

# **ECE 341**

## **Lecture # 17**

**Instructor: Zeshan Chishti**  
**zeshan@ece.pdx.edu**

**November 26, 2014**

**Portland State University**

# Lecture Topics

- The Memory System
  - Cache Memories
    - Cache Hits and Misses
    - Mapping Functions
      - Direct Mapping
      - Set-Associative Mapping
      - Fully-Associative Mapping
    - Replacement Algorithms
- Reference:
  - Chapter 8: Sections 8.6 and 8.7

# Cache Hit

- When the processor needs to access some data, that data may or may not be found in the cache
- If the data is found in the cache, it is called a [cache hit](#)
- Read hit:
  - The processor reads data from cache and does not need to go to memory
- Write hit:
  - Cache has a replica of the contents of main memory, both cache and main memory need to be updated. Two options:
    - Update the contents of cache and main memory simultaneously. This is called [write-through](#) protocol
    - Update the contents of cache, and mark the cache block as updated by setting a bit known as [dirty](#) or [modified](#) bit. The main memory contents are updated when this block is evicted from cache. This is called [write-back](#) protocol

# Cache Miss

- If the data is not found in the cache, it is called a cache miss
- Read miss:
  - The block containing the data is transferred from memory to cache
  - After the block is transferred, the desired data is forwarded to the processor.
  - The desired data may also be forwarded to the processor as soon as it is transferred without waiting for the entire cache block to be transferred. This is called [load-through](#) or [critical word first](#)
- Write-miss:
  - If *write-through* is used, then main memory contents are updated directly
  - If *write-back* is used, then the block containing the addressed word is first brought into the cache. The desired word is overwritten with new data

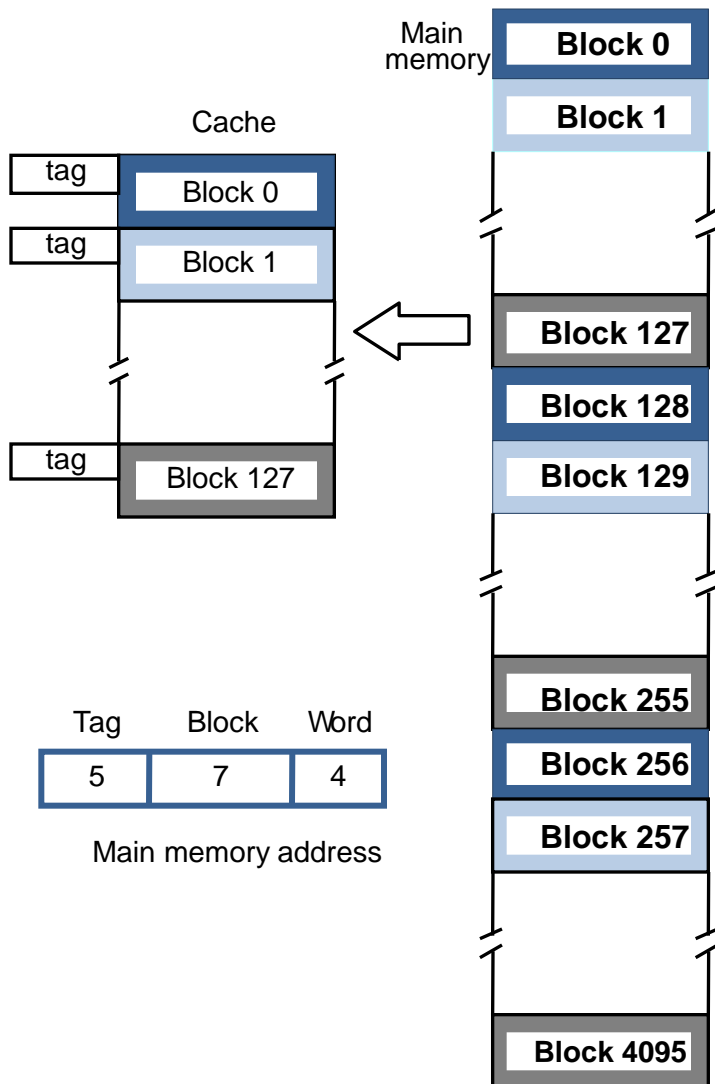
# Cache Mapping and Replacement Functions

