10 Projections of Planes

10.1 INTRODUCTION

Plane figures or surfaces have only two dimensions *i.e.* length, breadth and negligible thickness. Plane surfaces may be considered of infinite sizes. However for convenience, segments of planes are only considered in the solutions. Planes are represented in space by either of the following :

- Three non-collinear points, Fig. 10.1(*a*)
- A line and a point, Fig. 10.1(*b*)
- Two intersecting lines, Fig. 10.1(*c*)
- Two parallel lines, Fig. 10.1(*d*)
- A plane, Fig. 10.1(*e*)

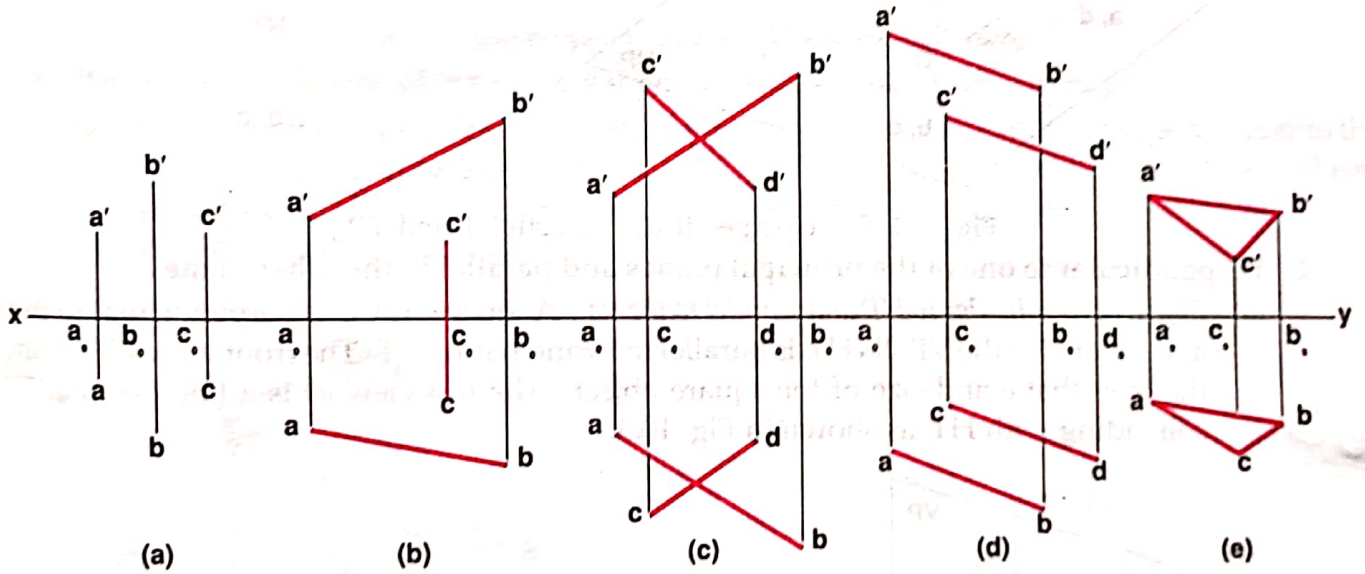


Fig. 10.1 Representation of planes

10.2 TYPES OF PLANES

Planes may be divided into two main types :

- Principal Planes
- Secondary Planes
- **Principal Planes.** The planes on which the projections are taken, called as principal planes. Example horizontal and vertical planes.
- **Secondary Planes.** Secondary planes are of two types :

(*i*) Perpendicular planes

(*ii*) Oblique planes

(*i*) *Perpendicular planes*. These planes can be divided into the following sub-types :

1. Perpendicular to both the principal planes
2. Perpendicular to one of the principal planes and parallel to the other plane
3. Perpendicular to one of the principal planes and inclined to the other plane

1. Perpendicular to both the principal planes. A square *ABCD* is perpendicular to both the principal planes. Its HT and VT are in a straight line perpendicular to *xy*, as shown in Fig. 10.2. The front view *b'c'* and top view *ab* of the square are both straight lines coinciding with VT and HT respectively.

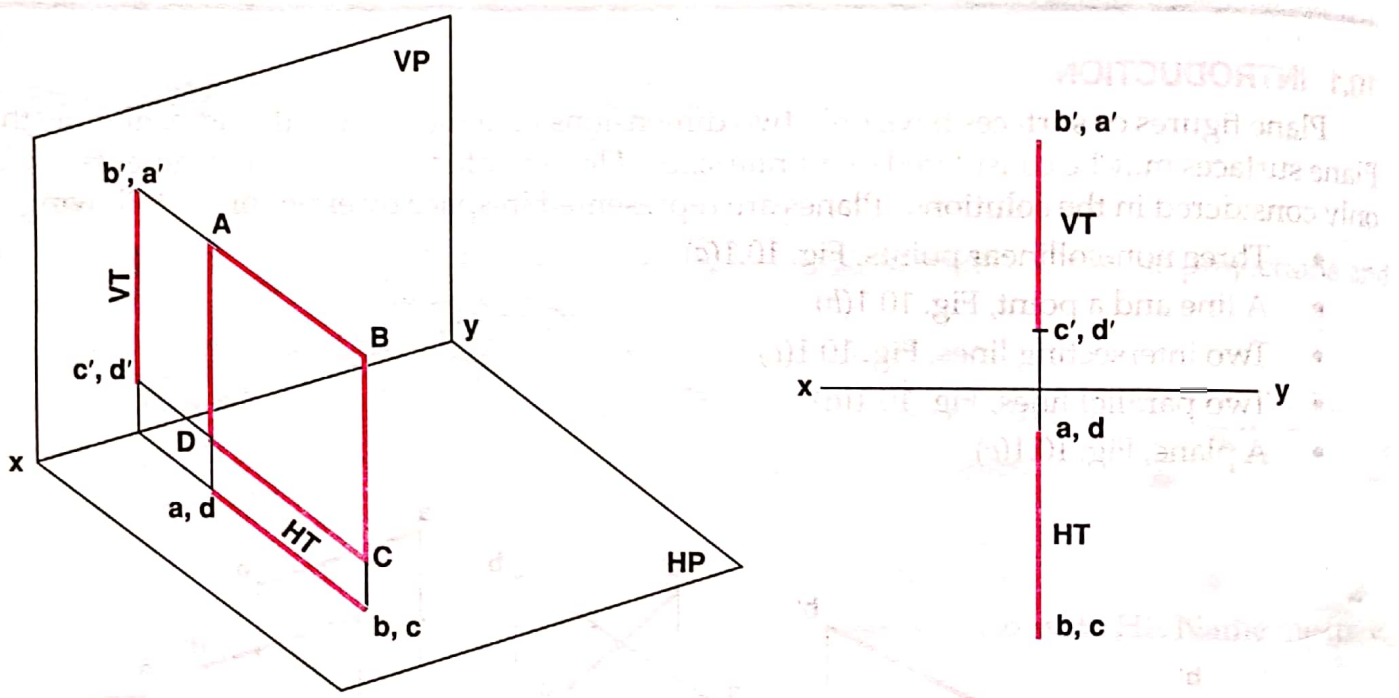


Fig. 10.2 Plane perpendicular to both HP and VP

2. Perpendicular to one of the principal planes and parallel to the other plane

(*a*) *Plane, perpendicular to HP and parallel to the VP*. A square *ABCD* is perpendicular to the HP and parallel to the VP. Its HT, is parallel to *xy* and has no VT. The front view *a'b'c'd'* shows the true shape and size of the square object. The top view *ab* is a line, parallel to *xy*, coinciding with HT, as shown in Fig. 10.3.

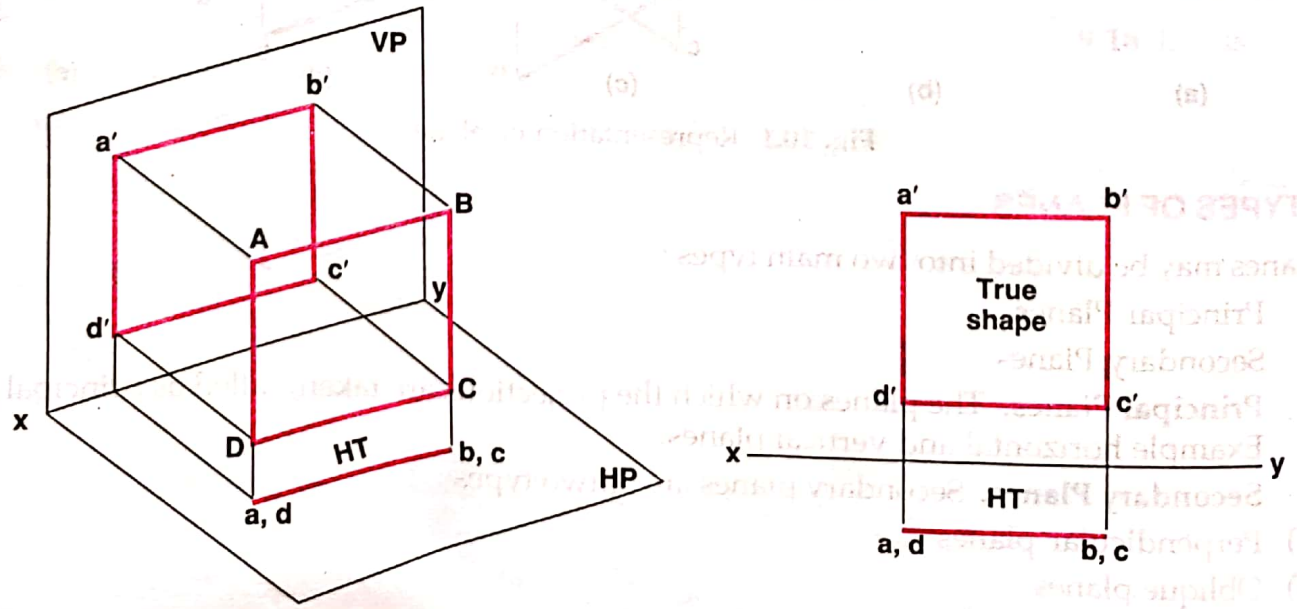


Fig. 10.3 Plane perpendicular to HP and parallel to VP

(*b*) *Plane, perpendicular to VP and parallel to the HP.* A square *ABCD* is perpendicular to the VP and parallel to the HP. Its VT is parallel to *xy* and has no HT. The top view *abcd* shows the true shape and size of the square object. The front view *d'c'* is a line, parallel to *xy*, coinciding with VT, as shown in Fig. 10.4.

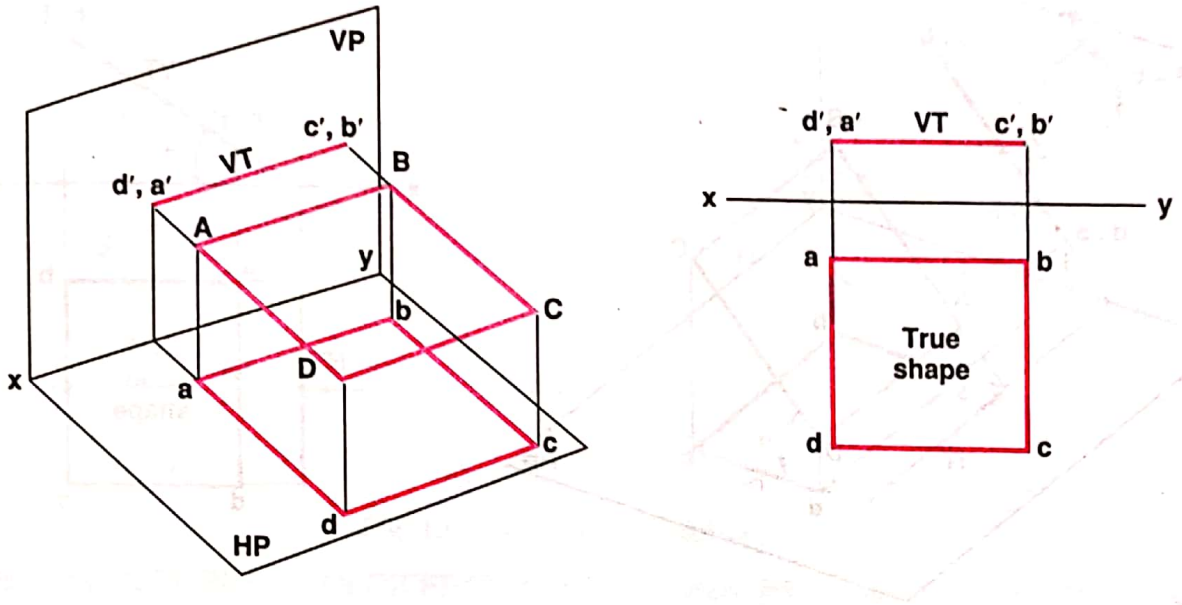


Fig. 10.4 Plane perpendicular to VP and parallel to HP

3. Perpendicular to one of the principal planes and inclined to the other plane

(*a*) *Plane, perpendicular to the HP and inclined to the VP.* A square *ABCD* is perpendicular to the HP and inclined at an angle ϕ to the VP. Its VT is perpendicular to *xy* and HT is inclined at angle ϕ to *xy*. Its top view *ab* is a line inclined at an angle ϕ to *xy* and the front view *a'b'c'd'* is smaller than square *ABCD*, as shown in Fig. 10.5.

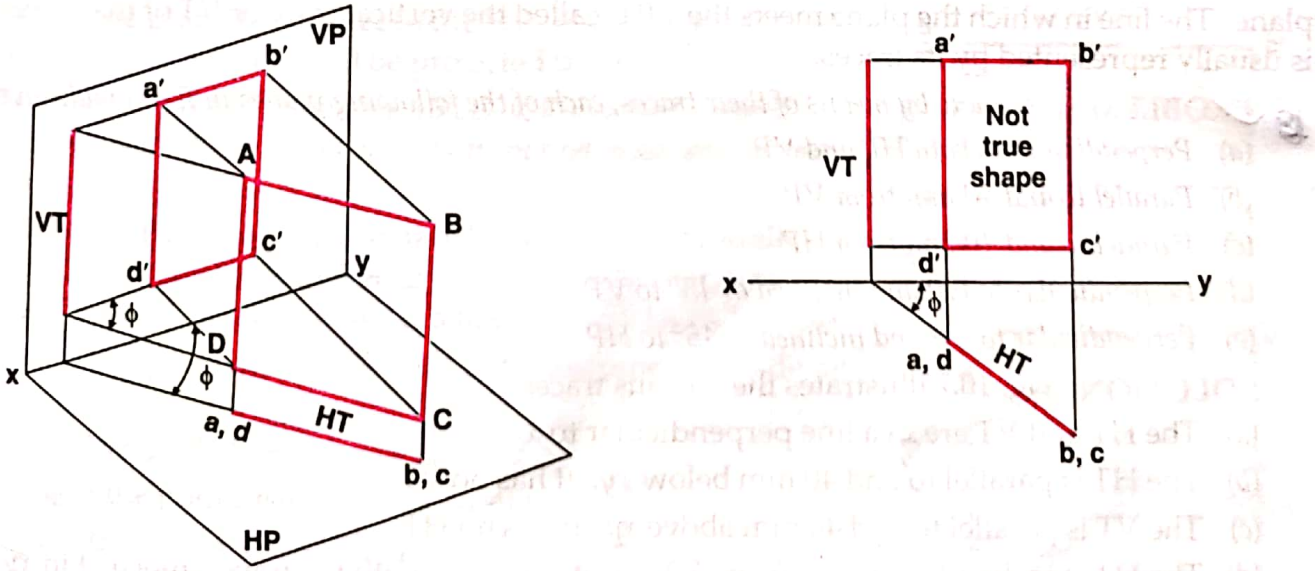


Fig. 10.5 Plane perpendicular to HP and inclined to VP

(*b*) *Plane, perpendicular to the VP and inclined to the HP.* A square *ABCD* is perpendicular to the VP and inclined at an angle θ to the HP. Its HT is perpendicular to *xy* and VT is inclined

at an angle θ to the *xy*. Its front view *d'c'* is a line inclined at an angle θ to *xy* and the top view *abcd* is a rectangle which is smaller than square *ABCD*, as shown in Fig. 10.6.

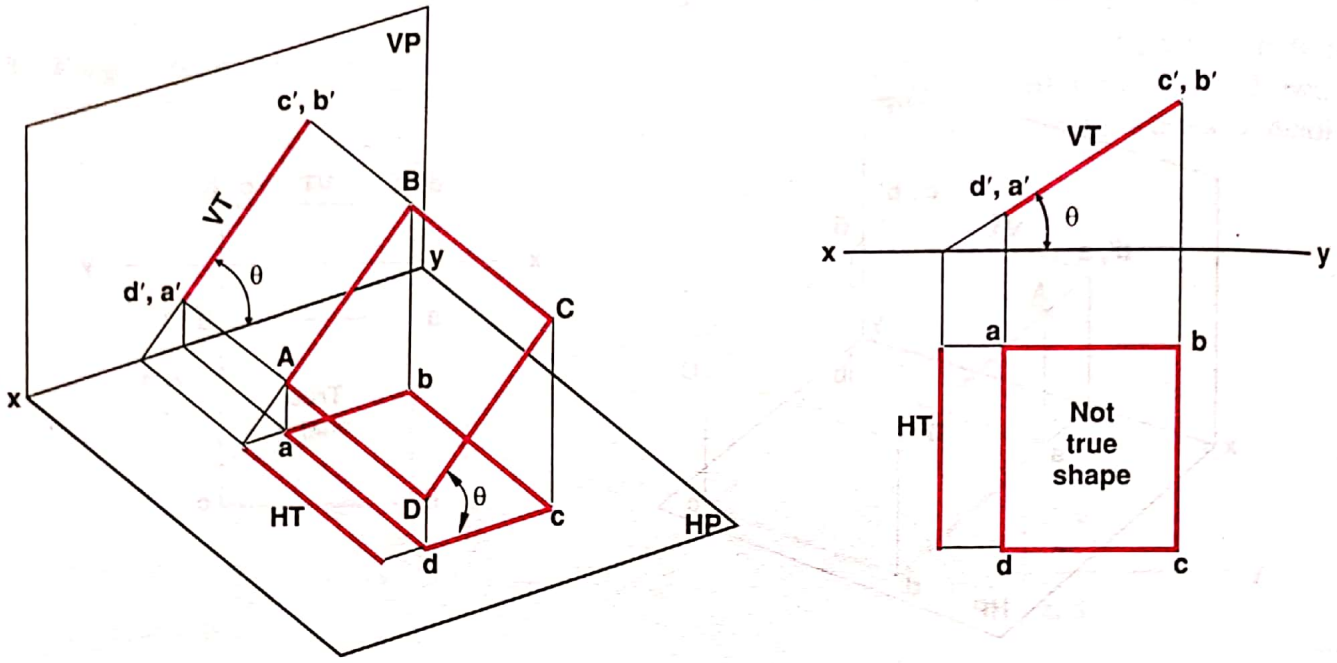


Fig. 10.6 Plane perpendicular to VP and inclined to HP

(*ii*) *Oblique Planes.* Planes which are inclined to both the principal planes are called oblique planes. Representation of oblique planes by their traces is too advanced to be included in this book.

10.3 TRACES OF PLANES

A plane extended if necessary, will meet the principal planes in lines. These lines are called the traces of the planes. The line in which the plane meets the HP is called the horizontal trace or HT of the plane. The line in which the plane meets the VP is called the vertical trace or VT of the plane. A plane is usually represented by its traces.

PROBLEM 10.1 *Show by means of their traces, each of the following planes in first quadrant only.*

(*a*) *Perpendicular to both HP and VP*

(*b*) *Parallel to and 40 mm from VP*

(*c*) *Parallel to and 40 mm from HP*

(*d*) *Perpendicular to HP and inclined at 45° to VP*

(*e*) *Perpendicular to VP and inclined at 45° to HP.*

SOLUTION. Fig. 10.7 illustrates the various traces.

(*a*) The HT and VT are in a line perpendicular to *xy*.

(*b*) The HT is parallel to and 40 mm below *xy*. It has no VT.

(*c*) The VT is parallel to and 40 mm above *xy*. It has no HT.

(*d*) The HT is inclined at 45° to *xy*, the VT is normal to *xy*, both the traces intersect in *xy*.

(*e*) The VT is inclined at 45° to *xy*, the HT is normal to *xy*, both the traces intersect in *xy*.

