9

Projections of Lines

9.1 INTRODUCTION

A line is the path of a moving point. It may be a curved line or a straight line. A straight line is the shortest distance between two points. The projections of a straight line can be drawn by joining the respective projections of its end points. The position of a straight line in space may be defined by

- the location of its end points or extremities from principal planes and its true length (TL).
- the location of one of its end points and inclination of the line with the principal planes.
- the location of its end points and distance between the end projectors etc.

9.2 POSITION OF A STRAIGHT LINE

Various positions, which a line in space can take with reference to the principal planes of projections are as follows :

(*i*) Line parallel to both HP and VP.

(*ii*) Line inclined to one plane and parallel to the other
- Line inclined to the HP and parallel to the VP.
- Line inclined to the VP and parallel to the HP.

(*iii*) Line perpendicular to one of the planes.
- Line perpendicular to the HP.
- Line perpendicular to the VP.

(*iv*) Line contained by one or both of the principal planes
- Line contained by the HP.
- Line contained by the VP.
- Line contained by both HP and VP.

(*v*) Line inclined to both HP and VP.

(*vi*) Line contained by a profile plane (PP) or line contained by a plane, perpendicular to both HP and VP.

9.3 LINE PARALLEL TO BOTH HP AND VP

Line *AB* parallel to both HP and VP is shown in Fig. 9.1 (*a*). Here ends *A* and *B* of a line *AB* are at equal distances from the HP and VP. When a line is parallel to any plane, its projections on that plane is a straight line of the same length. Its front view *a'b'* and top view *ab* are both parallel to *xy* and their lengths represent the true length (*AB* of a line).

Fig. 9.1 (*b*) shows the orthographic projections of the line *AB*.

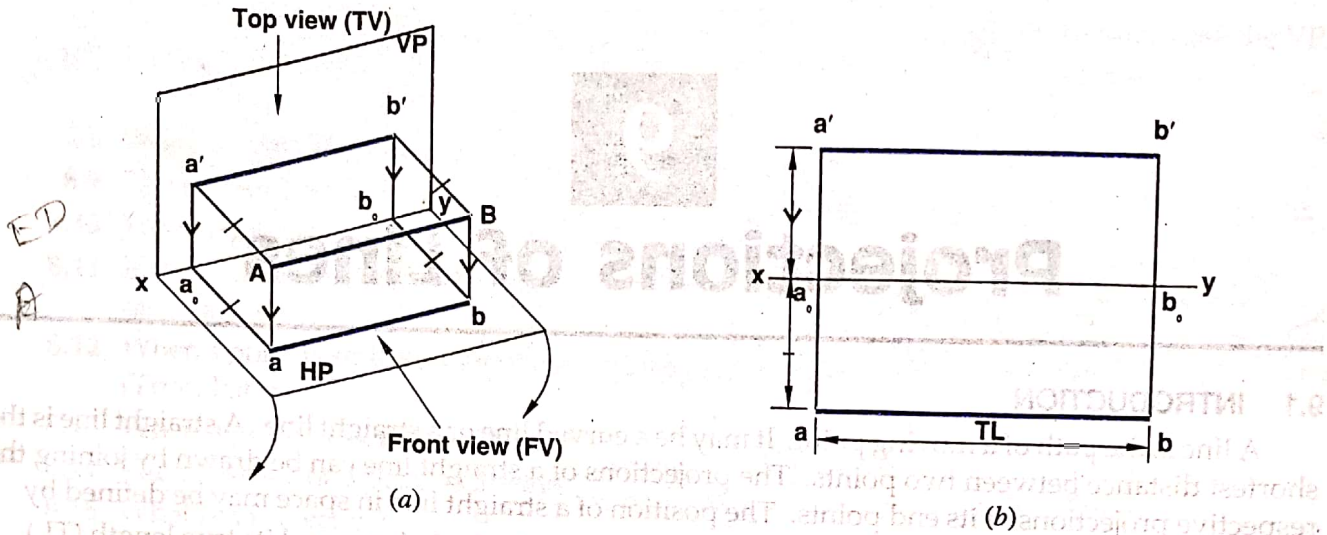


Fig. 9.1 Line parallel to both HP and VP

PROBLEM 9.1 *A line AB, 50 mm long has its end A 30 mm away from the HP and 20 mm away from the VP. The line is parallel to both HP and VP. Draw its projections in all the four quadrants.*

SOLUTION.

(*i*) Draw a *xy* line and draw a projector perpendicular to it.
(*ii*) On $a'a_0$, mark front view of *A* as *a'* at a distance equal to the distance of *A* from HP.
(*iii*) On aa_0, mark top view of *A* as *a* at a distance equal to the distance of *A* from VP.
(*iv*) Through *a'* and *a*, draw horizontal lines parallel to *xy* and mark each 50 mm true length, to locate *b'* and *b*.
(*v*) Join *a'* with *b'* and *a'* with *b*.
(*vi*) Then *a'b'* and *ab* are the required front view and top view, respectively.

Follow the above mentioned steps to draw the projections of a line in all the four quadrants. See Fig. 9.2.

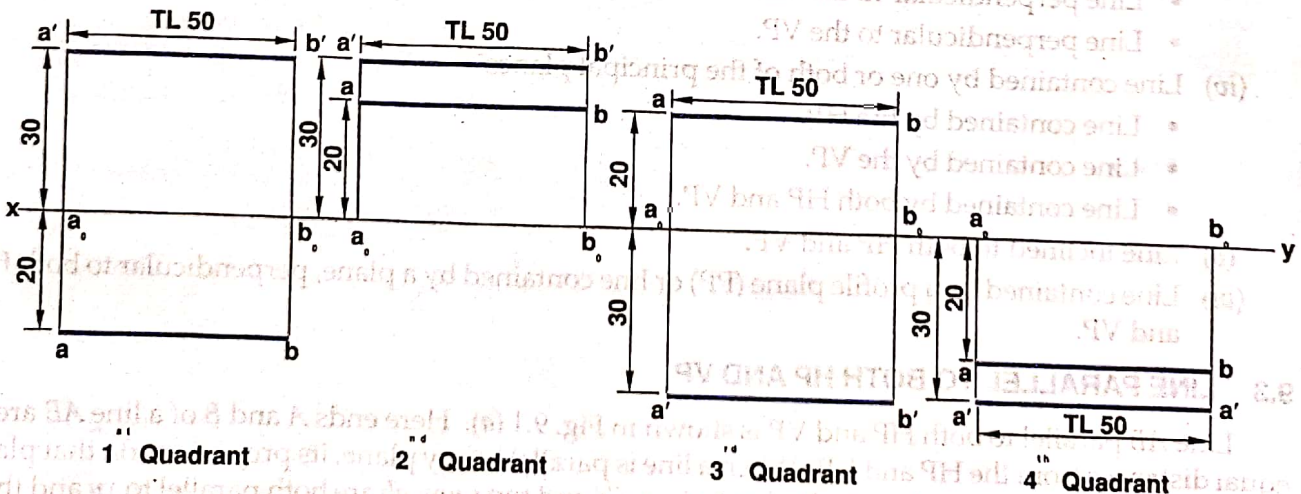


Fig. 9.2 Solution to problem 9.1

9.4 LINE INCLINED TO ONE PLANE AND PARALLEL TO THE OTHER

When a line is inclined to one plane and parallel to the other, its projections on the plane to which it is parallel gives the true length of a line and its projections on the plane to which it is inclined is a straight line, shorter than its true length but parallel to the *xy*.

(*a*) Line inclined to the HP and parallel to the VP

Fig. 9.3 (*a*) shows the line *AB*, inclined to the HP at an angle θ and parallel to the VP. Its front view *a'b'* of the line is inclined to the *xy* at an angle θ and the length of the front view (or elevation) is equal to its true length. Its top view (or plan) *ab* is shorter than the line *AB* and is parallel to the *xy*.

Fig. 9.3 (*b*) shows the orthographic projections of a line *AB*.

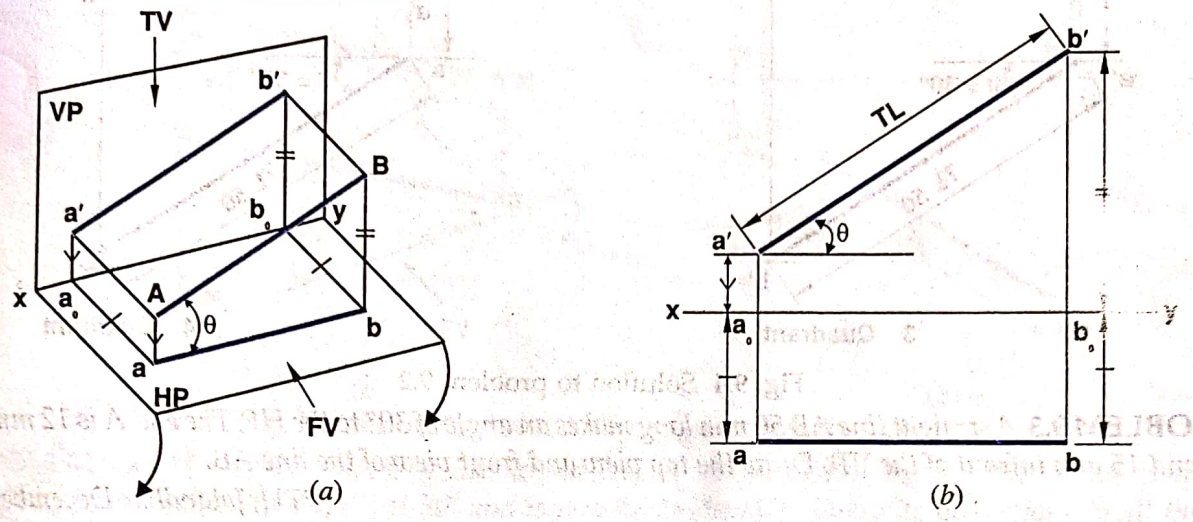


Fig. 9.3 Line inclined to the HP and parallel to the VP

PROBLEM 9.2 *A line AB 50 mm long, has its end A is 25 mm away from the HP and 15 mm away from the VP. The line is inclined to the HP at 30° and parallel to the VP. Draw its projections in all the four quadrants. Assume that the whole of the line lies in the same quadrant.*

SOLUTION.

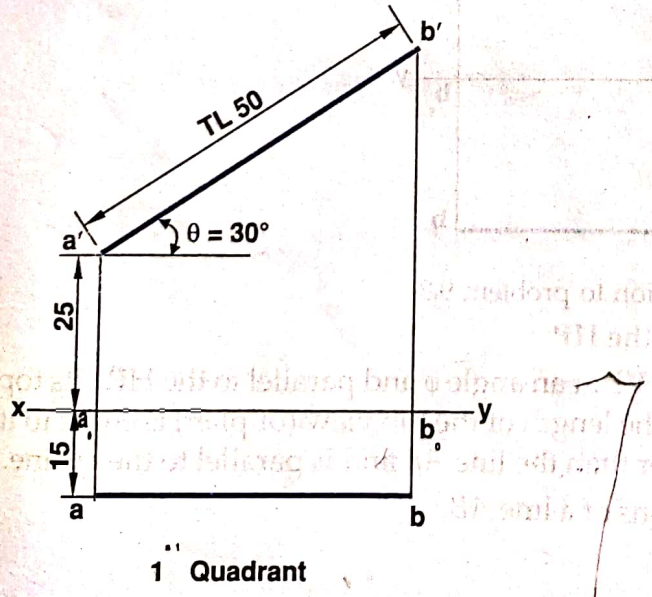
(*i*) Draw a *xy* line. At a convenient point on *xy*, draw the projector of end point A.

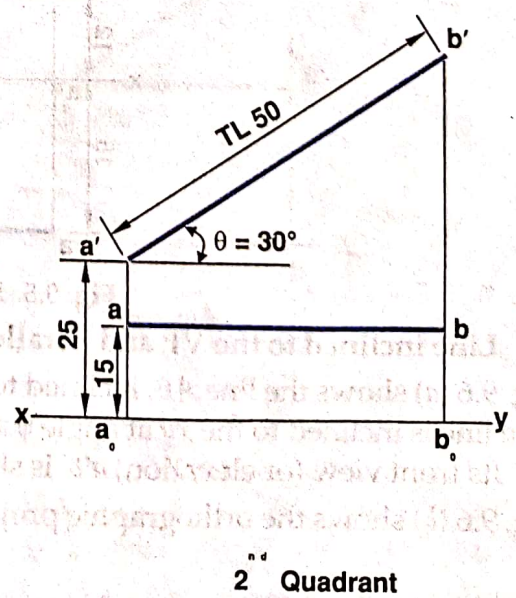
(*ii*) On this projector, draw front view *a'* and top view *a* on it.

(*iii*) Draw front view *a'b'* of the line at an angle 30° to *xy*. Cut *a'b'* = 50 mm (TL).

(*iv*) Project *a'* to *a* and *b'* to *b* on top view *ab*, which is a line parallel to *xy*.

Follow the above mentioned steps to draw the projections of a line in all the four quadrants. See Fig. 9.4.





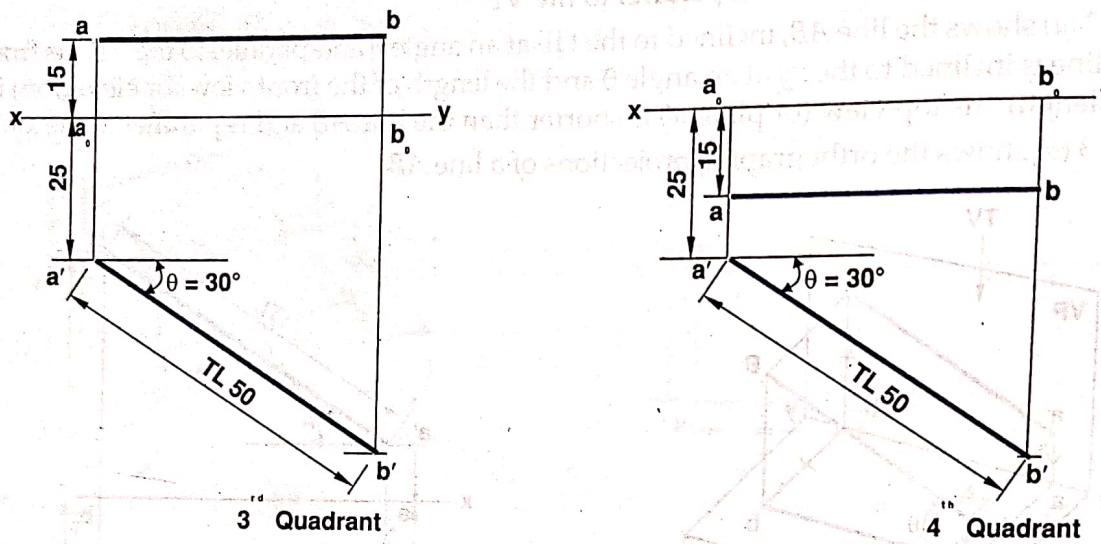


Fig. 9.4 Solution to problem 9.2

PROBLEM 9.3 *A straight line AB 50 mm long makes an angle of 30° to the HP. The end A is 12 mm above the HP and 15 mm infront of the VP. Draw the top view and front view of the line AB.*

(PTU, Jalandhar December 2007)

SOLUTION. Here, it is assumed that the line is inclined to the HP and parallel to the VP.

(*i*) Draw *xy* line. At a convenient distance on *xy*, draw the projector of end point *A*.

(*ii*) On this projector, draw front view *a'* and top view *a* on it.

(*iii*) Draw front view *a' b'* of the line at an angle of 30° to *xy*. Cut *a' b'* ≡ 50 mm (TL).

(*iv*) Project *a'* to *a* and *b'* to *b* on top view *ab*, which is a line parallel to *xy* line. See Fig. 9.5.

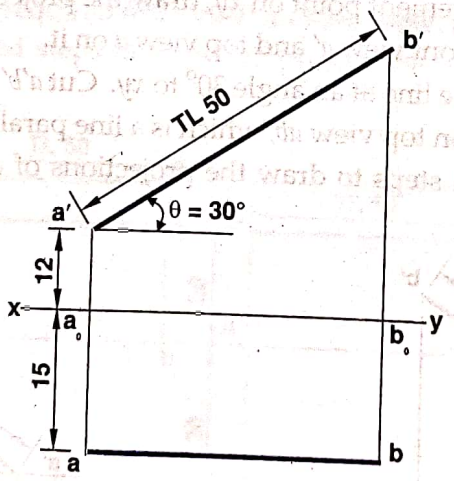


Fig. 9.5 Solution to problem 9.3

(*b*) Line inclined to the VP and parallel to the HP

Fig. 9.6 (*a*) shows the line *AB*, inclined to the VP at an angle ϕ and parallel to the HP. Its top view *ab* of the line is inclined to the *xy* at angle ϕ and the length of the top view (or plan) is equal to its true length. Its front view (or elevation) *a'b'* is shorter than the line *AB* and is parallel to the *xy* line.

Fig. 9.6 (*b*) shows the orthographic projections of a line *AB*.

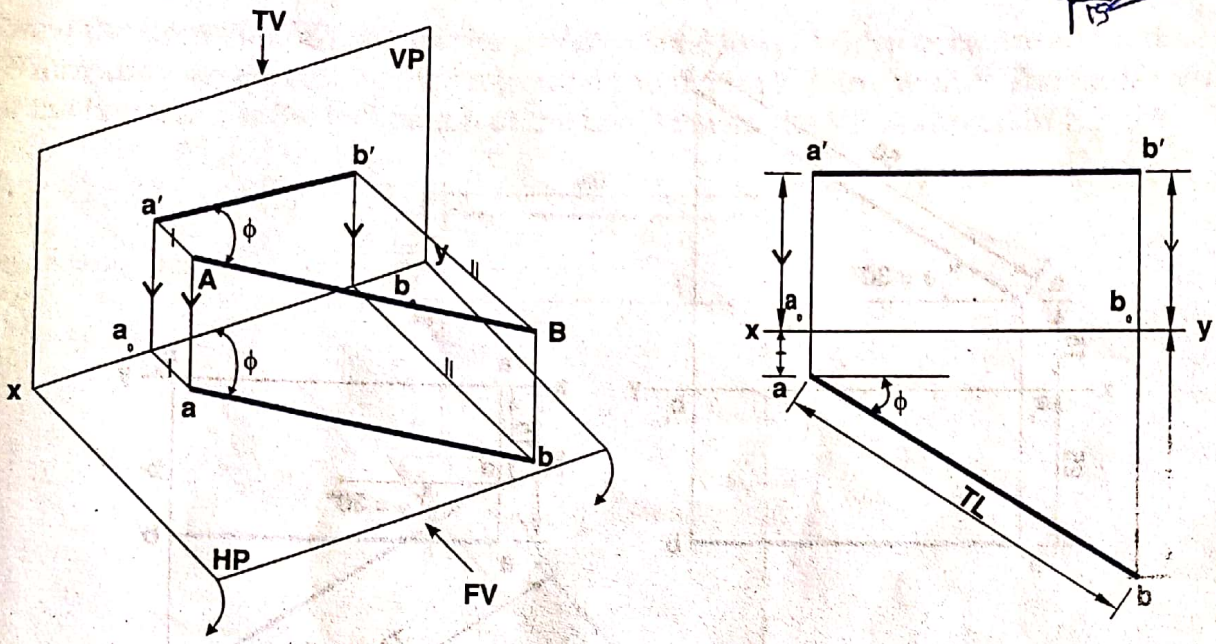


Fig. 9.6 Line inclined to the VP and parallel to the HP

PROBLEM 9.4 *A line AB 50 mm long, has its end A is 25 mm away from the HP and 15 mm away from the VP. The line is inclined to the VP at 30° and is parallel to the HP. Draw its projections in all the four quadrants. Assume that the whole of the line lies in the same quadrant.*

SOLUTION.

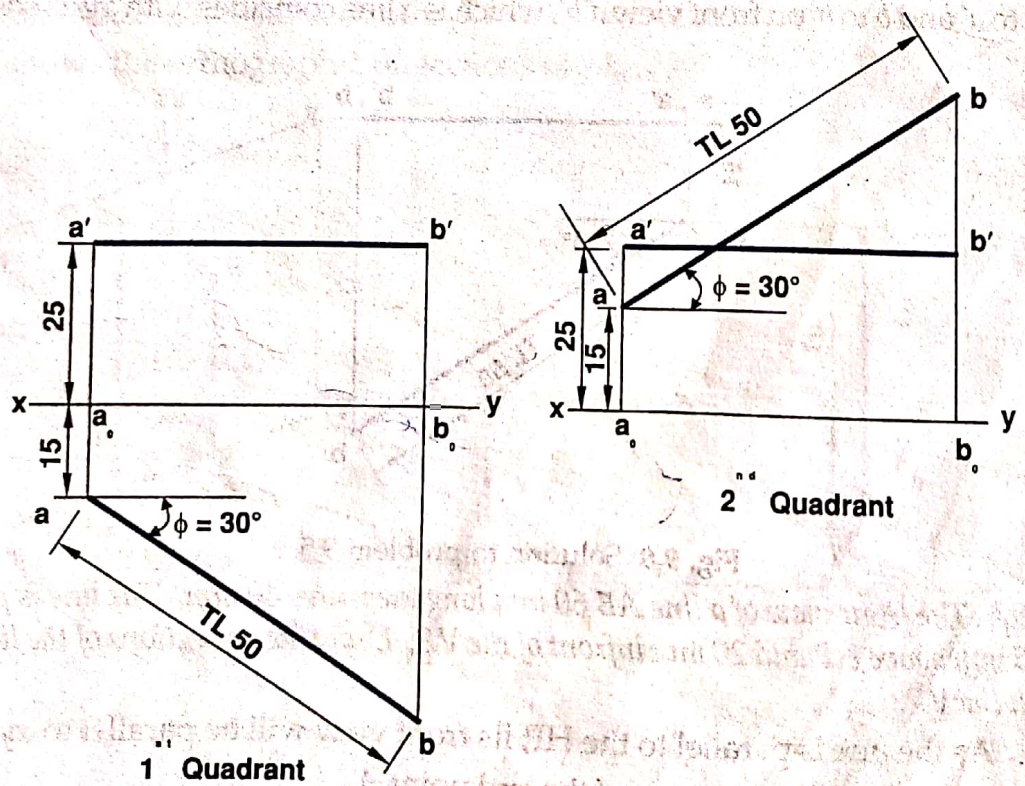
(*i*) Draw a *xy* line. At a convenient distance on *xy*, draw the projector of end point *A*.

(*ii*) On this projector, draw front view *a'* and top view *a* on it.

(*iii*) Draw top view *ab* of the line at an angle of 30° to *xy*. Cut *ab* = 50 mm (TL).

(*iv*) Project *a* to *a'* and *b* to *b'* on front view *a'b'*, which is a line parallel to *xy*.

Follow the above mentioned steps to draw the projections of a line in all the four quadrants. See Fig. 9.7.



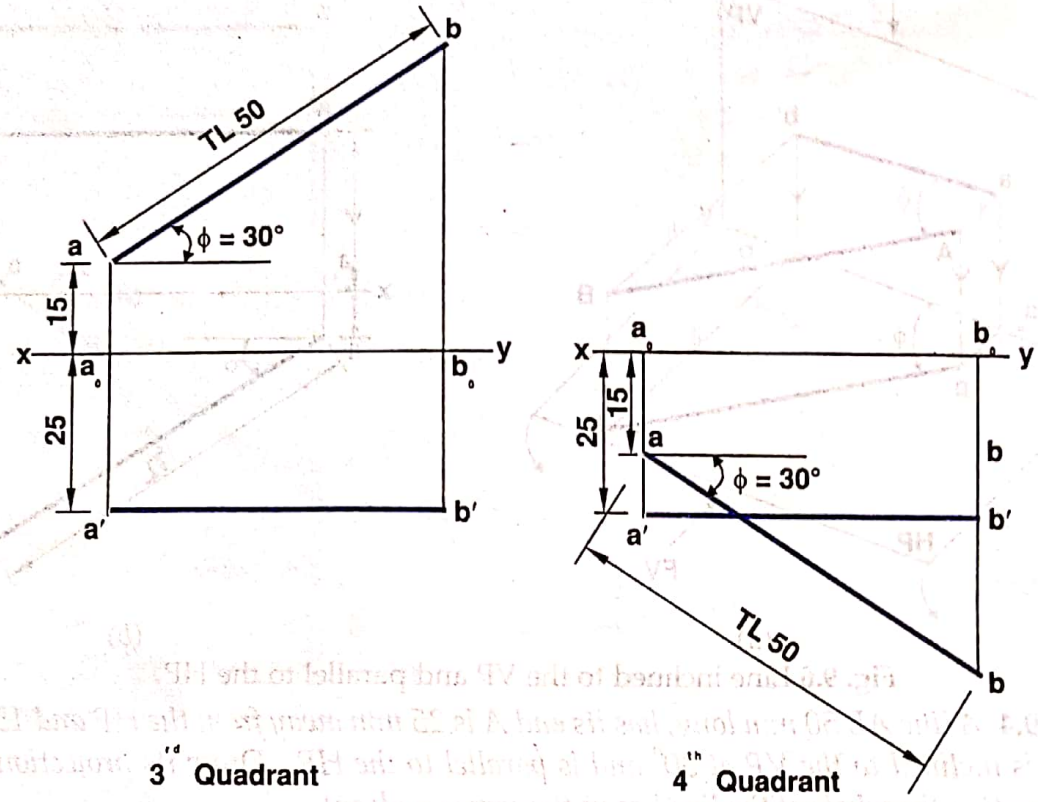


Fig. 9.7 Solution to problem 9.4

PROBLEM 9.5 *A line AB 50 mm long, has its end A is in HP and 15 mm infront of the VP. The line is inclined to the VP at 30° and is parallel to the HP. Draw its projections.*

SOLUTION.

(*i*) Draw a *xy* line. At a convenient distance on *xy*, draw the projector of end point *A*.

(*ii*) On this projector, draw front view *a′* and top view *a* on it.

(*iii*) Draw top view *ab* of the line at an angle of 30° to *xy*. Cut *ab* = 50 mm (TL).

(*iv*) Project *a* to *a′* and *b* to *b′* on front view *a′b′*, which is a line coincides with *xy* as shown in Fig. 9.8.

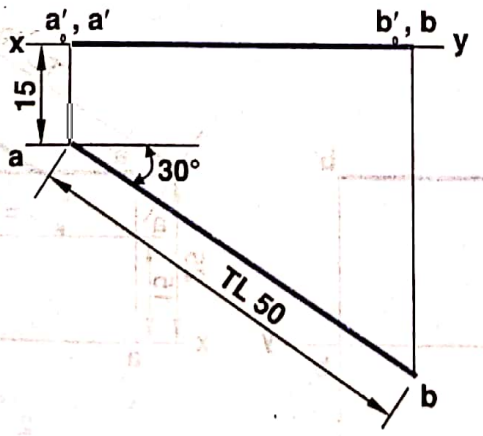


Fig. 9.8 Solution to problem 9.5

PROBLEM 9.6 *The front view of a line AB 60 mm long measures 40 mm. The line is parallel to the HP and the end A is 15 mm above HP and 20 mm infront of the VP. Draw the projections of the line and determine its inclination with the VP.*

SOLUTION. As the line is parallel to the HP, its front view will be parallel to *xy*.

(*i*) Draw front view *a′* and top view *a* of the end point *A*.

(*ii*) Draw the front view $a'b'$ 40 mm long and parallel to xy. With a as centre and radius equal to 60 mm, draw an arc cutting the projector through b' to b. Join a with b. Thus ab is the top view of the line and ϕ is the inclination of the line AB with the VP as shown in Fig. 9.9.

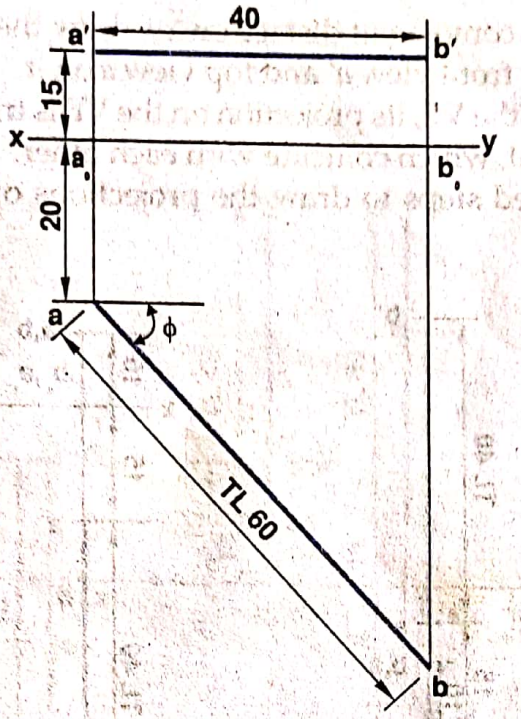


Fig. 9.9 Solution to problem 9.6

9.5 LINE PERPENDICULAR TO ONE OF THE PLANES

When a line is perpendicular to one of the principal planes, it will be parallel to the other.

(*a*) Line perpendicular to the HP

Fig. 9.10 (*a*) shows the pictorial view of a line AB, when it is perpendicular to the HP. Its front view $a'b'$, is true length of a line AB and perpendicular to xy. Its top view ab is a point where the projections of the two end points coincide with each other.

Fig. 9.10 (*b*) shows the orthographic projections of a line AB.

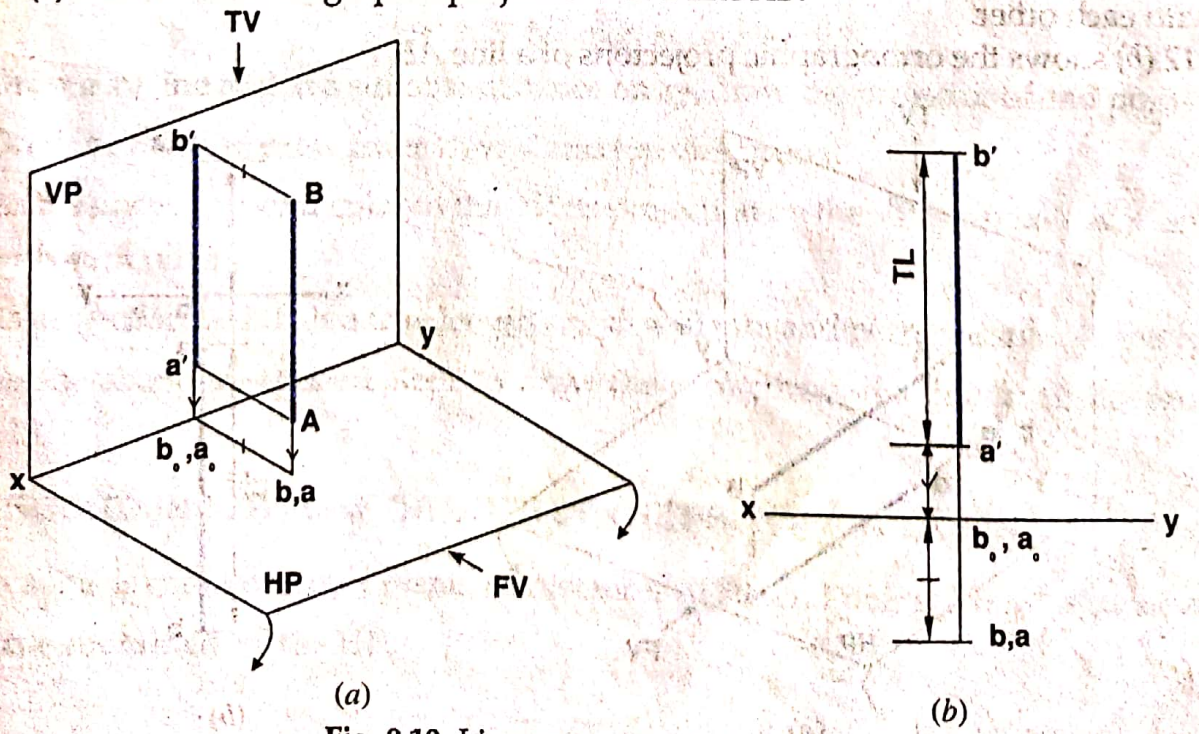


Fig. 9.10 Line perpendicular to the HP

PROBLEM 9.7 *A line AB 40 mm long, is perpendicular to the HP and its end A is 15 mm away from the HP and 10 mm away from the VP. Draw its projections in all the four quadrants. Assume that the whole of the line lies in the same quadrant.*

SOLUTION.

(*i*) Draw a *xy* line and at a convenient distace on *xy*, draw the projector of end point *A*.
(*ii*) On this projector, draw front view *a'* and top view *a* on it.
(*iii*) As the line is parallel to the VP, its projection on the VP is true length and perpendicular to *xy*.
(*iv*) Its top view *ab* is a point, which coincide with each other.