- Because a cache is smaller than main memory, only a subset of blocks in main memory can be held in the cache at a given time
- Mapping function:
  - The correspondence between main memory blocks and blocks in the cache is specified by a *mapping function*
  - It determines which memory addresses map to a given cache block
- Replacement Algorithm:
  - When cache is full, and a new block needs to be brought into the cache, another block needs to be removed from cache to create room for new block
  - The set of rules used to decide which block to remove constitutes a replacement algorithm

# Mapping Functions

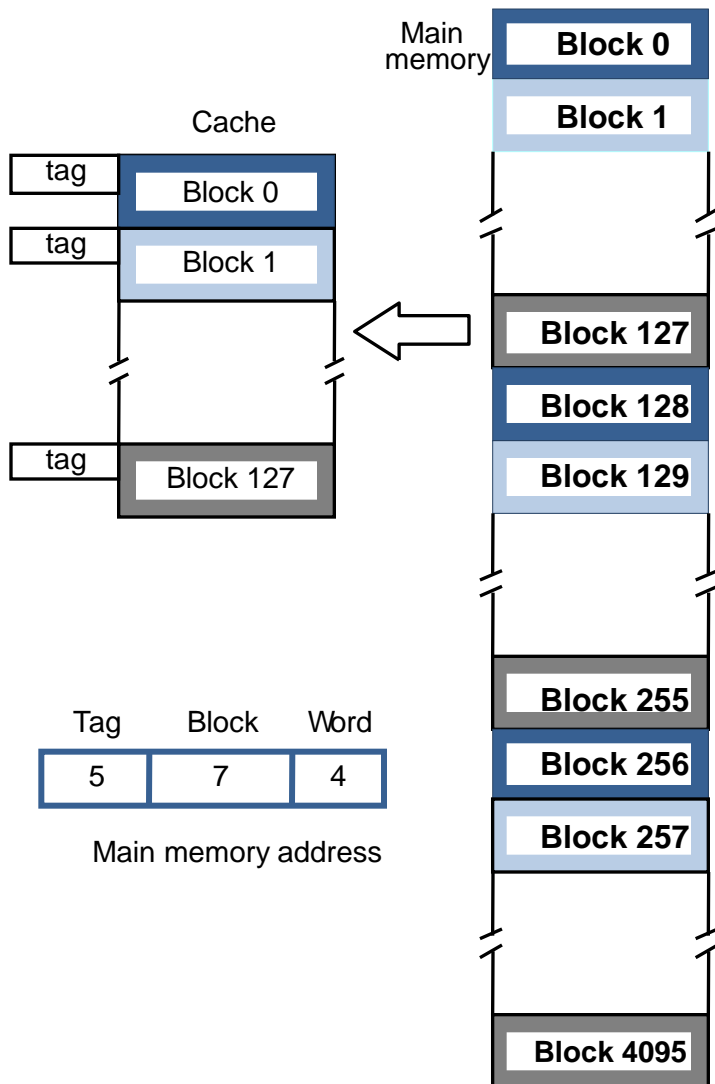
- Mapping function determines how memory blocks are placed in the cache
- A simple example:
  - Consider a cache consisting of 128 blocks of 16 words each
  - Total size of cache =  $128 * 16 = 2048$  or 2K words
  - Assume that main memory is addressable by a 16-bit address
  - Size of main memory =  $2^{16} = 64K$  words
  - Main memory has 4K blocks of 16 words each
- Three mapping functions:
  - Direct mapping
  - Set-Associative mapping
  - Fully-associative mapping.

# Direct Mapping



- Block  $j$  of main memory maps to block  $j \text{ modulo } 128$  of the cache
  - e.g., memory blocks 0, 128, 256,..... map to cache block 0, memory blocks 1, 129, 257,..... map to cache block 1
- Direct mapping allows only one of the memory blocks mapped to a cache block to be present in the cache at any time
- Even if cache is not full, there may be contention for cache blocks
  - e.g., what happens if memory block 0 and 128 are accessed by the processor?
- Direct mapping resolves contention by allowing new block to replace the old block,

# Direct Mapping (cont.)



- Memory address divided into 3 fields:
  - High order 5 bits determine which of the possible 32 memory blocks is currently mapped to the cache block. These bits are called “**tag**” bits
  - The next 7 bits determine the number of cache block
  - Low order 4 bits determine one of the 16 words in a block
- Simple to implement but not very flexible
  - What happens if memory blocks 0 and 128 are repeatedly accessed by the processor in an alternating fashion?



