The Resilience and Robustness (and Hopefully Responsibility) of Linear Algebra Lecture 1

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March 2024

7. [6 points] Write at least one complete sentence about the most interesting thing you learned this semester.

The most interesting thing I learned was nothing. This class was boring.

The following problem is from Tablet VAT 8389, from the Old Babylonian period, 2000–1600 BCE, as translated into English (Høyrup 2002):

> *The total area of two fields is 1800 sar [unit of area], the rent for one is 2 silà [unit of volume] of grain per 3 sar, for the other is 1 silà per 2 sar, and the total rent on the first exceeds that on the other by 500 silà.*

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This is a system of linear equations.

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For more information, see Grcar 2011.

Row reduction was known in the third century CE or earlier in China.

Gauss was far from the first European to use the method.

So, it is very much a misnomer to call it "Gaussian elimination."

For more information, see Grcar 2011.

Source: `https://xkcd.com/1838/`

Linear algebra:

so robust,

so resilient.

COURSE OUTLINE

- **Lecture 1**
  - **Signals and vector spaces**
  - **Linear combinations**
- Lecture 2
  - Inner product
  - Inner product as similarity
  - Moving average
  - Convolution and cross correlation
  - **2***D* convolution and cross correlation
- Lecture 3
  - Cosine similarity
  - Projection
  - Change of basis
  - Graph theory
- Lecture 4
  - Eigenvalues and eigenvectors
  - Principal component analysis
  - Responsibility vignette

Lecture Outline

Signals and vector spaces

Linear combinations

What is a vector?

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- the amount of red, green, and blue in a color pixel,
- a digital song, etc.

$\mathbb{R}^d$

Mathematicians label the ordered lists of d (real) numbers $\mathbb{R}^d$.

Written as columns and using set builder notation, we have

$$\mathbb{R}^d = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{pmatrix} \;\middle|\; x_1, x_2, \ldots, x_d \in \mathbb{R} \right\}.$$

OPERATIONS ON $\mathbb{R}^d$

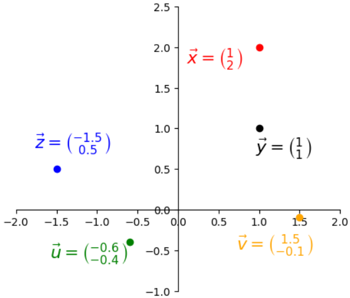
Two operations that mathematicians like to do to $\mathbb{R}^d$ are scalar multiplication and vector addition:

for $a \in \mathbb{R}$ and $\vec{x}, \vec{y} \in \mathbb{R}^d$

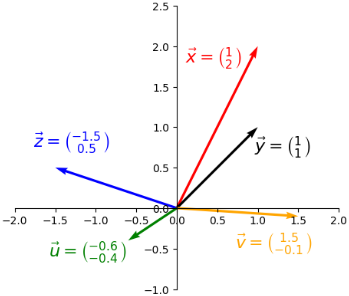
$$a \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{pmatrix} = \begin{pmatrix} ax_1 \\ ax_2 \\ \vdots \\ ax_d \end{pmatrix} \qquad \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_d \end{pmatrix} = \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \\ \vdots \\ x_d + y_d \end{pmatrix}.$$

These seemingly simple operations actually wield a lot of power.

Drawing vectors in $\mathbb{R}^2$ as points and arrows

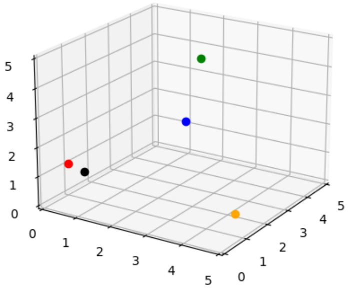


Vectors as points.

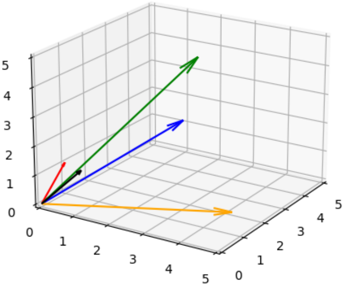


Vectors as arrows.

