DISCRETE STRUCTURES
Lecture 2. Rules of Inference

Bui Anh Tuan

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Contents

Examples

Example 2.1

1. People make mistakes
2. Mistakes are wrong
3. You are a person
4. You are wrong!

Example 2.2

1. Theists define God as all-powerful.
2. Therefore, God can lift any size rock.
3. Therefore, there can be no rock that is too big for God to lift
4. Therefore, God cannot create a rock that is too big for him to lift
5. Therefore, there is something that God can not do.
6. Therefore, God is not all-powerful.
7. Therefore, God does not exist.

Superman does not exist!

Example 2.3

If Superman were able and willing to prevent evil, he would do so. If Superman were unable to prevent evil, he would be impotent; if he were unwilling to prevent evil, he would be malevolent. Superman does not prevent evil. If Superman exists, he is neither impotent nor malevolent. Therefore, Superman does not exist.

Definitions

- By an argument, we mean a sequence of statements that ends with a conclusion.
- The conclusion is the last statement of the argument.
- The premises are the statements of the argument preceding the conclusion.
- By a valid argument, we mean that the conclusion must follow from the truth of the premises.
- Fallacy is form of incorrect reasoning which leads to invalid argument.

Modus Ponens - Law of detachment

If it is raining, then I will stay at home.	‖	$p \to q$
It is raining.	‖	p
$\therefore$ I will stay at home.	‖	$\therefore q$

The symbol $\therefore$ denotes *therefore*.

The propositions above the line are premises (hypotheses).

The proposition below the line is the conclusion.

The basis of the *modus ponens* is the tautology

$$((p \to q) \land p) \to q$$

Modus Ponens: Latin for *mode that affirms*.

Fallacy: Affirming the consequent

⚠

If it is raining, then I will stay at home.	$p \to q$
I stay at home.	q
$\therefore$ It is raining.	$\therefore p$

Modus Tollens

If there is no class today, then students will be happy.	$p \to q$
Students are not happy.	$\neg q$
$\therefore$ There are some classes today.	$\therefore \neg p$

The basis of the *modus tollens* is the tautology

$$((p \to q) \land \neg q) \to \neg p$$

Modus Tollens: Latin for *mode that denies*.

Fallacy: Denying the Hypothesis

⚠

If it is raining, then I will stay at home.	$p \rightarrow q$
It is not raining.	$\neg p$
$\therefore$ I won't be at home.	$\therefore \neg q$

The Addition

The rule of inference

$$\frac{p}{\therefore p \vee q}$$

is the rule of addition.

The basis of the rule of addition is the tautology

$$p \rightarrow (p \vee q)$$

Example 2.4

It is below freezing now. Therefore, it is either below freezing or raining now.

The Simplification

The rule of inference

$$\frac{p \land q}{\therefore p}$$

is the rule of simplification.

The basis of the rule of simplification is the tautology

$$p \land q \to p$$

Example 2.5

It is below freezing and raining now. Therefore, it is below freezing now.

The Simplification

The rule of inference

$$\begin{array}{c} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

is the rule of hypothetical syllogism.

The basis of the rule of hypothetical syllogism is the tautology

$$((p \to q) \wedge (q \to r)) \to (p \to r)$$

Example 2.6

Socrates is a man. All men are mortal. Therefore, Socrates is mortal.

The Disjunctive Syllogism

The rule of inference

$$\begin{array}{c} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

is the rule of Disjunctive Syllogism.

The basis of the rule of Disjunctive Syllogism is the tautology

$$((p \vee q) \wedge \neg p) \to q$$

The Conjunction

The rule of inference

$$\begin{array}{c} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

is the rule of Conjunction.

The basis of the rule of Conjunction is the tautology

$$((p) \wedge (q)) \rightarrow (p \wedge q)$$

The Resolution

The rule of inference

$$\begin{array}{c} p \vee q \\ \neg p \vee r \\ \hline \therefore q \vee r \end{array}$$

is the rule of Resolution.

The basis of the rule of Resolution is the tautology

$$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (p \vee r)$$

Using Rules of Inference to Build Arguments

Example 2.7

Show that the premises "It is not sunny this afternoon and it is colder than yesterday," "We will go swimming only if it is sunny," "If we do not go swimming, then we will take a canoe trip," and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset."

Let

- p = "It is sunny this afternoon,"
- q = "It is colder than yesterday,"
- r = "We will go swimming,"
- s = "We will take a canoe trip," and
- t = "We will be home by sunset."

Validate:

$$\begin{array}{l} \neg p \wedge q \\ r \rightarrow p \\ \neg r \rightarrow s \\ s \rightarrow t \\ \hline \therefore t \end{array}$$

Using Rules of Inference to Build Arguments

Step	Reason
1. $\neg p \wedge q$	Premise
2. $r \rightarrow p$	Premise
3. $\neg r \rightarrow s$	Premise
4. $s \rightarrow t$	Premise
5. $\neg p$	Simplification using (1)
6. $\neg r$	Modus Tollens using (2) and (5)
7. s	Modus Ponens using (3) and (6)
8. t	Modus Ponens using (4) and (7)

Q.E.D (quod erat demonstrandum - Thus it has been demonstrated)

Superman does not exist!

Example 2.8

If Superman were able and willing to prevent evil, he would do so. If Superman were unable to prevent evil, he would be impotent; if he were unwilling to prevent evil, he would be malevolent. Superman does not prevent evil. If Superman exists, he is neither impotent nor malevolent. Therefore, Superman does not exist.

- p = "Superman is able to prevent evil,"
- q = "Superman is willing to prevent evil,"
- r = "Superman prevents evil,"
- s = "Superman is impotent,"
- t = "Superman is malevolent," and
- k = "Superman exists."

Validate:

$$
\begin{array}{l}
p \wedge q \to r \\
\neg p \to s \\
\neg q \to t \\
\neg r \\
k \to (\neg s \wedge \neg t) \\
\hline
\therefore \neg k
\end{array}
$$