# Example: Direct-Mapped Cache

## Problem Statement:

A computer system uses **16-bit** memory addresses. It has a **512-byte** cache organized in a direct-mapped manner with **64 bytes** per cache block. Assume that the size of each memory word is 1 byte. The processor reads data from the following word addresses: **64, 128, 80, 576, 64**

For each address, indicate whether the cache access will result in a hit or a miss.

## Solution:

Block size = 64 bytes =  $2^6$  bytes =  $2^6$  words (since 1 word = 1 byte)

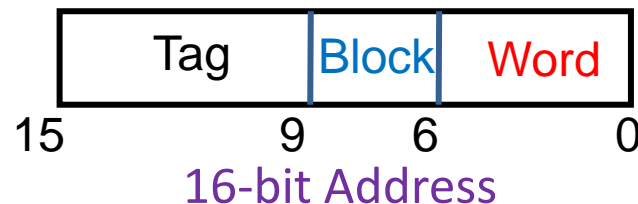
Therefore, **Number of bits in the *Word* field = 6**

Cache size = 512-bytes =  $2^9$   $\Rightarrow$  No. of cache blocks = Cache size / Block size =  $2^9/2^6 = 2^3$

Therefore, **Number of bits in the *Block* field = 3**

Total number of address bits = 16

Therefore, **Number of bits in the *Tag* field =  $16 - 6 - 3 = 7$**



# Example: Direct-Mapped Cache

Solution (cont.):

## Access # 1:

Address = (**64**)<sub>10</sub> = (00000000**001**000000)<sub>2</sub>

Tag = 0000000, Block = **001**, Word = **000000**

Since the cache is empty before this access, this will be a cache **miss**

After this access, Tag field for cache block **001** is set to **0000000**

| Block | Tag     |
|-------|---------|
| 000   | x       |
| 001   | 0000000 |
| 010   | x       |
| 011   | x       |
| 100   | x       |
| 101   | x       |
| 110   | x       |
| 111   | x       |

“x”  
denotes  
empty or  
invalid

# Example: Direct-Mapped Cache

Solution (cont.):

## Access # 1:

Address = (**64**)<sub>10</sub> = (00000000**001**000000)<sub>2</sub>

Tag = 0000000, Block = **001**, Word = **000000**

Since the cache is empty before this access, this will be a cache **miss**

After this access, Tag field for cache block **001** is set to **0000000**

## Access # 2:

Address = (**128**)<sub>10</sub> = (00000000**010**000000)<sub>2</sub>

Tag = 0000000, Block = **010**, Word = **000000**

Block 010 is empty before this access, this will be a cache **miss**

| Block | Tag     |
|-------|---------|
| 000   | x       |
| 001   | 0000000 |
| 010   | x       |
| 011   | x       |
| 100   | x       |
| 101   | x       |
| 110   | x       |
| 111   | x       |

“x”  
denotes  
empty or  
invalid

# Example: Direct-Mapped Cache

Solution (cont.):

## Access # 1:

Address = (**64**)<sub>10</sub> = (00000000**001****000000**)<sub>2</sub>

Tag = 0000000, Block = **001**, Word = **000000**

Since the cache is empty before this access, this will be a cache **miss**

After this access, Tag field for cache block **001** is set to **0000000**

## Access # 2:

Address = (**128**)<sub>10</sub> = (00000000**010****000000**)<sub>2</sub>

Tag = 0000000, Block = **010**, Word = **000000**

Block 010 is empty before this access, this will be a cache **miss**

After this access, Tag field for cache block **010** is set to **0000000**

| Block | Tag     |
|-------|---------|
| 000   | x       |
| 001   | 0000000 |
| 010   | x       |
| 011   | x       |
| 100   | x       |
| 101   | x       |
| 110   | x       |
| 111   | x       |

“x”  
denotes  
empty or  
invalid

|     |         |
|-----|---------|
| 000 | x       |
| 001 | 0000000 |
| 010 | 0000000 |
| 011 | x       |
| 100 | x       |
| 101 | x       |
| 110 | x       |
| 111 | x       |

# Example: Direct-Mapped Cache

Solution (cont.):

