Content Market Optimization Results

CSC494 Midterm Report

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1 Introduction

The purpose of this report is to detail progress made in understanding the empirical dynamics of a formalization of online social networks as content markets. Specifically, we evaluate the impact of market parameters in both the perfect and imperfect information markets, where imperfect information differs from perfect information in that producers do not have direct access to consumers' preferences, and judge how both the market structure and ending social welfare differ between the two market types. We also highlight some notable interactions between parameters in both market types.

The goal of this report is to provide a comprehensive foundation for future extensions and adaptations of the content market formalization to other multi-agent environments.

The report will proceed as follows: first, we'll describe the content market formalization, including the optimization problems involved for all the agents and the hyper-parameters. We will also briefly discuss some of the implementation details used for testing; specifically, we'll discuss the difference between local and global optimization and how this difference can be minimized by optimal initialization of the producers' topics produced. After this, we will discuss the structural properties of the perfect information market as well as the impact of varying the hyper-parameters. We'll then discuss the structure of the imperfect information market and how it differs from the perfect information market, again highlighting notable hyper-parameter impacts. Finally, we will focus in specifically on the cost of influence, the difference in total social welfare between the perfect and imperfect markets, and how it is impacted by hyper-parameter adjustments.

1.1 Content Market Formalization

The content market formalization views online social networks as markets where agents produce content for and consume content from other agents based on their own interests. There are also influencer agents, who do not produce or consume content for themselves but instead aggregate and re-share produced content based on consumer interests. In this manner, an influencer is a proxy for general consumer interests. Consumer agents receive utility for consuming content similar to their main interest within the market. In the formalization, 'consuming' content consists of allocating following rate to the producer of said content; consumers may also follow the influencer or content external to the market. Producer agents, in turn, receive utility based on the overall following, or social support, that their produced content attracts. Influencer utility is also based on their total following, though their task is not to produce content to

attract that following but to follow the most optimal producers so their re-shared content is of the most interest to consumers. Agents all act selfishly to maximize their own utility, but (as will be shown) iterative optimization of agent parameters (topics produced and following rates) leads to eventual market convergence.

1.1.1 Market Parameters and Hyper-parameters

For clarity in this section, market parameter names will be *italicized* while hyper-parameters will be **bolded**.

A content market is defined by a compact set of topics $\mathcal{T} \subset \mathbb{R}^N$. For the testing in this report, we fix $\mathcal{T} = [-1, 1]$. The closeness of topics $x, x' \in \mathcal{T}$ is given by the Euclidean distance $|x - x'|$.

A community member (consumer and producer) in the market is defined by a main interest $y \in \mathcal{T}$. The set of community members is denoted $C \subset \mathcal{T}$. The probability that y is interested in producing content of topic $x \in \mathcal{T}$ is given by

$$p(x|y) = f(|x - y|)$$

where $f : \mathbb{R}_+ \mapsto [0, 1]$ is strictly decreasing and continuous. We refer to f as the **production topic interest function**. The probability that y is interested in consuming content of topic $x \in \mathcal{T}$ is similarly given by

$$q(x|y) = g(|x - y|)$$

where $g : \mathbb{R}_+ \mapsto [0, 1]$ is also strictly decreasing and continuous. We refer to g as the **consumption topic interest function**. In most cases, we test with $f = g$ and refer to the function as the **agent topic interest function**.

For a consumer y and producer z, we let $\mu(z|y)$ be the rate at which y follows z. Then,

$$\mu(y) = (\mu(z|y))_{z \in C \setminus \{y\}}$$

is y's producer following rate vector of . In addition, we let $\mu(y_{infl}|y)$ be the rate with which y follows the influencer y_{infl}, and $\lambda(y)$ be the rate with which y follows content sources external to the community. Then, y's overall *consumer following rate vector* is given by

$$\Lambda(y) = (\mu(y), \mu(y_{infl}|y), \lambda(y))$$

In the content market formalization, consumers have a limited amount of attention defined by the **consumer attention bound** $M > 0$ such that for all $y \in C$,

$$\mu(y_{infl}|y) + \lambda(y) + \sum_{z \in C \setminus \{y\}} \mu(z|y) \leq M$$

In addition, we fix $\alpha > 0$ as the **delay sensitivity** of agents in the market: the discount rate of content utility based on how it is received by the consumer. We also fix $B_0 > 0$ as the **consumer external interest probability**, the probability that content produced outside of the community is of interest to consumers within the community.

The influencer has an *influencer following rate vector*

$$\mu_{infl} = (\mu_{infl}(y))_{y \in C}$$

where $\mu_{infl}(y)$ is the rate with which y_{infl} follows the producer $y \in C$. We also define an **influencer attention bound** $M_{infl} > 0$ such that

$$\sum_{z \in C} \mu_{infl}(z) \leq M_{infl}$$

Finally, for a producer $z \in C$, we let $x(z) \in \mathcal{T}$ be the *topic produced* by z. The formalization also defines $r_P > 0$ as the rate at which content is produced in the community and $r_0 > 0$ as the rate at which content is produced by sources external to the community, however for this testing we fix $r_P = r_0 = 1$.

1.1.2 Optimization Problems

All the agents in the market have a notion of utility. Community members receive utility for consuming content close to their main interests at a small delay and producing content that nets them a larger following. They therefore each have two optimization problems they wish to solve: choosing the best following rates and producing the best content. Influencers receive utility for re-sharing producer content that nets them a large following. They have a single optimization problem to choose the best following rates. We can iteratively optimize all these problems, and eventually, the market converges to an optimal state.

The key distinction between perfect and imperfect information is in the producer optimization problem. In perfect information, producers determine the utility of a topic by evaluating how much consumer and influencer social support the topic nets them. However, in imperfect information, we remove the knowledge of consumer social support and instead force producers to make decisions solely based on the influencer's support.

1.2 A Note on Optimization Techniques

In general, the optimizer used to perform the testing in this report performs local, rather than global, optimization. This is because global optimization was much slower, especially for running tests in the imperfect information market, so a local optimizer was used to allow for more tests to be performed. However, I did take some time to evaluate the difference between local and global optima using a basin-hopping algorithm. Overall, I found that when producers' topics produced are initialized to be equal to the producers' main interests, the ending market structure and total social welfare after optimizing with a local optimizer are approximately equal to those obtained by using a global optimizer. This effect holds regardless of the topic produced initialization used for the global optimization, and also holds for imperfect information. Notably, when topics produced are initialized to be far away in the topic space from producer main interests, the results of the local optimizer do not match the results of the global optimizer (and, specifically, are worse in terms of total social welfare), suggesting that there is some local optima that the local optimizer gets stuck in. From this, we can conclude that in general main interest initialization for producer topics produced is already pretty close to the optimal topics produced in both the perfect and imperfect information markets, so the use of a global optimizer in this case is not necessary.

2 Perfect Information Market

In the perfect information market, content producers have access to consumers' following habits and topic interests. Because of this, they can directly optimize their topics produced to be of

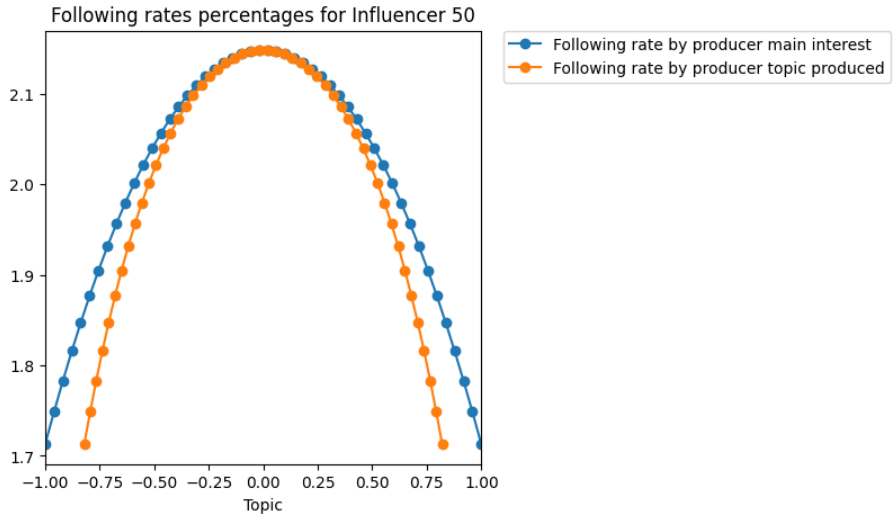

Figure 1: A plot of the influencer following rates at the end of optimization. The rates by producer main interest and ending topic produced are shown.

the most interest to the market's consumers. We'll discuss the typical dynamics of the perfect information market in terms of topics produced and the following rates before discussing the impact of the market hyper-parameters.

2.1 Structural Properties of Perfect Information

In general, we've found that the perfect information market highlights content about topics that are closer to the center of the topic space. The influencer typically deals the greatest proportion of its attention to producers in the center, and since each consumer deals attention to producers whose main interests are close to their own, on average consumers provide the most attention to producers closer to the center than those on the fringes of the market.

2.1.1 Influencer Following Rates

As shown in fig. 1, the influencer following rates typically converge to an inverse parabola centered at the center of the topic space. The greatest proportion of the influencer's attention is given to producers whose main interest and ending topic produced are in the center, with a drop-off on either side. This reflects that the producers in the center of the market likely have much more appeal to a larger proportion of the consumers in the market than those on the fringes; the influencer wishes to maximize $B(z|y)$ for the greatest number of consumers y and producers z. As will be shown, the slope of this curve can be impacted by the steepness of agent topic interest functions or other parameters.

2.1.2 Consumer Following Rates

In an aggregate form, consumers allocate attention similarly to the influencer. We can see in fig. 2 that producers whose topics produced or main interests are closer to the center of the market topic space receive more total attention from consumers than those on the fringes. We'll discuss consumer following rates more specifically, but the general idea is that while most consumers can receive some utility from the more central topics (depending on the steepness of

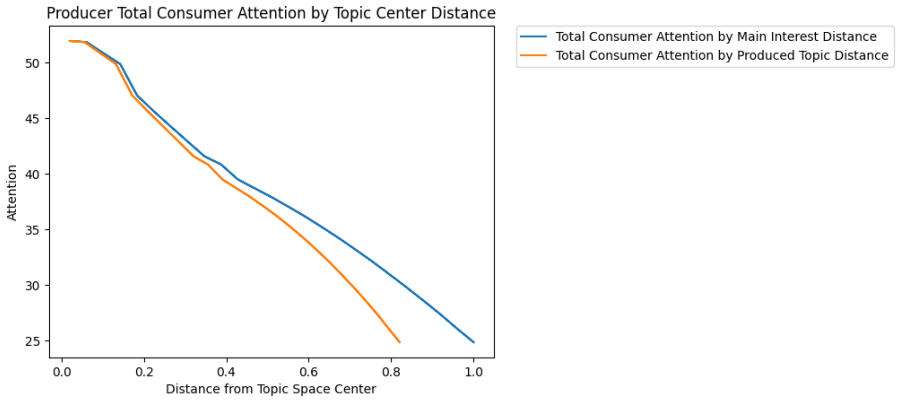

Figure 2: A plot of the total amount of attention producers receive based on the distance from the center of the topic space $\mathcal{T}$ of their main interests and ending topics produced.

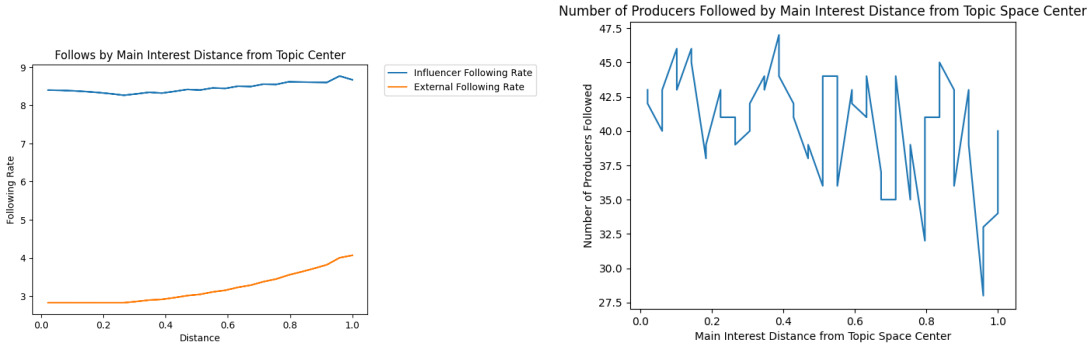

Figure 3: Left: Consumer following rates towards the influencer and external content based on their main interest's distance from the topic space center. Right: Number of producers followed by consumer main interest distance from the topic space center. Consumers towards the edges of the market pay more attention to the influencer and external content than those in the center; these consumers are also pickier in terms of the producers that they follow.

their interest functions), only the consumers near the edges of the topic space can receive much utility from topics on the edges of the topic space.

The reverse of this effect can be seen in fig. 3 when considering the consumer following rates towards the influencer or external content. Here, we see that consumers towards the edges of the topic space tend to follow the influencer and external content more than consumers in the center. These consumers are also somewhat pickier in terms of which producers they follow, generally following fewer producers than those in the center (though this correlation is not very strong). Consumers towards the edges of the market receive less utility from topics produced in the center, so they compensate by following the influencer and external content more. As will be discussed in section 2.2, the slopes of these curves are determined by the topic interest functions, consumer external interest, consumer delay sensitivities, and the number of agents in the market.

