a circular pipe of 30 mm diameter through the transition piece. Draw the projections and develop the lateral surface of the transition piece.*

SOLUTION.

(*i*) Draw the front view and top view of the given object.

(*ii*) Divide the top view of the circle into twelve equal parts.

(*iii*) The transition piece is composed of four isosceles triangles and four conical surfaces.

(*iv*) Begin the development along the line 1-A. The conical surfaces are developed by the

triangulization method as shown in Fig. 14.56. In the top view, join the division of the circle 1, 2, 3 etc. with the corner *a*, *b*, *c* etc. Project them in the front view. Obtain the true length of the sides of each triangle.

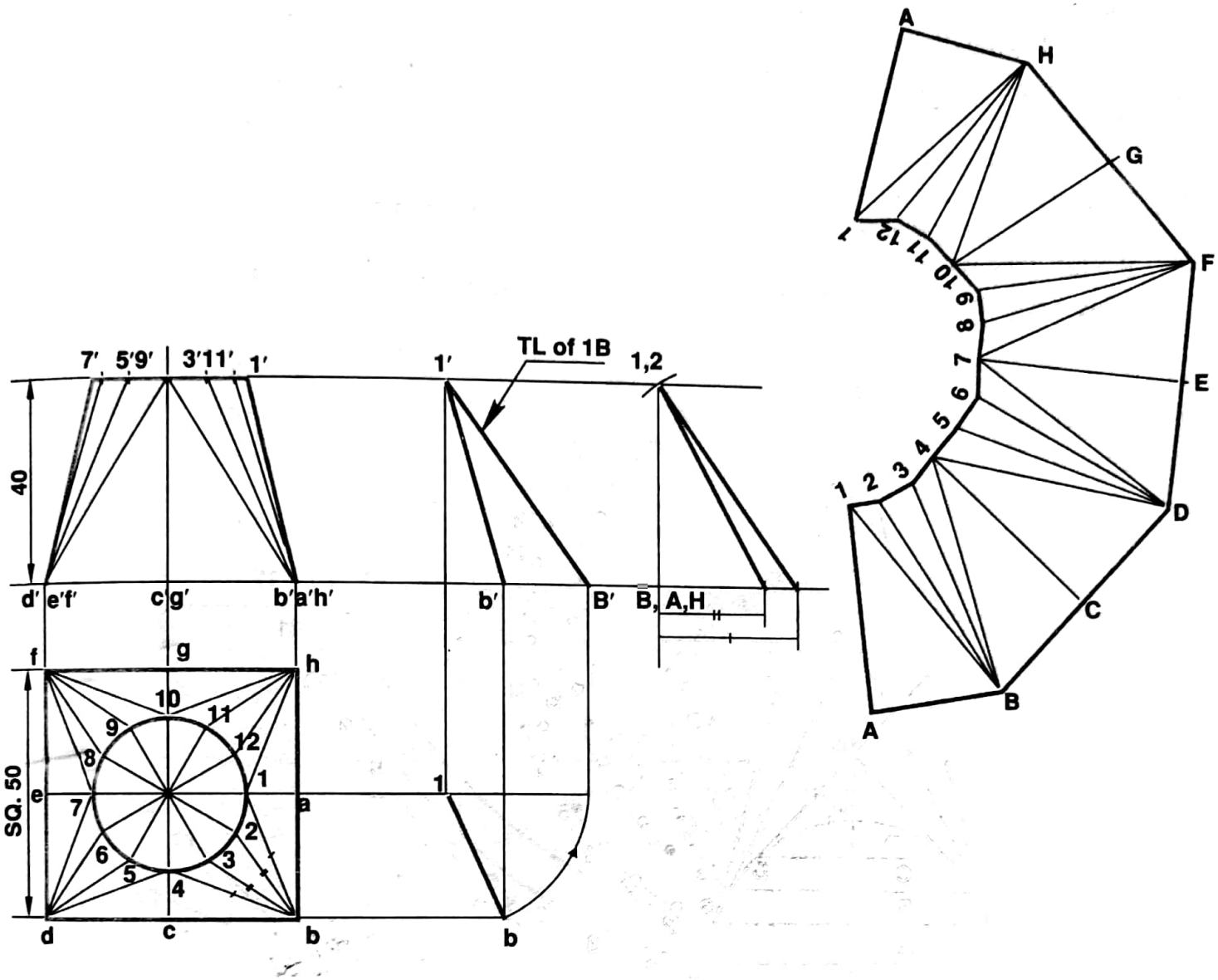


Fig. 14.56 Solution to problem 14.55

14.6 APPROXIMATE METHOD

The approximate method is used for double curved surfaces such as sphere etc.

