(Geometric) Camera Calibration

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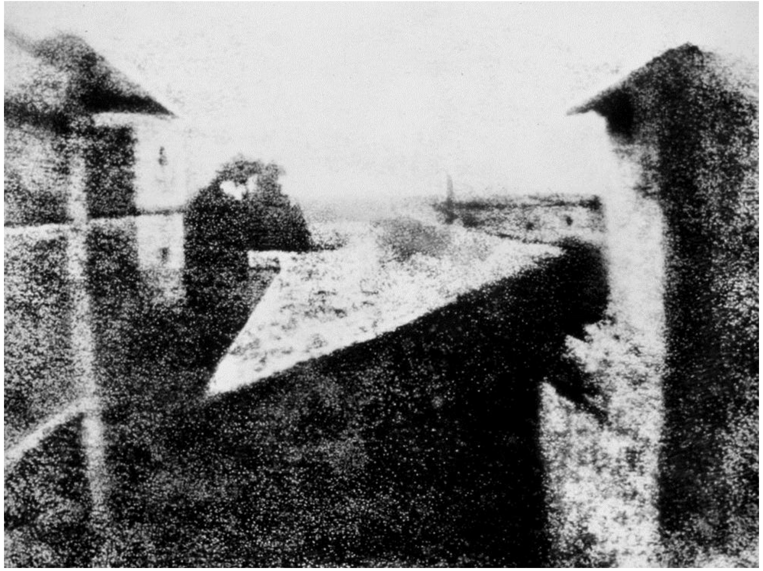
Camera Calibration

- **Cameras and CCDs**
- Aberrations
- Perspective Projection
- Calibration

Cameras

- First photograph due to Niepce (1826)



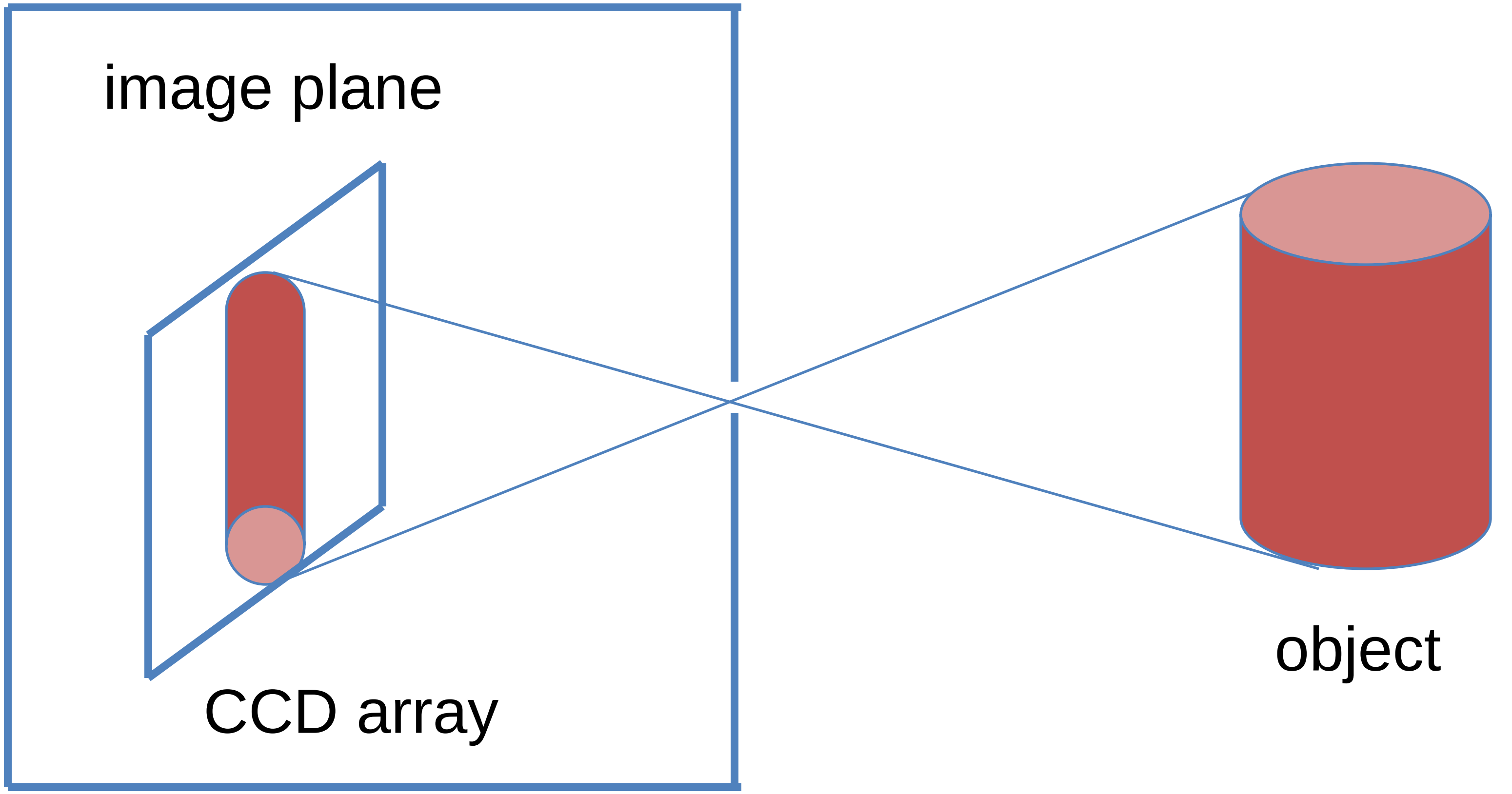
- http://www.hrc.utexas.edu/exhibitions/permanent/firstphotograph/process/#top