Follow the above mentioned steps to draw the projections of a line in all the four quadrants. See Fig. 9.11.

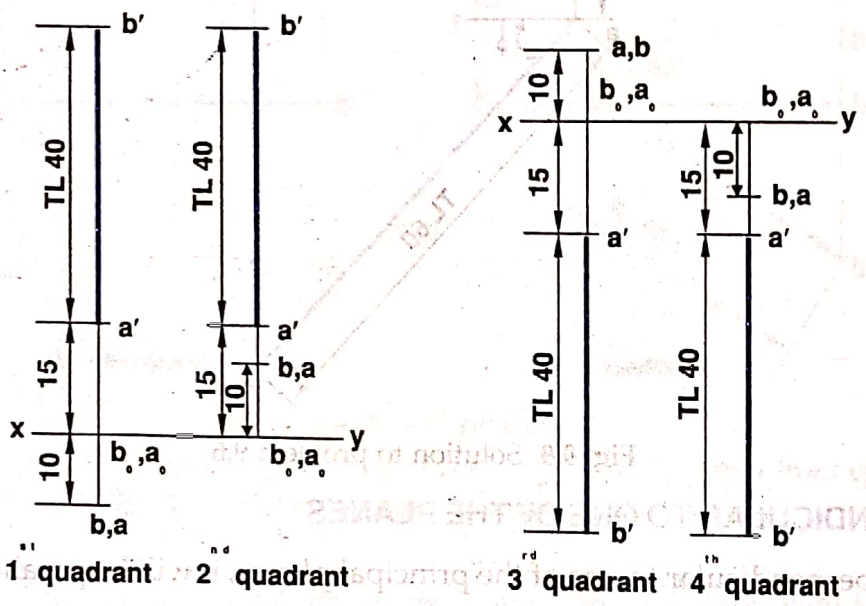


Fig. 9.11 Solution to problem 9.7

(*b*) Line perpendicular to the VP

Fig. 9.12 (*a*) shows the line *AB*, perpendicular to the VP. Its top view *ab*, is true length of a line *AB* and perpendicular to *xy*. Its front view *a'b'* is a point where the projections of the two end points coincide with each other.

Fig. 9.12 (*b*) shows the orthographic projectons of a line *AB*.

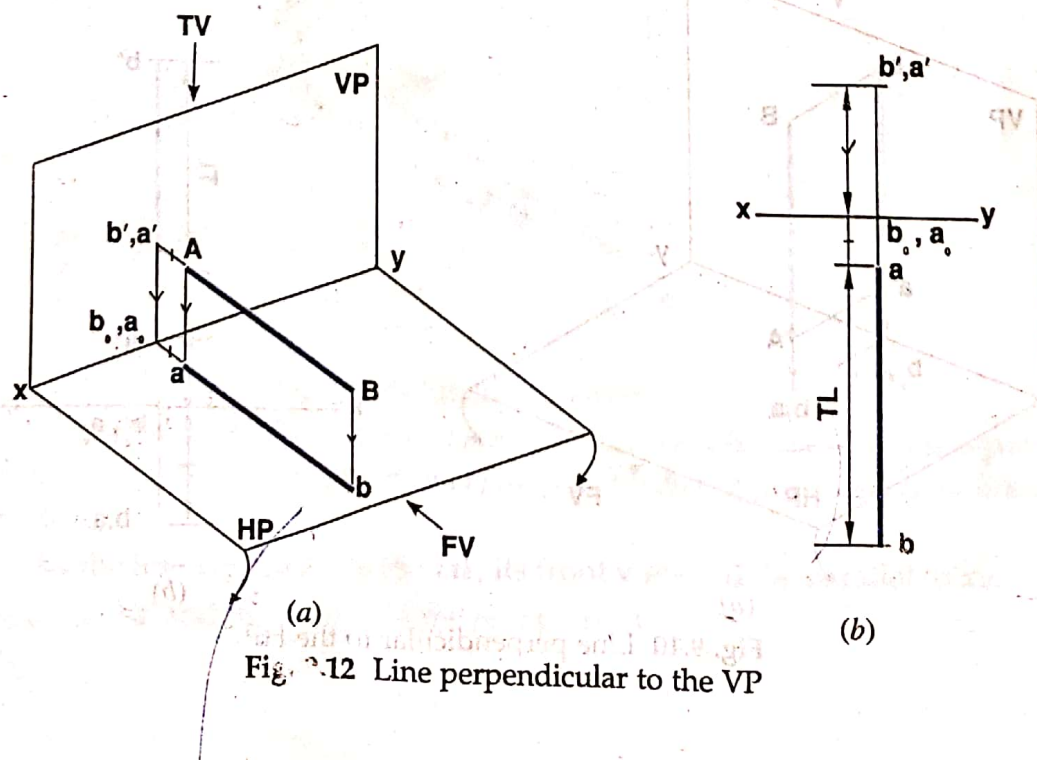
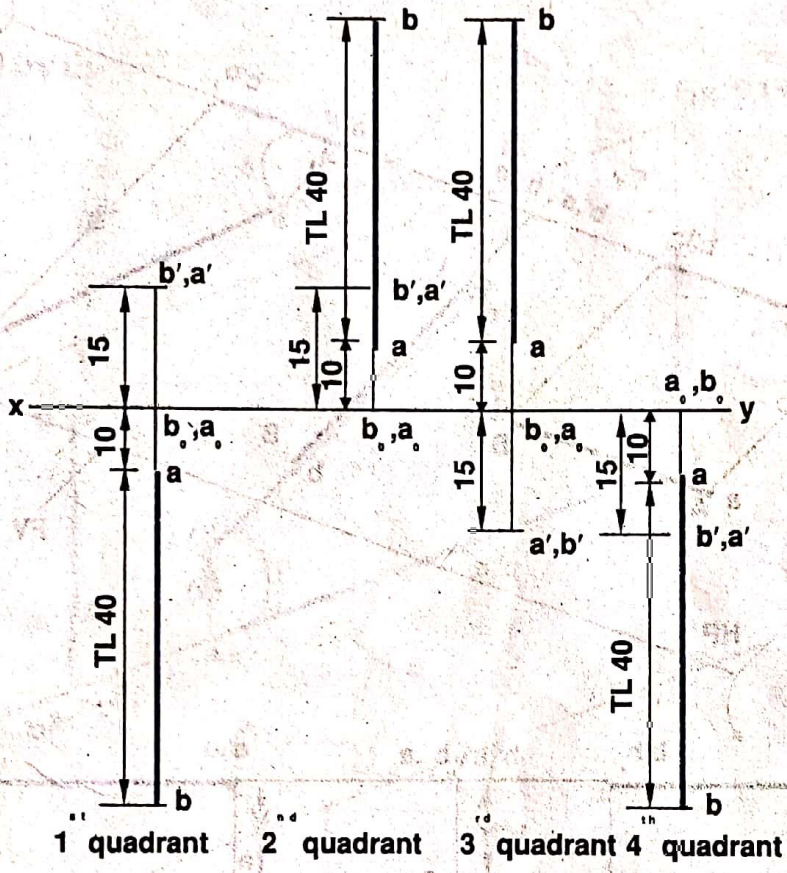


Fig. 9.12 Line perpendicular to the VP

PROBLEM 9.8 *A line AB 40 mm long, is perpendicular to the VP and its end A is 15 mm away from the HP and 10 mm away from the VP. Draw its projections in all the four quadrants. Assume that the whole of the line lies in the same quadrant.*

SOLUTION.



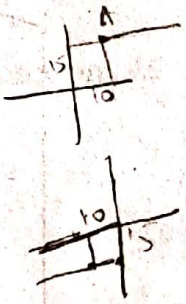


Fig. 9.13 Solution to problem 9.8

(*i*) Draw a *xy* line and at a suitable distance on *xy*, draw the projector of end point *A*.

(*ii*) On this projector, draw front view *a'* and top view *a* on it.

(*iii*) As the line is perpendicular to the VP, its projection on the VP is a point *i.e.*, *a'* and *b'* coincide with each other.

(*iv*) Its projection on HP, is a true length *i.e.*, *ab* = 40 mm and perpendicular to the *xy*.

Follow the above mentioned steps to draw the projections of a line in all the four quadrants. See Fig. 9.13.

9.6 LINE CONTAINED BY ONE OR BOTH OF THE PRINCIPAL PLANES

When a line is contained by a plane, its distances of the end points from the plane are zero.

(*a*) Line contained by the HP

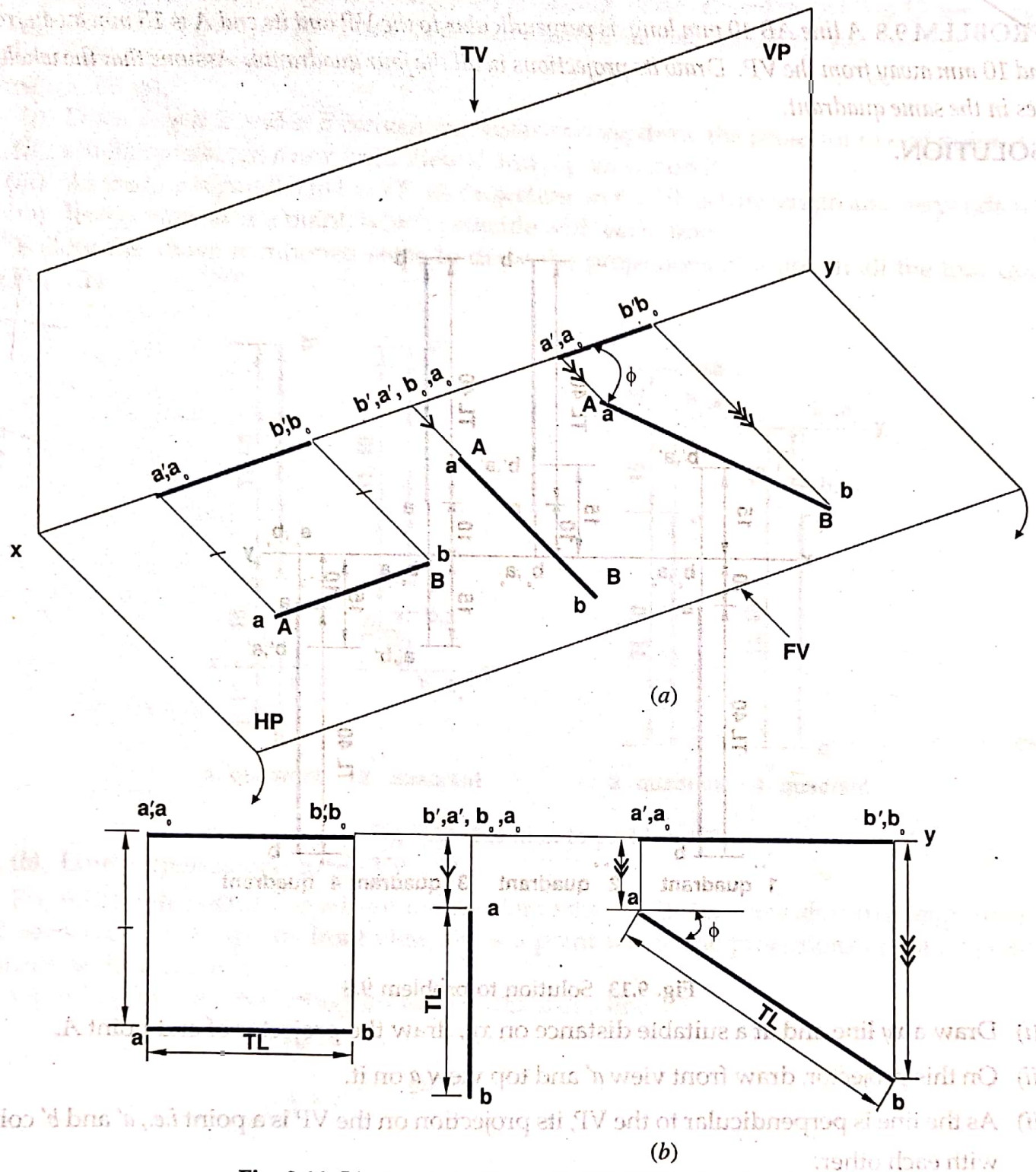


Fig. 9.14 Line contained by the HP (Different Cases)

Fig. 9.14 (*a*) shows the different cases of a line *AB* lying in the HP. In all these cases, the top view *ab* is inclined to the *xy* line at an angle ϕ, which is the same angle the line *AB* makes with the VP. Here top view *ab* is the line *AB* itself (True length of the line). The front view *ab* lies on *xy* line in all the cases.

Fig. 9.14 (*b*) shows the orthographic projections of a line *AB* in different cases.

PROBLEM 9.9 *A line EF, 50 mm long, is contained by HP and is inclined at 45° to the VP, has its end E 10 mm infront of the VP. Draw its projections.*

SOLUTION.

(*i*) Draw a *xy* line and at a convenient distance on *xy*, draw the projector of end point *E*.

(*ii*) On this projector, draw front view *e'* and top view *e* on it.

(*iii*) As the line is contained by the HP, so it will give the true length in the top view. Therefore,

through e draw a line inclined at 45° to the xy and along it mark a point f at a distance of 50 mm. Then ef is the top view of the given line EF (TL of the given line EF).

(*iv*) Project the front view $e'f'$, will be a line on the xy. See Fig. 9.15.

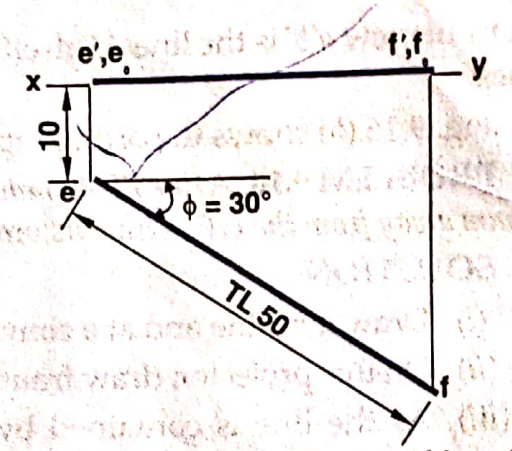


Fig. 9.15 Solution to problem 9.9

(*b*) Line contained by the VP

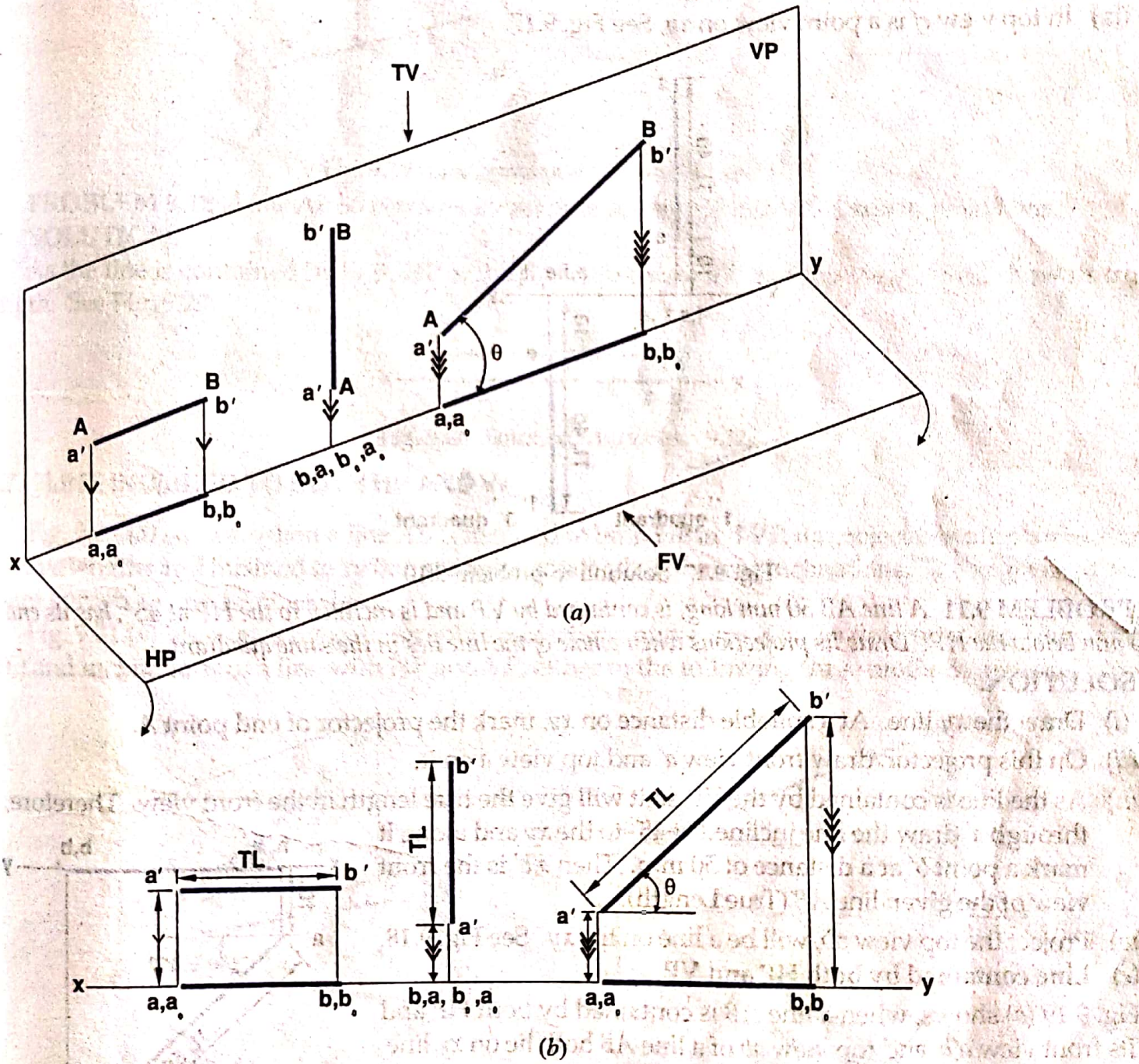


Fig. 9.16 Line contained by the VP (Different Cases)

Fig. 9.16 (*a*) shows the different cases of a line AB lying in the VP. In all these cases, the front view $a'b'$ is inclined to the xy line at an angle θ, which is the same angle the line AB makes with the HP. Here

the front view *a'b'* is the line *AB* itself (True length of the line). The top view *ab* lies on *xy* line in all the cases.

Fig. 9.16 (*b*) shows the orthographic projections of a line *AB* in different cases.

PROBLEM 9.10 *A line EF 40 mm long, is contained by VP and is perpendicular to the HP, has its end E 10 mm away from the HP. Draw its projections in 1st and 3rd quadrants only.*

SOLUTION.

(*i*) Draw a *xy* line and at a convenient distance on *xy*, draw the projector of end point *E*.

(*ii*) On this projector, draw front view *e'* and top view *e* on it.

(*iii*) As the line is contained by the VP, so its projections on the VP is the true length and perpendicular to *xy*.

(*iv*) In top view *ef* is a point view on *xy*, See Fig. 9.17.

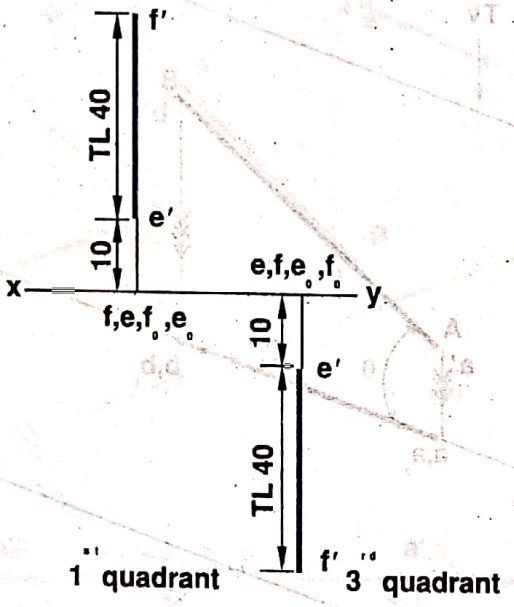


Fig. 9.17 Solution to problem 9.10

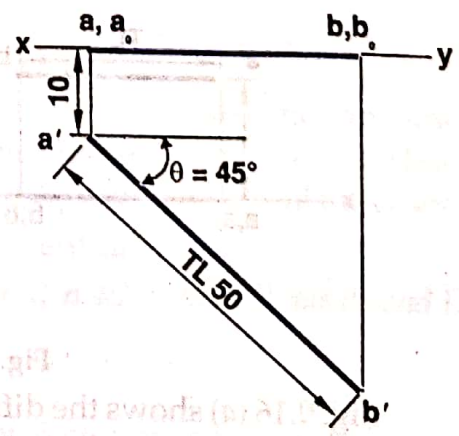
PROBLEM 9.11 *A line AB 50 mm long, is contained by VP and is inclined to the HP at 45°, has its end A 10 mm below the HP. Draw its projections when whole of the line lies in the same quadrant.*

SOLUTION.

(*i*) Draw the *xy* line. At a suitable distance on *xy*, mark the projector of end point *A*.

(*ii*) On this projector, draw front view *a'* and top view *a* on it.

(*iii*) As the line is contained by the VP, so it will give the true length in the front view. Therefore, through *a'* draw the line inclined at 45° to the *xy* and along it mark a point *b'* at a distance of 50 mm. Then *a'b'* is the front view of the given line *AB* (True Length).



(*iv*) Project the top view *ab*, will be a line on the *xy*. See Fig. 9.18.

(*c*) Line contained by both HP and VP

Fig. 9.19 (*a*) shows, when a line *AB* is contained by both HP and VP. Its front view *a'b'* and top view *ab* of a line *AB* both lie on *xy* line.

Fig. 9.18 Solution to problem 9.11

Both front view $a'b'$ and top view ab represent the true length of the line AB.

Fig. 9.19 (b) illustrates the orthographic projections of the line AB.

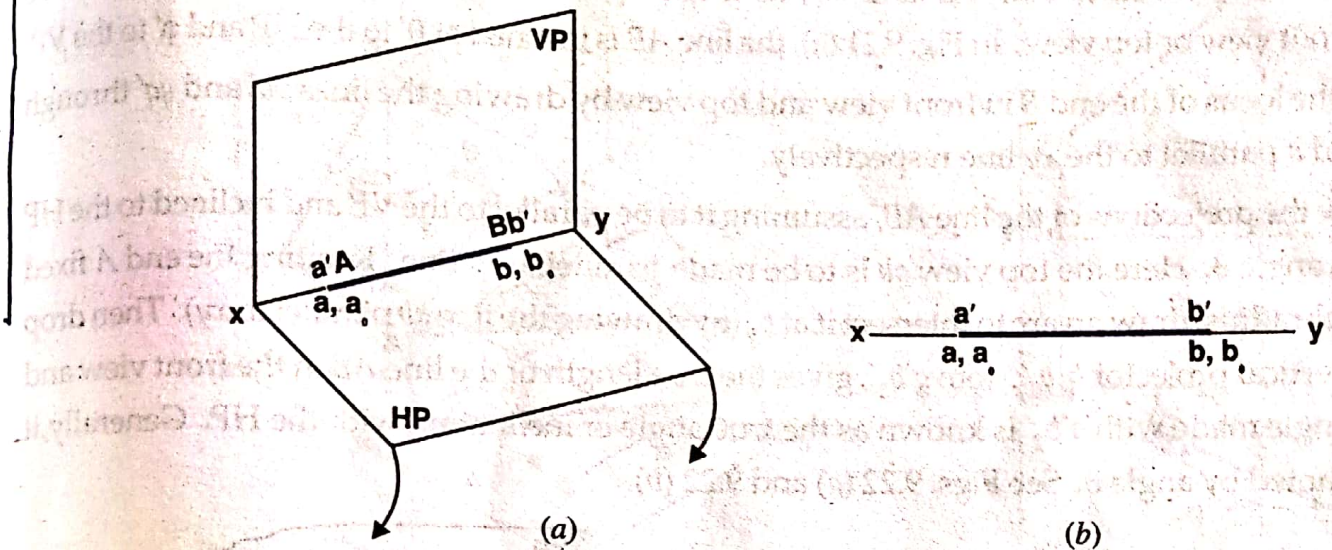


Fig. 9.19 Line contained by both HP and VP

PROBLEM 9.12 *A line AB 50 mm long is contained by both HP and VP. Draw its projections.*

SOLUTION.

As the line is contained by both HP and VP, its front view $a'b'$ and top view ab will show its true length. See Fig. 9.20.

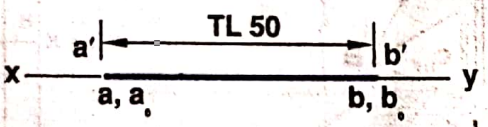


Fig. 9.20 Solution to problem 9.12

9.7 LINE INCLINED TO BOTH HP AND VP

Fig. 9.21 (a) shows, when a line AB is inclined to both HP and VP, its projections are shorter than the true lengths and inclined to xy line at angles greater than the true inclinations. The angle *i.e.*, θ' and ϕ' are known as apparent angles.

Fig. 9.21 (b) illustrates the orthographic projections of the line AB. To determine the true length (TL) and inclinations of a line with HP and VP, either of the following three methods are used here.

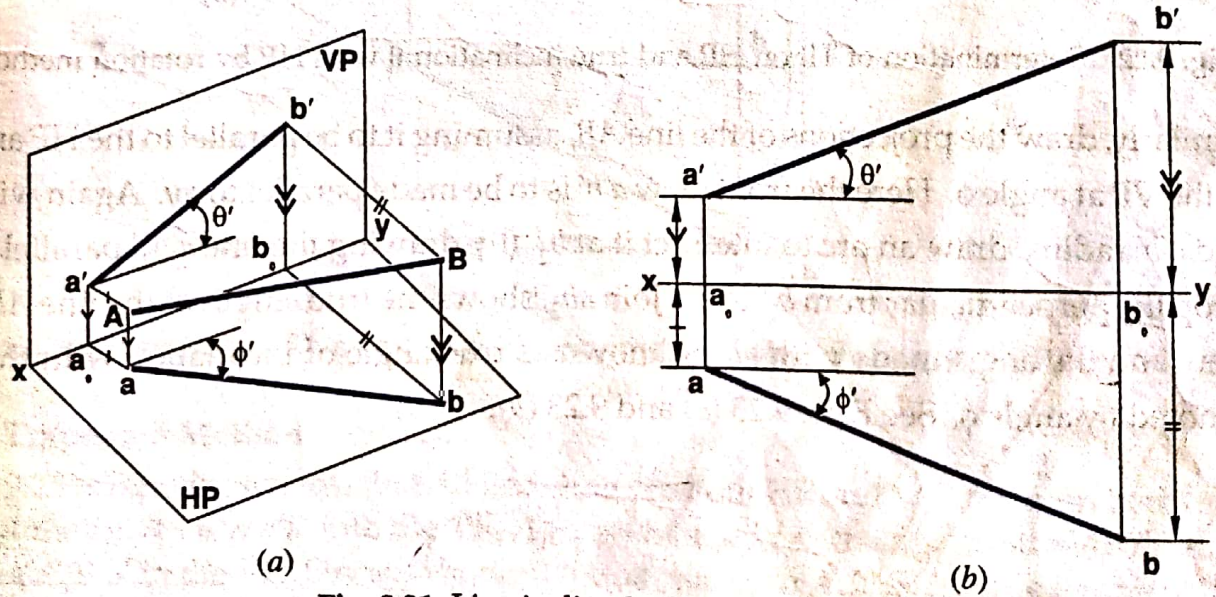


Fig. 9.21 Line inclined to both HP and VP

(*a*) Rotation Method

When a line is inclined to both HP and VP, its true lengths and true inclinations can neither be obtained in front view or top view. In Fig. 9.21 (*a*), the line *AB* is inclined at θ′ to the HP and ϕ′ to the VP.

(*i*) Fix, the locus of the end *B* in front view and top view by drawing the lines *pp′* and *qq′* through *b′* and *b* parallel to the *xy* line respectively.

(*ii*) Draw the projections of the line *AB*, assuming it to be parallel to the VP and inclined to the HP at an angle θ. Here the top view *ab* is to be made parallel to *xy* line. Keeping the end *A* fixed and *ab* radius, draw an arc to intersect it at b_1 (by drawing the line *ab* parallel to *xy*). Then drop the vertical projector b_1b_1'. Join $a'b_1'$, gives the true length of the line *AB* in the front view and the angle made with $a'b_1'$ is known as the true angle of inclination with the HP. Generally,it is denoted by angle θ. See Figs. 9.22 (*a*) and 9.22 (*b*).

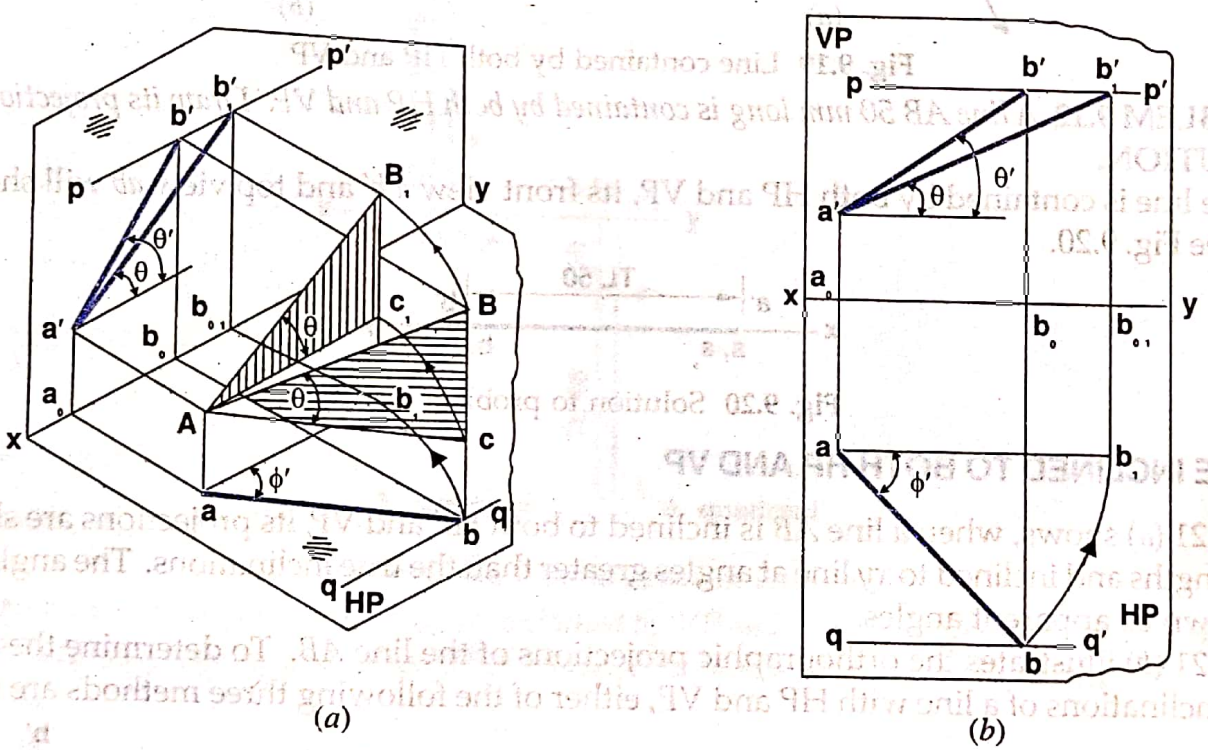
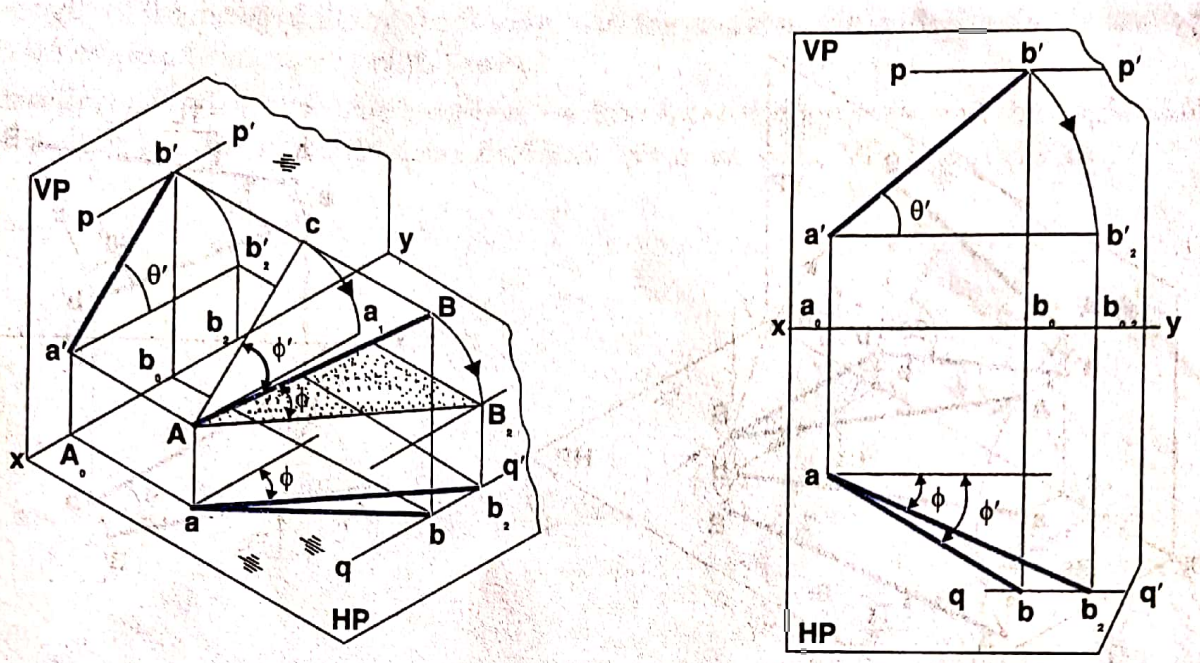


Fig. 9.22 Determination of TL on HP and true inclination θ with HP by rotation method

(*iii*) Similarly, draw the projections of the line *AB*, assuming it to be parallel to the HP and inclined to the VP at angle ϕ. Here the front view *a′b′* is to be made parallel to *xy*. Again with centre *a′* and *a′b′* radius, draw an arc to intersect it at b_2' (by drawing the line $a'b_2'$ parallel to *xy* line). Drop the perpendicular from b_2' to b_2. Join ab_2, shows the true length of the line *AB* in the top view and the angle made with ab_2 is known as true angle of inclination with the VP. It is denoted by angle ϕ. See Figs. 9.23 (*a*) and 9.23 (*b*).



