4 Superconductivity

(1) INTRODUCTION

The electrical resistivity of many metals and alloys drops suddenly to zero when their specimens are cooled to a sufficiently low temperature, often in the liquid helium range. This phenomenon was observed by Kammerlingh onnes in 1911 in mercury; the resistivity of mercury vanished completely below 4.2 K. The transition from normal conductivity occurred over a very narrow range of temperature (called Transition Width) of the order of 0.05 K for mercury.

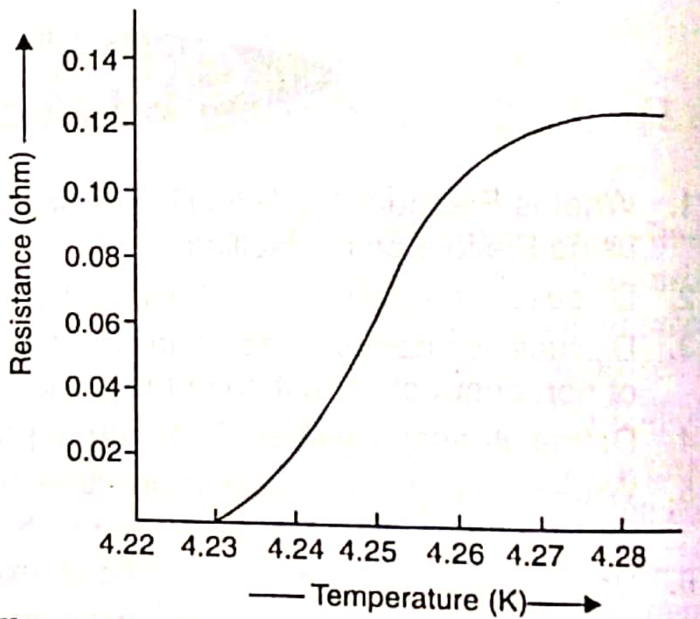

Fig. 1.

"The process of loss of resistance of certain materials when cooled below certain specific temperature is called Superconductivity and materials showing this property are called Superconductors".

"The minimum temperature to which material must be cooled so that its resistance becomes just zero is called critical temperature or superconducting transition temperature."

(2) EXPERIMENTAL FACTS

After the discovery of superconductivity, a large number of experiments were performed to find out, the dependence of superconductivity on external conditions like current, magnetic field, nature of substance, presence of impurity etc. The observations made from these experiments are depicted below, with their implications :

(*i*) The X-ray diffraction pattern is same both above and below the critical temperature, which shows that during superconducting transition, the crystal structure of the material remains same.

(*ii*) The change in intensity of diffracted X-rays was not noticeable, which indicates that during superconducting transition, the electronic structure remains almost same.

(*iii*) There is no appreciable change in the reflectivity of specimen either in visible or in infrared region.

(*iv*) The elastic and thermal expansion properties donot change with superconducting transition.

(*v*) The photoelectric properties are also unchanged, *i.e.* there is no change in the absorption of fast or slow electrons.

(*vi*) The electrical properties of material get totally changed *i.e.* above T_C resistance is more than zero and below T_C, resistance is zero.

(*vii*) The magnetic properties of the material undergo a change in the same way as electric properties. In a pure superconductor sample of large dimensions practically no magnetic flux is able to enter the metals which thus behaves as if it had zero permeability or strong diamagnetic susceptibility.

(*viii*) The specific heat changes discontinuously at the transition temperature. There is small change of volume at the transition in the presence of magnetic field.

(*ix*) All thermoelectric effects (Seebeck, Peltier, Thomson) disappear in the superconductivity state.

(*x*) The thermal conductivity changes discontinuously when the superconductivity is destroyed in magnetic field. It is lower in the superconducting state for pure metals but higher for certain alloys.

(3) EFFECT OF MAGNETIC FIELD ON SUPERCONDUCTIVITY

Consider a superconductor maintained at temperature less than T_C. If a magnetic field of sufficient magnitude is applied parallel to the length of superconductor then superconductivity can be destroyed even though temperature is still less than T_C. Thus original resistance of the wire gets restored and it starts behaving like a normal conductor.

"The minimum value of magnetic field at a given temperature, that must be applied across that material so that it loses superconducting behaviour is called critical magnetic field" and it is denoted by H_C. Alternatively critical field is the "maximum field in which a superconductor remains in superconducting state".

Critical field is a function of temperature. It decreases with increase in temperature and $T = T_C$, H_C becomes zero. Graph between H_C and temp. T is shown in fig. (2) for various substances. The variation is parabolic in nature. Hence empirical relation between H_C and temp. T is of the form

$$H_C(T) = H_C(0)\left(1 - \frac{T^2}{T_C^2}\right)$$

Where $H_C(0)$ is the value of critical field at 0K. It has a specific value for each material. This diagram may be realised as a phase diagram. The lower left region of the graph represents superconducting state and upper right region represents normal conducting state.

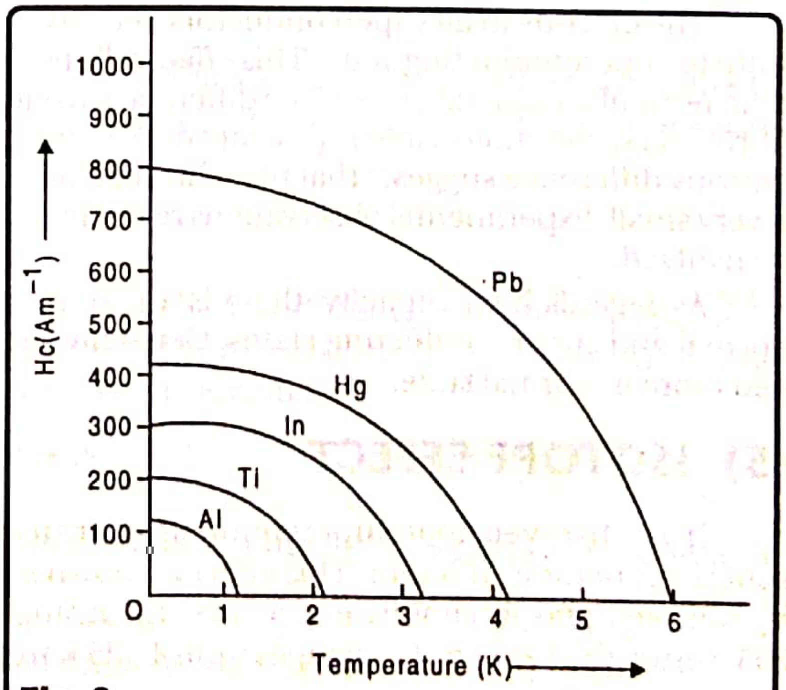

Fig. 2.

We know that when current is passed through a conductor it produces a magnetic field. If the magnetic field produced due to current is more than critical magnetic field H_C, then, as discussed above, the superconducting state of the material may be destroyed even though there is no external magnetic field around superconductor. This limits the maximum amount of current, that can be passed through a superconductor. This maximum allowable current is called critical current I_C.

Let r = radius of superconducting wire

I = Current passed

B = magnetic flux density at the surface of wire

H= magnetic field at surface of wire

∴ We know from lower classes that

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\therefore \quad H = \frac{B}{\mu_0} = \frac{I}{2\pi r}$$

When $I = I_C$ = critical current, then $H = H_C$ = critical field.

Thus above equation becomes

$$H_C = \frac{I_C}{2\pi r} \quad \text{...(1)}$$

This equation is called Silsbee's Rule. Silsbee suggested that the important factor in causing the destruction of superconductivity was the magnetic field associated with the current, rather than the value of current itself. Thus destruction of superconductivity is actually field controlled effect.

Note: If external electric field H is also applied transverse to the superconducting wire then expression for critical current is given by $I_C = 2\pi r\,(H_C - 2H)$

(4) ENTROPY AND HEAT CAPACITY

The entropy in all superconductors decreases when their specimens go from the normal state to superconducting state. This effect tells us that superconducting state is more ordered than normal conducting state. The difference in entropy of two states is very small, of the order of $10^{-3}\,N_A k_B$ per mole. Here N_A = avogadro's number and k_B is Boltzmann constant. The small entropy difference suggests that the rearrangement of system on becoming a superconductor is very small. Experimental observations reveal that only rearrangement of conducting electrons is involved.

As regards heat capacity, there is a marked difference between heat capacities in the normal and super conducting states. Generally specific heat is less in superconducting state and more in normal state.

(5) ISOTOPE EFFECT

It is observed that superconducting critical temperature for various isotopes of a superconductor is different. This effect is known as Isotope Effect. This effect was discovered by Maxwell and Reynolds. e.g. as average isotopic mass of mercury varies from 199.5 to 203.4 amu, the value of T_C changes from 4.185 K to 4.146 K.

If M = Isotopic mass

T_C = Critical temperature in Kelvin

Then experimentally it was found that $\dot{M}$ and T_C satisfy following empirical relation

$$\sqrt{M} \,.\, T_C = \text{constant} \qquad ...(2)$$

Let T_D = Debye temperature for the specimen (It is that temperature above which specific heat of a solid becomes constant *i.e.* Dulong Petit law is obeyed).

Then T_D is also related to isotopic mass by the relation

$$\sqrt{M}\ T_D = \text{constant} \qquad ...(3)$$

Divide (2) by (3) we get

$$\frac{T_C}{T_D} = \text{constant} \qquad ...(4)$$

This relation suggests that T_C depends on T_D. But T_D depends on speed of sound in that solid and we know that speed of sound in a solid is determined by lattice vibrations. Thus above relation suggests that lattice vibrations must have something to do with superconductivity. Hence electron lattice interactions must be taken into account in finding the explanation for superconductivity.

(6) EFFECT OF IMPURITIES

Addition of chemical impurities modifies nearly all superconducting properties (including T_C) and particularly magnetic properties.

(7) EFFECT OF STRESS

The transition temperature T_C can be changed by stress. Usually a stress, which increases the dimensions increases the transition temperature. Stress may also affect critical magnetic field upto some extent.

(8) EFFECT OF SIZE

Reduction of size of specimen below 10^{-4} cm modifies the superconducting properties in many respects. The relative magnetic permeability (μ_r) is found to be zero for sizes more than 10^{-4} cm but it is non zero for sizes less than 10^{-4} cm (approx.)

(9) EFFECT OF FREQUENCY

If alternating current of frequency upto 10^7 Hz is passed through a superconductor at $T < T_C$, resistance is still zero. But with further increase in frequency resistance comes into play. At frequency 10^{10} Hz, the resistance of specimen is considerable.

(10) EXISTENCE OF ENERGY GAP

We know that any process taking place naturally is accompanied by release of energy and this infact gives more stability to the system. When a superconductor specimen is maintained at temperature less than T_C, its resistance becomes zero naturally. It means that at temp. $T < T_C$ the superconducting state is more stable than normal state. The transition from normal to superconducting state is accompanied by release of finite amount of energy. In order to destroy superconductivity, this amount of energy should be given back to specimen externally. Thus the energy states of specimen in the superconducting state are separated by a finite gap from energy states of normal state. The lower states are superconducting states and higher states are normal states. With rise in temperature the energy gap decreases and above T_C the energy gap is destroyed and resistance of material becomes non zero.

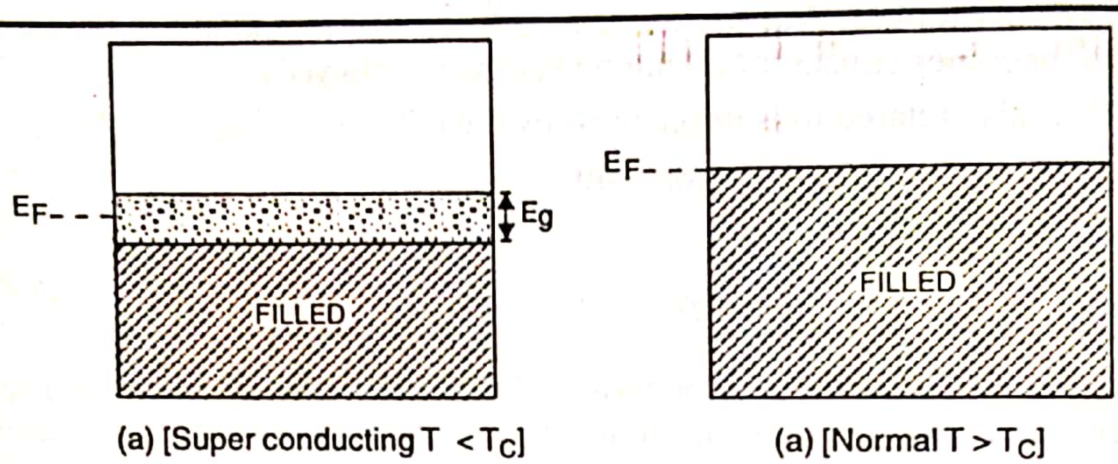

Fig. 3.