DRAWING VECTORS IN $\mathbb{R}^3$ AS POINTS AND ARROWS



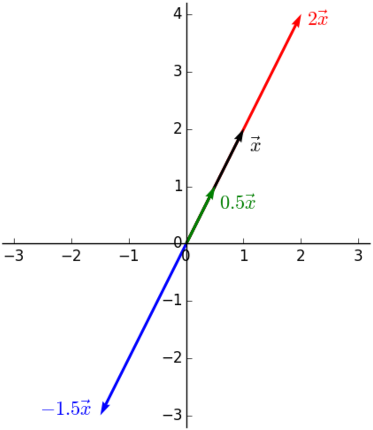
Vectors as points.



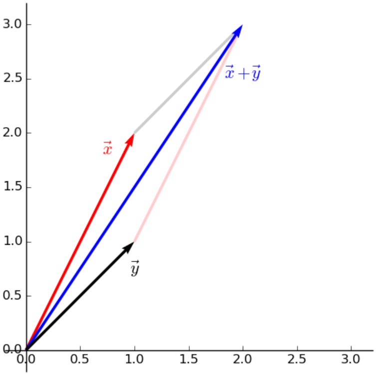
Vectors as arrows.

Arrows and scaling

$$\vec{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad 0.5\vec{x} = \begin{pmatrix} 0.5 \\ 1 \end{pmatrix} \qquad 2\vec{x} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \qquad -1.5\vec{x} = \begin{pmatrix} -1.5 \\ -3 \end{pmatrix}$$

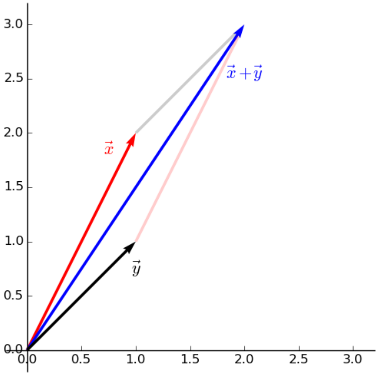


Arrows and adding



$$\color{red}{\begin{pmatrix}\mathbf{1}\\ \mathbf{2}\end{pmatrix}} + \begin{pmatrix}\mathbf{1}\\ \mathbf{1}\end{pmatrix} = \color{blue}{\begin{pmatrix}\mathbf{1}+\mathbf{1}\\ \mathbf{2}+\mathbf{1}\end{pmatrix}}.$$

Arrows and adding



$$\begin{pmatrix} \mathbf{1} \\ \mathbf{2} \end{pmatrix} + \begin{pmatrix} \mathbf{1} \\ \mathbf{1} \end{pmatrix} = \begin{pmatrix} \mathbf{1+1} \\ \mathbf{2+1} \end{pmatrix}.$$

I.e., tip-to-tail

(Switch to Matlab)

What is a vector?

The mathematically rigorous answer is to first define a vector space and then define vectors as elements of a vector space.

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A set V with linear operators

- for $\vec{u}, \vec{v} \in V, \quad \vec{u} + \vec{v} \in V$
- for $s \in \mathbb{R}$ and $\vec{v} \in V, \quad s\vec{v} \in V.$

is called a (real)[1] vector space if it also follows all of the axioms on the next slide.