(*a*) (*b*)

Fig. 9.23 Determination of TL on VP and true inclination ϕ with VP by rotation method

(*iv*) Fig. 9.24 shows the true length and true inclinations, when the line *AB* is inclined to both HP and VP.

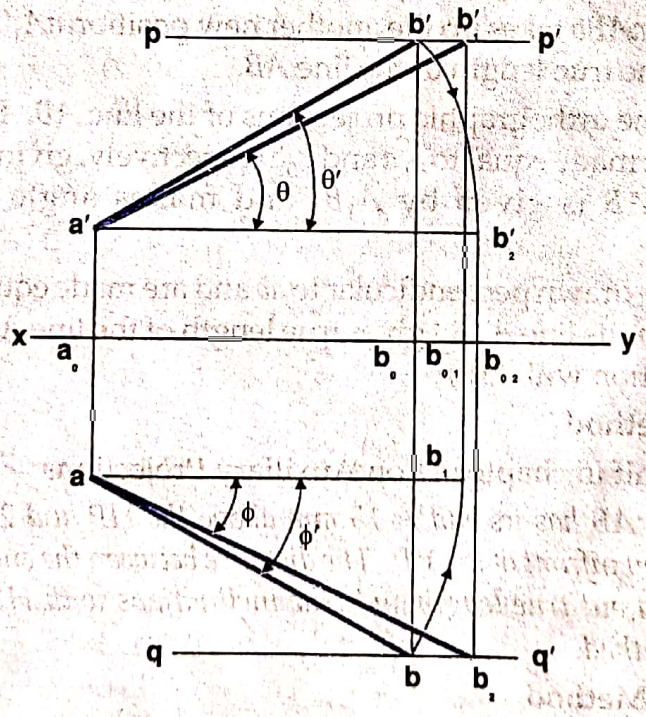


Fig. 9.24 Determination of TL on HP and VP and true inclination θ and ϕ with HP and VP respectively by rotation method

(*b*) Trapezoid Method

Fig. 9.25 (*a*) shows, when a line *AB* is inclined to both HP and VP. It is seen here that by rotating AB about the front view *a'b'* into the VP, A_1B_1 is obtained *i.e.*, the trapezoid *a'ABb'* takes up the new position $a'A_1B_1b'$ in the VP. Since it shows the true shape, so A_1B_1 is the true length of the line *AB*.

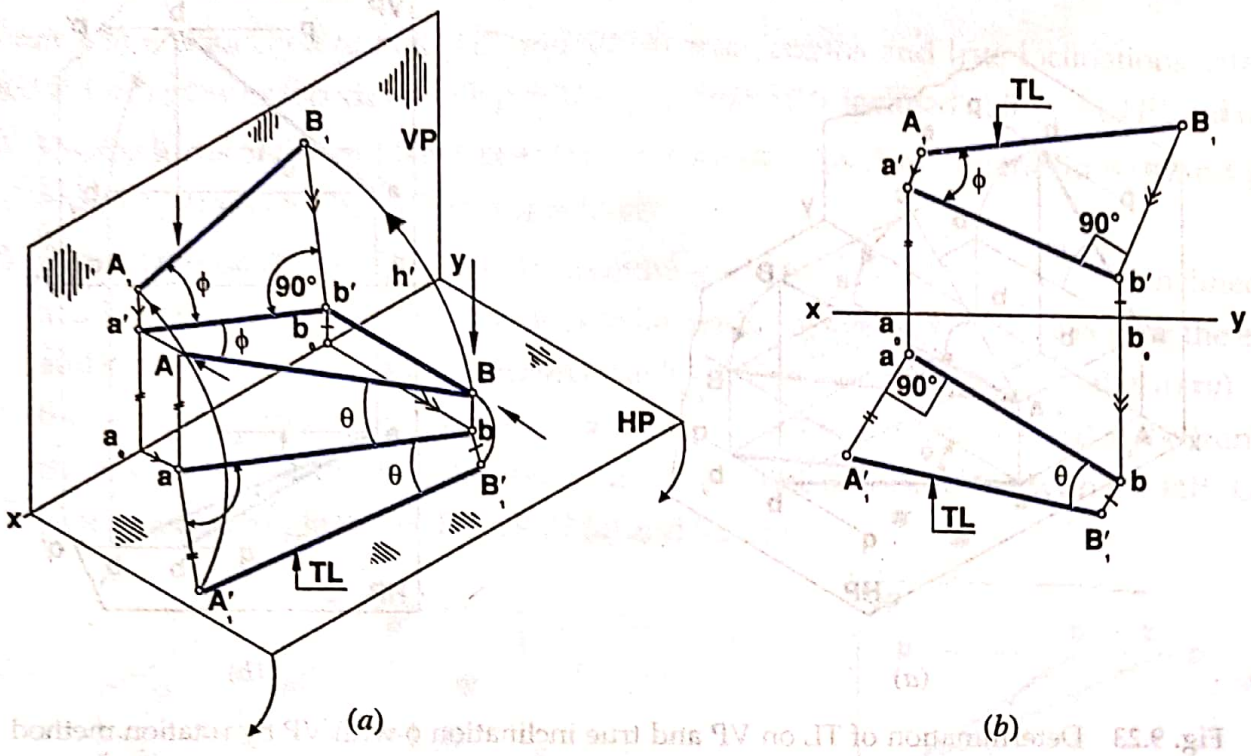


Fig. 9.25 Determination of TL on HP and VP and true inclination θ and ϕ with HP and VP respectively by trapezoid method

Similarly, the trapezoid *aABb* takes up the another new position $aA_1'B_1'b$ in the HP. Since it shows the true shape, so $A_1'B_1'$ is the true length of the line *AB*.

Fig. 9.25 (*b*) illustrates the orthographic projections of the line *AB*. Here $a'A_1$ and $b'B_1$ are drawn perpendicular to $a'b'$ and are made equal to a_0a and b_0b respectively, giving the trapezoid $a'A_1B_1b'$. The true length of the line *AB* is given by A_1B_1 and makes angle ϕ between $a'b'$ and A_1B_1. (True inclination with VP).

Similarly, aA_1' and bB_1' are drawn perpendicular to *ab* and are made equal to a_0a' and b_0b' respectively, giving the trapezoid $aA_1'B_1'b$. Its line $A_1'B_1'$ gives true length of the line *AB* and makes angle θ between *ab* and $A_1'B_1'$. (True inclination with HP)

(c) Auxiliary Plane Method

This method will be dealt in chapter 11 on Auxiliary Projections.

PROBLEM 9.13 *A line AB has its end A 15 mm above the HP and 20 mm infront of the VP, end B 40 mm above the HP and 50 mm infront of the VP. The distance between the end projectors is 45 mm. Draw the projections of the line and find out true length and true inclinations with HP and VP by using (i) Rotation Method and (ii) Trapezoid Method.*

SOLUTION. Rotation Method

(*i*) Draw the *xy* line and locate the projections of two ends of the line on two projectors drawn perpendicular to *xy* and 45 mm apart.

(*ii*) Join $a'b'$ and *ab* to obtain the front view and top view of the line respectively.

(*iii*) Since the angles θ and ϕ made by the line with the HP and VP respectively are to be found out, so find out true length (TL) of the line in front view (elevation) and top view (plan). Fix the locus of the end *B* in front view and top view by drawing lines pp' and qq' through b' and *b* parallel to the *xy* respectively.

(*a*) To do so, with centre *a* and radius *ab*, draw an arc to intersect it at b_1 (by drawing the line ab_1 parallel to *xy*). Then drop the perpendicular from b_1 to b_1'. Join $a'b_1'$, gives the true

length of the line AB in the front view and the angle made with the $a'b_1'$ is known as the true angle of inclination with the HP.

(*b*) Similarly, find out the true length of the line AB in the top view and the angle made with the ab_2 is known as the true angle of inclination with the VP. See Fig. 9.26.

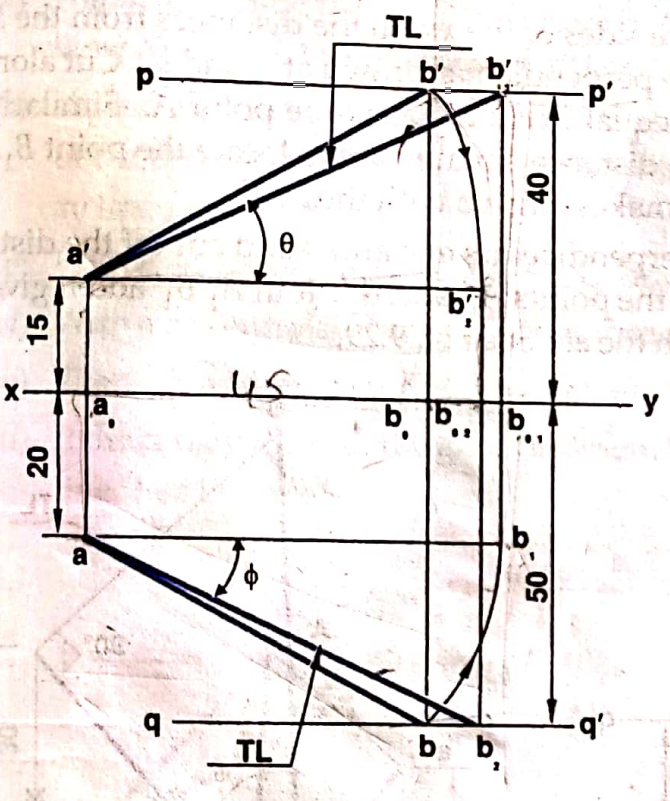


Fig. 9.26 Solution to problem 9.13 (rotation method)

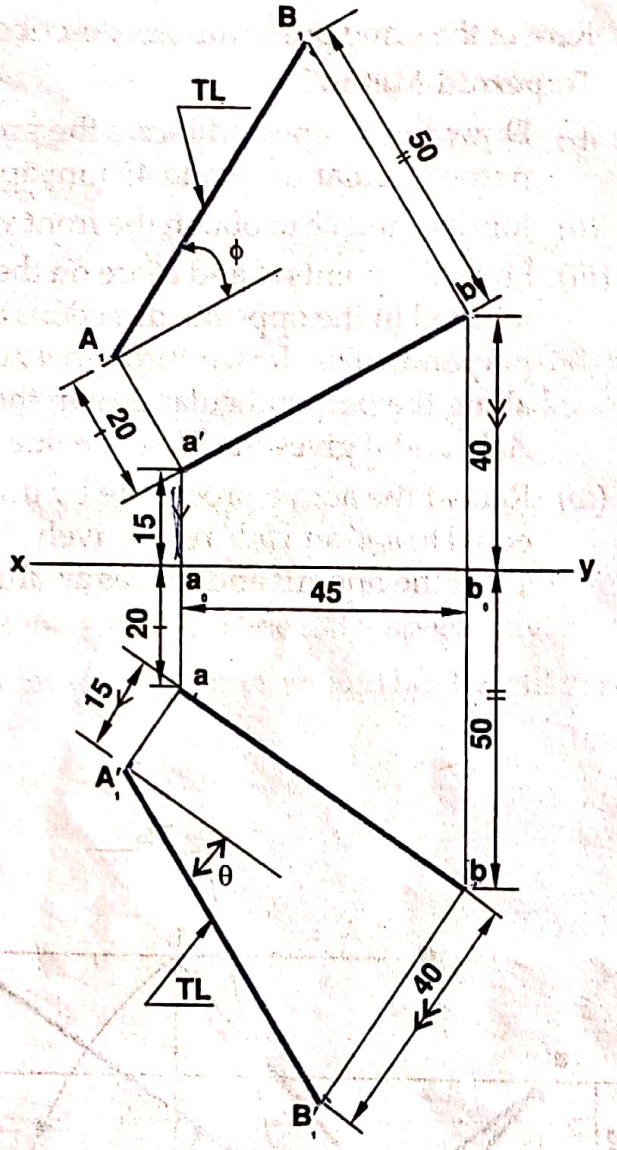


Fig. 9.27 Solution to problem 9.13 (trapezoid method)

Trapezoid Method

(*i*) Draw the xy line and locate the projections of two ends of the line on two projectors drawn perpendicular to xy and 45 mm apart.

(*ii*) Join $a'b'$ and ab to obtain the front view and top view of the line respectively.

(*iii*) Draw perpendiculars to the front view (elevation) $a'b'$ at the each end a' and b'. Cut along the perpendicular drawn through a', distance equal to a_0a to locate the point A_1. Similarly cut along the perpendicular drawn through b', distance equal to b_0b to locate the point B_1. Join A_1B_1, gives the true length of the line AB and makes angle ϕ with the $a'b'$.

(*iv*) Repeat the above procedure by drawing perpendiculars at a and b and mark along, the distances equal to a_0a' and b_0b' respectively, to obtain the points A_1' and B_1'. Join $A_1'B_1'$, gives the true length of the line AB and makes angle θ with the ab. See Fig. 9.27.

PROBLEM 9.14 *A line AB has its end A 15 mm above the HP and 20 mm infront of the VP, end B 40 mm below the HP and and 30 mm behind the VP. The distance between the end projectors is 45 mm. Draw the projections of the line and find out true length and true inclinations with HP and VP by using (i) Rotation Method and (ii) Trapezoid Method.*

SOLUTION. Rotation Method

Repeat the same procedure, as described in the previous problem 9.13. See Fig. 9.28.

Trapezoid Method

(*i*) Draw the *xy* line and locate the projections of two ends of the line on two projectors drawn perpendicular to *xy* and 45 mm apart.

(*ii*) Join *a'b'* and *ab* to obtain the front view and top view of the line respectively.

(*iii*) Since the points *a'* and *b'* are on the opposite sides of the *xy*, so the distances from the *xy* are marked in the opposite directions along the perpendicular drawn at *a'* and *b'*. Cut along the perpendicular drawn through *a'*, distance equal to a_0a to locate the point *A*. Similarly, cut along the perpendicular drawn through *b'*, distance equal to b_0b to locate the point B_1. Join A_1B_1 and it gives the *TL* of the line *AB* and makes angle ϕ with the *a'b'*.

(*iv*) Repeat the above procedure by drawing perpendiculars at *a* and *b* and cut off the distances equal to a_0a' and b_0b' respectively, to obtain the points A_1' and B_1'. Join $A_1'B_1'$ and it gives the TL of the line *AB* and makes an angle θ with the *ab*. See Fig. 9.29.

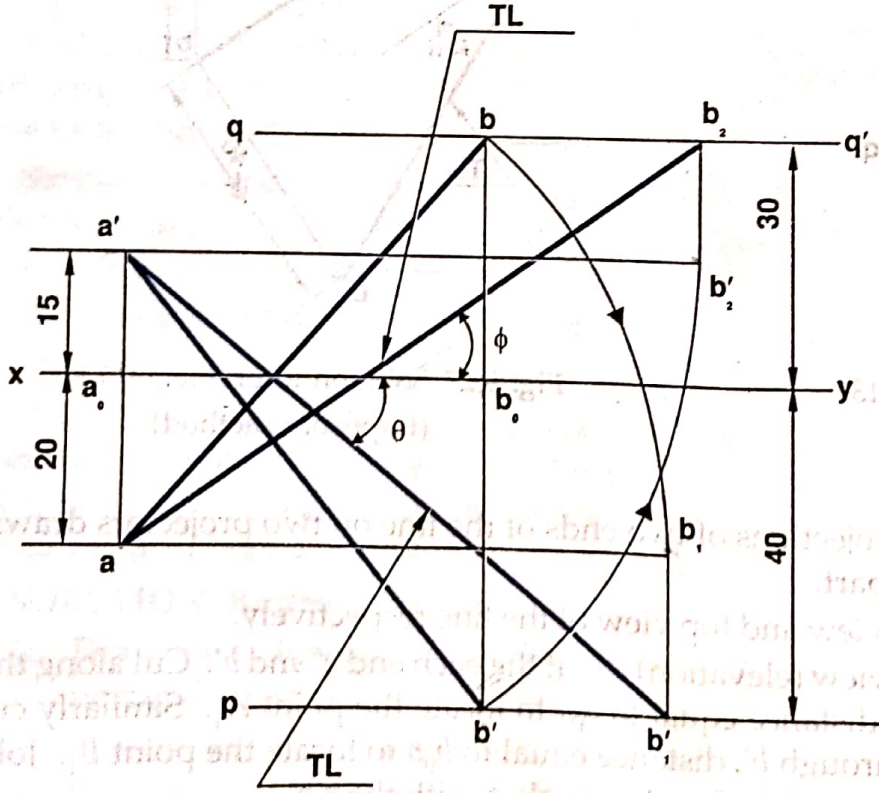


Fig. 9.28 Solution to problem 9.14 (rotation method)

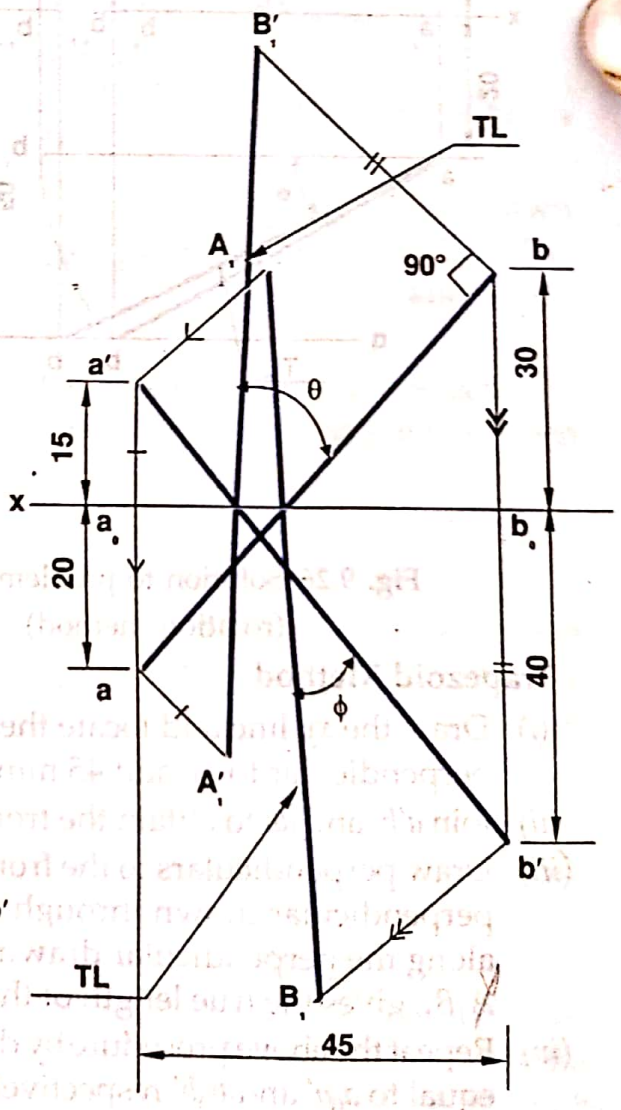


Fig. 9.29 Solution to problem 9.14 (trapezoid method)

PROBLEM 9.15 *A line AB, 50 mm long has its end A 15 mm away from HP and 20 mm away from the VP. It is inclined at 30° to the HP and 45° to the VP. Draw its projections in all the four quadrants. Assume that the whole of the line lies in the same quadrant.*

SOLUTION. First quadrant

(*i*) Draw the *xy* line and locate the front view (elevation) *a'* and top view (plan) *a* of the given point *A* on a projector drawn perpendicular to *xy*.

(*ii*) Assuming the line *AB* to be parallel to the VP and inclined to the HP at an angle 30°, draw its front view $a'b_1'$ (equal to *AB*) and project the top view ab_1.

(*iii*) Again assuming the line *AB* to be parallel to the HP and inclined at 45° to the VP, draw its top view ab_2 (equal to *AB*) and project the front view $a'b_2'$.

(*iv*) ab_1 and $a'b_2'$ are the lengths of *AB* in the top view and the front view respectively. Fix the locus of the end *B* in front view and top view by drawing *pp'* and *qq'* through *b'* and *b* parallel to the *xy* line respectively.

(*v*) With *a'* as centre and $a'b_2'$ as radius, draw an arc to intersect the locus *pp'* at *b'*. Join *a'b'*.

(*vi*) With *a* as centre and ab_1 as radius, draw an arc to intersect the locus *qq'* at *b*. Join *ab*.

(*vii*) Then *a'b'* and *ab* are the required front view and top view of the line *AB* respectively.

(*viii*) If the construction of the given problem is correct, join *b'* to *b*, which should be straight vertical line. See Fig. 9.30.

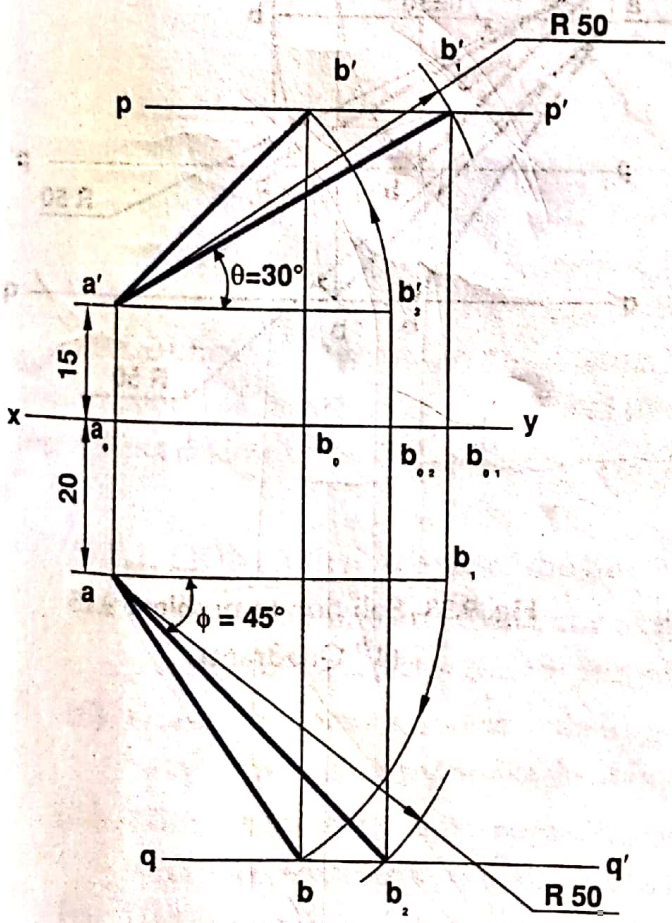


Fig. 9.30 Solution to problem 9.15 (1st Quadrant)

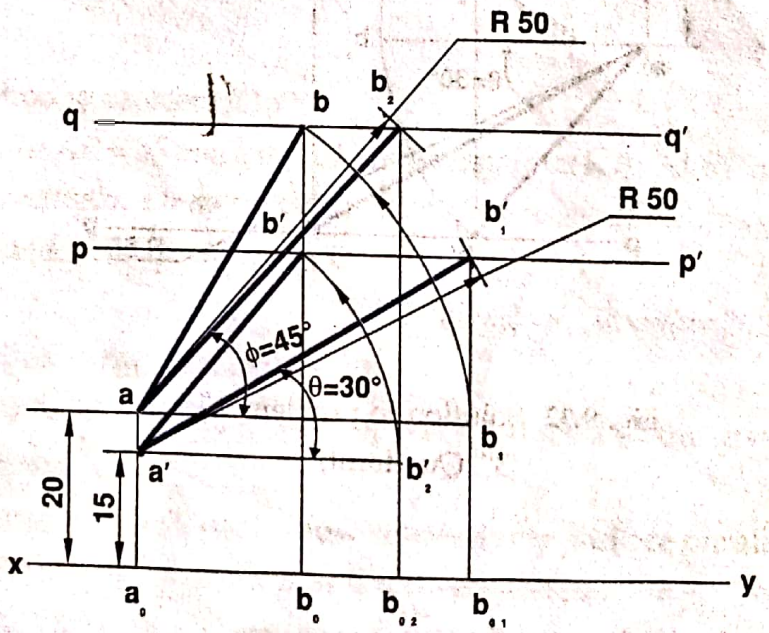


Fig. 9.31 Solution to problem 9.15 (2nd Quadrant)

Second Quadrant

If the line *AB* is located in the second quadrant, the front view and top view of the line will remain above the *xy* line. Using the same procedure as in problem 9.15 (first quadrant), the projections can be drawn. Fig. 9.31 shows the projections of a line AB in the second quadrant.

Third Quadrant

If the line *AB* is placed in the third quadrant, the front view will be below the *xy* line and the top view will be above the *xy* line. Fig. 9.32 shows the projections of a line *AB* in the third quadrant.

Fourth Quadrant

If the line *AB* is located in the fourth quadrant, the front view and top view will be below the *xy* line. Fig. 9.33 shows the projections of a line *AB* in the fourth quadrant.

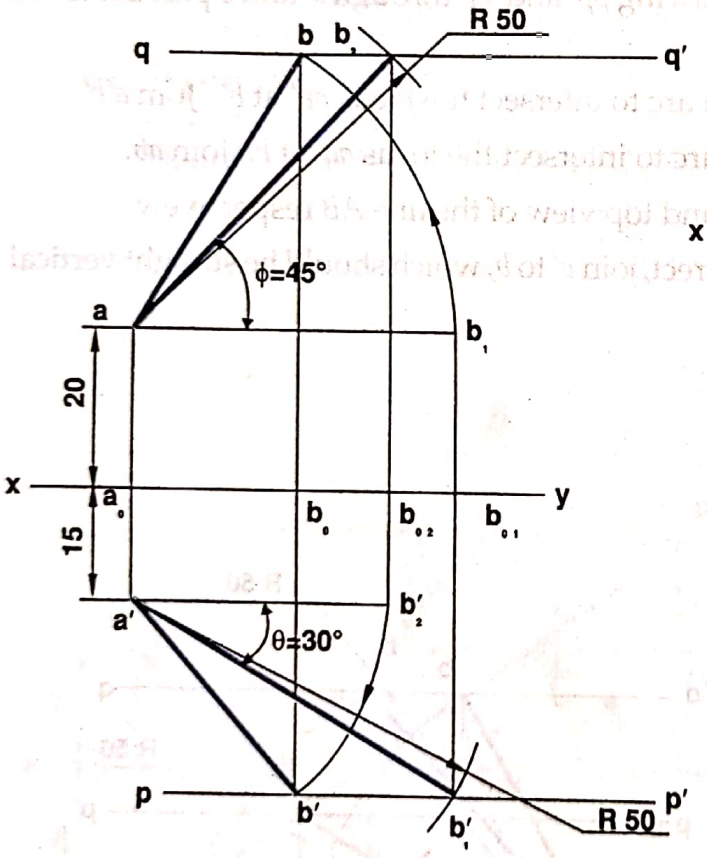


Fig. 9.32 Solution to problem 9.15 (3rd Quadrant)

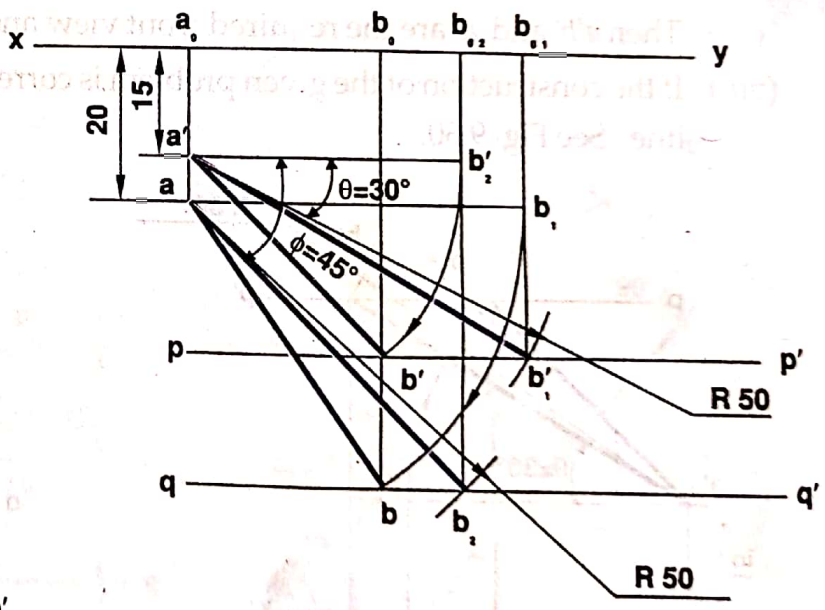


Fig. 9.33 Solution to problem 9.15 (4th Quadrant)

PROBLEM 9.16 *A straight line AB 60 mm long makes an angle of 45° to HP and 30° to the VP. The end A is 15 mm in front of VP and 25 mm above HP. Draw the projections of the line AB.*

(PTU, Jalandhar December 2004)

SOLUTION. The procedure to draw this problem has already been explained in previous problems. See Fig. 9.34.

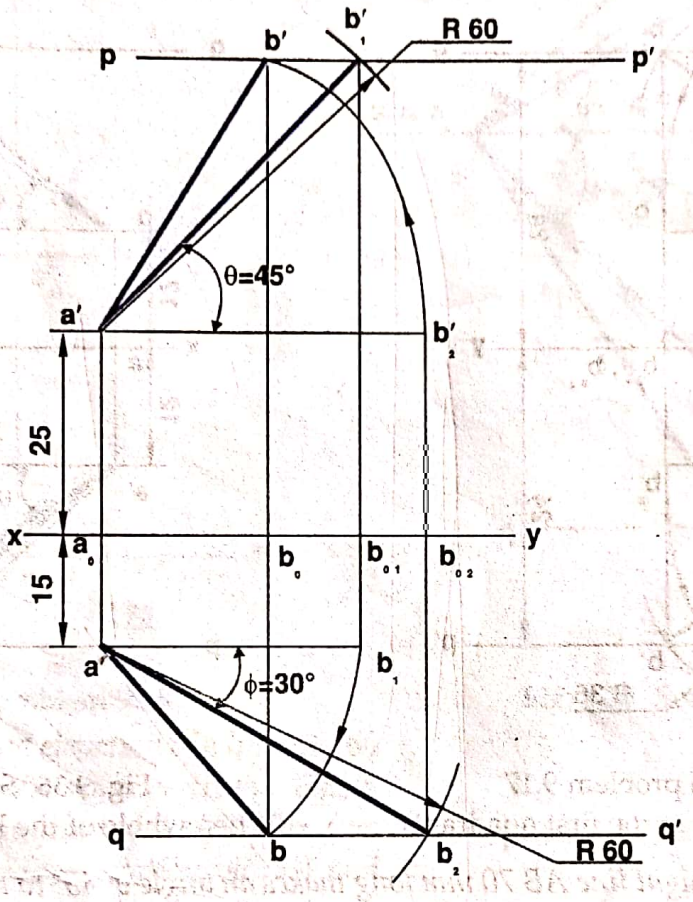


Fig. 9.34 Solution to problem 9.16

PROBLEM 9.17 *The end point A of a straight line AB = 36 mm long is 12 mm away from HP and VP and another point B is 24 mm away from HP and VP respectively. Draw the top view and front view of the straight line AB and determine the true inclination with HP and VP respectively.*

(PTU, Jalandhar December 2003)

SOLUTION. When whole of the line *AB*, lies in the first quadrant

(*i*) Draw the *xy* line and locate the front view (elevation) *a'* and top view (plan) a of the given point *A* on a projector drawn perpendicular to *xy*.