Development of sphere

The surface of a sphere can be approximately developed by dividing it into a number of parts. There are two methods in which a sphere can be developed :

(*a*) Zone Method

(*b*) Lune or Gore Method

(*a*) Zone Method

In this method, the sphere is cut into a number of zones and substituting each zone as a frustum of a right circular cone. Now developing each zone separately by radial line method as already been discussed in section 14.4.

PROBLEM 14.56 *Draw the development of a sphere of 50 mm diameter by zone method.*

(PTU, Jalandhar December 2004, May 2005)

SOLUTION.

(*i*) Draw the front view and top view of the sphere and in the front view, draw a number of lines parallel to the diameter passing through the points *A*, *B*, *C* and *D* respectively.

(*ii*) Project the points of intersection of these parallel lines in the top view and draw circles through the points of intersection of projectors with the diameters of the top view.

(*iii*) Join points *A* and *B*, *B* and *C*, *C* and *D*.

(*iv*) Extend *AB* to meet the axis produced at O_1, *BC* to meet at O_2 and *CD* to meet at O_3. Point O_1, O_2 and O_3 are the apexes of cones of which each zone is a frustum. The top zone is regarded as the full cone with apex at O_4.

(*v*) Now develop each zone assuming it to be the frustum of the cone, as shown in Fig. 14.57.

