TRUONG DAI HOC KHOA HOC TU NHIEN
KHTN
TP. HO CHI MINH

Integers

① Overflow

② Half / full adder

③ Signed / unsigned integers

Perform the following number conversions:

A. 0x39A7F8 to binary
   0011 1001 1010 0111 1111 1000
B. Binary 1100 1001 0111 1011 to hexadecimal
   C 9 7 B
C. 0xD5E4C to binary
   1101 0101 1110 0100 1100
D. Binary 10 0110 1110 0111 1011 0101 to hexadecimal
   2 6 E 7 B 5

Pentium FDIV bug: $475 million

$$\frac{4{,}195{,}835}{3{,}145{,}727} = 1.333739068902037589$$

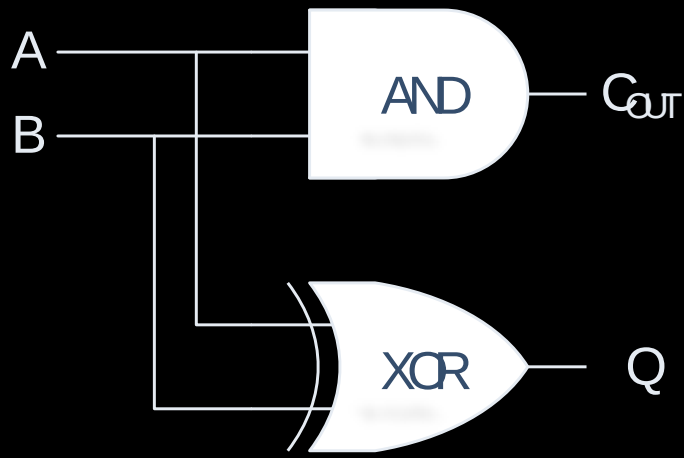
Ariane 5 Overflow bug: $7 billion

Half Adder

0	+	0	=	00
0	+	1	=	01
1	+	0	=	01
1	+	1	=	10

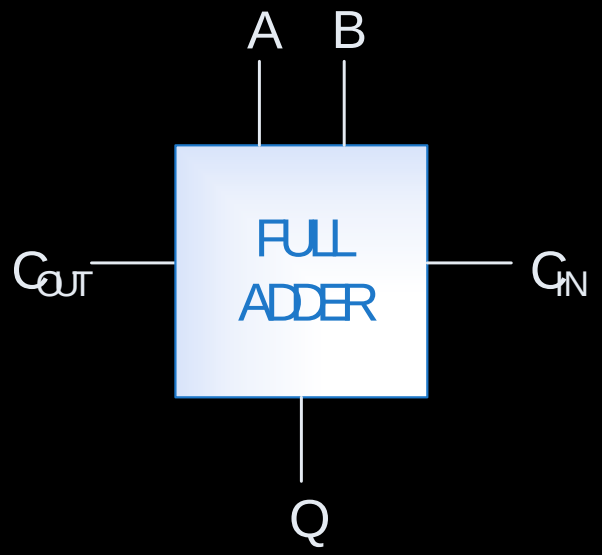
A	B	Q	C_{OUT}
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

