Problems: 1.3.10, 20; 1.4.12; 1.5.10; 1.6.16; 1.7.11

1.3.10 Given: Model 3: $\frac{d^2x}{dt^2} = 32.2 - \frac{0.00046}{D}\left(\frac{dx}{dt}\right)^2$

$r = 0.00046/D$
$b = \sqrt{32.2 r}$
$v = dx/dt$
$v(0) = 0$

Find: Show that $\frac{dx}{dt} = v = \sqrt{\frac{32.2}{r}}\,\frac{e^{bt}-1}{e^{bt}+1}$

Assumptions: None

Solution: $\frac{d^2x}{dt^2} = \frac{dv}{dt} = 32.2 - r v^2$ separable DE

$\Rightarrow \frac{dv}{32.2 - rv^2} = dt$ use PFE or table to integrate.

PFE (after slight rearrangement):

$$\frac{1}{r}\int\left(\frac{A}{\sqrt{\frac{32.2}{r}} - v} + \frac{B}{\sqrt{\frac{32.2}{r}} + v}\right)dv = \int dt$$

$$\sqrt{\frac{32.2}{r}}(A+B) + v(A-B) = 1 \Rightarrow A = B,\ A = \frac{1}{2}\sqrt{r/32.2}$$

$$\Rightarrow \frac{1}{2b}\int\left(\frac{1}{\sqrt{\frac{32.2}{r}} - v} + \frac{1}{\sqrt{\frac{32.2}{r}} + v}\right)dv = \int dt$$

$$\Rightarrow \frac{1}{2b}\ln\left|\frac{\sqrt{\frac{32.2}{r}} + v}{\sqrt{\frac{32.2}{r}} - v}\right| = t + C \quad \text{By } v(0) = 0 \Rightarrow C = 0$$

Exponentiate $\Rightarrow \frac{\sqrt{\frac{32.2}{r}} + v}{\sqrt{\frac{32.2}{r}} - v} = e^{2bt}$

Solve for $v \Rightarrow v = \sqrt{\frac{32.2}{r}}\,\frac{e^{2bt}-1}{e^{2bt}+1}$ 2?

1.3.20 Given: Models 2 and 3.
Data in Table 2.

Find: Write program that takes D as input and returns approvriate v_{term}.

Assumptions: None

Solution: see R code HW2.R

For $0.00025 < D < 0.004$, Model 2 is closer for smaller values and Model 3 closer for larger values.

1.4.12| Given: from 10) $P(0) = 1$
@ increments of $\Delta t = 1$, probability of doubling is 0.75
$P(t) = P(0)r^t$

Find: Tabulate population for t between 1 and 15.
Fit tabulation to model.

Assumptions: none

Solution: See HW2.R for tabulation.

For model fit, we can use Excel's solver (see HW2.xlsx) or linear least squares method:

$P(t) = r^t$ since $P(0) = 1$. P_i is ith data pt.

$\Rightarrow \ln P(t) = t \ln r$

$E(\ln r) = \sum_{i=1}^{15} (\ln P_i - t_i \ln r)^2 \equiv$ sum of squared error

$$\frac{dE}{d \ln r} = -2 \sum_{i=1}^{15} t_i (\ln P_i - t_i \ln r)$$
$$= 0 \text{ @ } \ln r = \sum_{i=1}^{15} t_i \ln P_i \Big/ \sum_{i=1}^{15} t_i^2$$

$$\Rightarrow r = \exp\left(\sum_{i=1}^{15} t_i \ln P_i \Big/ \sum_{i=1}^{15} t_i^2 \right)$$

1.5.10) See HW2.R

1.6.16| See HW2.R

1.7.11| Given: Inventory policy model:
r = rate items sold (6 days/week)
s = storage cost per item per day (7 days/week)
k = order cost
x = #items in order

Find: a) Make table like Table 1 covering two weeks.
b) Write function that outputs yearly cost based on iput order size.
c) Can function be used in prescriptive manner?

Assumptions: Day 1 falls on Monday.

Solution: $r = 20$, $s = 0.05$, $k = 100$, $x = 100$ per Table 1.

$x/r = 5$

1.7.11 (cont.)

	Day	Delivery	ordering cost	# in inventory	Carrying cost for this day
M	1	Yes	$100	100	$5
T	2	No	0	80	4
W	3	No	0	60	3
R	4	No	0	40	2
F	5	No	0	20	1
S	6	Yes	$100	100	5
S	7	No	0	100	5
M	8	No	0	80	4
T	9	No	0	60	3
W	10	No	0	40	2
R	11	No	0	20	1
F	12	Yes	$100	100	5
S	13	No	0	80	4
S	14	No	0	80	4

see HW2.R

It appears that optimum order size is 240.