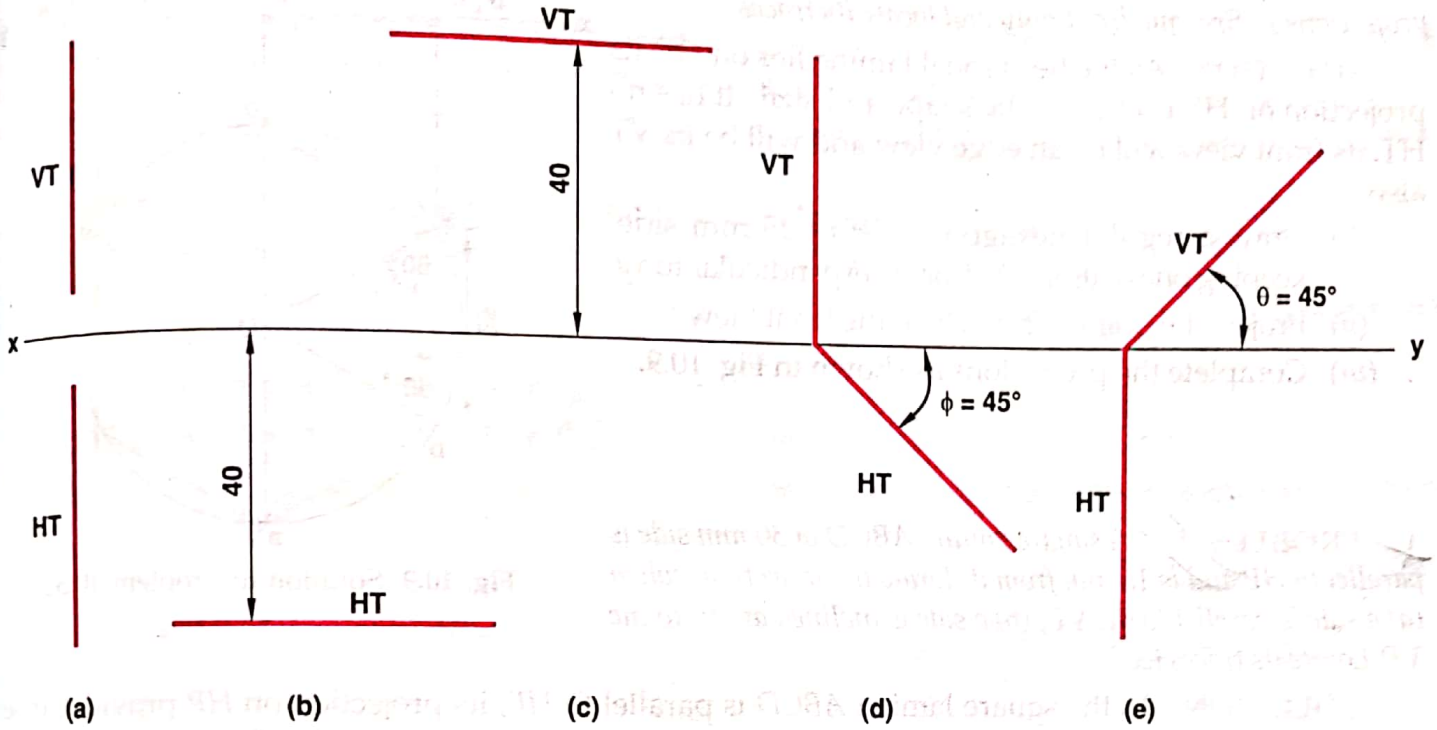


Fig. 10.7 Solution to problem 10.1

10.4 A SECONDARY PLANE IN DIFFERENT POSITIONS WITH RESPECT TO THE PRINCIPAL PLANES

Various positions, which a secondary plane in space can take with reference to the principal planes of projections and the corresponding projections are described in the subsequent paragraphs.

10.5 PROJECTIONS OF PLANE PARALLEL TO ONE OF THE PRINCIPAL PLANES

Projections of such plane can be drawn in one step. First draw the projection of the plane on that principal plane to which it is parallel, as that view will show true shape and size. The other view which will be a line, should then be projected from it.

(*a*) **When the plane is parallel to the HP.** The top view should be drawn first and front view will be projected from it.

PROBLEM 10.2 *A square lamina ABCD of 30 mm side is parallel to HP and is 15 mm from it. Draw its projections in first quadrant only and locate its traces.*

SOLUTION. Fig. 10.8, as the square lamina *ABCD* is parallel to HP, its projection on HP provides true shape and size. It has no HT, its front view will be an edge view *i.e.* a line and will be its VT also.

(*i*) Draw a square *abcd* of 30 mm side, keeping one side *ab* parallel to *xy*.
(*ii*) Keep 15 mm distance above *xy*.
(*iii*) Project the points *a'b'c'd'* in the front view.
(*iv*) In the front view *c'* overlaps point *b'* and point *d'* overlaps *a'*.
(*v*) Therefore *d'*, *c'* will be written before the front view *a'*, *b'* respectively. *e.g.*, in front view write *d'*, *a'* and *c'*, *b'*.

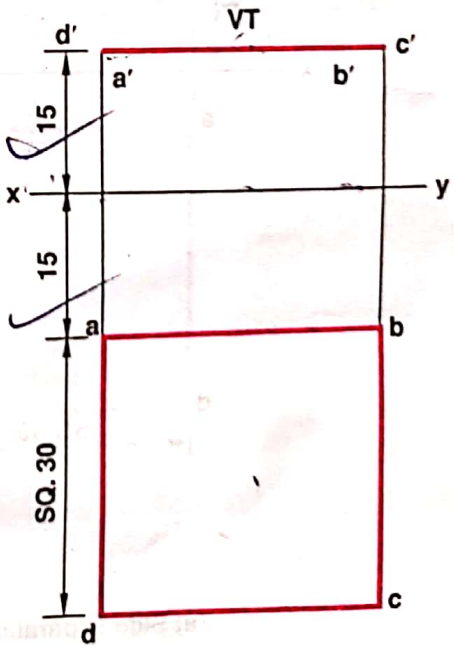


Fig. 10.8 Solution to problem 10.2

PROBLEM 10.3 *A regular hexagonal lamina, of side 25 mm side, rests on HP such that one of its sides is perpendicular to VP. Draw its*

projections in first quadrant only and locate its traces.

SOLUTION. As the hexagonal lamina lies on HP, its projection on HP provides true shape and size. It has no HT, its front view will be an edge view and will be its VT also.

(*i*) Draw a regular hexagon *abcdef* of 25 mm side, keeping one of its sides *cb* or *ef* perpendicular to *xy*.

(*ii*) Project the points *a'b'c'd'e'f'* in the front view.

(*iii*) Complete the projections as shown in Fig. 10.9.

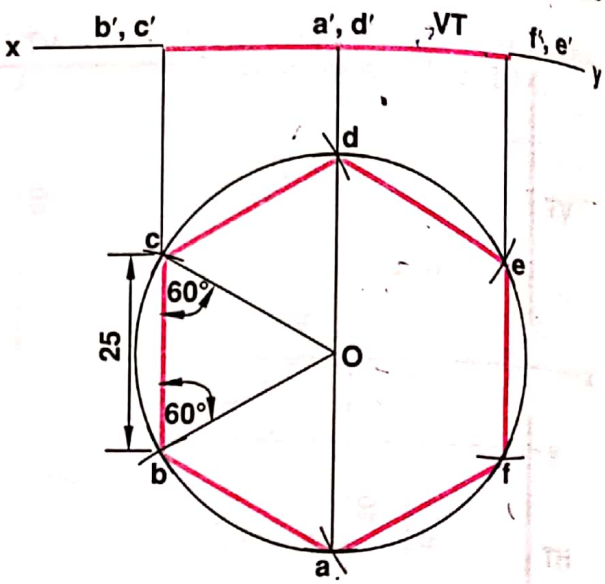


Fig. 10.9 Solution to problem 10.3

PROBLEM 10.4 *A square lamina ABCD of 30 mm side is parallel to HP and is 15 mm from it. Draw its projections when (a) a side is parallel to the VP; (b) a side is inclined at 30° to the VP. Locate its traces too.*

SOLUTION. As the square lamina *ABCD* is parallel to *HP*, its projection on *HP* provides true shape and size. It has no *HT*, its front view will be an edge view *i.e.* a line and will be its *VT* also.

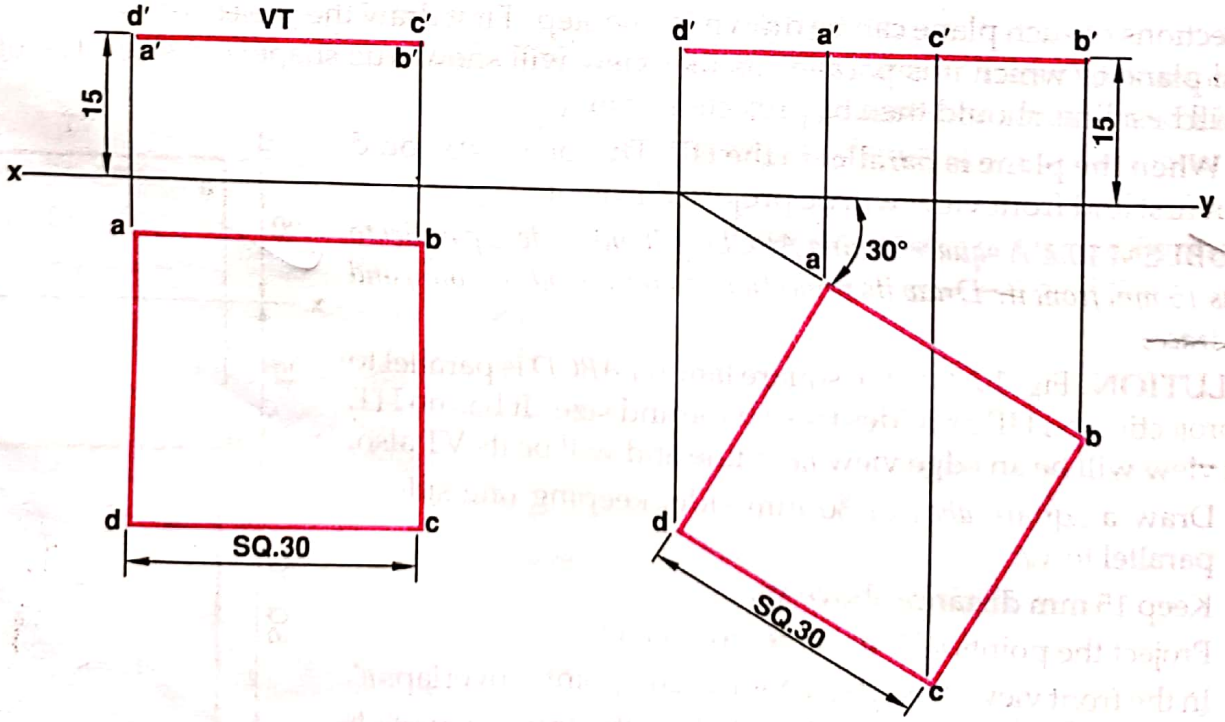
(*a*) (*i*) Draw a square *abcd* of 30 mm side, keeping one side *ab* parallel to *xy*.

(*ii*) Keep 15 mm distance above *xy*.

(*iii*) Project the points *a' b' c' d'* in the front view.

(*iv*) In the front view *c'* overlaps point *b'* and *d'* overlaps *a'*.

(*v*) Therefore *d', c'* will be written before the front view *a', b'* respectively. *e.g.*, in front view write *d', a'* and *c', b'*. See Fig. 10.10 (*a*).



(a) Side is parallel to VP

(b) Side is inclined at 30° to VP

Fig. 10.10 Solution to problem 10.4

(*b*) (*i*) Draw a square *abcd* of 30 mm side, keeping one side *ab* to be inclined at 30° to the *xy*.

(*ii*) Keep 15 mm distance above *xy*.

(*iii*) Project the points *a' b' c' d'* in the front view as shown in Fig. 10.33 (*b*)

PROBLEM 10.5 *A regular hexagonal lamina of side 25 mm is parallel to HP and 15 mm from it. Draw its projections when (a) a side is perpendicular to the VP; (b) a side is parallel to the VP; (c) a side is inclined at 45° to the VP. Locate its traces too.*

SOLUTION. As the hexagonal lamina is parallel to the *HP*., so its projection on *HP* provides true shape and size. It has no *HT*, its front view will be an edge view *i.e.*, a line and will be its *VT* also.

(*a*) (*i*) Draw a regular hexagon *abcdef* of 25 mm side, keeping one of its sides *ab* or *de* perpendicular to *xy*.

(*ii*) Keep 15 mm distance above *xy*.

(*iii*) Project the points *a' b' c' d' e' f'* in the front view as shown in Fig. 10.11 (*a*).

(*b*) (*i*) Draw a regular hexagon *abcdef* of 25 mm side, keeping one of its sides say *ab* or *de* parallel to *xy*.

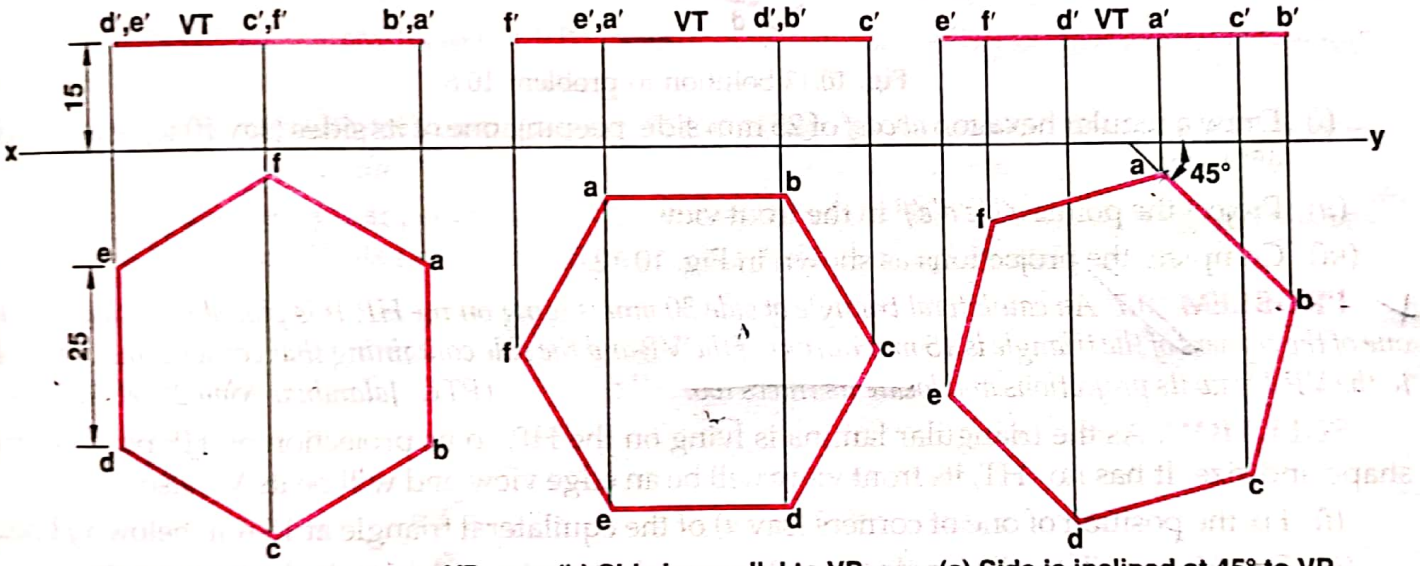


Fig. 10.11 Solution to problem 10.5

(*ii*) Keep 15 mm distance above *xy*.

(*iii*) Project the points *a' b' c' d' e' f'* in the front view as shown in Fig. 10.11 (*b*)

(*c*) (*i*) Draw a regular hexagon *abcdef* of 25 mm side, keeping one side say *ab* to be inclined at 45° to the *xy*.

(*ii*) Keep the distance of 15 mm above *xy*.

(*iii*) Project the points *a' b' c' d' e' f'* in the front view. See Fig. 10.11 (*c*)

PROBLEM 10.6 *A regular hexagonal lamina ABCDEF 25 mm side rests on HP such that one of its sides (say AF) inclined to VP at 45°. Draw its projections and locate its traces.*

SOLUTION. As the hexagonal lamina lies on HP, so its projection on HP provides true shape and size. It has no HT, its front view will be an edge view and will be its VT also.

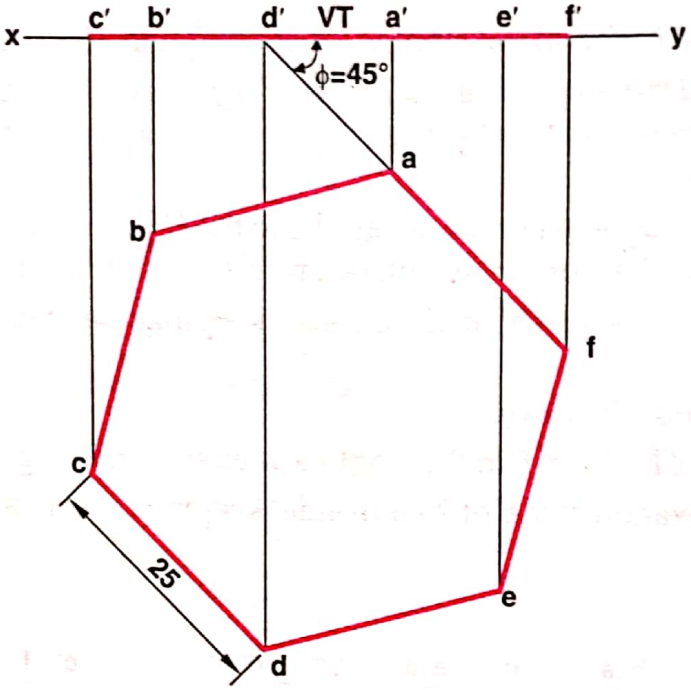


Fig. 10.12 Solution to problem 10.6

(*i*) Draw a regular hexagon *abcdef* of 25 mm side, peeping one of its sides (say *af*) to be inclined at 45° to *xy*.

(*ii*) Project the points *a'b'c'd'e'f'* in the front view.

(*iii*) Complete the projections as shown in Fig. 10.12.

PROBLEM 10.7 *An equilateral triangle of side 30 mm is lying on the HP. It is placed in such a way that one of the corners of the triangle is 15 mm infront of the VP and the side containing that corner is inclined at 45° to the VP. Draw its projections and locate its traces too.* *(PTU, Jalandhar May 2008, May 2015)*

SOLUTION. As the triangular lamina is lying on the HP, so its projection on HP provides true shape and size. It has no HT, its front view will be an edge view and will be its VT also.

(*i*) Fix the position of one of corners (say *a*) of the equilateral triangle at 15 mm below *xy* line..

(*ii*) Draw an equilateral triangle *abc* of 30 mm side, keeping side *ab* to be inclined at 45° to *xy*.

(*iii*) Project the points *a'b'c'* in the front view.

(*iv*) Complete the projections as shown in Fig. 10.13.

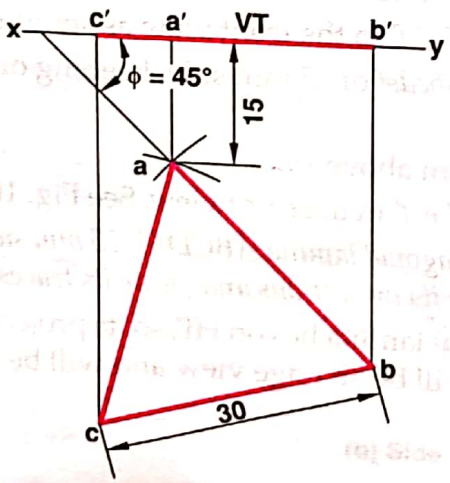


Fig. 10.13 Solution to problem 10.7

PROBLEM 10.8 *A regular pentagonal lamina ABCDE of 25 mm side, having one of its side is parallel to VP and 10 mm above HP. A corner opposite to this side is 12 mm infront of the VP. Draw the projections when the lamina is parallel to HP and locate its traces too.*

SOLUTION. As the pentagonal lamina is parallel to the HP, so its projections will start from the top view, where it will provid : true shape and size. It has no HT, its front view will be an edge view and will be its VT also.

(*i*) Fix the position of one of corners (say a) of the regular pentagonal at 12 mm below the xy line.

(*ii*) Draw a regular pentagonal abcde of 25 mm side, keeping side cd to be parallel to xy line.

(*iii*) Project the points a'b'c'd'e' in the front view at a distance of 10 mm above xy line.

(*iv*) Complete the projections as shown in Fig. 10.14

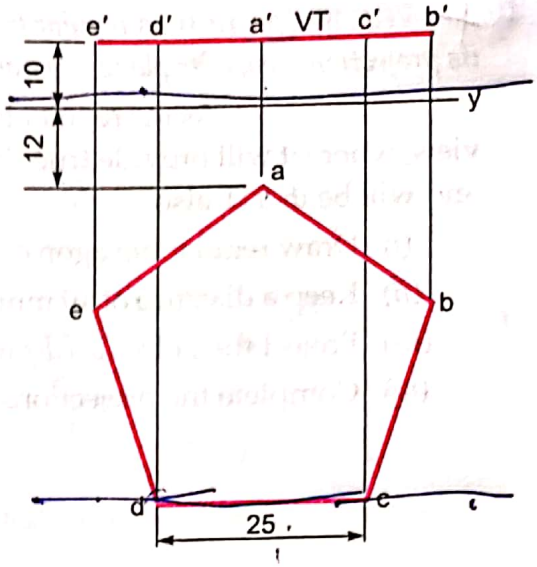


Fig. 10.14 Solution to problem 10.8

(*b*) **When the plane is parallel to the VP.** The front view should be drawn first and top view will be projected from it.

PROBLEM 10.9 *A regular pentagonal lamina ABCDE 25 mm side, has its side CD lies on HP. Draw its projections when its plane is parallel to and 15 mm infront of VP. Also locate its traces.*

SOLUTION. As the regular pentagon *ABCDE* is parallel to VP, its projection on VP provides true shape and size. It has no VT, its top view will be an edge view and will be its HT also.