XOR AND

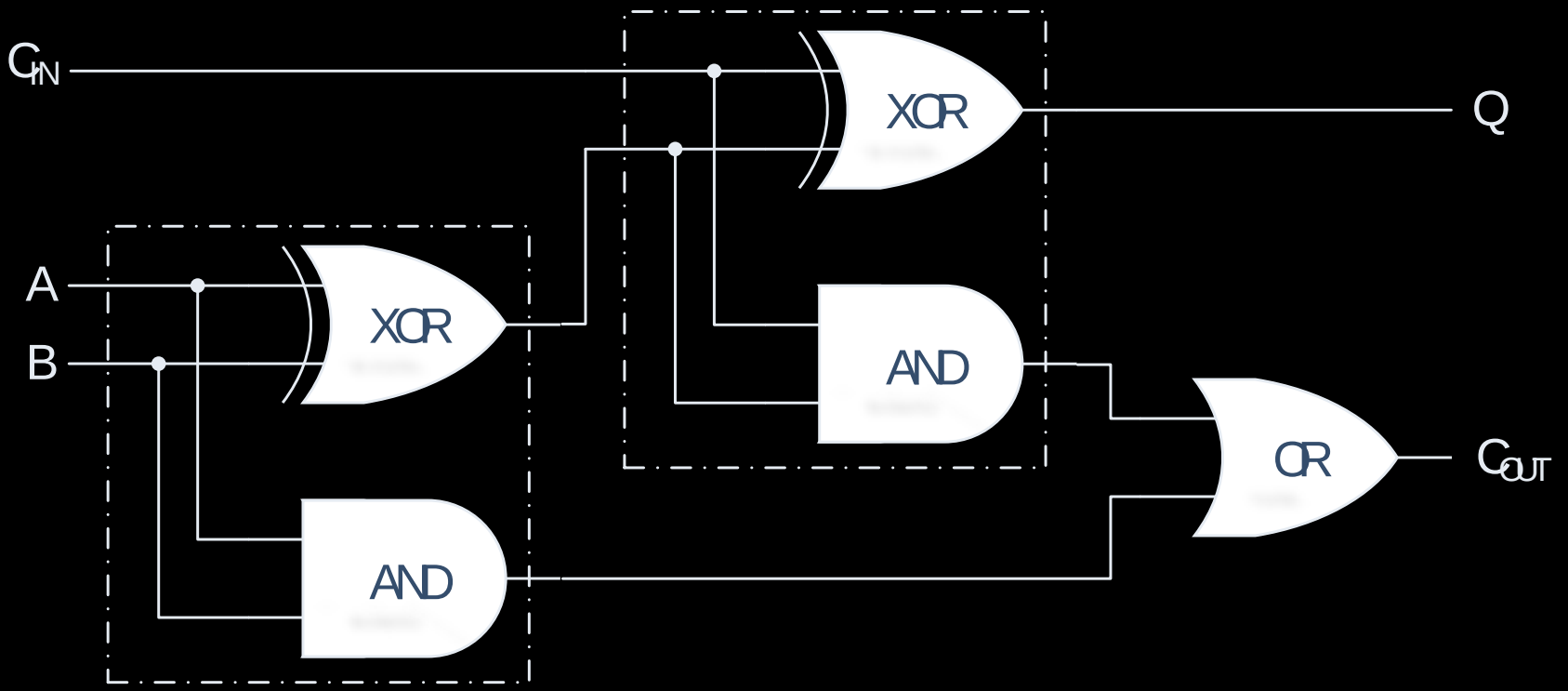


Full Adder

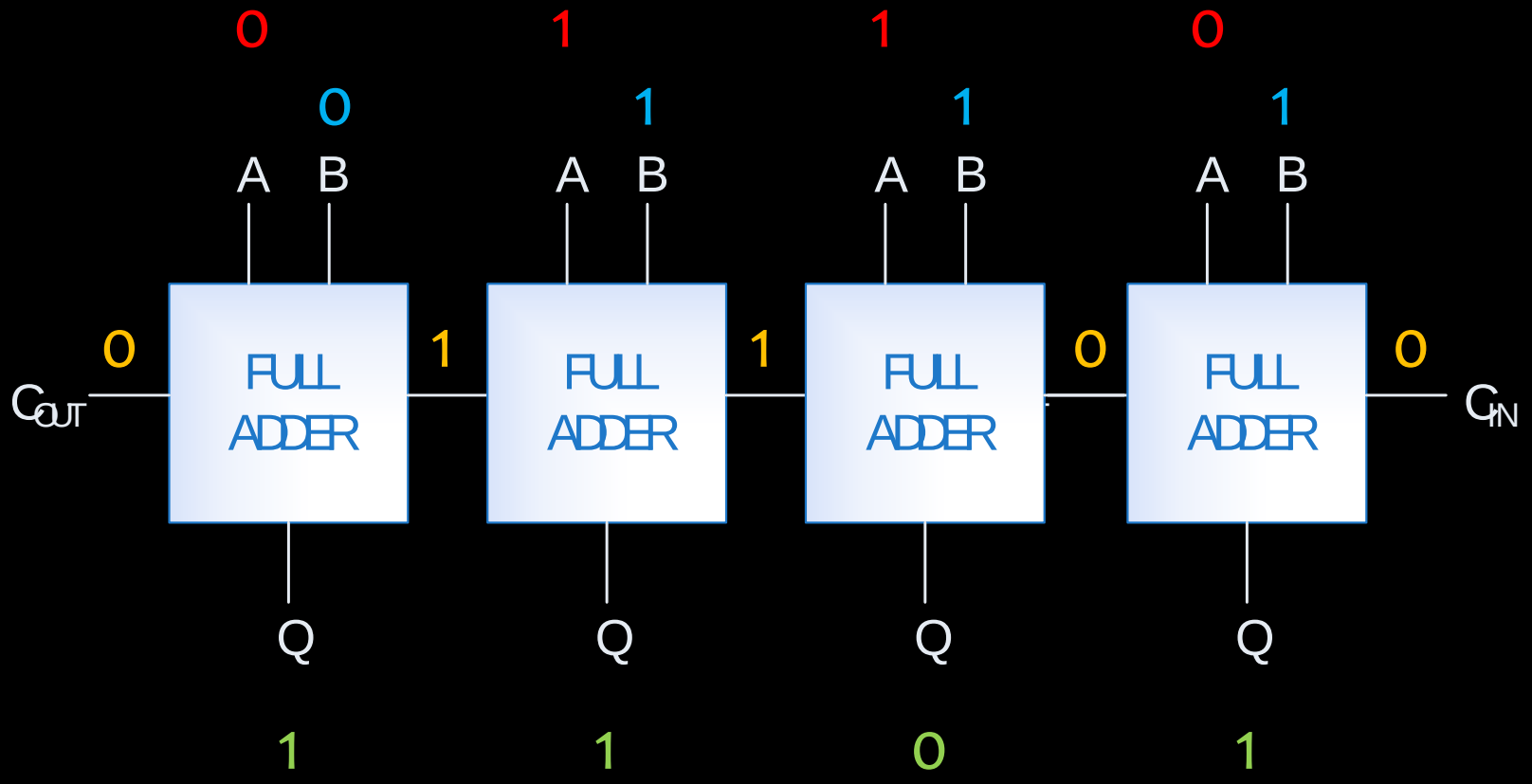
C_{IN}	A	B	Q	C_{OUT}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1