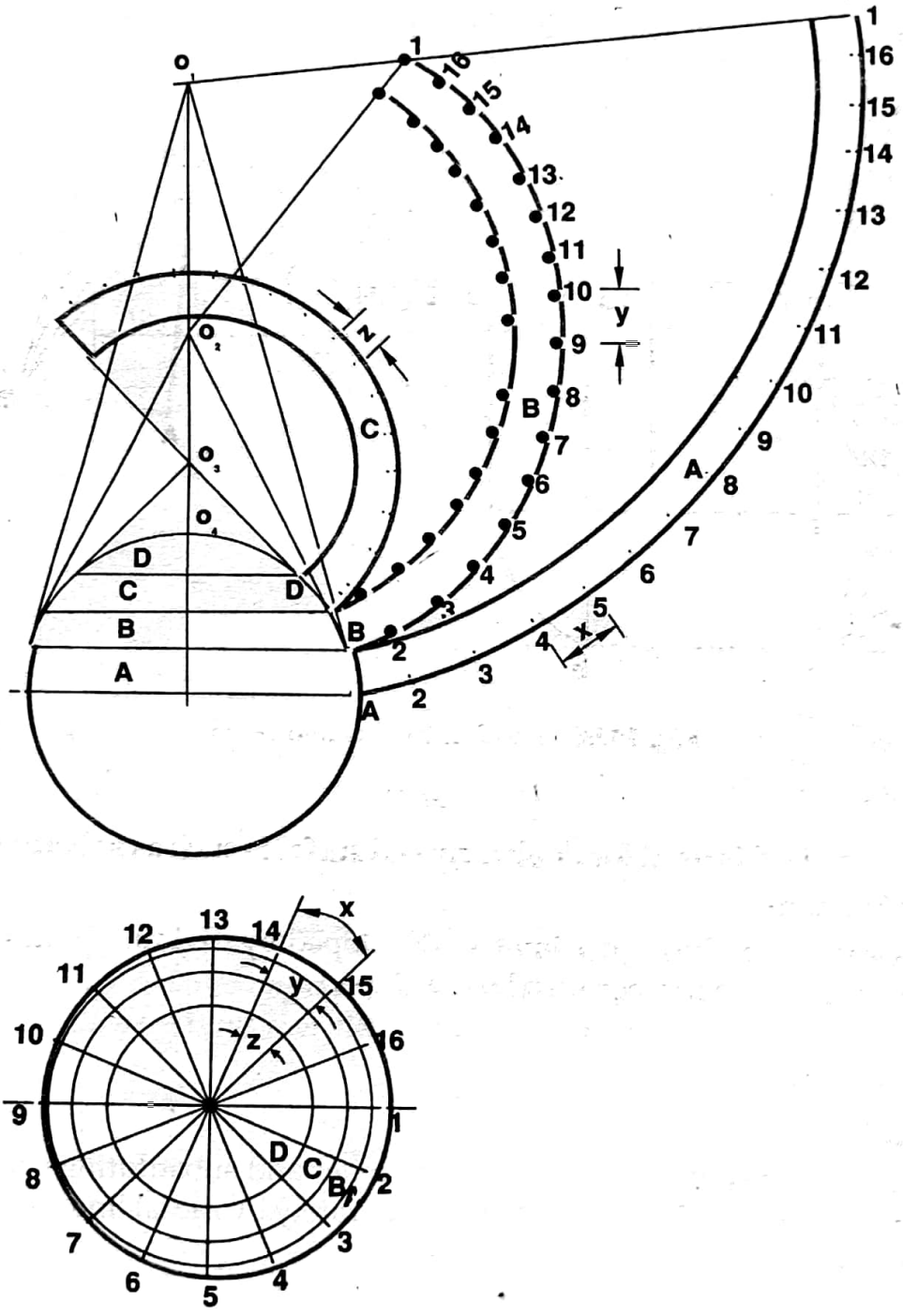


Fig. 14.57 Solution to problem 14.56

(b) Lune or Gore Method

In this method, the sphere is cut into a number of equal meridian sections called Lunes. A meridian section of the sphere so sectioned is developed assuming it to be an approximate cylinder. As all the meridian sections are equal, other sections are duplicated with the first developed section used as template.

PROBLEM 14.57 *Draw the development of a sphere of 50 mm diameter by lune method.*

SOLUTION.

(PTU, Jalandhar May 2013)

(*i*) Draw the top and front views of the sphere. Divide the front view of the sphere into sixteen equal parts.

(*ii*) Divide the half of the semicircle in a number of equal parts (say eight)

(*iii*) Develop the one-sixteenth of a sector of the sphere as shown in Fig. 14.58.

(*iv*) Draw the centre line of the sector *OBAO* and project points $a', b', c', d', e', f', g'$ and h' on this line.