**Access # 3:**

Address = (**80**)<sub>10</sub> = (00000000**001010000**)<sub>2</sub>

Tag = 0000000, Block = **001**, Word = **010000**

Tag for block 001 (0000000) matches address tag

=> this will be a cache **hit**

| Block | Tag     |
|-------|---------|
| 000   | x       |
| 001   | 0000000 |
| 010   | 0000000 |
| 011   | x       |
| 100   | x       |
| 101   | x       |
| 110   | x       |
| 111   | x       |

“x”  
denotes  
empty or  
invalid

# Example: Direct-Mapped Cache

Solution (cont.):

## Access # 3:

Address = (**80**)<sub>10</sub> = (00000000**001010000**)<sub>2</sub>

Tag = 0000000, Block = **001**, Word = **010000**

Tag for block 001 (0000000) matches address tag

=> this will be a cache **hit**

## Access # 4:

Address = (**576**)<sub>10</sub> = (00000001**001000000**)<sub>2</sub>

Tag = 0000001, Block = **001**, Word = **000000**

Tag for block 001 (0000000) does not match the address tag (0000001) => this will be a cache **miss**.

| Block | Tag     |
|-------|---------|
| 000   | x       |
| 001   | 0000000 |
| 010   | 0000000 |
| 011   | x       |
| 100   | x       |
| 101   | x       |
| 110   | x       |
| 111   | x       |

“x”  
denotes  
empty or  
invalid

# Example: Direct-Mapped Cache

Solution (cont.):

## Access # 3:

Address = (**80**)<sub>10</sub> = (00000000**001010000**)<sub>2</sub>

Tag = 0000000, Block = **001**, Word = **010000**

Tag for block 001 (0000000) matches address tag

=> this will be a cache **hit**

## Access # 4:

Address = (**576**)<sub>10</sub> = (00000001**001000000**)<sub>2</sub>

Tag = 0000001, Block = **001**, Word = **000000**

Tag for block 001 (0000000) does not match the address tag (0000001) => this will be a cache **miss**.

After this access, Tag field for cache block **001** is set to **0000001**

| Block | Tag     |
|-------|---------|
| 000   | x       |
| 001   | 0000000 |
| 010   | 0000000 |
| 011   | x       |
| 100   | x       |
| 101   | x       |
| 110   | x       |
| 111   | x       |

“x”  
denotes  
empty or  
invalid

|     |         |
|-----|---------|
| 000 | x       |
| 001 | 0000001 |
| 010 | 0000000 |
| 011 | x       |
| 100 | x       |
| 101 | x       |
| 110 | x       |
| 111 | x       |

# Example: Direct-Mapped Cache

Solution (cont.):

## Access # 5:

Address =  $(64)_{10} = (00000000001000000)_2$

Tag = 0000000, Block = 001, Word = 000000

Tag for block 001 (0000001) does not match the address tag (0000000) => this will be a cache **miss**.

Even though address 64 was previously loaded into the cache, this access is a miss due to the conflict between address 64 and 576 (both addresses map to the same block)