Finally, we can look directly at the consumer following rates. These results (fig. 4) can be summarized as follows: consumers pay the most attention to the producers whose main interests are closest to their own and the slopes of consumer attention profiles as producers' interests are

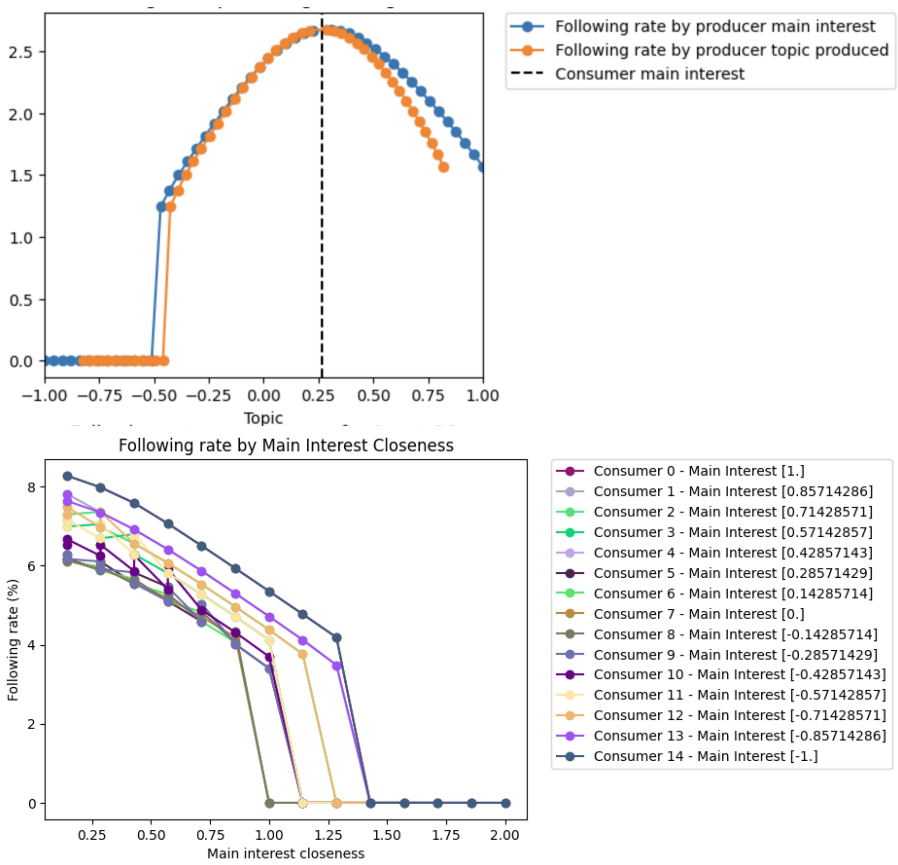

Figure 4: Top: An example attention profile of a consumer in the perfect information market with 50 agents. The curve peaks near the consumer's main interest. Bottom: Consumer-producer following rates based on the producer's main interest closeness to the consumer. For clarity, this graph comes from a test where fewer agents were present in the market, but the trends are similar regardless.

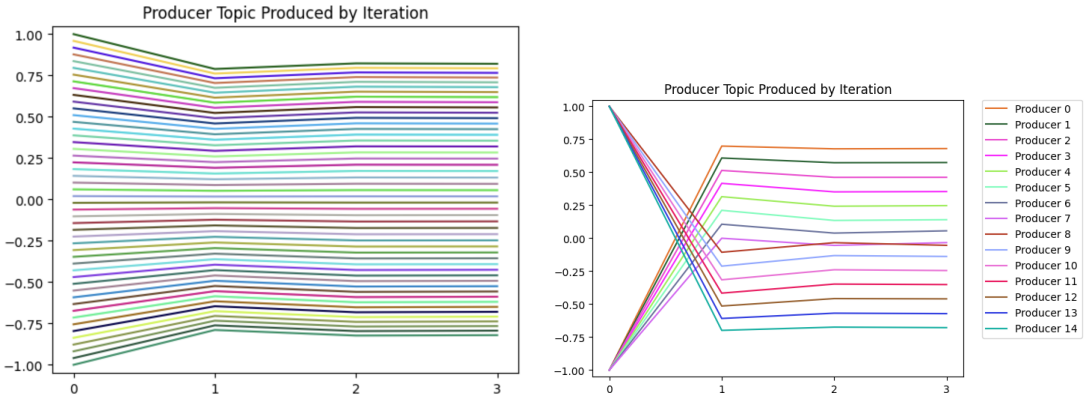

Figure 5: Producer topic produced by iteration in two example tests. In the right graph, the topics produced were initialized to be as far as possible from the producers' main interests, but in the end, they all still cross over to be closer.

farther from their own is steeper for consumers closer to the edge of the market (in other words, these consumers are pickier in choosing how much attention to use on each producer). The latter conclusion is seen in the graph of the following rates based on producer main interest closeness, as the slopes of the curve for agents more towards the edges of the markets are steeper than those in the center. This graph seems to suggest that the edge agents follow more producers, but that's not the case; the central agents simply have more overlapping points as there are more producers who have a close main interest to their own. However, it is notable that the final drop-off point at which each consumer stops giving any attention to producers is farther along in the topic space for the edge agents.

2.1.3 Producer Topics Produced

Finally, we can consider the ending topics produced by producers and how they compare to the producers' main interests. In general, the presence of the influencer and uniform distribution of the consumers act as a force driving producers to produce content closer to the center of the topic space. However, this drive is not enough to push producers a significant distance from their own main interests, and the ending arrangement of topics produced still always reflects the ordering of the main interests. We can see this by plotting how producers' topics produced change by optimization iteration. The effect holds even when the topics produced are initialized to be as far from the producers' main interests as possible; all producers cross their topics over to produce more close to their main interests. From this, we can conclude that the producers' ending topics produced are influenced by a balance between this competing central force and their own main interests. The strengths of these forces, then, can likely be affected by adjusting the influencer and consumer attention bounds and the topic interest functions.

2.2 Hyper-Parameter Impacts

With the general structural properties of the perfect information market established, we can discuss how these properties are affected by varying the market hyper-parameters. For each hyper-parameter, the notable impacts of variance in the hyper-parameter values will be highlighted, and the overall role of the hyper-parameter will be concluded.

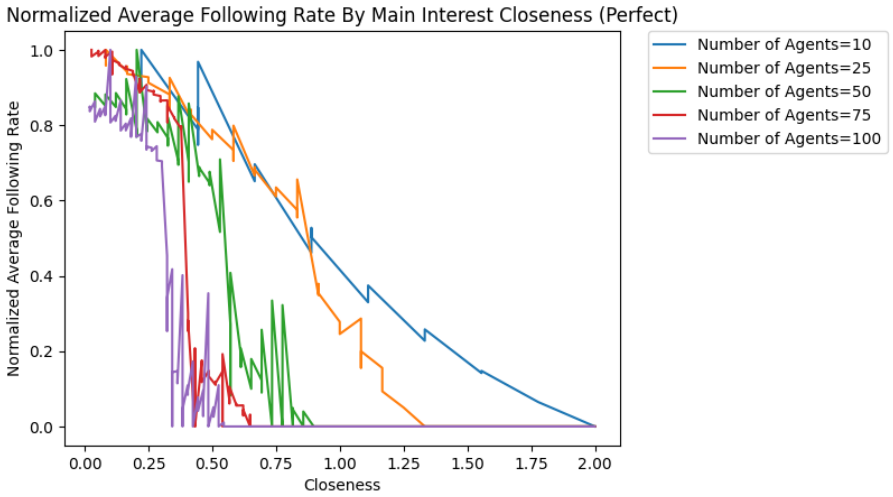

Figure 6: Normalized average following rate based on the closeness of producer and consumer main interests. When more agents are added, the consumer attentional profiles become steeper and more concentrated on the producers close to the consumer's main interest.

2.2.1 Number of Agents

In the perfect information market, adding more agents generally causes the consumers and influencers to be pickier in terms of their following rate allocations. Overall, as more agents are added we see that attention distributions become narrower and flatter.

1. Consumer Following Rates

   In fig. 6, we see that consumer following rate profiles become steeper as more agents are added. For any given consumer, there are more producers whose main interest is close to their own, so there's less reason for any of them to follow content that's further away. Graphs of the width of the nonzero area of the following rate vectors, and the standard deviation of the nonzero values, confirm this: in larger markets, the consumer following

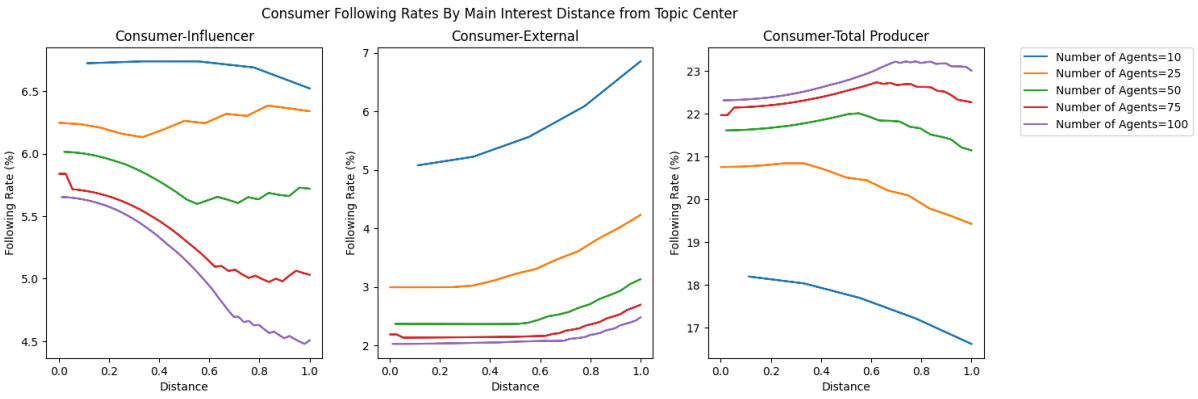

Figure 7: Influencer, External, and Total Producer following rates of consumers based on their main interest's distance from the topic space center.

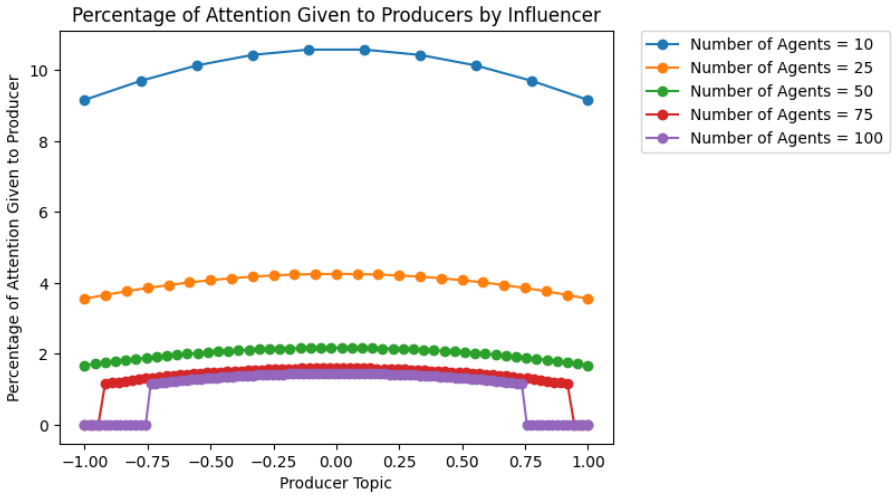

Figure 8: Influencer attention given to producers for each of the market sizes tested. Similar to consumers, the influencer develops a narrower and flatter attention profile in larger markets.

rates are distributed more narrowly, but among the producers that are followed the rate is approximately the same. Consumers in larger markets simply have no need to follow producers whose main interests and topics produced are not close to their main interests.

In fig. 7, we see a similar effect as in the standard market that consumers towards the edges of the market follow the influencer less and external content more, but the slopes of these curves are clearly impacted by the size of the market. Specifically, in larger markets the farther-out consumers "give up" on following the influencer much sooner; however, rather than compensating by greatly increasing attention towards external content, in larger markets they focus on the producers who are closer to them as more are present.

2. Influencer Following Rates

   In fig. 8, we see the obvious effect that any single producer receives less attention when there are more agents. In addition, we see similar to the consumer graph that the influencer develops a narrower and flatter attention profile in larger markets. For the influencer, there is still much more utility in following the more central producers, and since there are more central producers their following rate distribution will grow narrower. The main difference, then, from the changes in the consumer profiles is that the influencer's attention is always centered at the center of the market rather than on a main interest.

3. Topics Produced

   In fig. 9, we see that as more agents are present in the market the range of topics produced by producers widens. This reflects the fact that since more consumers are present in the market, there are specifically more consumers towards the edges of the market, so the producers whose main interests are there feel less pressure to move towards the center of the topic space. In addition, the consumers towards the edges of the market allocate more attention towards these edge producers as shown in fig. 7, further motivating the producers to widen their topic distribution.

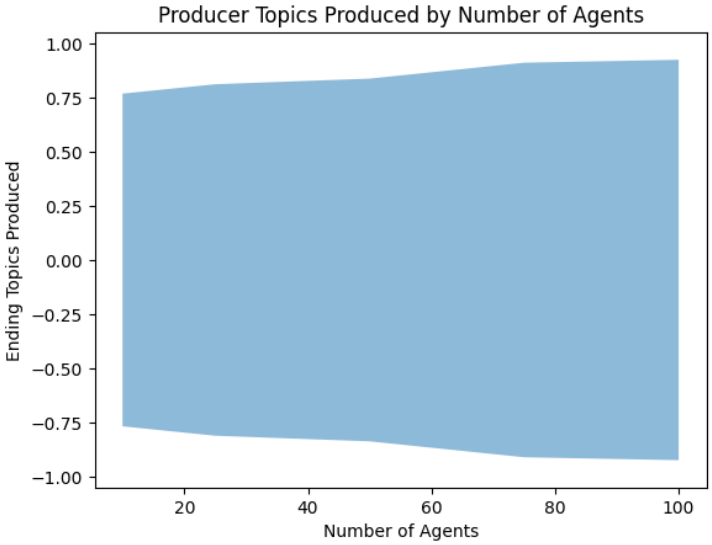

Figure 9: The region of the topic space that producers' produce topics in by the number of agents in the market.