Full Adder



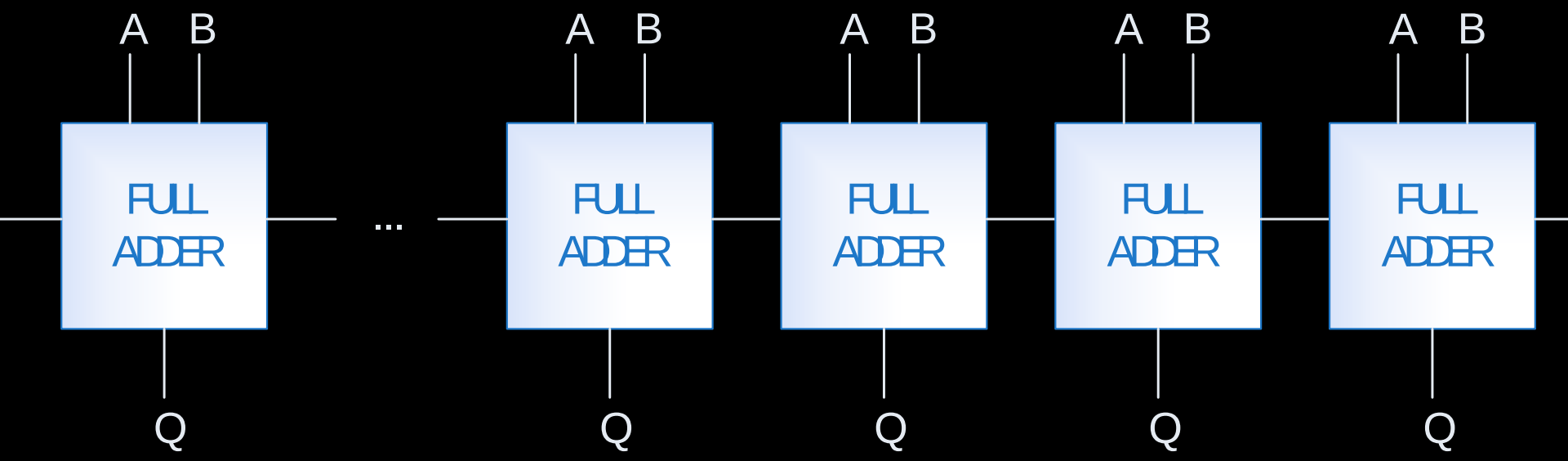
N-bit Full Adder



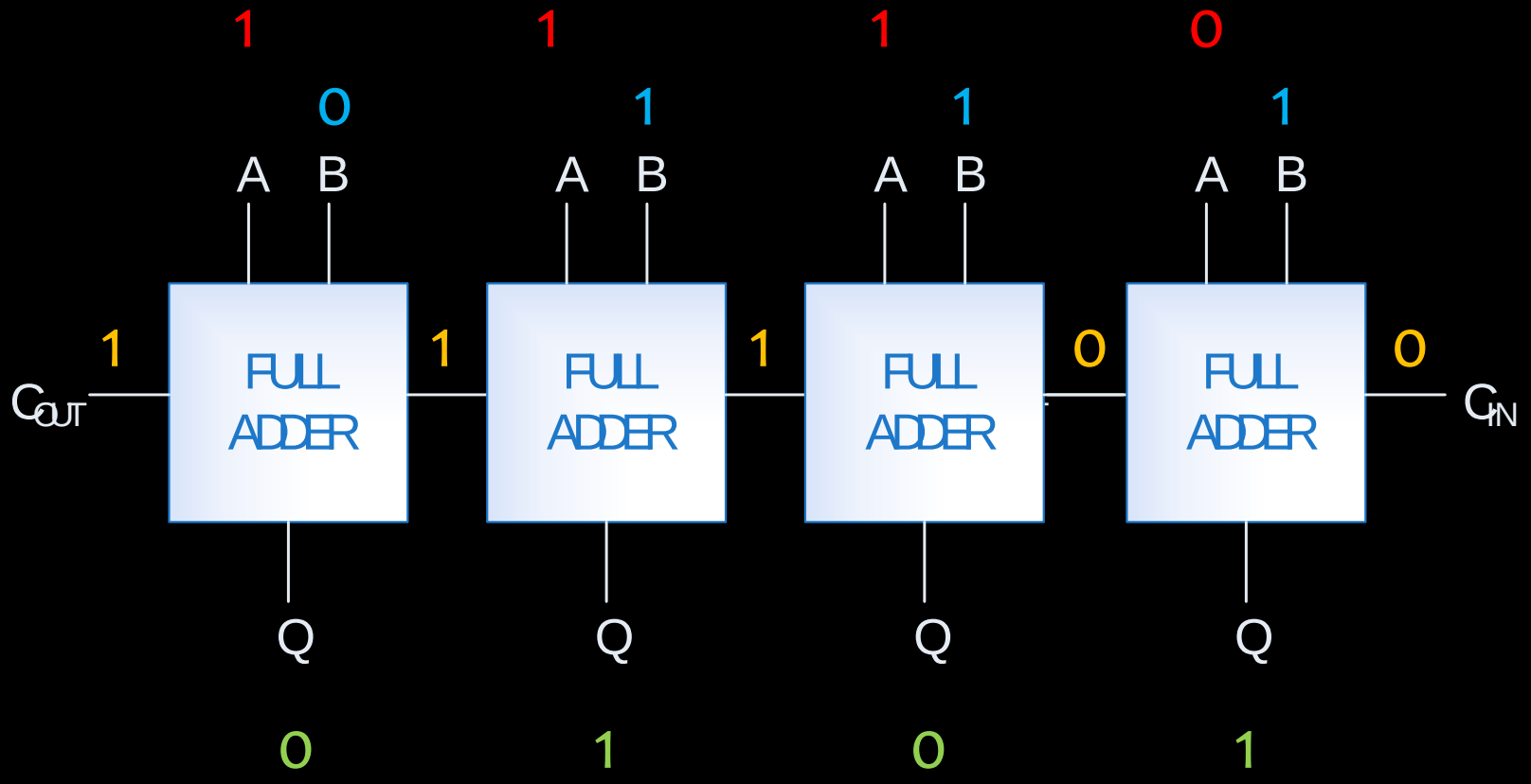
0 1 1 0 + 0 1 1 1 = 1 1 0 1

6 + 7 = 13

Ripple-Carry Adder

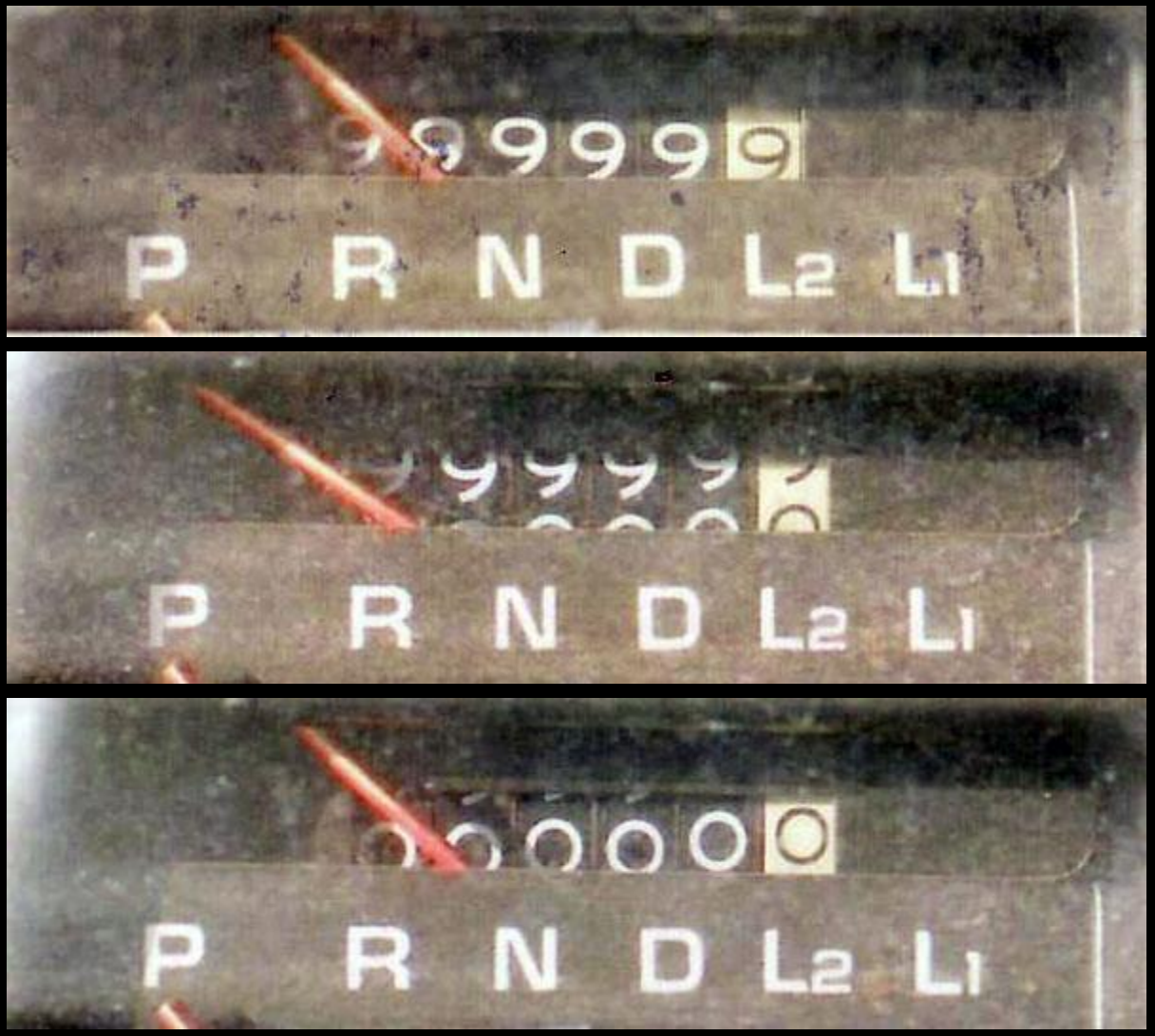


Arithmetic overflow



1 1 1 0 + 0 1 1 1 = 0 1 0 1

14 + 7 = 5


P R N D L2 L1
P R N D L2 L1
P R N D L2 L1

Register width

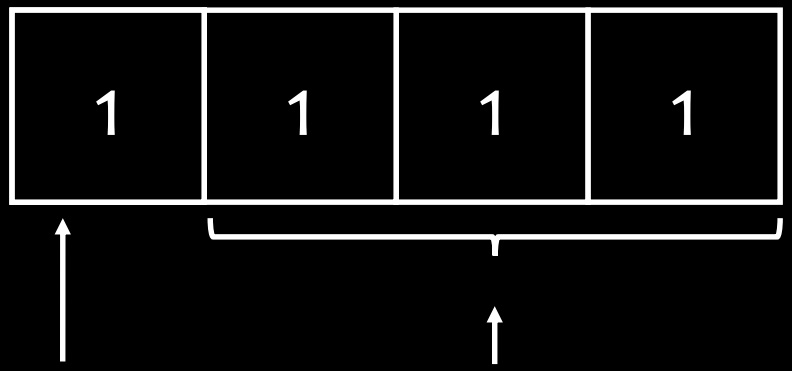
	Maximum representable value	
8 bits	2^8-1	255
16 bits	2^{16}-1	65,535
32 bits	2^{32}-1	4,294,967,295
64 bits	2^{64}-1	18,446,744,073,709,551,615