(*ii*) Draw two lines *pp'* and *qq'* lines 24 mm above and 24 mm below respectively and are parallel to *xy*. These lines represent the locus of end *B* in front view and top view respectively.

(*iii*) With *a'* as centre and ab_1' radius (36 mm), draw an arc to intersect the locus *pp'* at b_1'. Similarly, with *a* as centre and ab_2 radius (36 mm *TL*), draw an arc to intersect the locus *qq'* at b_2.

(*iv*) With *a* as centre and ab_1 radius, draw an arc to intersect the locus *qq'* at *b*. Join *ab*.

(*v*) With *a'* as centre and ab_2' radius, draw an arc to intersect the locus *pp'* at *b'*. Join *a'b'*.

(*vi*) Then *a'b'* and *ab* are the required front view and top view of the line AB respectively.

Here $a'b_1'$ makes true angle of inclination (θ) with HP and ab_2 makes true angle of inclination (ϕ) with VP. See Fig. 9.35.

When the whole of the line *AB*, lies in the third quadrant

Repeat the same procedure, as described above. See Fig. 9.36.

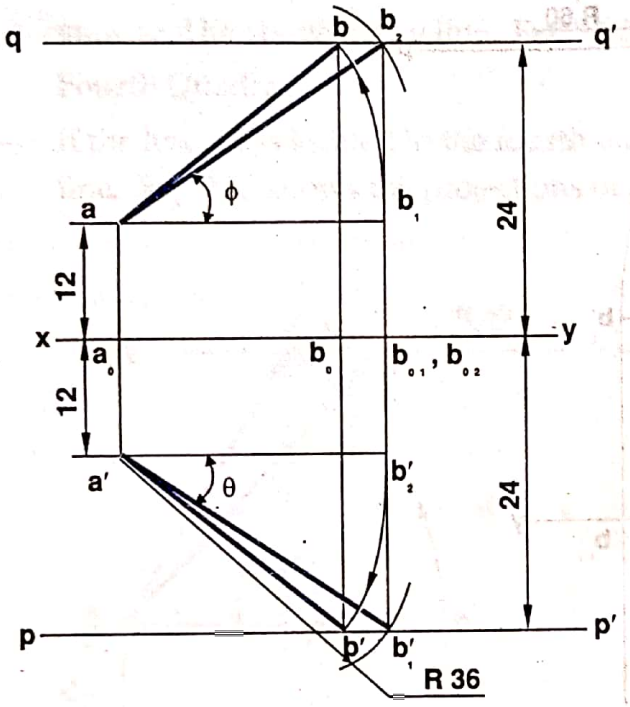


Fig. 9.35 Solution to problem 9.17
(When whole of the line lies in the first quadrant)

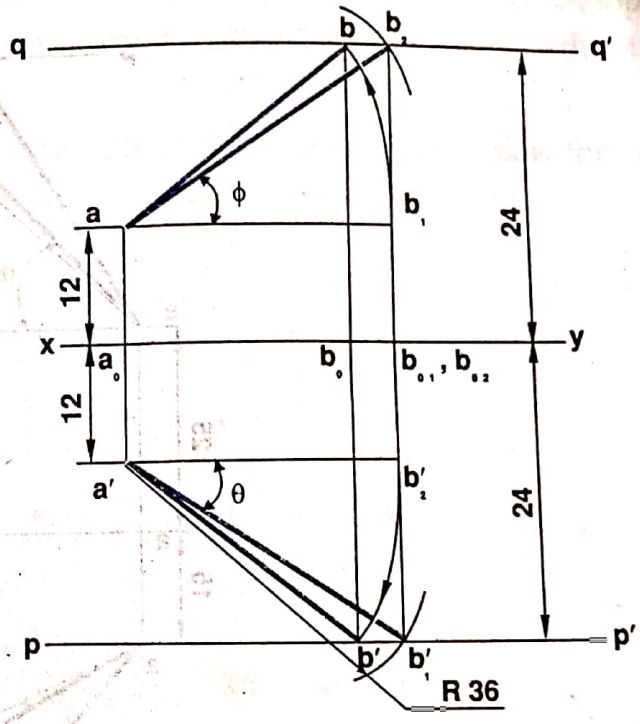


Fig. 9.36 Solution to problem 9.17
(When whole of the line lies in the third quadrant)

PROBLEM 9.18 *A straight line AB 70 mm long makes an angle of 45° to the HP and 30° to the VP. The end A is 15 mm infront of the VP and 20 mm above HP. Draw the plan and elevation of the line AB.*

(*PTU, Jalandhar May 2005, May 2008, May 2009, December 2010*)

SOLUTION. The procedure to draw this problem has already been explained in previous problems. See Fig. 9.37.

PROBLEM 9.19. *A line AB, 60 mm long, has its end A both in HP and VP. It is inclined at 45° to the VP and 30° to the HP. Draw its projections in first quadrant only.*

SOLUTION. The solution to this problem is self-explanatory. See Fig. 9.38.

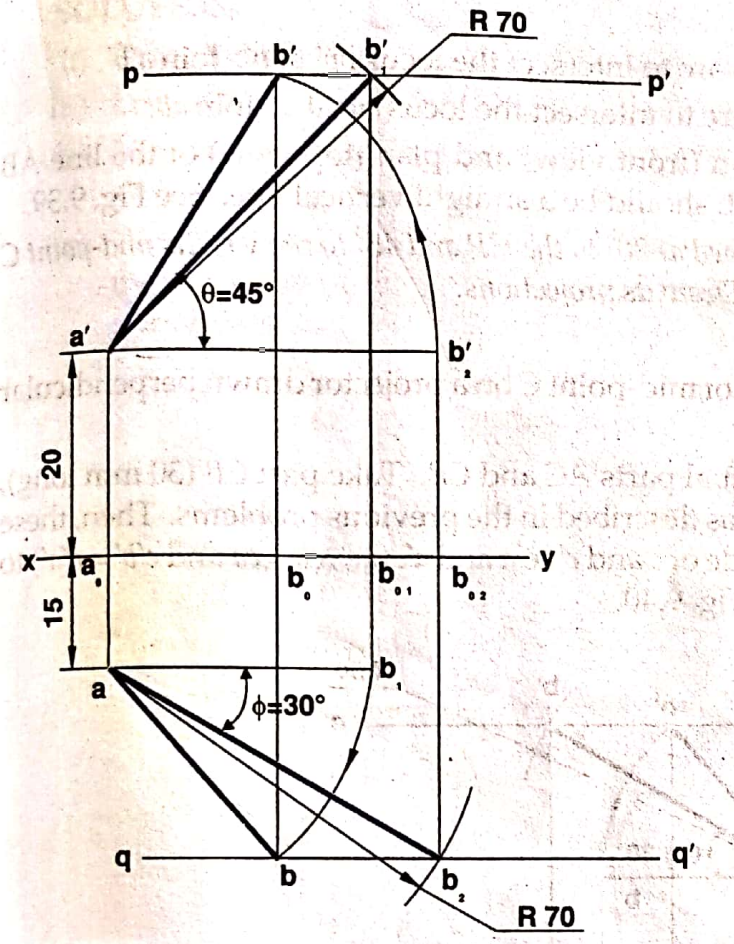


Fig. 9.37 Solution to problem 9.18

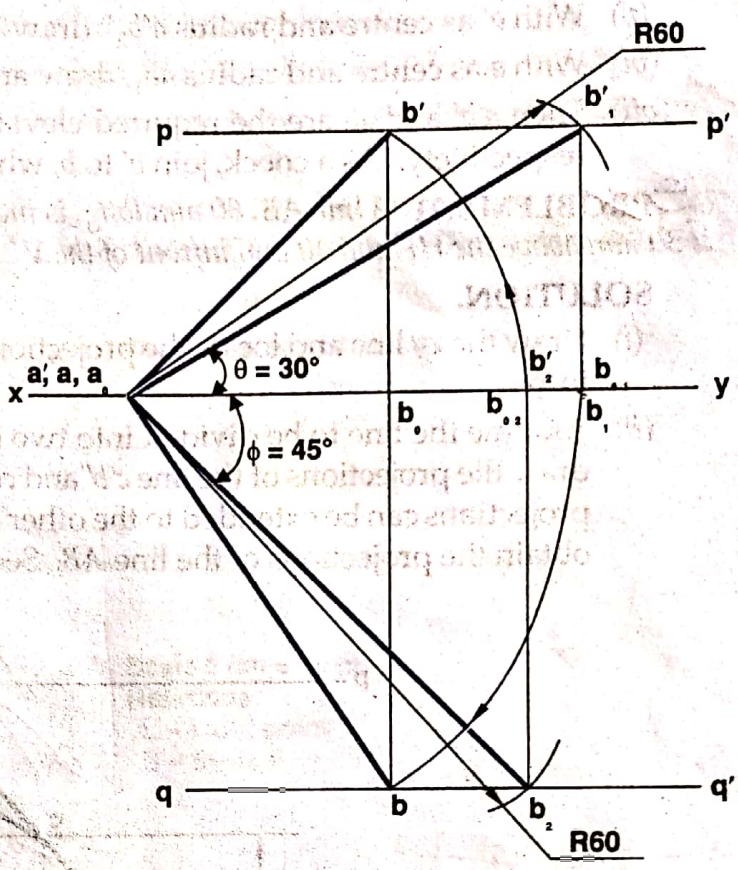


Fig. 9.38 Solution to problem 9.19

PROBLEM 9.20 *Plan and elevation of a line AB, 60 mm long, measure 50 mm and 40 mm respectively. End A is 15 mm above HP and 20 mm infront of the VP. Draw its projections and determine the true inclination with HP and VP respectively.*

SOLUTION.

(*i*) Draw the *xy* line and locate the projections of end A on a projector drawn perpendicular to *xy*.

(*ii*) Draw ab_1 equal to 50 mm parallel to *xy*. From b_1, draw a vertical projector. With *a'* as centre and radius 60 mm, draw an arc cutting the projector at b_1'. Join $a'b_1'$ and measure the inclination of the line $a'b_1'$ with *xy* to find out the true inclination with HP (angle θ).

(*iii*) Draw $a'b_2'$ equal to 40 mm parallel to *xy*. From b_2', draw a vertical projector. With *a* as centre and radius 60 mm, draw an arc cutting the projector at b_2. Join ab_2 and measure the inclination of the line ab_2 with *xy* to find out the true inclination with VP (angle ϕ).

(*iv*) Fix the locus of the end B in elevation and plan by drawing lines *pp'* and *qq'* through b_1'

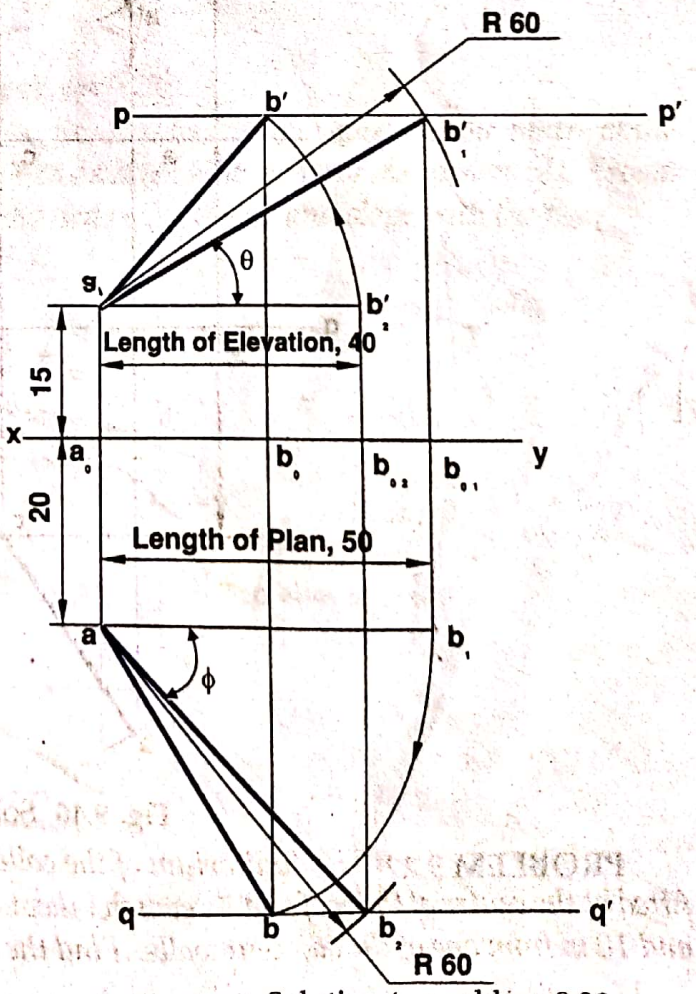


Fig. 9.39 Solution to problem 9.20

and b_2 parallel to the xy line respectively.

(*v*) With a' as centre and radius $a'b_2'$, draw an arc to intersect the locus pp' at b'. Join $a'b'$.

(*vi*) With a as centre and radius ab_1, draw an arc to intersect the locus qq' at b. Join ab.

(*vii*) Then $a'b'$ and ab are the required elevation (front view) and plan (top view) of the line AB respectively. As a check, join b' to b, which should be a straight vertical line. See Fig. 9.39.

PROBLEM 9.21 *A line AB, 60 mm long, is inclined at 30° to the HP and 45° to the VP. Its mid-point C is 30 mm above the HP and 40 mm infront of the VP. Draw its projections.*

SOLUTION.

(*i*) Draw the xy line and locate the projections of mid-point C on a projector drawn perpendicular to xy.

(*ii*) Assume the line to be divided into two equal parts AC and CB. Take part CB (30 mm long), draw the projections of the line $c'b'$ and cb, as described in the previous problems. Then, these projections can be extended to the other side of c and c' to a and a'. (*i.e.*, $cb = ca$ and $c'b' = c'a'$) to obtain the projections of the line AB. See Fig. 9.40.

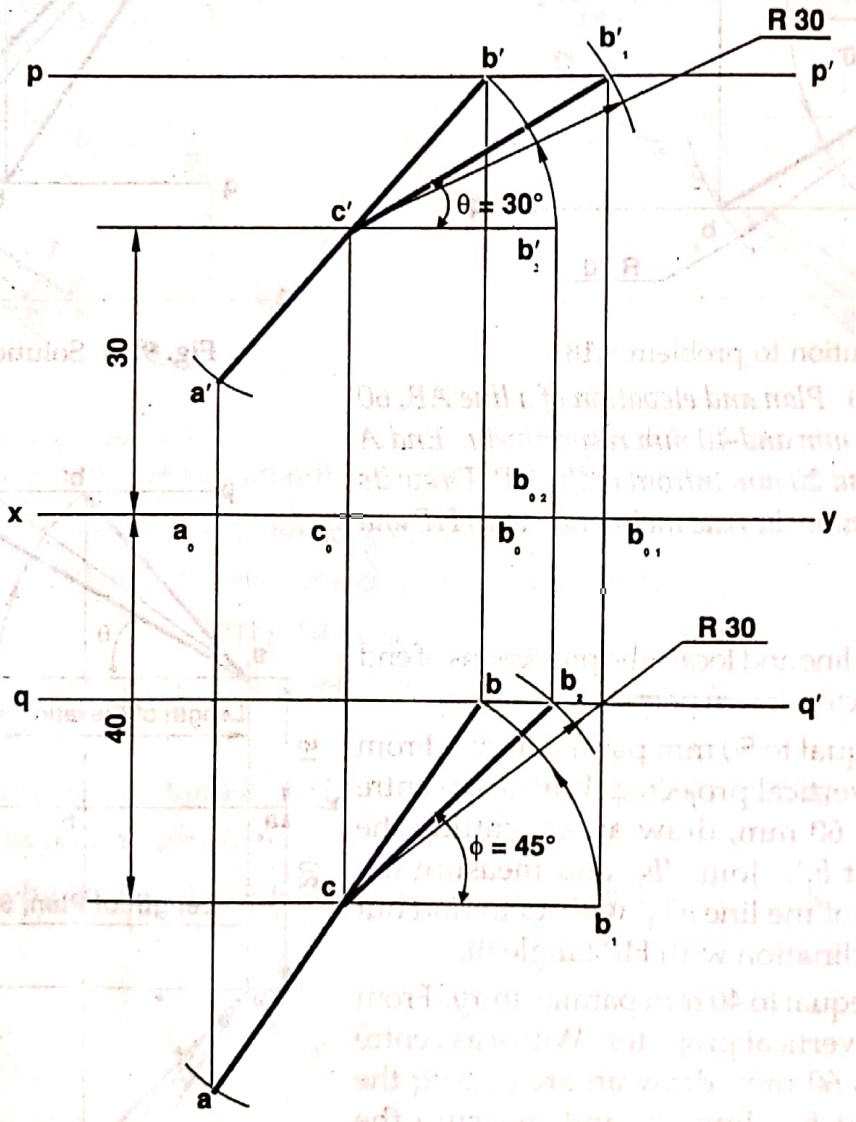


Fig. 9.40 Solution to problem 9.21

PROBLEM 9.22 *The auditorium of the college is 50 m long, 30 m wide and 15 m high. A light point is fitted at the centre of the roof and its switch is kept on one of the side walls of the auditorium, 2 m above the floor and 10 m from one of the adjacent walls. Find the actual distance between the light point and its switch.*

SOLUTION.

(*i*) Draw the front view and top view of the auditorium with suitable scale.

(*ii*) Locate the position of the light point in top view and front view as *l* and *l'* respectively.

(*iii*) Now locate the position of the switch in top view and front view as *s* and *s'* respectively. Join *s'l'* and *sl*.

(*iv*) With *s* as centre and *sl* as radius, rotate *sl* to sl_1 such that it is parallel to *xy*. Project l_1 to l_1' in the front view. Join $s'l_1'$ to obtain the required actual distance between the light point and the switch. See Fig. 9.41.

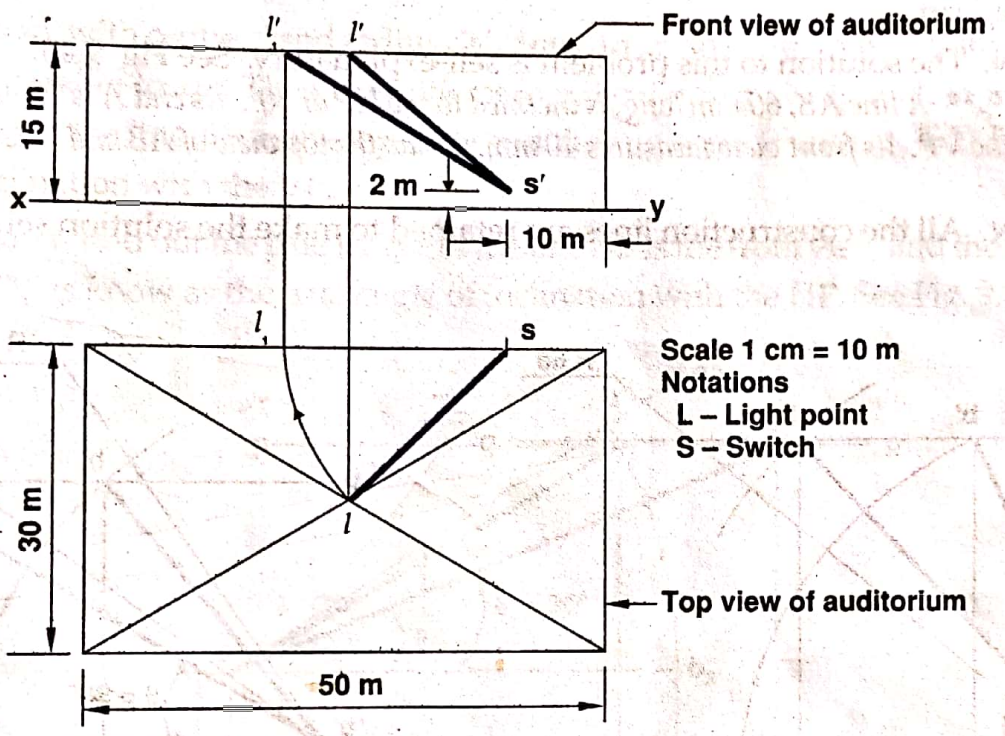


Fig. 9.41 Solution to problem 9.22

PROBLEM 9.23 *A room 10 m long, 6 m wide and 4 m high. An electric bulb hangs in the centre of the ceiling and 1 m below it. A thin straight wire connects the bulb to a switch kept in one of the corners of the room and 1.5 m above the floor. Draw the projections of the wire and determine its TL and slope with the floor.*

SOLUTION.

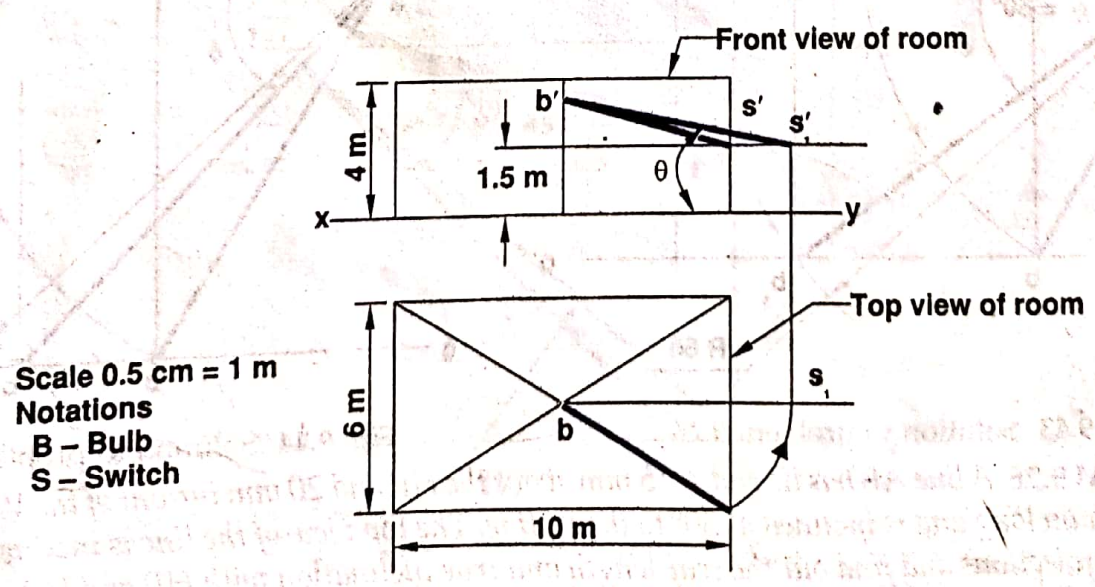


Fig. 9.42 Solution to problem 9.23

(*i*) Draw the front view and top view of the room with suitable scale.
(*ii*) Locate the position of the bulb in top view and front view as *b* and *b'* respectively.
(*iii*) Similarly, locate the position of the switch in top view and front view as *s* and *s'* respectively. Join *b's'* and *bs*.
(*iv*) With *b* as centre and *bs* as radius, rotate *bs* to bs_1, such that it is parallel to *xy*. Project s_1 to s_1' in the front view. Join $b's_1'$ to obtain the true length of the wire and measure the true angle of inclination of $b's_1'$ line with *xy*. See Fig. 9.42.

PROBLEM 9.24 *A line AB, 60 mm long, is inclined at 45° to the HP and its top view makes an angle of 60° with the VP. The end A is in the HP and 15 mm infront of the VP. Draw its front view and find its true inclination with the VP.*

SOLUTION. The solution to this problem is self-explanatory. See Fig. 9.43.

PROBLEM 9.25 *A line AB, 60 mm long, is inclined to the HP at 30°. Its end A is 10 mm above the HP and 15 mm infront of the VP. Its front view measures 40 mm. Draw the top view of AB and determine its inclination with the VP.*

SOLUTION. All the construction lines are retained to make the solution self-explanatory. See Fig. 9.44.

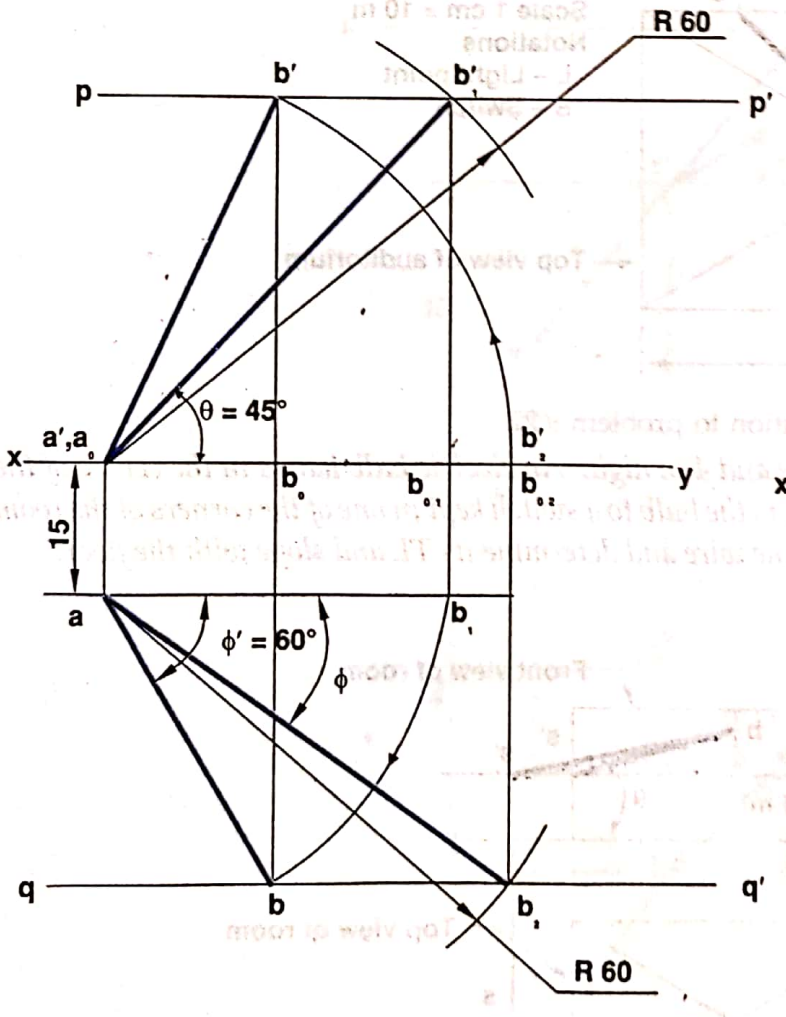


Fig. 9.43 Solution to problem 9.24

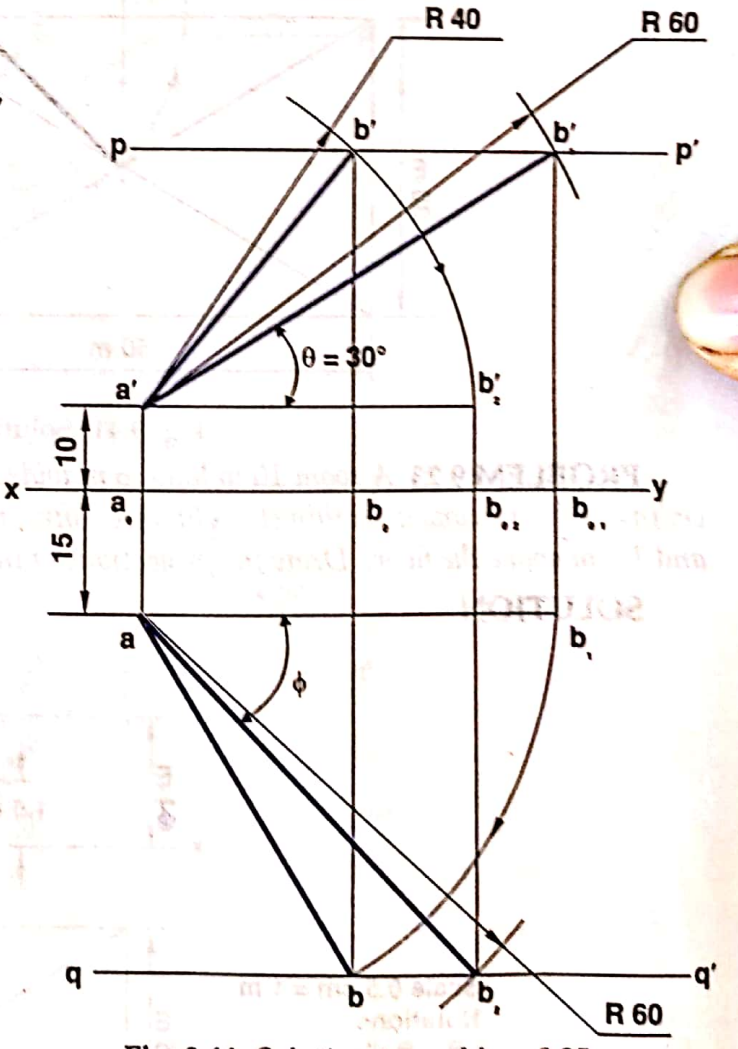


Fig. 9.44 Solution to problem 9.25

PROBLEM 9.26 *A line AB has its end A 15 mm above the HP and 20 mm infront of the VP. The front view of the line is 45 mm long and is inclined at 30° to the xy line. The top view of the line is inclined at 45° to the xy line. Draw its projections and find out the true length and true inclination with HP and VP.*

SOLUTION. (*i*) Draw the *xy* line and locate the projections of end *A* on a projector drawn perpendicular to *xy*.

(*ii*) Through a', draw a line at an angle of 30° to xy and mark the distance 45 mm to locate the point b'.

(*iii*) Since b' is being known, so $b'b$ can be drawn. Through a draw a line at an angle of 45° to intersect the second end projector at b. Then ab is the required top view.

(*iv*) Since the angles θ and ϕ made by the line with the *HP* and *VP* respectively are to be foundout. Fix the locus of the end *B* in front view and top view by drawing lines pp' and qq' through b' and b parallel to the xy respectively.

(*a*) To do so, with centre a' and radius $a'b'$, draw an arc to intersect it at b_2' (by drawing the line $a'b_2'$, parallel to xy). Then drop the perpendicular from b_2' to b_2. Join ab_2, gives the true length of the line *AB* in the top view and the angle made with ab_2 is known as true angle of inclination with the VP.

(*b*) Similarly, find out the true length of the line *AB* is the front view and the angle made with the $a'b_1'$ is know as the true angle of inclination with the HP. See Fig. 9.45.

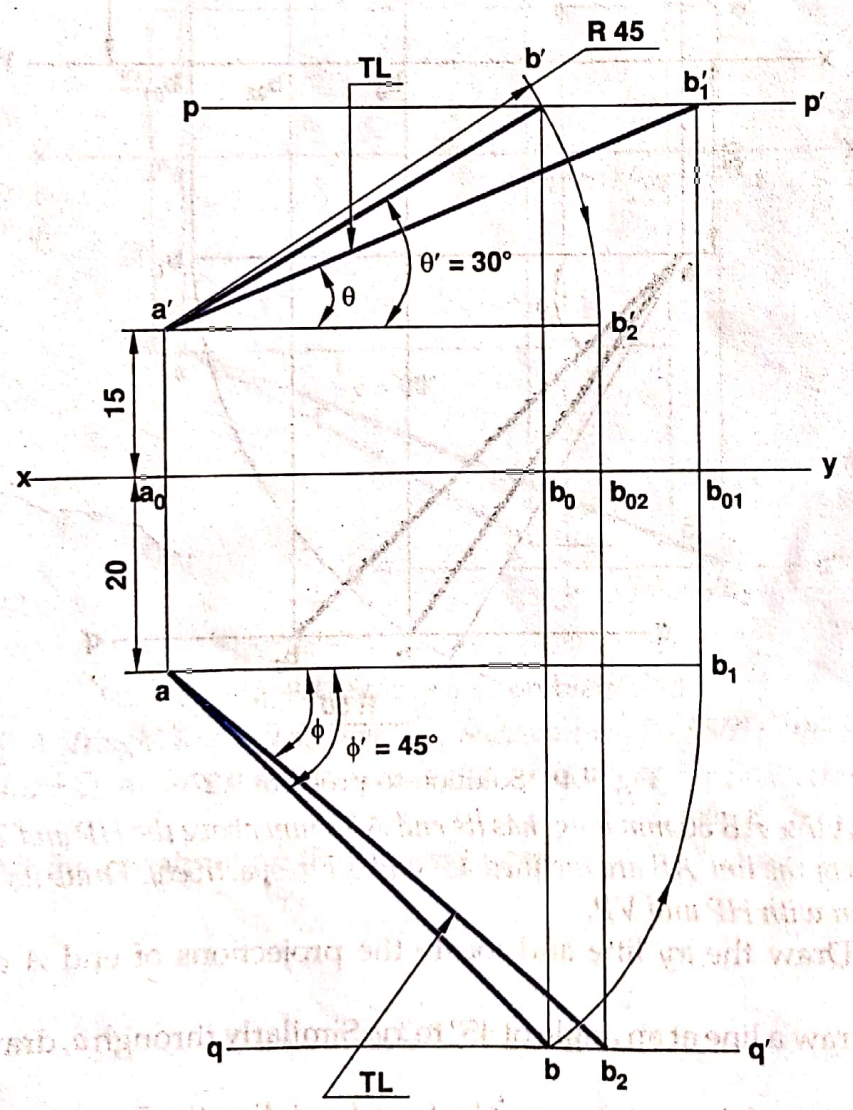


Fig. 9.45 Solution to problem 9.26

PROBLEM 9.27 *A line AB has its end A 20 mm infront of the VP and end B 55 mm above the HP. The line is inclined at 30° to the HP while its front view makes an angle of 45° to the xy line. Draw its projections, when its top view is 50 mm long. Find the true length and true angle of inclination with the VP.*

SOLUTION. The procedure to draw this problem has already been explained in previous problems. See Fig. 9.46.