At absolute 0K, all electrons are superconducting and at temperature above 0K some normal states are also filled.

This Band diagram is different from that of insulators and semi conductors in following ways :

(i) In insulators the lower band is called Valence Band and upper band is called Conduction Band. Electrons in conduction band are free and can take part in conduction while electrons in valence band are not free and cannot conduct. However in superconductors electrons in Normal as well as superconducting band are conducting electrons. The only difference is that normal electrons show resistance and superconductivity electrons donot show resistance.

(ii) The energy gap of superconductors depends on temperature. With increase in temperature it decreases and becomes zero at temperature T_C. However energy gap of insulators is nearly independent of the temperature.

The existence of energy gap in super conductors has been confirmed by a number of experiments. Same experiments have been employed for the experimental determination of its value. Theoretical value of the energy gap of specimen at 0K is given as :

$$E_g = 2\Delta = 2bk_B T_C \qquad ...(5)$$

Here $\Delta = bk_B T_C$ is called energy gap parameter. b is a constant, Its value usually is 1.4. The energy gap is related to fermi energy as $E_g \approx 10^{-4} E_F$.

(11) MICROWAVE AND INFRARED PROPERTIES

We know from preceeding discussion that at 0K the normal and superconducting states are separated by energy gap E_g. If a superconductor at 0K is irradiated by electromagnetic radiation of energy less than E_g, these radiations are unable to take electrons from superconducting to normal state. Hence resistance of specimen is still zero. On the other hand if energy of radiations is more than E_g, then electrons will make transition to normal state by absorbing this energy and hence superconductivity gets destroyed. Moreover due to radiations, the resistivity of sample approaches that of normal resistivity if energy of radiations is in the Infrared Region. Thus at these frequencies, there is no change in the resistivity as the temperature is raised through T_C. If radiations incident on the sample are in the microwave region then resistivity increases enormously. This is because of the fact that, microwaves can excite even low energy superconducting electrons to the normal state along with high energy superconducting electrons.

(12) THERMAL CONDUCTIVITY

Thermal conductivity of an ideal superconductor drops markedly when specimen shows transition from normal to super conducting state. This drop suggests that the normal electronic contribution is no longer fully added to the total thermal conductivity. The superconducting electrons possibly play no part in heat transfer.

On the other hand thermal conductivity of impure non ideal superconducting sample increases on making transition from normal to superconducting state. According to Hulm, this is due to the decreased scattering of lattice waves by electrons. It should be noted that only normal electrons can scatter photons.

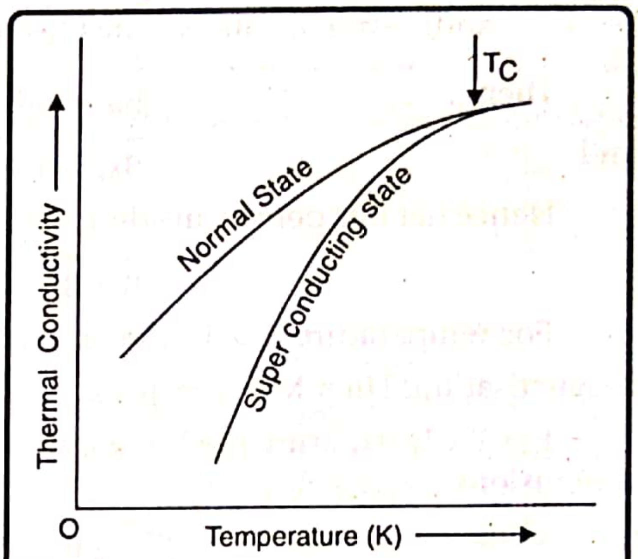

Fig. 4.

(13) FLUX EXPULSION (MEISSNER EFFECT)

If a superconducting sample is cooled in a longitudinal magnetic field from above critical temperature to below critical temperature [The critical temperature corresponding to the magnetic field in which specimen is placed. See equation (1)], then the lines of magnetic flux density $\vec{B}$ are pushed out of the body of the specimen at the transition provided the specimen of considerable dimensions (> 10^{-4} cm). This phenomenon is called Meissner Effect. The effect is of fundamental importance as it shows that a bulk superconductor behaves as if inside the specimen net magnetic flux density is zero.

Let $\vec{B}_a$ = applied magnetic flux density

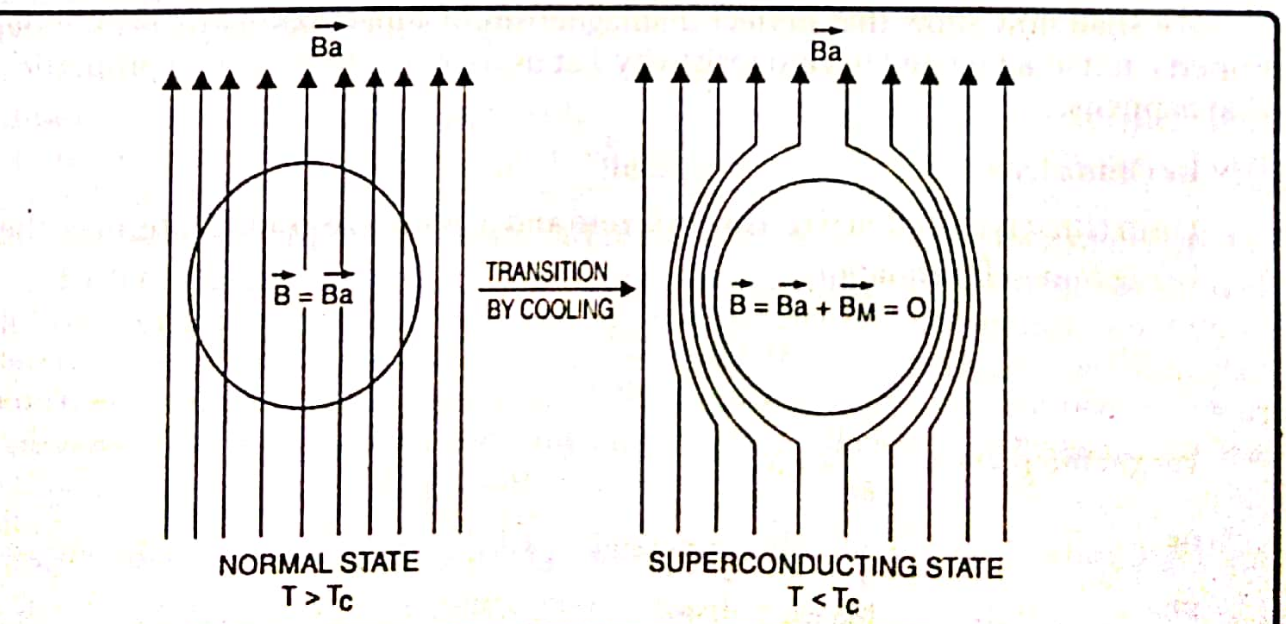

Fig. 5.

$\vec{B}_M$ = flux density induced in the specimen due to magnetisation

$\vec{M}$ = magnetisation vector

$\vec{B}$ = net flux density inside the sample

Then $\vec{B}_a = \mu_0 \vec{H}$ (where $\vec{H}$ = applied magnetic field)

and $\vec{B}_M = \mu_0 \vec{M}$

Hence net flux density inside specimen is

$$\vec{B} = \vec{B}_a + \vec{B}_M = \mu_0 (\vec{H} + \vec{M}) \quad ...(6)$$

For temperature $T > T_C$ material is in normal conducting state and shows almost no magnetisation. Thus $\vec{M} = 0 \Rightarrow \vec{B} = \mu_0 \vec{H} = \vec{B}_a$.

For $T < T_C$ experimentally it is found that net flux density inside sample is zero (provided dimensions are large)

Thus $\vec{B} = 0$

$$\Rightarrow \mu_0 (\vec{H} + \vec{M}) = 0$$

$$\Rightarrow \vec{M} = -\vec{H} \quad ...(7)$$

This means that when a specimen becomes a superconductor, it gets magnetised in a direction opposite to applied magnetic field $\vec{H}$ (characteristic property of Diamagnetic materials) such that magnitude of magnetisation vector is equal to magnitude of applied magnetic field $\vec{H}$.

But $\vec{M} = \chi_M \vec{H}$ ...(8)

Where χ_m = magnetic susceptibility.

Compare (7) and (8) we get

$$\chi_m = -1 \quad ...(9)$$

Thus when a bulky specimen becomes a superconductor, then, it shows perfect diamagnetism.

We shall now show that perfect diamagnetism of superconductors is an independent property, not at all related to zero resistivity. Let us try to relate these two properties and see what happens.

By Ohm's Law $\vec{J} = \sigma \vec{E}$ or $\vec{E} = \rho \vec{J}$ ...(10)

From (10) it is clear that if resistivity is zero and $\vec{J}$ is non-zero and finite, even then $\vec{E} = 0$.

Using Maxwell's equation

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

We get for $\vec{E} = 0$ as $\dfrac{-\partial \vec{B}}{\partial t} = 0$

or $\vec{B}$ = constant

or $\vec{B}_{initial} = \vec{B}_{Final}$

This means that if only zero resistivity is the only condition for a material to become superconductor, then the value of magnetic flux density inside it should not change when transition from normal to superconducting state takes place.

This implies that magnetic flux expulsion should not take place at all. However as discussed earlier (and as shown in figure (05)), all bulky superconductors show flux expulsion at superconducting transition.

Thus we conclude that for a material to become superconductor, there are two independent conditions:

(*i*) The resistivity should become zero

(*ii*) The material should show perfect diamagnetism.

(14) FLUX PENETRATION

In the preceeding article we discussed that field lines get expelled from a Bulky Superconductor. However if same experiment is performed with a thin super conducting film (of thickness << 10^{-4} cm) then field lines can penetrate through the thin film even when material is in superconducting state. This phenomenon is called Flux Penetration. Thus thin films donot exhibit Meissner Effect. The explanation of both Meissner Effect and flux penetration was successfully given with the help of London Equations.

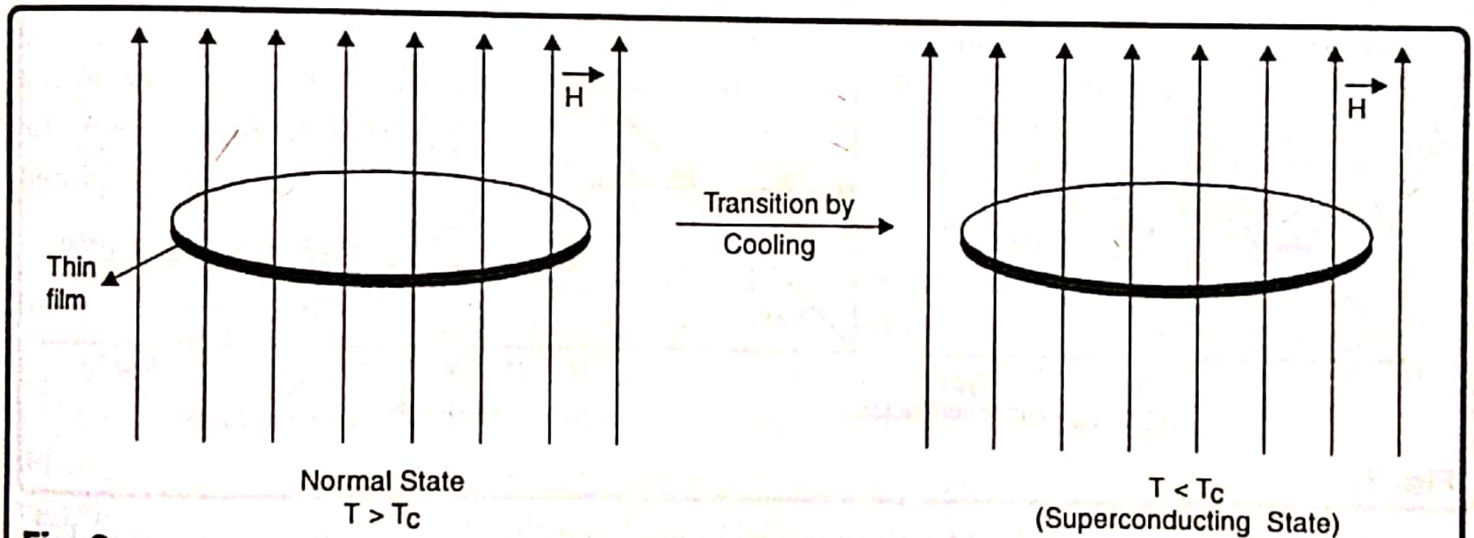

Fig. 6.