[1] replacing $\mathbb{R}$ some other field like $\mathbb{C}$ or $\mathbb{F}_q$ also yields a vector space

Requirements ("axioms")

$$s(\vec{u}+\vec{v}) = s\vec{u}+s\vec{v} \qquad (s+t)\vec{u} = s\vec{u}+s\vec{u} \qquad \text{(Distributive)}$$

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(Associative)

$$\vec{u}+\vec{v} = \vec{v}+\vec{u}$$

(Commutative)

Matrices

An $(m \times n)$-matrix of numbers is a table

$$\mathbf{M} = \begin{pmatrix} M_{1,1} & M_{1,2} & \dots & M_{1,n} \\ M_{2,1} & M_{2,2} & \dots & M_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ M_{m,1} & M_{m,2} & \dots & M_{m,n} \end{pmatrix}$$

where for each $1 \leq i \leq m$ and $1 \leq j \leq n$,

$$M_{i,j} \text{ is a number.}$$

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Matrices require:

number of rows $\times$ number of columns.

(Mnemonic device to remember: ROWℓ COL "roll coll")

Vector operations on matrices

Scalar multiplication of a matrix involves multiplying each entry of the matrix by the given scalar.

$$-\mathbf{2}\begin{pmatrix} \mathbf{3.6} & \mathbf{2} & \mathbf{1.1} \\ \mathbf{0} & \mathbf{0} & \mathbf{-0.3} \end{pmatrix} = \begin{pmatrix} \mathbf{-7.2} & \mathbf{-4} & \mathbf{-2.2} \\ \mathbf{0} & \mathbf{0} & \mathbf{0.6} \end{pmatrix}$$

Addition of two matrices (of the same size) involves adding each corresponding pair of entries.

$$\begin{pmatrix} \mathbf{3.6} & \mathbf{2} & \mathbf{1.1} \\ \mathbf{0} & \mathbf{0} & \mathbf{-0.3} \end{pmatrix} + \begin{pmatrix} \mathbf{-0.2} & \mathbf{0} & \mathbf{3.4} \\ \mathbf{-1} & \mathbf{1} & \mathbf{0} \end{pmatrix} = \begin{pmatrix} \mathbf{3.4} & \mathbf{2} & \mathbf{4.5} \\ \mathbf{-1} & \mathbf{1} & \mathbf{-0.3} \end{pmatrix}$$

Images as matrices

Above is an 800×850 matrix of numbers between $\mathbf{0}$ and $\mathbf{255}$.

- **0** appears as black.
- **255** appears as white.
- Values between **0** and **255** are used for grays.

WARNING:

Since the kinds of things we like to do to digital images are typically "vector-y"

(i.e., "linear things" like scalar multiplication and component-wise addition)

and not "matrix-y"

(i.e., "bilinear things"),

some people get angry if digital images are called "matrices."

(Switch to Matlab)

A tensor is a collection Γ of data indexed by ℓ numbers:

$$1 \le i_1 \le d_1, \quad \dots \ , \quad 1 \le i_\ell \le d_\ell$$

.

The valence is the number ℓ, i.e., the number of indices.

The dimensions are $d_1, \dots, d_\ell$.

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To be really mathematically correct, a tensor also requires associated multilinear operations.

An $\boldsymbol{m} \times \boldsymbol{n}$ matrix is a tensor of valence $\mathbf{2}$ with dimensions $\boldsymbol{m}$ and $\boldsymbol{n}$.

A vector with $\boldsymbol{n}$ entries is a tensor of valence $\mathbf{1}$ with dimension $\boldsymbol{n}$.

An $m \times n$ matrix is a tensor of valence $\mathbf{2}$ with dimensions m and n.

A vector with n entries is a tensor of valence $\mathbf{1}$ with dimension n.

Sometimes in data science, people use these facts to sound cooler.

RGB

One common way to encode color is with RGB
(red-green-blue), a triple of numbers,
either between **0** and **255** or **0** and **1**.

The first number tells “how much” red, the second green, the third blue.
All being **255** (or **1**) yields white.
All being **0** yields black.

Images as tensors[2]

Recall a color pixel in RGB is $\vec{p} = (r, g, b)$.

Put colors in a grid and you get a rectangular picture.

So in total

$$\text{row} \times \text{column} \times \text{color}$$

A color image is a tensor.

[2] same warning as before

(Switch to Matlab)

Lecture Outline

Signals and vector spaces

Linear combinations

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Any single vector addition

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Creating a grayscale image from color channels

Any single vector addition

Any scalar multiplication

Let V be a vector space.