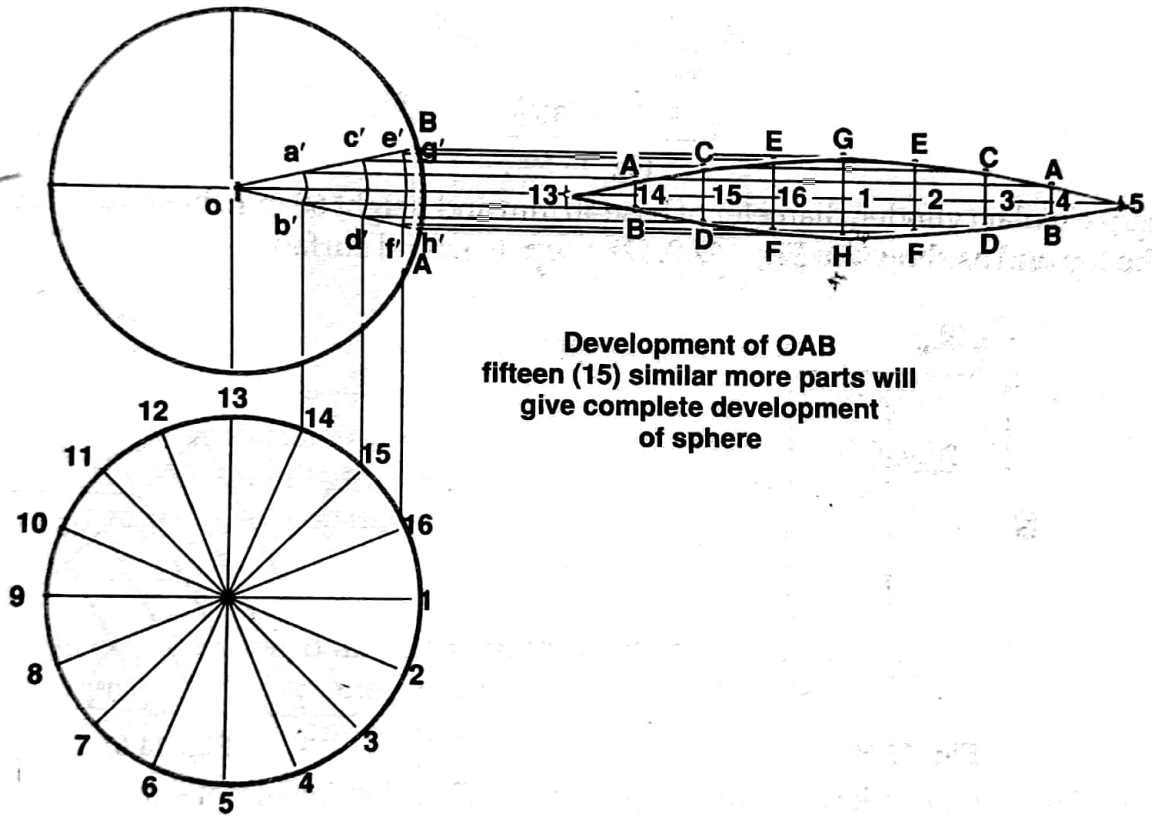


Fig. 14.58 Solution to problem 14.57

(*v*) Draw arcs $a' - b'$, $c' - d'$, $e' - f'$ and $g' - h'$ with centre O and radius equal to the points of intersection of projection of points a', c', e' and g'.

(*vi*) Draw a stretchout line 13 - 5 equal to the length of an arc of a semi circle. Divide it into eight equal parts and draw parallel lines through each division marks.

(*vii*) Join the points of intersections of the parallel lines with respective projectors by a smooth curve. This is the development of one sixteenth sector of the sphere.

EXERCISES

PARALLEL LINE METHOD

14.1 A right regular pentagonal prism, side of base 25 mm and height 50 mm, rests on its base in HP with one of its base edges perpendicular to the VP. A section plane perpendicular to the VP and inclined to the HP at 45° cuts the prism, bisecting its axis. Develop the lateral surface of the truncated prism.

14.2 A square prism is cut at top and bottom ends, as shown in Fig. 14.59. Develop its lateral surface.

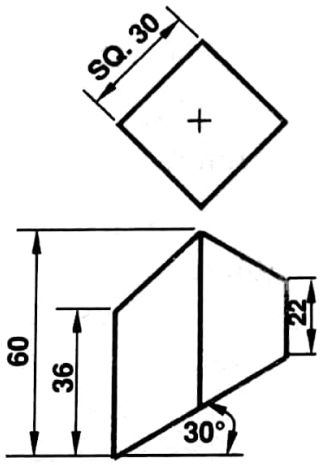


Fig. 14.59

14.3 A right circular cylinder, diameter of base 40 mm and height 60 mm, has a cut of 30 mm radius at the top end as shown in Fig. 14.60. Develop its lateral surface.