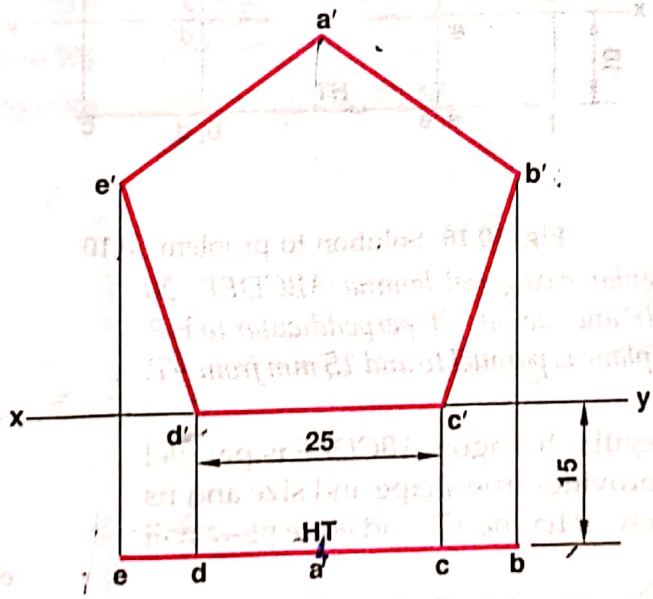


Fig. 10.15 Solution to problem 10.9

(*i*) Draw a regular pentagon *a'b'c'd'e'* of 25 mm side, keeping its side *c'd'* on *xy*.

(*ii*) Keep a distance of 15 mm below *xy*.

(*iii*) Project the points *abcde* in the top view.

(*iv*) Complete the projections as shown in Fig. 10.15.

PROBLEM 10.10 *A regular hexagonal lamina ABCDEF 25 mm side, has its side DE lies on HP. Draw its projections when its plane is parallel to and 10 mm infront of VP. Also locate its traces.*

SOLUTION. As the regular hexagon *ABCDEF* is parallel to VP, its projection will start from front view, where it will provide true shape and size. It has no VT, its top view will be an edge view or line and will be its HT also.

(*i*) Draw regular hexagon *a'b'c'd'e'f'* of 25 mm side, keeping its side *d'e'* on *xy*.

(*ii*) Keep a distance of 10 mm below *xy*.

(*iii*) Project the points *abcdef* in the top view.

(*iv*) Complete the projections as shown in Fig. 10.16.

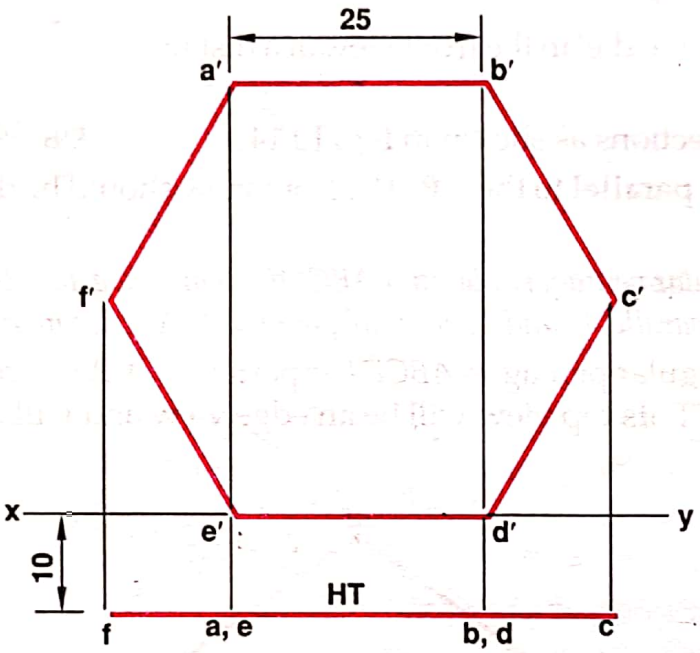


Fig. 10.16 Solution to problem 10.10

PROBLEM 10.11 *A regular hexagonal lamina ABCDEF 20 mm side, has its corner D in HP and the side EF perpendicular to HP. Draw its projections when its plane is parallel to and 15 mm from VP. Locate it traces too.*

SOLUTION. As the regular hexagon *ABCDEF* is parallel to VP, its projection on VP provides true shape and size and its top view will be an edge view. It has no VT and edge view will represent its HT.

(*i*) Draw a regular hexagon *a'b'c'd'e'f'* of 25 mm side, keeping its corner *d'* on *xy*.

(*ii*) Keep a distance of 15 mm below *xy*.

(*iii*) Project the points *abcdef* in the top view.

(*iv*) Complete the projections as shown in Fig. 10.17.

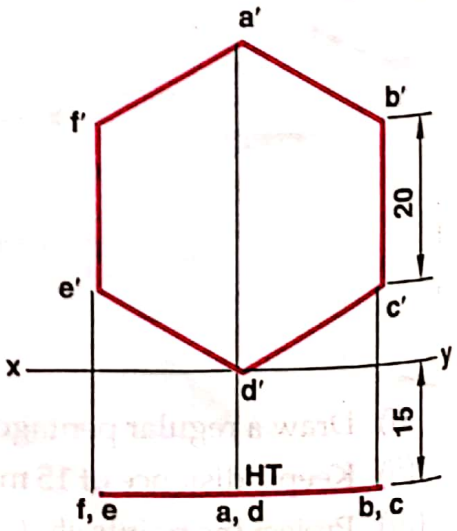


Fig. 10.17 Solution to problem 10.11

PROBLEM 10.12 *A square lamina ABCD of 20 mm side has its HT parallel to and 12 mm below xy line. It has no VT. Draw its projections when all the sides are equally inclined to the HP.* (PTU, Jalandhar December 2004, 2007)

SOLUTION.

(*i*) Draw a square *a'b'c'd'* of 20 mm side in the front view, keeping all its sides equally inclined to the HP.

(*ii*) Keep a distance of 12 mm below *xy*.

(*iii*) Project the points *abcd* in the top view.

(*iv*) Complete the projections as shown in Fig. 10.18.

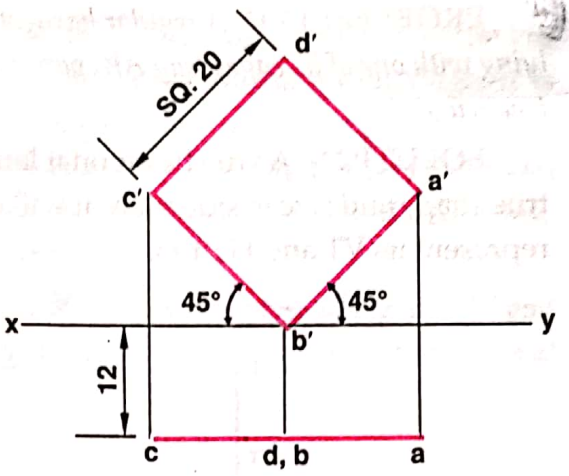


Fig. 10.18 Solution to problem 10.12

10.6 PROJECTIONS OF PLANE PERPENDICULAR TO BOTH HP AND VP

When a plane is perpendicular to both HP and VP, its projection will start from side view, where it gives the true shape and size. Then front and top views are to be projected from it.

PROBLEM 10.13 *A square lamina ABCD of 25 mm side is perpendicular to both HP and VP. Draw its projections in first quadrant only and locate its traces too.*

SOLUTION. As square lamina *ABCD* is perpendicular to both HP and VP, so it will give true shape and size in side view. It will have an edge view or line, both in front view and top view, which will represent its VT and HT too.

(*i*) Draw a square lamina *a"b"c"d"* of 25 mm side in side view.

(*ii*) Project the front and top views from it.

(*iii*) Complete the projections as shown in Fig. 10.19.

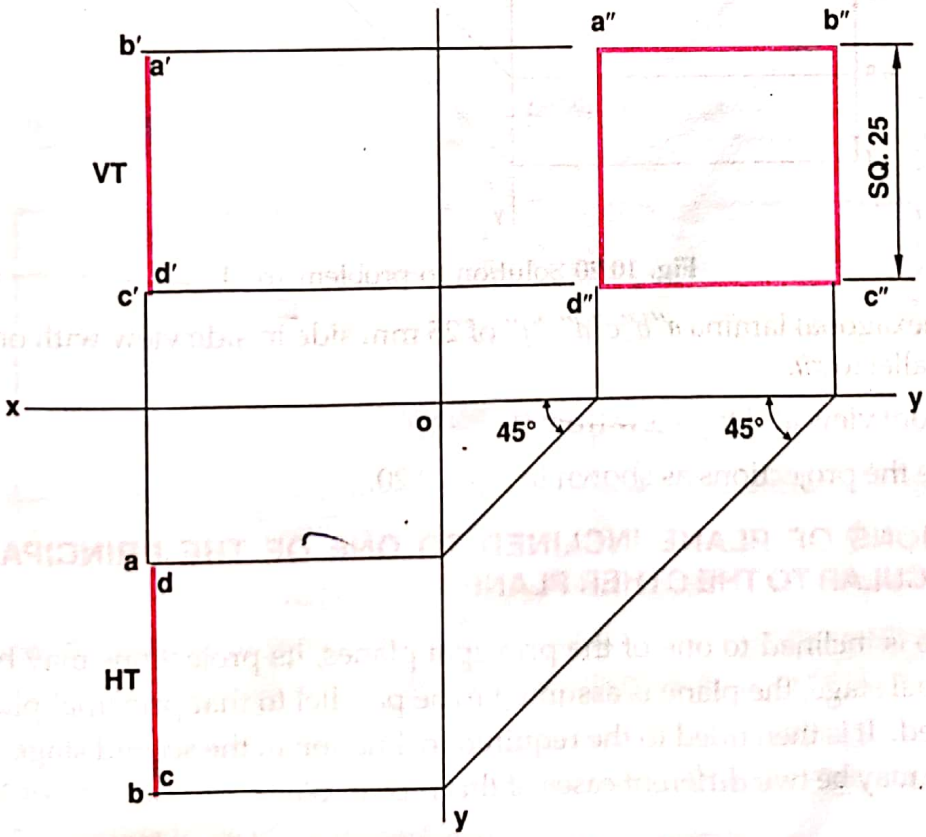


Fig. 10.19 Solution to problem 10.13

PROBLEM 10.14 *A regular hexagonal lamina ABCDEF 25 mm side is normal to both HP and VP. It is lying with one of its edges (say AB) parallel to HP and perpendicular to VP. Draw its projections and locate its traces too.*

SOLUTION. As the hexagonal lamina *ABCDEF* is normal to both HP and VP, so it will provide true shape and size in side view. It will have an edge view, both in front view and top view, which will represent its VT and HT too.

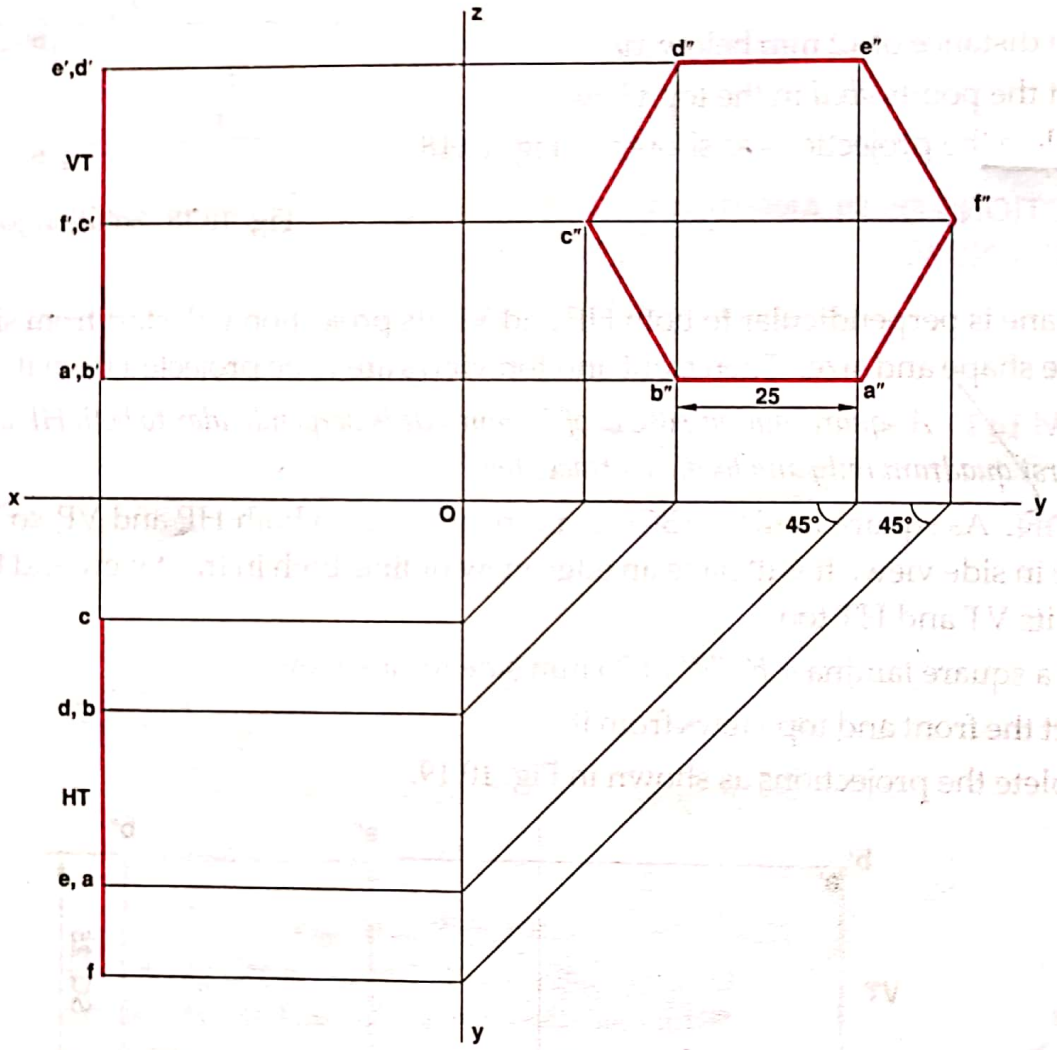


Fig. 10.20 Solution to problem 10.14

(*i*) Draw a hexagonal lamina *a"b"c"d"e"f"* of 25 mm side in side view with one of its edges (say *a"b"*) parallel to *xy*.

(*ii*) Project front view and top view from it.

(*iii*) Complete the projections as shown in Fig. 10.20.

10.7 PROJECTIONS OF PLANE INCLINED TO ONE OF THE PRINCIPAL PLANES AND PERPENDICULAR TO THE OTHER PLANE

When a plane is inclined to one of the principal planes, its projections may be obtained in two stages. In the initial stage, the plane is assumed to be parallel to that principal plane to which it has to be made inclined. It is then titled to the required inclination in the second stage and its projections are drawn. There may be two different cases of this type of plane.

(*a*) **Plane, inclined to the HP and perpendicular to the VP.** When a plane is inclined to the HP and perpendicular to the VP, its projections are drawn into two stages. In the initial stage, it is assumed to be parallel to the HP. Its top view will show the true shape and size in this position. The front view will be a line parallel to *xy*. In the second stage, tilt the front view so that it makes the given inclination to the *xy*. The new front view will be inclined to *xy* at the true inclination. In the top view the corners will move along their respective paths (parallel to *xy*).

PROBLEM 10.15 *A rectangular lamina ABCD of 50 mm × 30 mm is inclined to HP at 45° and perpendicular to VP. It rests on one of it sides say AB in HP. Draw its projections in first quadrant only and locate its traces too.*

SOLUTION.

(*i*) Draw a rectangular lamina abcd of 50 mm × 30 mm in the top view, assuming the lamina to be lying in HP and keep the side *ab* perpendicular to *xy*.

(*ii*) Project front view *a'b'c'd'* from top view, which will lie on *xy*.

(*iii*) Tilt the front view about the *a'b'*, so that it makes 45° angle with *xy* and name the points on it by adding suffix 1 to them.

(*iv*) Project the corresponding top view $a_1b_1c_1d_1$ by drawing vertical projectors through the points in second stage front view and horizontal projectors through the first stage top view as shown in Fig. 10.21.

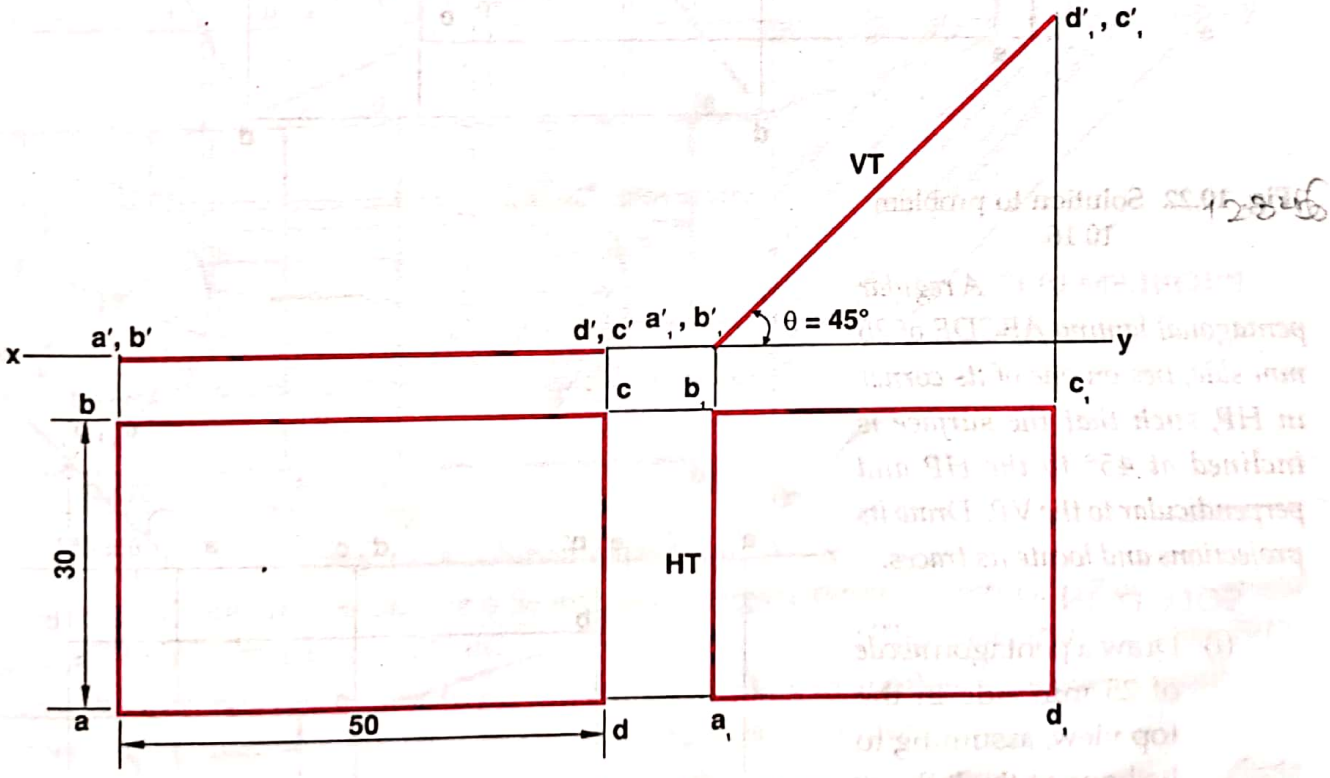


Fig. 10.21 Solution to problem 10.15

PROBLEM 10.16 *A regular pentagonal lamina ABCDE of 25 mm side has one side on the HP. Its surface is inclined at 45° to the HP and perpendicular to the VP. Draw its projections and show its traces.*

SOLUTION.

(*i*) Draw a pentagon *abcde* of 25 mm side in the top view with one side *ae* perpendicular to *xy*, assuming the lamina to be lying in the HP.

(*ii*) Project front view *a'b'c'd'e'* from top view to the *xy*.

(*iii*) Tilt the front view about the side *a'e'*, so that it makes 45° angle with *xy* and name the points on it by adding suffix 1 to them.

(*iv*) Project the corresponding top view $a_1b_1c_1d_1e_1$ and traces as shown in Fig. 10.22.