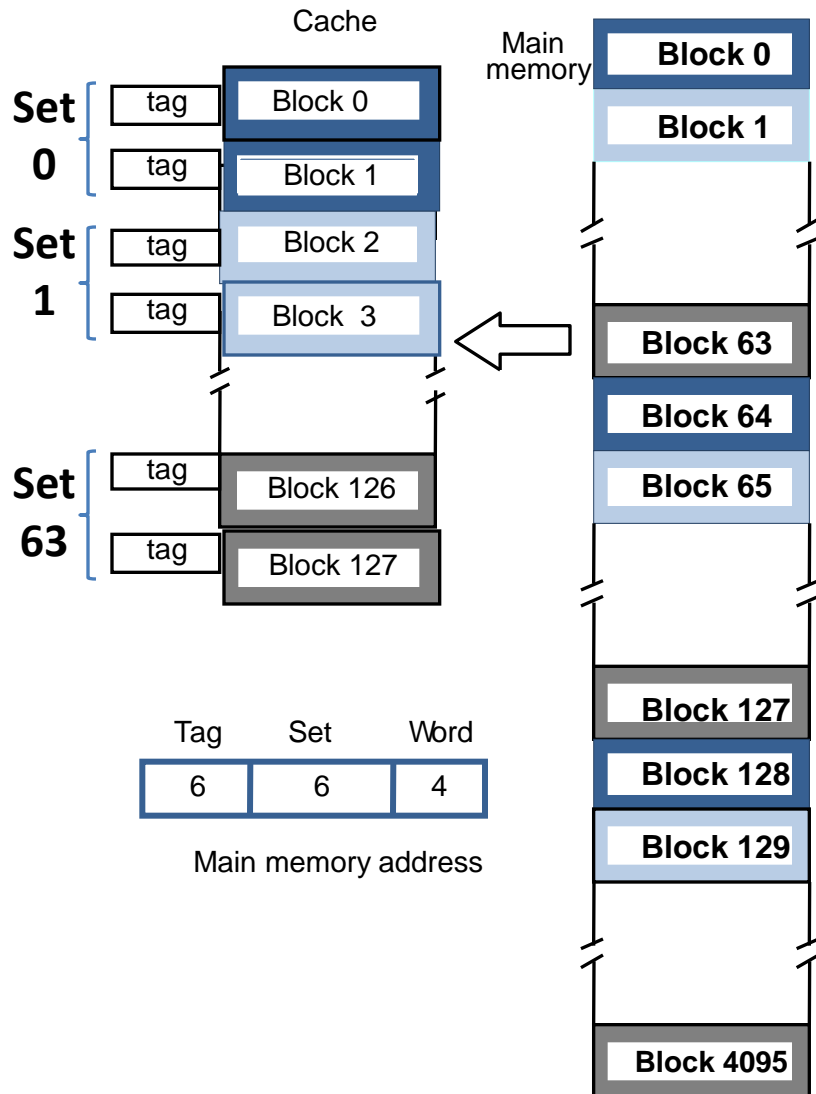
After this access, Tag field for cache block 001 is set to 0000000

| Block | Tag     |
|-------|---------|
| 000   | x       |
| 001   | 0000000 |
| 010   | 0000000 |
| 011   | x       |
| 100   | x       |
| 101   | x       |
| 110   | x       |
| 111   | x       |