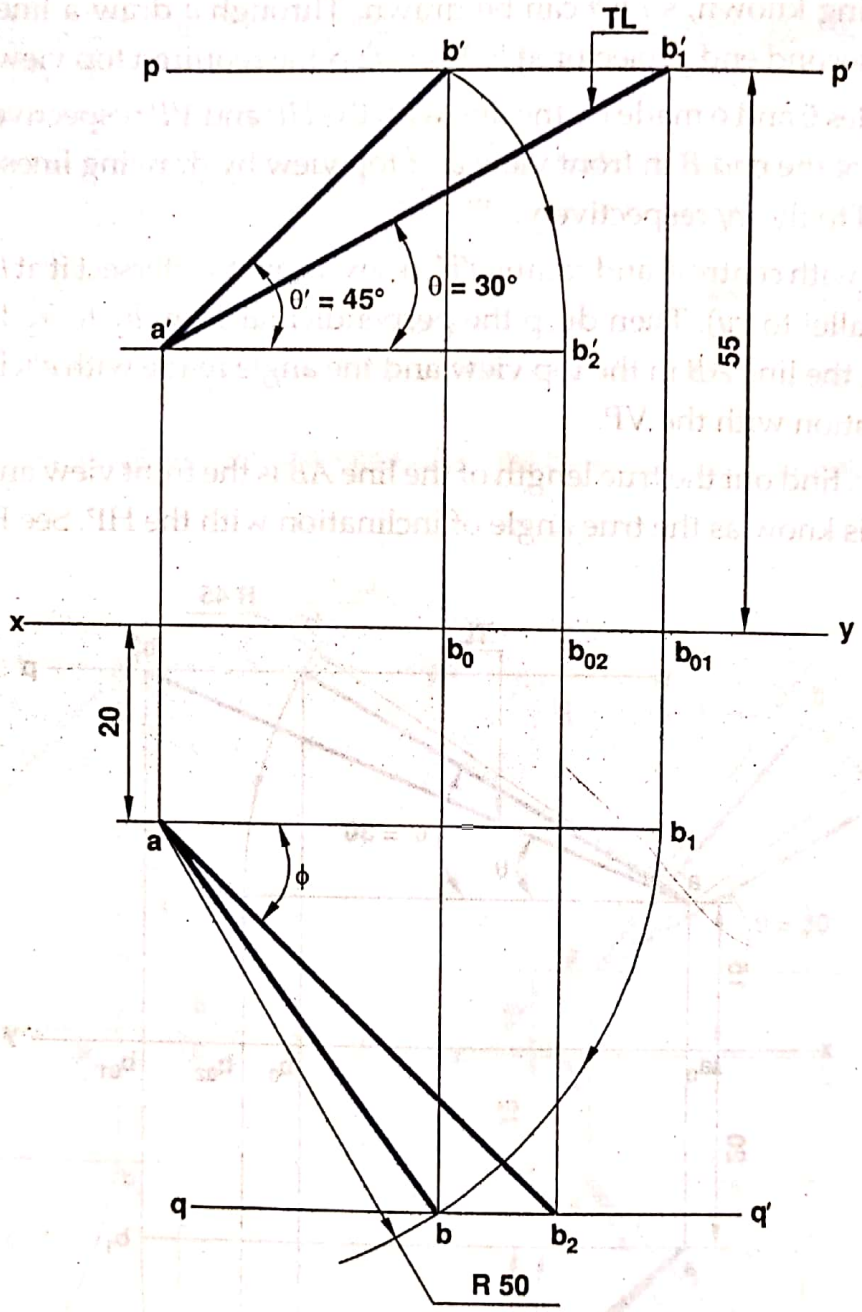


Fig. 9.46 Solution to problem 9.27

PROBLEM 9.28 *A line AB 60 mm long, has its end A 15 mm above the HP and 20 mm infront of the VP. The front and top views of the line AB are inclined 45° and 30° respectively. Draw its projections and find the true angles of inclination with HP and VP.*

SOLUTION. (*i*) Draw the *xy* line and locate the projections of end *A* on a projector drawn perpendicular to *xy*.

(*ii*) Through *a′*, draw a line at an angle of 45° to *xy*. Similarly through *a*, draw a line at angle of 30° to *xy*.

(*iii*) Since the location of the end *B* cannot be found out directly. For this take any point C on the line *AB*. Then *a′c′* and *ac* can be drawn.

(*iv*) Since the angles θ and ϕ made by the part line or full length line with the *HP* and *VP* respectively will remain same, so either of θ or ϕ for AC line should be found. To do so, with centre *a′* and radius *a′c′*, draw an *arc* to intersect it at c_2'. From c_2', draw a vertical projector. Through *c* draw

a line parallel to xy, which will intersect at a point c_2. Join ac_2 and extend ac_2 to b_2 (True length of the line). Measure the inclination of the line ab_2 with xy to find the true inclination with VP (angle ϕ).

(*v*) Repeat the same procedure as described in the previous problems. See Fig. 9.47.

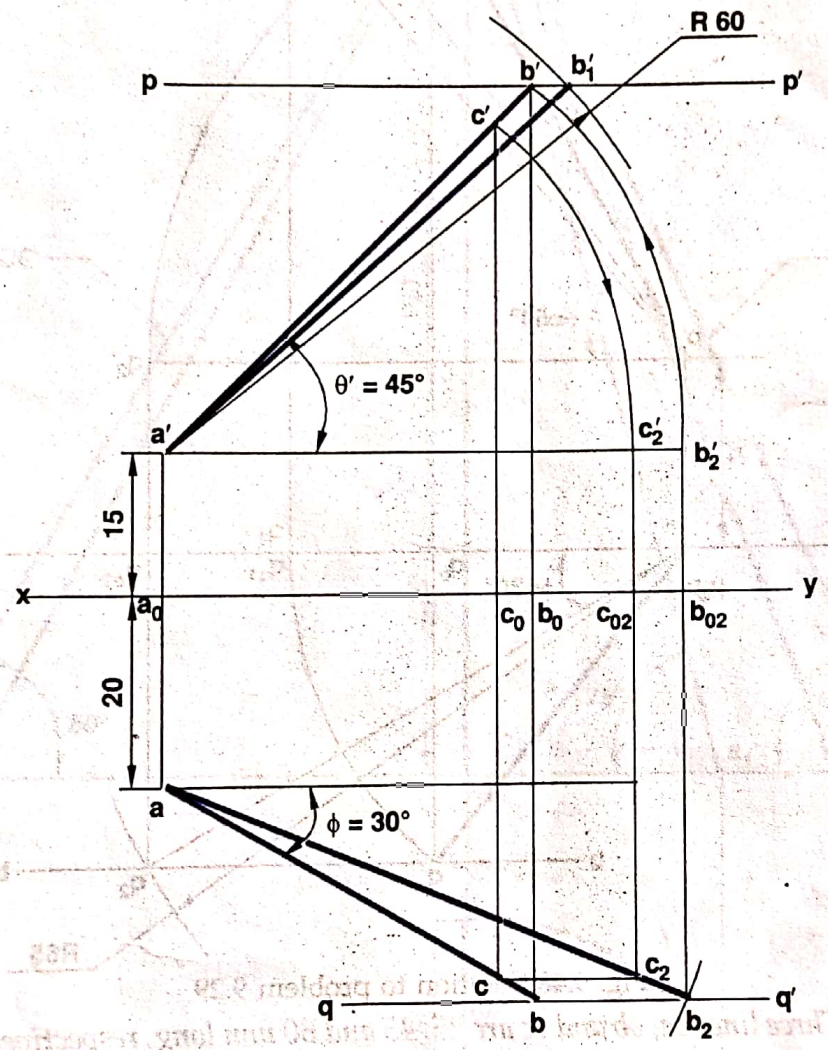


Fig. 9.47 Solution to problem 9.28

PROBLEM 9.29 *A straight line PQ 65 mm long makes an angle of 50° to the HP and 30° to the VP. The end P of the straight line PQ lies in VP and is 20 mm above HP. Draw the projections of the line PQ.*

(PTU, Jalandhar December 2005, May 2011)

SOLUTION. All the construction lines are retained to make the solution self–explanatory. See Fig. 9.48.

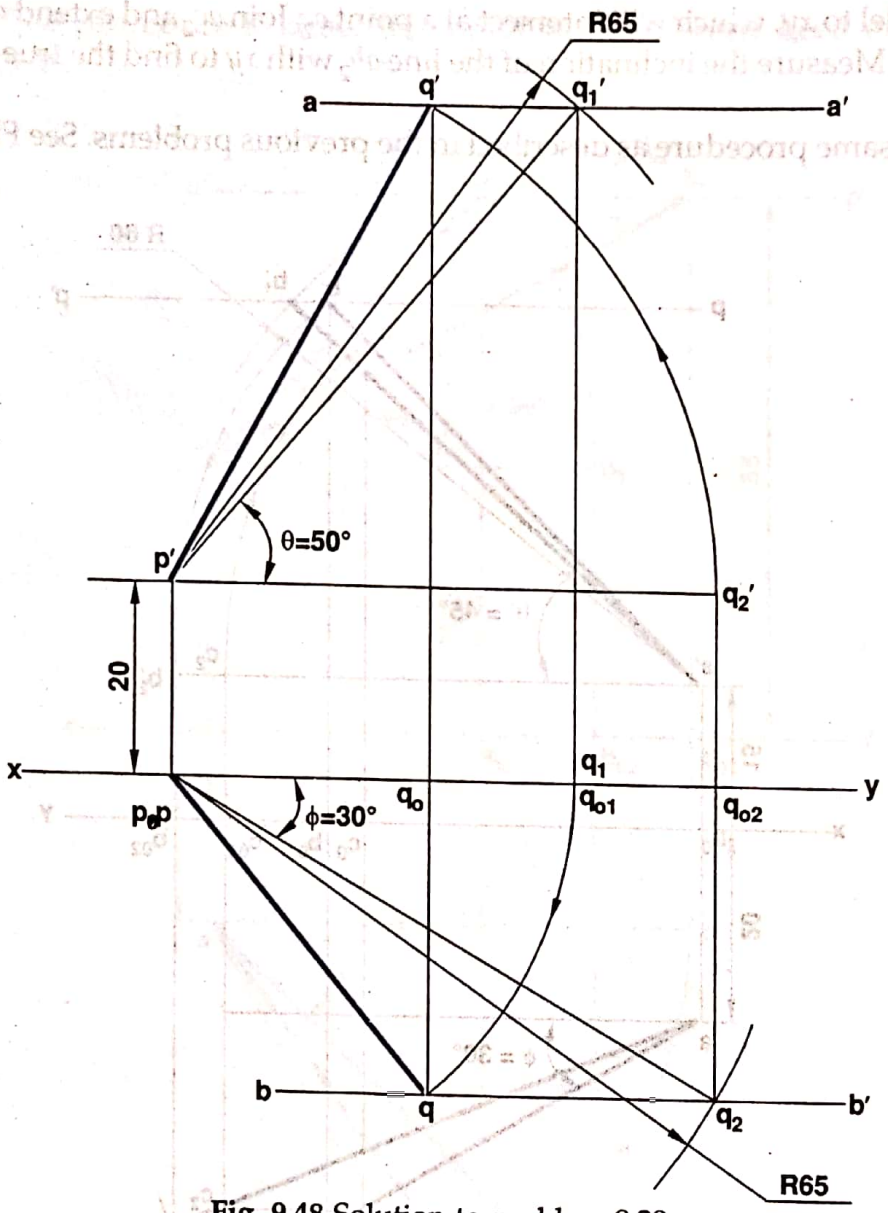


Fig. 9.48 Solution to problem 9.29

PROBLEM 9.30 *Three lines oa, ob and oc are 25, 45 and 60 mm long, respectively. Each makes an angle of 120° with the other two. The shortest line being vertical, the figure represents the top view of the three rods OA, OB and OC where each A, B and C are on the ground, while O is 100 mm above it. Draw the front view and determine the true length of each rod and its inclination with the ground.*

(PTU, Jalandhar December 2005)

SOLUTION. (*i*) Draw lines *oa*, *ob* and *oc* meeting at point *o*, an angle of 120° each. Keep the line oa to be vertical. Mark the distances of 25 mm, 45 mm and 60 mm long for *oa*, *ob* and *oc* respectively.

(*ii*) Draw the projections from points *a*, *b* and *c* to meet *xy* line at *a'*, *b'* and *c'* respectively.

(*iii*) Project *o* to *o'* and set-off a distance of 100 mm above *xy* line.

(*iv*) Join *a'*, *b'* and *c'* with *o'*.

(*v*) Since none of the three lines of the rod represents true lengths, so true lengths can be found out by rotation method as described in the previous problems. See. Fig. 9.49.

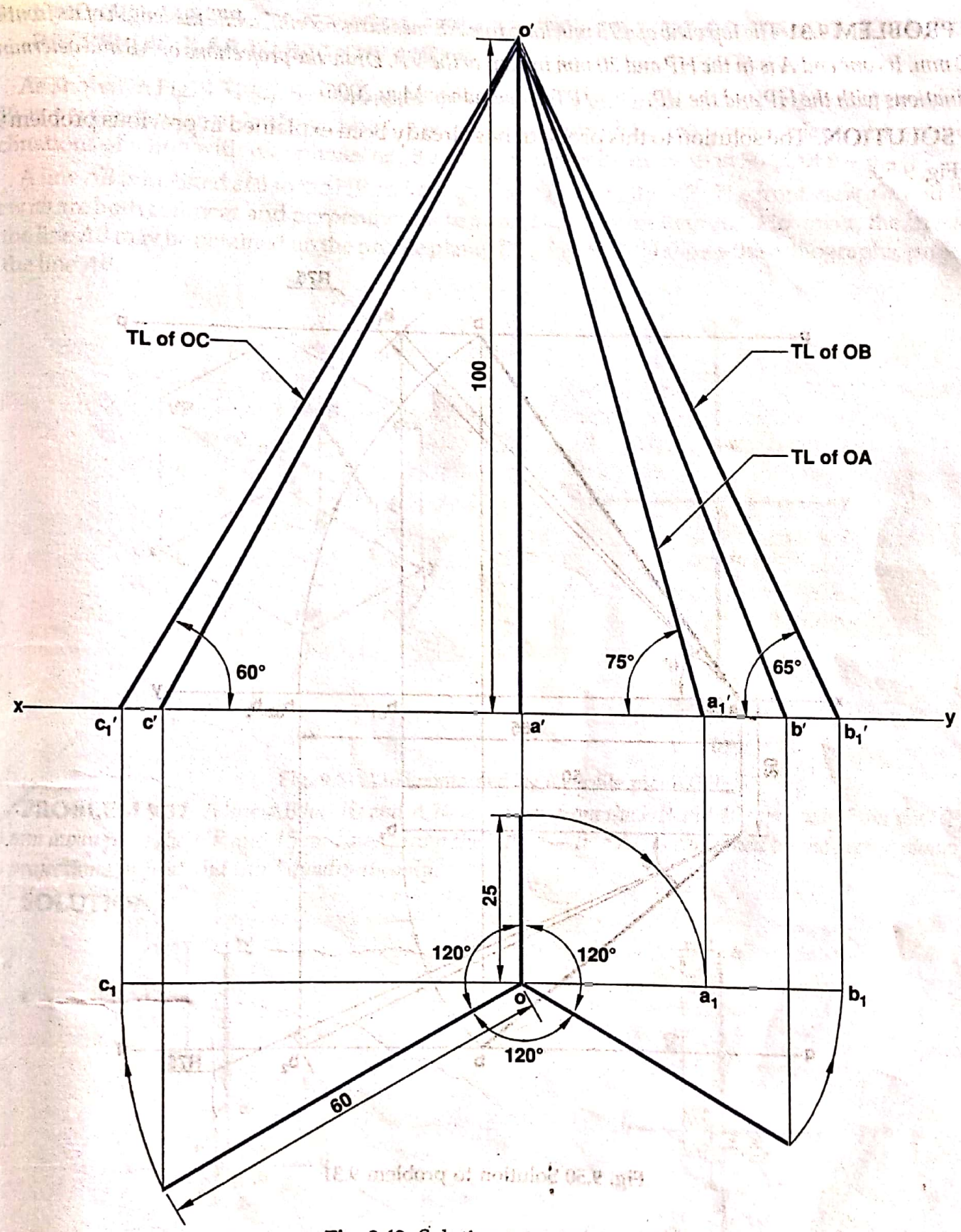


Fig. 9.49 Solution to problem 9.30

PROBLEM 9.31 *The top view of a 75 mm long line AB measures 65 mm, while the length of its front view is 50 mm. Its one end A is in the HP and 20 mm infront of the VP. Draw the projections of AB and determine its inclinations with the HP and the VP.* *(PTU, Jalandhar May 2006)*

SOLUTION. The solution to this problem has already been explained in previous problem 9.20. See Fig. 9.50.

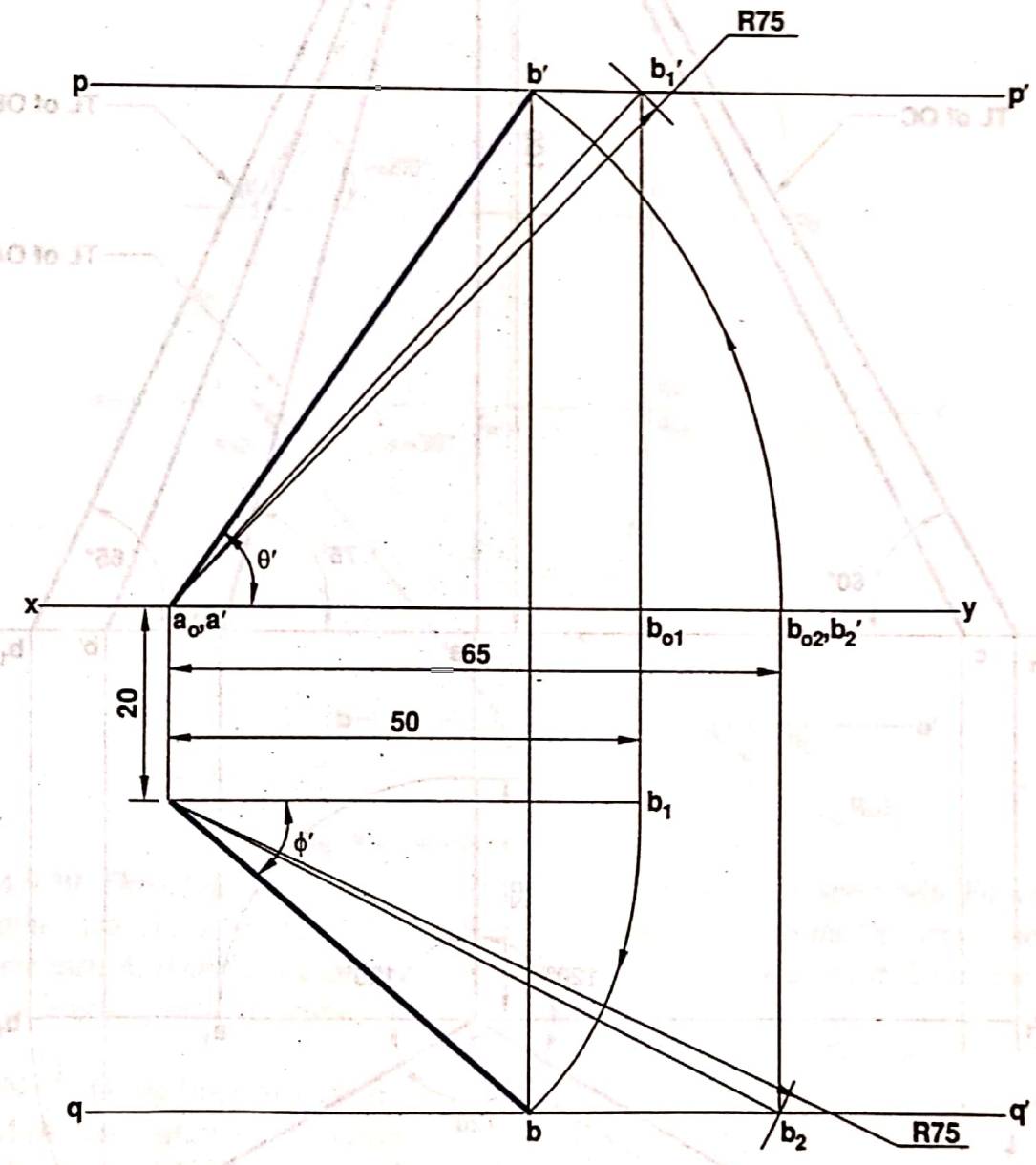


Fig. 9.50 Solution to problem 9.31

9.8 LINE CONTAINED BY A PROFILE PLANE (PP) OR LINE CONTAINED BY A PLANE, PERPENDICULAR TO BOTH HP AND VP

As shown in Fig. 9.51 (*a*), two principal planes are at right angles to each other and profile plane (PP) is perpendicular to both of them. For a line contained by the profile plane, the sum of the inclinations of a line with two planes *i.e.*, θ and ϕ can never be more than 90°, but $\theta + \phi = 90°$.

A line *AB* is inclined at θ to the HP and ϕ (equal to $90° - \theta$) to the VP. The front view *a'b'* and the top view *ab* are both collinear and perpendicular to *xy* and are shorter than *AB*. However, the true length of the line *AB* may be obtained on the profile plane (PP). Fig. 9.51 (*b*) shows the orthographic projections of the line *AB*.

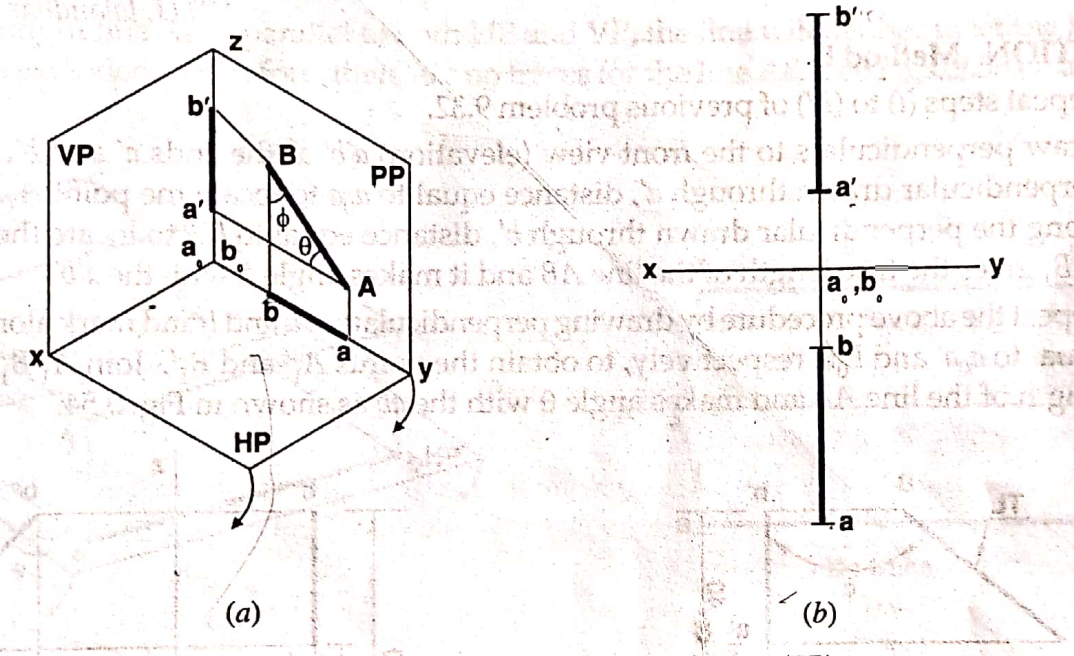


Fig. 9.51 Line contained by a profile plane (PP)

PROBLEM 9.32 *A line AB has its end A 20 mm away from the HP and 40 mm away from the VP, end B 50 mm away from the HP and 15 mm away from the VP. The line AB is contained by the profile plane. Draw its projections in first and third quadrants only.*

SOLUTION.

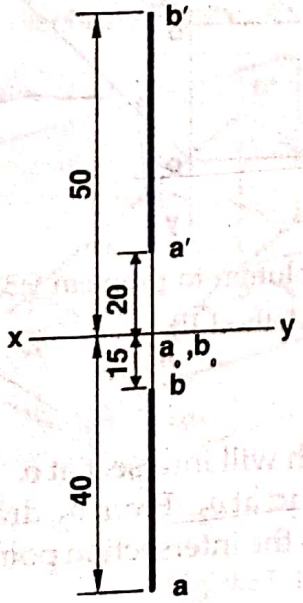


Fig. 9.52 Solution to problem 9.32 (1st quadrant)

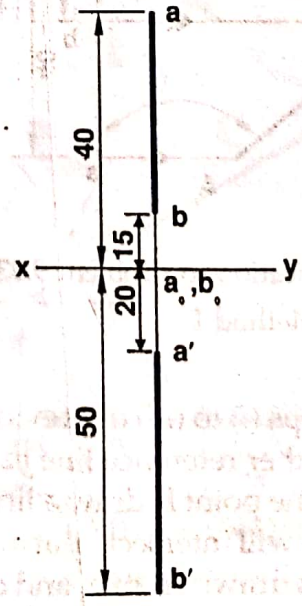


Fig. 9.53 Solution to problem 9.32 (3rd quadrant)

As the line *AB* is contained by the profile plane, both front view (elevation) *a'b'* and top view (plan) *ab* are collinear and perpendicular to the *xy*.

(*i*) Draw the *xy* line and locate the projections of two ends of the line on the projectors perpendicular to *xy*.

(*ii*) Mark along the projector the front view *a'b'* and top view *ab* of the two ends of the line AB.

(*iii*) Join *a'b'* and *ab*. Then *a'b'* and *ab* are the required elevation (front view) and plan (top view) of the line AB respectively as shown in Fig. 9.52.

See Fig. 9.53, follow the above mentioned steps to draw the projections of a line in third quadrant.

PROBLEM 9.33 *Line AB lies in profile plane. A is 40 mm infront of VP and 20 mm above HP. End B is 20 mm infront of VP and 40 mm above HP. Find its true length and inclinations.*

(PTU, Jalandhar December 2002)

SOLUTION. Method I

(*i*) Repeat steps (*i*) to (*iii*) of previous problem 9.32.

(*ii*) Draw perpendiculars to the front view (elevation) *a'b'* at the ends *a'* and *b'*. Cut along the perpendicular drawn through *a'*, distance equal to a_0a to locate the point A_1. Similarly, cut along the perpendicular drawn through *b'*, distance equal to b_0b to locate the point B_1. Join A_1B_1 gives the true length of the line *AB* and it makes angle ϕ with the *a'b'*.

(*iii*) Repeat the above procedure by drawing perpendiculars at *a* and *b* and mark along the distances equal to a_0a' and b_0b' respectively, to obtain the points A_1' and B_1'. Join $A_1'B_1'$ gives the true length of the line *AB* and makes angle θ with the *ab* as shown in Fig. 9.54.

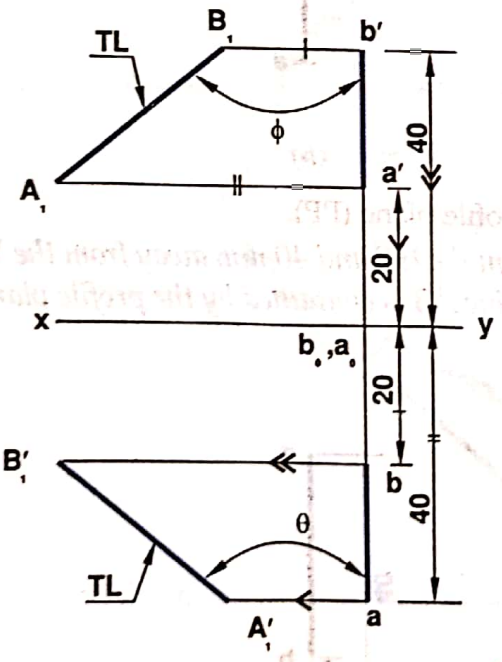


Fig. 9.54 Solution to problem 9.33 (Method I)

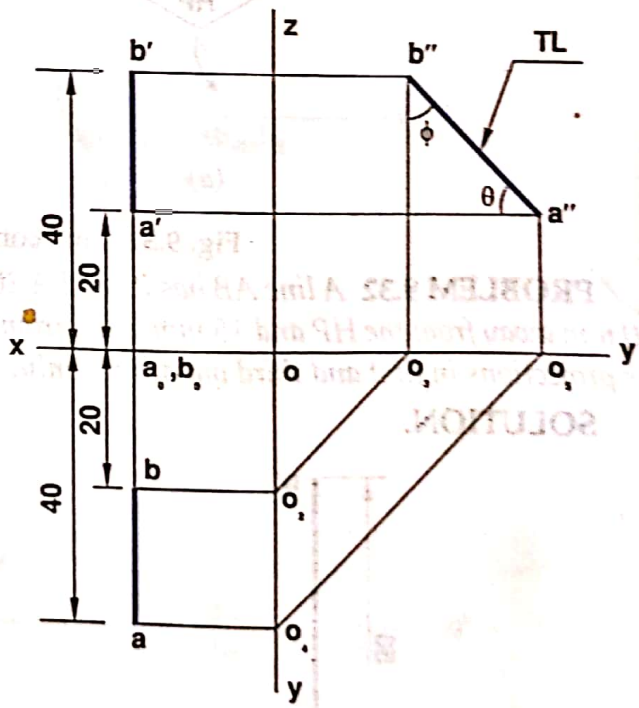


Fig. 9.55 Solution to problem 9.33 (Method II)

Method II

(*i*) Repeat steps (*i*) to (*iii*) of previous problem 9.32.

(*ii*) Draw another reference line *yz* perpendicular to *xy*, which will intersect at *o*.

(*iii*) Through the point *b*, draw a line parallel to *xy* to intersect *yz* at o_2. From o_2, draw an angle of 45°, which will intersect *xy* at o_3. Locate the side view *b"* as the intersection point between the projectiors drawn from o_3 and *b'*.

(*iv*) Repeat the above procedure for the end point *A*.

(*v*) Join *a''b''*, gives the true length of the line *AB* and makes true inclination θ and ϕ with *oy* and *oz* reference lines respectively. See Fig. 9.55.

9.9 TRACES OF A LINE

The point of intersection of a given line, produced or extended if necessary, meets the principal planes are called its traces. The point of intersection of a given line with the HP is called the horizontal trace. It is usually denoted by HT or H. The point of intersection of a given line with the VP is called the vertical trace and is usually denoted by VT or V'. The different positions of a line in space are as follows :

(*a*) Line parallel to both HP and VP

When a straight line *AB* is parallel to both HP and VP, the line will neither meet the HP nor VP, even when it is extended. Therefore, there are no traces for the line *AB*. See Figs. 9.56 (*a*) and 9.56 (*b*).