(15) PERSISTENT CURRENTS

Consider a super conductor in the form of a ring. Place it in external magnetic field. Then cool it from temperature higher than T_C to a temperature sufficiently lower than T_C. Then suddenly switch off the magnetic field. Then due to electromagnetic induction an induced current is produced in the ring. Since temperature is less than T_C so resistance of material is practically zero. It has been observed that his current continues to persist with undiminished strength for days or even years. Such currents are called persistent currents and are quite expected of superconductors e.g. A current of several hundred amperes was passed through a lead ring and for over a year its strength remained undiminished. Attempts have been made to use these currents in the relation with decay formula to place an upper limit on the value of R

$$I(t) = I_0\, e^{-Rt/L}$$

of super conductor. Accordingly the resistivity of superconducting lead is certainly less than 10^{-20} Ωcm.

(16) TYPE I AND II SUPERCONDUCTORS

We know that inside a superconductor net magnetic flux density is zero.

i.e. $$\vec{B} = \mu_0 (\vec{M} + \vec{H}) = 0$$

or $$\vec{M} = -\vec{H} \quad ...(11)$$

Where symbols have their usual meanings.

In magnitude we have M = H

Thus in a superconductor placed in external field $\vec{H}$ the spontaneous magnetisation causes appearence of non zero magnetic dipole moment in such a way that magnetisation vector is always equal and opposite to the applied magnetic field $\vec{H}$ (The situation is similar to static friction versus applied force).

Thus if a graph is plotted between M and H (ignoring their directions), it is a straight line passing through origin and having slope unity (in SI system).

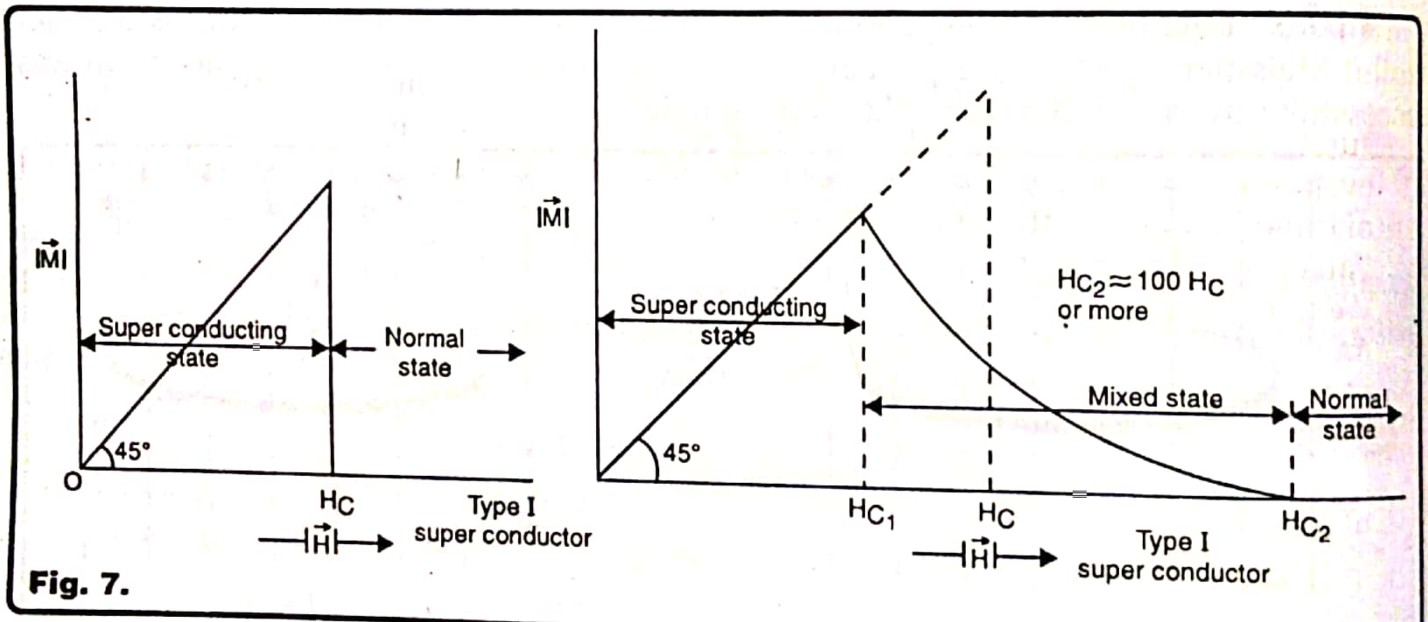

Fig. 7.

Thus with increase in H, M will increase accordingly so that it remains equal to H in magnitude. However when applied field $|\vec{H}|$ equals critical field then material loses its superconducting behaviour. Thus for $H > H_C$, M becomes zero.

We expect M to become zero as soon as M exceeds H_C (See Fig. 7(*a*)) so that Meissner effect is obeyed strictly, and transition from superconducting to normal state is abrupt. This was experimentally observed for a number of superconductors. "Such superconductors for which M becomes zero sharply as H exceeds H_C are called type I or soft superconductors."

However for certain superconductors the graph between M and H was found to be similar to that shown in fig. 7 (*b*). In such materials, there are two critical magnetic fields H_{C_1} and H_{C_2}. For $H < H_{C_1}$, M remains equal to H and perfect diamagnetism is exhibited. The material is thus totally in superconducting state in this region. For $H > H_{C_1}$, M does not drop to zero abruptly but it happens gradually *i.e.* beyond H_{C_1}, some electrons start behaving like normal electrons and some still behave as superconducting electrons. This gradual decrease of M continues upto second critical field H_{C_2}. At $H = H_{C_2}$, M becomes zero. The region between H_{C_1} & H_{C_2} thus represents a mixed state where material is partially superconducting and partially normal conductor. This state is also called Vortex state. For $H > H_{C_2}$ material becomes a normal conductor.

"Such materials in which M decreases slowly with increase in magnetic field beyond first critical field are called Hard or Type II superconductors."

Type I represents ideal superconductors and obey Meissner Effect strictly while type II superconductors are non ideal and donot obey Meissner Effect strictly. These materials exhibit irreversible magnetisation i.e. Hysteresis.

Type I or soft superconductors are usually pure specimens of some elements and values of H_C for them are always too low to have any useful technical application in coils for superconducting magnets, because even low magnetic fields can easily destroy superconductivity.

Type II or Hard superconductors are usually alloys or transition metals with very high values of electrical resistivity in the normal state. These materials can withstand large magnetic fields without loosing superconductivity. Hence these materials are technically very useful materials in contrast to type I superconductors.

(17) THERMODYNAMICS OF SUPERCONDUCTING TRANSITION

The Meissner Effect suggests that the transition between the normal and the superconducting state is thermodynamically reversible under the conditions of slow evaporation. This is because the superconducting currents donot die away with the production of Joule Heat, when the superconductivity is destroyed by the application of a magnetic field. Van Laer and Keerom has confirmed this nature of superconducting transition experimentally. We may therefore apply thermodynamics to the transition and thereby obtain an expression for the difference in the entropy between the normal and the superconducting states in terms of critical field H_C versus T. We shall restrict ourselves to Type I superconductors only with complete Meissner effect so that B = 0 inside the superconductor.

Let G, U denote Gibb's free energy per unit volume and internal energy per unit volume of the specimen respectively.

Let S = entropy per unit volume of specimen

Let $\vec{\mu}$ = induced magnetic dipole moment of sample

μ_0 = permeability of free space

$\vec{H}$ = applied magnetic field

$\vec{M}$ = Magnetisation Vector, V = Volume of Specimen

Thus potential energy per unit volume due to magnetisation of sample is

$$\frac{U_p}{V} = \frac{-\vec{\mu}\cdot\vec{B}}{V} \qquad \text{(By definition pot. energy} = -\vec{\mu}\cdot\vec{B})$$

$$= -\frac{\mu_0\,\vec{\mu}\cdot\vec{H}}{V} \qquad (\because \vec{B} = \mu_0\vec{H})$$

$$= -\mu_0\,\vec{M}\cdot\vec{H} \qquad \left(\because \frac{\vec{\mu}}{V} = \vec{M}\right)$$

$$= -\mu_0 MH\cos 180° \qquad (\because \vec{M} \text{ and } \vec{H} \text{ are in opposite directions})$$

$$= \mu_0 MH$$

If T = absolute temperature of the superconductor, then the Gibb's free energy per unit volume of the specimen is given by

$$G = U - TS - \mu_0 MH \quad ...(12)$$

Thus differential change in free energy per unit volume is given by

$$dG = dU - TdS - SdT - \mu_0 MdH - \mu_0 HdM \quad ...(13)$$

But by first law of thermodynamics

$$dQ = dU + PdV + (-\mu_0 HdM)$$

or $$TdS = dU - \mu_0 HdM \quad (\because dV = 0 \text{ for a solid specimen } \& \; TdS = dQ)$$

Hence $$dU - TdS - \mu_0 HdM = 0 \quad ...(14)$$

Put in (13), we get

$$dG = -SdT - \mu_0 MdH \quad ...(15)$$

***(i)* When material is in the Normal State.** We assume that transition between normal and superconducting state takes place at constant temperature. Thus $dT = 0$.

Also in the normal state, there is no spontaneous magnetisation of the sample *i.e.* $M = 0$

Hence (15) becomes

$$dG_N(T, H) = 0$$

Here subscript N indicates normal state and T, H indicates variables on which G depends.

Integrating above equation from 0 to H, we get

$$\int_0^H dG_N(T, H) = 0$$

$$\Rightarrow \quad G_N(T, H) - G_N(T, 0) = 0$$

$$\Rightarrow \quad G_N(T, H) = G_N(T, 0) \quad ...(16)$$

Thus free energy of a normal conductor is independent of the magnetic field at a given temperature.

***(ii)* When material is in superconducting state.** Here also we assume that

$$T = \text{constant} \Rightarrow dT = 0$$

Also for Type I superconductors, Meissner effect is obeyed. Hence $M = -H$.

Thus equation (15) gives

$$dG_S(T, H) = -\mu_0(-H)\, dH = \mu_0 HdH \quad ...(17)$$

Integrating above equation from 0 to H, we get

$$G_S(T, H) - G_S(T, 0) = \frac{\mu_0 H^2}{2}$$

or $$G_S(T, H) = G_S(T, 0) + \frac{\mu_0 H^2}{2} \quad ...(18)$$

Thus when a superconducting material is placed in magnetic field, its free energy increases.

At the critical field H_C, however, the energies of the normal and superconducting states must be equal to each other ($\because$ because at H_C and constant temperature T, the two states are in equilibrium).

Thus, $$G_S(T, H_C) = G_N(T, H_C)$$

or $$G_S(T,0) + \frac{\mu_0 H_C^2}{2} = G_N(T,0) \qquad \text{(Using (16) and (18))}$$

$\Rightarrow$ $$G_N(T,0) - G_S(T,0) = \frac{\mu_0 H_C^2}{2} \qquad ...(19)$$

Right side of above equation is always positive. Thus we conclude that $G_N(T,0) > G_S(T,0)$.

This means that free energy of Normal conductor at a given temperature is more than free energy of superconductor. The result also predicts that the superconducting and normal states are in equilibrium along the $H_C - T$ curve (called critical field curve) and the quantity $\frac{\mu_0 H_C^2}{2}$ is a direct measure of Stabilization Energy Density of the superconductor. Typical values of stabilization energy density is 10^4 erg/cm^3 or 10^{-19} erg per conduction electron.

(iii) **Change in entropy.** If H is kept constant, then from equation (15), we can write

$$(dG)_H = -S\,(dT)_H \qquad (\because dH = 0 \text{ for } H = \text{constt.})$$

or $$S = -\left(\frac{dG}{dT}\right)_H \qquad ...(20)$$

Let S_N, S_S represent entropy per unit volume in the normal and superconducting states respectively. Then change in entropy during superconducting transition is given as

$$S_N - S_S = -\left(\frac{dG_N}{dT}\right)_H + \left(\frac{dG_S}{dT}\right)_H$$

$$= -\frac{d}{dT}(G_N - G_S)_H = -\frac{d}{dT}\left(\frac{\mu_0 H_C^2}{2}\right)_H$$

$$= -\mu_0 H_C \frac{dH_C}{dT} \qquad ...(21)$$

[Note H_C depends on temperature of specimen]

The quantity H_C is a decreasing function of temperature T of specimen (See fig. (2) and equation (1). Thus $\frac{dH_C}{dT}$ is always negative.

Thus from (21), we conclude that

$S_N - S_S$ is always positive or $S_N > S_S$.

Thus entropy in the normal state is always more than entropy in superconductivity state or superconductivity state is more ordered than the normal state. Further $\frac{dH_C}{dT}$ is never zero. It means that there is a finite entropy change, when the superconducting transition takes place in the presence of magnetic field. Further a finite entropy change means that there is a latent heat associated with the superconducting transition.