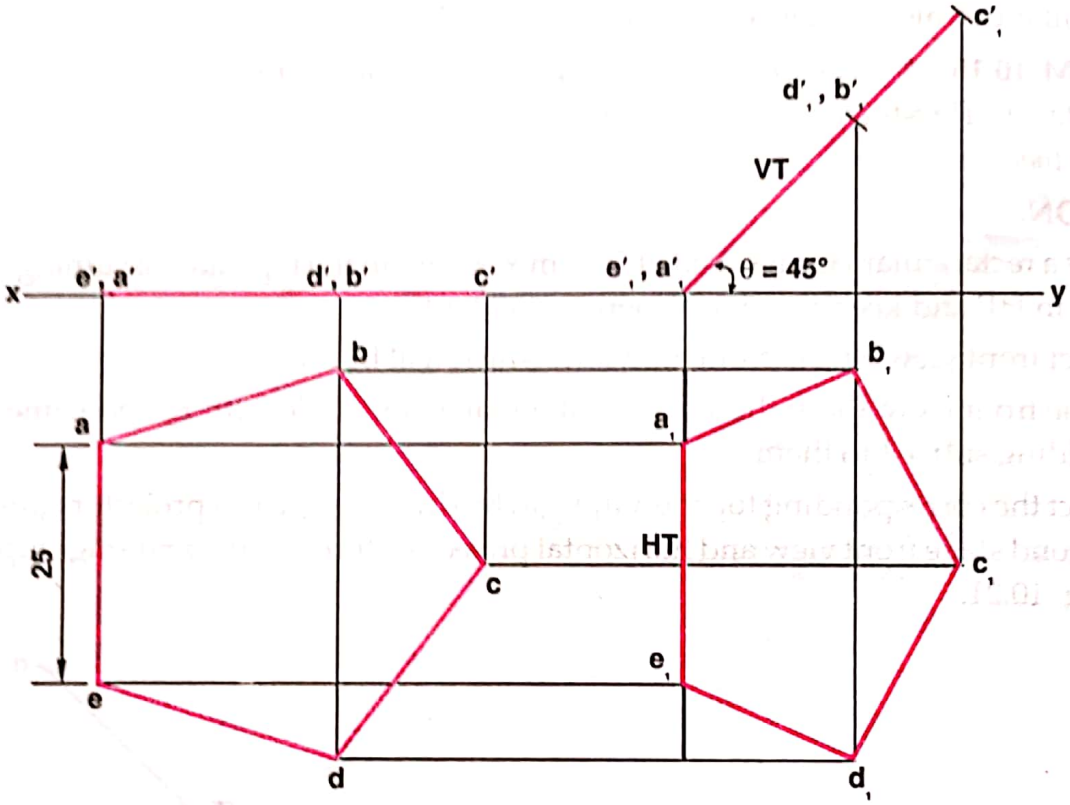


Fig. 10.22 Solution to problem 10.16

PROBLEM 10.17 *A regular pentagonal lamina ABCDE of 25 mm side, lies on one of its corner in HP, such that the surface is inclined at 45° to the HP and perpendicular to the VP. Draw its projections and locate its traces.*

SOLUTION.

(*i*) Draw a pentagon *abcde* of 25 mm side in the top view, assuming to be lying in the HP.

(*ii*) Project front view *a'b'c'd'e'* from top view to the *xy*.

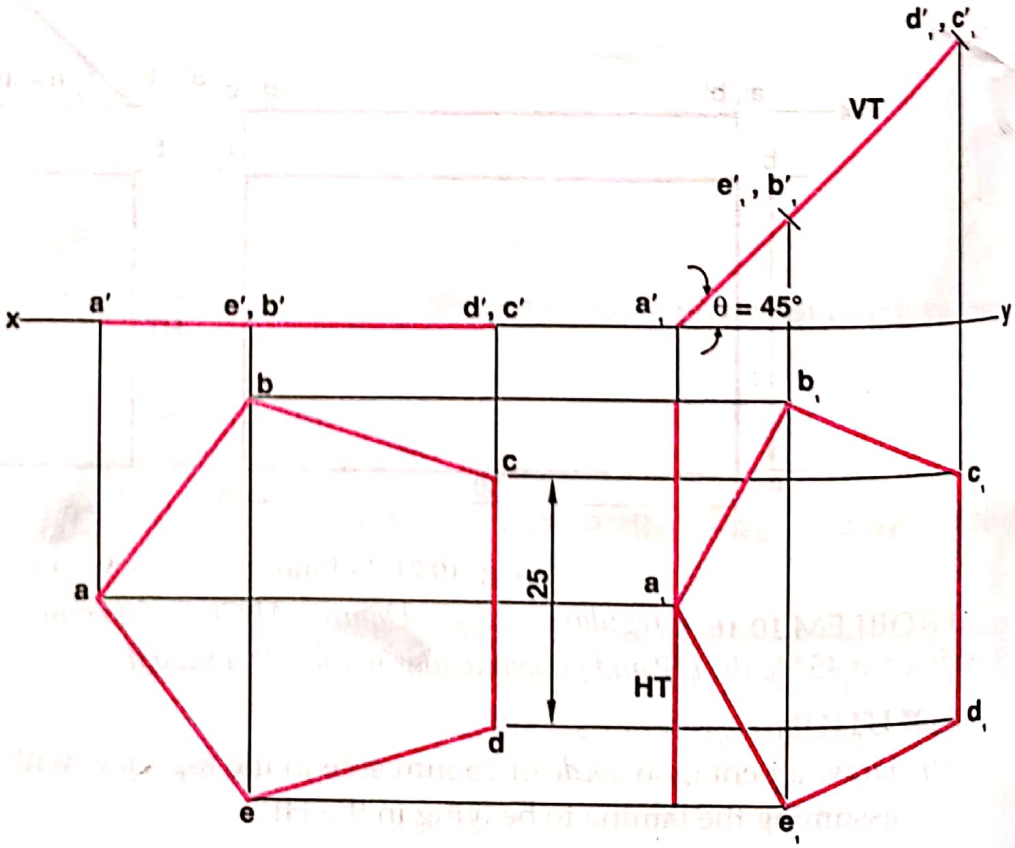


Fig. 10.23 Solution to problem 10.17

(iii) Tilt the front view about the corner a', so that it makes an angle of 45° to xy and name all the points by adding suffix 1 to them.

(iv) Project the corresponding top view and traces as shown in Fig. 10.23.

PROBLEM 10.18 *A square lamina of 70 mm sides has a circular hole of 40 mm diameter centrally placed. The lamina is resting on one of its sides on HP, this side being perpendicular to VP. The surface of the lamina is inclined at 30° to HP. Draw the projections of the lamina. Also draw its side view.*

(PTU, Jalandhar May 2001)

SOLUTION.

(i) Draw a square *abcd* of 70 mm side in the top view, keeping side *cd* perpendicular to xy.

(ii) Draw a hole of ϕ 40 mm in the centre of the square. Divide the circular hole into sixteen equal parts.

(iii) Project the corresponding front view for the square lamina along with the circular hole.

(iv) Tilt the front view at angle of 30° to xy.

(v) Project the corresponding front and side views as shown in Fig. 10.24.

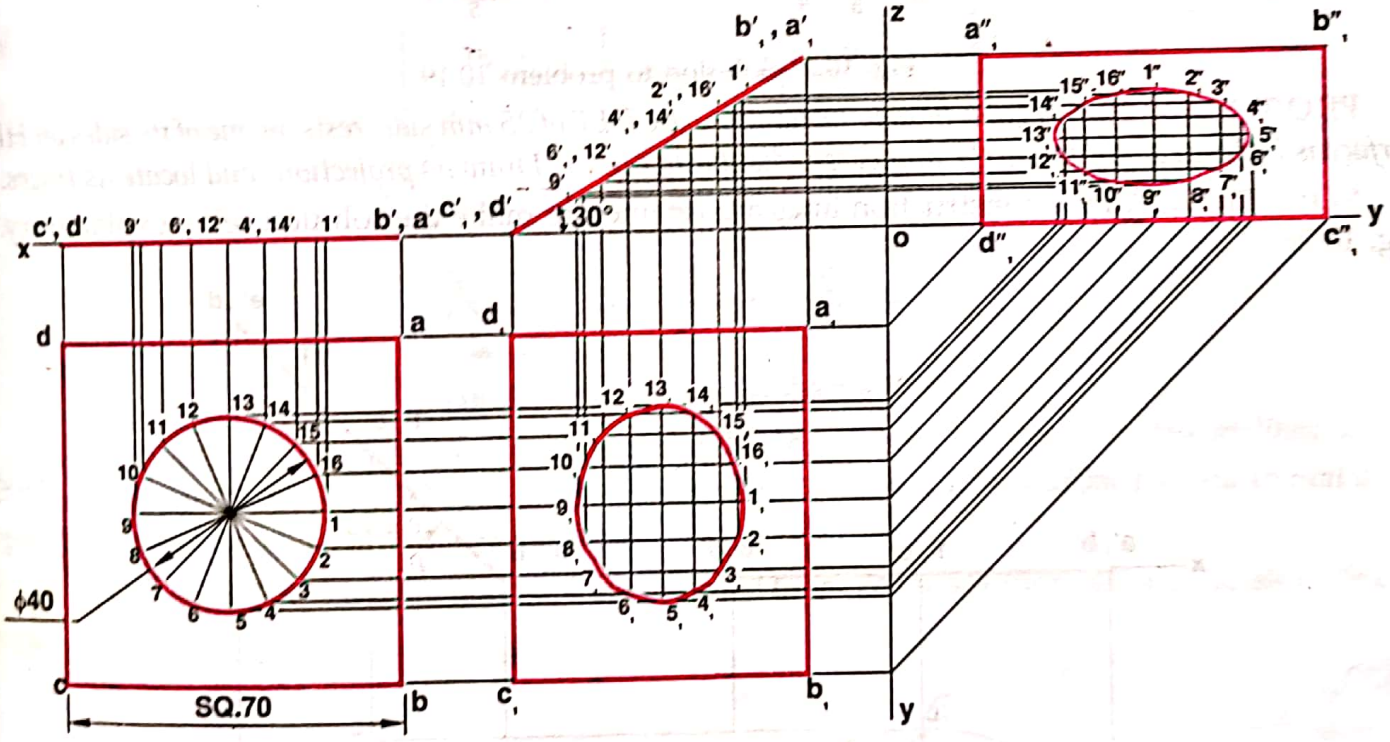


Fig. 10.24 Solution to problem 10.18

PROBLEM 10.19 *A this circular plate of ϕ 50 mm and negligible thickness rests on HP on its rim and makes an angle of 45° to HP. Draw its projections.* *(PTU, Jalandhar December 2015)*

SOLUTION. A circle has no corners to project from one view to another. However a number of points say sixteen (equal distances apart) may be marked on its circumference.

(i) Assume the circular plate to be lying in HP, draw its projections. The top view will be a circle of diameter 50 mm and front view will be an edge view on xy.

(ii) Divide the circumference of the circle into sixteen equal parts. Project these points in the front view.

(iii) Tilt the front view about the point 9′ and makes an angle of 45° to xy. Name all the points on it by adding suffix 1 to them.

(*iv*) Draw its corresponding top view. See Fig. 10.25.

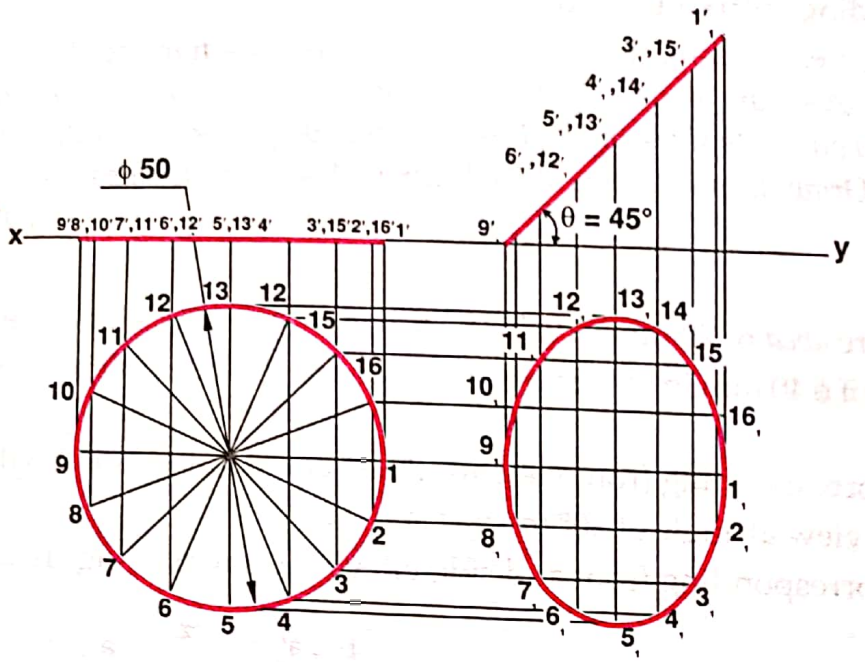


Fig. 10.25 Solution to problem 10.19

PROBLEM 10.20 *A regular hexagonal lamina ABCDEF of 25 mm side, rests on one of its sides on HP. Its surface is inclined at 45° to the HP and perpendicular to the VP. Draw its projections and locate its traces.*

SOLUTION. All the construction lines are retained to make the solution self – explanatory. See Fig. 10.26.

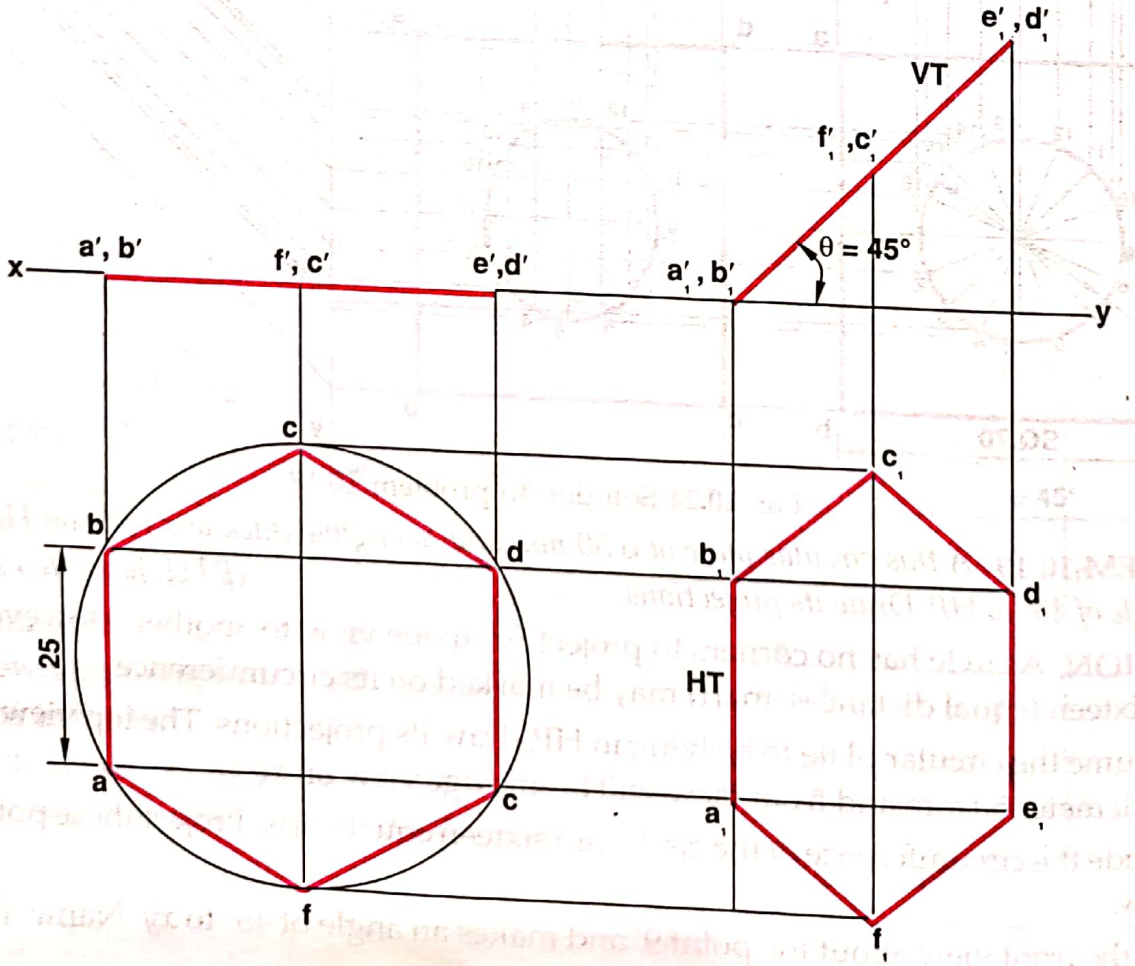


Fig. 10.26 Solution to problem 10.20

PROBLEM 10.21 *A regular pentagonal lamina ABCDEF of 25 mm side, lies on one of its corner in HP such that the surface is inclined at 45° to the HP and perpendicular to the VP. Draw its projections and locate its traces.*

SOLUTION. (*i*) Draw a hexagon abcdef of 25 mm side in the top view, assuming to be lying in the HP and keep the side bc or ef to be parallel to the xy line.

(*ii*) Project the front view a'b'c'd'e'f' from top view.

(*iii*) Tilt the front view about the corner a' so that it makes an angle of 45° to xy and name all the points by adding suffix 1 to them.

(*iv*) Project the corresponding top view and traces as shown is Fig. 10.27.

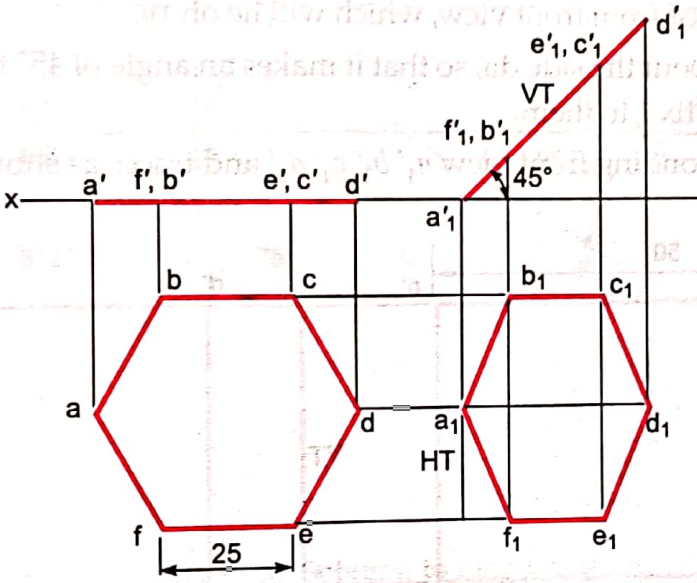


Fig. 10.27 Solution to problems 10.21

PROBLEM 10.22 *A regular pentagonal lamina ABCDE of 25 mm side has its surface inclined at 30° to the HP and perpendicular to the VP. One of its side is parallel to VP and 10 mm above HP and 15 mm infront of VP. Draw its projections and locate its traces too.*

SOLUTION. All the construction lines are retained to make the solution self–explanatory. See Fig. 10.28.

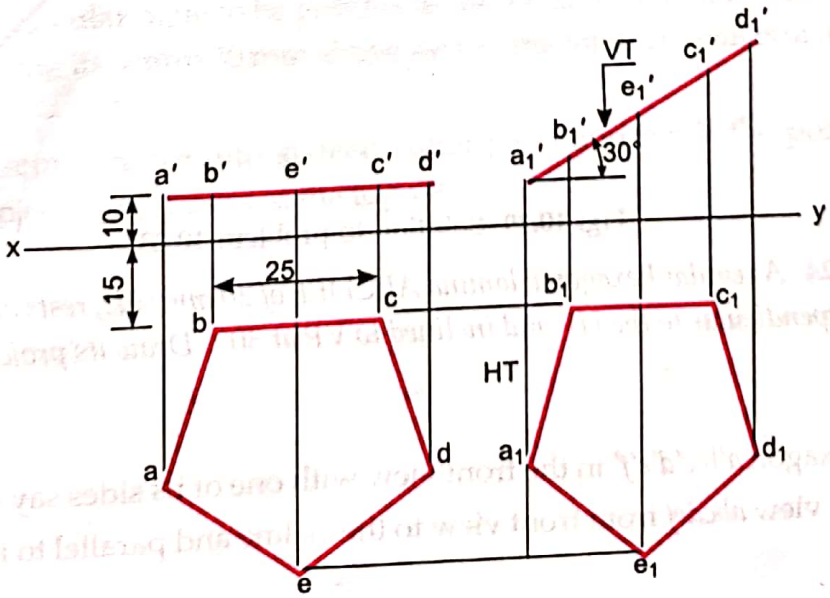


Fig. 10.28 Solution to problem 10.22

(*b*) **Plane, inclined to the VP and perpendicular to the HP.** When a plane is inclined to the VP and perpendicular to the HP, its projections are drawn into two stages. In the initial stage, it is assumed to be parallel to the VP and its front view will show the true shape and size in this position. The top view will be a line parallel to *xy*. In the second stage, tilt the top view so that it makes the given inclination to the *xy*. The new front view will be projected from it.

PROBLEM 10.23 *A rectangular lamina ABCD of 50 mm × 30 mm is inclined to the VP at 45° and perpendicular to HP. Its one of the sides say AD lies in VP. Draw its projections and locate its traces too.*

SOLUTION.

(*i*) Draw a rectangular lamina *a'b'c'd'* of 50 mm × 30 mm in the front view.

(*ii*) Project top view *abcd* from front view, which will lie on *xy*.

(*iii*) Tilt the top view about the side da, so that it makes an angle of 45° to *xy* and name the points on it by adding suffix 1 to them.

(*iv*) Project the corresponding front view $a_1'\ b_1'\ c_1'\ d_1'$ and traces as shown in Fig. 10.29.