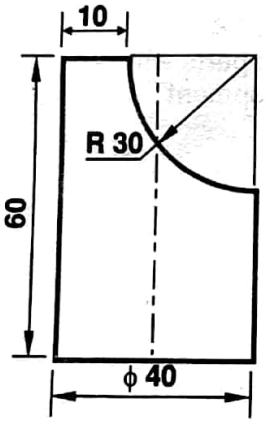


Fig. 14.60

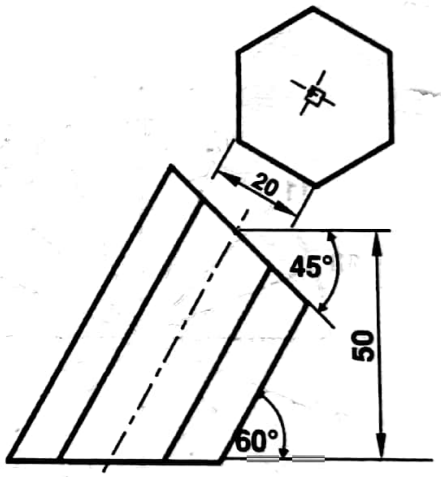


Fig. 14.61

14.4 Fig. 14.61 shows two views of an oblique prism. Develop its lateral surface.

14.5 Develop the lateral surface of an oblique cylinder as shown in Fig. 14.62.

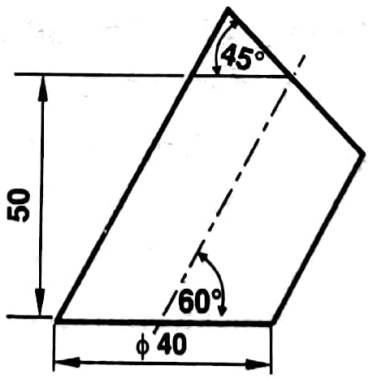


Fig. 14.62

14.6 Develop the lateral surface of the Y-piece formed by the three cylindrical pipes of 30 mm diameter, as shown in Fig. 14.63.

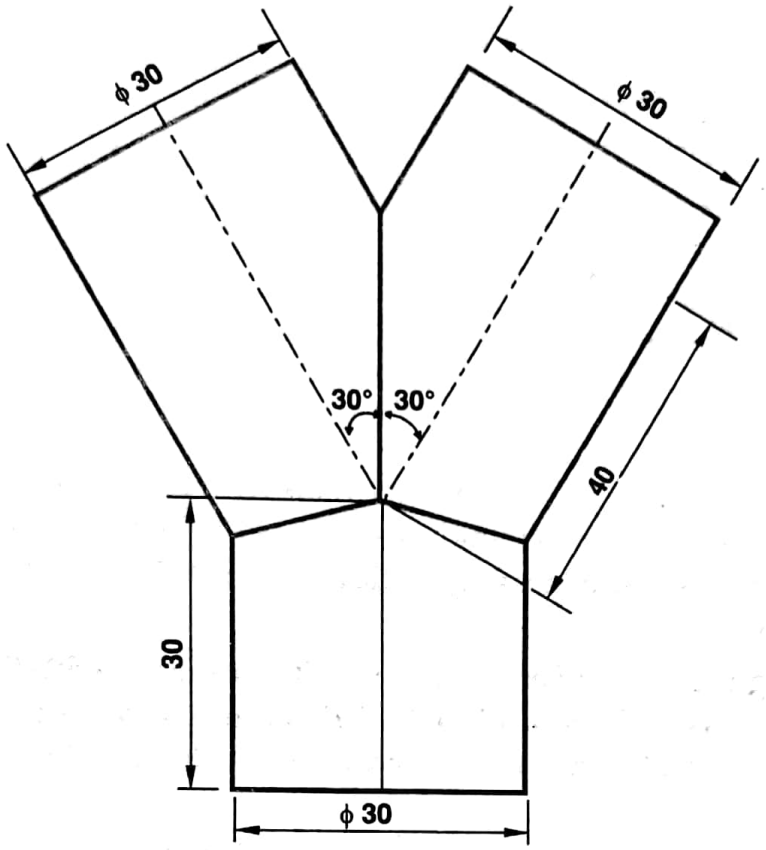


Fig. 14.63

14.7 A right regular square prism, side of base 30 mm and height 50 mm, rests on its base on HP such that its vertical faces are equally inclined to the VP. A horizontal circular hole of diameter 30 mm drilled centrally through it such that the axis of the hole cuts the diagonally opposite vertical edges. Develop its lateral surfaces.

RADIAL LINE METHOD

14.8 A right circular cone, diameter of base 40 mm and height 60 mm rests on its base in HP. The front view is cut by a plane passing through the mid-point of the axis at an angle of 45° to the HP. Draw the development of the truncated cone.