A linear combination of vectors $\vec{v}_1, \ldots, \vec{v}_n \in V$ is

$$s_1\vec{v}_1 + \ldots + s_n\vec{v}_n = \sum_{i=1}^{n} s_i\vec{v}_i$$

for some choice of scalars $s_1, \ldots, s_n \in \mathbb{R}$.

Representing data as linear combinations of special vectors can be very informative.

The upper left pixel of Pete Dog has the values **120, 101, 68**.

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We can think of this is a linear combination of

$$\begin{pmatrix}\mathbf{1}\\\mathbf{0}\\\mathbf{0}\end{pmatrix} \text{ pure red}, \quad \begin{pmatrix}\mathbf{0}\\\mathbf{1}\\\mathbf{0}\end{pmatrix} \text{ pure green}, \quad \text{and} \begin{pmatrix}\mathbf{0}\\\mathbf{0}\\\mathbf{1}\end{pmatrix} \text{ pure blue},$$

where the coefficient of the first vector tells us how much red is at the location (i.e., $\mathbf{120/255}$),

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the coefficient of the second vector tells us how much green is at the location (**101/255**), and

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where the coefficient of the first vector tells us how much red is at the location (i.e., **120/255**),

the coefficient of the second vector tells us how much green is at the location (**101/255**), and

the last coefficient tells us how much blue is there (**68/255**).

ELECTROMAGNETIC SPECTRUM

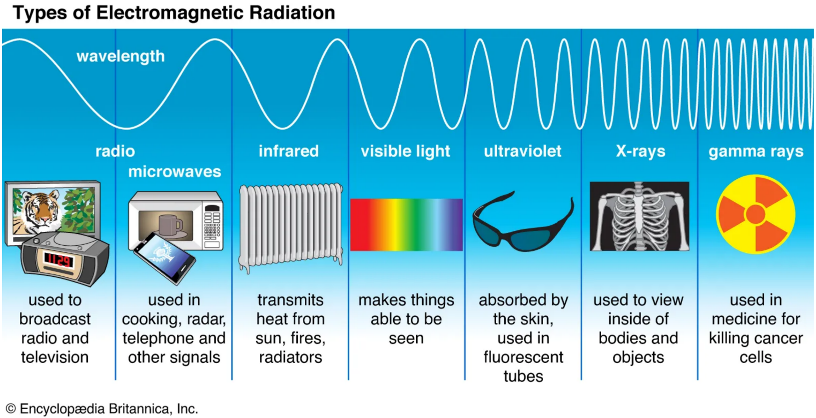

Image source: https://www.britannica.com/science/electromagnetic-spectrum

Multi- and hyperspectral imaging

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Full color image: **3** wavelength bands measured per location

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Hyperspectral image: $\approx \mathbf{100+}$ wavelength bands measured per location

Each results in an $\boldsymbol{m} \times \boldsymbol{n} \times \boldsymbol{w}$ tensor, where
$\boldsymbol{m}$ and $\boldsymbol{n}$ are the number of rows and columns in each slice
(i.e., representing the geographical location of the data reading)
and $\boldsymbol{w}$ is the number of wavelength bands measured.

Hyperspectral data cube with $w = \mathbf{700}$. Dr. Nicholas M. Short, Sr., NASA
`https://commons.wikimedia.org/wiki/File:HyperspectralCube.jpg`

Hyperspectral data cube with $w = \mathbf{700}$. Dr. Nicholas M. Short, Sr., NASA
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This is known as false color, where color is added to aid human visualization of data not representing the visible spectrum.

Hyperspectral data cube with $w = \mathbf{700}$. Dr. Nicholas M. Short, Sr., NASA
`https://commons.wikimedia.org/wiki/File:HyperspectralCube.jpg`

Each location is represented by a vector with w entries (here **700**) called a voxel or hypervoxel.