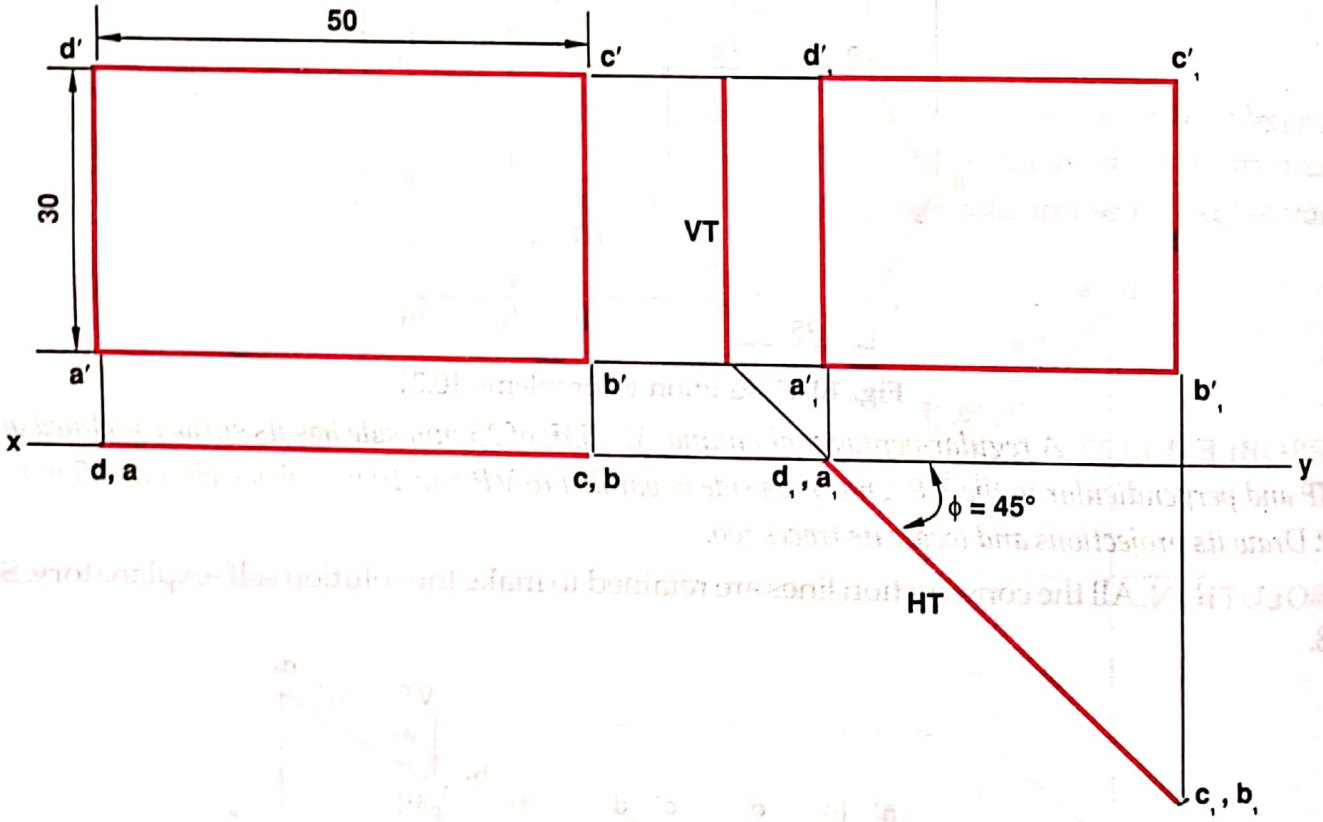


Fig. 10.29 Solution to problem 10.23

PROBLEM 10.24 *A regular hexagonal lamina ABCDEF of 20 mm side, rests on HP on one of its sides say BC such that it is perpendicular to the HP and inclined to VP at 30°. Draw its projections and locate its traces too.*

SOLUTION.

(*i*) Draw a hexagon *a'b'c'd'e'f'* in the front view with one of its sides say (*b'c'*) on the *xy*.

(*ii*) Project top view *abcdef* from front view to the *xy* line and parallel to it.

(*iii*) Tilt the top view so that it makes an angle of 30° to *xy* and name the points on it by adding suffix 1 to them.

(*iv*) Project the corresponding front view and traces as shown in Fig. 10.30.

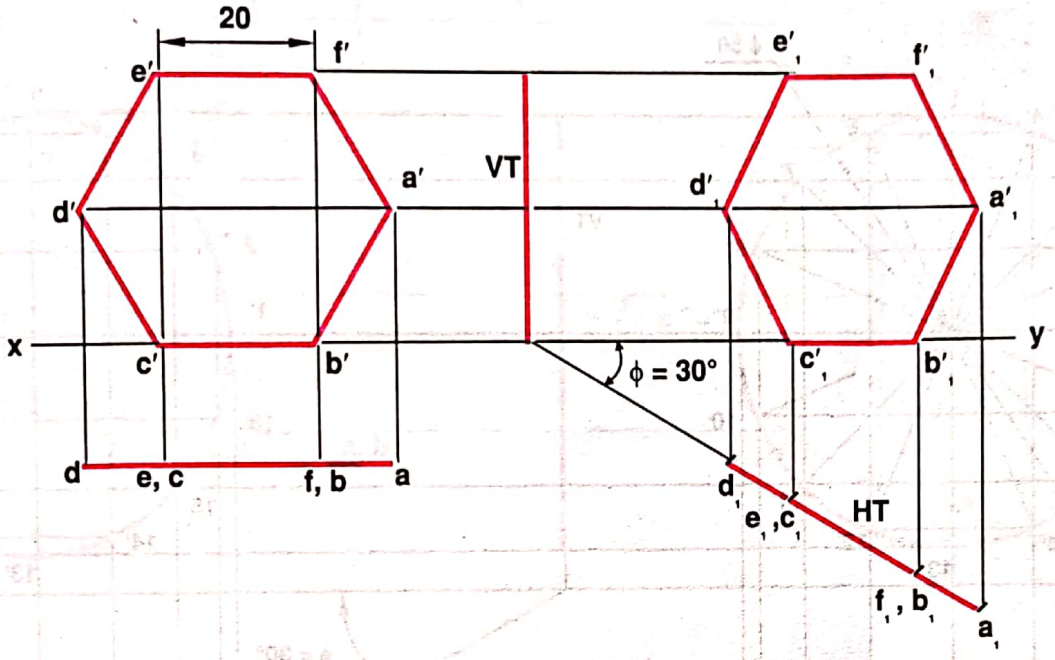


Fig. 10.30 Solution to problem 10.24

PROBLEM 10.25 *A thin circular plate of 50 mm diameter is held such that its plane is inclined at 30° to VP and perpendicular to HP. A point on its circumference is 20 mm infront to VP and 30 mm above HP. Draw its projections and locate its traces too.*

SOLUTION. A circle has no corners to project from one view to another. However a number of points say sixteen (equal distances apart) may be marked on its circumference.

(*i*) Assuming the circular plate to be parallel to the VP, draw its projections. The front view will be a circle, having its centre 30 mm above *xy* and the top view will be a line, parallel to and 20 mm below *xy*.

(*ii*) Divide the circumference into sixteen equal parts and mark the points as shown in the Fig. 10.31. Project these points in the top view.

(*iii*) Draw the top view in the new position, so that it makes an angle of 30° to *xy* and the circumference still 20 mm away from it. Name the points on it by adding suffix 1 to them.

(*iv*) Draw its corresponding front view and traces too. Join a freehand curve through the sixteen points $1_1'2_1'3_1'$ .... etc. This will be an ellipse.

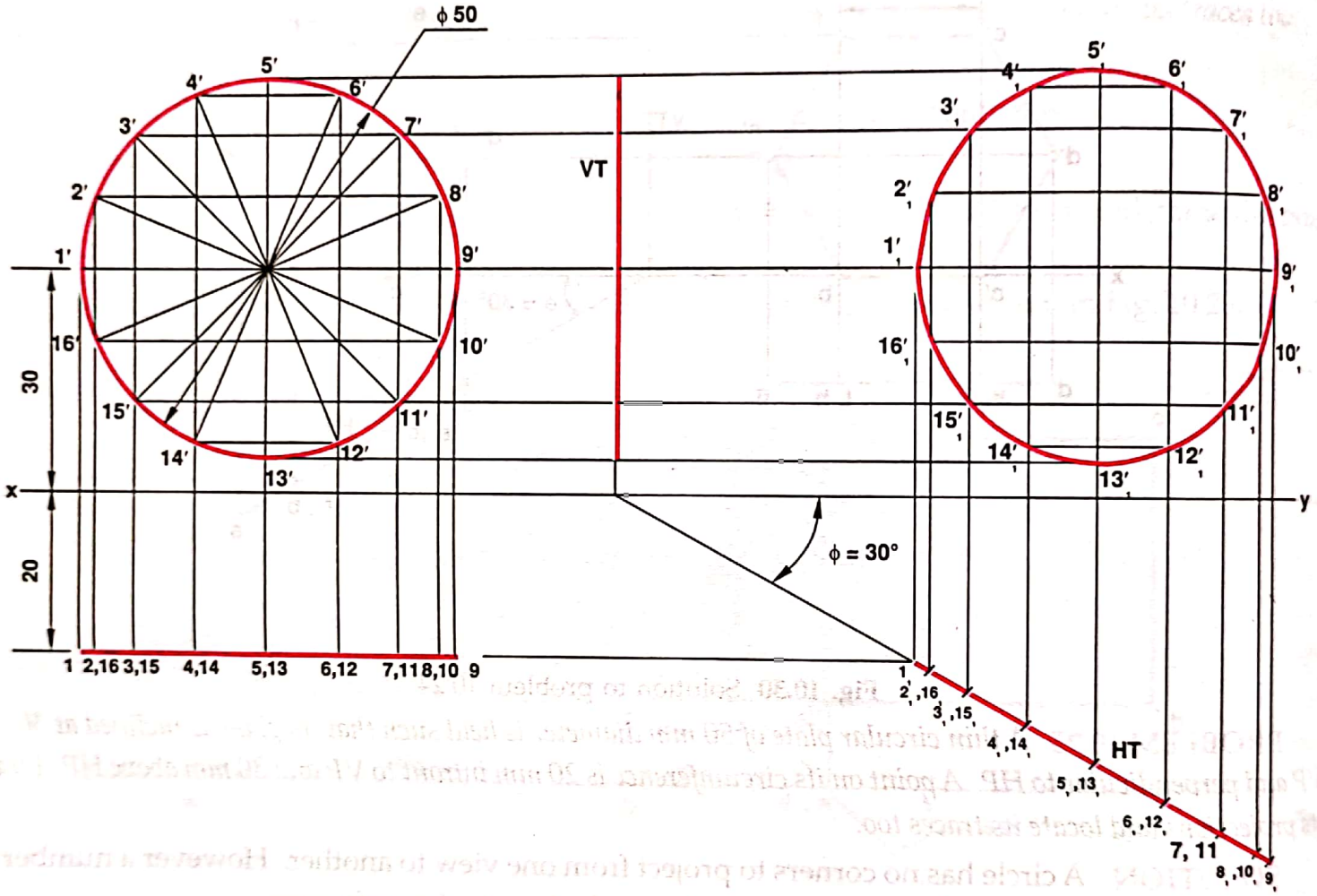


Fig. 10.31 Solution to problem 10.25

PROBLEM 10.26 *A regular hexagonal thin plate of 40 mm side has a circular hole of ϕ 40 mm diameter in its centre. It is resting on one of its corners in HP. Draw its projections when the plate surface is vertical, inclined at 30° to the VP and locate its traces too.*

SOLUTION.

(*i*) Draw a hexagon *a'b'c'd'e'f'* of 40 mm side in the front view, keeping one of its corner say *c'* in HP.

(*ii*) Draw a hole of ϕ 40 mm in the centre of the hexagon. Divide the circular hole into sixteen equal parts.

(*iii*) Project the corresponding top view for the hexagonal plate along with the circular hole.

(*iv*) Tilt the top view at angle of 30° to *xy*.

(*v*) Project the corresponding front view and traces as shown in Fig. 10.32.

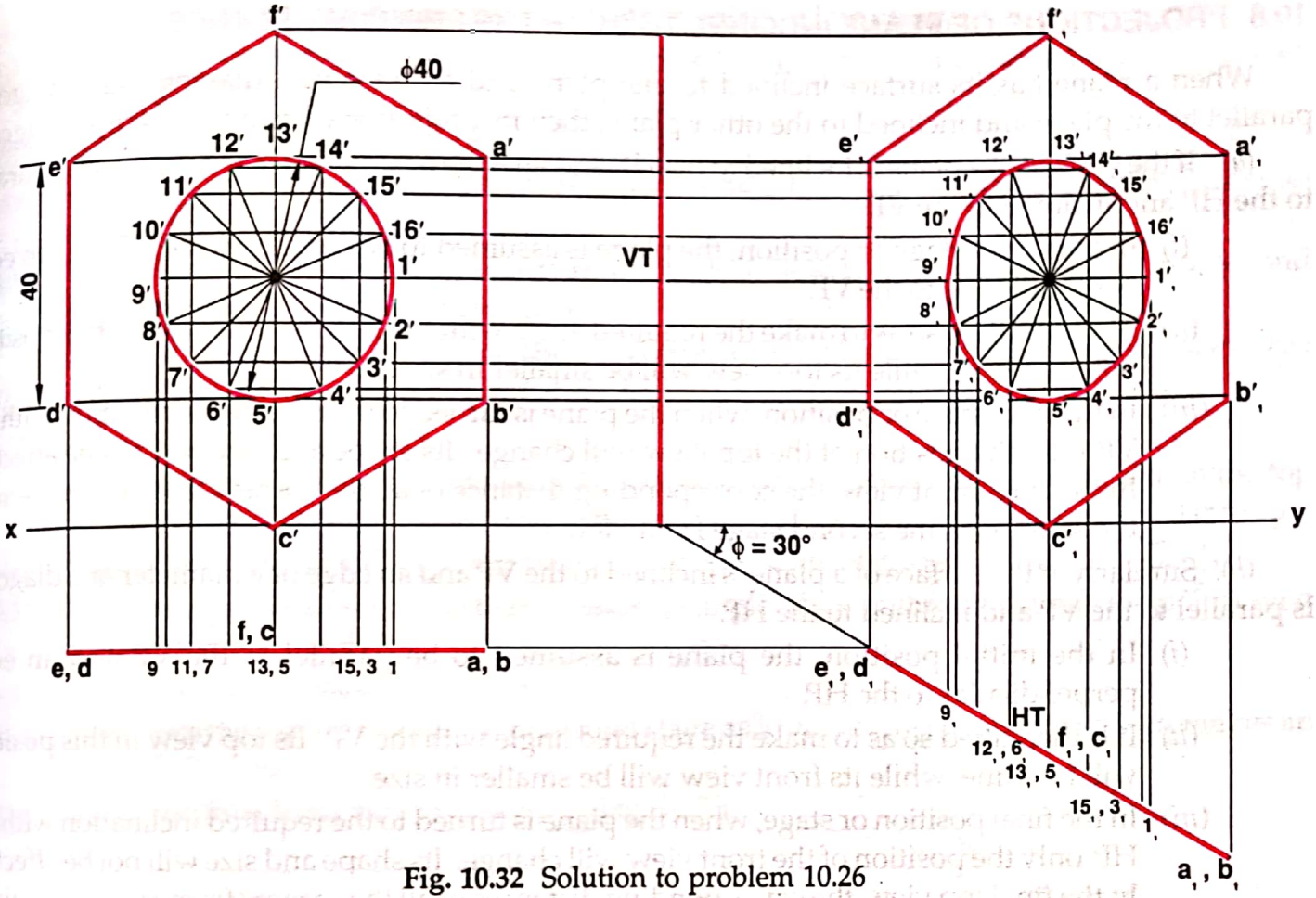


Fig. 10.32 Solution to problem 10.26

PROBLEM 10.27 *Draw the projections of a regular pentagonal lamina ABCDE of 25 mm side, having one of its sides AB in the VP and with its surface inclined at 60° to the VP. Locate its traces too.*

SOLUTION. Method of drawing projections have already been explained in previous problems. See Fig. 10.33.

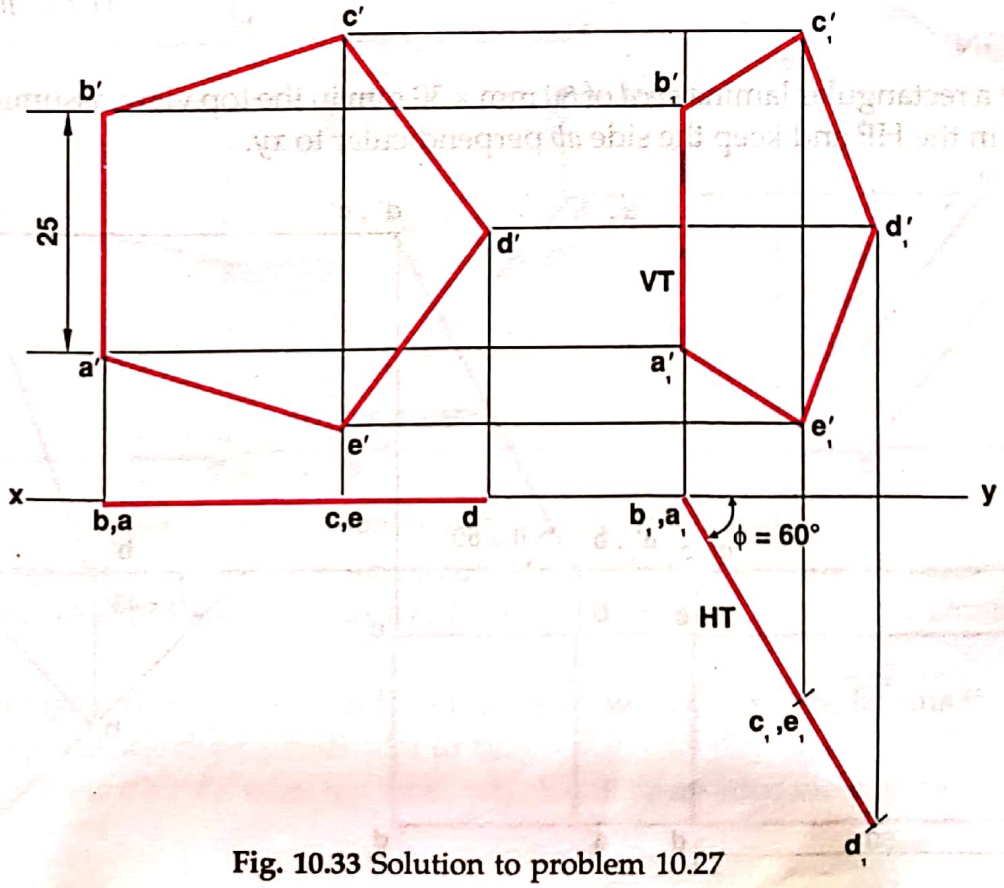


Fig. 10.33 Solution to problem 10.27

10.8 PROJECTIONS OF PLANE INCLINED TO BOTH THE PRINCIPAL PLANES

When a plane has its surface inclined to one plane and an edge or a diameter or a diagonal parallel to the plane and inclined to the other plane, then its projections are drawn in three stages.

(*a*) If the surface of a plane is inclined to the HP and an edge or a diameter or a diagonal is parallel to the HP and inclined to the VP.

(*i*) In the initial stage or position, the plane is assumed to be parallel to the HP and an edge perpendicular to the VP.

(*ii*) It is then tilted so as to make the required angle with the HP. Its front view in this position will be a line, while its top view will be smaller in size.

(*iii*) In the final stage or position, when the plane is turned to the required inclination with the VP, only the position of the top view will change. Its shape and size will not be affected. In the final front view, the corresponding distance of all the corners from *xy* will remain the same as in the second stage front view.

(*b*) Similarly, if the surface of a plane is inclined to the VP and an edge or a diameter or a diagonal is parallel to the VP and inclined to the HP.

(*i*) In the initial position, the plane is assumed to be parallel to the VP and an edge perpendicular to the HP.

(*ii*) It is then tilted so as to make the required angle with the VP. Its top view in this position will be a line, while its front view will be smaller in size.

(*iii*) In the final position or stage, when the plane is turned to the required inclination with the HP, only the position of the front view will change. Its shape and size will not be affected. In the final top view, the corresponding distances of all the corners from *xy* will remain as in the second stage top view.

PROBLEM 10.28 *A rectangular lamina ABCD of 60 mm × 30 mm, has its side AB in HP and inclined at 45° to VP and the plane of the lamina is inclined at 60° to the HP. Draw its projections.*

(PTU, Jalandhar May 2014)

SOLUTION.

(*i*) Draw a rectangular lamina *abcd* of 60 mm × 30 mm in the top view, assuming the lamina to be lying in the HP and keep the side *ab* perpendicular to *xy*.

