(1) $f(n) = n \log n \quad g(n) = n + \log n$

$n \log n$ grows faster than $n + \log n$

$$\lim_{n \to \infty} \frac{n \log n}{n + \log n} = \lim_{n \to \infty} \frac{\log n}{1 + \frac{\log n}{n}} = \infty$$

$\therefore \; f(n) = \omega(g(n))$ - tighter, more precise

$f(n) = \Omega(g(n))$

(2) $f(n) = 2^n \quad g(n) = 2^{2n}$

g grows faster than f

$$\lim_{n \to \infty} \frac{2^n}{2^{2n}} = \lim_{n \to \infty} \frac{2^n}{2^n \cdot 2^n} = \lim_{n \to \infty} \frac{1}{2^n} = 0$$

$\therefore \; f(n) = o(g(n))$ more precise

$f(n) = O(g(n))$

(3) $f(n) = 3^n \quad g(n) = 3^{n+2}$

$f(n)$ and $g(n)$ are same order

$$\lim_{n \to \infty} \frac{3^n}{3^{n+2}} = \frac{3^n}{3^n \cdot 9} = \lim_{n \to \infty} \frac{1}{9} = \frac{1}{9}$$

$f(n) = \Theta(g(n))$

$f(n) = O(g(n))$

$f(n) = \Omega(g(n))$

(4) $f(n) = 6n^3 + 7n^2 + 2$ $g(n) = n^3$

both are cubic so they are similar

$f(n) = \Theta(g(n))$ - more precise
$f(n) = O(g(n))$
$f(n) = \Omega(g(n))$

(5) $f(n) = n^{50}$, $g(n) = n!$

the factorial grows faster than the polynomial

$f(n) = o(g(n))$ * more precise
$f(n) = O(g(n))$

(2) $T(n) = O(n^4 \log n)$ and $T(n) = \omega(n^2 \log^2 n)$

$n^2 \log n$ $\quad$ n^3 $\quad$ $n^{2.2}$ $\quad$ $n^2 \log^2 n$ $\quad$ $n^4 \log n$

(3) time $t = \dfrac{\text{\# operations}}{\text{second}}$, $\Theta(n^4)$

$time_2$ = time for $n = 200$

$time_1$ = time for $n = 100$

$\therefore$

$\dfrac{time_2}{time_1} = \dfrac{\#\,ops_2}{\#\,ops_1} = \dfrac{200^4}{100^4}$; $time_1 = 3$

$time_2 = 3\left(\dfrac{200}{100}\right)^4 = 3.16 = 48$ seconds

(4) Alg X $\Theta(2^n)$

Alg Y $\Theta(\sqrt{n})$

for Alg X

$$\text{Time}_x = \frac{\#\text{ ops}}{\text{speed}} = \frac{2^n}{400{,}000{,}000} \text{ seconds}$$

$\text{time}_x = 1$ second

$$1 = \frac{2^n}{400{,}000{,}000} \Rightarrow 2^n = 400 \cdot 10^6 \Rightarrow n = \frac{\log(400 \cdot 10^6)}{\log 2}$$

$n = 28.575$, largest input possible for Alg X is 28

for Alg Y

$$1 = \frac{\sqrt{n}}{400 \cdot 10^6} \Rightarrow \sqrt{n} = 400 \cdot 10^6 \Rightarrow n = 160 \cdot 10^{15}$$

largest input for Alg Y $= 160 \cdot 10^{15}$

(5)

(1) for (i=1; i<=n; i*=3) → $\Theta(\log n)$
for (j=0; j n/5; +=2) → $\Theta(n)$
for (k=n; → $\Theta(n)$
op

$T(n) = \Theta(\log n) \cdot \Theta(n) \cdot \Theta(n) = \Theta(n^2 \log n)$

(2) for (i=1; i<n; i+=3)
{
for (j=4; j<10; j++)
op
for (k=4; k<n/2; k++)
op
}

$T(n) = \Theta(n)(\Theta(1) + \Theta(n)) = \Theta n + \Theta(n^2) = \Theta(n^2)$

(3) for (i=1; i<n; i+=3) $\Theta(n)$
{
for (j=1; j<n; j*=2) $\Theta(\log n)$
op
x = n * n
while (x>1)
{ op; x--; }

$T(n) = \Theta(n)(\Theta(\log n) + \Theta(n^2)) = \Theta(n) \cdot \Theta(\log n)$
$+ \Theta(n) \cdot \Theta(n^2) = \Theta(n \log n) + \Theta(n^3) = \Theta(n^3)$

(4)
```
for (i=10; i<=3*n; i++)
{
    for (j=i; j>=5; j--)
        op;
}
```

The inner for loop executes "op" $i-4$ times and depends on the value of i

$\therefore$ the total # of steps is the sum of all the inner steps over all values of i

$$T(n) = \sum_{i=10}^{3n} (i-4) = \left(\sum_{i=10}^{3n} i\right) - 4\sum_{i=10}^{3n} 1 =$$

$$= \sum_{i=1}^{3n} i - \sum_{i=1}^{9} i - 4(3n-9) =$$

$$= \frac{3n(3n+1)}{2} - \frac{9 \cdot 10}{2} - 12n + 36 =$$

$$= \frac{9n^2 + 3n - 24n}{2} - 9 = \frac{9n^2 - 21n}{2} - 9$$

$\therefore T(n) = \Theta(n^2)$