128 bits	2^{128}-1	340 billion billion billion billion

2005

2012

Negative numbers?

7 -7



Sign-and-Magnitude

0x00 = 0 0 0 0 0 0 0 0$_2$ is non-negative, because the sign bit is 0

0x7F = 0 1 1 1 1 1 1 1$_2$ is non-negative

0x85 = 1 0 0 0 0 1 0 1$_2$ is negative

0x80 = 1 0 0 0 0 0 0 0$_2$ is negative

8 bits

Binary				Decimal
0	0	0	0	+0
0	0	0	1	+1
0	0	1	0	+2
0	0	1	1	+3
0	1	0	0	+4
0	1	0	1	+5
0	1	1	0	+6
0	1	1	1	+7
1	0	0	0	-0
1	0	0	1	-1
1	0	1	0	-2
1	0	1	1	-3
1	1	0	0	-4
1	1	0	1	-5
1	1	1	0	-6
1	1	1	1	-7

$$\begin{array}{r} 0\ 1\ 0\ 0 \\ +\ 1\ 0\ 1\ 1 \\ \hline 1\ 1\ 1\ 1 \end{array} \qquad \begin{array}{r} 4 \\ +\ -3 \\ \hline -7 \end{array}$$

math is cumbersome

Binary				Decimal
0	0	0	0	0
0	0	0	1	+1
0	0	1	0	+2
0	0	1	1	+3
0	1	0	0	+4
0	1	0	1	+5
0	1	1	0	+6
0	1	1	1	+7
1	0	0	0	-8
1	0	0	1	-7
1	0	1	0	-6
1	0	1	1	-5
1	1	0	0	-4
1	1	0	1	-3
1	1	1	0	-2
1	1	1	1	-1

$$\begin{array}{r} 0\ 1\ 0\ 0 \\ +\ 1\ 1\ 0\ 1 \\ \hline 0\ 0\ 0\ 1 \end{array} \qquad \begin{array}{r} 4 \\ +\ -3 \\ \hline 1 \end{array}$$