(iv) **Change in Heat Capacity.** The heat capacity per unit volume is given as

$$C = T\frac{dS}{dT}$$

Thus change in heat capacity at the superconducting transition is given as

$$C_S - C_N = T\frac{d}{dT}(S_S - S_N)$$

$$= T\frac{d}{dT}\left(\mu_0 H_C \frac{dH_C}{dT}\right) \qquad \text{(using 21)}$$

$$= \mu_0 T \left[\left(\frac{dH_C}{dT} \right)^2 + H_C \frac{d^2 H_C}{dT^2} \right]$$

At $T = T_C$, $H_C = 0$

Thus $$C_S - C_N = \mu_0 T_C \left(\frac{dH_C}{dT} \right)^2 \quad ...(22)$$

This relation is called Rutgers formula. It shows that there is a discontinuity in heat capacity at T_C.

We know that H_c varies with temperature as follows :

$$H_C(T) = H_C(0) \left[1 - \frac{T^2}{T_C^2} \right]$$

$\therefore$ $$\frac{dH_C}{dT} = H_C(0) \left[-\frac{2T}{T_C^2} \right]$$

and $$\frac{d^2 H_C}{dT^2} = H_C(0) \left[\frac{-2}{T_C^2} \right]$$

Put these values in equation (22), we get

$$C_S - C_N = \mu_0 T \left[\left(\frac{-2 H_C(0).T}{T_C^2} \right)^2 + (H_C(0))^2 \left(1 - \frac{T^2}{T_C^2} \right) \left(\frac{-2}{T_C^2} \right) \right]$$

$$= \mu_0 T [H_C(0)]^2 \left[\frac{4T^2}{T_C^4} - \frac{2}{T_C^2} + \frac{2T^2}{T_C^4} \right]$$

$$= 2\mu_0 T [H_C(0)]^2 \left[\frac{3T^2 - T_C^2}{T_C^4} \right] \quad ...(23)$$

From equation (23), we see that if $T > \frac{T_C}{\sqrt{3}}$ then $C_S - C_N > 0$ *i.e.* specific heat capacity will be more in the super conducting state than in normal state.

Also for $T < \frac{T_C}{\sqrt{3}}$ we get $C_S - C_N < 0$

It means that specific heat of super conducting state will be less than that of normal state. This variation is shown in fig. (8).

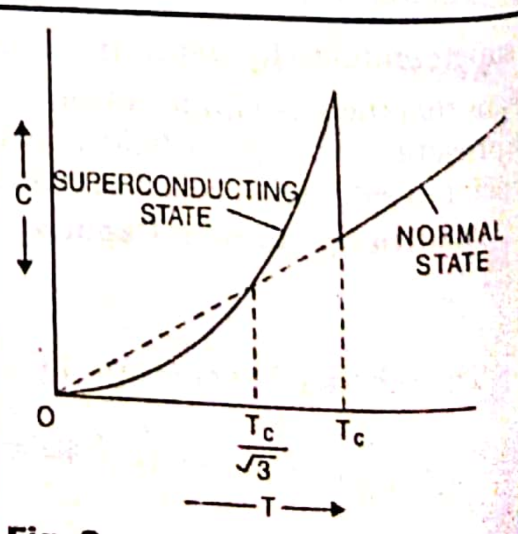

Fig. 8

(18) LONDON EQUATIONS

We have seen that Meissner Effect in superconductors can be explained if we assume $\chi_m = -1$ or $\vec{M} = -\vec{H}$. However this is a very drastic assumption. Moreover, it tends to cut off further discussion of superconductors, particularly the flux penetration through thin films. To get around this difficulty, F. London and H. London in 1935 proposed a modification of the electrodynamic equations. Two new equations have come out of their work, which can explain Meissner Effect as well as flux penetration. We shall now derive these equations.

Let m = mass of electron

n_s = number of superconducting electrons per unit volume

n_n = number of normal electrons per unit volume

$-e$ = charge on electron

$\vec{V}_S$ = drift velocity of superconducting electrons,

$\vec{E}$ = Electric field.

The equation of motion of the electron inside a conductor is given by Newton's second law as

$$m\frac{d\vec{V}}{dt} = -e\vec{E} + \text{Resistive Force} \quad ...(24)$$

However for superconducting electrons there is no resistance. Hence resistive force is zero. Thus above equation becomes

$$m\frac{d\vec{V}_S}{dt} = -e\vec{E}$$

But drift velocity and current density are related as

$$\vec{J}_s = -n_s e\vec{V}_s$$

Negative sign shows that current density is opposite to drift of electrons.

From above equation, we get

$$\frac{d\vec{J}_s}{dt} = -n_s e\frac{d\vec{V}_s}{dt} = -n_s e\left(\frac{-e\vec{E}}{m}\right) \quad \text{(using 24)}$$

or

$$\frac{d\vec{J}_s}{dt} = \frac{n_s e^2}{m}\vec{E} \quad ...(25)$$

Equation (25) is called London's first equation.

Taking curl of equation (25) and interchanging the order of space and time differentiation we get

$$\frac{d}{dt}(\vec{\nabla}\times\vec{J}_s) = \frac{n_s e^2}{m}\vec{\nabla}\times\vec{E}$$

or

$$\frac{d}{dt}(\vec{\nabla}\times\vec{J}_s) = -\frac{n_s e^2}{m}\frac{d\vec{B}}{dt} \quad \left(\because \vec{\nabla}\times\vec{E} = \frac{-d\vec{B}}{dt}\right)$$

Integrating w.r.t. time we get

$$\vec{\nabla}\times\vec{J}_s = -\frac{n_s e^2}{m}\vec{B} \quad ...(26)$$

The magnetic flux density is related to vector potential $\vec{A}$ as follows

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

Put in (26), we get

$$\vec{\nabla} \times \vec{J}_s = \frac{-n_s e^2}{m} \vec{\nabla} \times \vec{A}$$

or

$$\vec{J}_s = \frac{-n_s e^2}{m} \vec{A} \quad ...(27)$$

This equation is called London's second equation.

Importance of London Equations

(*i*) Conductors obey Ohm's law, which is given in vector form as

$$\vec{J} = \sigma \vec{E} \quad ...(28)$$

Here σ is electrical conductivity. Comparision of (27) and (28) reveals that London's 2nd equation is a replacement of Ohm's Law for superconductors *i.e.* superconductors donot obey Ohm's law. The electrical density depends upon vector potential rather than electric field.

(*ii*) We can explain both Meissner Effect as well as flux penetration using London equations as follows :

By Ampere's circuital law in differential form, we have

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}_s$$

Taking curl on both sides, we get

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \mu_0 (\vec{\nabla} \times \vec{J}_s)$$

or

$$\vec{\nabla} (\vec{\nabla} . \vec{B}) - \nabla^2 \vec{B} = \mu_0 (\vec{\nabla} \times \vec{J}_s)$$

or

$$-\nabla^2 \vec{B} = \mu_0 (\vec{\nabla} \times \vec{J}_s) \qquad (\because \vec{\nabla} . \vec{B} = 0)$$

or

$$-\nabla^2 \vec{B} = \mu_0 \left(\frac{-n_s e^2}{m} \vec{B} \right)$$

or

$$\nabla^2 \vec{B} = \frac{1}{\lambda^2} \vec{B} \quad ...(29)$$

Here

$$\lambda = \sqrt{\frac{m}{\mu_0 n_s e^2}} \quad ...(30)$$

λ is called penetration depth. Its value ranges from 300 Å to 5000 Å. It is a characteristic of the material.

Equation (29) in one dimension becomes

$$\frac{d^2 B}{dx^2} = \frac{1}{\lambda^2} B$$

The solution of this equation is given by

$$B = B_0 e^{-\frac{x}{\lambda}} \quad ...(31)$$

Here x is the distance from the surface of the specimen and B_0 is magnetic flux density at the surface of the superconductor. This equation tells that magnetic field inside the sample decreases exponentially with the distance from the surface.

If thickness x of material is much less than the penetration depth *i.e.*

$$x << \lambda \quad \text{or} \quad \frac{x}{\lambda} \approx 0$$

Then (31) ⇒ $\qquad B \approx B_0$

i.e. magnetic flux density every where inside the superconductor is constant and is equal to the value at the surface of superconductor. This explains flux penetration.

If thickness x of sample is much greater than penetration depth *i.e.* $x >> \lambda$ than

$$e^{-x/\lambda} \approx e^{-\infty} = 0.$$

Thus (31) ⇒ $\qquad B \approx 0$

Thus magnetic flux density inside the sample will be zero *i.e.* Meissner Effect (Perfect diamagnetism) will be exhibited by the material.

Thus London equations explain flux penetration and Meissner effect successfully.

(19) PENETRATION DEPTH

The flux density inside a superconductor changes as

$$B = B_0\, e^{-x/\lambda} \qquad \text{...(32)}$$

If we put $x = \lambda$ then $\qquad B = \dfrac{B_0}{e}$

Thus penetration depth can be defined as the distance inside the sample, which can reduce magnetic flux density to $\frac{1}{e}$ of its value at the surface.

It is given as $\qquad \lambda(0) = \sqrt{\dfrac{m}{\mu_0 n_s e^2}}$ (at temp. 0K)

Since λ contains n_s, and n_s varies with temperature. Thus penetration depth is a function of the temperature of the sample.

For $T > T_C$, $\quad n_s = 0 \quad$ (no electron is superconducting)

Thus $\quad \lambda = \infty$

at $T = 0$, $\quad n_s$ = maximum $\quad$ (all electrons are superconducting)

∴ $\quad \lambda$ = minimum

Thus in general λ increases with increase in temperature of the sample.

The empirical relation between λ and temperature is

$$\lambda(T) = \frac{\lambda(0)}{\left[1 - \left(\dfrac{T}{T_C}\right)^4\right]^{1/2}} \qquad \text{...(33)}$$

Where $\lambda(0)$ = Penetration depth at 0K and $\lambda(T)$ is penetration depth at TK.

Penetration depth also varies with applied magnetic field H. According to Pippard, there is only a few percent change of λ with H, even upto H_C. The empirical relation is of the form.

$$\lambda(H) = \lambda(H = 0)\left[1 + 0.02\,\frac{H^2}{H_C^2}\right] \qquad \text{...(34)}$$

Where $\lambda(H)$ is penetration depth in the presence of field and $\lambda(0)$ is penetration depth in the absence of field.

(20) BCS THEORY

The quantum theory of super conductivity was given by Bardeen, Cooper and Schrieffer in 1957. This theory has wide range of applicability from He^3 atoms in condensed phase to type I and type II metallic super conductors and to high temperature super conductors based on cuprate ions.

A solid is composed of a lattice of positive ions occupying definite sites with some binding energy and valence electrons move through the lattice randomly (*i.e.* having no correlation within themselves). Due to randomness of motion these valence electrons collide with lattice ions and loose their kinetic energy as heat. Thus cause of resistance is collision of electrons with ions. The path of one such randomly moving electron and making frequent collisions is shown by dotted line in fig. (9).

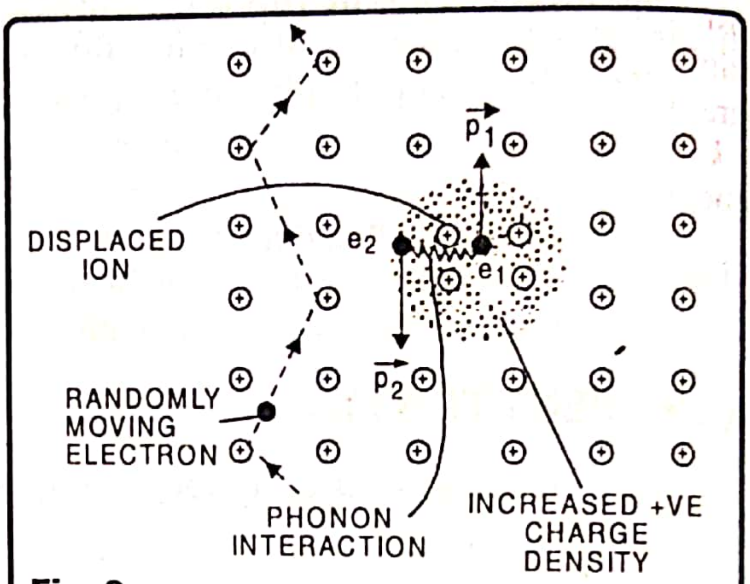

Fig. 9

According to BCS theory superconductivity is due to the attractive interaction between electrons at very low temperature. Due to this interaction electrons move in pairs called cooper pairs. The two electrons in cooper pairs move coherently through lattice in such a way that they do not suffer any collision with lattice ions. Due to this resistance becomes zero and material becomes super conductor.