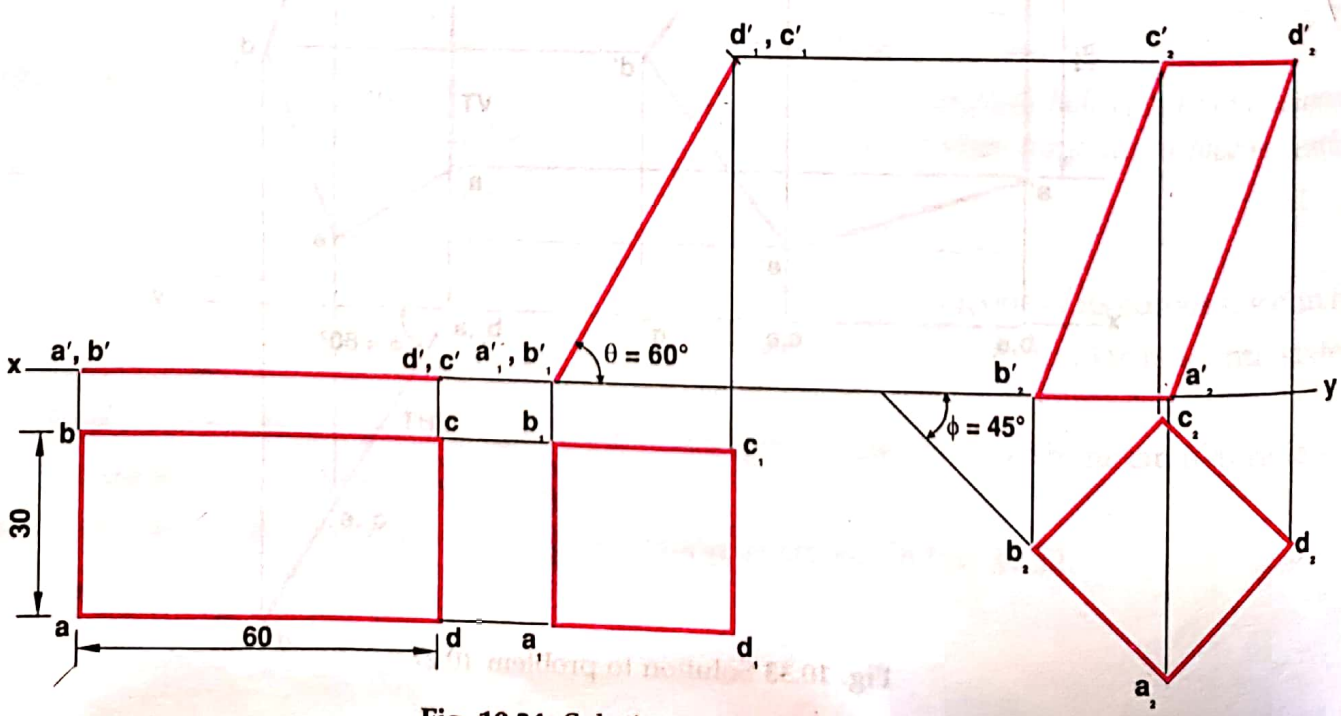


Fig. 10.34 Solution to problem 10.28

(ii) Project front view $a'b'c'd'$ from top view, which will be a line on xy.

(iii) Tilt the front view about the $a'b'$ so that it makes 60° angle with xy and name the points on it by adding suffix 1 to them.

(iv) Project the corresponding top view $a_1\, b_1\, c_1\, d_1$.

(v) Reproduce the top view $a_1\, b_1\, c_1\, d_1$ of second stage as $a_2\, b_2\, c_2\, d_2$ such that the side a_2b_2 makes an angle of 45° to xy.

(vi) Project the final front view upwards from this top view and horizontally from the second stage front view, as shown in Fig. 10.34.

PROBLEM 10.29 *A rectangular lamina ABCD of 60 mm ×30 mm has its side AB on ground and inclined at 60° to VP and plane of the lamina is inclined at 45° to the HP. Draw its projections in third angle.*

SOLUTION.

(i) Draw two lines xy and gl, a suitable distance apart. Draw a rectangular lamina *abcd* in the top view, assuming the lamina to be lying in the ground and keep the side *ab* perpendicular to xy.

(ii) Project front view $a'b'c'd'$ from top view, which will be an edge view on gl.

(iii) Tilt the front view about $a'b'$ so that it makes an angle of 45° with gl and name the points on it by adding suffix 1 to them.

(iv) Project the corresponding top view.

(v) Reproduce the top view $a_1\, b_1\, c_1\, d_1$ of second stage as $a_2\, b_2\, c_2\, d_2$ such that the side a_2b_2 makes an angle of 60° to xy.

(vi) Project the final front view as shown in Fig. 10.35.

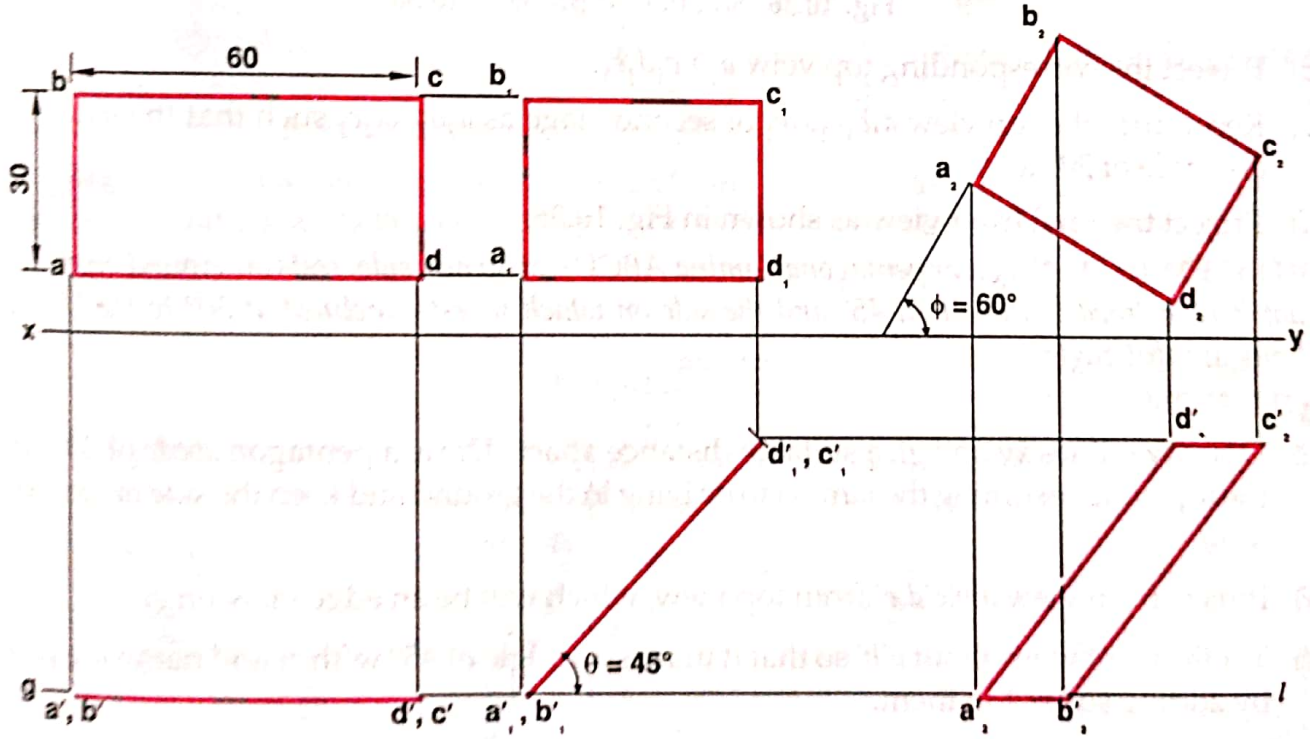


Fig. 10.35 Solution to problem 10.29 (third angle)

PROBLEM 10.30 *A regular pentagonal lamina ABCDE of 30 mm side, rests on HP on one of its sides such that it is inclined to the HP at 45° and the side on which it rests, inclined at 30° to the VP. Draw its projections.*

SOLUTION.

(i) Draw a pentagon *abcde* of 30 mm side in the top view, assuming the lamina to be lying in the HP and keep the side *ab* perpendicular to xy.

(ii) Project front view $a'b'c'd'e'$ from top view, which will be an edge view on xy.

(*iii*) Tilt the front view about $a'b'$ so that it makes 45° angle with xy and name the points on it by adding suffix 1 to them.

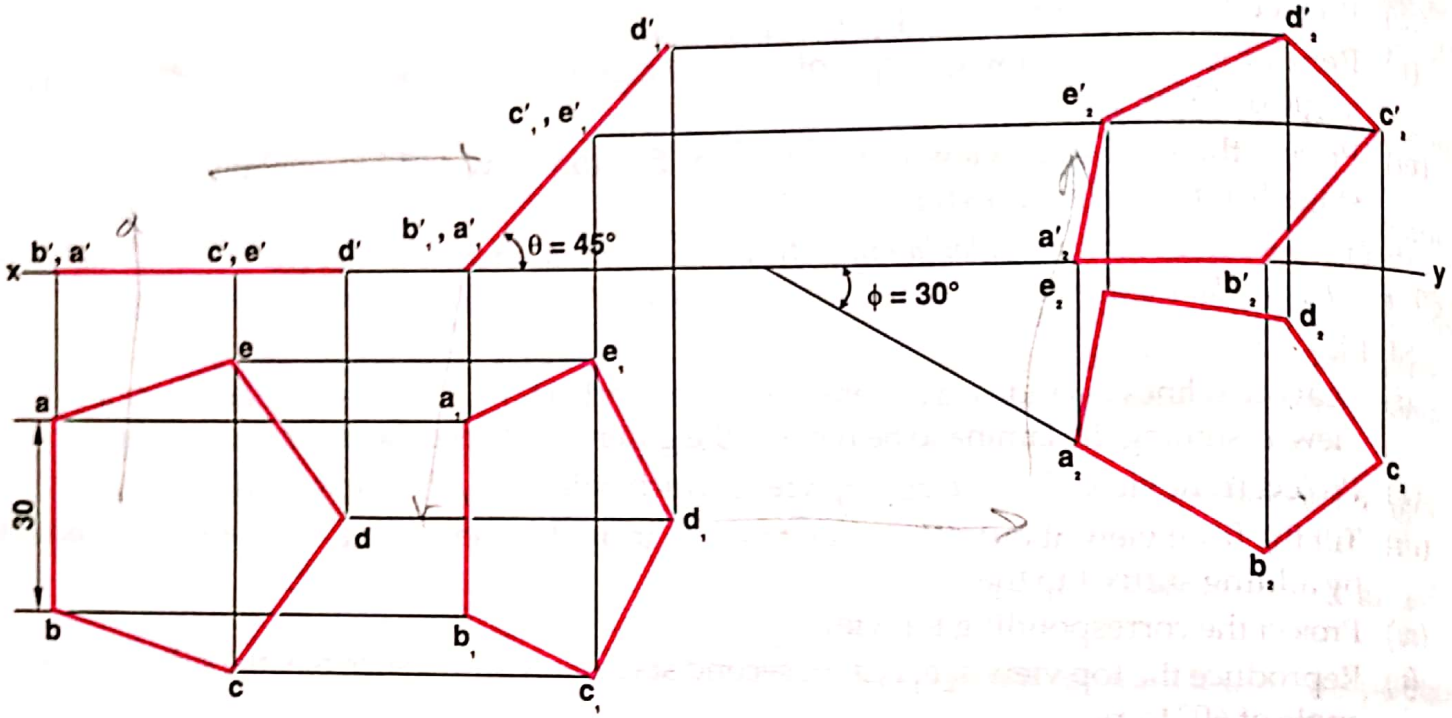


Fig. 10.36 Solution to problem 10.30

(*iv*) Project the corresponding top veiw $a_1b_1c_1d_1e_1$.

(*v*) Reproduce the top view $a_1b_1c_1d_1e_1$ of second stage as $a_2b_2c_2d_2e_2$ such that the side a_2b_2 makes an angle of 30° to xy.

(*vi*) Project the final front view as shown in Fig. 10.36.

PROBLEM 10.31 *A regular pentagonal lamina ABCDE of 30 mm side, rests on ground on one of its sides such that it is inclined to the HP at 45° and the side on which it rests, inclined at 30° to the VP. Draw its projections in third angle.*

SOLUTION.

(*i*) Draw two lines xy and gl, a suitable distance apart. Draw a pentagon $abcde$ of 30 mm side in the top view, assuming the lamina to be lying in the ground and keep the side an perpendicular to xy.

(*ii*) Project front view $a'b'c'd'e'$ from top view, which will be an edge view on gl.

(*iii*) Tilt the front view about $a'b'$ so that it makes an angle of 45° with gl and name the points on it by adding suffix 1 to them.

(*iv*) Project the corresponding top view.

(*v*) Reproduce the top view $a_1b_1c_1d_1e_1$ of second stage as $a_2b_2c_2d_2e_2$ such that the side a_2b_2 makes an angle of 30° to xy.

(*vi*) Project the final front view as shown in Fig. 10.37.

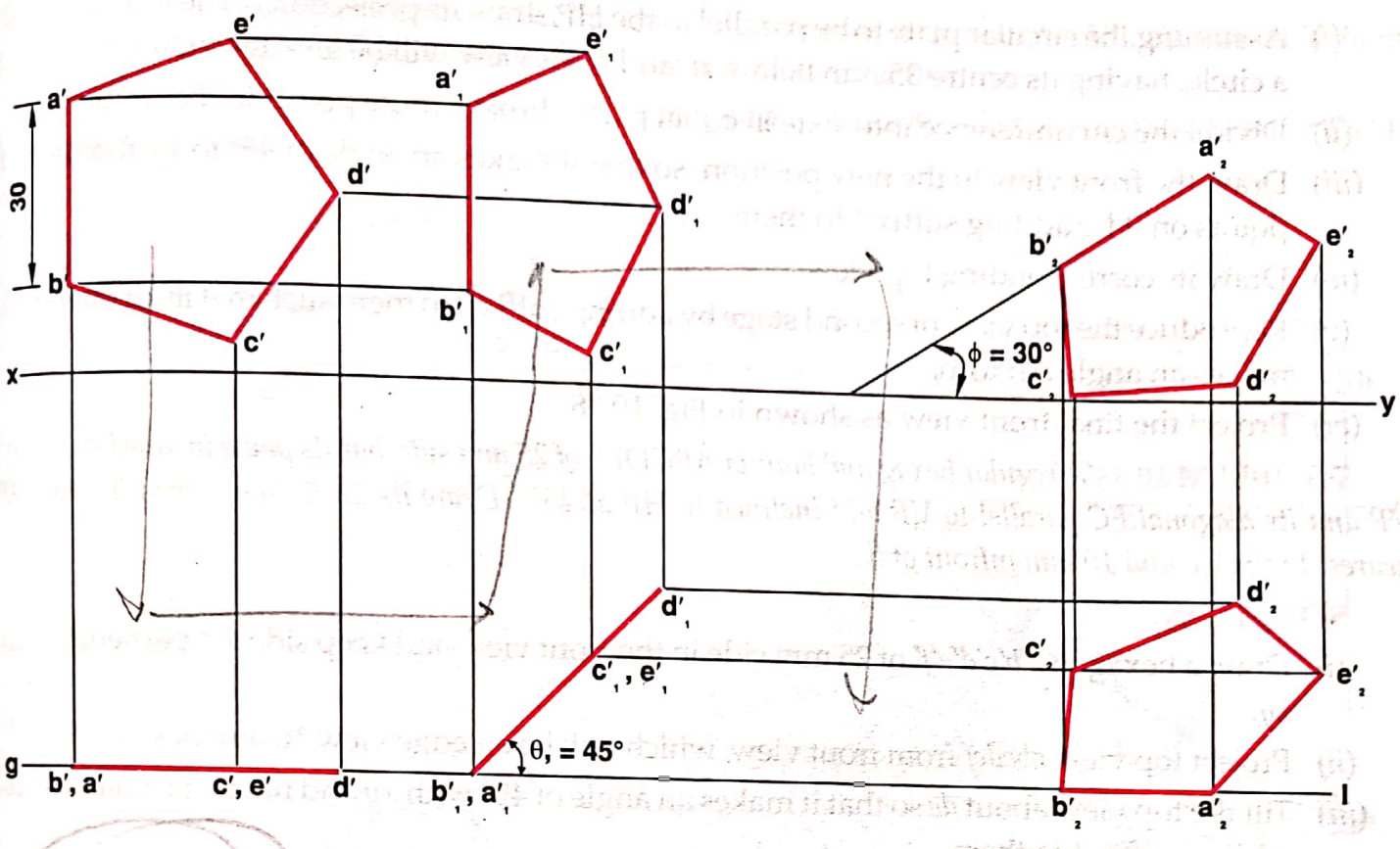


Fig. 10.37 Solution to problem 10.31 (third angle)

PROBLEM 10.32 *A thin circular plate of ϕ 50 mm and negligible thickness rests on HP on its rim and makes an angle of 45° to it. One of its diameters is inclined to VP at 30°. Draw its projections keeping distance of the centre of the circular plate 35 mm infront of the VP.*

SOLUTION. A circle has no corners to project from one view to another. However a number of points say sixteen (at equal distances apart) may be marked on its circumference.

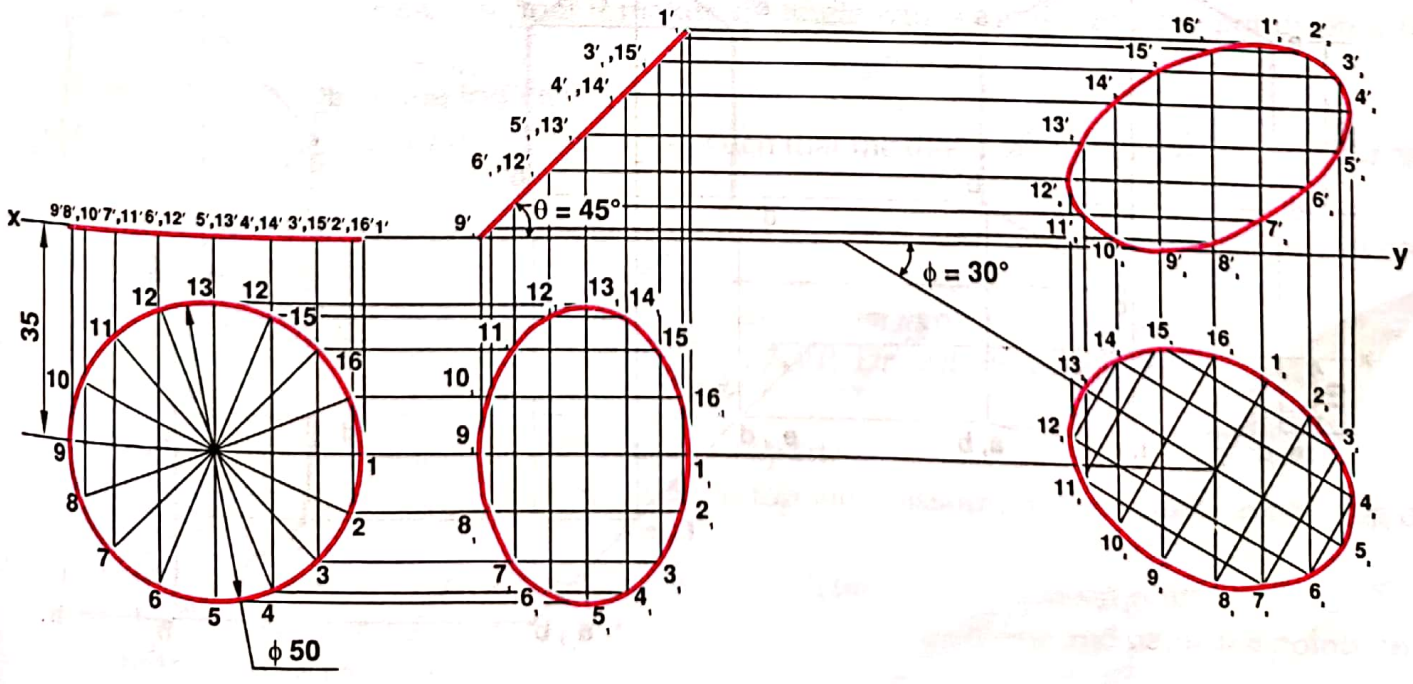


Fig. 10.38 Solution to problem 10.32

(*i*) Assuming the circular plate to be parallel to the HP, draw its projections. The top view will be a circle, having its centre 35 mm below *xy* and front view will be an edge view on *xy*.

(*ii*) Divide the circumference into sixteen equal parts. Project these points in the front view.

(*iii*) Draw the front view in the new position, so that it makes an angle of 45° to *xy* and name the points on it by adding suffix 1 to them.

(*iv*) Draw its corresponding top view.

(*v*) Reproduce the top view of second stage by adding suffix 2 to them such that its diameter $13\frac{2}{2}$ makes an angle 30° to *xy*.

(*vi*) Project the final front view as shown in Fig. 10.38.

PROBLEM 10.33 *A regular hexagonal lamina ABCDEF of 25 mm side, has its plane inclined at 45° to the VP and its diagonal EC parallel to VP and inclined to HP at 45°. Draw its projections when its side DE is nearest to the VP and 10 mm infront of it.*

SOLUTION.

(*i*) Draw a hexagon *a'b'c'd'e'f'* of 25 mm side in the front view and keep side *d'e'* perpendicular to *xy*.

(*ii*) Project top view *abcdef* from front view, which will be an edge view 10 mm below *xy*.

(*iii*) Tilt the top view about *de* so that it makes an angle of 45° with *xy* and name the points on it by adding suffix 1 to them.

(*iv*) Projecting the corresponding front view $a'_1b'_1c'_1d'_1e'_1f'_1$.

(*v*) Reproduce the front view $a'_1b'_1c'_1d'_1e'_1f'_1$ of second stage as $a'_2b'_2c'_2d'_2e'_2f'_2$ such that the diagonal $c'_2f'_2$ makes an angle of 45° to *xy*.

(*vi*) Project the final top view as shown in Fig. 10.39.