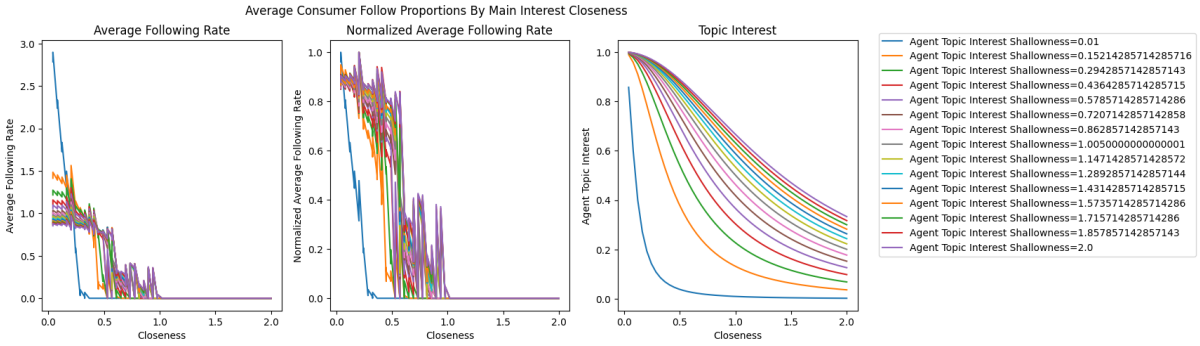

Figure 10: The impact of topic interest function shallowness on consumer-producer following rate distributions. The rightmost graph shows the actual topic interest functions used.

2.2.2 Topic Interest Functions

In general, varying the production topic interest functions on their own does not significantly impact consumer or influencer following rate distributions compared to varying the consumption and production topic interest functions together (and having them be equal). The only aspect of market structure in which varying the functions separately has an interesting impact is with producer topics produced. Because of this, our discussion of consumer and influencer following rates here will be based on tests that varied both types of topic interest function together, but for topics produced, we'll discuss varying them separately.

1. Consumer Following Rates

   We can see in fig. 10 the clear impact of the slope of the agent topic interest function (this effect, in fact, is primarily generated by the consumer topic interest function): the steepness of the consumer following rate distributions mirrors the steepness of the topic

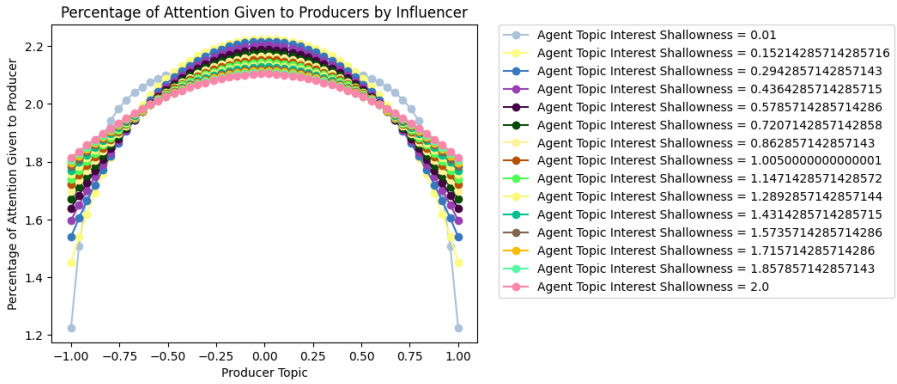

Figure 11: Influencer following rates by agent topic interest shallowness

interest functions. This relationship is unsurprising, as the topic interest functions directly determine the utility that consumers receive from the content they consume. The impact of the topic interest functions on consumer-influencer and consumer-external following rates is not significant. This result is surprising but makes sense given that between a steeper and shallower interest function, the same overall utility can be achieved just by varying which producers are followed; the loss of interest in producers that are further away is compensated by the gain of interest in producers that are closer, for instance.

2. Influencer Following Rates

   We see in fig. 11 that shallower agent topic interest functions lead to a flatter influencer attention profile. This makes sense, as if agents are interested in a wider variety of topics then the influencer can get more utility from promoting topics that are further away from the center of the topic space. In a sense, the influencer's attention distribution acts as a sort-of average for all agent topic interests, and as agent topic interest distributions widen, this average flattens out.

3. Topics Produced

   The producer topics produced is the area where varying consumption and the production topic interest function have differing effects. In figure fig. 12, we see that when only consumption interest distributions are widened the producers end up producing a wider range of topics than when production interest distributions or overall agent interest distributions are widened. It seems, then, that the production distribution is more impactful in determining the topic that producers actually end up producing.

2.2.3 Influencer Attention Bound

1. Consumer Following Rates

   In fig. 13, we see that as the influencer bound grows, all consumers follow the influencer more greatly while they follow producers and external content less. The impact of consumer main interest distance from the topic space center is the same regardless of the influencer attention bound, so the graphs in fig. 7 are not shown here. This result makes sense: as the influencer bound grows larger than the consumer bound, it becomes possible for consumers to gain greater utility from consuming content through the influencer since the delay function decays exponentially as the following rates increase. Notably, the

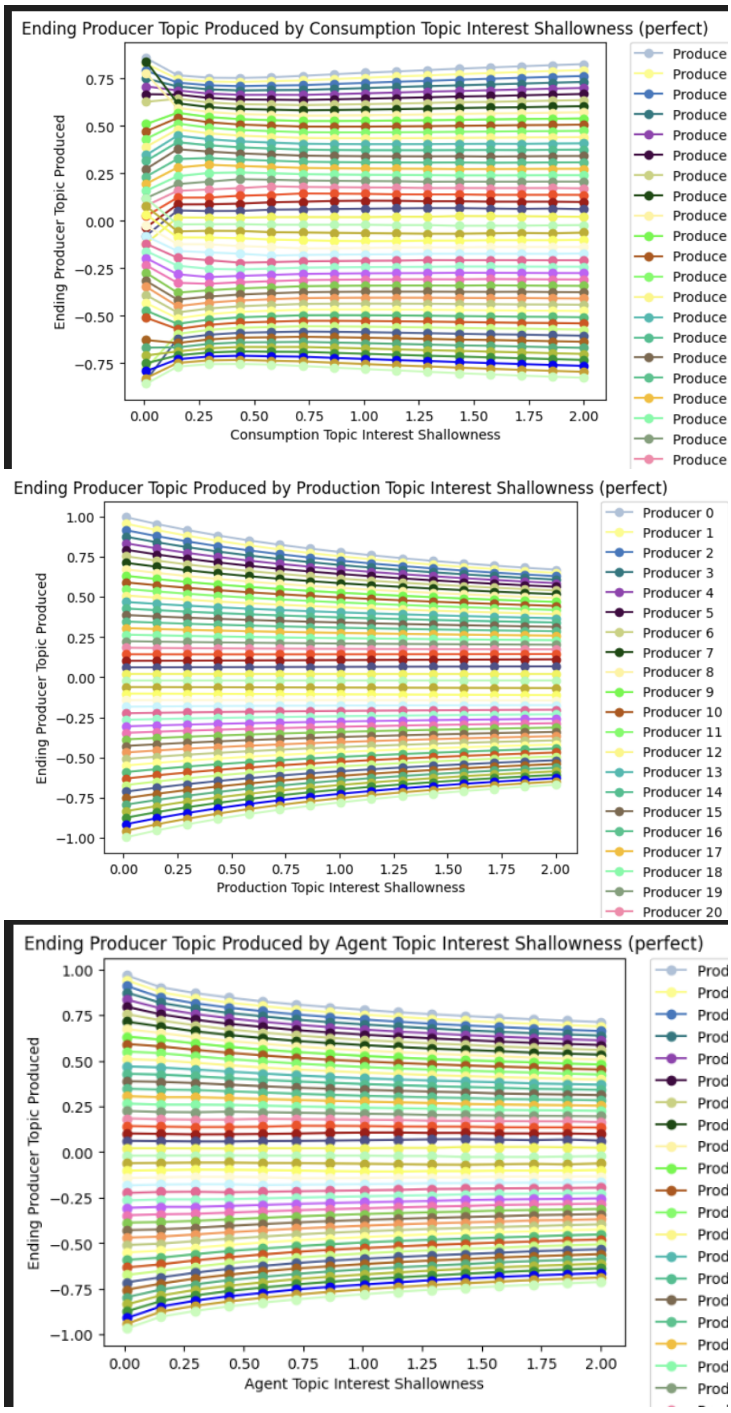

Figure 12: Top: Producer topics produced as consumption topic interest distributions are widened. Middle: Producer topics produced as production topic interest distributions are widened. Bottom: Producer topics produced as both consumption and production topic interest distributions are widened.

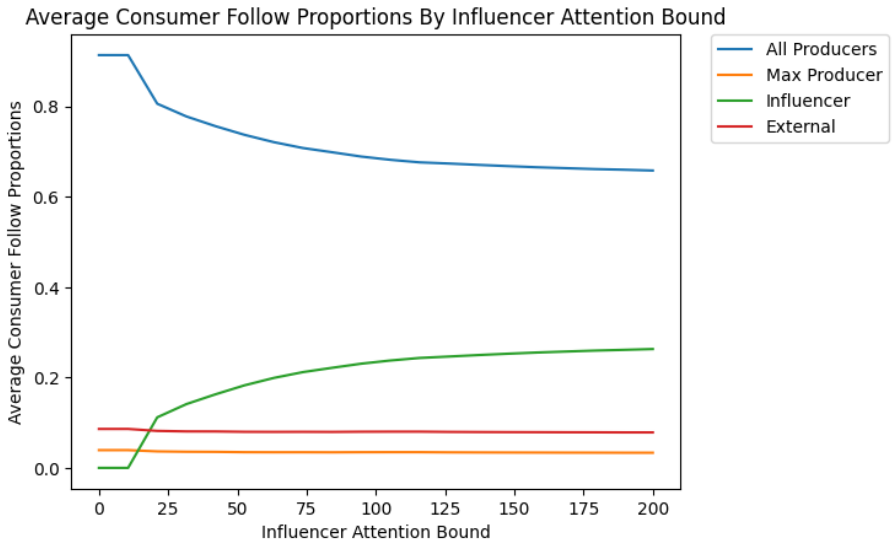

Figure 13: Consumer-Influencer, Consumer-External, and Total Consumer-Producer following rates based on consumer main interest distance from the topic space center as the influencer bound grows.

trend in this figure appears only once the influencer attention bound reaches a threshold of around 12; below this point, it seems there is no utility to be gained from following the influencer, so consumers don't bother.

In fig. 14, we see that as the influencer bound increases, the average distribution of nonzero consumer-producer following rates becomes narrower and flatter (lower standard deviation). This complements the result from figure fig. 13, as if consumers are choosing to follow producers less (in favor of following the influencer more), they will be pickier and follow a narrower grouping of producers.

2. Influencer Following Rates

   We see in fig. 15 that the influencer's following rate distribution widens as their attention bound increases until it encompasses the entire topic space (all the producers). At this point, then, further increases in the attention bound only cause the distribution to flatten out. Basically, with an infinite following rate the influencer would follow all producers equally; when that is not the case, they will prioritize the producers in the center of the market, as these producers appeal to the greatest proportion of the consumers and therefore following them provides the greatest utility to the influencer.

3. Topics Produced

   Finally, we see in item 3 that an increased following rate causes producers to cluster their topics produced more towards the topic space center, though this effect is not significant. As mentioned previously, ending topics produced reflect a balance between producer topic interest, consumer interest, and influencer interest; it seems that an increased attention bound slightly moves the balance in favor of influencer interest, but not to a significant extent. In perfect information, producers still receive direct utility from consumers, so the impact of influencer attention is not as great as it will be in the imperfect information market, as will be shown in section 3.1.2.

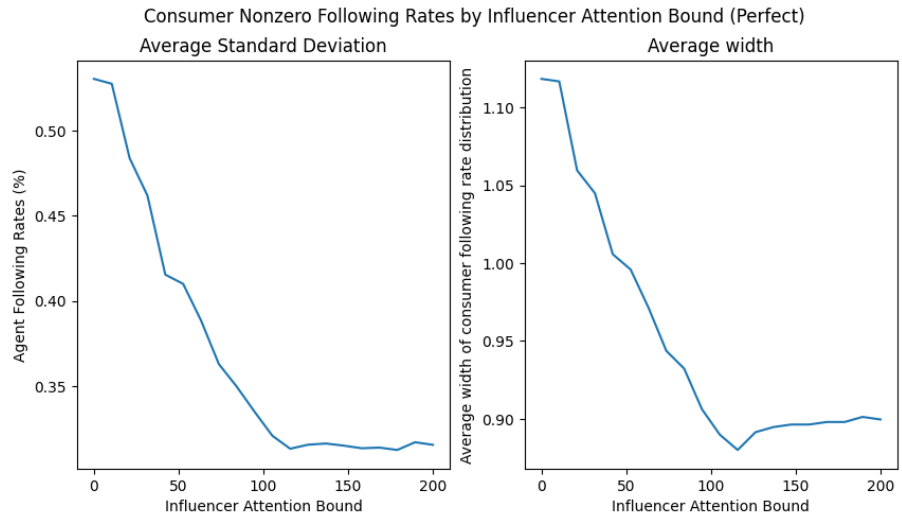

Figure 14: Average Standard Deviation and average topic space with of consumer following rates based on the influencer attention bound. As the influencer attention bound increases, consumers follow a narrower segment of the topic space more evenly.

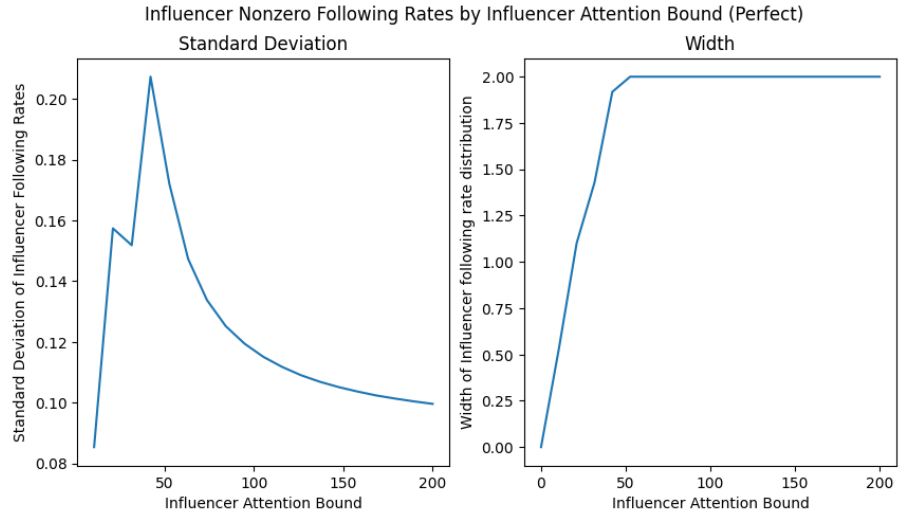

Figure 15: Standard deviation (steepness) and width of influencer following rate distribution based on the influencer attention bound.