Consider the example shown in fig. (9). An electron e_1 is moving with momentum $\vec{p}_1$ through the lattice. Because of Coulomb's interaction, the surrounding ions will be attracted towards e_1. The extent of displacement of positive ions depends on cohesive energy of lattice and will be more if cohesive energy is small. Due to displacement of ions, the positive charge density increases around e_1. If another electron e_2 happens to pass closely to e_1 then (*i*) e_1 will be shielded from e_2 due to condensation of ions around it, which decreases coulomb repulsion between e_1 & e_2. (*ii*) due to higher positive charge density around e_1; e_2 will be attracted in that direction. The second electron e_2 prefers to remain in the region of increased positive charge density and for this it will move in the same way (after entering the +ve charge cloud) as the electron e_1 is moving. It appears as if two electrons are bonded to each other and behave like a single particle called cooper pair. The two electrons in cooper pair are exchanging phonon (A phonon is a quantum of vibration energy of lattice ions) through lattice ions. Hence the attractive interaction between two electrons takes place indirectly through lattice ions. The phonon exchange between two electrons is shown in figure (5.10). Here $\vec{q}$ is momentum imparted by first electron to lattice, which is eventually given to second electron of copper pair. Cooper pair is a quasi particle. The possibility of attraction between e_1 and e_2 is more if two electrons are moving with high speed in opposite direction. This is because if e_1 is fast, it will distort the lattice and produce condensed charge condition and immediately moves out of this region. The ions on the

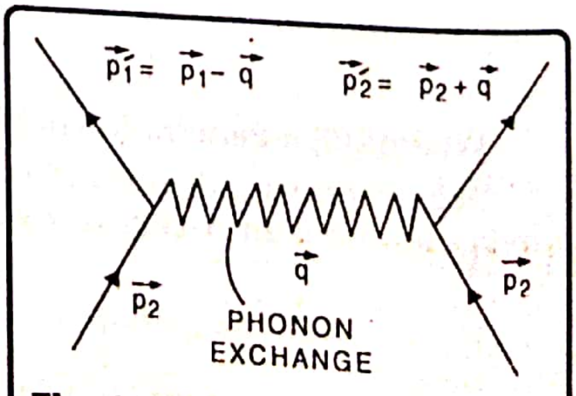

Fig. 10

other hand take large time to restore their position. Hence increased positive charge density remains there for some time. Due to this e_2 will feel only attraction from increased positive charge density and direct coulomb repulsion between e_1 and e_2 will disappear. This repulsion will further decrease if e_2 is moving in opposite direction to e_1 because in that case time of overlap of wave packets of e_1 and e_2 is very small. Since fastest moving electrons are those whose energy lies close to Fermi energy, thus cooper pair formed is most stable (possesses maximum binding energy) if electrons forming it lie close to fermi surface on energy band diagram. In fact only those electrons can form cooper pair, whose energy lies in the range $E_F - k_B T_D$ to $E_F + k_B T_D$ where E_F is Fermi Energy, k_B is Boltzmann constant and T_D is Debye temperature.

Some characteristics of cooper pairs are given below :-

(*i*) The two electrons in a cooper pair have opposite momenta and opposite spin [represented as $\vec{p}_1 \uparrow$ and $\vec{p}_2 \downarrow$]. If magnitude of momenta is also equal then net current density is zero and if $p_1 \neq p_2$ then net current density is non zero.

(*ii*) The mass of a cooper pair is $2m^*$, where m^* is effective mass of electron.

(*iii*) Charge on cooper pair is $-2e$.

(*iv*) The spin of cooper pair is zero. Hence cooper pair behaves like Boson (does not obey Pauli's Exclusion Principle).

(*v*) Volume occupied by cooper pair is 10^6 times the volume occupied by free electron.

(*vi*) Binding energy of cooper pair is of the order of $k_B T_D$ ($\approx 10^{-3}$ eV $- 10^{-4}$ eV). This energy corresponds to a temperature range of about 10K – 1K. Hence super conductivity is a low temperature phenomenon. At high temperature, thermal energy easily breaks the bonded electrons of cooper pair and material behaves like a normal conductor.

(*vii*) If external magnetic flux density $\vec{B}$ is applied across a super conductor then magnetic force on two electrons is in opposite directions $[\vec{F} = -e(\vec{V} \times \vec{B})]$. It means that magnetic field tends to break up bond in a copper pair and hence tends to destroy super conductivity. Alternatively we can easily convince ourselves that when a normal conductor is placed in external magnetic field and super conducting transition is achieved by fastly cooling the sample, then magnetic field lines will be pushed out by the electrons forming cooper pair. This shows that super conducting material must become diamagnetic at super conducting transition.

(*viii*) The binding energy of cooper pair is given approximately as $E_g \approx 3.53\, k_B T_C$ where k_C = Boltz,amm constant

From above discussion, we conclude that BCS theory explains super conductivity exhaustively.

(21) IMPLICATIONS OF BCS THEORY

Following are implications of BCS theory :

(*i*) **BCS Ground State.** The energy band diagram of superconductor is shown in the figure (9). According to BCS theory there is an attractive attraction between two electrons forming a cooper pair. Hence it contributes a negative energy term $-U_a$ to the copper pair, in addition to kinetic energy term

$$E_k = \frac{1}{2}mu^2 + \frac{1}{2}mu^2 = mu^2.$$

Thus total energy of electrons in cooper pair is

$$E = E_k - U_a = mu^2 - U_a.$$

While electrons in normal state have only kinetic energy and no attraction energy term. Thus energy per two electrons in a normal conductor is $E' = E_k = mu^2$.

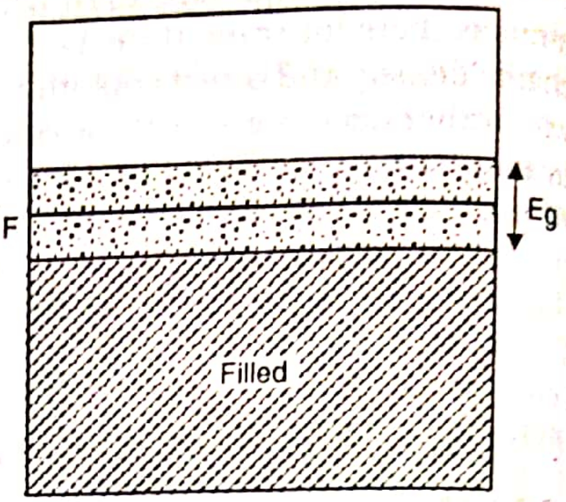

Fig. 11

Clearly $E' > E$ i.e. cooper pair is more stable than normal electrons. In order to break cooper pair an energy equal to $+U_a$ must be provided externally. This means that in the absence of any external excitation, the energy state of superconducting electrons (cooper pairs) are separated from energy states of normal electrons by a finite energy gap E_g. This set of filled energy states of a super conductor is called BCS ground state. The fermi level has energy E_F and lies symmetrically between the energy gap of BCS ground state and normal state. The Fermi energy of electron moving in 3-D is given by

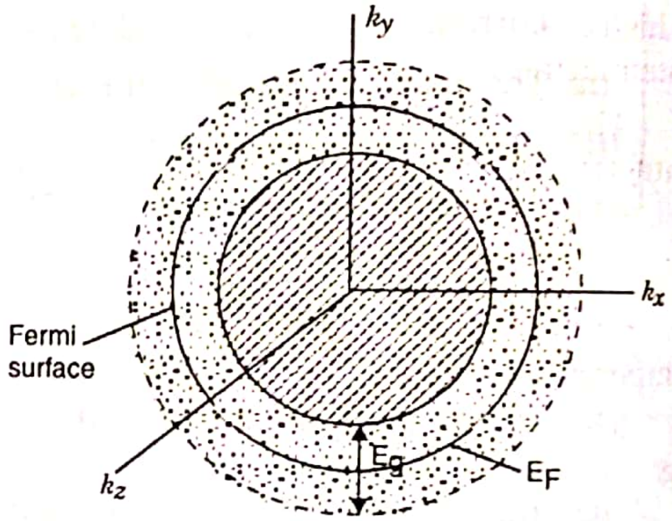

Fig. 12

$$k_x = \frac{2\pi p_x}{h}$$

$$E_F = \frac{p^2}{2m^*} = \frac{\hbar^2 k^2}{2m^*}$$

$$\left(\because p = \frac{h}{\lambda} = \frac{h}{2\pi} \times \frac{2\pi}{\lambda} = \hbar k\right)$$

or $$E_F = \frac{\hbar^2}{2m^*}\left(k_x^2 + k_y^2 + k_z^2\right)$$ (where $\vec{k} = k_x\hat{i} + k_y\hat{j} + k_z\hat{k}$)

or $$k_x^2 + k_y^2 + k_z^2 = \frac{2m^*}{\hbar^2}E_F$$

This is the equation of a sphere with centre at origin and radius $\frac{\sqrt{2m^* E_F}}{\hbar}$. Hence in momentum space (space described by k_x, k_y, k_z), fermi surface becomes a spherical system as shown in Fig. (12).

There are two features of BCS ground state.

(*a*) The total energy of BCS state is lower with respect to the Fermi Surface. Thus BCS ground sate is more stable than the fermi state.

(*b*) The one particle states are occupied in pairs ; if a wave vector $\vec{k}$ and spin up is occupied, then the wave vector with spin $-\vec{k}$ and spin down is also occupied. Conversely if a wave vector $\vec{k}$ and spin up is empty, then a wave vector $-\vec{k}$ with spin down is also empty.

***(ii)* Coherence Length.** The electrons in cooper pair cannot move coherently upto infinite distance. But they can maintain their coherence upto certain maximum distance called coherence length. It is denoted by ξ. It is a measure of the distance, with in which, the energy gap parameter Δ does not change very much in the varying magnetic field. Coherence length depends upon the temperature of the specimen. Its value at 0K is given as

$$\xi_0 = \frac{\hbar V_F}{2E_g}$$

Here $\hbar = \frac{h}{2\pi}$ is called Planck's Reduced Constant. V_F is velocity of an electron having energy equal to Fermi Energy.

(22) HIGH TEMPERATURE SUPERCONDUCTORS (HTSC)

HTSC denote the superconducting oxides with high transition temperatures accompanied by high critical magnetic field (H_C) and high critical current density (J_C). HTSC have substantial potential industrial importance.

The compound $La_2\,CuO_4$ is insulator while the compound $La_2\,M_x\,Cu\,O_{4-y}$ became Antiferromagnetic. Then following compounds were prepared viz

$La_{2-x}\,Ba_x\,Cu\,O_{4-y}$
$La_{2-x}\,Sr_x\,Cu\,O_{4-y}$ | Here x, y denote relative concentration $0 \le x, y \le 1$

With these changes, the materials became superconductors having high transition temperature $T_C = 35$ K.

Then other samples were prepared, in which Lanthanum was replaced by other elements e.g.

$YBa_2\,Cu_3\,O_{6+x}$ and $Bi\,.\,Sr_2\,Cu_3\,O_{6+x}$ | These are called 1, 2, 3 superconductors and $T_C = 92$ K for these

$Tl_2\,Ba_2\,Cu_1\,O_{6+x}$	(2, 2, 1 super conductor, $T_c = 85$ K)
$Bi_2\,Sr_2\,Ca_1\,Cu_2\,O_{6+x}$	(2, 2, 1, 2 super conductor, $T_C = 80$ K)
$Br_2\,Sr_2\,Ca_2\,Cu_3\,O_{6+x}$	(2, 2, 2, 3 superconductor, $T_C = 110$ K)
$Tl_2\,Ba_2\,Ca\,Cu_2\,O_{6+x}$	(2, 2, 1, 2 superconductor, $T_C = 108$ K)
$Tl_2\,Ba_2\,Ca_2\,Cu_3\,O_{6+x}$	(2, 2, 2, 3 superconductor, $T_C = 125$ K)

Thus the take rare alkaline earth metals for the formation of these compounds, which are oxygen deficient, and chemical bonds are non saturated in them for $x \neq 0$ and saturated when $x = 0$.

In these materials element copper is present. So Cu and O play important role in these materials. Further, all materials have tetra orthorhombic structure.

All these materials are anisotropic i.e. value of T_C is not same for all crystal axes or planes for these materials. Thus superconductivity is a "Layer Phenomenon".

Further the value of critical temperature is a function of concentration parameter x. Initially as x

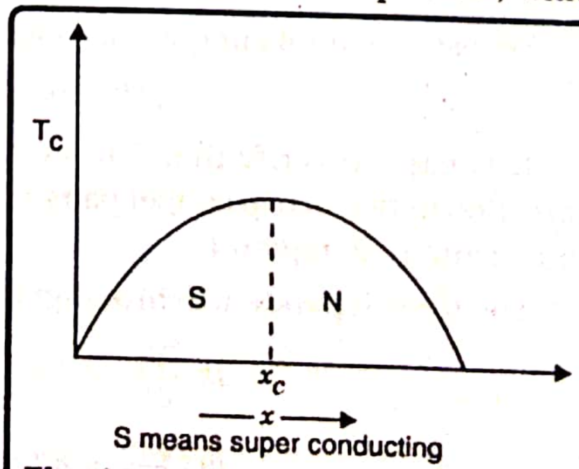

Fig. 13

increases, the value of T_C also increases. But beyond a certain value x_C, the critical temperature is found to decrease again and reaches 0K when $x = 1$.