Two's Complement

8 bits

	0	0	0	0	0	0	0	1	1
+	1	1	1	1	1	1	1	1	+ -1
1	0	0	0	0	0	0	0	0	= 0
	0	0	0	0	0	0	1	0	2
+	1	1	1	1	1	1	1	0	+ -2
1	0	0	0	0	0	0	0	0	= 0
	0	0	0	0	0	0	1	1	3
+	1	1	1	1	1	1	0	1	+ -3
1	0	0	0	0	0	0	0	0	= 0

Two's Complement Negatives

MSB have same value, but negative weight

1 0 1 0 $-1*2^3 + 0*2^2 + 1*2^1 + 0*2^0 = -6_{10}$

0 0 1 0 $-0*2^3 + 0*2^2 + 1*2^1 + 0*2^0 = 2_{10}$

1 1 1 0 $-1*2^3 + 1*2^2 + 1*2^1 + 0*2^0 = -2_{10}$

8 bits

0x00 =	$0\ 0\ 0\ 0\ 0\ 0\ 0\ 0_2$	0
0x7F =	$0\ 1\ 1\ 1\ 1\ 1\ 1\ 1_2$	127
0x85 =	$1\ 0\ 0\ 0\ 0\ 1\ 0\ 1_2$	-123
0x80 =	$1\ 0\ 0\ 0\ 0\ 0\ 0\ 0_2$	-128

$\underbrace{\sim x}_{\text{one's complement}} + 1 = -x$

two's complement

C data type	No. of bit	Minimum	Maximum
char	8	-128	127
unsigned char	8	0	255
short	16	-32,768	32,767
unsigned short	16	0	65,535
int	32	-2.147 Billion	2.147 Billion
unsigned	32	0	4.295 Billion
long	64	-9.2 Quintillion	9.2 Quintillion
unsigned long	64	0	18.4 Quintillion
long long	64	-9.2 Quintillion	9.2 Quintillion
unsigned long long	64	0	18.4 Quintillion

1 1 1 1 1 1 1 1

Bits are unchanged, just interpreted differently!

If you mix unsigned and signed in a single expression, then signed values implicitly cast to unsigned

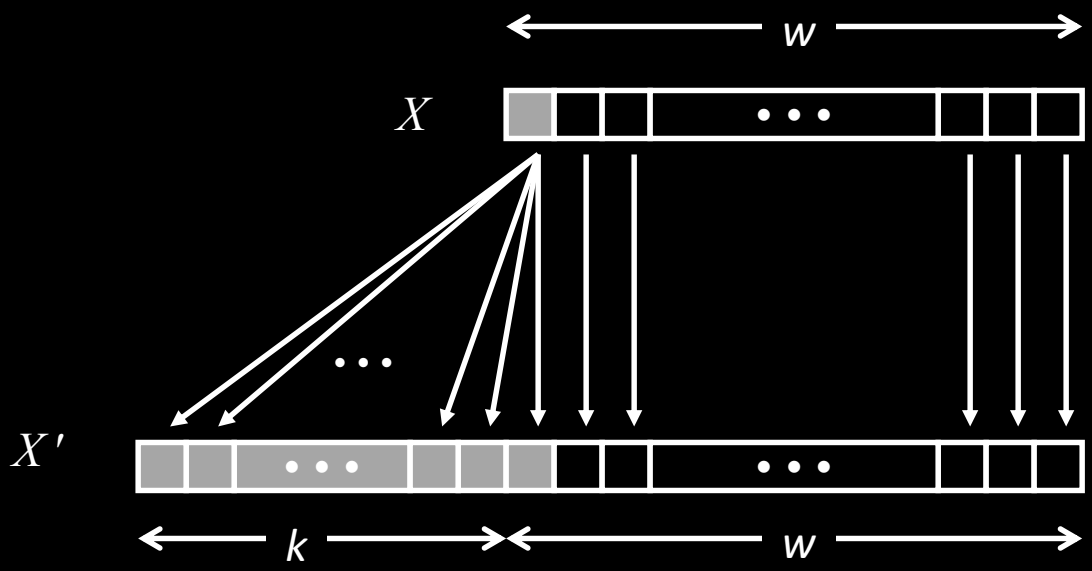
Constant$_1$	Constant$_2$	Relation	Evaluation
0	0U	==	unsigned
-1	0	<	signed
-1	0U	>	unsigned
2,147,483,647	-2,147,483,648	>	signed
2,147,483,647U	-2,147,483,648	<	unsigned
-1	-2	>	signed
(unsigned) -1	-2	>	unsigned
2,147,483,647	2,147,483,648U	<	unsigned
2,147,483,647	(int) 2,147,483,648U	>	signed

Sign Extension

Task:

Given w-bit signed integer x,
convert it to w+k-bit integer *with same value*

Rule: Make k copies of sign bit:



Sign Extension Example

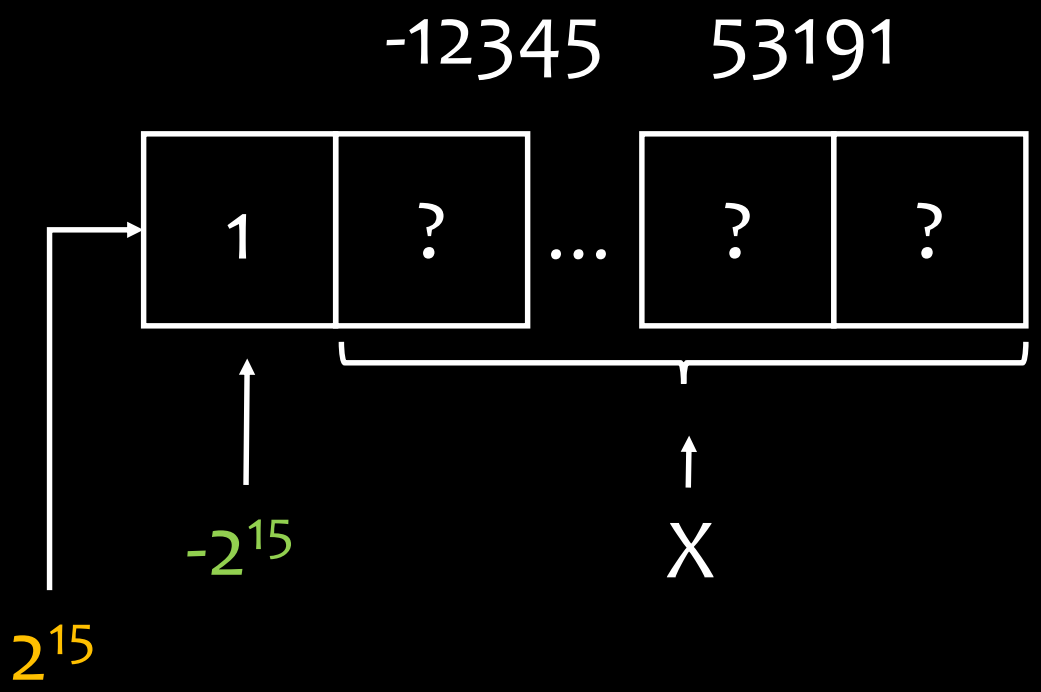
Converting from smaller to larger integer data type

C automatically performs sign extension

```
short int  x = 12345;
int        ix = (int)x;
short int  y = -12345;
int        iy = (int)y;
```

x	Decimal	Hex	Binary
x	12345	30 39	00110000 01101001
ix	12345	00 00 30 39	00000000 00000000 00110000 01101101
y	-12345	CF C7	11001111 11000111
iy	-12345	FF FF CF C7	11111111 11111111 11001111 11000111

```
short int v = -12345;
unsigned short uv = (unsigned short)v;
printf("v = %d, uv = %u\n", v, uv);
```

