University System of Georgia

Georgia Gwinnett College

Applied Math

Project 1

Topic: How many hours should we sleep to not wake up tired?

Course: Mathematical Modeling

Student:
Anh Thu Nguyen

Instructor:
Dr. Junkoo Park

May 18, 2023

Contents

I Sleep Stages

I.I Number of Stages

The human body cycles through two phases of sleep, (1) rapid eye movement (REM) and (2) non-rapid eye movement (NREM) sleep, which is further divided into three stages, N1-N3.

N1, N2, N3, and REM. Stages N1 to N3 are considered non-rapid eye movement (NREM) sleep, with each stage a progressively deeper sleep. Approximately 75% of sleep is spent in the NREM stages, with the majority spent in the N2 stage. A typical night's sleep consists of 4 to 5 sleep cycles, with the progression of sleep stages in the following order: N1, N2, N3, N2, REM. A complete sleep cycle takes roughly 90 to 110 minutes. The first REM period is short, and, as the night progresses, longer periods of REM and decreased time in deep sleep (NREM) occur.[1]

Stage	**Percent**	**Amount of Time**
N1 (Light Sleep)	5%	1-5 minutes
N2 (Deeper Sleep)	45%	25 minutes
N3 (Deepest Sleep)	25%	30-60 minutes
REM	25%	$\geq$10 minutes

Table 1: Time Period for Each Stage

I.II Activities in Each Stage

- **Stage 1:** Light sleep. This is a short stage, usually no more than 5% of your total sleep, which begins right after you fall asleep.
- **Stage 2:** Deeper sleep. This stage is deeper and makes up about 45% of all the time you spend sleeping (this number goes up as you get older). Research indicates this stage is key in memory storage and learning.
- **Stage 3:** Deepest sleep. This stage makes up about 25% of the time you spend sleeping. This stage is the most important to how your body recovers and maintains itself because the brain prioritizes this stage in people with sleep deprivation. It's very hard to wake someone up from this stage, and they'll usually feel foggy or confused for up to 30 minutes after waking up.
- **REM sleep:** REM stands for "rapid eye movement." This stage is when you dream. When a person is in REM sleep, you can see their eyes moving beneath their eyelids.

I.III Sequence of Sleep Stages

1. Sleep begins with NREM stage 1 sleep.
2. NREM stage 1 progresses into NREM stage 2.
3. NREM stage 2 is followed by NREM stage 3.
4. NREM stage 2 is then repeated.

5. Finally, you are in REM sleep.[4]

II Setting Up Model

II.I Theory

After understanding how each stage works, we need to answer the question

When should we wake up?

During deep sleep (stage 3 and REM), your cells repair and rebuild, and hormones are secreted to promote bone and muscle growth. Your body also uses deep sleep to strengthen your immunity so you can fight off illness and infection. Therefore, an alarm going off when a person is in one of the deeper stages of sleep may lead to grogginess or difficulty waking up.[3]

Waking up *at the end of the cycle*, when sleep is lightest, may be best to help the person wake feeling more rested and ready to start the day.

II.II Recommendations

Many organizations around the world have their own sleep duration recommendations, overall, they are all very similar, and often reference the recommendations from the US.[2]

National Sleep Foundation (US)		**AASM/SRS (US)**	
Age group	**Rec. Hours**	**Age group**	**Rec. Hours**
Newborns (0–3 months)	14–17 hours	Newborns (0–3 months)	Not included
Infants (4–11 months)	12–15 hours	Infants (4–11 months)	12–16 hours
Toddlers (1–2 years)	11–14 hours	Toddlers (1–2 years)	11–14 hours
Preschoolers (3–5 years)	10–13 hour	Preschoolers (3–5 years)	10–13 hours
Children (6–13 years)	9–11 hours	Children (6–12 years)	9–12 hours
Teenagers (14–17 years)	8–10 hours	Teenagers (13–17 years)	8–10 hours
Young adults (18–25 years)	7–9 hours	Adults (18–60 years)	$\geq$7 hours
Adults (26–64 years)	7–9 hours	Adults (26–64 years)	Not included
Older adults ($\geq$65 years)	7–8 hours	Older adults ($\geq$65 years)	Not included

Table 2: Sleep duration recommendations in the US

According to tables above and the theory, we can calculate the average number of cycles that different age group needs. The results must be an integer since our theory said that we need to complete a full cycle, and results have to meet the recommendation hours for each group as well. Most data shows that every cycle takes about 90 minutes, so in this case we assume *1 cycle = 90 minutes*.

National Sleep Foundation (US)		
Age group	**Rec. Hours**	**Rec. Cycles**
Newborns (0–3 months)	16.5 hours	11 cycles
Infants (4–11 months)	13.5 hours	9 cycles
Toddlers (1–2 years)	12 hours	8 cycles
Preschoolers (3–5 years)	12 hours	8 cycles
Children (6–13 years)	10.5 hours	7 cycles
Teenagers (14–17 years)	9 hours	6 cycles
Young adults (18–25 years)	9 hours	6 cycles
Adults (26–64 years)	7.5 hours	5 cycles
Older adults (≥65 years)	7.5 hours	5 cycles

Table 3: Sleep cycles recommendations

II.III Formulation

Let n be the recommendation cycle of each age group, which we have calculated above. Since the stage 1 of the cycle only starts when we officially enter our sleep, so it might take an amount of time C to get to it.

$$T = 90 \times n + C$$

- T: Hours need to sleep
- n: Number of cycles needed
- C: Amount of time to sleep (to start stage 1)

II.IV Other Factors

At most dosages, alcohol typically causes a decrease in sleep onset, that is the amount of time it takes to fall asleep, and the higher the levels of alcohol the deeper the sleep. However, this is offset by fragmented and disrupted sleep the later part of the night.