Many of the properties of these high temperature superconductors are identical to those of conventional low T_C superconductors. These include the existence of energy gap over the entire fermi surface below T_C and Josephson effect. However they differ in certain properties from low T_C superconductors. *e.g.* Isotopic Effect in high T_C superconductors is almost nil. Further they have particularly low value of coherence length.

(23) JOSEPHSON EFFECT

Consider a thin insulating material sandwiched between two superconducting materials. The junction formed in this case is called 'Josephson Junction' or 'Weak Link'.

Under suitable conditions, super conducting electron pairs (cooper pairs) can show tunneling through the insulating layer. As a result of this, *d.c.* or *a.c.* current starts passing through junction. This effect is called Josephson Effect. It is of two types :-

(*i*) ***d.c.* Josephson Effect.** A *d.c.* current flows across the junction in the absence of any electric field (voltage) or magnetic field. This effect is called *d.c.* Josephson Effect.

(*ii*) ***a.c.* Josephson Effect.** When a *d.c.* voltage is applied across the junction, then a.c. current of very high frequency (radio frequency) starts flowing across the junction. This effect is called *a.c.* Josephson Effect. This effect has been used in finding value of $\hbar / e$ more precisely.

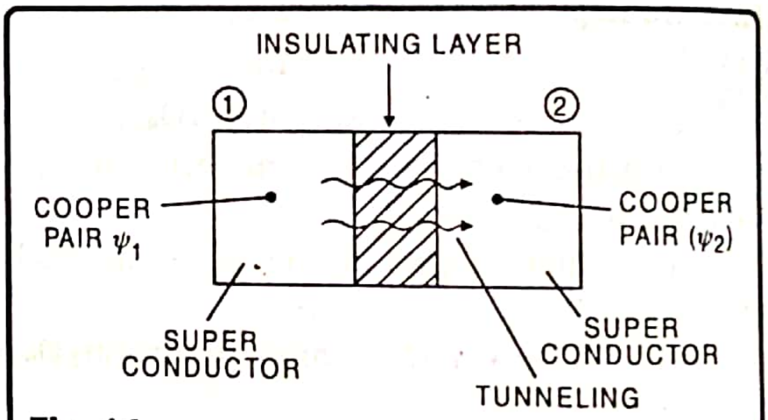

Fig. 14

***d.c.* Josephson Effect.** Let ψ_1 is wave function of cooper pair on one side of junction and ψ_2 is wave function on other side of the junction. We assume that potential on both sides of insulating layer is zero (no potential difference). The time evolution of probability amplitude is governed by time dependent wave function. The Hamiltonian contains

$$H = \text{Total energy} + \text{applied voltage contribution}$$
$$= \text{Total energy} \qquad (\because \text{ applied voltage} = 0)$$

We assume total energy of a cooper pair of the form

$$H = \text{total energy} = \hbar T \qquad \ldots(35)$$

It is easy to verify that T has dimensions of frequency. T represents the strength of interaction between two cooper pairs across the junction *i.e.* it is a measure of leakage of ψ_1 into region 2 and ψ_2 in region 1.

The time dependent Schrondigner equation applied to ψ_1 and ψ_2 gives

$$i\hbar \frac{\partial \psi_1}{\partial t} = \hbar T \psi_2 \qquad \ldots(36)$$

and

$$i\hbar \frac{\partial \psi_2}{\partial t} = \hbar T \psi_1 \qquad \ldots(37)$$

where $$\hbar = \frac{h}{2\pi}$$

Suppose $n_1(t)$ and $n_2(t)$ represent density of cooper pairs in region 1 and 2 at time t. Due to tunneling their values change with time. We assume that ψ_1, ψ_2 are of the following form

$$\left.\begin{aligned}\psi_1 &= A\sqrt{n_1}\, e^{i\theta_1} \\ \psi_2 &= A\sqrt{n_2}\, e^{i\theta_2}\end{aligned}\right\} \quad \text{...(38)}$$

Where constant A takes care of dimensions of ψ_1 and ψ_2 so that $|\psi_1|^2, |\psi_2|^2$ represent probability density. A is so chosen that ψ_1, ψ_2 are normalised.

From (38), we have

$$\frac{\partial\psi_1}{\partial t} = A\left[\frac{1}{2\sqrt{n_1}}\frac{\partial n_1}{\partial t}e^{i\theta_1} + \sqrt{n_1}\, i\, e^{i\theta_1}\frac{\partial\theta_1}{\partial t}\right]$$

$$\therefore \quad -iT\psi_2 = A\left[\frac{1}{2\sqrt{n_1}}\frac{\partial n_1}{\partial t} + i\sqrt{n_1}\frac{\partial\theta_1}{\partial t}\right]e^{i\theta_1} \quad \text{...(39) using (36)}$$

Similarly $$-iT\psi_1 = A\left[\frac{1}{2\sqrt{n_2}}\frac{\partial n_2}{\partial t} + i\sqrt{n_2}\frac{\partial\theta_2}{\partial t}\right]e^{i\theta_2} \quad \text{...(40)}$$

Multiply equation (39) by $\frac{1}{A}\sqrt{n_1}\, e^{-i\theta_1}$, we get

$$\begin{aligned}\frac{1}{2}\frac{\partial n_1}{\partial t} + in_1\frac{\partial\theta_1}{\partial t} &= -iT\frac{\sqrt{n_1}}{A}e^{-i\theta_1}\psi_2 \\ &= -iT\frac{\sqrt{n_1}}{A}e^{-i\theta_1}A\sqrt{n_2}\, e^{i\theta_2} \quad \text{(using 38)} \\ &= -iT\sqrt{n_1 n_2}\, e^{-i(\theta_1-\theta_2)}\end{aligned}$$

or $$\frac{1}{2}\frac{\partial n_1}{\partial t} + in_1\frac{\partial\theta_1}{\partial t} = -iT\sqrt{n_1 n_2}\, e^{i\delta} \quad \text{...(41)}$$

Where $$\delta = \theta_2 - \theta_1$$

Since θ_1, θ_2 are functions of time, hence δ is in general a function of time.

Similarly if we multiply equation (40) by $\frac{1}{A}\sqrt{n_2}\, e^{-i\theta_2}$, and simplify, we get

$$\frac{1}{2}\frac{\partial n_2}{\partial t} + in_2\frac{\partial\theta_2}{\partial t} = -iT\sqrt{n_1 n_2}\, e^{-i\delta} \quad \text{...(42)}$$

Put $e^{i\delta} = \cos\delta + i\sin\delta$ in (41), we get

$$\frac{1}{2}\frac{\partial n_1}{\partial t} + in_1\frac{\partial\theta_1}{\partial t} = -iT\sqrt{n_1 n_2}\,(\cos\delta + i\sin\delta)$$

Equate real and imaginary terms, we get

$$\frac{1}{2}\frac{\partial n_1}{\partial t} = T\sqrt{n_1 n_2}\sin\delta$$

or $$\frac{\partial n_1}{\partial t} = 2T\sqrt{n_1 n_2}\sin\delta \quad \text{...(43)}$$

and $$n_1 \frac{\partial \theta_1}{\partial t} = -T\sqrt{n_1 n_2}\cos\delta$$

or $$\frac{\partial \theta_1}{\partial t} = -T\sqrt{\frac{n_2}{n_1}}\cos\delta \qquad ...(44)$$

Similarly from equation (42), we can obtain

$$\frac{\partial n_2}{\partial t} = -2T\sqrt{n_1 n_2}\sin\delta \qquad ...(45)$$

and $$\frac{\partial \theta_2}{\partial t} = -T\sqrt{\frac{n_1}{n_2}}\cos\delta \qquad ...(46)$$

We assume that superconductors 1 and 2 are identical. Thus $n_1 \approx n_2$. In this case equations (44) and (46) give us

$$\frac{\partial \theta_1}{\partial t} = \frac{\partial \theta_2}{\partial t} = -T\cos\delta$$

or $$\frac{\partial \theta_2}{\partial t} - \frac{\partial \theta_1}{\partial t} = -T\cos\delta + T\cos\delta$$

$\Rightarrow$ $$\frac{\partial \theta_2}{\partial t} - \frac{\partial \theta_1}{\partial t} = 0$$

$\Rightarrow$ $$\frac{\partial}{\partial t}(\theta_2 - \theta_1) = 0$$

or $$\frac{\partial \delta}{\partial t} = 0$$

$\Rightarrow$ $$\delta = \text{constant} \qquad ...(47)$$

Also equations (43) and (45) give us (using $n_1 \approx n_2$)

$$\frac{\partial n_1}{\partial t} = -\frac{\partial n_2}{\partial t} = 2n_1 T\sin\delta \qquad ...(48)$$

However n_1, n_2 are number of cooper pairs per unit volume in superconductors 1 & 2 respectively. Thus $\frac{\partial n_1}{\partial t}$ represents rate of flow of cooper pairs from superconductor 1 to 2.

Similarly $\frac{\partial n_2}{\partial t}$ represents rate of flow of cooper pairs from superconductor 2 to 1.

Let J is current density across the Josephson junction, then by above discussion we can write.

$$J \propto \frac{\partial n_1}{\partial t} \qquad \left(\text{or equivalently } J \propto \frac{\partial n_2}{\partial t}\right)$$

or $$J \propto 2Tn_1\sin\delta$$

or $$J = 2TCn_1\sin\delta$$

Where C = constant of proportionality

or $$J = J_0\sin\delta \qquad ...(49)$$

where $J_0 = 2TCn_1$ represents peak value of current density. From (47), we see that δ is constant. Hence J is non oscillating or *d.c.* current density. Thus we conclude that when no voltage is applied across the junction, then a *d.c.* current flows across the junction giving rise to *d.c.* Josephson Effect.

It should be noted that $J_0 = 2TCn_1$ or $J_0 \propto T$ *i.e.* J_0 depends on strength of interaction across the junction. If junction is thick, then interaction across the junction is zero. Hence $T = 0$ which means $J_0 = 0$ or $J = 0$. Thus *d.c.* Josephson effect will vanish in that case.

With no voltage applied across the junction a *d.c.* current I_C or $-I_C$ will flow across the junction depending on value of $\delta = \theta_2 - \theta_1$. If voltage is applied, then current will increase linearly with voltage as shown in fig. (15).

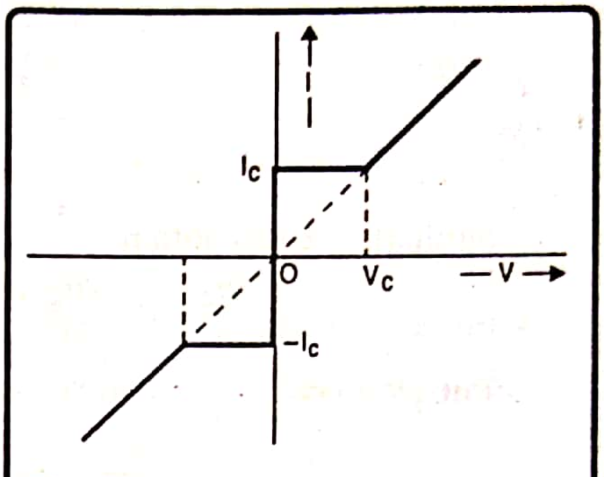

Fig. 15

a.c. Josephson Effect. Let a *d.c.* voltage V is applied across the junction. Due to this voltage, the additional energy acquired by a cooper pair is qV, where $q = -2e$ is charge on cooper pair. Thus difference in energy of cooper pair across the junction due to applied voltage is

$$E_2 - E_1 = qV = -2eV$$

or

$$E_1 - E_2 = 2eV$$

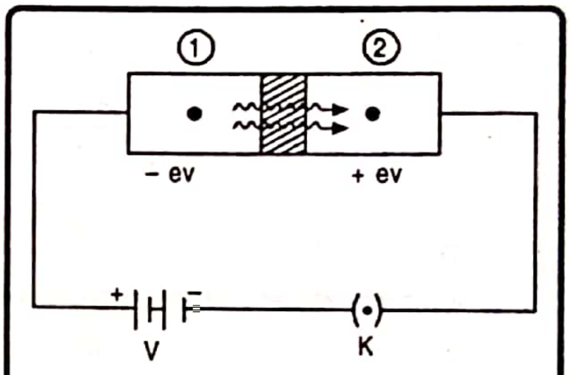

Fig. 16

For simplicity we assume potential at the centre of insulating layer to be zero. So that potential in region 1 is $\frac{V}{2}$ and in region 2 is $-\frac{V}{2}$ (so that p.d. $= \frac{V}{2} - \left(\frac{-V}{2}\right) = V$). Hence potential energy of cooper pair on one side of junction is $q\left(\frac{V}{2}\right) = -eV$ and on other side of junction is $q\left(\frac{-V}{2}\right) = eV$. Hence time dependent Schroendinger equation for region 1 and 2 is of the form

$$i\hbar \frac{\partial \psi_1}{\partial t} = \hbar T \psi_2 - eV\psi_1 \qquad \text{...(50)}$$

and

$$i\hbar \frac{\partial \psi_2}{\partial t} = \hbar T \psi_1 + eV\psi_2 \qquad \text{...(51)}$$