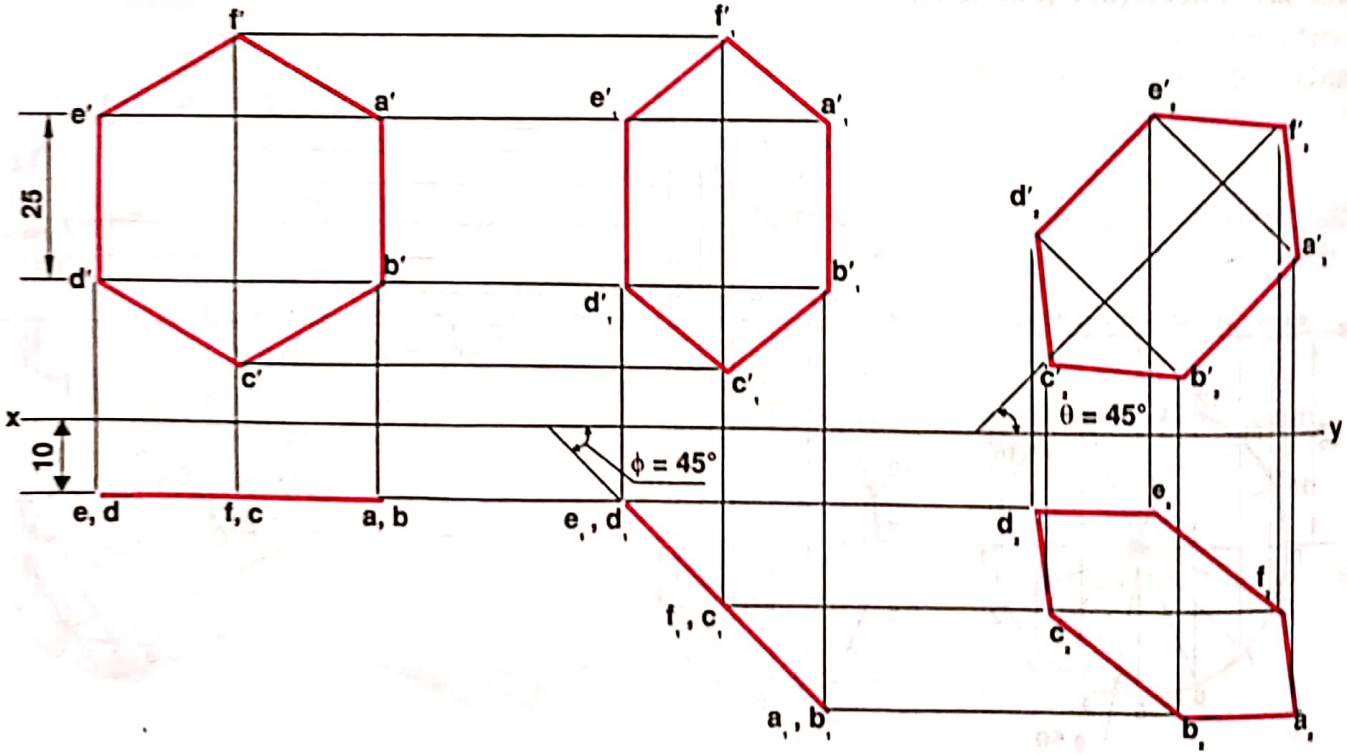


Fig. 10.39 Solution to problem 10.33

PROBLEM 10.34 *A square lamina ABCD of 25 mm side, rests on its corner C in HP. Its plane is inclined at 45° to the HP and diagonal DB inclined at 30° to the VP. Draw its projections.*

(PTU, Jalandhar May 2006, December 2007, December 2014)

SOLUTION.

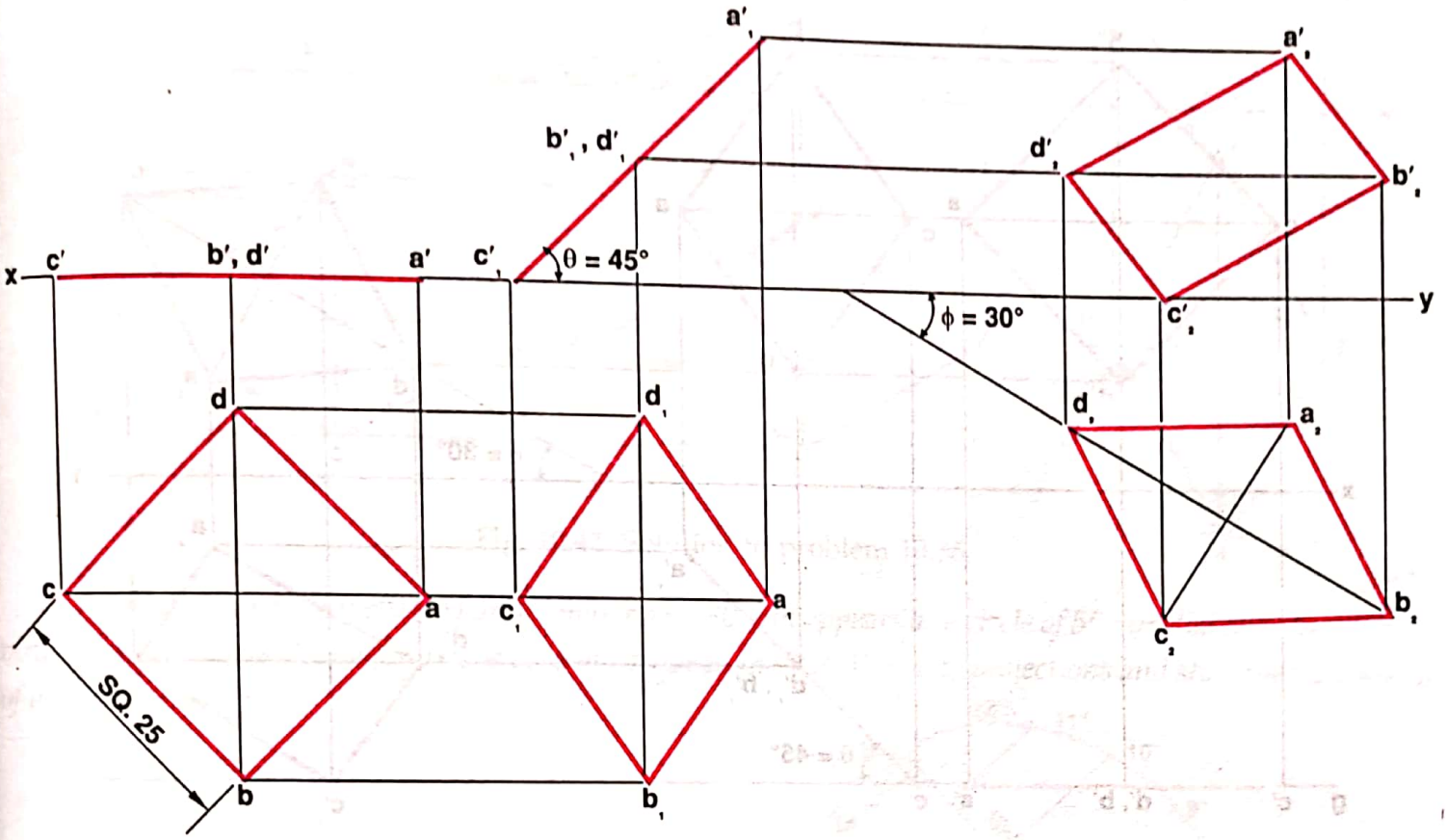


Fig. 10.40 Solution to problem 10.34

(*i*) Draw a square lamina *abcd* of 25 mm side in top view, assuming the lamina to be lying in the HP.

(*ii*) Project front view *a'b'c'd'* from top view, which will be a line on *xy*.

(*iii*) Tilt the front view about *c'* so that it makes 45° angle with *xy* and name the points on it by adding suffix 1 to them.

(*iv*) Project the corresponding top view $a_1b_1c_1d_1$.

(*v*) Reproduce the top view $a_1b_1c_1d_1$ as $a_2b_2c_2d_2$ such that the diagonal d_2b_2 makes an angle of 30° to *xy*.

(*vi*) Project the final front view upwards from this top view and horizontally from the second stage front view as shown in Fig. 10.40.

PROBLEM 10.35 *A square lamina ABCD of 25 mm side, rests on its corner C on ground plane. Its plane is inclined at 45° to the HP and diagonal DB inclined at 30° to the VP. Draw its projections in third angle.*

SOLUTION.

(*i*) Draw two lines *xy* and *gl*, a suitable distance apart.

(*ii*) Draw a square lamina *abcd* of 25 mm side in top view, assuming the lamina to be lying on ground plane.

(*iii*) Project front view *a'b'c'd'* from top view, which will be an edge view on *gl* line.

(*iv*) Tilt the front view about *c'*, so that it makes angle of 45° with *gl* line and name the points on it by adding suffix 1 to them.

(*v*) Project the corresponding top view $a_1b_1c_1d_1$.

(*vi*) Reproduce the top view $a_1b_1c_1d_1$ of second stage as $a_2b_2c_2d_2$ such that the diagonal d_2b_2 is inclined at 30° to *xy*.

(*vii*) Project the final front view as shown in Fig. 10.41.

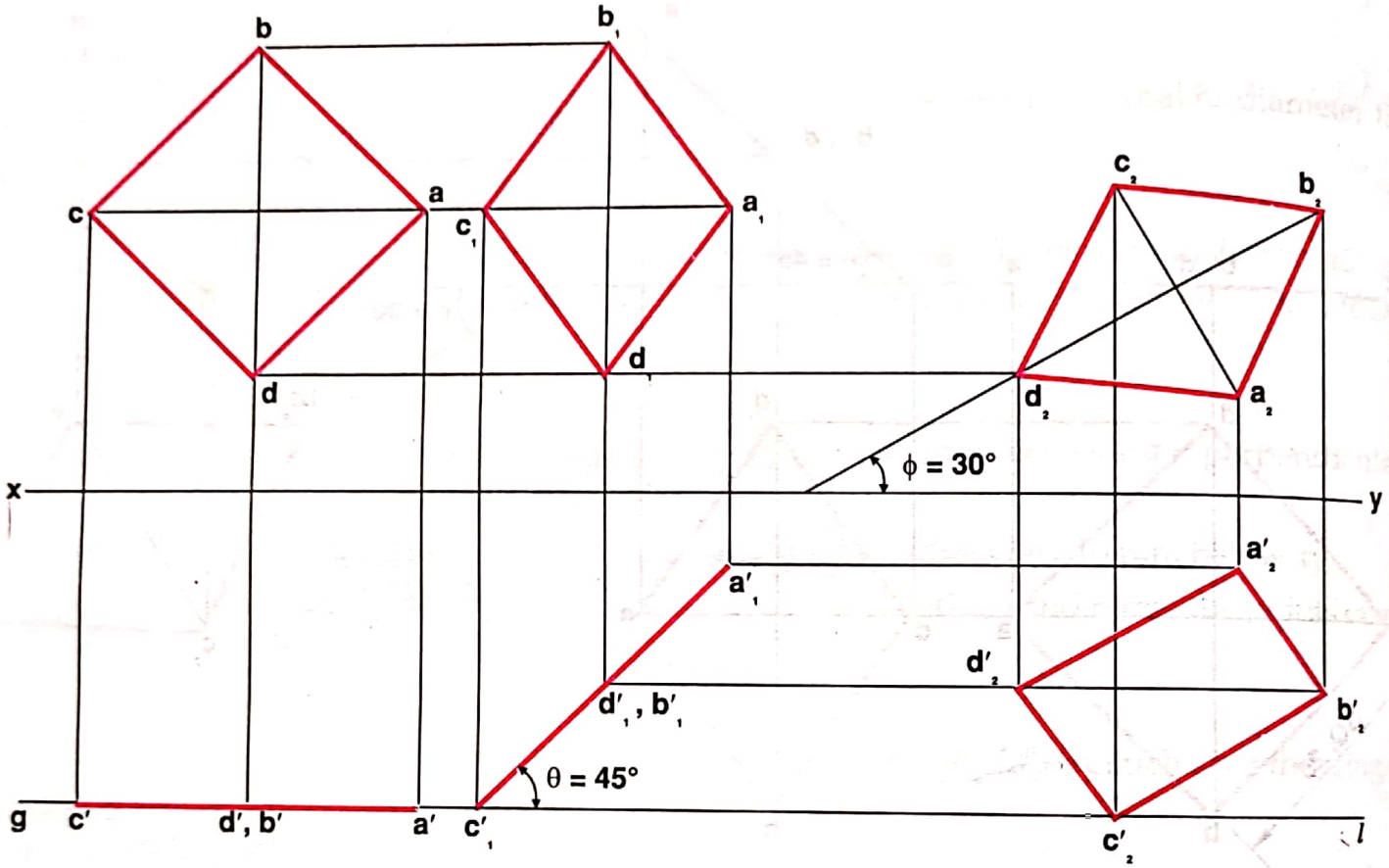


Fig. 10.41 Solution to problem 10.35 (third angle)

PROBLEM 10.36 *Draw the projections of a rhombus having diagonals 100 mm and 50 mm long. The bigger diagonal is inclined at 30° to the HP with one of the end point in HP and the smaller diagonal is parallel to both the planes.* *(PTU, Jalandhar May 2013)*

SOLUTION.

(*i*) Draw the top view *abcd* of the rhombus, such that its bigger diagonal is parallel to the *xy* line.

(*ii*) Project the corresponding front view *a'b'c'd'* in *xy*.

(*iii*) Tilt the front view in such a way that the end point *c'* in the HP and the bigger diagonal *c'd'* makes an angle of 30° to the HP and name the points on it by adding suffix 1 to them.

(*iv*) Project the corresponding top view $a_1b_1c_1d_1$.

(*v*) Reproduce the top view such that b_1d_1 is parallel to both the planes (*i.e.*, *xy* line) and name the points on it by adding suffix 2 to them.

(*vi*) Project the final front view upwards from this top view and horizontally from the second stage front view, as shown in Fig. 10.42.

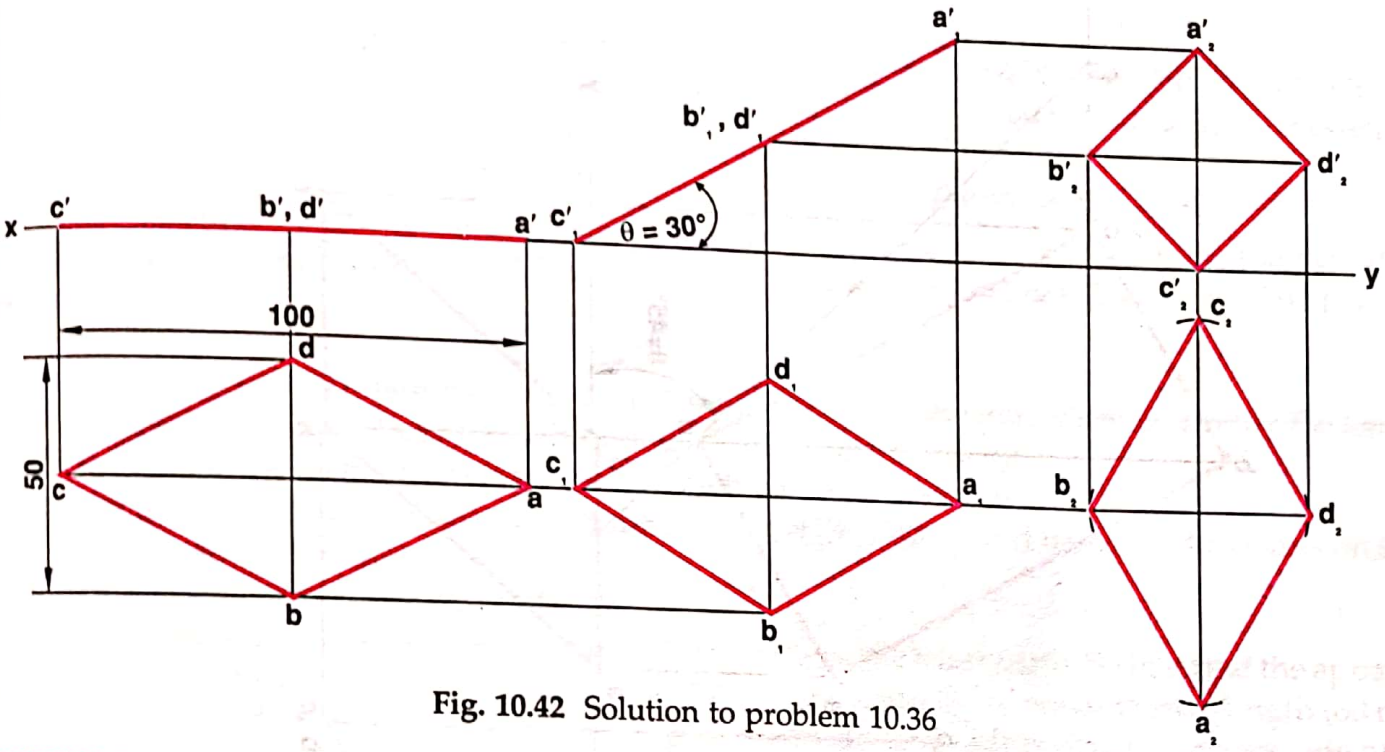


Fig. 10.42 Solution to problem 10.36

PROBLEM 10.37 *An ellipse having major axis 80 mm appears as a circle of 55 mm diameter in the front view. The top view of the major axis is perpendicular to the VP. Draw its projections and state the inclination of its surface with HP.*

