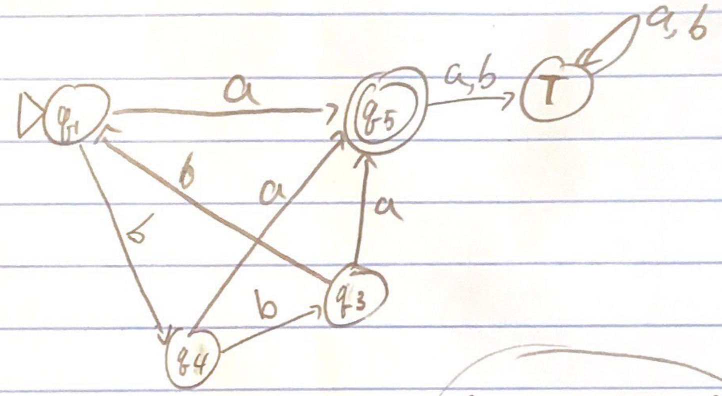
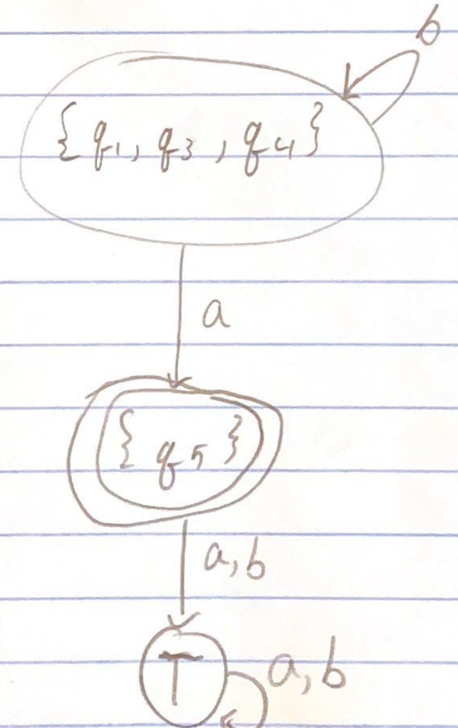
Assignment 3 Gabriele Nicula

1) First we remove unreachable states

q_2 and q_6 are unreachable because each state only has one incoming transition from each other.



		a	b
G_1	q_1	G_2	G_1
	q_3	G_2	G_1
	q_4	G_2	G_1
G_2	$- q_5$	G_3	G_3
G_3	$-$ T	G_3	G_3



$L = \{b^n a : n \geq 0\}$

Strings accepted:

$\{a, ba, bba, bbba, \dots\}$

2)

a) Regular expression for $R^{(0)}_{(44)}$ on $L^{(0)}_{(44)}$

$L^{(0)}_{(44)}$ = all non-empty strings formed with $\{a, b, c\}$ symbols

$R^{(0)}_{(44)} = (a+b+c)(a+b+c)^*$

b) Regular expression for $R^{(0)}_{(12)}$ on $L^{(0)}_{(12)}$

$L^{(0)}_{(12)} = \{a\}$
$R^{(0)}_{(12)} = a$

c) Regular expression for $R^{(4)}_{(12)}$ on $L^{(4)}_{(12)}$

$L^{(4)}_{(12)} = \{a, aba, ababa, \ldots\}$
$R^{(4)}_{(12)} = a(ba)^*$

d) Regular expression for $R^{(0)}_{(11)}$ on $L^{(0)}_{(11)}$

$L^{(0)}_{(11)} = \{\emptyset\}$
$R^{(0)}_{(11)} = \emptyset$

e) Regular expression for $R^{(4)}_{(11)}$ on $L^{(4)}_{(11)}$

$L^{(4)}_{(11)} = \{ab, abab, ababab \ldots\}$
$R^{(4)}_{(11)} = ab(ab)^*$

3)

a) Regular expression for $\Sigma = \{0,1\}$ with at most 1 pair of consecutive ones

$L = \{ \lambda, 0, 01, 10, 11, 00, 001, 100, 010, 1001, \ldots \}$

$RE = (0+10)^*(\lambda+11)(0+01)^*$

The $(0+10)^*$ and $(0+01)^*$ allow for any combination of 0's and 1's without any adjacent 1's in the string. $(\lambda+11)$ allows either no pair of consecutive ones or just one. This pair can either be at the beginning, middle, or end of the string.

b) Not containing 010 as a substring.

$L = \{ \lambda, 0, 01, 10, 101, 1101, 01101, \ldots \}$

$RE = 1^*0^*(111^*0^*)^*(1+\lambda)$

The regular expression allows for any number of 1's to come before any number of 0's. Once the first 0 is seen, the next 1 that comes after must be followed by atleast another 1 to avoid 010. This can be concatenated and can end in a 0 or 1.

4) Simplify $S \to a \mid aA \mid B \mid C$
$A \to aB \mid \lambda$
$B \to Aa$
$C \to cCD$
$D \to ddd \mid Cd$

a) Remove useless productions

$S \to a \mid aA \mid B \mid \cancel{C}$
$A \to aB \mid \lambda$
$B \to Aa$
$\cancel{C \to cCD}$ cannot derive terminal string
$D \to ddd \mid \cancel{Cd}$

grammar becomes

$S \to a \mid aA \mid B$
$A \to aB \mid \lambda$
$B \to Aa$
$\cancel{D \to ddd}$ cannot be reached

grammar becomes

$S \to a \mid aA \mid B$
$A \to aB \mid \lambda$
$B \to Aa$

b) $\lambda \notin L(G)$

Remove the λ productions

$S \to a \mid aA \mid B$
$A \to aB \mid \lambda$
$B \to Aa$

grammar becomes

$S \to a \mid aA \mid B$
$A \to aB$
$B \to Aa \mid a$

c) Remove the unit productions

S production rule contains a unit production $S \to B$ which can be removed and replaced with the B production rule.

grammar becomes

$S \to a \mid aA \mid Aa$
$A \to aB$
$B \to Aa \mid a$

5) Right linear grammar using S, A, B as variables

$S \to aA \mid \lambda$

$A \to aB$

$B \to aB \mid b$

$L = \{\lambda, aab, aaab, aaaab, \ldots\}$

The grammar is right-linear because all productions have variables on the right.