Typically, the linear information we seek from a hyperspectral data cube comes from linear operations on the voxels.

Similar to with RGB, the w entries of each hypervoxel tells you about the “amount” of the corresponding wavelength band at that location,

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Similar to with RGB, the $\boldsymbol{w}$ entries of each hypervoxel tells you about the "amount" of the corresponding wavelength band at that location,

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However, the power comes from representing each voxel as a linear combination of a different set of vectors than the ones with one $\mathbf{1}$ and six hundred ninety nine $\mathbf{0}$'s.

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Now imagine you have a similar vector $\vec{y}$ representing pure pavement and another $\vec{z}$ representing soil.

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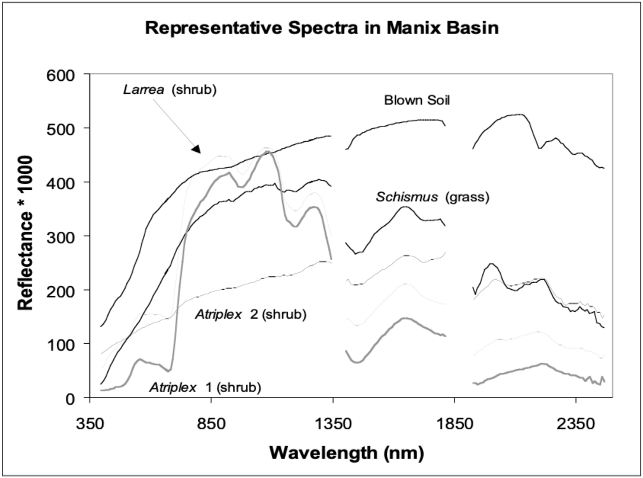
If a voxel can be written as a linear combination like

$$0.7\vec{x} + 0.1\vec{y} + 0.2\vec{z},$$

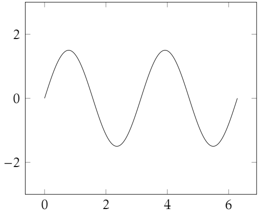
then you might assume that location has a lot of grass, a bit of pavement, and some soil, maybe a patch of grass with a curb at the edge.

Typically, the endmembers are more specific than just “grass.”

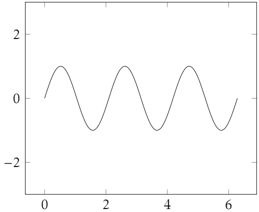


Representative spectra (endmembers) in the Manix Basin spectral library. Okin et al. 1999

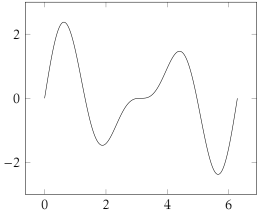
It can also be beneficial to think of music vectors as being linear combinations of vectors representing pure tones.



$1.5\sin(2x)$



$\sin(3x)$



$1.5\sin(2x) + \sin(3x)$

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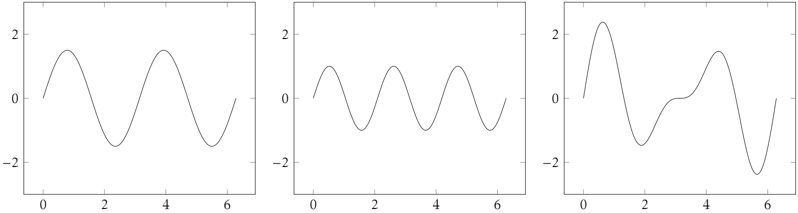
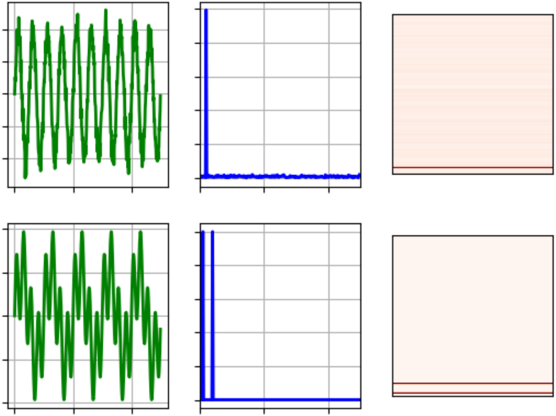


