

ECE341 Homework No. 7 Solution

Problem No. 1

The following tables show the predictions made by the two branch predictors for each branch instance:

(a) 1-bit branch predictor, which starts in the LNT state

Branch Instance	Current State	Prediction	Actual Outcome	Next State
1	LNT	NT	T	LT
2	LT	T	T	LT
3	LT	T	T	LT
4	LT	T	NT	LNT
5	LNT	NT	NT	LNT
6	LNT	NT	T	LT
7	LT	T	T	LT
8	LT	T	T	LT
9	LT	T	NT	LNT
10	LNT	NT	NT	LNT

Branch prediction accuracy = $6/10 = 60\%$

(b) 2-bit branch predictor, which starts in the SNT state

Branch Instance	Current State	Prediction	Actual Outcome	Next State
1	SNT	NT	T	LNT
2	LNT	NT	T	ST
3	ST	T	T	ST
4	ST	T	NT	LT
5	LT	T	NT	SNT
6	SNT	NT	T	LNT
7	LNT	NT	T	ST
8	ST	T	T	ST
9	ST	T	NT	LT
10	LT	T	NT	SNT

Branch prediction accuracy = $2/10 = 20\%$

Problem No. 2

Clock Rate “R” = 2.8 GHz = $2.8 * 10^9$ Hz

The processor incurs a stall of 1-cycle whenever a *Load* instruction is immediately followed by a dependent instruction.

Therefore: Stall frequency = %age of *load* instructions * %age of *load* instruction followed by a dependent instruction = $0.35 * 0.4 = 0.14$

Stall penalty = 1 cycle

δ = Stall frequency * Stall penalty = $0.14 * 1 = 0.14$

Throughput $P_p = R / (1 + \delta) = (2.8 * 10^9) / (1 + 0.14) = 2.456 * 10^9$ instructions per second

Problem No. 3

Percentage of branch instructions in the program = 20%

Percentage of branch instructions that are taken = 70%

For processor R_2 :

Since the branch predictor always predicts a “not-taken” branch, it will incur a penalty on branches that are “taken”. Thus the pipeline will have to stall on every “taken” branch.

$$\begin{aligned}\text{Stall frequency} &= \text{Percentage of branch instructions} * \text{Percentage of branches that are taken} \\ &= 0.2 * 0.7 = 0.14\end{aligned}$$

Stall penalty = 2 cycles (Branch outcome computed in stage-3)

$$\delta = \text{Stall frequency} * \text{Stall penalty} = 0.14 * 2 = 0.28$$

For processor R_3 :

The dynamic branch predictor incurs a penalty whenever it predicts a branch incorrectly; causing the pipeline to stall.

$$\begin{aligned}\text{Stall frequency} &= \text{Percentage of branch instructions} * \text{Percentage of incorrect branch predictions} \\ &= 0.2 * (1 - \text{Branch Prediction Accuracy})\end{aligned}$$

Stall penalty = 2 cycles (Branch outcome computed in stage-3)

$$\delta = \text{Stall frequency} * \text{Stall penalty} = 0.2 * (1 - \text{Branch Prediction Accuracy}) * 2 = 0.4 * (1 - \text{Branch Prediction Accuracy})$$

- (a) Since both R_2 and R_3 have the same clock rates, their throughputs would match, if both of them have the same number of stall cycles per instruction (δ):

$$0.4 * (1 - \text{Branch Prediction Accuracy}) = 0.28$$

$$1 - \text{Branch Prediction Accuracy} = 0.7$$

$$\text{Branch Prediction Accuracy} = 0.3 \text{ or } \mathbf{30\%}$$

- (b) Both the processors have the same clock rate “R”.

For processor R_2 :

$$\text{Throughput } P_p = R / (1 + \delta) = R / (1 + 0.28) = 0.781 R$$

For processor R_3 :

$$\delta = 0.4 * (1 - \text{Branch Prediction Accuracy}) = 0.4 * (1 - 0.9) = 0.04$$

$$\text{Throughput } P_p = R / (1 + \delta) = R / (1 + 0.04) = 0.962 R$$

$$\text{Speedup for } R_2 \text{ relative to } R_1 = 0.962R / 0.781R = \mathbf{1.231}$$

Problem No. 4

Clock Rate “R” = 3 GHz = $3 * 10^9$ Hz

No branch prediction is used.

The compiler is able to fill 80% of the branch delay slots with useful instructions. Therefore 80% of the branches will have no stall penalty.

The remaining 20% delay slots are filled with NOP instructions. Therefore, 20% of the branches will incur a stall penalty.

$$\begin{aligned}\text{Stall frequency} &= \text{\%age of branch instructions} * \text{\%age of delay slots filled with NOP instructions} \\ &= 0.2 * 0.2 = 0.04\end{aligned}$$

Since branch outcome is computed in the decode stage (stage-2):

Stall penalty = 1 cycle

$$\delta = \text{Stall frequency} * \text{Stall penalty} = 0.04 * 1 = 0.04$$

$$\text{Throughput } P_p = R / (1 + \delta) = (3 * 10^9) / (1 + 0.04) = \mathbf{2.885 * 10^9 \text{ instructions per second}}$$

Problem No. 5

$$\text{Clock Rate "R"} = 2.4 \text{ GHz} = 2.4 * 10^9 \text{ Hz}$$

We need to consider stall cycles due to both the cache misses and branch mispredictions.

For cache misses:

$$\text{Percentage of Load and Store instructions} = 20\% + 10\% = 30\%$$

$$\text{Percentage of instruction fetches that miss in the cache} = 1\%$$

$$\text{Percentage of data accesses that miss in the cache} = 2\%$$

$$\text{Cache miss penalty} = 10 \text{ cycles}$$

All the instructions need to access the cache for instruction fetches, whereas only the Load and Store instructions need to access the cache for data accesses.

Therefore:

$$\delta_{\text{miss}} = \text{stall frequency} * \text{stall penalty} = (0.01 + (0.3 * 0.02)) * 10 = 0.16$$

For branch mispredictions:

$$\text{Percentage of branch instructions} = 20\%$$

Since branch computation is carried out in "Decode" stage (stage-2):

$$\text{Stall penalty} = 1 \text{ cycle}$$

Assume that the branch prediction accuracy is represented by "A", then:

$$\text{Stall frequency} = \text{\%age of branch instructions} * \text{\%age of branches which are mispredicted} = (0.2)(1 - A)$$

Therefore:

$$\delta_{\text{branch_penalty}} = \text{stall frequency} * \text{stall penalty} = (0.2)(1 - A) * 1 = 0.2 - 0.2A$$

Adding the two different sources of stall cycles:

$$\text{Total stall cycles per instruction } \delta = \delta_{\text{miss}} + \delta_{\text{branch_penalty}} = 0.16 + 0.2 - 0.2A = 0.36 - 0.2A$$

Throughput "P_p" is given by:

$$P_p = R / (1 + \delta) = (2.4 * 10^9) / (1 + 0.36 - 0.2A) = (2.4 * 10^9) / (1.36 - 0.2A)$$

Since the required throughput is 2 billion instructions per second:

$$2 * 10^9 = (2.4 * 10^9) / (1.36 - 0.2A)$$

$$1.36 - 0.2A = 2.4 / 2 = 1.2$$

$$A = (1.36 - 1.2) / 0.2 = 0.8$$

Therefore, the required minimum branch prediction accuracy is **80%**