“x”  
denotes  
empty or  
invalid

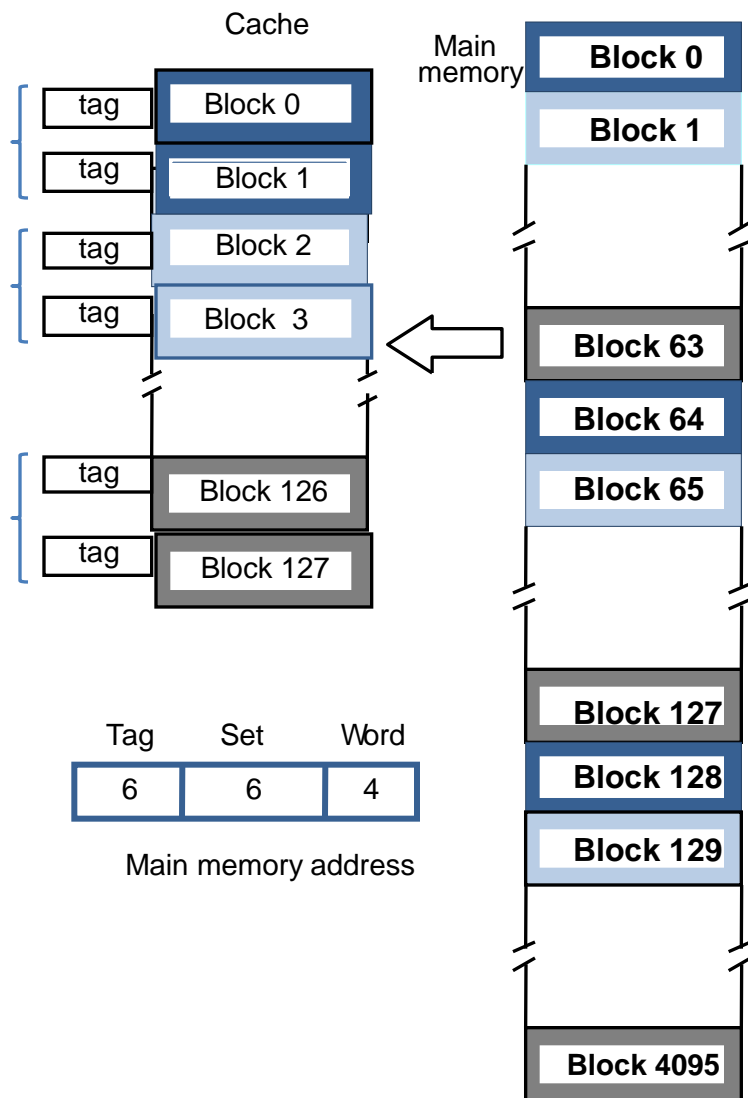


# Set-Associative Mapping



- Cache blocks are grouped into **sets**
- The number of blocks per set is called **associativity** of the cache
- Set-Associative mapping allows a block of main memory to be mapped to any block of a specific set
- Example (2-way set-associative cache):
  - Divide the cache into 64 sets, with two blocks per set
  - Memory block 0, 64, 128, ..... etc. map to set 0, and they can occupy either of the two cache blocks in set 0
  - Memory block 1, 65, 129, ..... etc. map to set 1, and they can occupy either of the two cache blocks in set 1

# Set-Associative Mapping (cont.)



- Memory address is divided into 3 fields:
  - High order 6 bits determine the **tag** field
  - Next 6 bits determine the set number
  - Low order 4 bits determine one of the 16 words in a block
- Cache associativity (number of blocks per cache set) is a design parameter
  - One extreme is to have one block per set, the same as direct mapping
  - Other extreme is to have all the blocks in one set, requiring no set bits (fully associative mapping)

# Example: Set-Associative Cache

## Problem Statement:

Repeat the previous problem, assuming that the cache is organized as a 2-way set-associative cache.

## Solution:

Block size = 64 bytes =  $2^6$  bytes =  $2^6$  words (since 1 word = 1 byte)

Therefore, **Number of bits in the *Word* field = 6**

Cache size = 512-bytes =  $2^9$  bytes

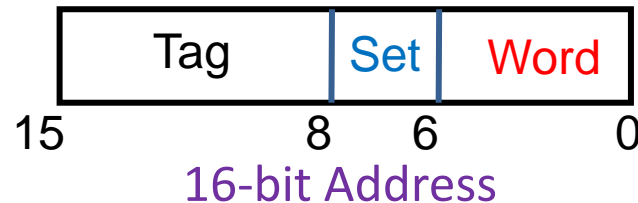
No. of cache blocks per set = 2 (2-way set-associative)

No. of sets = Cache size / (Block size \* Blocks per set) =  $2^9 / (2^6 * 2) = 2^2 = 4$

Therefore, **Number of bits in the *Set* field = 2**

Total number of address bits = 16

Therefore, **Number of bits in the *Tag* field = 16 - 6 - 2 = 8**



# Example: Set-Associative Cache

Solution (cont.):

**Access # 1:**

Address = (**64**)<sub>10</sub> = (000000000**01**000000)<sub>2</sub>

Tag = 00000000, Set = **01**, Word = 000000

Since the cache is empty before this access, this will be a cache **miss**

After this access, Tag field for way-**0** in set **01** is set to 00000000

| Set | Way | Tag      |
|-----|-----|----------|
| 00  | 0   | x        |
|     | 1   | x        |
| 01  | 0   | 00000000 |
|     | 1   | x        |
| 10  | 0   | x        |
|     | 1   | x        |
| 11  | 0   | x        |
|     | 1   | x        |

# Example: Set-Associative Cache

Solution (cont.):

## Access # 1:

Address = (**64**)<sub>10</sub> = (00000000**01**000000)<sub>2</sub>

Tag = 00000000, Set = **01**, Word = **000000**

Since the cache is empty before this access, this will be a cache **miss**

After this access, Tag field for way-**0** in set **01** is set to **00000000**

| Set | Way | Tag      |
|-----|-----|----------|
| 00  | 0   | x        |
|     | 1   | x        |
| 01  | 0   | 00000000 |
|     | 1   | x        |
| 10  | 0   | x        |
|     | 1   | x        |
| 11  | 0   | x        |
|     | 1   | x        |

## Access # 2:

Address = (**128**)<sub>10</sub> = (00000000**10**000000)<sub>2</sub>

Tag = 00000000, Set = **10**, Word = **000000**

Set 10 is empty before this access, this will be a cache **miss**

# Example: Set-Associative Cache

Solution (cont.):