$1.5\sin(2x)$ $\quad$ $\sin(3x)$ $\quad$ $1.5\sin(2x)+\sin(3x)$

We could represent the mixture of two pure tones above as a vector like

$$\begin{pmatrix} 0 & 0 & 1.5 & 1 & 0 & \dots & 0 \end{pmatrix},$$

where there is a 1.5 in the spot for $\sin(2x)$ and 1 in the spot for $\sin(3x)$ but zeros in spots corresponding to $\sin(kx)$ for $k \neq 2, 3$.

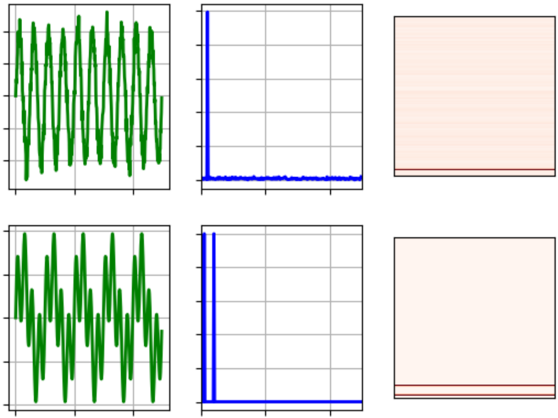


Left-to-right:

Visualization of sound vector in green.

Visualization of the coefficients of the sound as a linear combination of sinusoids in blue.

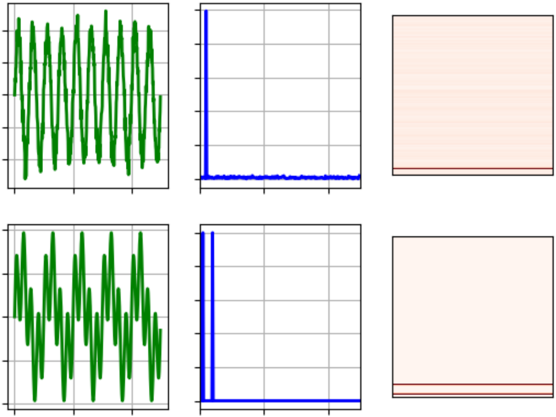
A blown-up heat map of the linear combination coefficients.



In the top row, we have a pure tone with static noise.

There's one big number which is the coefficient of the dominant tone.

Then there are many varying small coefficients representing the noise.



In the bottom row, we have a mixture of two tones like before.

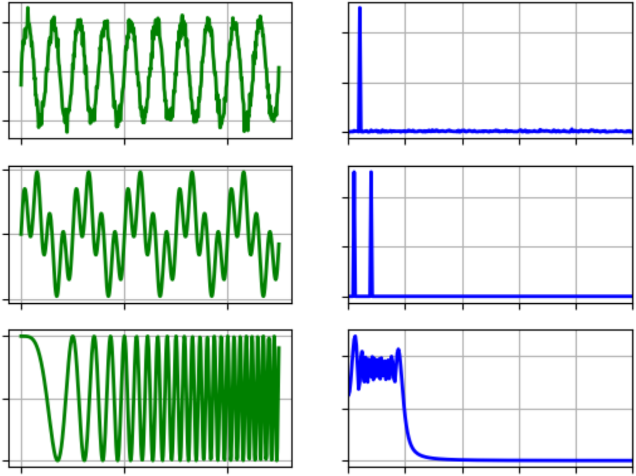
Two big numbers which are the coefficients of the two tones.

Then all other coefficients are zero.