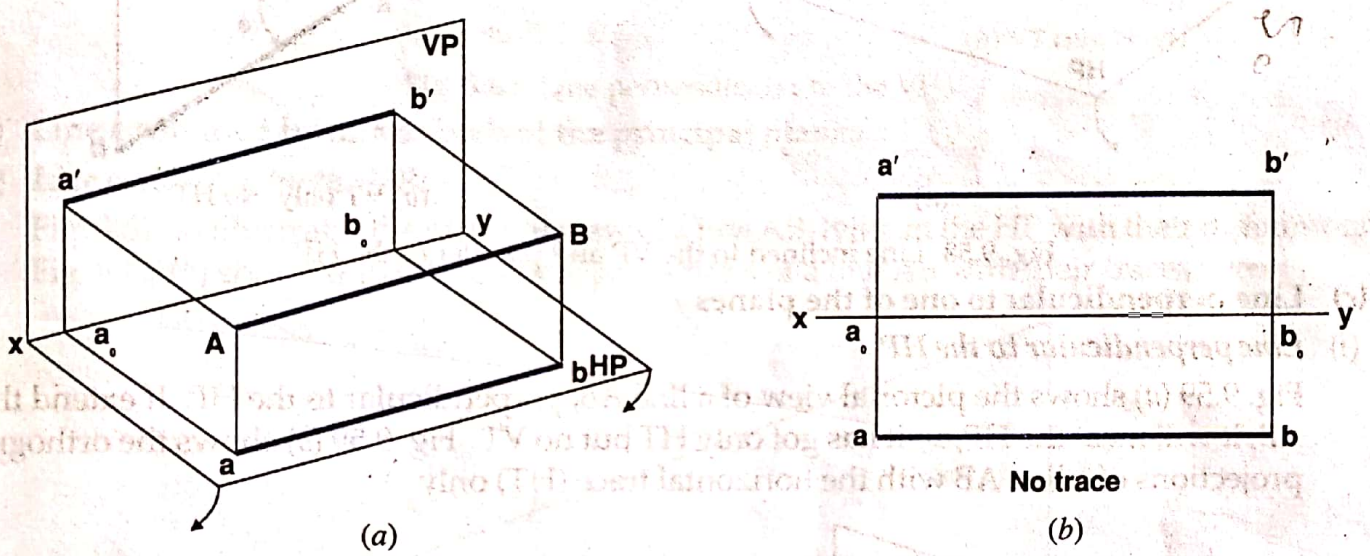


Fig. 9.56 Line parallel to both HP and VP

(*b*) Line inclined to one plane and parallel to the other

(*i*) *Line inclined to the HP and parallel to the VP*

Fig. 9.57 (*a*) shows, when a line *AB* is inclined to the HP and parallel to the VP. The line *BA* when extended will meet the HP at the HT, so it has got only HT, but no VT. Fig. 9.57 (*b*) shows the projection of a line *AB* with the horizontal trace (HT) only.

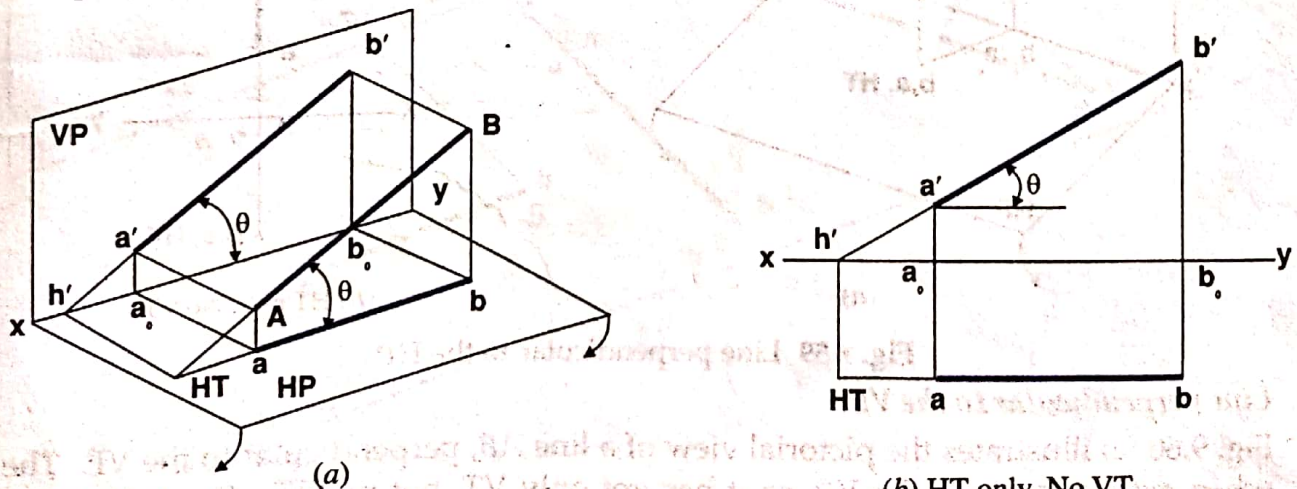
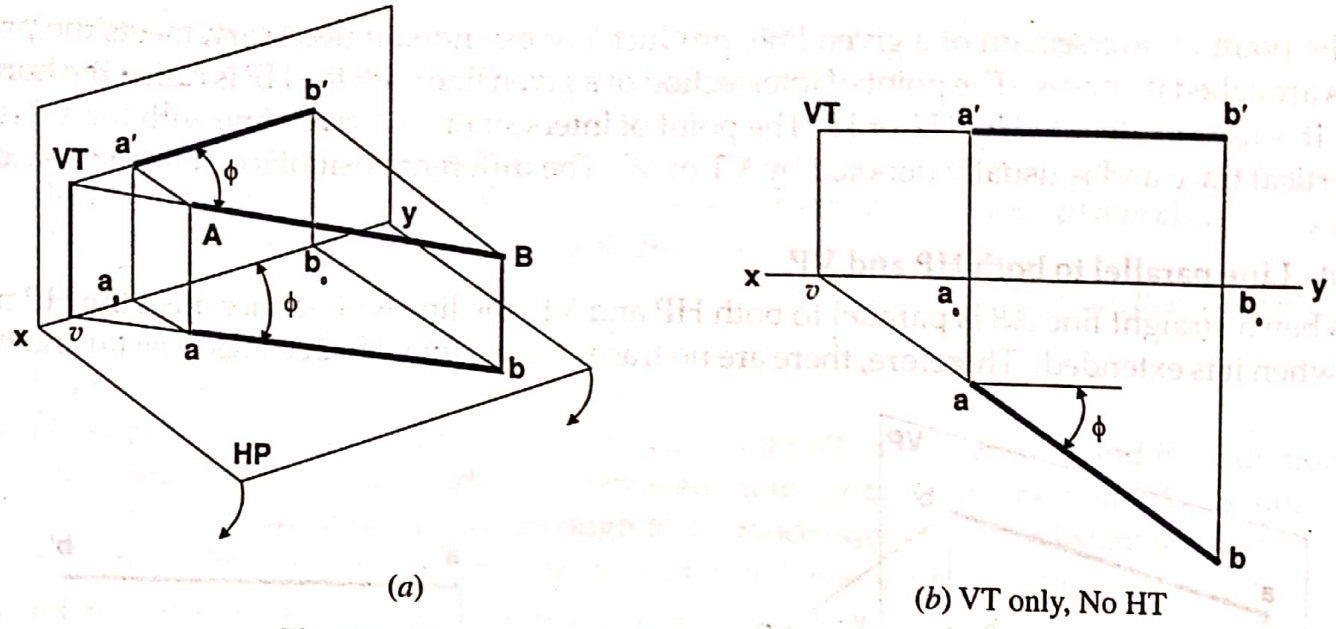


Fig. 9.57 Line inclined to the HP and parallel to the VP

(*ii*) ***Line inclined to the VP and parallel to the HP***

Fig. 9.58 (*a*) shows, when a line *AB* is inclined to the VP and parallel to the HP. The line *BA* when produced meets the VP at the VT, so it has got only VT, but no HT. Fig. 9.58 (*b*) shows the projections of a line *AB* with the vertical trace (VT) only.



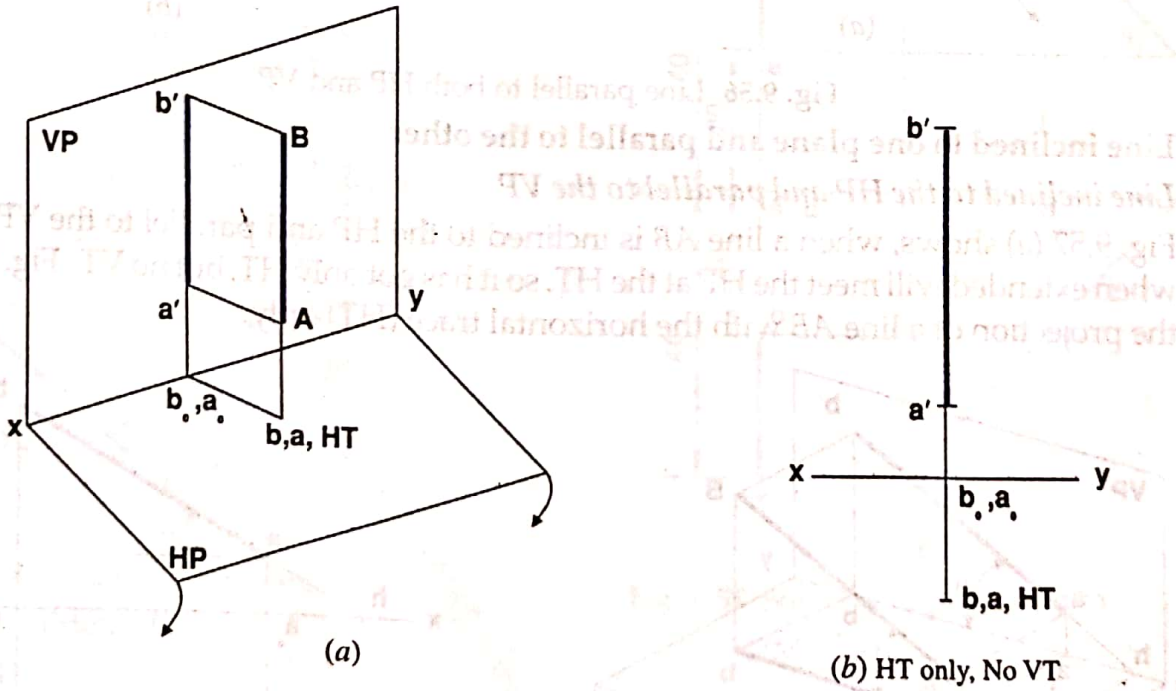
(*a*) (*b*) VT only, No HT

Fig. 9.58 Line inclined to the VP and parallel to the HP

(*c*) Line perpendicular to one of the planes

(*i*) ***Line perpendicular to the HP***

Fig. 9.59 (*a*) shows the pictorial view of a line *AB*, perpendicular to the HP. If extend the line *AB*, it will meet the HP, so it has got only HT but no VT. Fig. 9.59 (*b*) shows the orthographic projections of a line AB with the horizontal trace (HT) only.

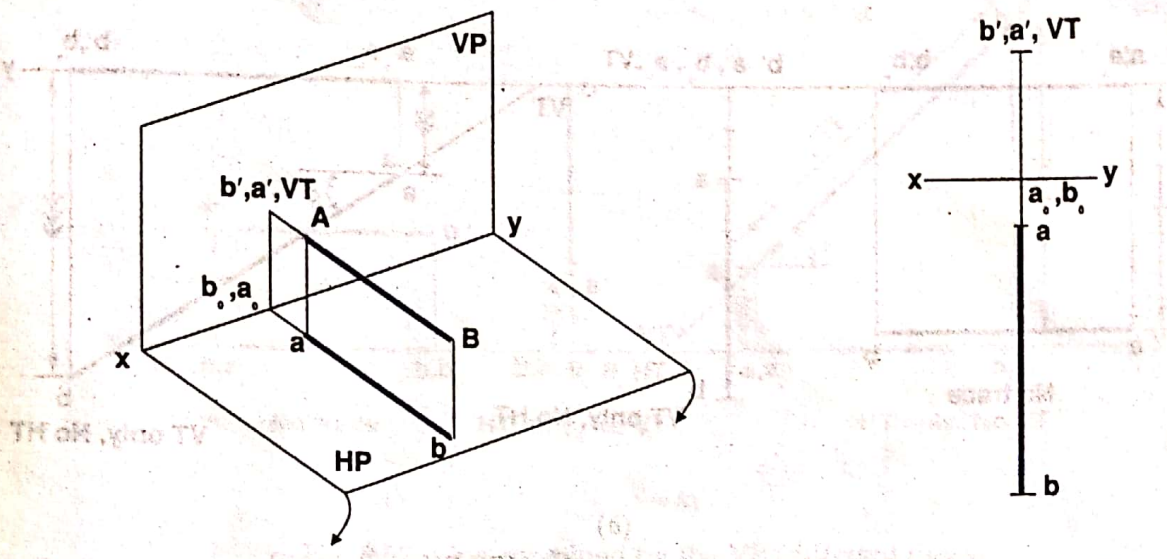


(*a*) (*b*) HT only, No VT

Fig. 9.59 Line perpendicular to the HP

(*ii*) ***Line perpendicular to the VP***

Fig. 9.60 (*a*) illustrates the pictorial view of a line *AB*, perpendicular to the VP. The line *BA* when extended meets the VP, so it has got only VT, but no HT. Fig. 9.60 (*b*) shows the orthographic projections of a line *AB* with the vertical trace (VT).



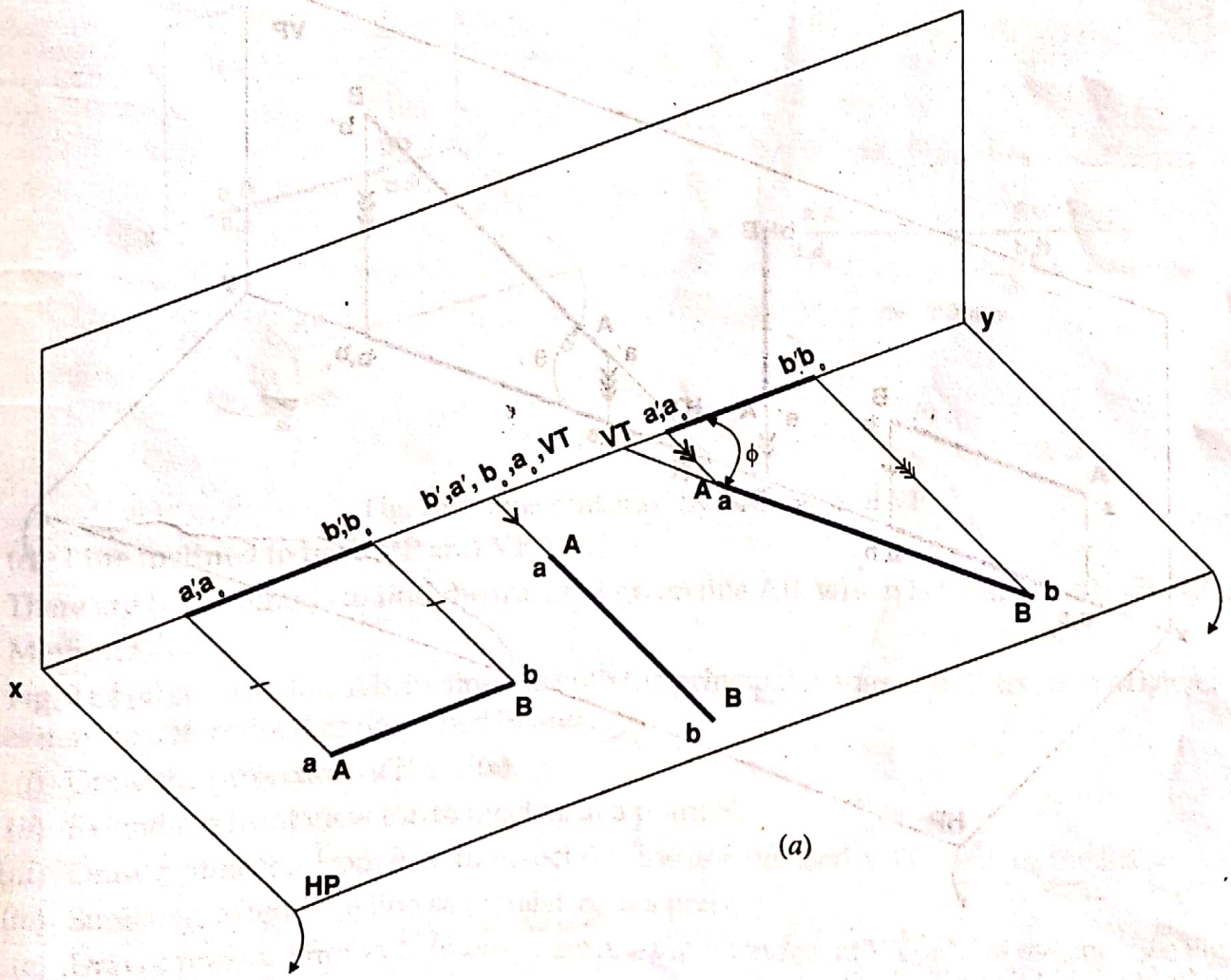
(*a*) (*b*) VT only, No HT

Fig. 9.60 Line perpendicular to the VP

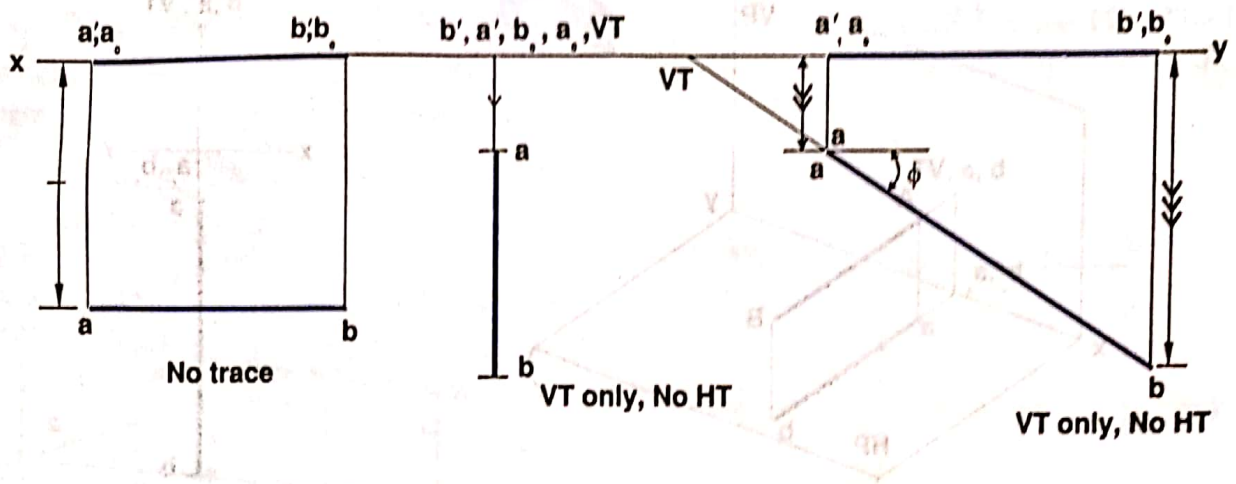
(*d*) Line contained by one or both of the principal planes

(*i*) *Line contained by the HP*

Fig. 9.61 (*a*) illustrates the pictorial view of a line AB, lying in the HP with their different cases. Fig. 9.61 (*b*) shows the orthographic projections of a line AB with their traces.



(*a*)

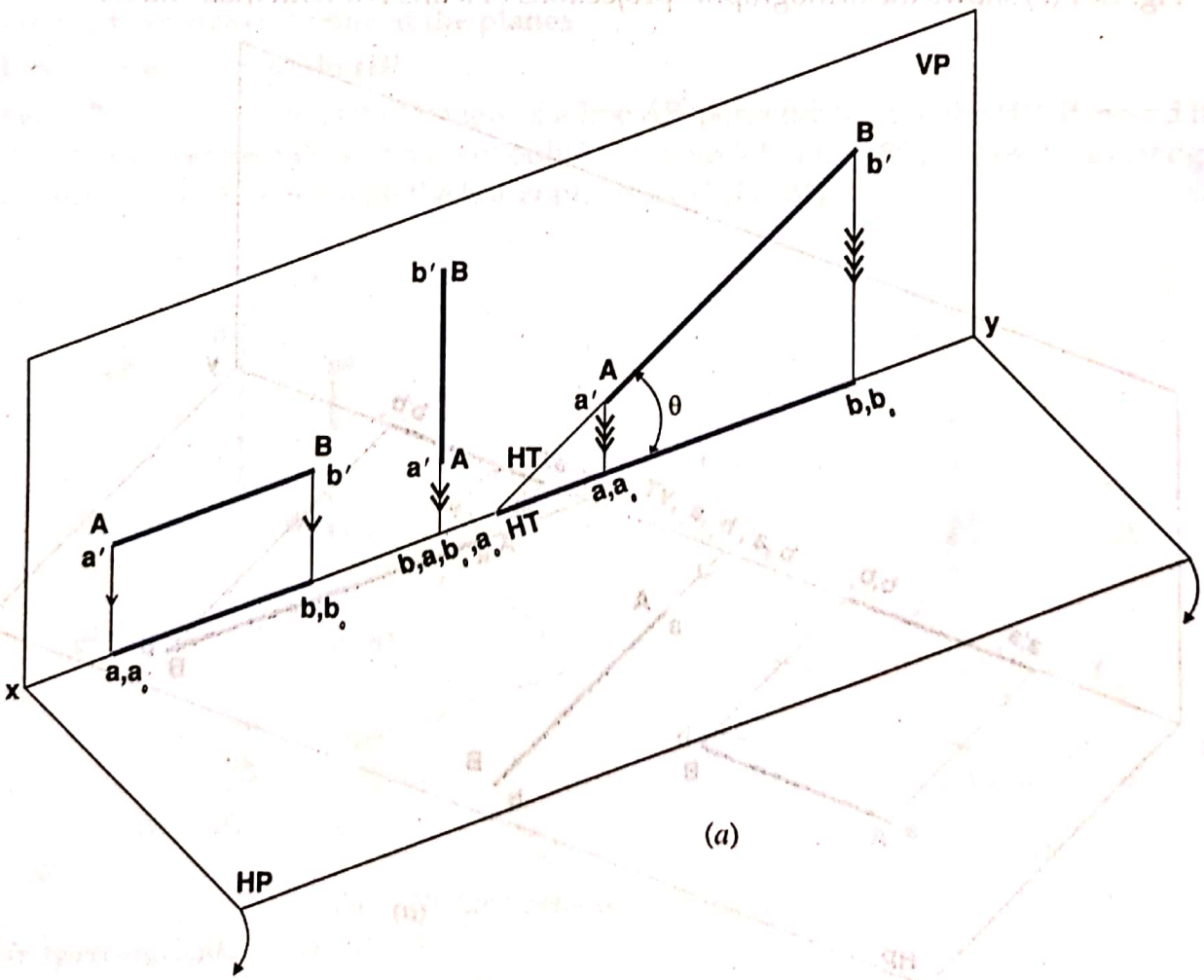


(*b*)

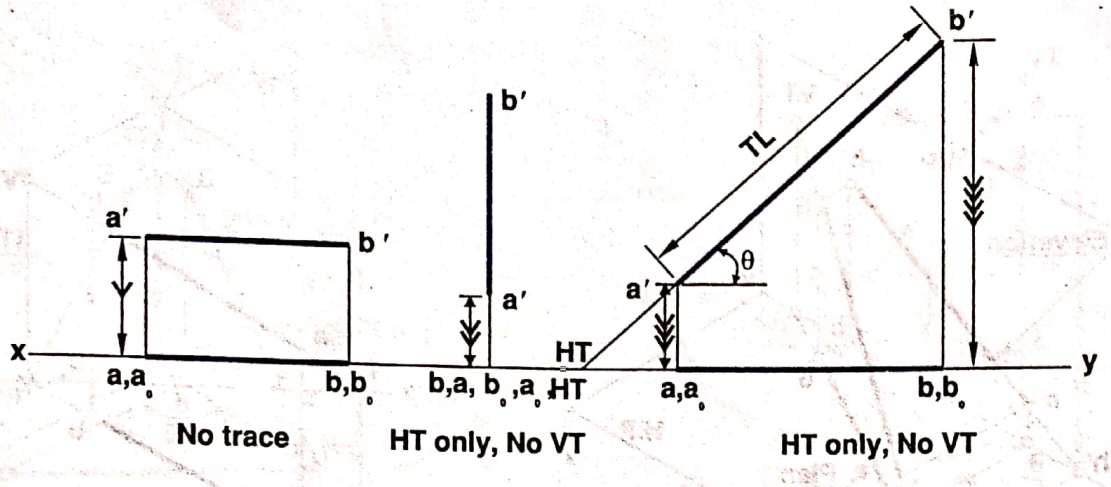
Fig. 9.61 Line contained by the HP (different cases)

(*ii*) ***Line contained by the VP***

Fig. 9.62 (*a*) shows the different cases of a line *AB*, lying in the VP. In these cases, it may have only one trace or no trace. Fig. 9.62 (*b*) shows the projections of a line *AB* with their traces.



(*a*)



(*b*)

Fig. 9.62 Line contained by the VP (different cases)

(*iii*) ***Line contained by both HP and VP***

See Fig. 9.63 (*a*), since the line *AB* is contained by both HP and VP, so it will not have any trace. Fig. 9.63 (*b*) shows the projections of a line *AB* with no trace.

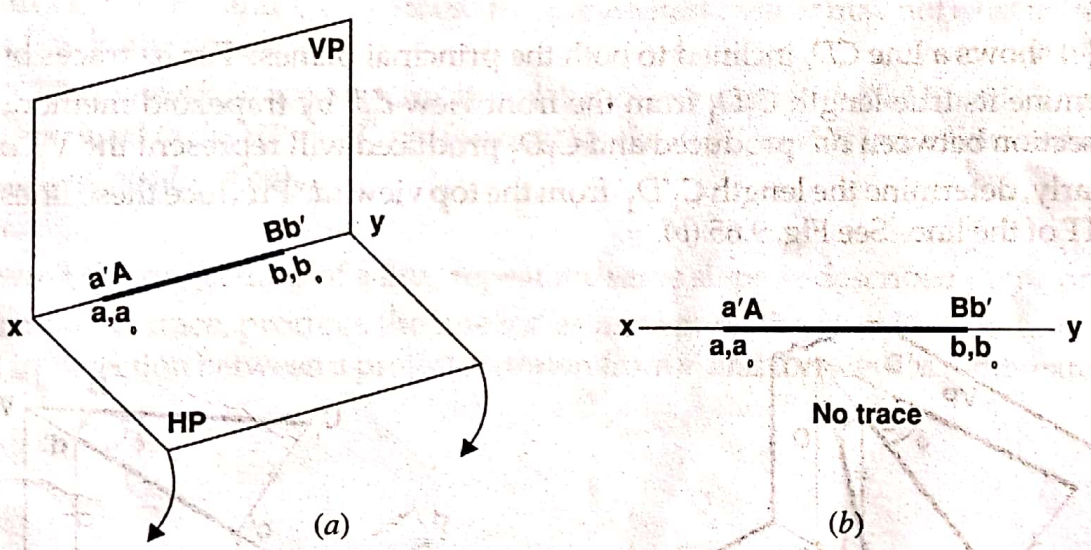


Fig. 9.63 Line contained by both HP and VP

(*e*) Line inclined to both HP and VP

There are two methods to find the traces of given line AB, which is inclined to both HP and VP.

Method I

Fig. 9.64 (*a*) shows a line AB, inclined to both the principal planes and its traces marked clearly. Its traces may be determined as described below :

(*i*) Draw the projections *a'b'* and *ab*.

(*ii*) Extend the front view *b'a'* to meet *xy* at a point *h'*.

(*iii*) Draw a projector from *h'* to intersect the line *ba* extended at HT or H of the line.

(*iv*) Similarly, extend the line *ba* to meet *xy* at a point v.

(*v*) Draw a projector from v to intersect the line *b'a'* extended, at VT or V' of the line. See Fig. 9.64 (*b*).

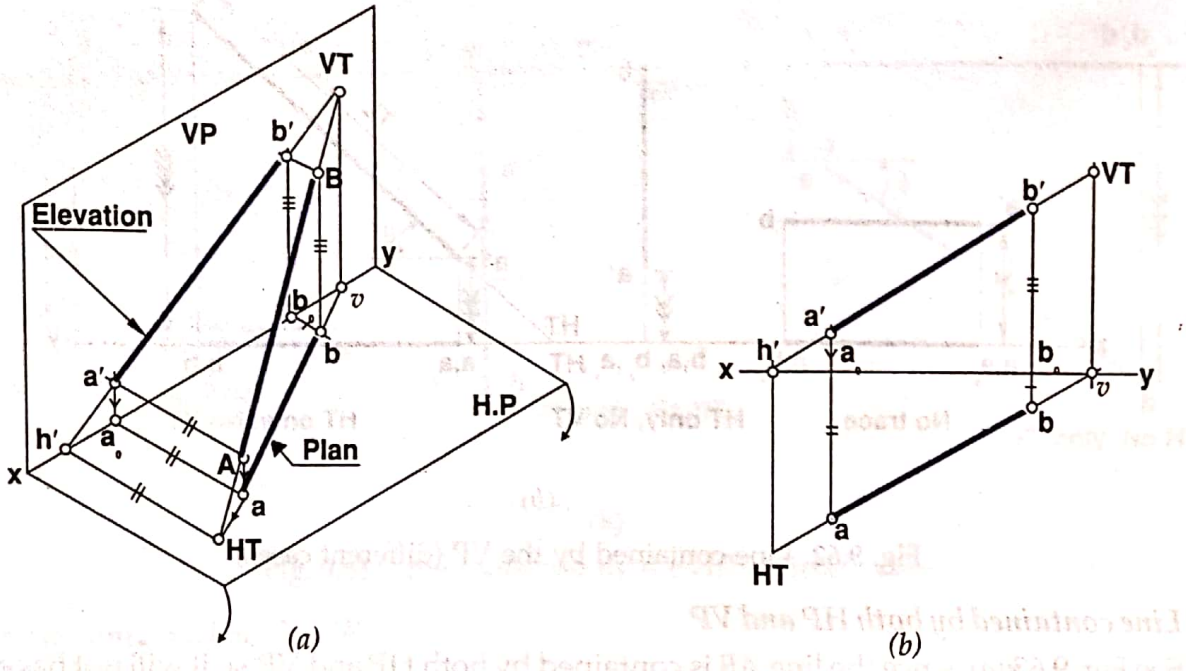


Fig. 9.64 To determine the traces of a line inclined to both HP and VP (Method I)

Method II

Fig. 9.65 (*a*) shows a line *CD*, inclined to both the principal planes. For its traces by this method,

(*i*) Determine its true length C_1D_1 from the front view *c'd'* by trapezoid method. The point of intersection between *c'd'* produced and C_1D_1 produced will represent the VT of the line.

(*ii*) Similarly, determine the length $C_1'D_1'$ from the top view *cd*. Produce these lines to intersect at the HT of the line. See Fig. 9.65 (*b*).

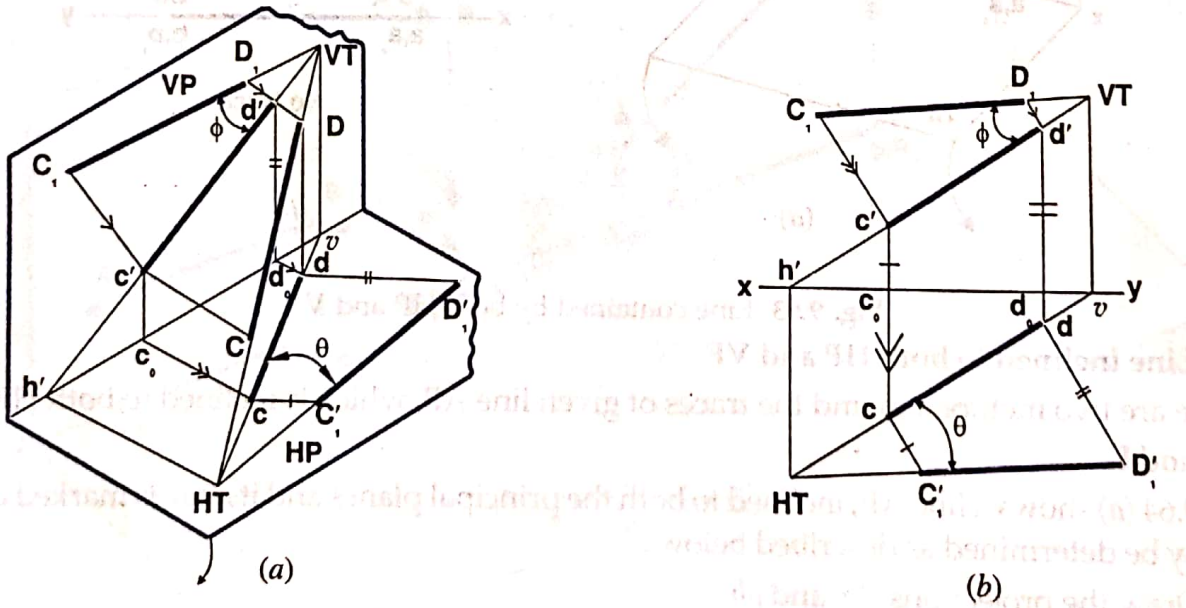


Fig. 9.65 To determine the traces of a line inclined to both HP and VP (Method II)

(f) Line contained by a profile plane

As shown in Fig. 9.66 (*a*), when a line *AB* is placed in such a way that the sum of its inclinations with both HP and VP is 90°, then it is not possible to find the traces by the method I. Method II must be employed to find its traces as shown in Fig. 9.66 (*b*).