## Access # 1:

Address = (**64**)<sub>10</sub> = (00000000**01**000000)<sub>2</sub>

Tag = 00000000, Set = **01**, Word = **000000**

Since the cache is empty before this access, this will be a cache **miss**

After this access, Tag field for way-**0** in set **01** is set to **00000000**

| Set | Way | Tag      |
|-----|-----|----------|
| 00  | 0   | x        |
|     | 1   | x        |
| 01  | 0   | 00000000 |
|     | 1   | x        |
| 10  | 0   | x        |
|     | 1   | x        |
| 11  | 0   | x        |
|     | 1   | x        |

## Access # 2:

Address = (**128**)<sub>10</sub> = (00000000**10**000000)<sub>2</sub>

Tag = 00000000, Set = **10**, Word = **000000**

Set 10 is empty before this access, this will be a cache **miss**

After this access, Tag field for way-**0** in set **10** is set to **00000000**

|    |   |          |
|----|---|----------|
| 00 | 0 | x        |
|    | 1 | x        |
| 01 | 0 | 00000000 |
|    | 1 | x        |
| 10 | 0 | 00000000 |
|    | 1 | x        |
| 11 | 0 | x        |
|    | 1 | x        |

# Example: Set-Associative Cache

Solution (cont.):

**Access # 3:**

Address = (**80**)<sub>10</sub> = (00000000**01010000**)<sub>2</sub>

Tag = 00000000, Set = **01**, Word = **010000**

Tag for way-0 in set 01 (00000000) matches  
address tag => this will be a cache **hit**

| Set | Way | Tag      |
|-----|-----|----------|
| 00  | 0   | x        |
|     | 1   | x        |
| 01  | 0   | 00000000 |
|     | 1   | x        |
| 10  | 0   | 00000000 |
|     | 1   | x        |
| 11  | 0   | x        |
|     | 1   | x        |

# Example: Set-Associative Cache

Solution (cont.):

## Access # 3:

Address = (**80**)<sub>10</sub> = (00000000**01010000**)<sub>2</sub>

Tag = 00000000, Set = **01**, Word = **010000**

Tag for way-0 in set 01 (00000000) matches address tag => this will be a cache **hit**

| Set | Way | Tag      |
|-----|-----|----------|
| 00  | 0   | x        |
|     | 1   | x        |
| 01  | 0   | 00000000 |
|     | 1   | x        |
| 10  | 0   | 00000000 |
|     | 1   | x        |
| 11  | 0   | x        |
|     | 1   | x        |

## Access # 4:

Address = (**576**)<sub>10</sub> = (00000010**01000000**)<sub>2</sub>

Tag = 00000010, Set = **01**, Word = **000000**

Neither of the tags in set 01 match the address tag (00000010) => this will be a cache **miss**.



# Example: Set-Associative Cache

Solution (cont.):

## Access # 3:

Address = (**80**)<sub>10</sub> = (00000000**01010000**)<sub>2</sub>

Tag = 00000000, Set = **01**, Word = **010000**

Tag for way-0 in set 01 (00000000) matches address tag => this will be a cache **hit**

| Set | Way | Tag      |
|-----|-----|----------|
| 00  | 0   | x        |
|     | 1   | x        |
| 01  | 0   | 00000000 |
|     | 1   | x        |
| 10  | 0   | 00000000 |
|     | 1   | x        |
| 11  | 0   | x        |
|     | 1   | x        |

## Access # 4:

Address = (**576**)<sub>10</sub> = (00000010**01000000**)<sub>2</sub>

Tag = 00000010, Set = **01**, Word = **000000**

Neither of the tags in set 01 match the address tag (00000010) => this will be a cache **miss**.