Put

$$\psi_1 = A\sqrt{n_1}\, e^{i\theta_1}$$

and

$$\psi_2 = A\sqrt{n_2}\, e^{i\theta_2} \qquad \text{...(52)}$$

Differentiate ψ_1 w.r.t. t, we get

$$\frac{\partial \psi_1}{\partial t} = \frac{A}{2\sqrt{n_1}} \frac{\partial n_1}{\partial t} e^{i\theta_1} + A\sqrt{n_1}\, i\, e^{i\theta_1} \frac{\partial \theta_1}{\partial t}$$

or

$$-iT\psi_2 + i\frac{eV}{\hbar}\psi_1 = A e^{i\theta_1}\left(\frac{1}{2\sqrt{n_1}} \frac{\partial n_1}{\partial t} + i\sqrt{n_1} \frac{\partial \theta_1}{\partial t}\right) \qquad \text{(using 50)}$$

Multiply by $\frac{\sqrt{n_1}\, e^{-i\theta_1}}{A}$, we get

$$\frac{1}{2}\frac{\partial n_1}{\partial t} + i n_1 \frac{\partial \theta_1}{\partial t} = -iT\frac{\sqrt{n_1}}{A} e^{-i\theta_1} \psi_2 + \frac{ieV}{\hbar} \frac{\sqrt{n_1}}{A} e^{-i\theta_1} \psi_1$$

$$= -i\frac{T\sqrt{n_1}}{A}e^{-i\theta_1}.A\sqrt{n_2}\,e^{i\theta_2}+\frac{i\,eV}{\hbar}\frac{\sqrt{n_1}}{A}e^{-i\theta_1}.A\sqrt{n_1}\,e^{i\theta_1}$$

$$= -iT\sqrt{n_1 n_2}\,e^{i\delta}+\frac{i\,eV}{\hbar}n_1 \qquad ...(53)$$

Similarly, we can obtain

$$\frac{1}{2}\frac{\partial n_2}{\partial t}+in_2\frac{\partial \theta_2}{\partial t} = iT\sqrt{n_1 n_2}\,e^{-i\delta}-\frac{ieV\,n_2}{\hbar} \qquad ...(54)$$

Put $e^{i\delta} = \cos\delta + i\sin\delta$ in (53) and (54) and equate real and imaginary parts, we get

$$\frac{\partial n_1}{\partial t} = 2T\sqrt{n_1 n_2}\sin\delta \qquad ...(55)$$

$$\frac{\partial \theta_1}{\partial t} = -T\sqrt{\frac{n_2}{n_1}}\cos\delta+\frac{eV}{\hbar} \qquad ...(56)$$

$$\frac{\partial n_2}{\partial t} = -2T\sqrt{n_1 n_2}\sin\delta \qquad ...(57)$$

$$\frac{\partial \theta_2}{\partial t} = -T\sqrt{\frac{n_1}{n_2}}\cos\delta-\frac{eV}{\hbar} \qquad ...(58)$$

For identical super conductors in region (1) & (2), $n_1 \approx n_2$, we get from (56) & (58) as:-

$$\frac{\partial \theta_1}{\partial t} = -T\cos\delta+\frac{eV}{\hbar}$$

and

$$\frac{\partial \theta_2}{\partial t} = -T\cos\delta-\frac{eV}{\hbar}$$

$\Rightarrow$

$$\frac{\partial}{\partial t}(\theta_2-\theta_1) = -\frac{2eV}{\hbar}$$

or

$$\frac{\partial\delta}{\partial t} = -\frac{2eV}{\hbar}$$

$\Rightarrow$

$$\partial\delta = -\frac{2eV}{\hbar}\partial t$$

$\Rightarrow$

$$\int_0^t \partial\delta = -\frac{2eV}{\hbar}\int_0^t \partial t$$

$\Rightarrow$

$$\delta(t)-\delta(0) = -\frac{2eV}{\hbar}t \qquad ...(59)$$

Also under the condition $n_1 \approx n_2$, (55) and (57) give us

$$\frac{\partial n_1}{\partial t} = \frac{-.\partial n_2}{\partial t} = 2T\,n_1\sin\delta$$

or

$$\frac{\partial n_1}{\partial t} = 2Tn_1\sin\left[\delta(0)-\frac{2eV}{\hbar}t\right] \qquad \text{(using (59))}$$

The current density is given as

$$J \propto \frac{\partial n_1}{\partial t}$$

or

$$J = C\frac{\partial n_1}{\partial t}$$

$$= 2T\, n_1\, C \sin\left[\delta(0) - \frac{2eV}{\hbar}t\right]$$

$$= J_0 \sin[\delta(0) - \omega t] \qquad ...(60)$$

where $J_0 = 2Tn_1\,C$ is peak value of current density

And $$\omega = \frac{2eV}{\hbar} \qquad ...(61)$$

From equation (60), we see that current flowing through junction is alternating and its angular frequency is $\frac{2eV}{\hbar}$. This gives rise to *a.c.* Josephson effect. It should be noted that applied *d.c.* voltage produces *a.c.* current. If a *d.c.* voltage of 1μV is applied then

$$\nu = \frac{\omega}{2\pi} = \frac{2eV}{\hbar} = \frac{eV}{\pi\hbar} = 483.6\,\text{MHz}.$$

Thus frequency of a.c. is very high.

From equation (61) we have $\hbar\omega = 2eV$.

Since $\hbar w$ is energy of phonon. It means that whenever a cooper pair crosses the junction, a phonon of energy $2eV$ is emitted or absorbed. Further from (61) we have $\frac{\hbar}{e} = \frac{2V}{\omega}$. Since V and ω are accurately known, hence equation (61) can be used for determining $\frac{\hbar}{e}$ accurately.

(24) FLUX QUANTISATION

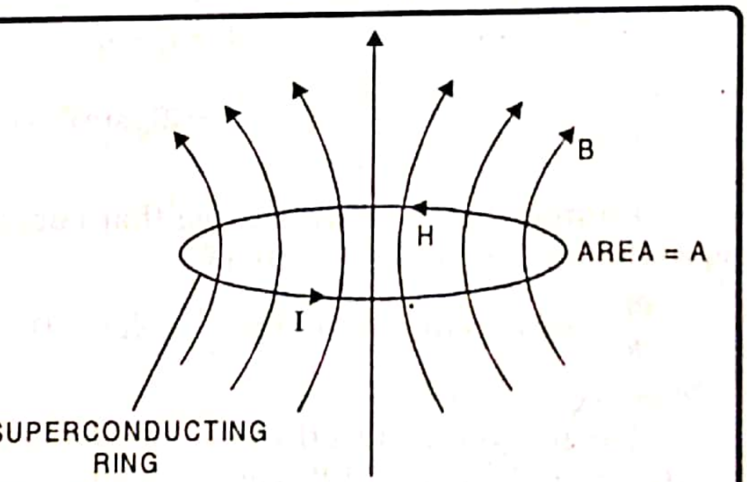

Fig. 17

Figure (17) shows a super conducting ring carrying current I. This current produces magnetic field of flux density B. If A is the area of ring, then magnetic flux associated with the ring is ϕ = BA. If we change the current in the ring, then magnetic flux linked with the ring will also change. However according to Faraday's Law of e.m. induction, whenever there is change in magnetic flux, there will be an induced current in the ring which will try to keep the magnetic flux through ring constant. In case of normal conductors (possessing resistance) the induced current is always less than change in applied current so that total current and total flux will show a finite and non zero change. (*e.g.* if applied current is increased from 5*m*A to 6*m*A then magnitude of induced current is less than change in current *i.e.* $I_{induced} < 6mA - 5mA = 1mA$). However the resistance of a super conductor is zero. Hence when ring is super conducting, then induced current is always exactly equal to current changed through the ring. In other words change in flux due to applied current will cancel out change in flux due to induced current. Hence magnetic flux through the coil is permanently trapped.

However the phase of wave function of cooper pairs in the ring must be continuous around the ring. This is possible only if magnetic flux linked with coil is quantized. It is found that magnetic flux through a super conducting ring is always an integral multiple of $\frac{h}{2e}$

i.e.
$$\phi = n\left(\frac{h}{2e}\right) = n\phi_0 \quad ...(62)$$
$$n = 1, 2, 3, 4,$$

Here $\phi_0 = \frac{h}{2e} = 2.068 \times 10^{-15}\, Tm^2$ is flux quantum and is called 'Fluxoid' or 'Fluxon'. Flux quantisation is a beautiful example of long range quantum effect, in which coherence of a superconducting state extends over a ring or a solenoid.

(25) SQUID

The SQUID is an aconym for Super conducting Quantum Interference Device. The device may be configured for measuring those quantities which change with magnetic flux. It can be operated on *a.c.* or *d.c.* It works on the principle that maximum super current in a ring is a function of magnetic flux. A SQUID consists of two Josephson junctions P & Q connected in parallel to a voltage source (See Fig. (18)). A magnetic field of flux density $\vec{B}$ is applied normal to the plane of circuit (shown by $\oplus$ sign.)

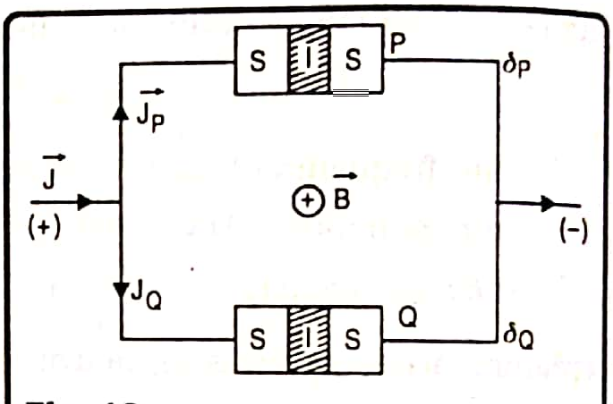

Fig. 18

Let ϕ = magnetic flux linked with the interior to circuit. Let J_P and J_Q are current density through Josephson junction P & Q and J is total current density. Then it can be proved that

$$J = J_P + J_Q$$
$$= 2J_0 \sin\delta_0 \cos\left(\frac{e\phi}{\hbar}\right) \quad \text{(where } J_0 \text{ is Peak current density)} \quad ...(63)$$

From equation (63), we see that current variation in (63) shows maxima for

$\frac{e\phi}{\hbar} = n\pi$ and minimum for $\frac{e\phi}{\hbar} = (2n+1)\pi/2$

where $n = 0, 1, 2, 3, 4 ....$

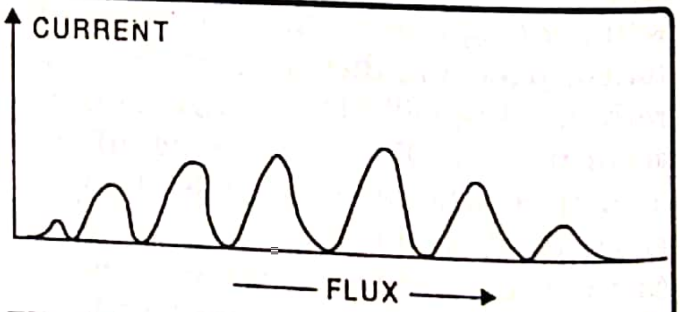

Fig. 19

The magnitude flux through the ring varies as shown in the figure (19). Hence super current passing through the circuit is a measure of magnetic flux (or magnetic flux density). Hence the device behaves as a sensitive magnetometer. SQUIDs have been used to measure magnetic fields in mouse brains to test whether there in enough magnetism to attribute to their navigational ability to an internal compass. High temperature super conductor SQUIDS are being used by US Navy to detect mines and submarines.

(26) APPLICATIONS OF SUPERCONDUCTORS

Superconductors are useful in a number of applications. A few are listed below :

(*i*) Generation and transmission of electric power.

(*ii*) Medical diagnosis.

(*iii*) Highly powerful strong field superconductors

(*iv*) Super computers.
(*v*) Magnetically levitating world's fastest trains.
(*vi*) Magnetic energy storage devices.
(*vii*) Electromagnetic shielding.
(*viii*) Superconducting transformers.
(*xi*) In medical industry as Superconducting Quantum Interferometers (SQUIDS).

