# Mathematical Calculations for PINNs in Fluid Dynamics

### 1. \*\*Synthetic Data Generation (Fluid Dynamics)\*\*

- You generate synthetic data for velocity (u, v) and pressure (p) fields using simple functions.

- \*\*Velocity field equations\*\*:

u(x, y, t) = sin(π \* x) \* cos(π \* y) \* exp(-t)

v(x, y, t) = -cos(π \* x) \* sin(π \* y) \* exp(-t)

- \*\*Pressure field equation\*\*:

p(x, y) = cos(π \* x) \* cos(π \* y)

### 2. \*\*Neural Network Architecture for PINN\*\*

- The neural network takes three inputs (x, y, t) and predicts three outputs (velocity components u, v, and pressure p).

- The layers use \*\*tanh\*\* activation, common for approximating smooth functions like those in fluid dynamics.

### 3. \*\*Physics-Informed Loss Function (Navier-Stokes Equations)\*\*

The loss function is based on enforcing physical laws using the \*\*Navier-Stokes equations\*\* and \*\*continuity equation\*\*.

- \*\*Continuity equation\*\* (ensures incompressibility of fluid):

∂u/∂x + ∂v/∂y = 0

- \*\*Momentum equations\*\* (Navier-Stokes):

- x-momentum equation:

u ∂u/∂x + v ∂u/∂y + ∂p/∂x - ν (∂²u/∂x² + ∂²u/∂y²) = 0

- y-momentum equation:

u ∂v/∂x + v ∂v/∂y + ∂p/∂y - ν (∂²v/∂x² + ∂²v/∂y²) = 0

- \*\*Data loss\*\*:

data\_loss = Σ [(u\_true - u\_pred)² + (v\_true - v\_pred)² + (p\_true - p\_pred)²]

- \*\*Physics-based loss\*\* (enforcing physical constraints):

physics\_loss = Σ [(continuity\_loss)² + (momentum\_x)² + (momentum\_y)²]

- \*\*Total loss\*\*:

total\_loss = data\_loss + physics\_loss

### 4. \*\*Gradient Calculations\*\*

Using TensorFlow's `GradientTape`, you calculate the following partial derivatives required for the Navier-Stokes equations:

- ∂u/∂x, ∂u/∂y, ∂v/∂x, ∂v/∂y

- ∂p/∂x, ∂p/∂y

### 5. \*\*Training the Model\*\*

The model is trained using synthetic data and the custom physics-informed loss function. The model adjusts its parameters to minimize both data loss and the physics-based loss.

### 6. \*\*Predictions and Visualizations\*\*

- \*\*Predicted velocity field\*\*:

Velocity vectors (u\_pred, v\_pred) are plotted as quivers on the (x, y)-plane.

- \*\*Predicted pressure field\*\*:

The predicted pressure p\_pred is visualized as a color-mapped scatter plot over the (x, y)-plane.