Assignment 1 Gabriele Nicula

1) Prove: (A): $\overline{\bigcup_{i=1}^{n} S_i} = \bigcap_{i=1}^{n} \overline{S_i}$ and

(B): $\overline{\bigcap_{i=1}^{n} S_i} = \bigcup_{i=1}^{n} \overline{S_i}$ Using induction

(A) Base case: $n=2$ $P(2)$: $\overline{S_1 \cup S_2} = \overline{S_1} \cap \overline{S_2}$.
True by DeMorgan's laws.

Assume $P(n)$ is true

$P(n)$: $\overline{\bigcup_{i=1}^{n} S_i} = \bigcap_{i=1}^{n} \overline{S_i}$

$P(n+1)$: $\overline{\bigcup_{i=1}^{n+1} S_i} = \bigcap_{i=1}^{n+1} \overline{S_i} \Leftrightarrow \overline{\left(\bigcup_{i=1}^{n} S_i\right) \cup S_{n+1}} = \bigcap_{i=1}^{n+1} \overline{S_i} \Leftrightarrow$

DeMorgan's
$\Leftrightarrow \left(\overline{\bigcup_{i=1}^{n} S_i}\right) \cap \overline{S_{n+1}} = \bigcap_{i=1}^{n+1} \overline{S_i} \Leftrightarrow$

I.H.
$\Leftrightarrow \left(\bigcap_{i=1}^{n} \overline{S_i}\right) \cap \overline{S_{n+1}} = \bigcap_{i=1}^{n+1} \overline{S_i} \Leftrightarrow$

$\Leftrightarrow \bigcap_{i=1}^{n+1} \overline{S_i} = \bigcap_{i=1}^{n+1} \overline{S_i}$, true because it is an identity

By induction $P(n)$ is true for all $n \geq 2$

1)B) Similar to A, proof by induction

Base Case: $n=2$ $P(2)$: $\overline{\bigcap_{i=1}^{2} S_i} = \bigcup_{i=1}^{2} \overline{S_i} \Leftrightarrow$

$\Leftrightarrow \overline{S_1 \cap S_2} = \overline{S_1} \cup \overline{S_2}$; true by DeMorgan's law

Assume $P(n)$ is true

$P(n)$: $\overline{\bigcap_{i=1}^{n} S_i} = \bigcup_{i=1}^{n} \overline{S_i}$

$P(n+1)$: $\overline{\bigcap_{i=1}^{n+1} S_i} = \bigcup_{i=1}^{n+1} \overline{S_i} \Leftrightarrow \overline{\left(\bigcap_{i=1}^{n} S_i\right) \cap S_{n+1}} = \bigcup_{i=1}^{n+1} \overline{S_i} \Leftrightarrow$

DeMorgan's on the lefthand side

$\Leftrightarrow \overline{\left(\bigcap_{i=1}^{n} S_i\right)} \cup \overline{S_{n+1}} = \bigcup_{i=1}^{n+1} \overline{S_i} \Leftrightarrow$

$\overset{\text{I.H.}}{\Leftrightarrow} \left(\bigcup_{i=1}^{n} \overline{S_i}\right) \cup \overline{S_{n+1}} = \bigcup_{i=1}^{n+1} \overline{S_i} \Leftrightarrow$

$\Leftrightarrow \bigcup_{i=1}^{n+1} \overline{S_i} = \bigcup_{i=1}^{n+1} \overline{S_i}$, true because it is an identity.

By induction $P(n)$ is true for all $n \geq 2$

2) Prove that $S_1 \cup S_2 - (S_1 \cap \overline{S_2}) = S_2$

Let $T = S_1 \cup S_2$ and $V = S_1 \cap \overline{S_2}$

Then, $T - V = T \cap \overline{V}$ which means

$(S_1 \cup S_2) - (S_1 \cap \overline{S_2}) = (S_1 \cup S_2) \cap (\overline{S_1 \cap \overline{S_2}})$ De Morgan $\Leftrightarrow$ RHS

$\Leftrightarrow (S_1 \cup S_2) - (S_1 \cap \overline{S_2}) = (S_1 \cup S_2) \cap (\overline{S_1} \cup S_2) \Leftrightarrow$

Distributive law on righthand side

$\Leftrightarrow (S_1 \cup S_2) - (S_1 \cap \overline{S_2}) = (S_1 \cap \overline{S_1}) \cup S_2 \Leftrightarrow$

$\Leftrightarrow (S_1 \cup S_2) - (S_1 \cap \overline{S_2}) = (\emptyset) \cup S_2 \Leftrightarrow$

$\Leftrightarrow (S_1 \cup S_2) - (S_1 \cap \overline{S_2}) = S_2$ true by equivalence

3) Give a description of the language generated by the grammar.

$S \to aaA$ $\quad \{\lambda, aab, aabaab, aabaabaab, ...\}$

$A \to bS$

$S \to \lambda$ $\quad L = \{(aab)^n : n \geq 0\}$

4) Give a description of the language generated by the grammar.

$S \to Aa$ $\quad L = \emptyset$, No finite sentences can

$A \to B$ $\quad$ be generated by the grammar

$B \to Aa$ $\quad$ because the grammar is infinitely recursive.

5) Find grammars for the following languages with alphabet $\Sigma = \{a, b\}$

a) All strings with at least two a's

$\{aa, aba, aab, baa, abba, abab, aaab, aaabba, \ldots\}$

$S \to AA \mid AS$
$A \to BaB$
$B \to bB \mid \lambda$

b) All strings with no more than three a's

$\{\lambda, a, b, ab, abba, ba, baaa, abab, \ldots\}$

$S \to A \mid B \mid C \mid D$
$A \to BaB$
$B \to bB \mid \lambda$
$C \to AA$
$D \to CA$