14.9 Develop the lateral surface of the conical part as shown in Fig. 14.64.

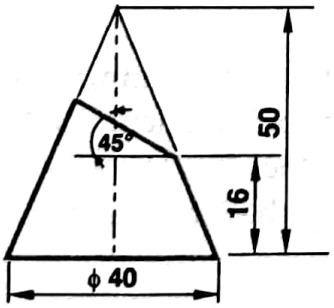


Fig. 14.64

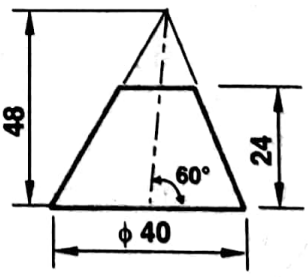


Fig. 14.65

14.10 Develop the lateral surface of the oblique cone as shown in Fig. 14.65.

14.11 Develop the lateral surface of the cone , having a circular hole cut in it as shown in Fig. 14.66.

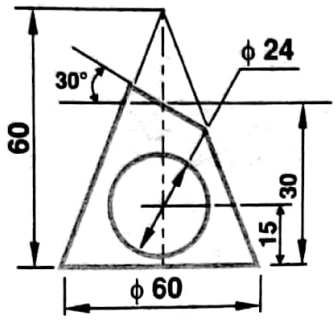


Fig. 14.66

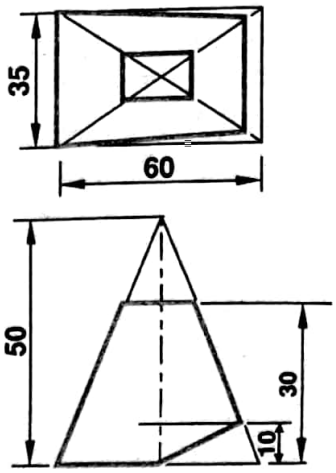


Fig. 14.67

14.12 Develop the lateral surface of the object as shown in Fig. 14.67.

14.13 A right regular pentagonal pyramid, edge of base 25 mm and height 50 mm, rests on its base in HP with one of its base edges perpendicular to the VP. Draw its projections and develop its lateral surface.

14.14 A right rectangular pyramid of base 50 mm × 30 mm and height 60 mm, rests on its base in HP with one of its base side parallel to the VP. A section plane perpendicular to the VP and inclined to the HP at 30° cuts the pyramid, bisecting its axis. Develop the lateral surface of the truncated pyramid.

14.15 A right regular hexagonal pyramid, edge of base 25 mm and height 50 mm rests on its base in HP. A hole of 15 mm diameter is drilled through the pyramid at right angles to its axis and at a height of 15 mm from the base. Develop the lateral surface of the pyramid.

14.16 A right regular hexagonal pyramid, edge of base 25 mm and height 50 mm, is lying on one of its triangular faces on HP with its axis parallel to the VP. A section plane perpendicular to the HP and inclined to the VP at 30° cuts te pyramid bisecting its axis. Develop the lateral surface of the truncated pyramid.

14.17 Develop the lateral surface of an oblique cone, diameter of base 40 mm and height 40 mm, having its axis inclined at 60° to its base.

14.18 A right rectangular pyramid of base 45 mm × 35 mm height 60 mm, rests on its base in HP with one of its base sides parallel to VP. A section plane perpendicular to the VP and inclined at 30° to the HP cuts the pyramid, bisecting its axis. Develop the lateral surface of the truncated pyramid.

14.19 A right regular hexagonal pyramid, edge of base 25 mm, height 60 mm, rests on its base on HP with one of its base edges parallel to VP. A section plane perpendicular to HP and inclined to the VP at 30° cuts the pyramid and is 10 mm away from the axis. Draw its top view, sectional front view and true shape of the section. Also develop the lateral surface of the truncated pyramid.

14.20 A right circular cone, diameter of base 50 mm and height 65 mm, rests on its base in HP. A section plane perpendicular to the HP and inclined to the VP at 45° cuts the cone and is 12 mm away from the axis. Draw its top view, sectional front view and true shape of the section. Also develop the lateral surface of the truncated cone.

14.21 A right regular pentagonal prism, side of base 25 mm and height 60 mm, rests on an edge of its base on HP, such that one of its base corners lies on HP and its axis is inclined at 45° to the HP and parallel to the VP. A section plane perpendicular to the HP and inclined to the VP at 45° cuts the prism bisecting its axis. Draw its top view and sectional front view. Also develop the lateral surface of the truncated prism.

TRIANGULATION METHOD

14.22 An air conditioning duct of a square cross-section 60 mm × 60 mm connects a circular pipe of 30 mm diameter through the transition piece. Draw the projections and develop the lateral surface of the transition piece.