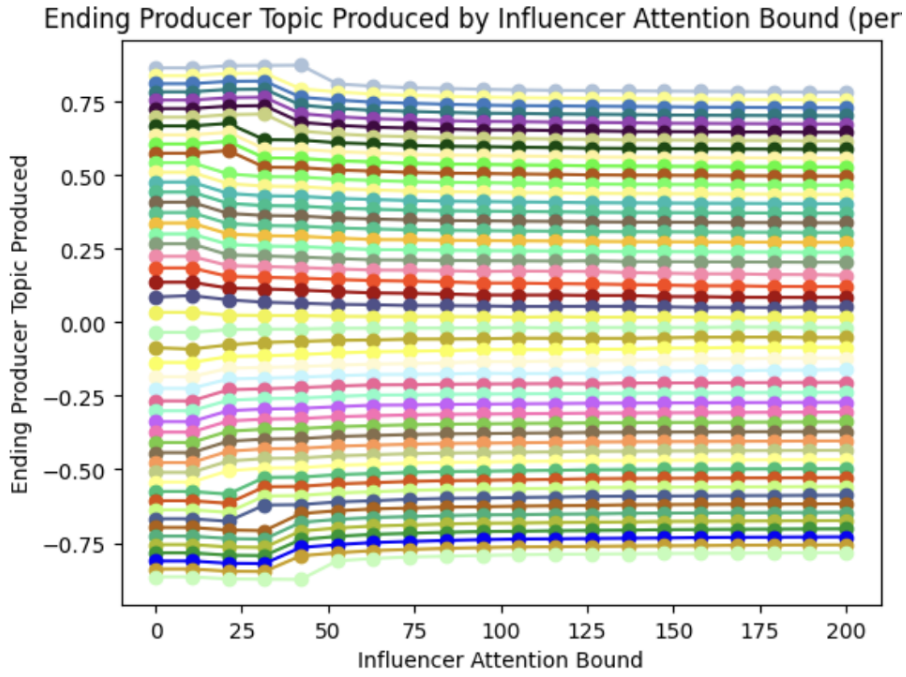

Figure 16:

2.2.4 Consumer Attention Bound

Outside of the case when the consumer attention bound is at or very close to zero, its impact is only really on consumer following rates. When the bound is at zero, all the optimization problems become flat, as they all contain indicators for whether consumers follow producers, which cannot happen if the consumers have no attention. As the bound grows beyond this, they switch to prioritizing the producers closer to their own main interests. We can see this effect in fig. 17 and fig. 18. It appears in these charts that for a consumer bound that is very close to zero (1, to be specific), the consumers choose to distribute attention equally among the producers; this effect is likely less a reflection of the market dynamics and more just the fact that, in this testing, consumer following rates were initialized to be equally distributed among the producers. It's most likely that a consumer attention bound of 1 does not make the optimization problems significantly less flat, so there simply isn't enough of a force to drive consumers to optimize their following rates based on main interest closeness. The more valid effect is that at a bound of 10, the consumers choose not to follow the producers at all and instead focus on the influencer and external content, likely because with limited attention capacity following those sources provides more utility than allocating all attention to the closest producers would. The other major impact of increasing the consumer attention bound is that their following rate distributions become wider and flatter, though the peak is always at the producers whose main interest is closest to the consumers' own.

2.2.5 Consumer External Interest

Similarly to the consumer attention bound, consumer external interest (in the perfect information market) only really impacts the consumer following rate distributions; the influencer following rates and producer topics produced are unaffected by varying this hyper-parameter.

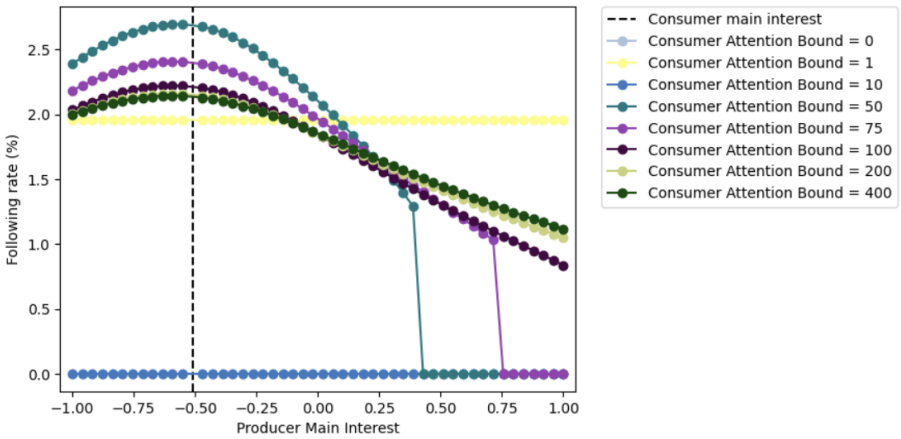

Figure 17: An example consumer's following rates by producer topic for each of the tested consumer attention bounds. The flat non-zero line for an attention bound of 1 is more likely a reflection of the initialization of the consumer rates, as the optimization problems would still all be basically flat in this case.

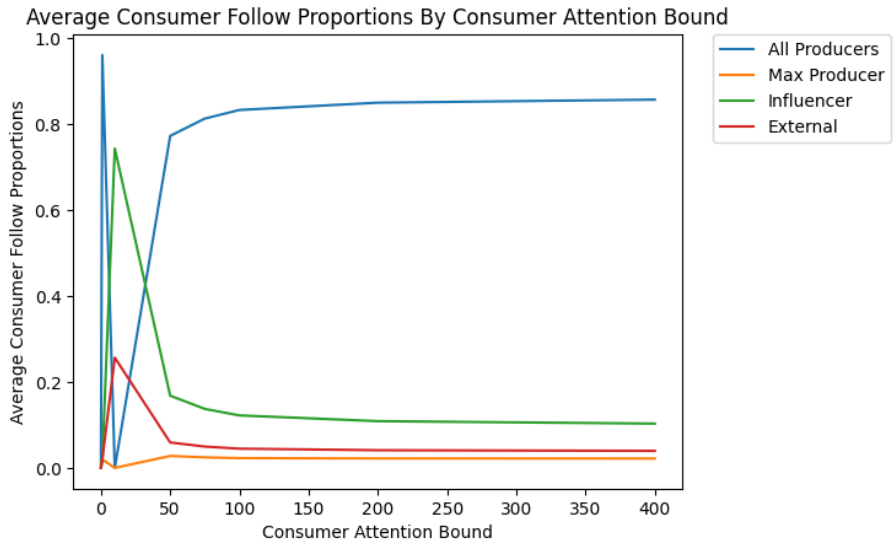

Figure 18: Average consumer-producer, consumer-influencer, and consumer-external follow proportions based on the consumer attention bound.

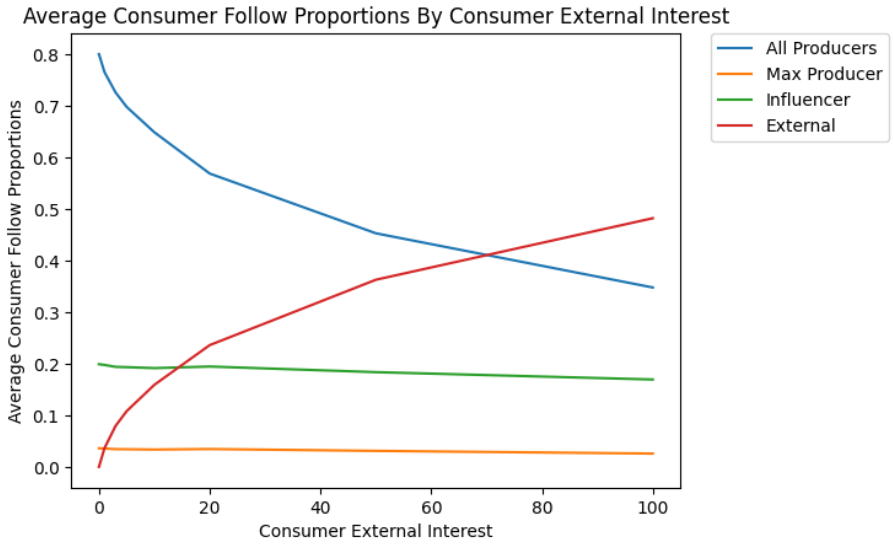

Figure 19:

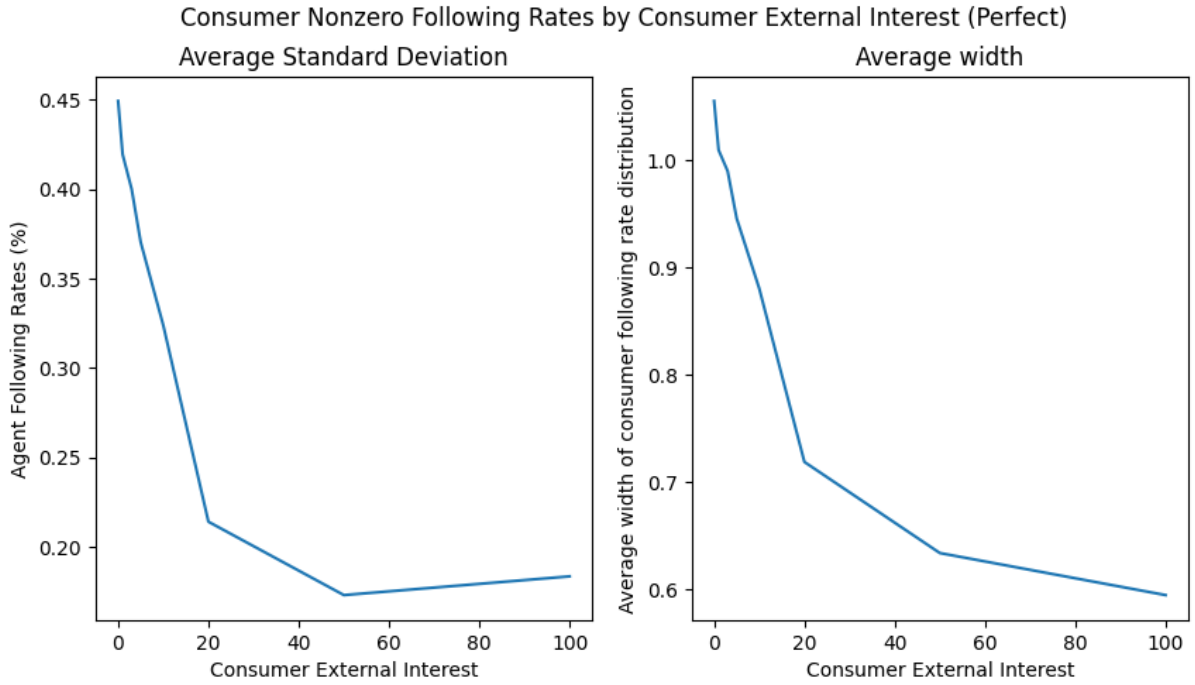

Figure 20: The standard deviation (steepness) and width of nonzero consumer-producer following rate distributions based on consumer external interest.

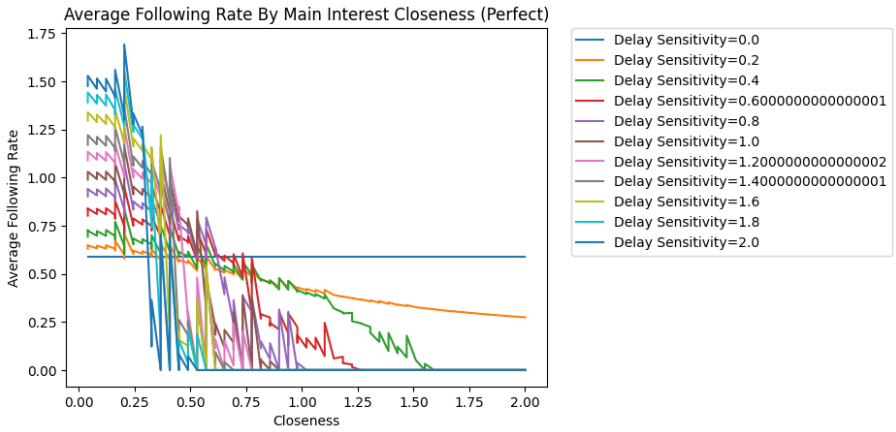

Figure 21: Average consumer-producer following rates by producer main interest closeness, based on delay sensitivity. Increasing delay sensitivity increases the slope of this curve.

Specifically, we see in fig. 19 that as external interest grows consumers devote much less attention to producers and much more to external content, though attention towards the influencer, on average, remains about the same. Attention towards the closest producer also remains about the same, so the major reduction in producer attention is for producers whose main interests are further away from the consumers' own, an effect we can see in fig. 20. These results are unsurprising: as consumers gain more utility from consuming external content, they devote more attention to this content and compensate by being much pickier in terms of which producers they follow. The interesting point, then, is that they do not draw attention away from the influencer. We will see in section 3.2.3 and **??** whether this means for the impact that consumer external interest has on the imperfect information market.

2.2.6 Delay Sensitivity

Delay sensitivity is an interesting parameter in that it is the only one for which an increase in its value leads to decreased total social welfare in the perfect information market. This is due to the presence of $e^{-\alpha}$ in the utility equations, as an increase in the value of α brings this discount multiplier closer to zero.

1. Consumer Following Rates

   The major impact of increased delay sensitivity on consumers in the perfect information market is that their following rates become more narrowly distributed; as shown in fig. 21, they tend to pay attention to a narrower set of producers more intensely. This makes sense: when delay sensitivity is increased, in order to receive the same amount of utility from a given producer, a consumer has to allocate more attention to that producer. In general, then, they will be forced to prioritize the producers who they were receiving the most utility from in the first place, the ones whose main interests are closest to their own. This means that the widths of their following rate distributions will be lessened, so the attention drop-off will be steeper as shown in the figure.