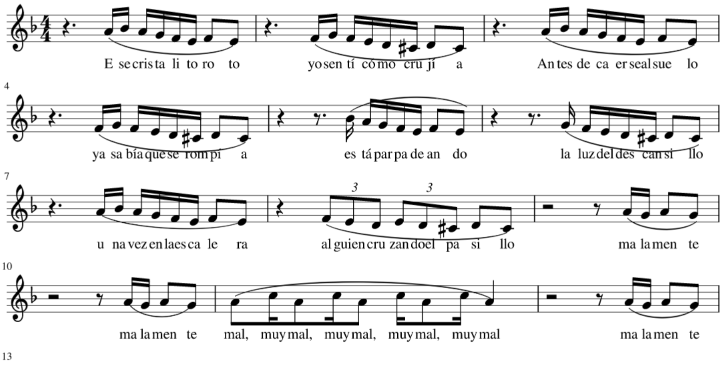
It would be very boring if there was only one sound all of the time.

Sounds typically change over time.

MESSY



Sheet music



Sheet music. Source: `https://musescore.com/user/28683468/scores/5328385`

We would like a visualization like sheet music, where the horizontal direction is time and the vertical frequency/tone.

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Visualize the coefficients of the linear combination as a vertical strip with colors corresponding to different values.

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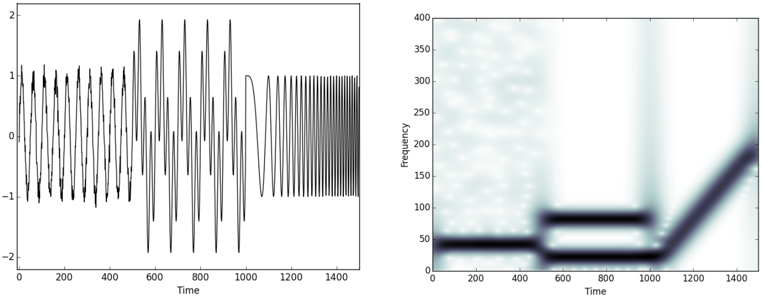
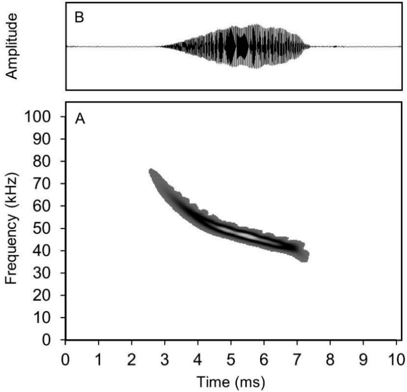
Visualize the coefficients of the linear combination as a vertical strip with colors corresponding to different values.

Glue the vertical strips together horizontally to create a $\mathbf{2}\boldsymbol{D}$ heat map of all of the linear combination coefficients over short time frames.

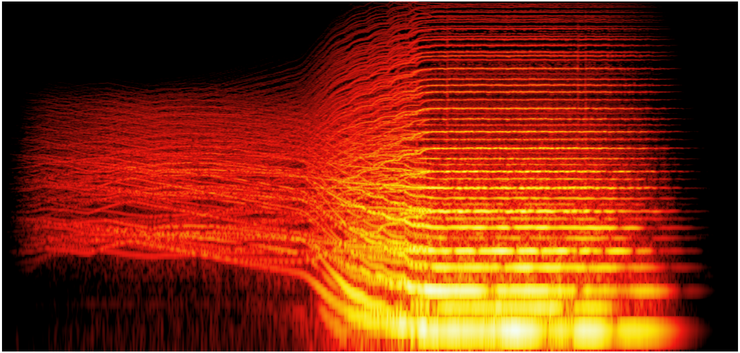


Left: A signal with three components over time. Right: Its spectrogram.