14.23 Develop the lateral surface of a transition piece to join coaxial circular and hexagonal duct holes 55 mm apart. Given the diameter of the circular hole as 40 mm and side of the hexagonal hole as 40 mm.

14.24 Develop the inside pattern of a sheet metal tray as shown in Fig. 14.68.

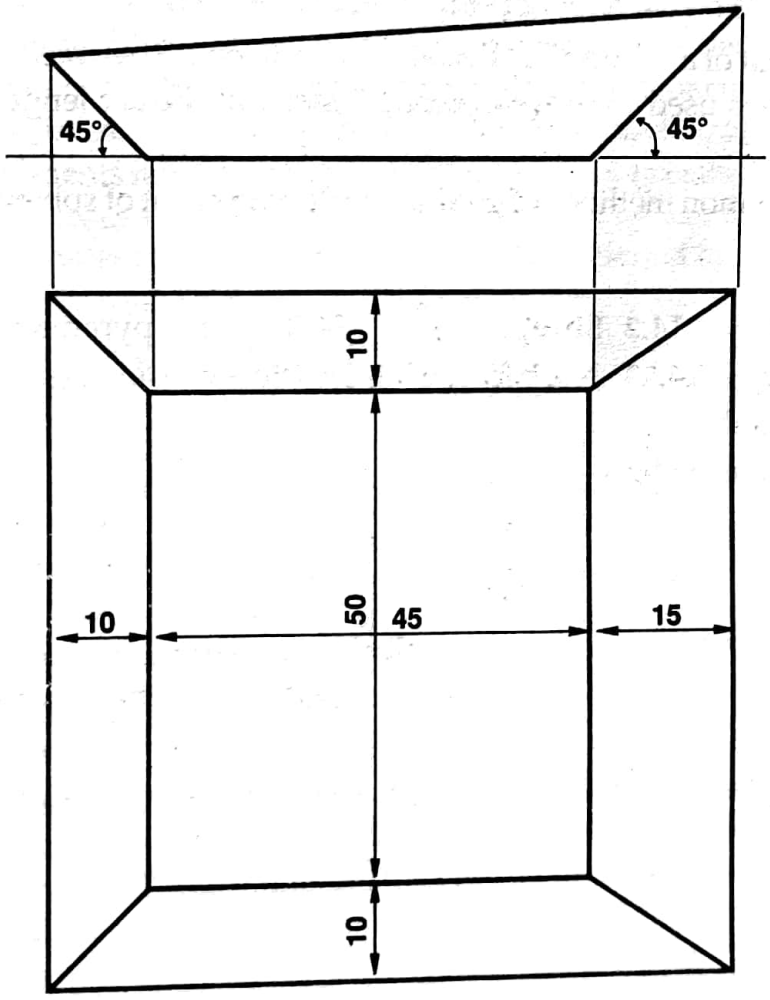


Fig. 14.68

APPROXIMATE METHOD

14.25 Draw the development of a sphere of 60 mm diameter by (nts, have to be located. Due method.

...e., a
...ugh these
...intersection of any
...cutting planes. These
...Certain important points
...ound knowledge of projections of

OBJECTIVE QUESTIONS

14.1 The complete layout of a 3 - D object on a 2- D sheet is known as ..... of surfaces.

14.2 The general application of development is used in the ..... metal work.

14.3 Radial line method is used for developing ..... and ..... objects.

14.4 Every line on the development must show the ..... length of the corresponding line on the surface to be developed.

14.5 What do you mean by development of surfaces ?

14.6 What are different methods of development of surfaces ?

14.7 Give practical examples of development of surfaces.

14.8 ..... method is used for the development of sphere.

14.9 Distinguish between single curved surface and double curved surface.

14.10 What is the importance of development of surface is a manufacturing industry ?

14.11 What is the shape obtained by development of a cone ?

14.12 Explain the method used for development of a right cone.

14.13 In development of a cone and cylinder the chord length between two successive point on the circumference is used for development, instead of the arc length. Is it a correct approach? Comment.

14.14 Name two common methods of getting the development of sphere.

ANSWERS

14.1 Development **14.2** Sheet **14.3** Cones, pyramids **14.4** True

14.8 Approximate **14.13** No, because it is less time consuming.

14.14 (i) Zone Method

(ii) Lune or Gore method