   We see also in fig. 22 that consumers follow the influencer and external content more heavily when their delay sensitivity increases, though the rate of this increase, especially

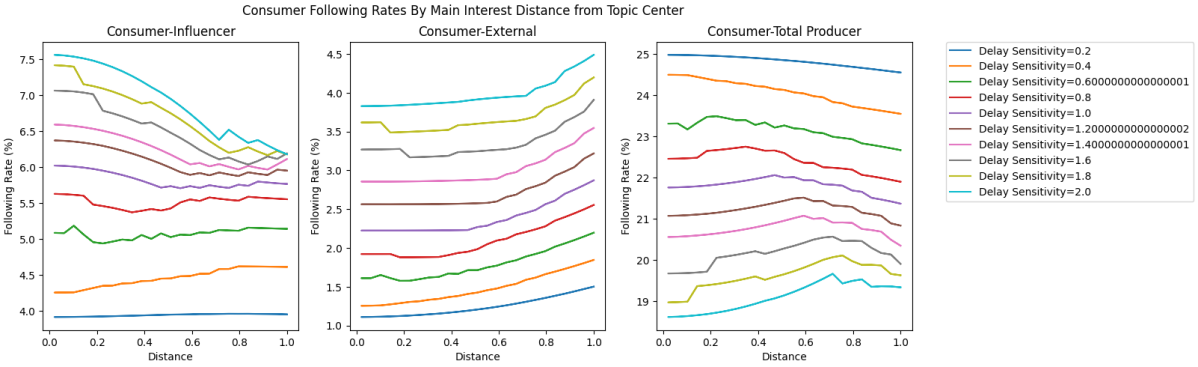

Figure 22: The rates of consumers following the influencer, external content, and producers in general by their main interests' distance from the topic space center and by their delay sensitivity.

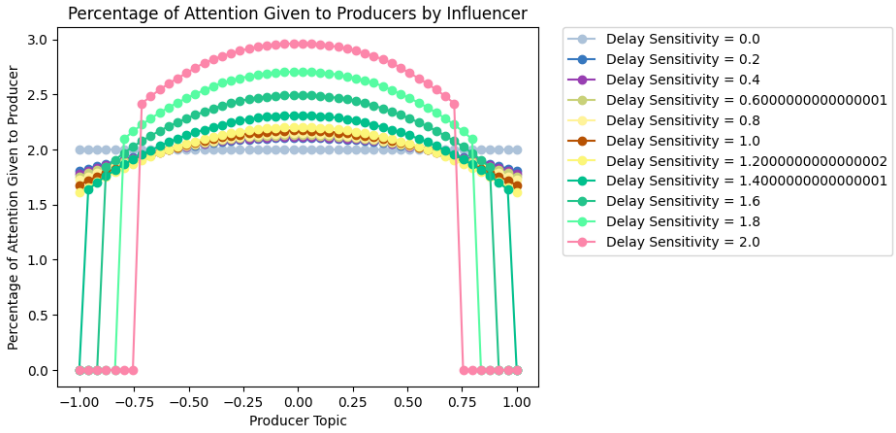

Figure 23: Influencer following rates in the perfect information market based on consumer delay sensitivity.

for the influencer, is not uniform across the topic space. Consumers to the edges of the topic space seem to eventually experience diminishing marginal returns from further following the influencer as delay sensitivity increases as compared to those in the center of the market. As will be discussed, this is likely because the increased delay sensitivity causes the influencer to have a narrower attention profile, meaning consumers on the edges of the market receive less interest utility from following them.

2. Influencer Following Rates

   Similarly to the consumers, we see in fig. 23 that increasing the delay sensitivity of consumers causes the influencer to develop a narrower attention profile, though as always this attention is centered on the topic space center. As delay sensitivity increases, a larger following rate is required to achieve the same amount of utility from a given producer, so the influencer prioritizes attention to the producers who are followed by the widest range of consumers: the ones whose main interests are in the center of the market.

3. Topics Produced

   Finally, in fig. 24 we see that increasing the delay sensitivity of consumers leads producers

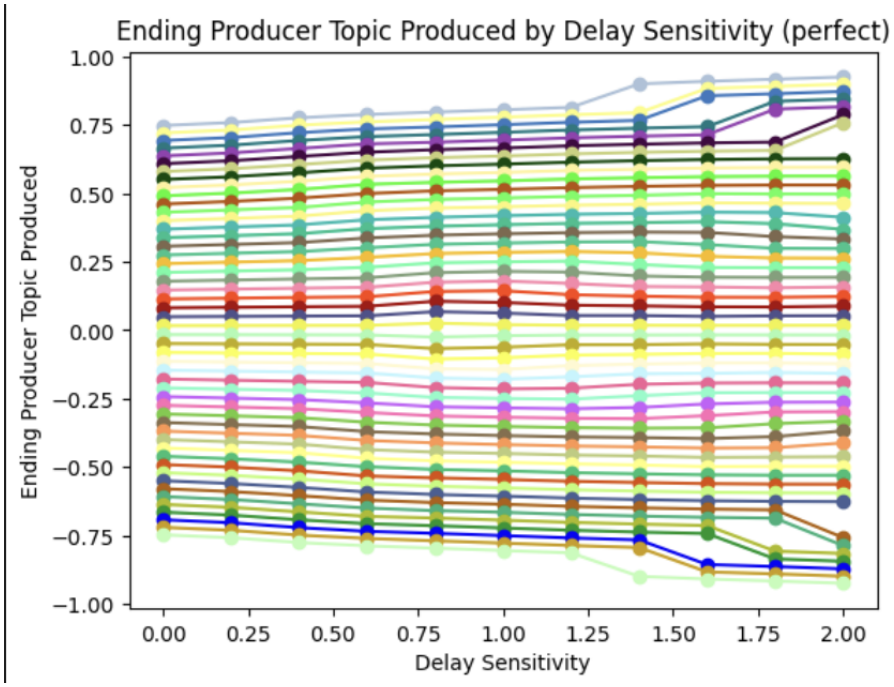

Figure 24:

to eventually produce a more dispersed set of topics, with the effect occurring the earliest and in the most pronounced manner for producers at the edges of the market. What's likely occurring is that as consumers and the influencer narrow their attention profiles, the producers feel less of an urge to appeal to a wider variety of consumers so are not as heavily driven to produce content that is more in the center of the topic space.

2.2.7 Parameters with Similar Impacts

- Increasing the number of agents or the steepness of agent topic interest curves causes consumer following rate distributions to become narrower and steeper
- Increasing the number of agents, the shallowness of consumption topic interest curves, or the delay sensitivity causes producer topic produced to move outwards
- Increasing the shallowness of consumption topic interest curves or increasing the consumer attention bound causes consumer following rate distributions to widen and flatten
- Increasing the influencer attention bound, the consumer external interest, or the delay sensitivity causes consumer following rates to grow narrower

Overall, the strongest links between hyper-parameters seem to be the number of agents with the topic interest functions and the number of agents with the attention bounds. It's possible that all the structural variance from changing the number of agents could be reproduced simply by varying the topic interest functions and the attention bounds.

3 Imperfect Information: Structural Differences

In the imperfect information market, producers do not have access to consumer following rates or topic interest functions. Instead, they are aware only of the influencer's attention distribution and must use the influencer as a proxy for consumer interests (in combination with their own topic interests) to decide what content to produce. Since it is not optimized directly for their interests, the consumers generally do not receive as much utility from the content that they consume, so total social welfare is generally lower; the difference in social welfare between perfect and information is known as the 'cost of influence'. In this section, we will evaluate just the structural differences between the perfect and imperfect information markets in terms of following rates and topics produced, considering the impacts of various hyper-parameters. Then, in section 4, we'll focus in on the cost of influence and how it, too, is impacted by the market hyper-parameters.

As a general note: it was not possible to test the imperfect information market with a similar size as the perfect information market. From tests varying the number of agents, it seems the increase in optimization time is near-exponential as more agents are added. This is because the calculation of the imperfect producer utility requires, for each producer, full reruns of the influencer optimization process so that the producer can evaluate how the influencer rates would change based on the topic produced. In reality, this should also be done for the perfect information producers, but it has not seemed necessary and does not significantly improve the optimization process. In imperfect information, though, not including this recalculation of influencer rates causes the producers to simply never adjust their topic produced. Overall, this efficiency limitation prevents testing to the extent that would be desired, and future work could definitely focus on re-implementing the optimization process to run more efficiently.

3.1 Comparison of Structural Properties

In general, we find that the imperfect information market accurately approximates the dynamics of the perfect information market. This is helped by the fact that the only real difference between the two is the producer optimization problem; so long as that is approximated accurately, the rest of the market structure will converge to about the same as the perfect information market, too.

3.1.1 Producer Topics Produced

First, we see in fig. 25 that the imperfect information producer topics are generally very close to those in the perfect information market, with the difference almost always staying within 3% of the width of the topic space. It's possible that the aberration seen in the producer with main interest 1 is due to a local optimum, as this test was not run with a global optimizer; tests of the imperfect information market with a global optimizer do not feature such aberrations, but these tests were only able to be performed with a smaller market. Overall, this result gives confidence that the producers are able to properly use the influencer's attention as a proxy for consumer interests and optimize their content produced to be as good as the content produced in the perfect information market.

3.1.2 Influencer Following Rates

We see in fig. 26 that the influencer following rates in the imperfect information market nearly perfectly approximate those in the perfect information market. The larger difference in the

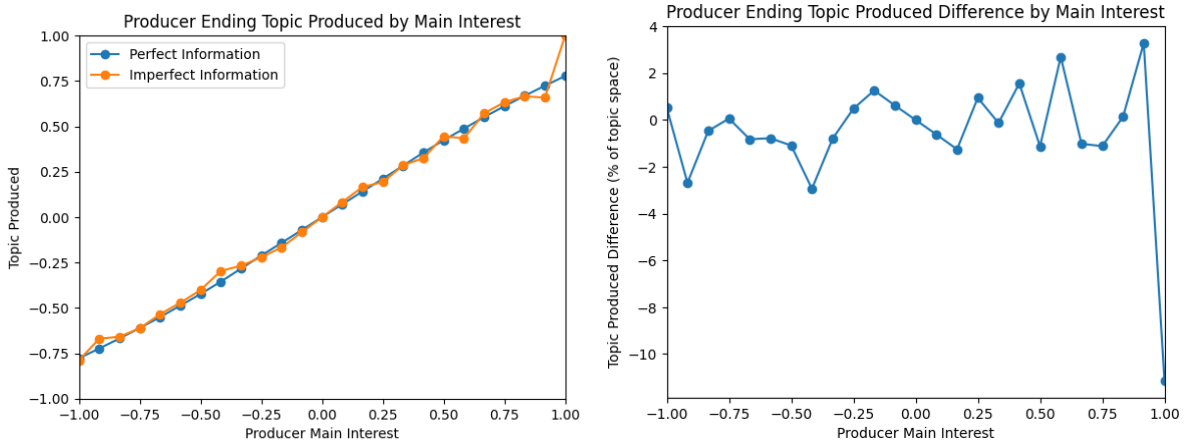

Figure 25: Left: Perfect vs Imperfect Information topics produced by producer main interest. Right: The difference between perfect and imperfect information topics produced by producer main interest, as a percentage of the width of the topic space.

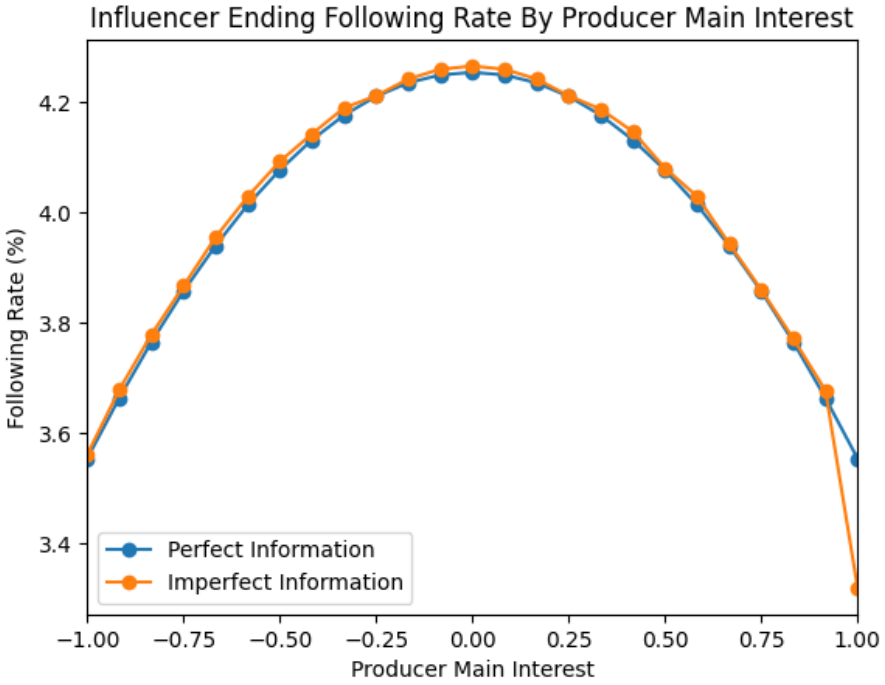

Figure 26: Influencer following rates based on producer main interests in perfect and imperfect information markets.