After this access, **Tag field for way-1 in set 01 is set to 00000010**

|    |   |          |
|----|---|----------|
| 00 | 0 | x        |
|    | 1 | x        |
| 01 | 0 | 00000000 |
|    | 1 | 00000010 |
| 10 | 0 | 00000000 |
|    | 1 | x        |
| 11 | 0 | x        |
|    | 1 | x        |

# Example: Set-Associative Cache

Solution (cont.):

## **Access # 5:**

Address = (**64**)<sub>10</sub> = (00000000**01**000000)<sub>2</sub>

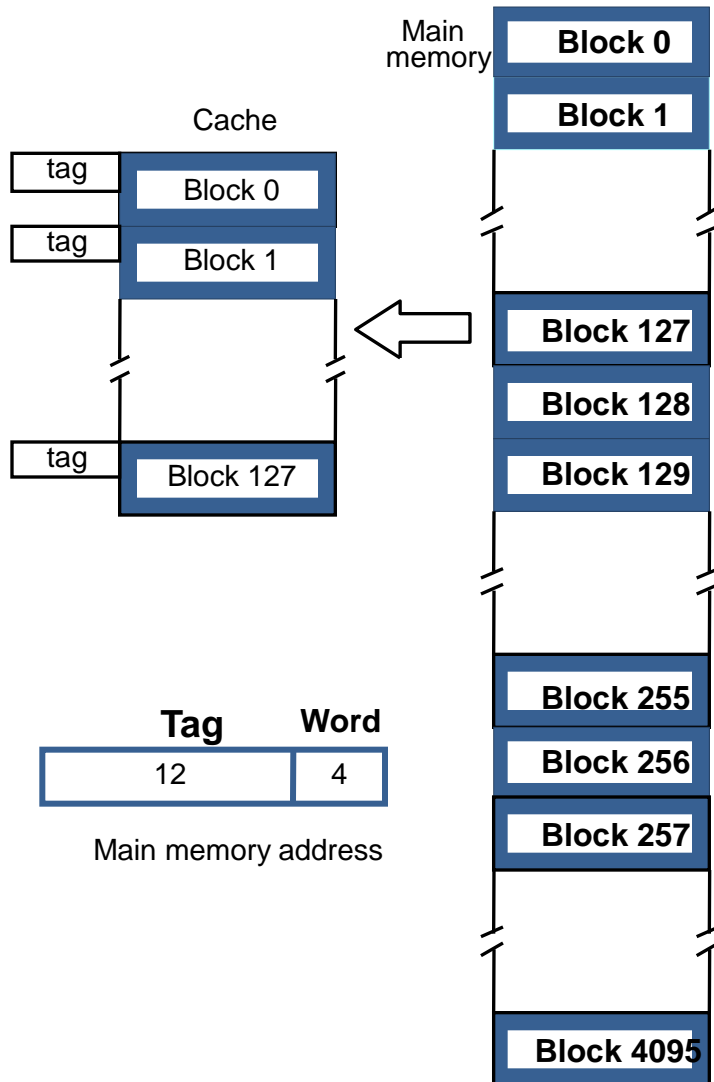
Tag = 00000000, Set = **01**, Word = **000000**

Tag for way-0 in set 01 (00000000) matches  
address tag => this will be a cache **hit**

By using a set-associative cache (as opposed to a direct-mapped cache), this access was converted from a miss to a hit.

| Set | Way | Tag      |
|-----|-----|----------|
| 00  | 0   | x        |
|     | 1   | x        |
| 01  | 0   | 00000000 |
|     | 1   | 00000010 |
| 10  | 0   | 00000000 |
|     | 1   | x        |
| 11  | 0   | x        |
|     | 1   | x        |

# Fully Associative Mapping



- Main memory block can be mapped into **any** cache block
- Memory address is divided into two fields:
  - High order 12 bits or tag bits identify which one of the 4096 memory blocks is mapped to a given cache block
  - Low order 4 bits determine of the 16 words within a block
- Complete freedom in choosing the cache location in which to place a memory block
  - Flexible, and uses cache space efficiently
- Higher complexity than direct-mapped cache because of the need to search all 128 tags to determine whether a given block is in the cache. This is called **associative search**

# Replacement Algorithm

- When the cache is full and some new data needs to be brought into the cache, some existing cache block needs to be replaced
- Replacement algorithm decides which block to choose among all candidates
- For direct-mapped cache:
  - New block can be placed in only one cache location
  - Replacement algorithm is trivial, since there is only 1 replacement candidate
- For set-associative cache:
  - New block can be placed in any of the multiple ways in the set
  - Number of replacement candidates is equal to the associativity of the cache
  - Replacement algorithm should choose the block, which is least likely to be accessed in future
  - **Least Recently Used (LRU) Replacement Algorithm**
    - High probability that blocks accessed recently will be accessed soon
    - Replace the block which has gone the longest time without being accessed