Example 1. *For a certain specimen, the critical fields are 2.8 × 10⁵A/m 5.4 × 10⁵ A/m for 14K and 13K respectively. Calculate the transition temperature and critical fields at 0K and 5.0K.*

Solution. Given $H_C(14K) = 2.8 \times 10^5 \text{ A}/m$

and $H_C(13K) = 5.4 \times 10^5 \text{ A}/m$

Now $$H_C T = \left[1 - \frac{T^2}{T_C^2}\right]$$

Hence $$H_C(14) = H_C(0)\left[1 - \frac{14^2}{T_C^2}\right] \quad ...(i)$$

and $$H_C(13) = H_C(0)\left[1 - \frac{13^2}{T_C^2}\right] \quad ...(ii)$$

Divide (*i*) by (*ii*) we get

$$\frac{H_C(14)}{H_C(13)} = \frac{T_C^2 - 14^2}{T_C^2 - 13^2}$$

or $$\frac{2.8 \times 10^5}{5.4 \times 10^5} = \frac{T_C^2 - 14^2}{T_C^2 - 13^2}$$

or $$T_C = 14.96 \text{ K}$$

Put value of T_C in (*i*) we get

$$H_C(0) = 2.25 \times 10^6 \text{ A}/m$$

Now $$H_C(5.0\text{ K}) = H_C(0) \times \left[1 - \frac{(5.0)^2}{T_C^2}\right] = 2.25 \times 10^5 \left[1 - \frac{(5.0)^2}{(14.96)^2}\right]$$

$$= 2.0 \times 10^6 \text{ A}/m$$

Example 2. *The critical temperature T_C for H_g with isotopic was 199.5 is 4.185 K. Calculate its critical temperature, when isotopic mass changes to 203.4.*

Solution. We know $T_C\sqrt{M}$ = constant

$$\Rightarrow \quad \frac{T_{C_1}}{T_{C_2}} = \sqrt{\frac{M_2}{M_1}}$$

$$\Rightarrow \quad T_{C_2} = T_{C_1}\sqrt{\frac{M_1}{M_2}} = 4.185\sqrt{\frac{199.5}{203.4}} = 4.14 \text{ K}$$

Example 3. *Calculate the critical current density for 1mm diameter wire of lead at 4.2 K. Assume the critical magnetic field depends upon T. Given T_C for lead is 7.18 K and $H_c(0)$ for lead is 6.5 × 10⁴ A/m.*

Solution. We know $$H_C(T) = H_C(0)\left[1 - \frac{T^2}{T_C^2}\right]$$

Put $T = 4.2$ K, $T_C = 7.18$ K, $H_C(0) = 6.5 \times 10^4$ A/m.

We get $$H_C = 6.5 \times 10^4\left[\frac{(7.18)^2 - (4.2)^2}{(7.18)^2}\right] = 4.28 \times 10^4 \text{ A}/m$$

Now $$H_C = \frac{I_C}{2\pi r} \quad \text{...(}i\text{)}$$

and $$J_C = \frac{I_C}{\pi r^2} = \frac{1}{\pi r^2}(2\pi r H_C) \quad \text{(using (}i\text{))}$$

$$= \frac{2H_C}{r} = \frac{4H_C}{D} \quad \text{where } D = 2r = \text{diameter}$$

Hence $$J_C = \frac{4 \times 4.28 \times 10^4}{10^{-3}} = 1.712 \times 10^8 \text{ A}/m^2$$

Example 4. *The London Penetration depth for a sample at 6 K & 7K are 41.2 nm and 180.3 nm respectively. Calculate its transition temperature as well as the penetration depth at 0K.*

Solution. We know $$\lambda(T) = \frac{\lambda(0)}{\sqrt{1 - \left(\frac{T}{T_C}\right)^4}} \quad \text{...(}i\text{)}$$

Given $\lambda(6) = 41.2\ nm$, $\lambda(7.1) = 180.3\ nm$

From (i) $$\frac{\lambda(6)}{\lambda(7)} = \sqrt{\frac{1 - \left(\frac{6}{T_C}\right)^4}{1 - \left(\frac{7}{T_C}\right)^4}}$$

$$\Rightarrow \quad \left(\frac{42.1}{180.3}\right)^2 = \frac{T_C^4 - (6)^4}{T_C^4 - (7)^4}$$

Solving we get $T_C = 7.04$ K

Again from (i) we have

$$\lambda(5) = \frac{\lambda(0)}{\sqrt{1 - \left(\frac{5}{T_C}\right)^4}}$$

$$\Rightarrow \quad \lambda(0) = 42.1\ nm\sqrt{1 - \left(\frac{5}{7.04}\right)^4} = 28.32\ nm$$

Example 5. *Estimate the penetration depth for Sn from the following data at 2.3 K: $T_C = 3.7$ K, density = 7.3 g/cm³, Atomic Weight = 118.7, Effective mass = 1.9 m_0, where $m_0 = 9.1 \times 10^{-31}$ kg is rest mass of electron.*

Solution. Number of atoms per mole = 6.023×10^{23}

$\therefore$ 118.7 g (1 mole) of Sn contains $= 2 \times 6.023 \times 10^{23} = 12.046 \times 10^{23}$ electrons

($\because$ Valency of Sn is 2 so one atom provides 2 free electrons)

Let N = Total free electrons inside material

n_s = number of free electrons per unit volume

$$\therefore \quad n_s = \frac{N}{\text{Volume}} \text{ (assume the specimen is purely superconducting)}$$

$$\Rightarrow \quad n_s = \left(\frac{M}{\text{Volume}}\right) \times \left(\frac{12.46 \times 10^{23}}{M_a}\right)$$

$\because$ If M = mass of specimen M_a = Molar mass then number of moles are $\frac{M}{M_a}$ and number of electrons will be $\left(\frac{M}{M_a}\right) \times 12.46 \times 10^{23}$

$$= \frac{\rho}{M_a} \times 12.43 \times 10^{23} \quad \left(\because \frac{M}{V} = \rho\right)$$

$$= \frac{7.3\ g/cm^3 \times 12.43 \times 10^{23}}{118.7 g}$$

$$= 7.40 \times 10^{22}\ cm^{-3} = 7.4 \times 10^{28}\ m^{-3}$$

m = mass of electron $= 1.9 \times 9.1 \times 10^{-31}\ kg$

The penetration depth is given by

$$\lambda\ (0) = \sqrt{\frac{m}{n_s e^2 \mu_0}} = \sqrt{\frac{1.9 \times 9.1 \times 10^{-31}}{7.4 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 4\pi \times 10^{-7}}}$$

$$= 269\ \text{Å}$$

The penetration depth of temperature T is given as

$$\lambda(T) = \frac{\lambda(0)}{\sqrt{1 - \left(\frac{T}{T_C}\right)^4}}$$

Given T = 2.3 K, T_C = 3.7K also $\lambda(0)$ = 269Å.

$$\therefore \quad \lambda(T) = \frac{269\text{Å}}{\sqrt{1 - \left(\frac{2.3}{3.7}\right)^4}} = 291\ \text{Å Ans.}$$

Example 6. *A type I superconductor with T_C = 7K has slope $\frac{dB_C}{dT} = -25 mT/K$ at T_C. Estimate its critical field at 6K. Calculate the jump in specific heat at T_C.*

Solution. We know
$$H_C(T) = H_C(0) \left[1 - \frac{T^2}{T_C^2}\right]$$

Multiply by μ_0 and use $B = \mu_0 H$, we get

$$B_C(T) = B_C(0) \left[1 - \frac{T^2}{T_C^2}\right] \quad \ldots(i)$$

Differentiate w.r.t. T, we get

$$\frac{dB_C}{dT} = -B_C(0) \left[\frac{2T}{T_C^2}\right] \quad \ldots(ii)$$

Given when $T = T_C = 7K$ then $\dfrac{dB}{dT} = -25mT/K$

Put in (*ii*) ⇒ $\quad -25mT/K = -\dfrac{2B_C(0)}{(7)}$

or $\quad B_C(0) = 87.5mT \quad$ ...(*iii*)

∴ From (*i*) $\quad B_C(6) = 87.5\,mT\left[1 - \dfrac{6^2}{7^2}\right] \quad$ (using (*iii*))

$$= 23.21\,mT$$

Also jump in specific heat is given by the expression

$$C_S - C_N = \mu_0 T_C \left(\frac{dH_C}{dT}\right)^2 = \mu_0 T_C \times \frac{1}{\mu_0^2}\left(\frac{dB_C}{dT}\right)^2 \quad (\because B = \mu_0 H)$$

$$= \frac{T_C}{\mu_0}\left(\frac{dB_C}{dT}\right)^2 = \frac{7}{4\pi \times 10^{-7}} \times \left(-25 \times 10^{-3}\right)^2 = 3.482\ JK^{-1}m^{-3}$$

(Remember we defined thermodynamic variables per unit volume of the sample)

Exercise 1. Determine critical field required to destroy super conductivity at 6K in Pb whose critical temperatrue is 7.19K and $B_C(0) = 0.803$ Tesla. [Ans. 0.0244T]

Exercise 2. For a certain super conductor the critical fields at 14K & 13K are 1.4×10^5 A/m and 4.2×10^5 A/M respectively. Find critical temperature. [Ans. 14.47 K]

Exercise 3. Calculate coherence length of Al, whose energy gap is 3.8×10^{-4} *e*V and fermi velocity is 1.9×10^8 cm/s. [Ans. 164.7×10^{-6} cm]

Exercise 4. Calculate the wavelength energy, which can break a copper pair in a specimen, whose critical temperature is 0.56 K. Given $E_g = 3.52\, k_B T_C$. [Ans. 3.9×10^{-3}m]

Exercise 5. The London penetration depth of Pb at 0K is 391Å. Calculate its value at 2K. [Ans. 392.3 Å]

SHORT ANSWER TYPE QUESTIONS

Q. 1. What is the difference between type I and type II superconductors ?

Ans. Type I superconductors show Meissner effect strictly and posses only one critical field. These materials do not show Hysteresis loss. On the other hand type II materials donot obey Meissner effect strictly. They possess two distinct critical fields and show Hysterisis loss.

Q. 2. What factors can affect the superconducting behaviour of a superconductor ?

Ans. The superconducting behaviour can be affected by

(*i*) temperature of the substance

(*ii*) the magnetic field applied across substance

(*iii*) the current passed through it.

Q. 3. What is Josephson Effect ?

Ans. When a very thin layer of an insulator is sandwiched between two superconductors and a voltage is applied across the two superconductors, a current starts flowing through the insulating layer. This effect is called Josephson Effect. The current produced in the thin layer has both a.c. and d.c. components. The d.c. component of current persists even when the applied voltage is removed, while a.c. component of current dies very quickly when applied voltage is removed.

Q. 4. What are SQUIDS ?

Ans. SQUID is an acronym for superconducting Quantum Interference Device. SQUID is a superconducting loop, which has weak link, that can measure change in magnetic flux within in the loop. SQUIDS are very sensitive magnetometers and find applications in various electric and magnetic field measurements.

Q. 5. Good conductors need not be good super conductors. Why ?

Ans. Lattice Vibrations play an important role in super conductivity. The materials in which amplitude of lattice vibration is large are good super conductors. The atoms of good conductors like copper, silver, sodium etc. have large cohesive energy and hence small amplitude of lattice vibrations. Hence they need not be good super conductors.

QUESTIONS

1. What do you understand by type I and type II superconductors ? Give BCS theory of superconductivity.
2. Why good conductors are not good superconductors ?
3. Write a short note on superconductivity.
4. What is superconductivity ? List the important applications of superconductors.
5. Explain Meissner Effect.
6. Distinguish between type I and type II superconductors. Briefly discuss the theory of superconductivity.
7. State and explain Meissner Effect. How do London equations account for this effect ?
8. Treating the transition from the normal state to the superconducting state as reversible thermodynamic process, find the discontinuity in specific heat of a superconductor at its critical temperature.
9. Enumerate the factors, which can lead to the destruction of superconductivity.
10. Write short notes on
    (*i*) Persistent current in a superconductor
    (*ii*) Type I and Type II superconductors.
11. What is Meissner Effect ? Prove that the Meissner Effect and the disappearance of resistivity in a superconductivity are two independent and essential properties of the superconducting state.
12. Outline some experimental facts about superconductivity.
13. What is isotopic effect in superconductors.
14. Mention some important properties, which undergo a change during superconducting transition.
15. Explain the concept of energy gap in superconductors.
16. Derive London equations and discuss how its solution explain Meissner Effect.
17. List some high temperature superconductors.
18. Differentiate between Soft and Hard Superconductors.
19. Describe the effect of magnetic field on a superconductor.
20. Define London penetration depth and write the importance.