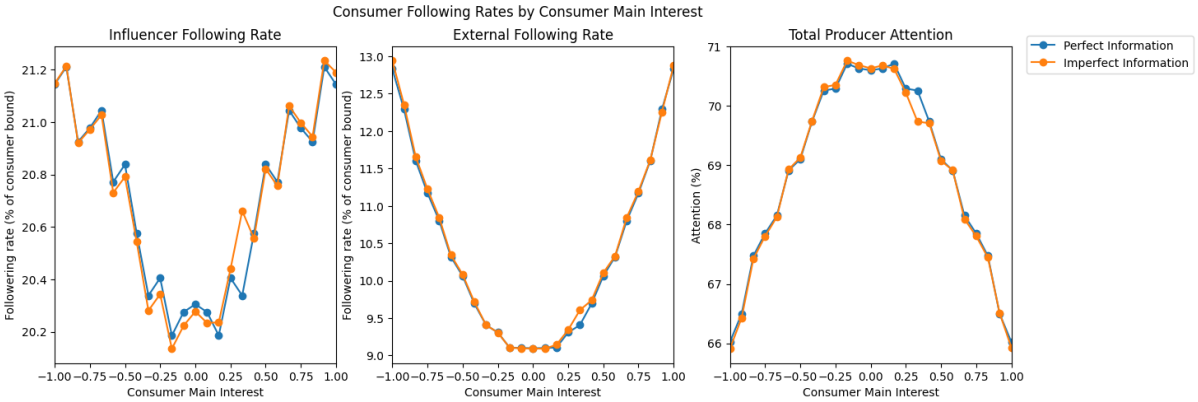

Figure 27: Consumer-influencer, consumer-external, and consumer-producer following rates in the perfect and imperfect information markets based on consumer main interests.

producer with main interest 1.00 is due to that producer not shifting their topic inward as the others do; it's possible that this difference does indeed reflect a slight difference between local and global optimization, as other testing (of smaller markets) with basinhopping did not have this anomaly. Overall, though, the influencer seems able to allocate their attention just as effectively in the imperfect information market as they do in the perfect information market.

3.1.3 Consumer Following Rates

Similarly to the influencer, consumer following rates are generally about the same in the perfect and imperfect information markets. In fig. 27, we see that the proportion of attention allocated by consumers throughout the topic space towards the influencer, external content, and producers, in general, does not vary significantly between perfect and imperfect information. Similarly, in fig. 28, we see that, on average, the consumer-producer following rate distributions do not vary between perfect and imperfect information. Since an average could be concealing variance among the consumers, we also have fig. 29 to validate that in general, the magnitude of the difference between the perfect and imperfect information following rate vectors was low. We also see for the specific consumer highlighted that the overall shape of the attention distribution is the same; the major difference, then, is just that the imperfect information distribution seems slightly less smooth. A similar evaluation of the vectors' cosine similarity suggests the same. Overall, although the following rates of consumers in the imperfect information market do not exactly match those in the perfect information market, they are generally almost the same. This result fits with the fact that the consumer utility calculation is the same between perfect and imperfect information, so the slight differences in following rates can be explained by the variance in producer topics produced in imperfect information.

3.2 Parameter Impacts on Structural Properties

When comparing following rate vectors between perfect and imperfect information, three metrics are generally used: average magnitude of difference, i.e. $\|\Lambda(y) - \Lambda(y)'\|$; average difference of norms, i.e. $\|\Lambda(y)\| - \|\Lambda(y)'\|$, and average cosine similarities, i.e. $\frac{\Lambda(y)\cdot\Lambda(y)'}{\|\Lambda(y)\|\|\Lambda(y)'\|}$. Analyzing variation in all these metrics gives a pretty complete picture of how the two vectors differ. For producers, we simply look at the average difference in topics as a percentage of the topic space.

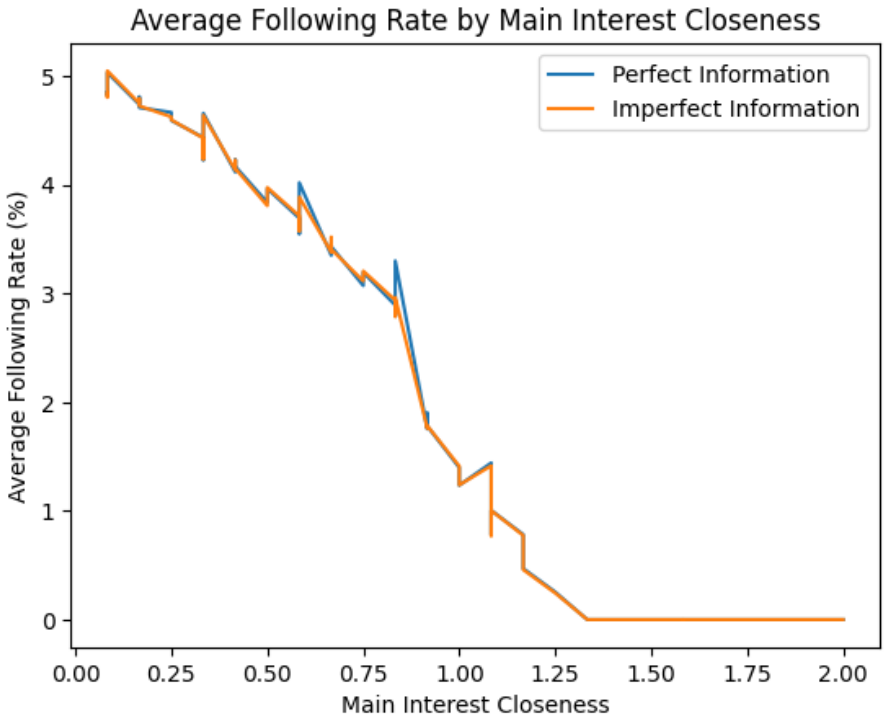

Figure 28: Average consumer following rates based on producer main interest closeness, compared between perfect and imperfect information. On average, consumer-producer following rates are the same between the two market types.

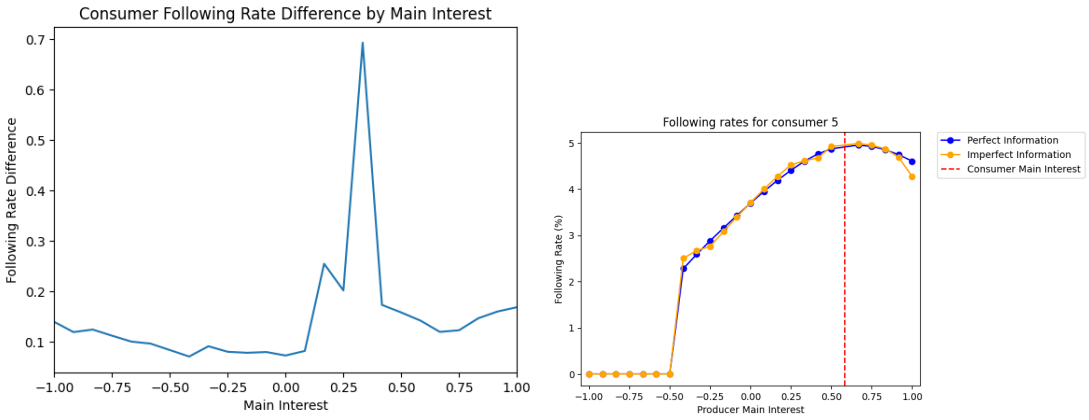

Figure 29: Left: The magnitude of the difference (i.e. $\|\Lambda(y) - \Lambda(y)'\|$) between the perfect and imperfect information consumer following rate vectors based on the consumer's main interest. Right: An example consumer following rate distribution compared between perfect and imperfect information markets.

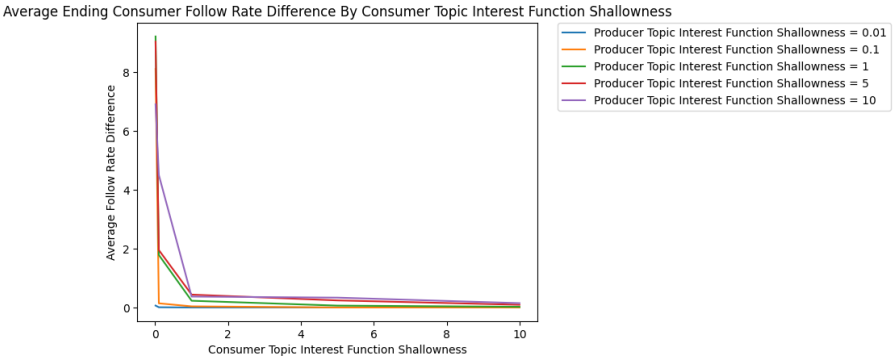

Figure 30: Consumer following rate normed difference between perfect and imperfect information based on consumer and producer topic interest function shallowness. The same trend occurs for the differences of norms and the cosine similarities.

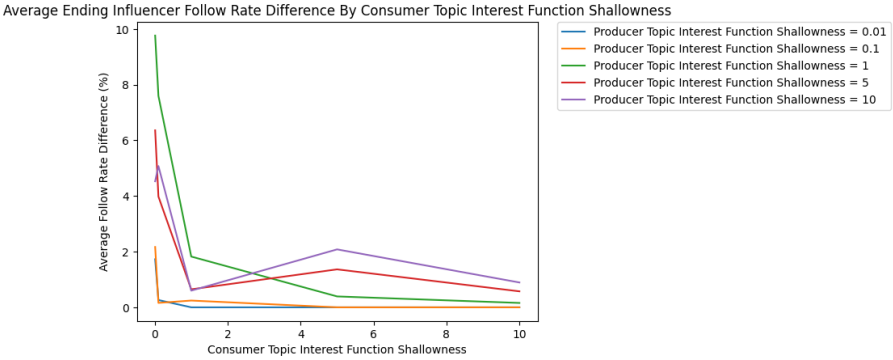

Figure 31: Influencer following rate normed difference between perfect and imperfect information based on consumption and production topic interest function shallowness.

3.2.1 Topic Interest Functions

For this test, the consumption and production topic interest functions were varied in parallel to see their interaction. Overall, what was found is that the perfect and imperfect information market structures will be approximately the same so long as the consumption topic interest function is not incredibly steep. This holds regardless of the production topic interest function, though the differences were additionally lessened when the production topic interest function was shallow.

We can see this effect for the consumers, influencer, and producers in fig. 30, fig. 31, and fig. 32, respectively. The reason for this effect is likely that when the consumption topic interest function is very steep, consumers tend to not follow the influencer, as they are really only focused on content that is very close to their main interest. Because of this, in this case the influencer cannot serve as a useful proxy for consumer interests, meaning the producers will be unable to properly optimize their content produced based on consumers' interests. A less steep consumption interest function allows the influencer to better act as a proxy for consumer interests and reduces the structural differences between perfect and imperfect information.

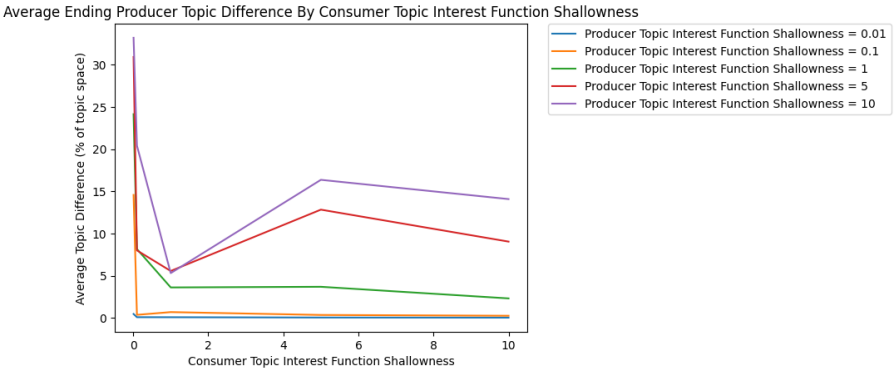

Figure 32: Producer topic produced difference between perfect and imperfect information based on consumption and production topic interest function shallowness.

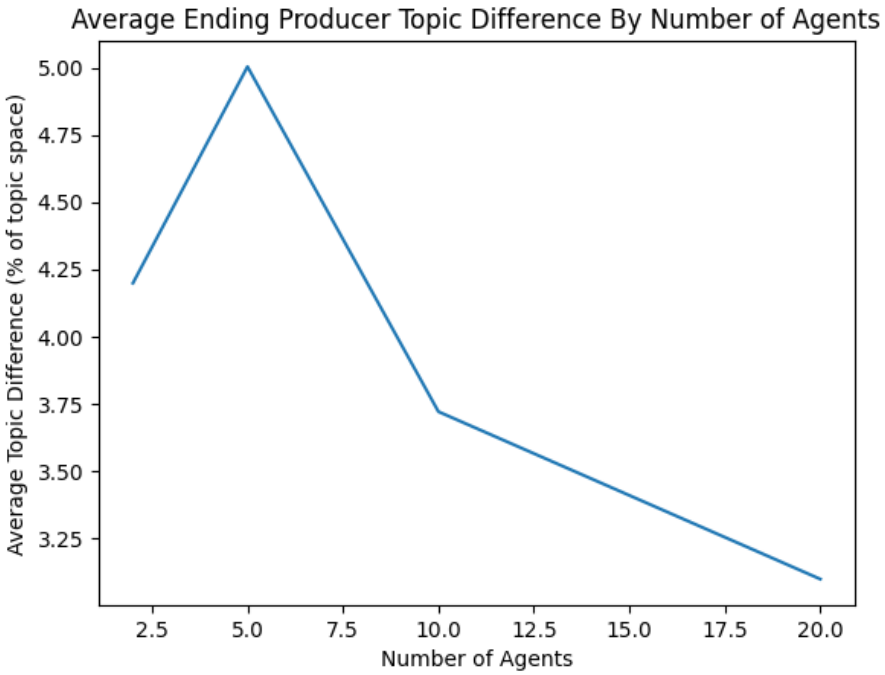

Figure 33: Producer average topic divergence between perfect and imperfect information by the number of consumer/producer agents in the market.

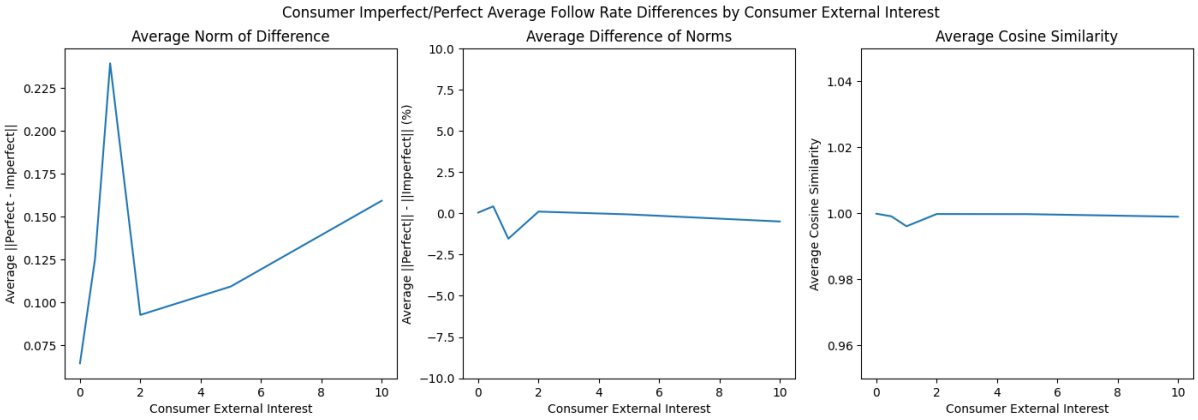

Figure 34: Consumer perfect-imperfect divergence by consumer external interest.