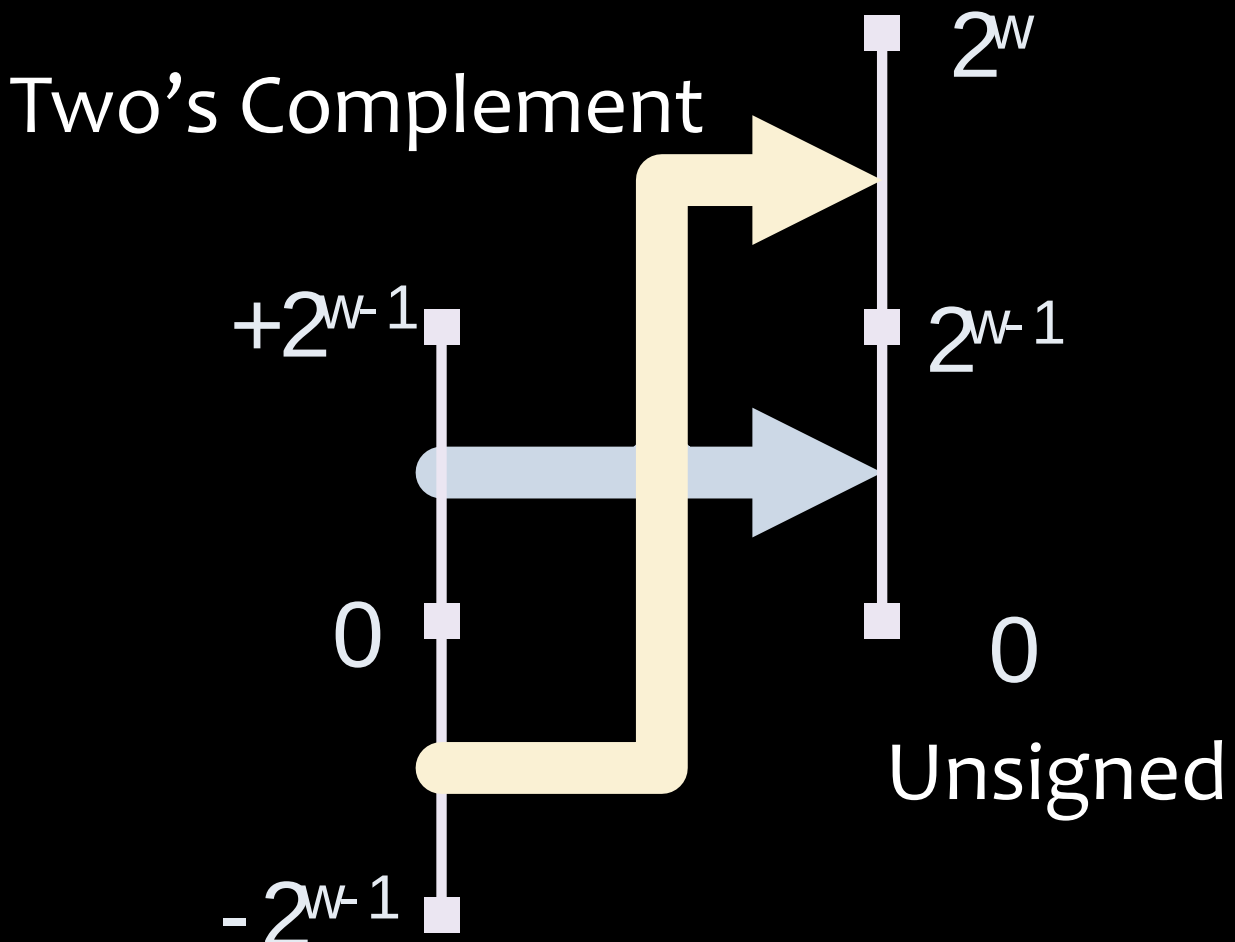


$v = -2^{15} + X$

$uv = 2^{15} + X$

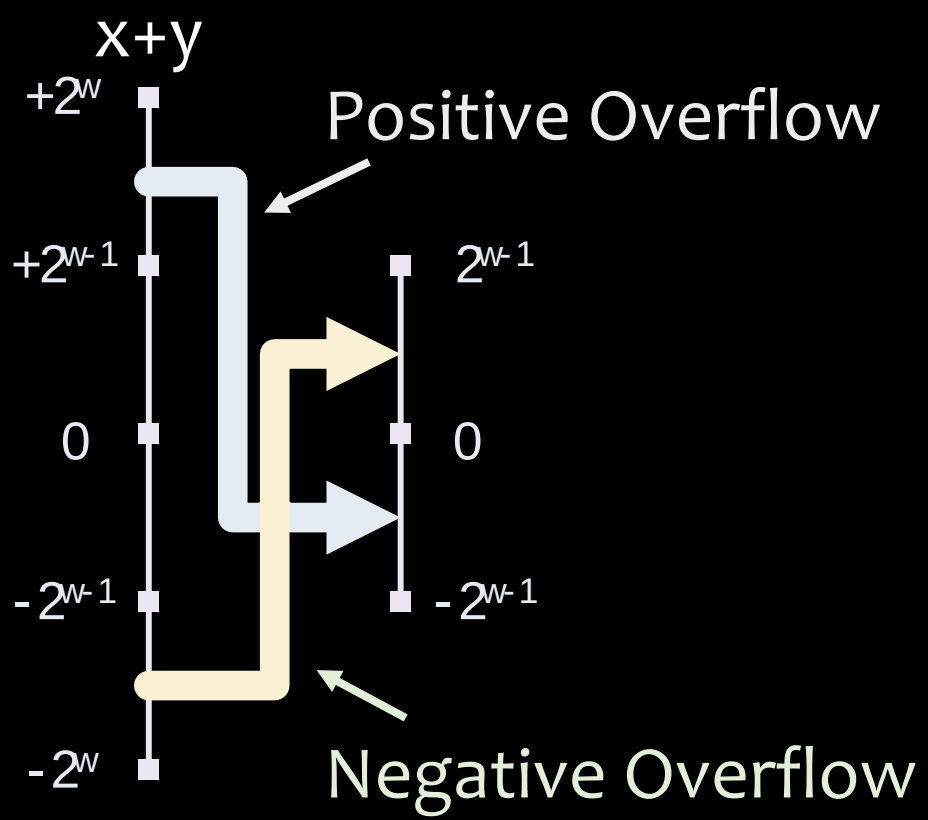
$= 2^{15} + v - (-2^{15})$

$= v + 2^{16}$


Two's Complement
2^w
$+2^{w-1}$
2^{w-1}
0
0
Unsigned
-2^{w-1}

Overflow

exceed the range of the representation



Underflow?

Summary

- Signed / Unsigned integers
- Adder
- Overflow

“There are 10 kinds of people in this world: those who can count in binary and those who can’t.”