Bat chirp as an amplitude graph (top) and as a spectrogram (bottom). Source: Erin E. Fraser et. al. Bat Echolocation Research: A handbook for planning and conducting acoustic studies, 2020

THX Deep Note



THX Deep Note Spectrogram. Image source:
`https://beautifulspectrograms.tumblr.com/`

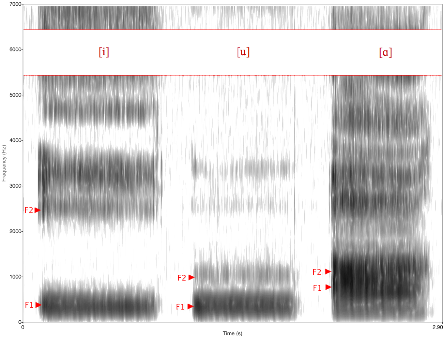
YouTube spectrograms

THX Deep Note
`https://www.youtube.com/watch?v=mjnM-Iw_pfo`

Aphex Twin –
$\boldsymbol{\Delta M i^{-1} = -\alpha \sum_{n=1}^{N} D_i[n] \left[\sum_{j \in C[i]} F_{ji}[n-1] + F\,\mathbf{ext}_i[n^{-1}]\right]}$
`https://www.youtube.com/watch?v=M9xMuPWAZW8`

Wisp – Hnipian
`https://beautifulspectrograms.tumblr.com/` and
`https://www.youtube.com/watch?v=4zMoVPMQIak`

FORMANT ANALYSIS

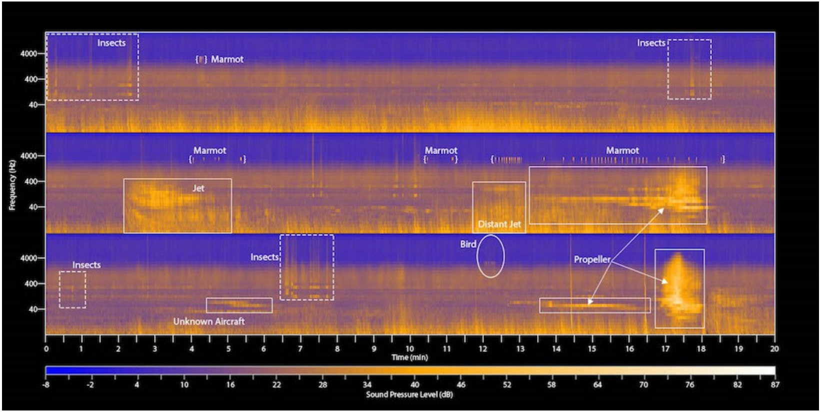


(In English) the two lowest frequency bands are sufficient to determine the vowel.

These are called the *formants*.

Spectrogram of the American English vowels [i, u, a] as pronounced by a native from Louisiana demonstrating the frequencies of first and second formants. Source: `http://en.wikipedia.org/wiki/File:Spectrogram_-iua-.png`

SOUNDSCAPE



Soundscape of Mount Rainier, showing marmot, bird, insect and aircraft noises.
Source: `https://en.wikipedia.org/wiki/File:Mount_Rainier_soundscape.jpg`

Some uses in apps

Spotify

Part of Spotify's recommender system algorithm involves mapping songs to spectrograms to determine genres.

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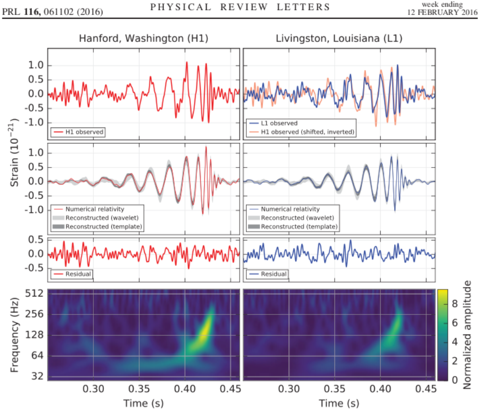
Shazam

The first step in Shazam's algorithm (called "perceptual hashing") to determine what song is being played is to compute a spectrogram.

Gravitational waves



Abbott et al. 2016 Physical Review Letters