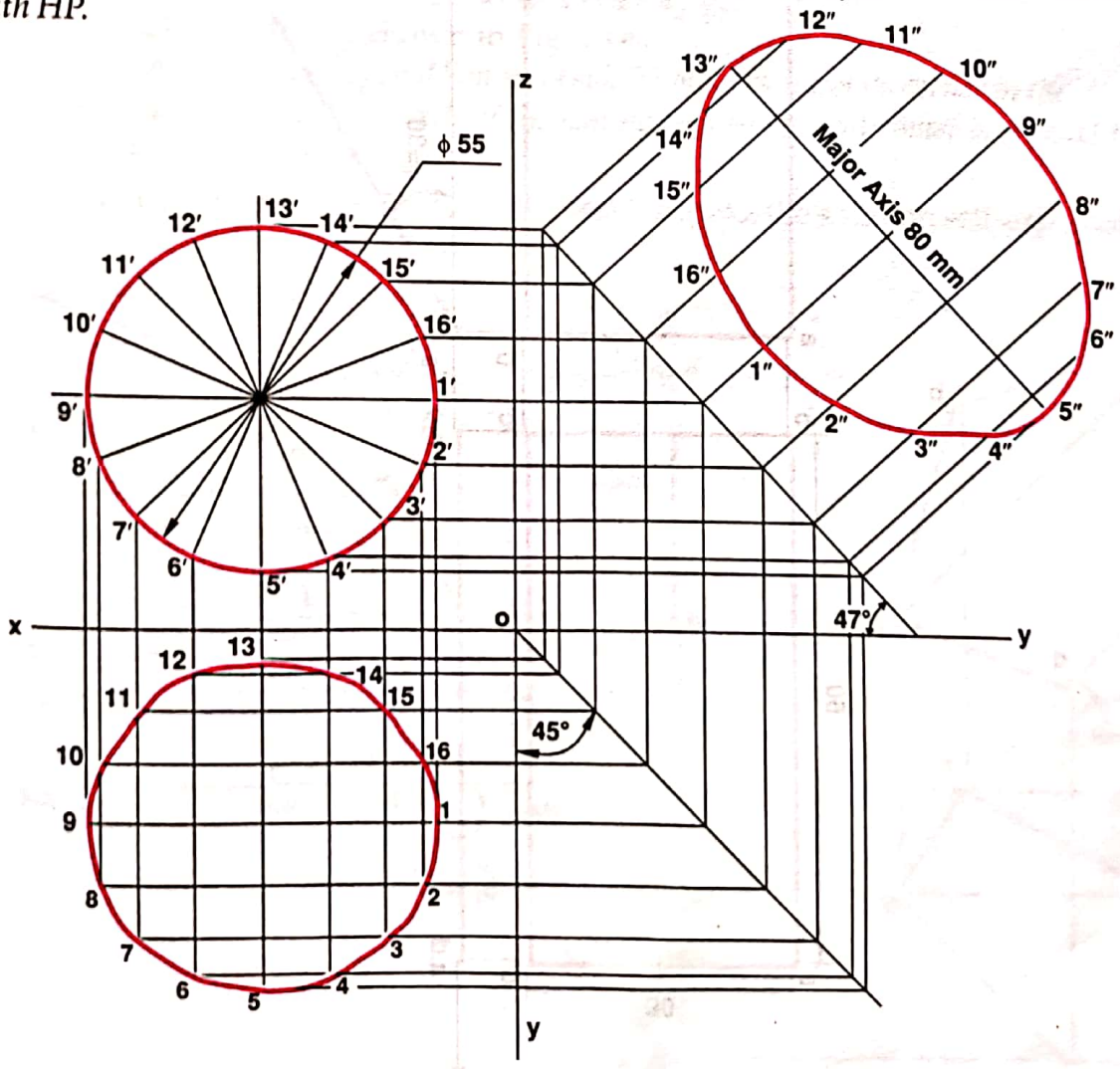


Fig. 10.43 Solution to problem 10.37

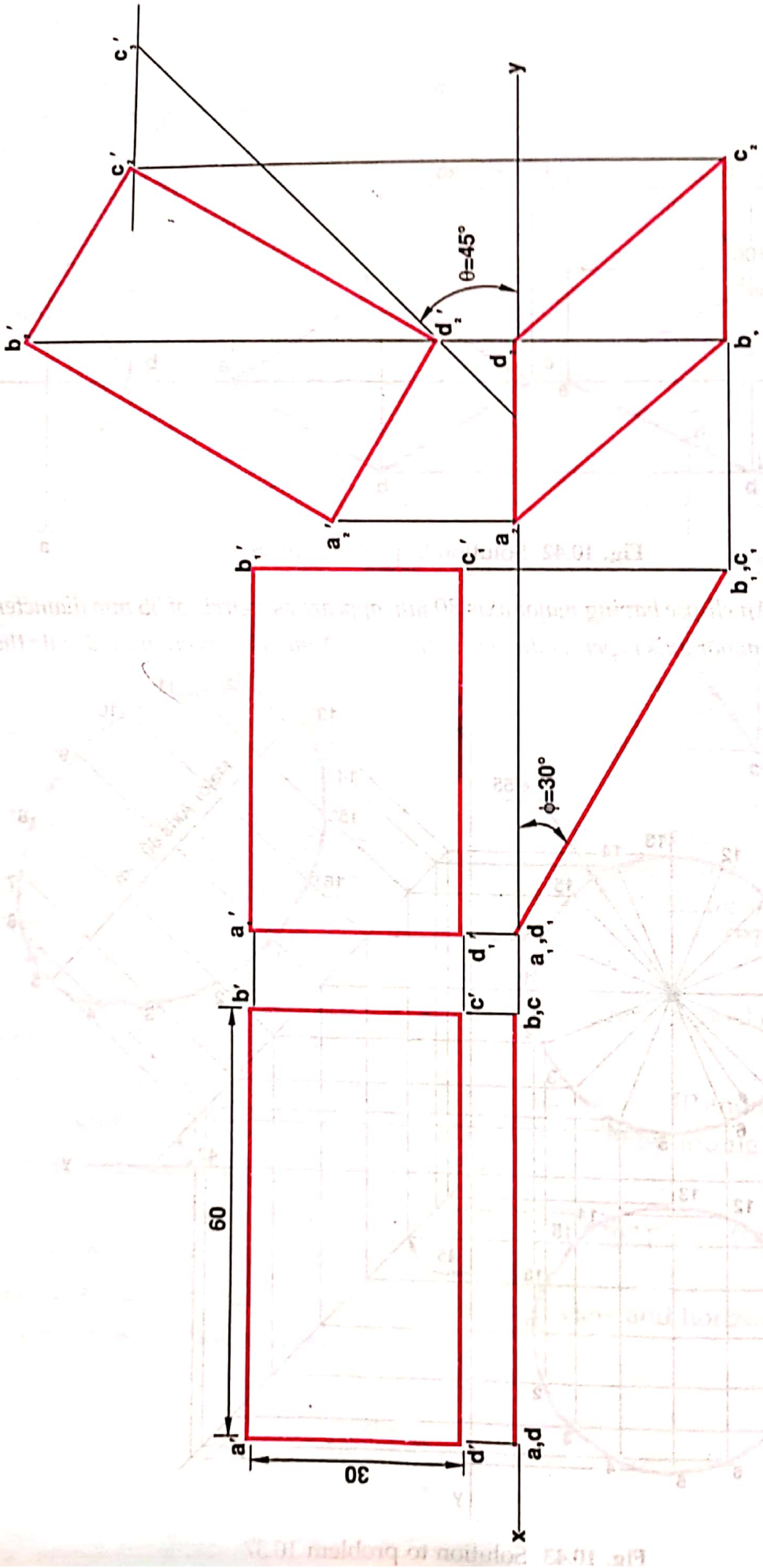


Fig. 10.44 Solution to problem 10.38

SOLUTION.

(*i*) Draw the front view of a circle of 55 mm diameter and divide it into sixteen equal parts.
(*ii*) Draw the side view by taking any point 13″ and cut 13″–5″ equal to 80 mm and measure the inclination of the line 13″–5″ with *xy* line.
(*iii*) Complete the top view from front view and side view as shown in Fig. 10.43.

PROBLEM 10.38 *A rectangular lamina ABCD of 60 mm × 30 mm rests on its shorter side in the VP and the surface makes 30° angle with the VP. The longer side of the plane is inclined at 45° to the HP. Draw its projections.*

SOLUTION.

(*i*) Draw a rectangular lamina *a′ b′ c′ d′* of 60 mm × 30 mm in the front view, assuming the lamina to be lying in the *VP* and keep the side *a′ d′* perpendicular to *xy*.
(*ii*) Project top view *abcd* from front view, which will be a line on *xy*.
(*iii*) Tilt the top view about *ad* so that it makes 30° angle with *xy* and name all the points on it by adding suffix 1 to them.
(*iv*) Project the corresponding front view.
(*v*) The front view of longer side of lamina does not give the true length. So first find the apparent angle at which the longer side to be inclined. Draw a line $d_2' c_3'$ equal to true length (60 mm) and inclined at 45° to *xy*. Draw an *arc* with centre d_2' and radius equal to longer side of the second stage ($d_1' c_1'$), to interrect the horizontal line at c_2'. This is the apparent angle of inclination and is greater than 45°.
(*vi*) Reproduce the front view along the longer edge $d_2' c_2'$.
(*vii*) Project the final top view as shown in Fig. 10.44.

PROBLEM 10.39 *A regular pentagonal lamina of side 25 mm has one of its corner in the VP and the plane of lamina in inclined at 45° to the VP. The side of the lamina opposite to that comer is parallel to the VP and inclined at 45° to the HP. Draw its projections.*

SOLUTION. All the construction lines are retained to make the solution self-explanatory. See Fig. 10.45.

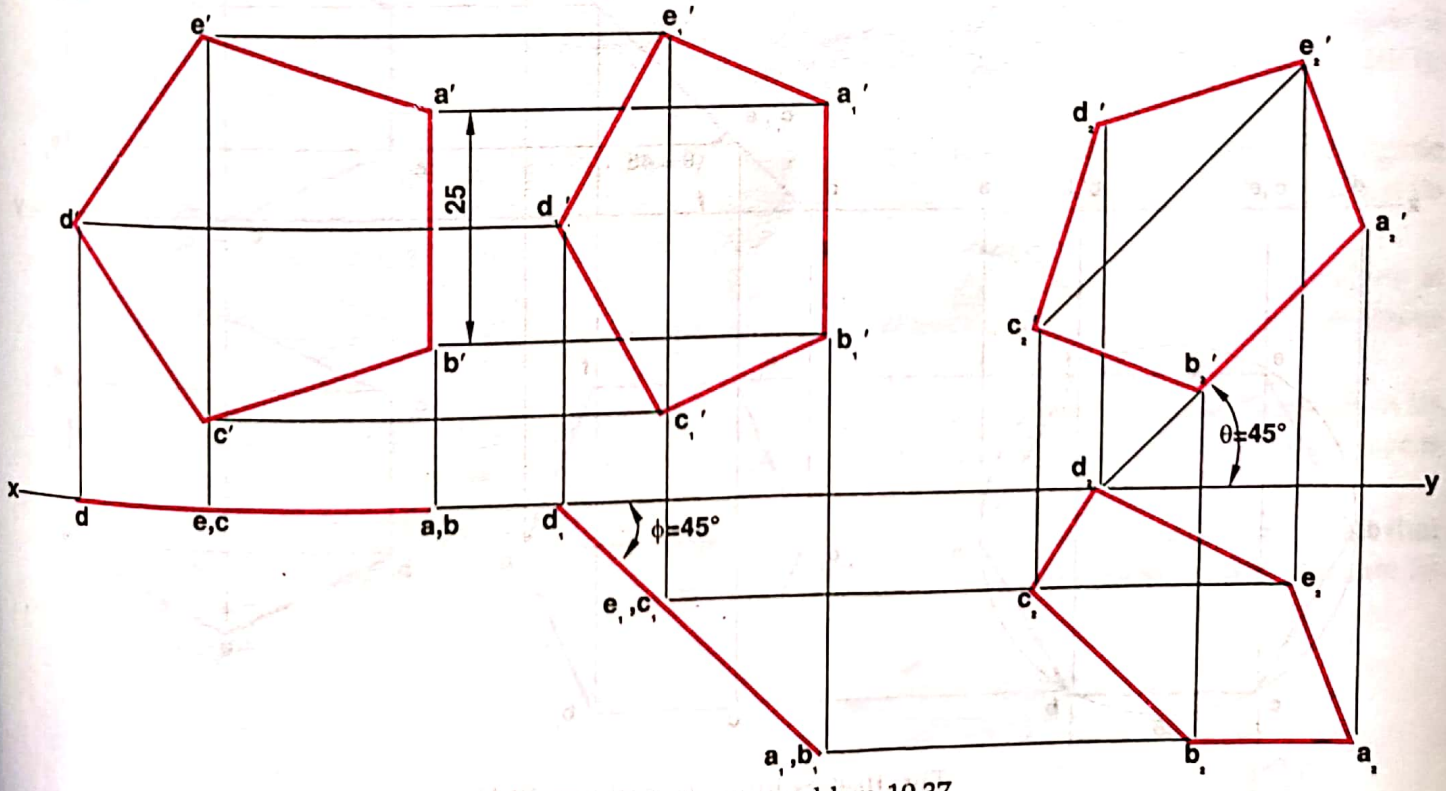


Fig. 10.45 Solution to problem 10.37

PROBLEM 10.40 *An equilateral triangular thin plate ABC of 25 mm side lies on one of its side in VP. Draw the projections of the plate when the plate surface is vertical and inclined at 45° to the VP. One of the sides of triangular plate is inclined at 30° to the HP.*

SOLUTION. All the construction lines are retained to make the solution self – explanatory. See Fig. 10.46.

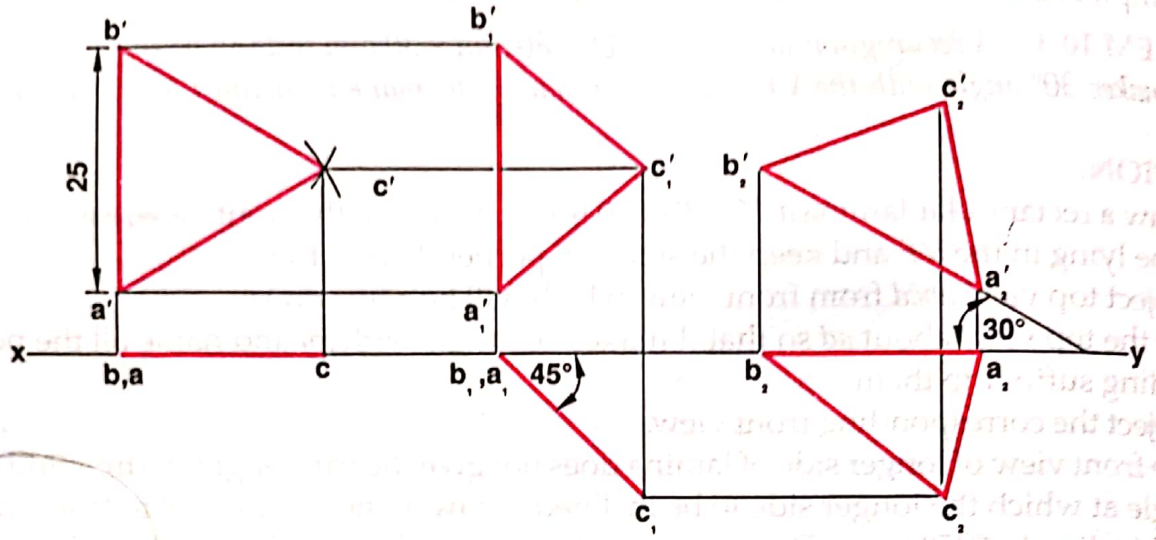


Fig. 10.46 Solution to problem 10.40

PROBLEM 10.41 *A regular hexagonal lamina ABCDEF 25 mm side has one of its corners (say D) on the HP, with the surface inclined at 45° to the HP and the top view of the diagonal through that corner is perpendicular to the VP. Draw its projections.*

SOLUTION. All the construction lines are retained to make the solution self– explanatory. See Fig. 10.47.

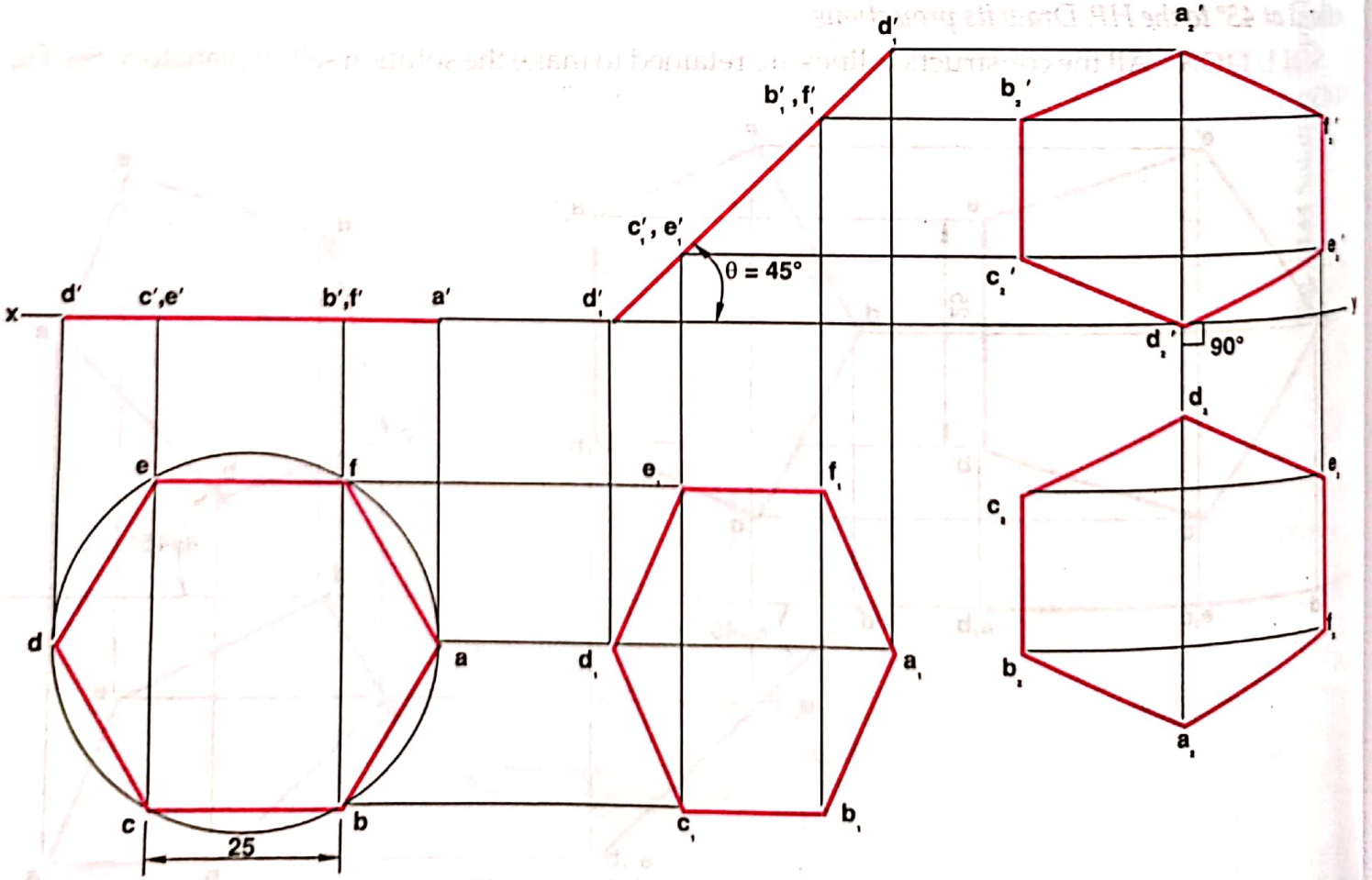


Fig. 10.47 Solution to problem 10.41

EXERCISES

TRACES OF PLANES

10.1 Show by means of their traces, each of the following planes in (*i*) first quadrant and (*ii*) third quadrant only

(*a*) Perpendicular to both HP and VP

(*b*) Parallel to and 30 mm from VP

(*c*) Parallel to and 30 mm from HP

(*d*) Perpendicular to HP and inclined at 30° to VP

(*e*) Perpendicular to VP and inclined at 30° to HP

LAMINA PARALLEL TO ONE OF THE PLANES

10.2 An equilateral triangle *ABC* of 40 mm side is parallel to HP with one of its sides inclined at 30° to VP and the end of the side nearer to VP is 20 mm infront of the VP and 25 mm above the HP. Draw its projections and locate its traces too.

10.3 A regular pentagonal lamina *ABCDE* of 25 mm side, has its corner *A* in HP and the side *CD* parallel to the HP. Draw its projections when its plane is parallel to VP. Also locate its traces.

10.4 A regular hexagonal lamina ABCDEF 20 mm side, has its corner A in HP and the side EF perpendicular to HP. Draw its projections when its plane is parallel to and 20 mm from VP. Locate it traces too.

10.5 A square lamina ABCD of 25 mm side is parallel to HP and is 10 mm from it. Draw its projections in first quadrant only and locate its traces.

LAMINA INCLINED TO ONE OF THE PLANES

10.6 A regular hexagonal lamina *ABCDEF* of 25 mm side, has its corner *A* in HP. Its plane is inclined at 45° to the HP and perpendicular to the VP. Draw its projections and locate its traces.

10.7 Draw the projections and traces of a thin circular sheet of 50 mm diameter and of negligible thickness, when its plane is inclined at 45° to HP and is perpendicular to VP. A point on its rim, nearest to the VP is 20 mm above the HP.

10.8 A regular pentagonal lamina *ABCDE* of 25 mm side, has its side *BC* in HP. Its plane is perpendicular to HP and inclined at 45° to the VP. Draw its projections and locate its traces too.

10.9 A regular hexagonal thin plate of 50 mm side has a circular hole of 50 mm diameter in its centre. It is resting on one of its corners in HP. Draw its projections when the plate surface is vertical and inclined at 30° to the VP.

10.10 A regular hexagonal lamina *ABCDEF* of 25 mm side, rests on one of its sides on HP such that it is perpendicular to VP and inclined to the HP at 45°. Draw its projections and locate its traces too.

10.11 A regular pentagonal lamina *ABCDE* of 25 mm side, has its side *BC* in HP. Its plane is perpendicular to the HP and inclined at 30° to the VP. Draw its projections and locate its traces.

10.12 A thin circular plate of ϕ 50 mm has a square hole of 25 mm side, cut centrally through it. Draw its projections when the plate is resting on HP with its surface inclined at 30° to the HP and an edge of square hole is perpendicular to VP.

LAMINA INCLINED TO BOTH HP AND VP

10.13 A square lamina *ABCD* of 30 mm side, rests on one of its corners on ground. Its plane is inclined at an angle of 45° to the HP and diagonal *BD* inclined at 60° to the VP and parallel to the HP. Draw its projections.

10.14 A regular pentagonal lamina *ABCDE* of 25 mm side has its corner *A* in HP. Its side *CD* parallel to the HP and inclined at 45° with the VP. The plane of the pentagon makes an angle of 30° with the HP. Draw its projections.

10.15 An equilateral triangular thin plate *ABC* of 65 mm side has a circle inscribed in it. Draw the projections, when its plane is vertical and inclined at 30° to the VP and one of the sides of the triangle is inclined at 45° to the HP.

10.16 A thin circular plate ϕ 60 mm appears as an ellipse in the front view, having its major axis 60 mm long and minor axis 40 mm long. Draw its top view when the major axis of the ellipse is horizontal.

10.17 A thin triangular sheet *ABC* has its sides *AB* = 50 mm, *BC* = 45 mm and *CA* = 35 mm. Draw its projections when its side *AB* in VP and inclined at 30° to HP, while its surfaces makes an angle of 45° with the VP.

10.18 A square lamina ABCD of sides 40 mm has one corner A and the VP and another corner D on the HP. The edge AB makes 30° to the HP and 60° to the VP. Draw the projections when the edge DC makes an angle of 30° to the HP. Draw the projections of the square lamina.

10.19 A regular hexagonal lamina of 30 mm sides is standing on an edge on the ground which makes 30° to the VP and the plane itself is at 60° to the ground. Draw the projections of the lamina.

10.20 A circular lamina of 20 mm radius appears as an ellipse of 40 mm major axis and 25 mm minor axis in the view from above. Draw the projections of the lamina.

OBJECTIVE QUESTIONS

10.1 Distinguish between plane and lamina ?

10.2 What is the trace of a plane ?

10.3 When a plane is perpendicular to both the principal planes, its traces are .....

10.4 What is an oblique plane ?

10.5 When a plane is perpendicular to the principal plane, its projection on the plane is .....

10.6 The traces of planes are .....

10.7 What is an edge view of a plane ?

10.8 The lines in which the plane meet the principal planes are called ..... of the plane.

10.9 Define and classify planes.

10.10 What is the difference between a quadirateral and a polygon ?

ANSWERS

10.3 Perpendicular to *xy*

10.5 Line

10.6 Straight lines

10.8 Traces