4.1.10, 14, 4.2.24, 25, 4.3.5, 4.4.17, 4.5.11

4.1.10 Given: X workers earn \$14 per run, x.
Y workers earn \$13 per run, y.
Material cost is $y^3+x^2-8xy+600$

Find: How many workers of each class to minimize cost? Min 3 each.

Assumptions: none

Solution: $C = 14x + 13y + y^3 + x^2 - 8xy + 600$

$\frac{\partial C}{\partial x} = 14 + 2x - 8y \quad = 0 \;@\; x = 4y - 7$

$\frac{\partial C}{\partial y} = 13 + 3y^2 - 8x \quad = 0 \;@\; x = \frac{13}{8} + \frac{3}{8}y^2$

$\Rightarrow \quad 4y - 7 = \frac{13}{8} + \frac{3}{8}y^2$

$3y^2 - 32y + 69 = 0$

$(3y-23)(y-9) = 0 \Rightarrow y = \frac{23}{3} \text{ or } 9$

$\Rightarrow x = \frac{92}{3} \text{ or } 29$

since we can't have fractions, round first point to (31, 8).

$\Rightarrow \quad C(31, 8) = 627$
$\quad\quad C(29, 9) = 605.$

Check borders since they may have min w/o critical point.

For $y = 3 \Rightarrow C = 14x + x^2 - 24x + 39 + 27 + 600$
$= x^2 - 10x + 666$

$\frac{dC}{dx} = 2x - 10 = 0 \;@\; x = 5$

$\Rightarrow C(5, 3) = 641$

For $x = 3 \Rightarrow C = y^3 + 13y - 24y + 651$
$= y^3 - 9y + 651$

$\frac{dC}{dy} = 3y^2 - 9 = 0 \;@\; y = \cancel{\sqrt{3}} \; \cancel{< 3}$

Therefore, $x = 29$ and $y = 9$ for cost of \$605.

4.1.14 Given: Exhaustive search procedure finds max of $f(x)$ on interval $[0,1]$ by dividing interval into N partition points, evaluating $f(x)$ @ each and selecting max.

Find: Write function using procedure to find max of $f(x) = |10 \sin 4x|$ on $0 \le x \le 1$

Assumptions: none

Solution: see HW5.R

4.2.24 Given: General 2×2 LP problem:

$$\max\ P = c_1 x + c_2 y$$
$$x, y \geq 0$$
$$a_1 x + b_1 y \leq d_1$$
$$a_2 x + b_2 y \leq d_2$$

w/ possibility that constraint lines don't intersect.

Find: Write program to solve problem. Apply to Example 1:

$$\max\ P = 1000x + 500y$$
$$4x + y \leq 10$$
$$18x + 15y \leq 66$$

Assumptions: none

Solution: See HW5.R

4.2.25 Given: Program from exercise 24.

Find: How 1% increases or decreases in coefficients c_1, c_2 of LP problem affect maximum.

Assumptions: none

Solution: See HW5.R ⇒ 1% changes in c_1 had greatest effect. This is the largest coeff.

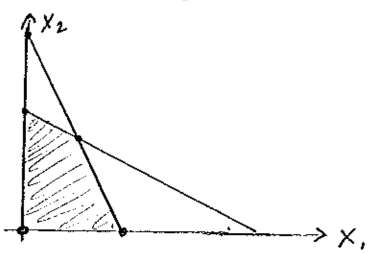
4.3.5 Given: $P = 5x_1 + 6x_2$; $x_1, x_2 \geq 0$

$$2x_1 + 4x_2 \leq 24$$
$$6x_1 + 3x_2 \leq 30$$

Find: max P graphically.
Corner points corresponding to BFSs (0, 0, 24, 30), and (0, 6, 0, 12) from examples.

Assumptions: none

Solution:



corners:	P:
$(0,0)^*$	0
$(0,6)^*$	36
$(8/3, 14/3)$	$\boxed{124/3 = 41.\overline{3}}$
$(5, 0)$	25

*corresponds to BFSs

4.4.17 Given: Traveling salesman problem

Find:
a) Generate 10 random points in plane. Compute distances between ea. pair.
b) Generate 100 random tours. Compute length of ea. & avg.
c) Plot pts. & try to find better tour by trial & error.
d) Develop method to replace random method.

Assumptions: none

Solution: See HW5.R

4.5.11) Given: Northwest corner rule f/ transportation problem.

Find: Write program to carry out rule with input: 2D matrix of costs, 1D array of supplies, 1D array of demands, and with output: 2D array whose nonzero entries are circled amounts in BFS obtained by NW corner rule.

Assumptions: none

Solution: see HW5.R