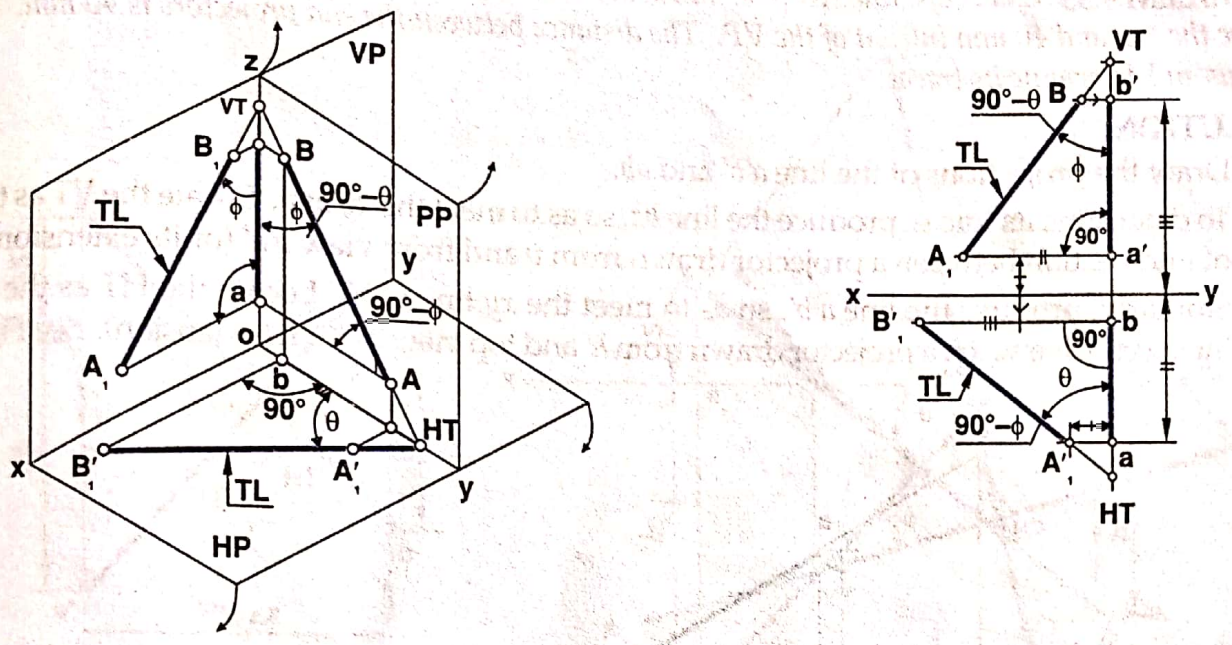


Fig. 9.66 To determine the traces of a line contained by a profile plane

Note : To have the complete knowledge of projections of lines, the student must acquaint himself or herself with TL, θ, ϕ, HT and VT. All these five parameters may or may not exist in the particular position of a line in the space.

PROBLEM 9.34 *A line AB 50 mm long, has its end A is 25 mm away from the HP and 15 mm away from the VP. The line is inclined to the HP at 30° and is parallel to the VP. Draw its projections and determine its traces in first and second quadrant only.*

SOLUTION.

(*i*) For drawing the projections of a line, repeat the same steps as described in the problem 9.2.

(*ii*) To determine its trace, produce the line *b'a'* so as to meet the *xy* at *h'*. Locate the HT as the point of intersection between a projector drawn from *h'* and top view *ba* (or its extension). See Fig. 9.67.

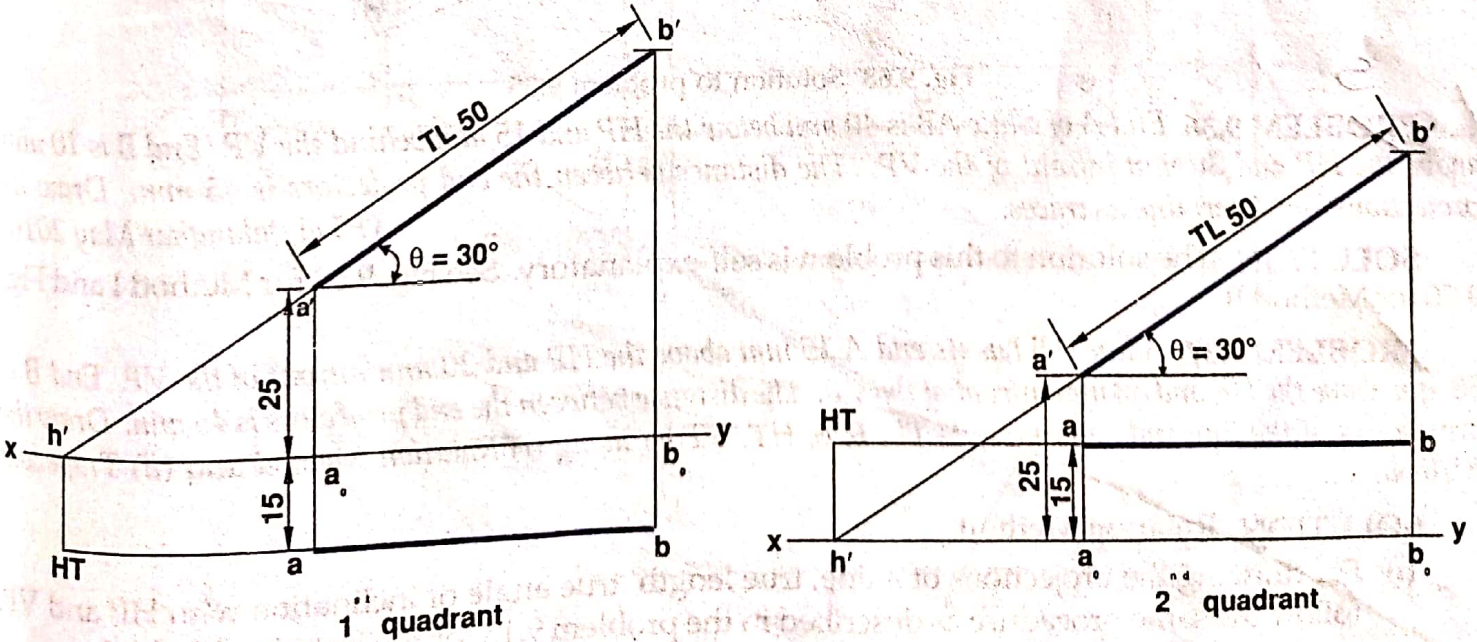


Fig. 9.67 Solution to problem 9.34

PROBLEM 9.35 *End A of a line AB is 30 mm above the HP and 10 mm infront of the VP and end B is 15 mm above the HP and 40 mm infront of the VP. The distance between the end projectors is 40 mm. Draw its projections and determine its traces.*

SOLUTION.

(*i*) Draw the projections of the line *a'b'* and *ab*.

(*ii*) To determine its traces, produce the line *ba*, so as to meet the *xy* at *v*. Locate the VT as the point of intersection between a projector drawn from *v* and front view *b'a'* (or its extension).

(*iii*) Similarly, produce the line *a'b'*, so as to meet the *xy* line at *h'*. Locate the HT as the point of intersection between a projector drawn from *h'* and top view *ab* (or its extension). See Fig. 9.68.

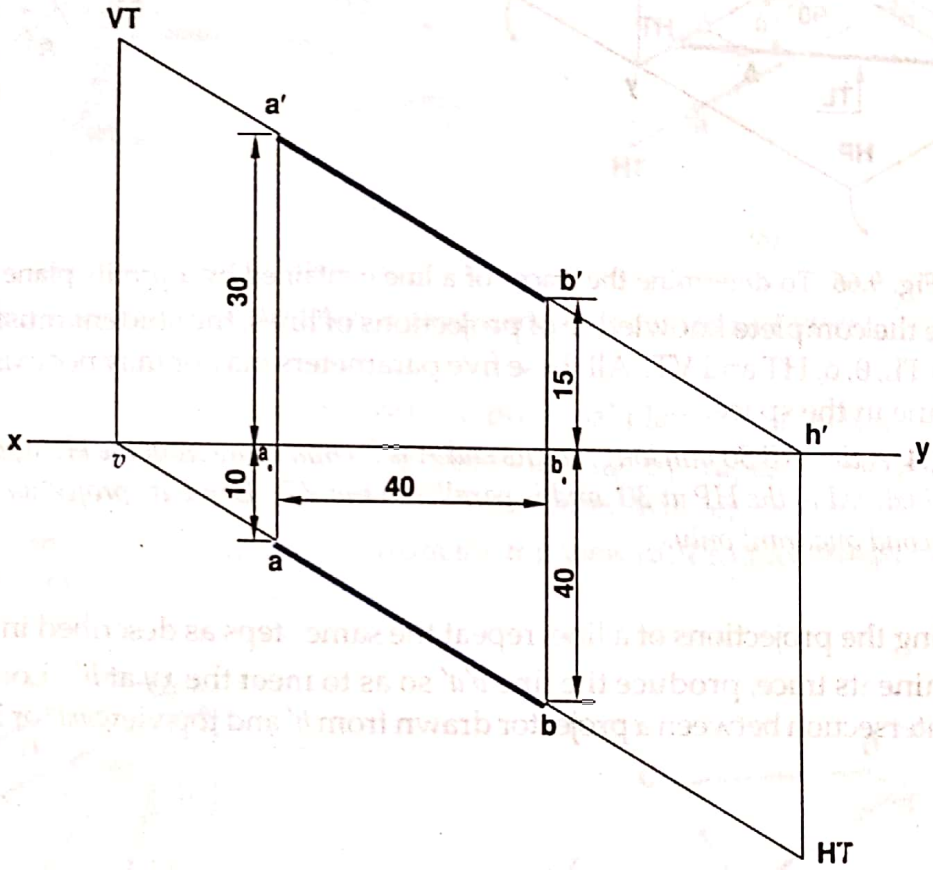


Fig. 9.68 Solution to problem 9.35

PROBLEM 9.36 *End A of a line AB is 40 mm below the HP and 15 mm behind the VP. End B is 10 mm above the HP and 30 mm infront of the VP. The distance between the end projectors is 45 mm. Draw its projections and determine its traces.* (*PTU, Jalandhar May 2010*)

SOLUTION. The solution to this problem is self-explanatory. See Fig. 9.69 for Method I and Fig. 9.70 for Method II.

PROBLEM 9.37 *A line AB has its end A 15 mm above the HP and 20 mm infront of the VP. End B is 30 mm above the HP and 50 mm infront of the VP. The distance between the end projectors is 45 mm. Draw the projections of the line and determine its TL, θ, ϕ, HT, VT by using* (*i*) *Rotation Method* and (*ii*) *Trapezoid Method.*

SOLUTION. Rotation Method

(*i*) For drawing the projections of a line, true length, true angle of inclination with HP and VP, follow the same procedure as described in the problem 9.13 for the Rotation Method.

(*ii*) To determine its traces, produce the line *ba*, so as to meet *xy* at *v*. Locate the VT as the point of intersection between a projector drawn from *v* and front view *b'a'* (or its extension).

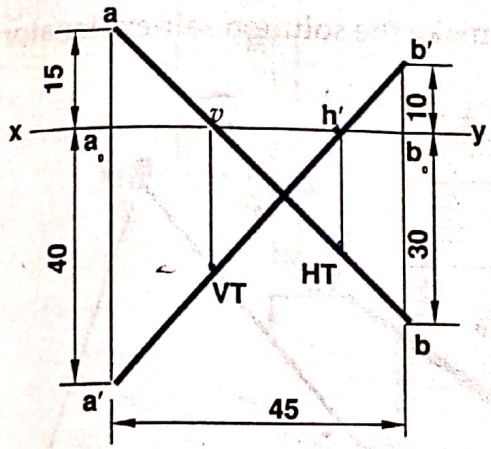


Fig. 9.69 Solution to problem 9.36 (Method I)

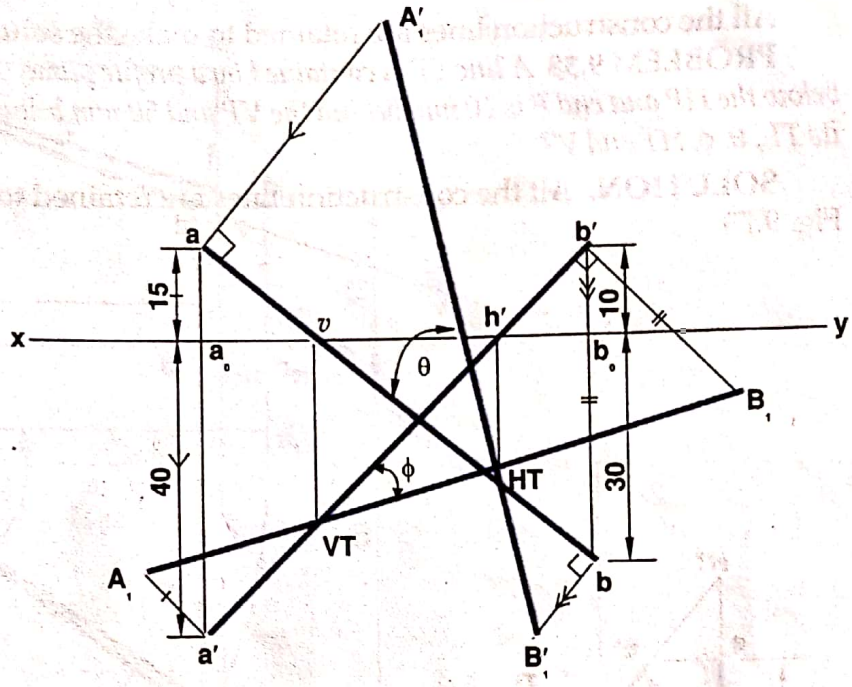


Fig. 9.70 Solution to problem 9.36 (Method II)

(*iii*) Similarly, produce the line $b'a'$, so as to meet xy at h'. Locate the HT as the point of intersection between a projector drawn from h' and top view ba (or its extension) as shown in Fig. 9.71.

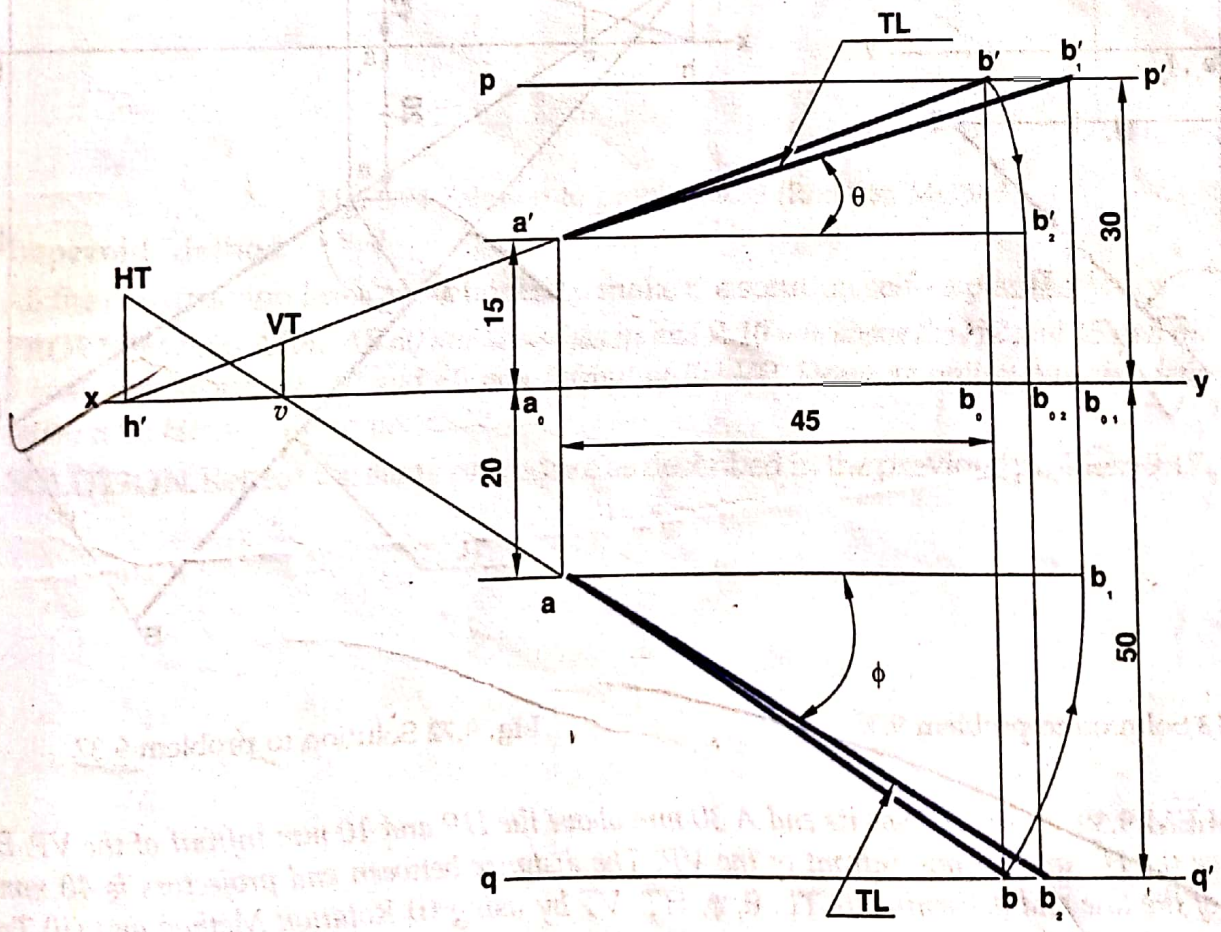


Fig. 9.71 Solution to problem 9.37 (rotation method)

Trapezoid Method

All the construction lines are retained to make the solution self-explanatory. See Fig. 9.72.

PROBLEM 9.38 *A line EF is contained by a profile plane. Its end E is 45 mm behind the VP and 10 mm below the HP and end F is 10 mm behind the VP and 50 mm below the HP. Draw its projections and determine its TL, θ, ϕ, HT and VT.*

SOLUTION. All the construction lines are retained to make the solution self-explanatory. See Fig. 9.73

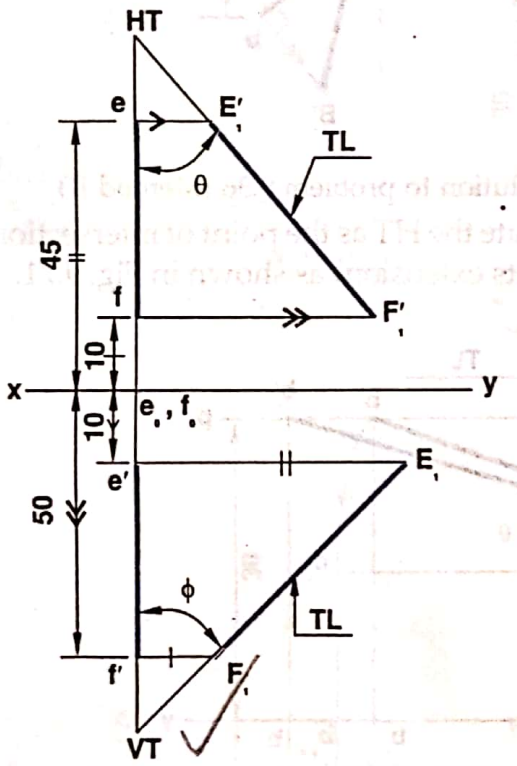


Fig. 9.73 Solution to problem 9.38

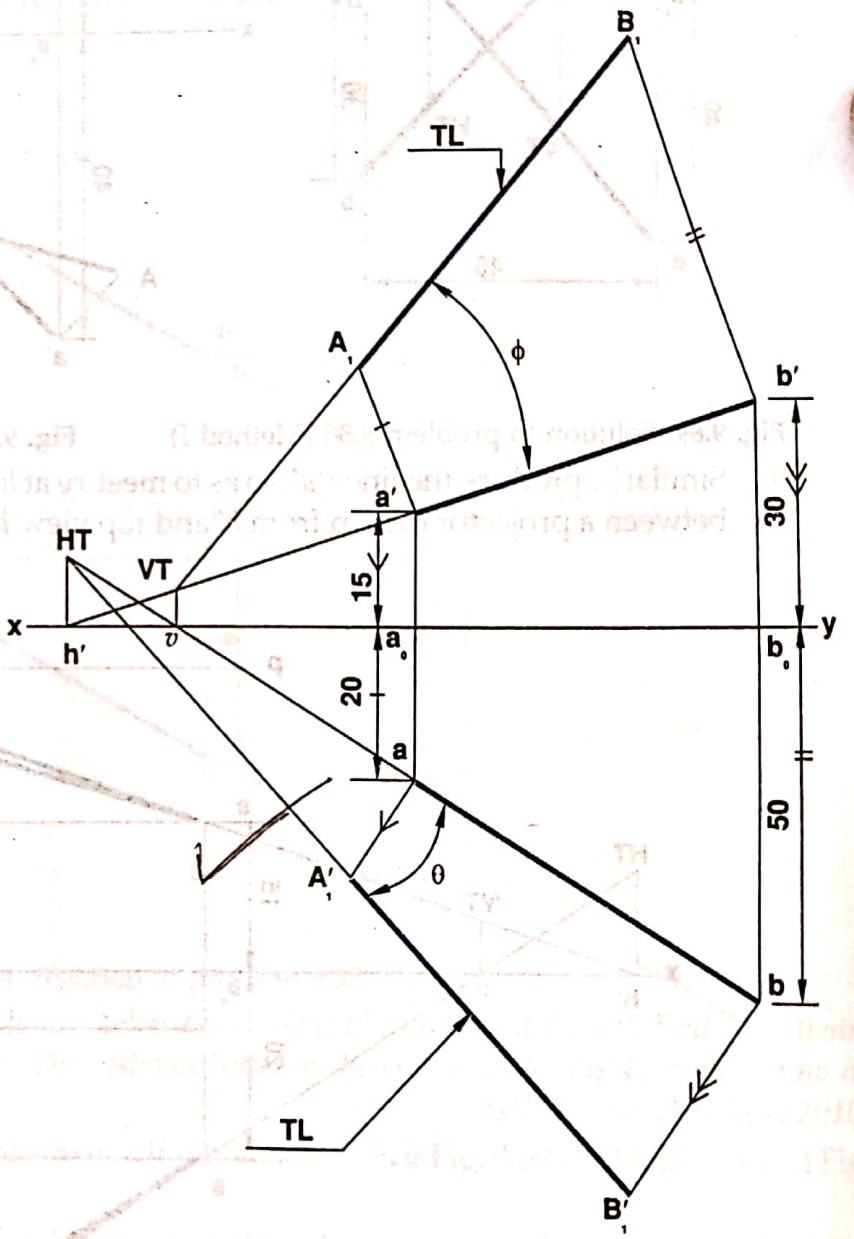


Fig. 9.72 Solution to problem 9.37

PROBLEM 9.39 *A line AB has its end A 30 mm above the HP and 10 mm infront of the VP. End B is 15 mm above the HP and 40 mm infront of the VP. The distance between end projectors is 40 mm. Draw projections of the line and determine its TL, θ, ϕ, HT, VT by using (i) Rotation Method and (ii) Trapezoid Method.*

SOLUTION. Rotation Method

Repeat the same procedure as described in the previous problems. See Fig 9.74.

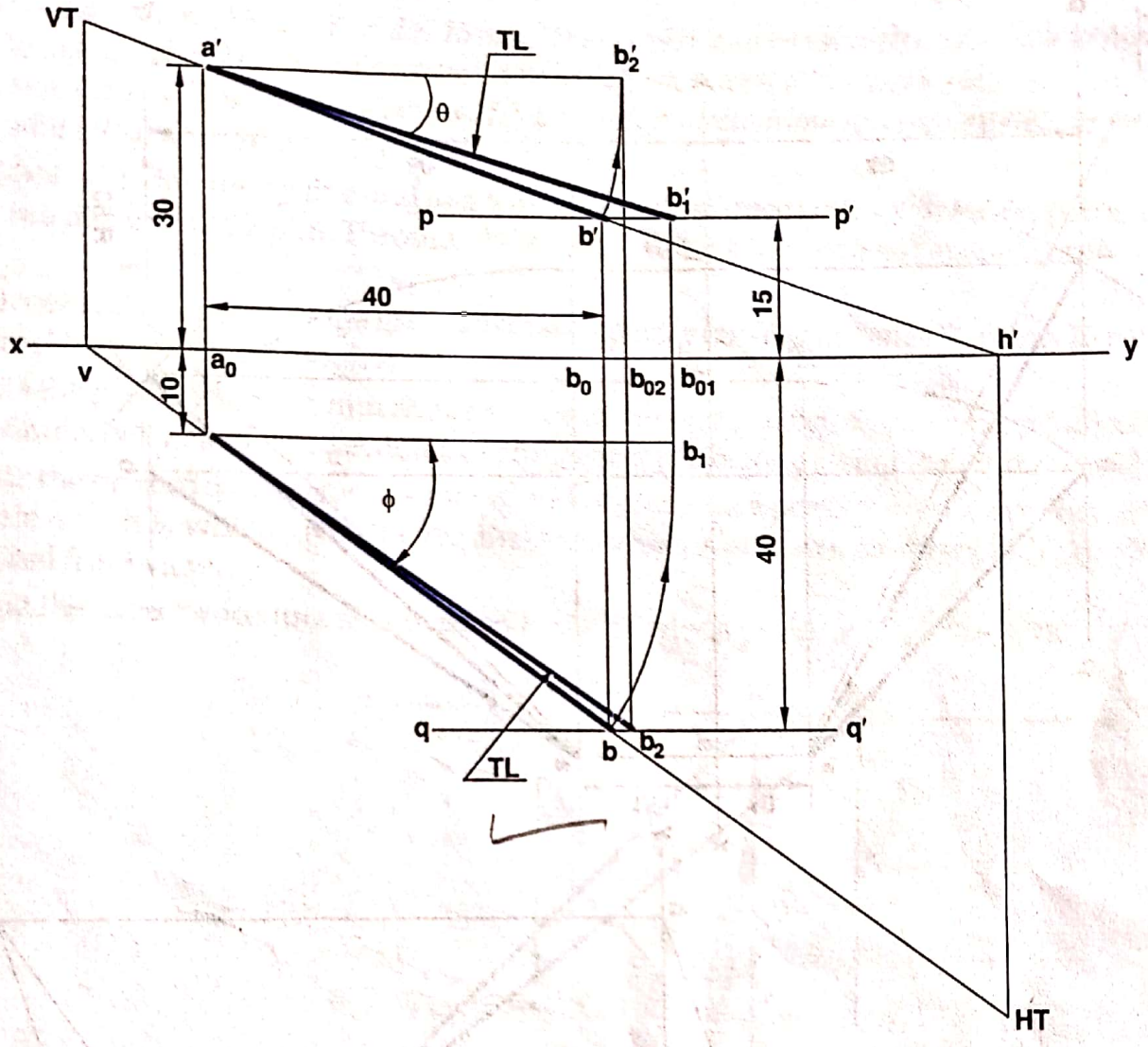


Fig. 9.74 Solution to problem 9.39 (Rotation Method)

Trapezoid Method

All the construction lines are retained to make the solution self–explanatory. See Fig. 9.75.

PROBLEM 9.40 *A line AB 60 mm long, has its end A 10 mm above the HP and 15 mm infront of the VP. and B is 40 mm above the HP and 60 mm infront of the VP. Draw its projections and determine the true inclination with HP and VP respectively.*

SOLUTION. Repeat the same procedure as described in the previous problem 9.17. See Fig. 9.76.

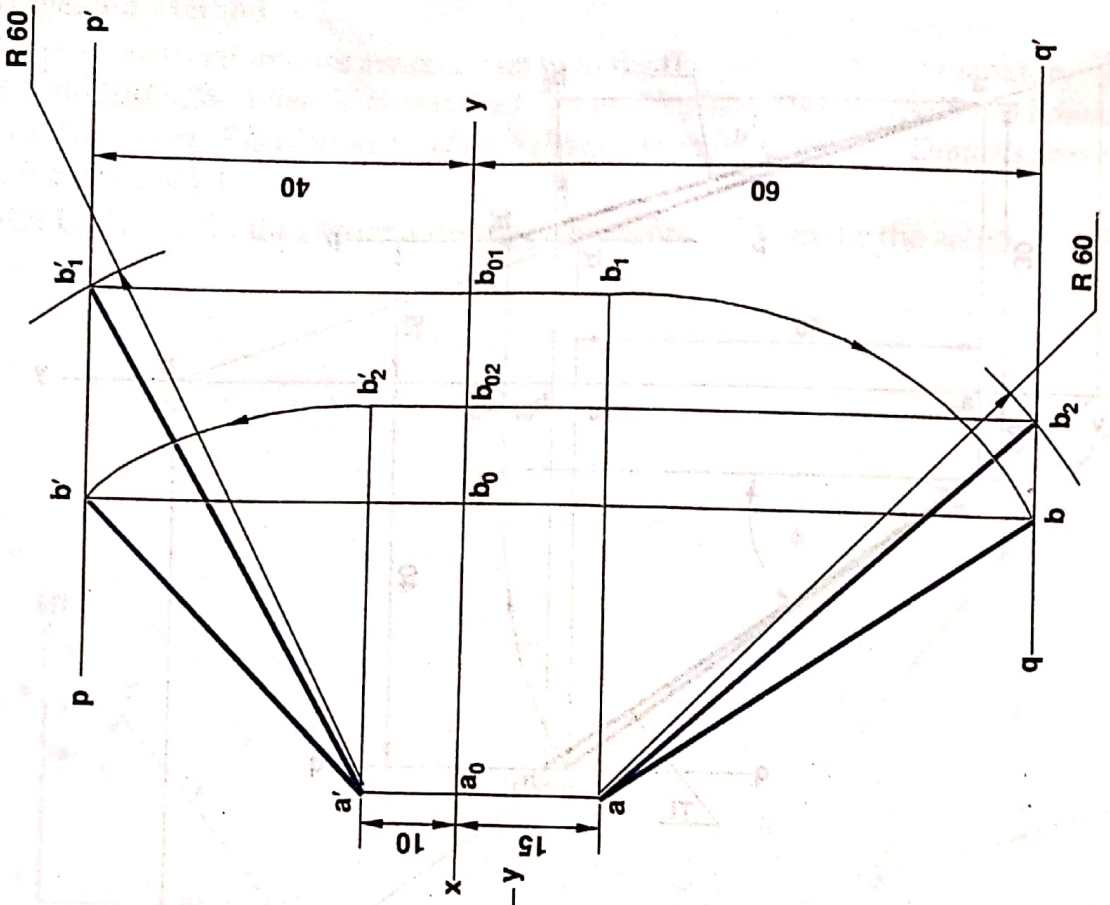


Fig. 9.76 Solution to problem 9.40

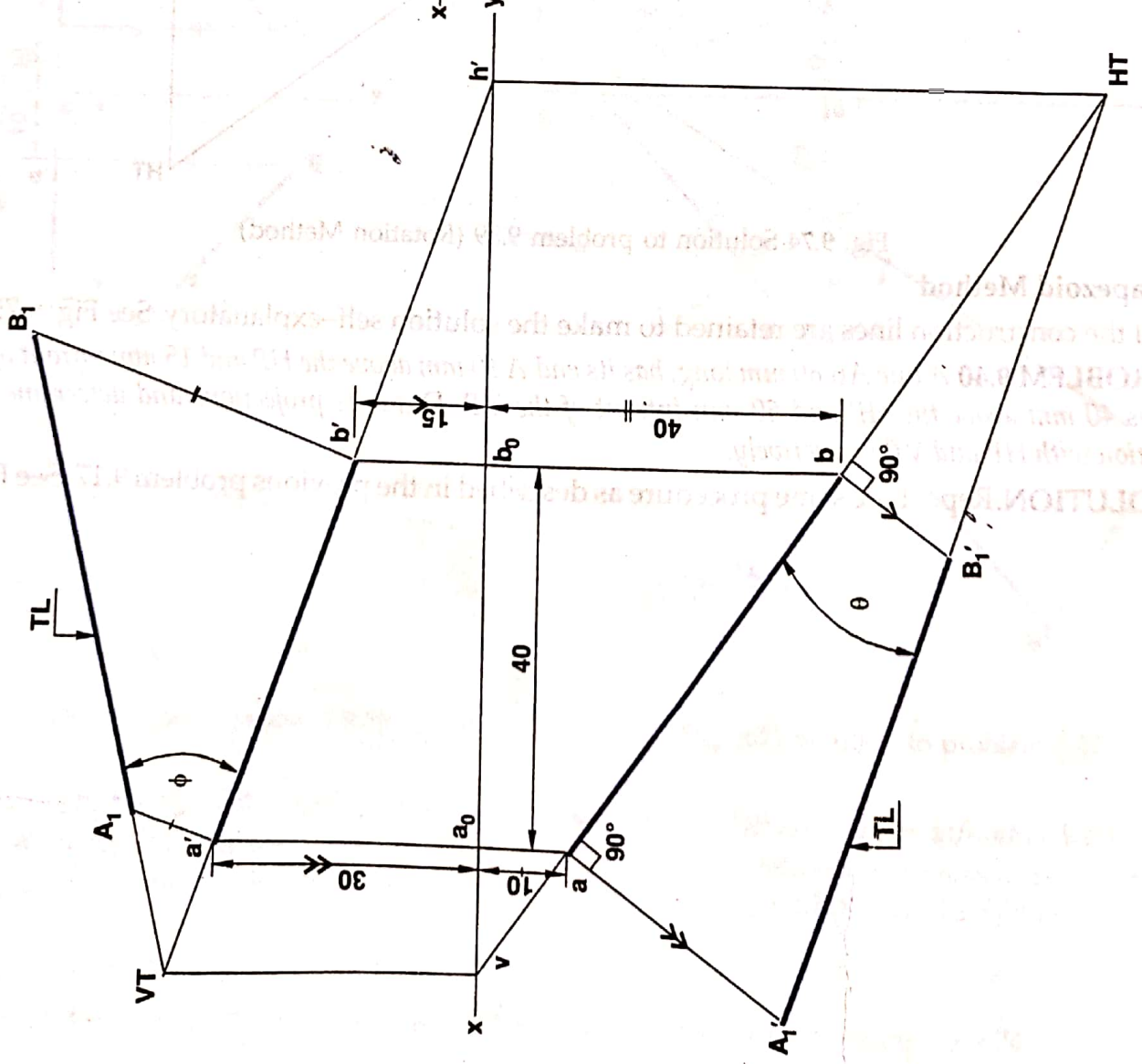


Fig. 9.75 Solution to problem 9.39 (Trapezoid Method)

PROBLEM 9.41 *A line AB has its end A 15 mm above the HP and end B is 60 mm infront of the VP. The front view of the line is inclined at 45° to the xy line. The horizontal trace (HT) of the line is 12 mm infront of the VP and its vertical trace (VT) is 15 mm below the HP. Draw its projection and determine its TL and true angle of inclination with HP and VP respectively.*

SOLUTION. (*i*) Draw the *xy* line and locate the horizontal trace (*HT*) by drawing perpendicular to the *xy* line at a distance of 12 mm. Through the point *h'*, draw a line inclined at angle of 45° to the *xy* line.