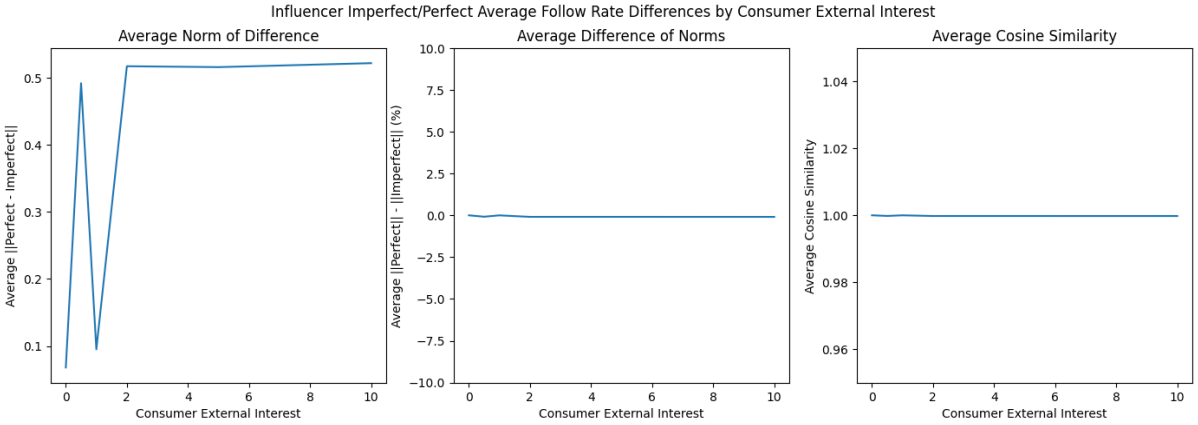

Figure 35: Influencer perfect-imperfect divergence by consumer external interest.

3.2.2 Number of Agents

In fig. 33, we see that increasing the size of the market leads to a small but noticeable decrease in the difference between topics produced by perfect and imperfect information producers. This is the only major impact of adding agents to the market in terms of the perfect-imperfect difference (aside from optimization time, which is much longer in imperfect information as agents are added). However, it's hard to know if this effect would generalize to larger markets, so not much can be concluded here.

3.2.3 Consumer External Interest

Consumer external interest has a noticeable impact on the differences between consumer following rates, influencer following rates, and producer topics produced between perfect and imperfect information.

In fig. 34, we see that this divergence peaks when the external interest is around 1 before dropping rapidly, though it begins to increase again as external interest grows. Additional investigation suggests, though, that the shape of this curve is heavily influenced by the influencer attention bound, so nothing really can be concluded from this graph other than that consumer

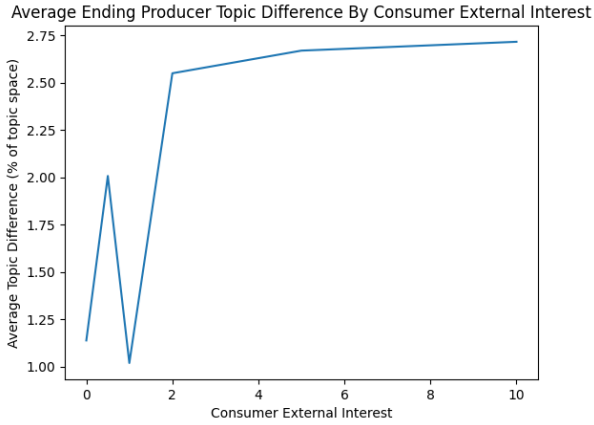

Figure 36: Producer perfect-imperfect divergence by consumer external interest.

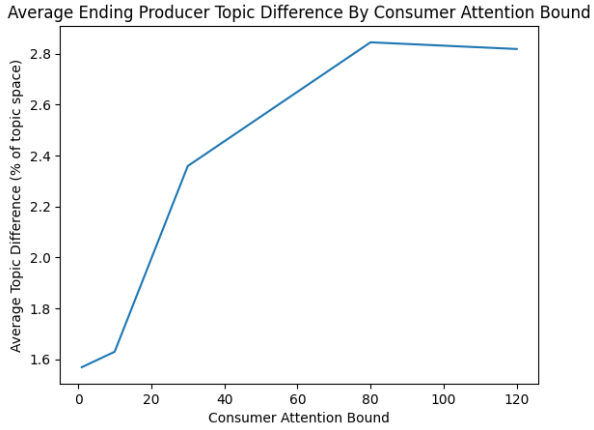

Figure 37: Producer topic divergence between perfect and imperfect information by consumer attention bound.

external interest never leads to very high divergence.

In fig. 35 and fig. 36, we see that values of consumer external interest below 1 lead to some strange variation in the differences between perfect and imperfect information, but once external interest reaches around 2 these differences plateau. Overall, in both cases the differences remain small regardless of the value of the consumer external interest.

3.2.4 Consumer Attention Bound

The only salient impact of varying the consumer attention bound on the perfect-imperfect market divergence is the producer topics produced; with higher consumer attention bounds, the imperfect producer's topics are generally farther away from the perfect producers, as seen in fig. 37. What's likely happening is that as the consumer attention bound grows, the consumers tend to slightly decrease the proportion of attention given to the influencer, as shown in fig. 18; this means, then, that the influencer will not serve as well as a proxy for consumer interests, making the optimization task of producers more difficult. Overall, though, this difference is not very significant.

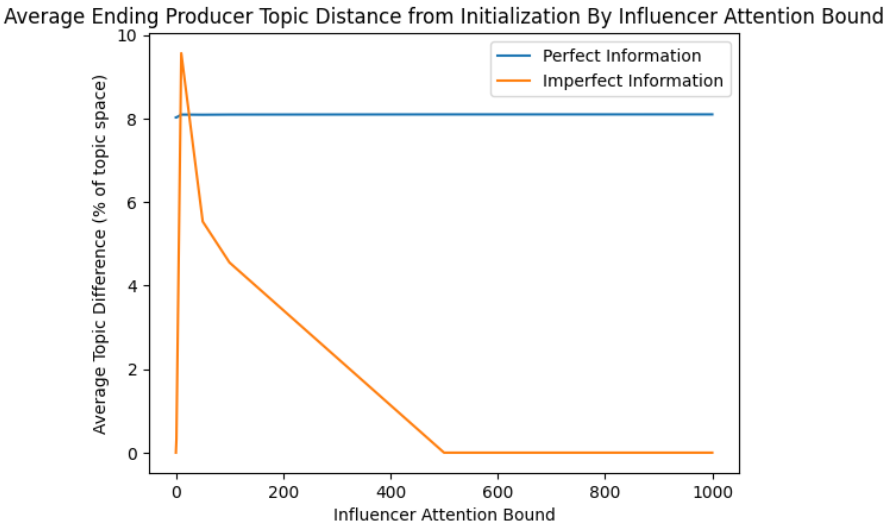

Figure 38: Producer topic produced distance from what it was initialized as before optimization. A distance of 0 suggests that the producer optimization problem was flat.

3.2.5 Influencer Attention Bound

In this experiment, the influencer attention bound was varied. What was found was that the difference in market dynamics between perfect and imperfect information is greatest when the influencer has a very low or very high attention bound. In either of these cases, the producer optimization problem becomes flat; when the influencer has no attention then producers receive no social support, regardless of topic produced, and when the influencer has a very large amount of attention then producers all receive an equal amount of social support, regardless of topic produced. This effect was seen in the perfect information market in fig. 15; as the bound increased, the width of the nonzero rates approached the entire topic space and the standard deviation of following rates approached zero. When the producer optimization problem is flat, producers simply end up producing whatever topic they were initialized to produce.

We can see this effect in fig. 38, as when the influencer bound is 0 or when it is very high the producers' topics remain at what they were initialized as. What this suggests is that it is perhaps not beneficial, in terms of minimizing the difference between perfect and imperfect information markets, to increase the influencer bound infinitely, as it seems that the limited attention capacity is necessary for the influencer to take on the role of providing an accurate proxy for consumer interests.

3.2.6 Delay Sensitivity

Varying the delay sensitivity of the consumers does not impact the difference between consumer and influencer following rates between the perfect and imperfect information markets, though the difference in topics produced by producers is lessened when delay sensitivity is at or above 1.

In fig. 39 we see the most notable impact of the consumer delay sensitivity on the perfect-imperfect information difference: the topics produced. It's clear that increasing delay sensitivity from 0 to 1 leads to lower divergence between perfect and imperfect information topics produced, with the effect most notable towards the edges of the topic space. This is likely because, as

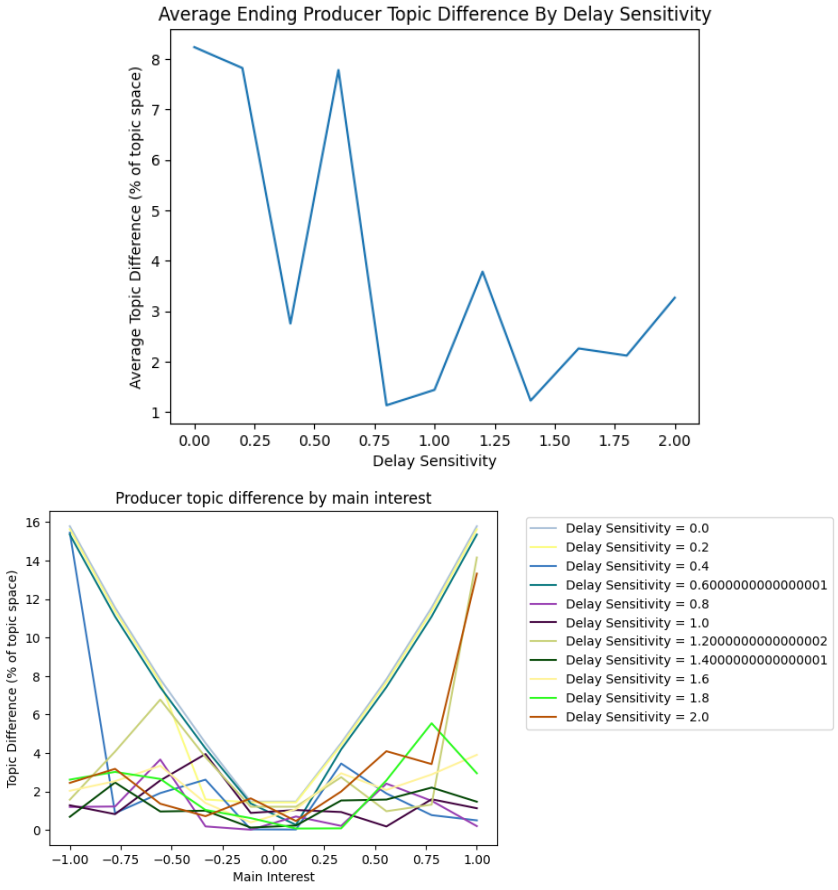
Average Ending Producer Topic Difference By Delay Sensitivity
Average Topic Difference (% of topic space)
Delay Sensitivity
Producer topic difference by main interest
Topic Difference (% of topic space)
Main Interest
Delay Sensitivity = 0.0
Delay Sensitivity = 0.2
Delay Sensitivity = 0.4
Delay Sensitivity = 0.6000000000000001
Delay Sensitivity = 0.8
Delay Sensitivity = 1.0
Delay Sensitivity = 1.2000000000000002
Delay Sensitivity = 1.4000000000000001
Delay Sensitivity = 1.6
Delay Sensitivity = 1.8
Delay Sensitivity = 2.0

Figure 39:

shown in section 2.2.6, lower delay sensitivity makes the influencer have a narrower attention profile, and in this case, the producers towards the edges of the topic space will have lower following rates and will receive less information about consumer interests.

4 The Cost of Influence: How is total social welfare different?

The cost of influence measures the divergence between perfect and imperfect information purely in terms of total social welfare. This means that it ignores some of the structural differences highlighted in the previous section so long as consumers are able to adapt their following rates properly. In general, it reflects the efficacy of the influencer in converting consumer interests to the producers to aid in their content optimization. In this section, we'll analyze the factors underlying the cost of influence and discuss how it can be minimized.

A high cost of influence occurs when imperfect information producers are unable to optimize their topics toward consumer interests. The ability of imperfect information producers to optimize their topics produced is based on how responsive the influencer is in its following rate allocation to changes in topics. If the influencer pays a similar amount of attention to all topics, perhaps due to the consumers not following it or because it simply develops a flat attention profile, then the imperfect producer optimization problem will become flat; changing topics will not seem to bring about an increase in utility, so the producers won't bother.

In this section, we'll show that among all the hyper-parameters, the only two that have a significant impact on the cost of influence in isolation are the influencer attention bound and the consumer delay sensitivity. However, additional investigation reveals that other hyper-parameters, such as the number of agents and the consumer external interest, are able to modulate the impact of the influencer attention bound on the cost of influence. The same is likely true for delay sensitivity.

It's also important to note that in the 'worst case', where either the consumers don't follow the influencer at all, the influencer doesn't follow the producers at all, and/or the influencer follows all producers approximately equally, the cost of influence could be arbitrarily high, since the producer optimization problem becomes completely flat. Practically, in these cases the cost of influence is determined by the initialization of the producer topics produced: the further the topics produced are from the producer and consumer main interests, the higher the cost will be. For demonstration purposes here, we show results that were obtained with producer topics produced initialized to be as far away from the main interests as possible within the market; this magnifies the slope of the cost of influence curve and best demonstrates how, with small adjustments to the hyper-parameters, the cost can be brought down to almost zero.

