The heat equation including effects of convection can be written as

$$c_P \rho \left(\frac{\partial T}{\partial t} + \boldsymbol{v} \cdot \nabla T \right) - \nabla \cdot (k \nabla T) = s, \tag{0.1}$$

where c_P is the specific heat, ρ the density, $\boldsymbol{v}$ the velocity, and k the thermal conductivity of the carrier medium, respectively. The heat exchange wqith the ambient medium can roughly be described by

$$-\boldsymbol{n} \cdot (k \nabla T) = h \, (T - T_a) \tag{0.2}$$

with $\boldsymbol{n}$ the outer normal at the contact with ambient medium, h the heat transfer coefficient between carrier body and ambient medium and T_a the ambient temperature. The boundary conditions (0.2) are of Robin type

$$\boldsymbol{n} \cdot (\mathbf{A} \nabla T) + \alpha T = g_R \tag{0.3}$$

with $\mathbf{A} = k\mathbf{I}$, $\alpha = h$ and $g_R = hT_a$. The general form of an elliptic Operator reads

$$-\nabla \cdot (\boldsymbol{A} \nabla u) + \boldsymbol{b} \cdot \nabla u + cu = f \tag{0.4}$$

with a positive definite matrix-valued function $\mathbf{A}$, a vector-valued function $\boldsymbol{b}$ and scalar-valued functions a, f. In its weak form (0.4) rewrites to

$$\int\limits_{\Omega} \nabla \varphi_i \cdot (\mathbf{A} \nabla u) \; dV + \int\limits_{\Omega} \varphi_i \, \boldsymbol{v} \cdot \nabla u \, dV + \oint\limits_{\partial\Omega} \varphi_i \, (\alpha u) \; dS = \int\limits_{\Omega} \varphi_i f \; dV + \oint\limits_{\partial\Omega} \varphi_i g_R \, dS, \tag{0.5}$$

where Robin boundary conditions (0.3) has been imposed on the whole boundary, $\partial\Omega$. Thus the FEM formulation for the time-dependent spatial temperature distribution $\boldsymbol{T}$ at DOFs of (0.1) is

$$\mathbf{M}(c_P \rho) \frac{d\boldsymbol{T}}{dt} + (\mathbf{D}(c_P \rho \boldsymbol{v}) + \mathbf{S}(k) + \mathbf{M}_b(h)) \, \boldsymbol{T} = \boldsymbol{b}(f) + \boldsymbol{b}_b(hT_a), \tag{0.6}$$

with stiffness matrix $\boldsymbol{S}$, damping matrix $\mathbf{D}$, mass matrix $\boldsymbol{M}$, right-hand side vector $\boldsymbol{b}$ and their boundary analogues indicated by a subscript b.