(*ii*) Mark vertical trace (*VT*) of the line such that its perpendicular distance from *v* is 15 mm of the inclined line passing through *h'*.

(*iii*) Locate the projection *a'* (15 mm above *HP*) of end *A* on a projector drawn perpendicular to *xy* and similarly locate the projection *a* of end *A* on a projector passing through *HT* and *v*.

(*iv*) Locate the projection *b* (60 mm infront of *VP*) of end *B* on a projector drawn perpendicular to *xy*. Join *a* with *b*, which gives the required top view. Also mark the point *b'*. Then *a'b'* is the required front view.

(*v*) Repeat the same procedure as described in the previous problems. See Fig. 9.78.

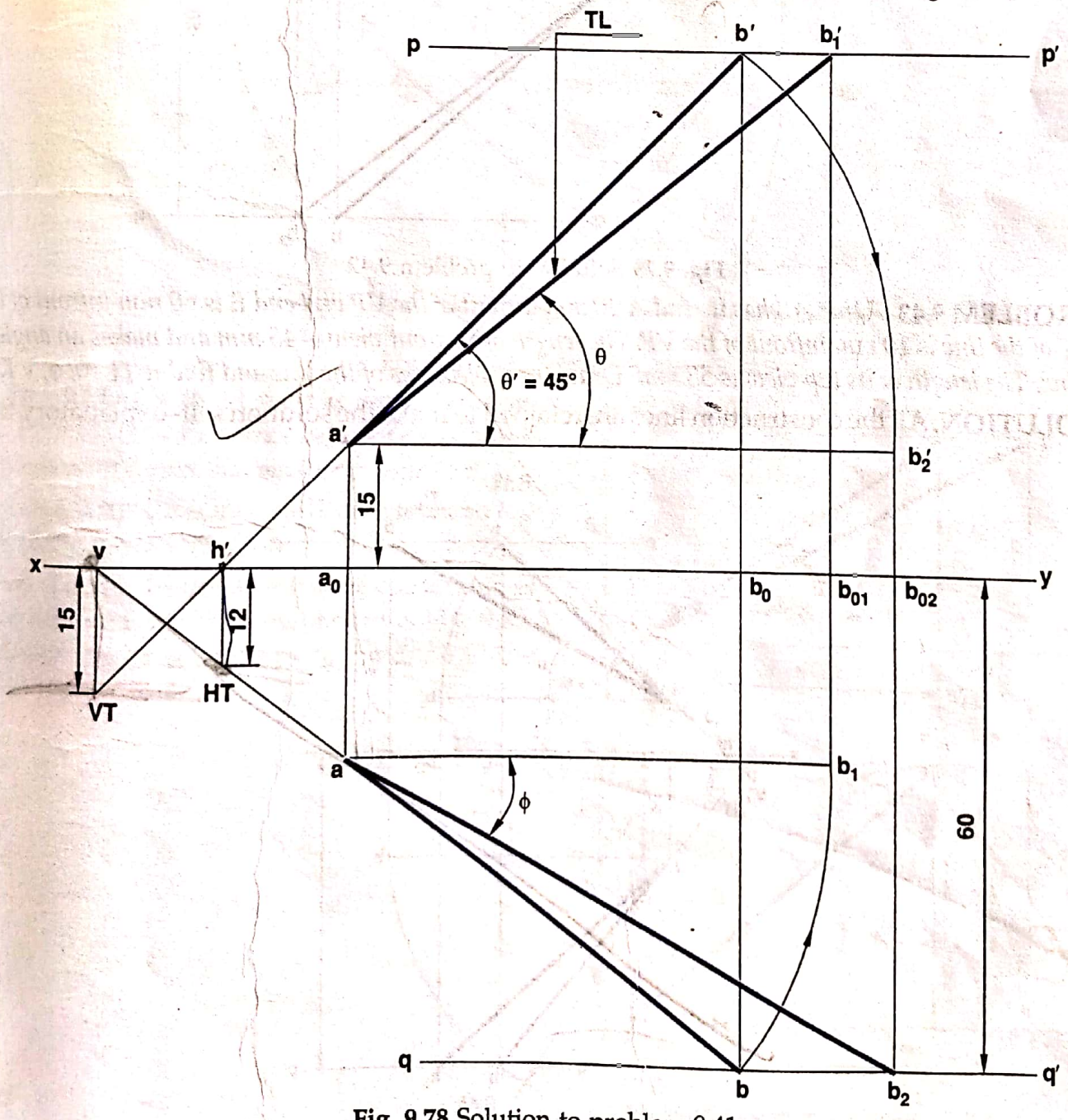


Fig. 9.78 Solution to problem 9.41

PROBLEM 9.42 *A line AB has its end A 15 mm above the HP and 20 mm infront of the VP. End B is 50 mm infront of the VP. The vertical trace (VT) is 7 mm above the HP. Draw the projections of the line if the distance between end projectors is 45 mm and find its TL, θ, ϕ, HT.*

SOLUTION. The solution to this problem is self–explanatory. See Fig. 9.79.

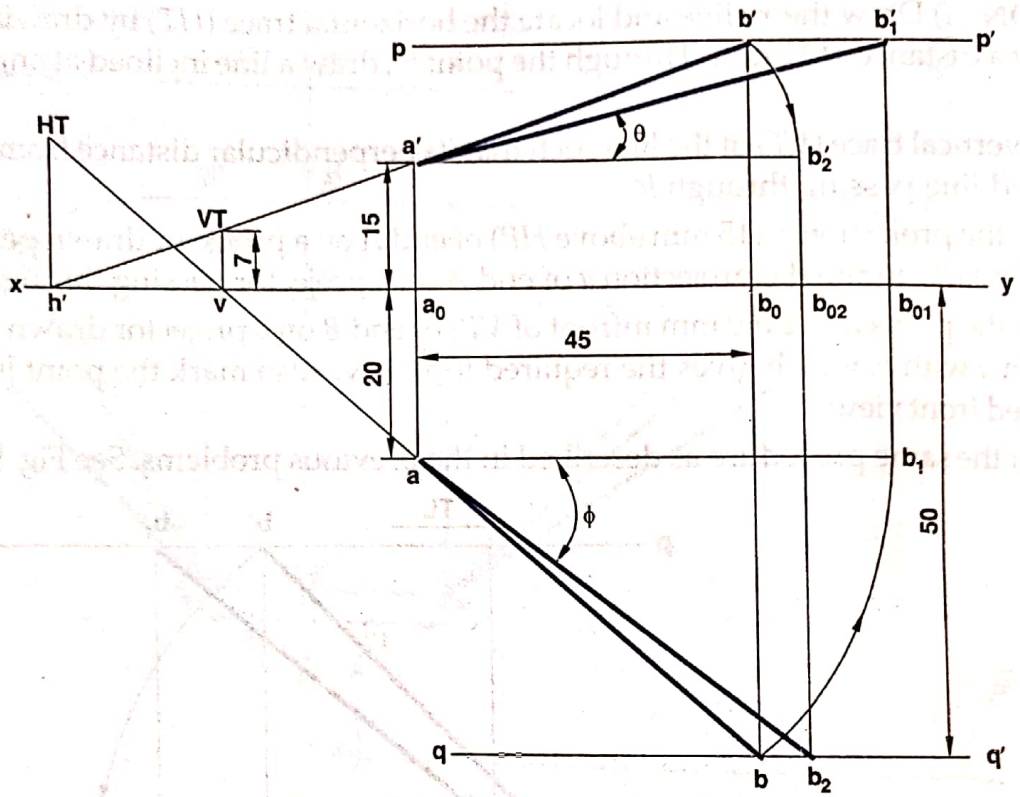


Fig. 9.79 Solution to problem 9.42

PROBLEM 9.43 *A line AB has its end A 20 mm infront of the VP and end B is 60 mm infront of the VP. The HT of the line is 10 mm infront of the VP. The length of its front view is 45 mm and makes an angle of 45° to xy line. The length of its top view is 55 mm. Draw the projections of the line and find it TL, θ, ϕ, VT.*

SOLUTION. All the construction lines are retained to make the solution self–explanatory. See Fig. 9.80

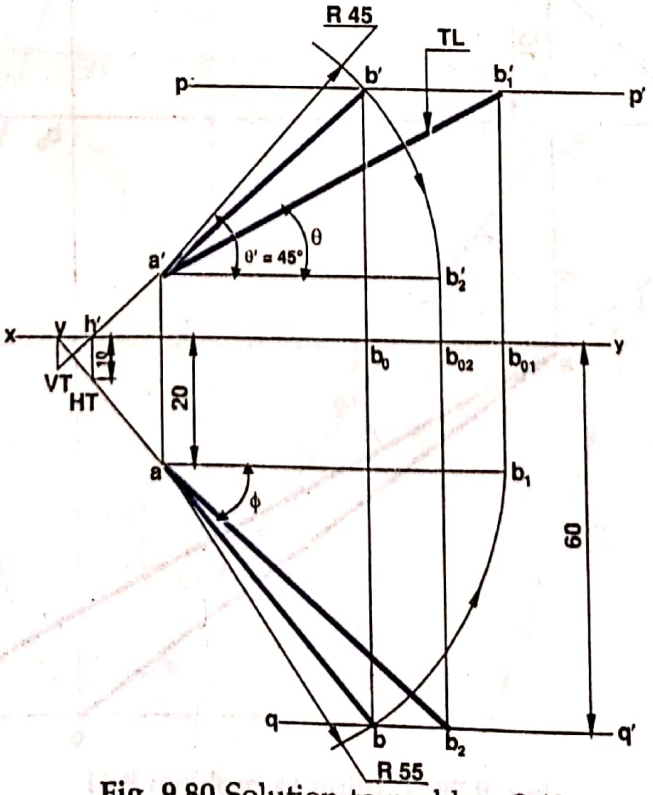


Fig. 9.80 Solution to problem 9.43

PROBLEM 9.44 *The front view of a line AB measures 65 mm and makes an angle of 45° with xy. A is in the HP and the VT of the line is 15 mm below the HP. The line is inclined at 30° to the VP. Draw the projections of AB and find its true length and inclination with the HP. Also locate the HT.* (*PTU, Jalandhar May 2006*)

SOLUTION. All the construction lines are retained to make solution self-explanatory. See Fig. 9.81.

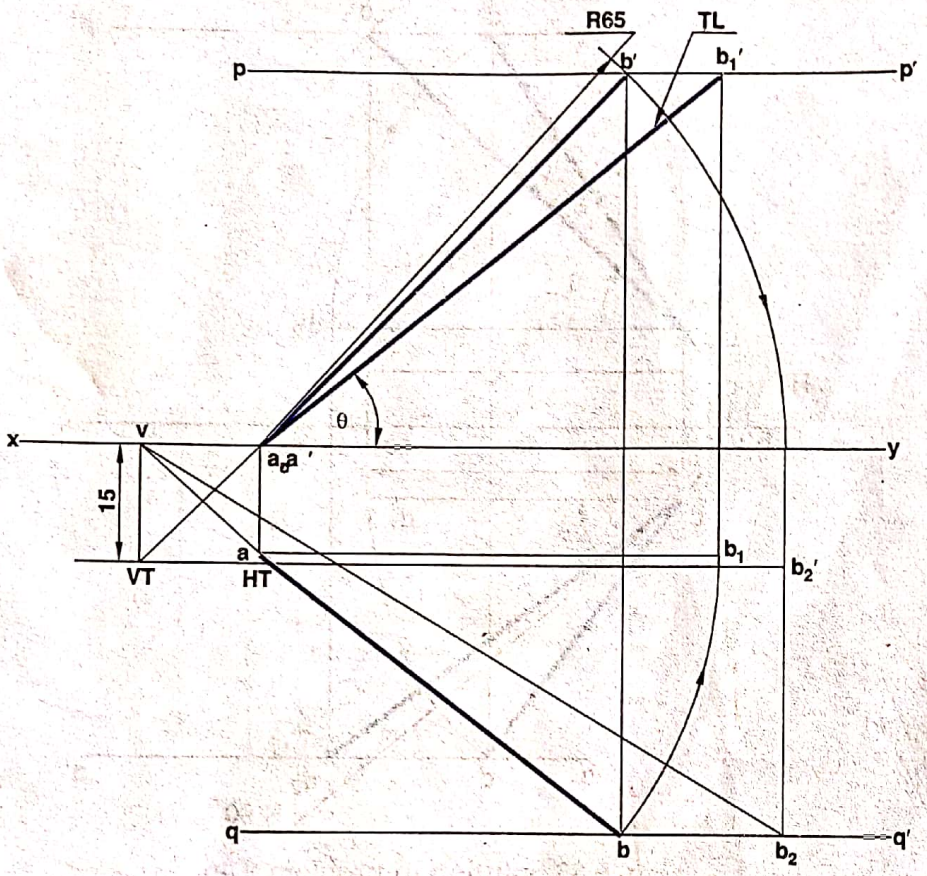


Fig. 9.81 Solution to problem 9.44

PROBLEM 9.45 *A room is 6m × 5m × 4m high. Find the actual distance between a top corner and the bottom corner diagonally opposite to it.*

SOLUTION. (*i*) Draw the front view and top view of the room with suitable scale.

(*ii*) Locate the positions of the corner points in the font and top view as shown is Fig. 9.82. Complete the projections as being done in the previous problems.

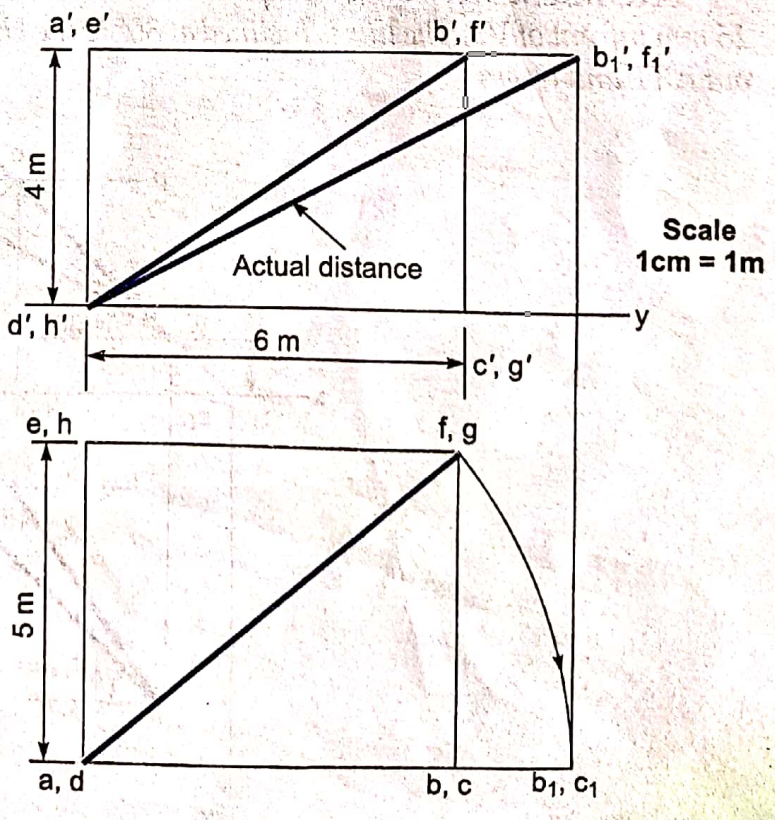


Fig. 9.82 Solution to problem 9.45

PROBLEM 9.46 *A line CD 90 mm long measures 72 mm in elevation and 65 mm in plan. Draw the projections of the line where point C is 20 mm above HP and 15 mm in front of VP. Also find out its traces.*
(PTU, Jalandhar May 2011)

SOLUTION. The solution to this problem has already been explained in previous problems. See Fig. 9.83.

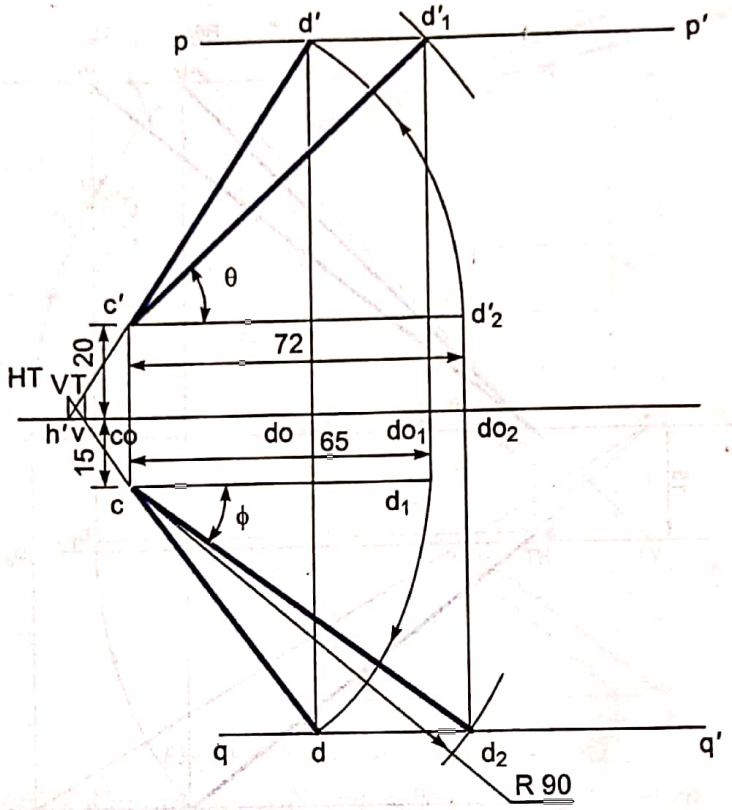


Fig. 9.83 Solution to problem 9.46

PROBLEM 9.47 *A line AB appears as 60 mm in front view. The end A is 15 mm above HP while end B is 25 mm infront of VP. The line is inclined at 30° to the HP and 45° to the VP. Draw the projections of line and find out it TL and traces.*

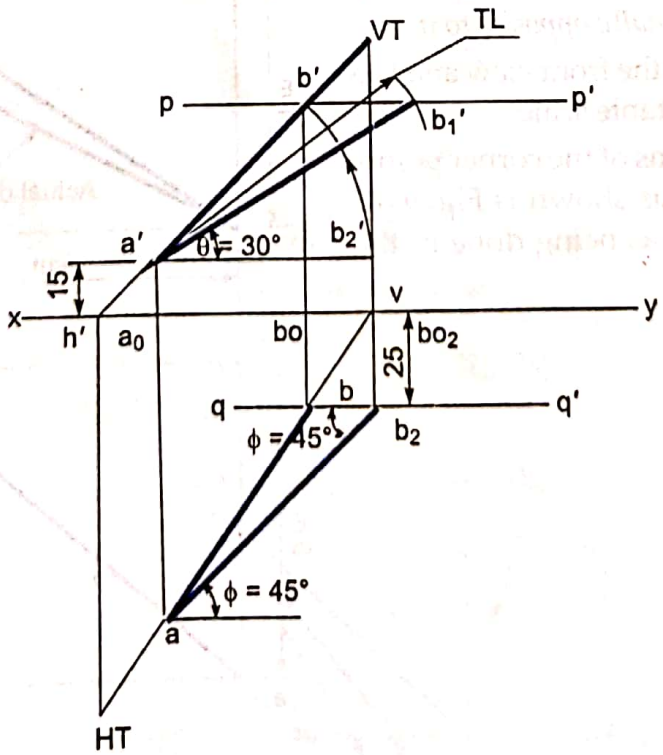


Fig. 9.84 Solution to problem 9.47

SOLUTION. The solution to this problem has already been explained in previous problems. See Fig. 9.84.

PROBLEM 9.48 *A line AB measuring 70 mm long has one of its ends 50 mm infront of VP and 15 mm above HP. The top view of the line is 55 mm long. The other end is 15 mm infront of VP and is first quadrant. Draw the projections of line and find out its true inclination with HP and VP and traces.*

SOLUTION. All the construction lines are retained to make the solution self–explanatory. See Fig. 9.85.

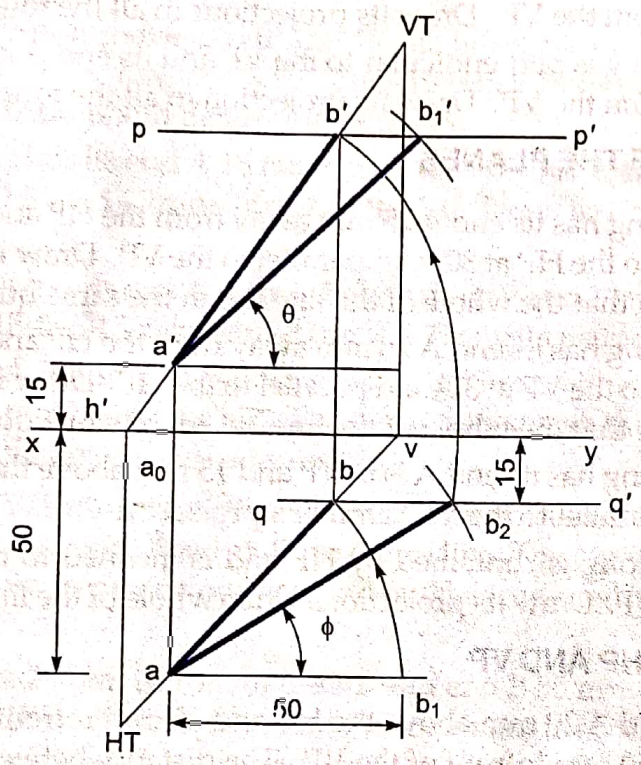


Fig. 9.85 Solution to problem 9.48

PROBLEM 9.49 *A line AB has its end A in HP and 50 mm infront of VP. Its front view is inclined at 55° to xy line and has a length of 70 mm. The other end B is in VP. Draw the projections of a line and find out its true inclinations with HP and VP.*

SOLUTION. The solution to this problem is self–explanatory. See Fig. 9.86.

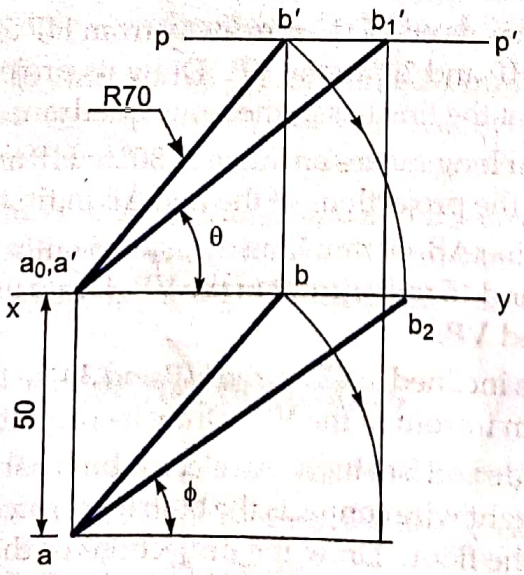


Fig. 9.86 Solution to problem 9.49

EXERCISES

PARALLEL OR PERPENDICULAR LINES

9.1 A line AB, 60 mm long has its end A 25 mm away from the HP and 20 m from the VP. The line is parallel to both HP and VP. Draw its projections in all the four quadrants.

9.2 A line AB, 40 mm long is perpendicular to the HP and its end A is 20 mm away from the HP and 15 mm away from the VP. Draw its projections in all the four quadrants.

9.3 A line AB, 40 mm long is perpendicular to the VP and its end A is 20 mm away from the HP and 15 mm away from the VP. Draw its projection in all the four quadrants.

LINE INCLINED TO ONE OF THE PLANES

9.4 A line AB, 60 mm long has its end A 25 mm away from the HP and 15 mm away from the VP. The line is inclined to the HP at 30° and parallel to the VP. Draw its projections in all the four quadrants. Assume that the whole of the line lies in the same quadrant.

9.5 A line AB, 60 mm long has its end A 20 mm away from the HP and 15 mm away from the VP. The line is inclined to the VP at 30° and parallel to the HP. Draw its projections in all the four quadrants. Assume that the whole of the line lies in the same quadrant.

9.6 A line AB, 50 mm long has its end A is in VP and 15 mm above the HP. The line is inclined to the HP at 30° and parallel to the VP. Draw its projections.

9.7 A line AB, 50 mm long is contained by HP and is inclined to the VP at 45°, has its end A 10 mm behind the VP. Draw its projections when whole of the line lies in the same quadrant.

LINE INCLINED TO BOTH HP AND VP

9.8 A line AB has its end A 20 mm above the HP and 25 mm infront of the VP. End B is 40 mm above the HP and 50 mm infront of the VP. The distance between end projectors is 45 mm. Draw the projections of the line and find out TL, θ, ϕ, HT and VT by using (*i*) Rotation Method and (*ii*) Trapezoid Method.

9.9 A line AB has its end A 10 mm above the HP and 15 mm infront of the VP. End B is 35 mm below the HP and 30 mm behind the VP. The distance between the end projectors is 40 mm. Draw the projections of the line and find out TL, θ, ϕ, HT and VT by using (*i*) Rotation Method and (*ii*) Trapezoid Method.

9.10 A line CD, 60 mm long has its end C 15 mm away from HP and 25 mm away from the VP. It is inclined at 45° to the HP and 30° to the VP. Draw its projections in all the four quadrants. Assume that the whole of the line lies in the same quadrant.

9.11 A straight line AB 50 mm long makes an angle of 30° to HP and 45° to the VP. The end A is in both HP and VP. Draw the projections of the line AB in first quadrant only.

9.12 Plan and elevation of a line AB, 60 mm long measure 54 mm and 43 mm respectively. End A is 10 mm above the HP and 15 mm infront of the VP. Draw its projections and determine true inclinations with HP and VP.

9.13 A line AB, 70 mm long is inclined at 45° to the HP and 30° to the VP. Its mid-point C is 30 mm above the HP and 40 mm infront of the VP. Draw its projections.

9.14 A room 8 m long, 6 m wide and 5 m high. An electric bulb hangs in the centre of the ceiling and 1 m below it. A thin straight wire connects the bulb to a switch kept in one of the corners of the room and 1.5 m above the floor. Draw the projections of the wire and determine its TL and slope with the floor.

9.15 A line AB 65 mm long is inclined at 30° to the HP and its top view makes an angle of 45° with the VP. The end A is in the HP and 15 mm infront of the VP. Draw its front view and find out its true inclination with the VP.

9.16 A line AB has its end A 20 mm away from the HP and 40 mm away from the VP. End B is 50 mm away from the HP and 15 mm away from the VP. The line AB is contained by profile plane. Draw its projections in first quadrant and third quadrant only. Also find out its TL, θ, ϕ, HT and VT.

9.17 End A of a line AB is 25 mm above the HP and 15 mm infront of the VP. End B is 10 mm above the HP and 40 mm infront of the VP. The distance between the end projectors is 40 mm. Draw its projections and determine its traces.

9.18 A line AB, 50 mm long has its end A 15 mm above the HP and 10 mm infront of the VP. End B is 40 mm above the HP and 45 mm infront of the VP. Draw its projections and determine θ and ϕ.

9.19 A line AB, 60 mm long has its end A 40 mm below the HP and 10 mm behind the VP. End B is 10 mm below the HP and 45 mm behind the VP. Draw its projections.

9.20 A line AB has its end A 10 mm infront of the VP and 15 mm above the HP. End B is 35 mm behind the VP and 40 mm below the HP. The distance between the end projectors is 40 mm. Draw its projections and determine its traces too.

9.21 A line AB has its end A 20 mm above the HP and 25 mm infront of the VP. The front view of the line is 50 mm long and is inclined at 30° to the *xy* line. The top view of the line is inclined at 45° to the *xy* line. Draw its projections and find out the true length and true inclination with HP and VP.

9.22 A line AB has its end A 20 mm infront of the VP and end B 55 mm above the HP. The line is inclined at 45° to the HP while its front view makes an angle of 30° to the xy line. Draw its projections, when its top view is 50 mm long. Find the true length and true angle of inclination with the VP.

PROJECTION OF LINES USING TRACES

9.23 A line AB has its end A 15 mm above the HP and end B is 60 mm infront of the VP. The front view of the line is inclined at 30° to the *xy* line. The horizontal trace (HT) of the line is 10 mm infront of the VP and its vertical trace (VT) is 15 mm below the HP. Draw its projection and determine its TL and true angle of inclination with HP and VP respectively.

9.24 A line AB has its end A 15 mm above the HP and 20 mm infront of the VP. End B is 50 mm infront of the VP. The vertical trace (VT) is 10 mm above the HP. Draw the projections of the line if the distance between end projectors is 45 mm and find its TL, θ, ϕ, HT.

9.25 A line AB has its end A 20 mm infront of the VP and end B is 60 mm infront of the VP. The HT of the line is 10 mm infront of the VP. The length of its front view is 50 mm and makes an angle of 30° to *xy* line. The length of its top view is 55 mm. Draw the projections of the line and find it TL, θ, ϕ, VT.

OBJECTIVE QUESTIONS

9.1 A straight line is defined as the ..... distance between two points or extremities.

9.2 When a line is perpendicular to one of the planes, it is ..... to the other plane.

9.3 When a line is inclined to ..... and parallel to ....., its top view represents the true length of the line.

9.4 When a line is inclined to HP and parallel to VP, the inclination of the front view with xy represents its ..... .

9.5 When a line is perpendicular to the VP, its ..... trace will coincide with ..... of the line.

9.6 When a line is contained by a profile plane, the sum of the angles of the inclination with the HP and VP is equal to ..... .

9.7 The trace of a line is a ..... .

9.8 A straight line will represent its true length in that plane to which it is ..... .

9.9 Define a straight line.

9.10 What are the apparent angles of inclinations ?

9.11 What do you mean by the trace of a line ?

9.12 Draw the projections of two parallel lines.

9.13 A line lies in a profile plane with equal elevation and plan length. Draw its projections and give the magnitude of angle θ and ϕ.

9.14 What is the true length of a line ?

9.15 When both the views of a line coincide with xy line, the line is lying on _____.

9.16 When a line is parallel to both HP and VP. It has _____ trace.

9.17 The side view of a line is a _____.

9.18 A straight line is generated as the _____ of a moving point.

9.19 Draw free hand the trace of a line when it is parallel to VP and inclined to HP. Name the trace.

ANSWERS

9.1 Shortest **9.2** Parallel **9.3** VP, HP **9.4** True inclination

9.5 Vertical, Front view **9.6** 90° **9.7** Point **9.8** Parallel

9.15 Both on HP and VP **9.16** No **9.17** Point **9.18** Locus

9.19. Horizontal trace (HT)