4.1 The Influencer Attention Bound: Too Little or Too Much

fig. 40 clearly demonstrates the relationship between the cost of influence and the influencer attention bound. When the influencer bound is near zero, the influencer is basically unable to allocate attention to the producers, so they do not receive any meaningful signals about how to optimize the content produced. Similarly, when the influencer bound is very high, the influencer tends to develop a flatter attentional profile that makes it so that producers, again, do not receive meaningful signals about how to optimize their content produced. Again, as noted earlier, the peaks of this curve could be arbitrarily high depending on the initialization conditions. From the graph here, we can see that there appears to be a 'Goldilocks' range wherein the cost of influence is consistently very low.

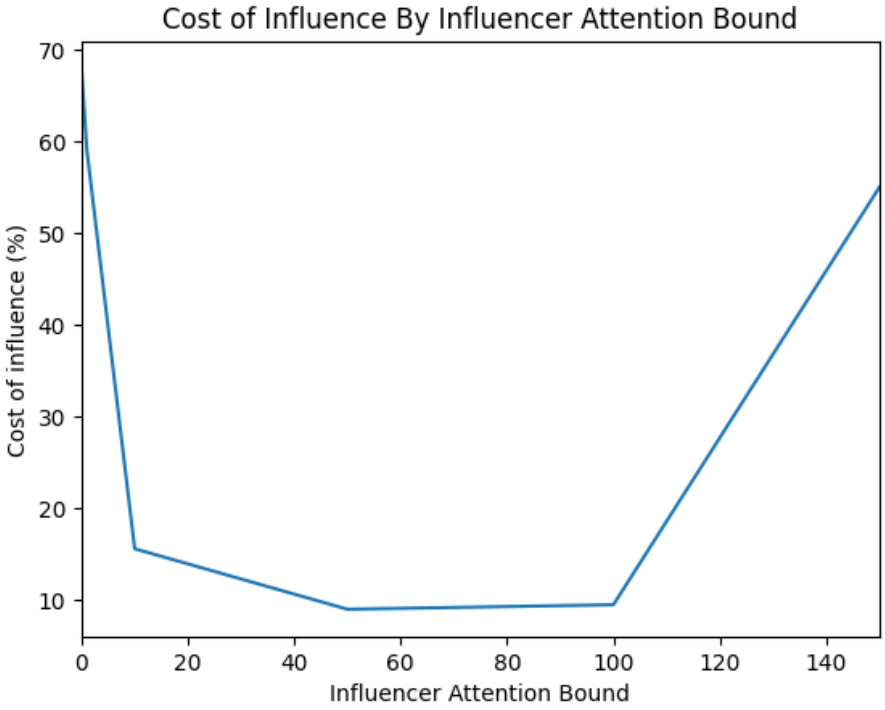

Figure 40: The cost of influence based on the influencer attention bound. We see that the curve peaks both near zero and for high values.

4.1.1 Relationship to other parameters

The immediate question from the general result presented above is what the 'Goldilocks' range is. From testing with varying other parameters in addition to the influencer attention bound, it is clear that this range is influenced by the other parameters included in this section's analysis. Recall from section 2.2.3 that a major effect of increasing the influencer attention bound is widening and flattening the influencer attention profile. The increased cost of influence comes when the influencer profile flattens. So, hyper-parameters whose impact is to narrow or steepen the influencer attention profile, such as the number of agents (see fig. 8), can shift or widen the range of influencer attention bounds that result in a lower cost of influence, an effect we can see to some extent in fig. 41. The same effect also holds to some extent with the agent topic interest functions, as seen in fig. 42.

A particularly interesting interaction between the influencer bound, another parameter, and the cost of influence is with the consumers' external interest. In fig. 43, we see clearly that increasing the consumer external interest diminishes the effect of a low influencer bound until it is unnoticeable compared to larger influencer bounds. What's happening here is that, as can be seen in fig. 44, when the influencer bound is near zero and external interest is high the consumers in both perfect and imperfect information devote almost all their attention towards external content. Since this pattern occurs in both market types, and since the cost of influence is determined only by the difference in total consumer utility, this means that a high external interest will lead to a low cost of influence even when the influencer bound is near zero.

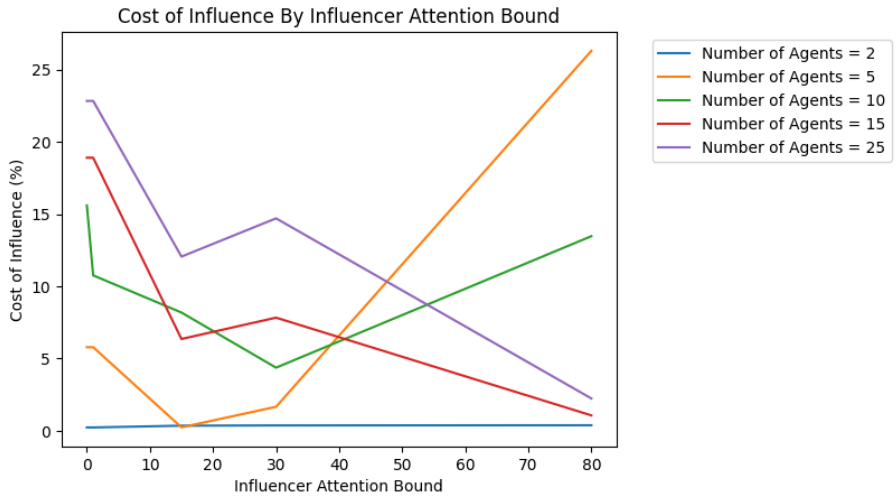

Figure 41: Cost of influence by influencer attention bound for several market sizes.

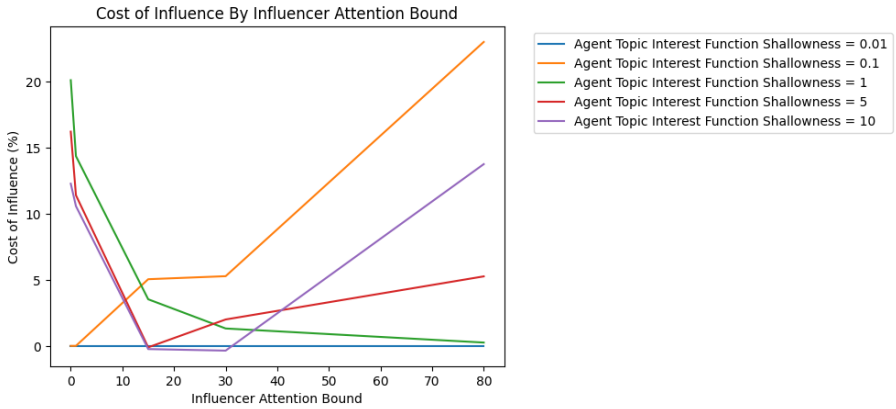

Figure 42: Cost of influence by influencer attention bound for several agent topic interest function slopes.

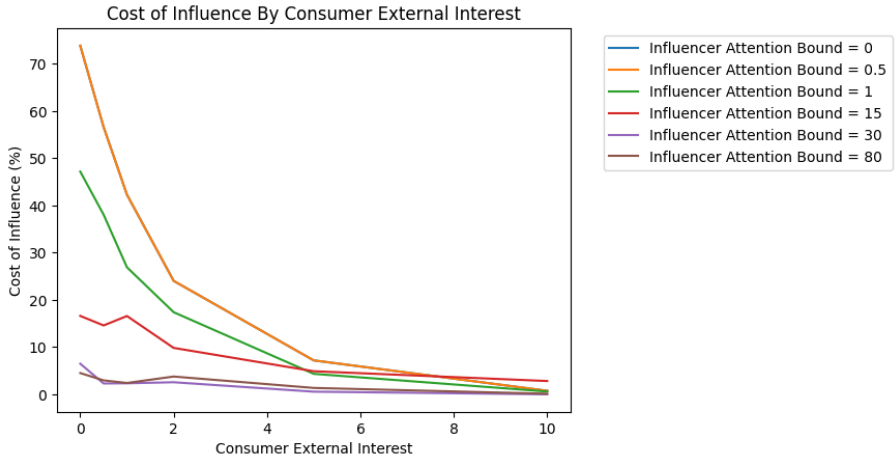

Figure 43: Cost of influence by consumer external interest, for several influencer attention bounds.

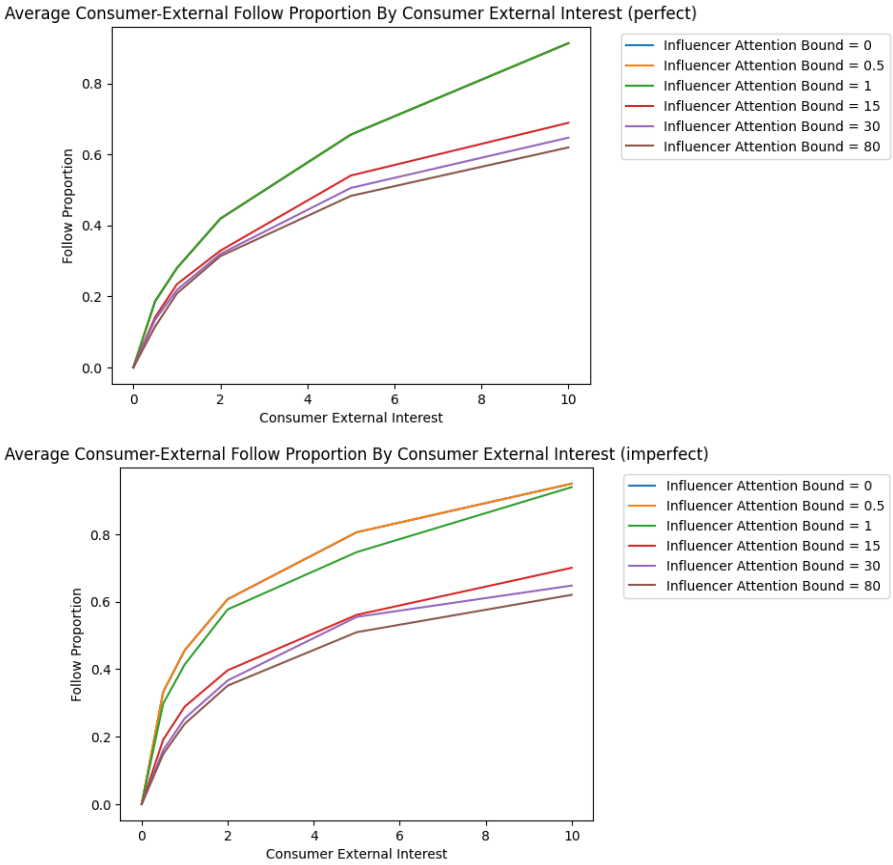

Figure 44: Consumer-External follow proportions by consumer external interest for several influencer bounds in perfect (top) and imperfect (bottom) information.

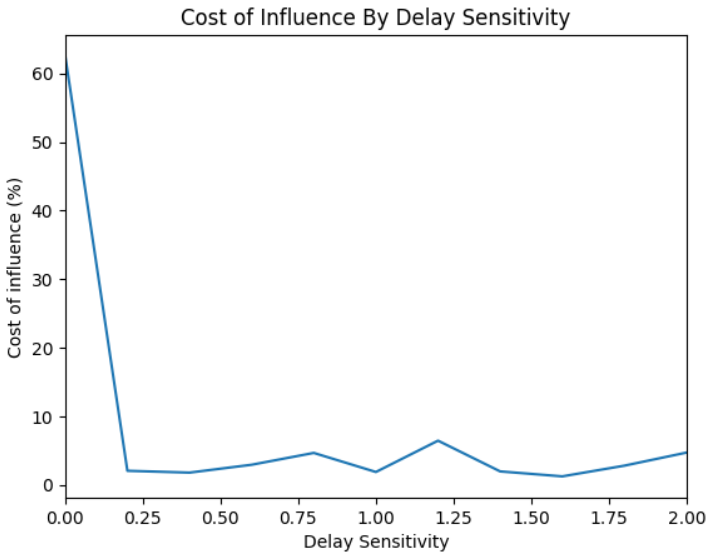

Figure 45: The cost of influence when varying consumer delay sensitivity

4.2 Delay Sensitivity

In fig. 45, we see that the consumer delay sensitivity approaching zero leads to a very high cost of influence, similar to when the influencer attention bound approached zero. This effect makes sense as we saw in fig. 23 that as delay sensitivity approaches zero, the influencer attention profile becomes increasingly flatter and wider. This means, then, that the imperfect information producers will not receive significantly different utility for producing different topics, so their optimization landscape will be mostly flat. Of course, the delay sensitivity impacts the perfect information producers as well, but since those producers receive both direct and indirect utility, their optimization landscape will not be as flat as the imperfect producers' and so they will still be able to optimize their topics produced.

5 Conclusion

This report has evaluated the empirical structural properties of the content market formalization. In so doing, we have found that in general, so long as the influencer attention bound and delay sensitivity are nonzero the cost of influence, the difference in total social welfare between perfect and imperfect information, is minimal. Within that bound, it is possible to further reduce the other structural differences (the following rates and topics produced), for instance by ensuring that the consumer topic interest function is not overly steep, keeping consumer external interest around 2, and keeping the consumer attention bound low relative to the influencer attention bound. In general, though, comparative evaluation of the perfect and imperfect information markets shows that the imperfect information market is very well able to approximate the perfect information market, and the same hyper-parameter impacts on the structural properties of the market found in perfect information apply to the imperfect information market too. In evaluating these hyper-parameter impacts, we found a wide variety of effects on the consumer and influencer following rates and the producer topics produced, as well as some indications of parameters with correlated effects.

The major remaining unresolved question from this report is why a continuous increase of the influencer attention bound can eventually lead to an increase in the cost of influence and structural differences between the perfect and imperfect information markets when the source paper suggested that as the influencer attention bound approaches infinity the perfect and imperfect total social welfare should converge to the same value. The fact that increasing the cost of influence leads to a flatter influencer attention profile does not seem surprising, but in imperfect information, it means that producers are unable to optimize their topics produced. It's possible, though, that the size of the market and the value of the consumer attention bound were not sufficiently high for the proposition in the paper to apply. Because of this possibility, future work with this formalization should focus on improving the efficiency of the optimization process so that tests featuring larger markets can be evaluated and more testing can be performed using global optimization algorithms.