During the years 2019, 2020 and 2021, Sleep Cycle's sleep survey users on average tagged alcohol in their sleep notes around 2.5 percent of their total number of sleeps. The sleep notes feature within the Sleep Cycle app allows users to tag a number of activities (such as alcohol consumption, exercise, etc.) that they have undertaken before bedtime. This lets our users draw their own conclusions on whether a particular activity led to a poorer or improved quality of sleep and can also let them see certain patterns over time.

The sleep records tagged with alcohol showed **no large difference in average bedtime** (the time the user went to sleep). It's important to note that we can't draw conclusions on individual sleep quality and data on the quantity of alcohol consumed or the time of day is not available. However, it's fascinating to learn that the data shows that the average sleep duration increases **(11-15 minutes)** and perhaps somewhat surprisingly a slightly better morning mood is noted by our users (up between 1.2-2.3 percent).[5]

According to the data we have, our formula will be adjusted to fit who uses alcohol before sleep.

$$T = 90 \times n + C + 15$$

- T: Hours need to sleep
- n: Number of cycles needed
- C: Amount of time to sleep (to start stage 1)
- sleep duration increases **15 minutes**

II.V Strength and Weakness

Strength: The formula is easy to calculate, everyone can estimate sleeping time and wake up time for themselves. The program below can help you with that.

Weakness: Since we created our formula based on different article and research, we did not have data to estimate the accurate of the model. Also, this is the recommend hours each age group should sleep, there are a lot of factors that affect sleep condition everyday. Hence, the amount of time for each cycle and amount of time to actually enter sleep stages are also estimated, there will have some errors. As long as we could collect more data, we could adjust the formula further on.

III Programming

```
1  import java.util.*;
2
3  public class Main
4  {
5    public static void main(String[] args) {
6        String name;
7        final double cyclePeriod = 90.0;
8        int age, numberOfCycle, enterSleep, totalHour, totalMinute;
9        int startHour, startMinute, wakeHour, wakeMinute;
10       double totalSleep;
11       boolean alcohol;
12       Scanner input = new Scanner(System.in);
13
14       System.out.println("Sleeping Hours Calculator");
15     System.out.println("What's your name? ");
16     name =  input.nextLine();
17     System.out.println("How old are you? (please type negative number of months
     if a baby is younger than 1 year-old)");
18     age = input.nextInt();
19     System.out.println("What time do you go to bed? (9:30pm -> 21 30) ");
20     startHour = input.nextInt(); startMinute = input.nextInt();
21     System.out.println("Estimate how long does it take you enter sleep? (in
     minutes) ");
22     enterSleep = input.nextInt();
23     System.out.println("Do you drink alcohol? (True/False) ");
24     alcohol = input.nextBoolean();
```

```
25
26      if (age >= -11){
27          if (age <= -4)
28              numberOfCycle = 9;
29          else if (age <= 0)
30              numberOfCycle = -11;
31          else if (age <= 5)
32              numberOfCycle = 8;
33          else if (age <= 13)
34              numberOfCycle = 7;
35          else if (age <= 25)
36              numberOfCycle = 6;
37          else
38              numberOfCycle = 5;
39      } else numberOfCycle = 5;
40
41    if (alcohol)
42        totalSleep = cyclePeriod * numberOfCycle + enterSleep + 15;
43    else
44        totalSleep = cyclePeriod * numberOfCycle + enterSleep;
45
46    totalHour = (int) Math.floor(totalSleep/60);
47    totalMinute = (int) totalSleep - totalHour*60;
48
49    wakeHour = startHour + totalHour;
50    wakeMinute = startMinute + totalMinute;
51    if (wakeMinute > 60){
52        wakeMinute -= 60;
53        ++wakeHour;
54    }
55    if (wakeHour > 24)
56        wakeHour -= 24;
57
58    System.out.println("------------Sleeping Hours Calculator------------");
59    System.out.println("| Name: " + name);
60    System.out.println("| Age: " + age);
61    System.out.println("| Alcohol: " + alcohol);
62    System.out.println("| Bed time: " + startHour + ":" + startMinute);
63    System.out.println("| Wake up time: " + wakeHour + ":" + wakeMinute);
64    System.out.println("| Total sleeping hours: " + totalHour + " hours " +
    totalMinute + " minutes");
65    System.out.println("-------------------------------------------------");
66    if(alcohol)
67        System.out.println("!!! It is recommended that you should NOT drink 4-6
    hours prior to your sleep.");
68    System.out.println("**Thank you for using calculator**");
69
70  }
71 }
```

You can try the program by copying and pasting source code to a compiler.

References

[1] Aakash K. Patel; Vamsi Reddy; Karlie R. Shumway; John F. Araujo. National Library of Medicine. [Physiology, sleep stages]. 2022.

[2] Sampasa-Kanyinga H Chaput JP, Dutil C. Nat Sci Sleep. [Sleeping hours: what is the ideal number and how does age impact this?]. 2018.

[3] Jon Johnson. Medical News Today. [What are the best times to sleep and wake up?]. 2020.

[4] National Institute of Neurological Disorders and Stroke. Brain Basics: Understanding Sleep. [Sleep stages and mechanisms]. 2023.

[5] ScienceDaily. Alcoholism: Clinical & Experimental Research. [Reviewing alcohol's effects on normal sleep]. 2013.