Digital Camera vs. "Film" Camera

- Charge-Coupled Device (CCD)
  - Image plane is a CCD array instead of film
  - CCD arrays are typically ¼ or ½ inch in size
  - CCD arrays have a pixel resolution (e.g., 640x480, 1024x1024)
  - CCD Cameras have a maximum "frame rate", usually determined by the hardware and bandwidth

- Number of CCDs
  - 3: each CCD captures only R, G, or B wavelengths
  - 1: the single CCD captures RGB simultaneously, reducing the resolution by 1/3 (kinda)
- Video
  - Interlaced: only "half" of the horizontal lines of pixels are present in each frame
  - Progressive scan: each frame has a full-set of pixels

The simplest 1-CCD camera in town



Exposures

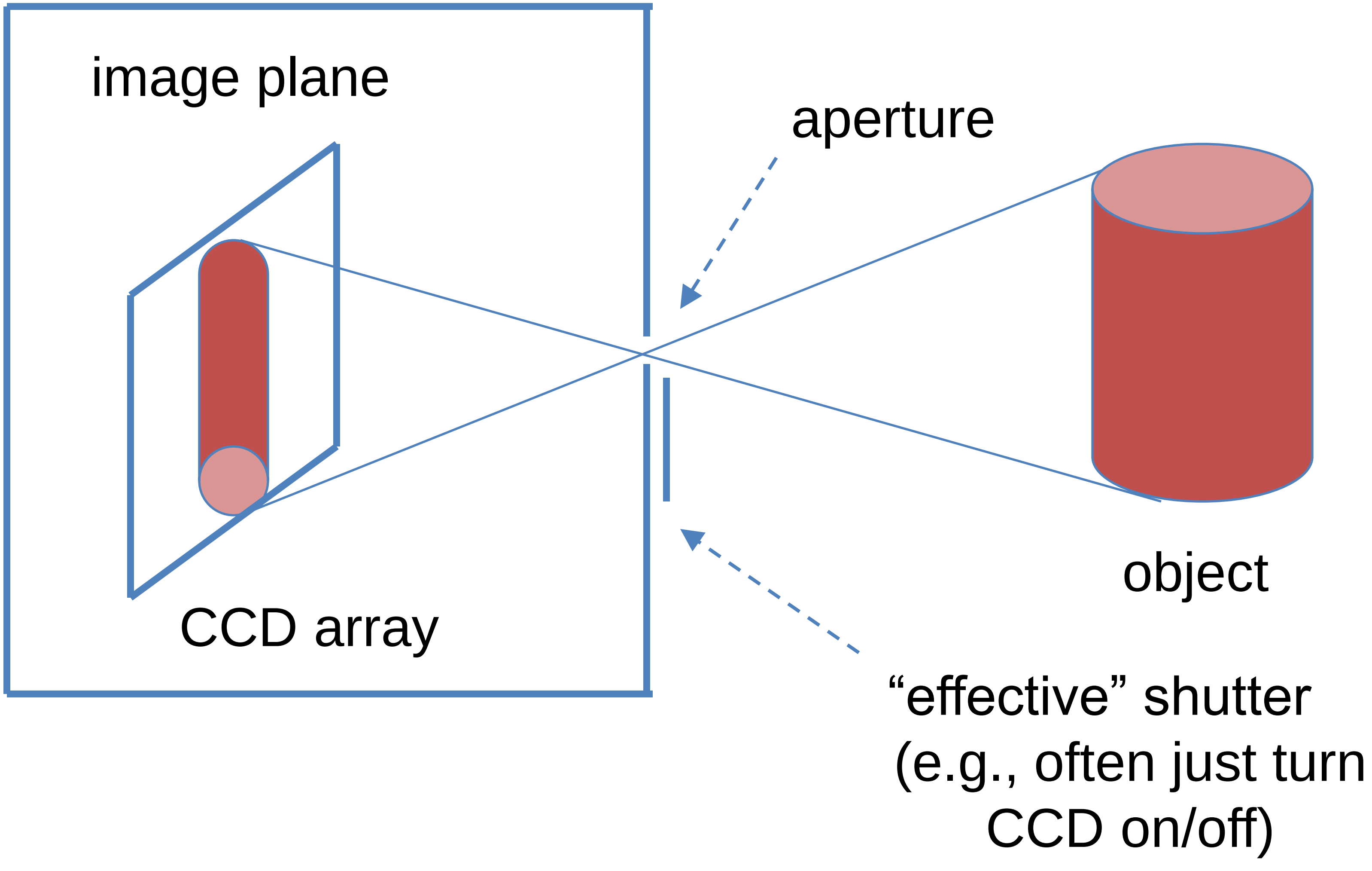


image plane
aperture
CCD array
object
"effective" shutter
(e.g., often just turn
CCD on/off)

Exposures

- An “exposure” is when the CCD is exposed to the scene, typically for a brief amount of time and with a particular set of camera parameters
- The characteristics of an “exposure” are determined by multiple factors, in particular:
  - Camera aperture
    - Determines amount of light that shines onto CCD
  - Camera shutter speed
    - Determines time during which aperture is “open” and light shines on CCD

Camera Calibration

- Digital Cameras and CCDs
- **Aberrations**
- Perspective Projection
- Calibration

Aberrations

- A "real" lens system does not produce a perfect image

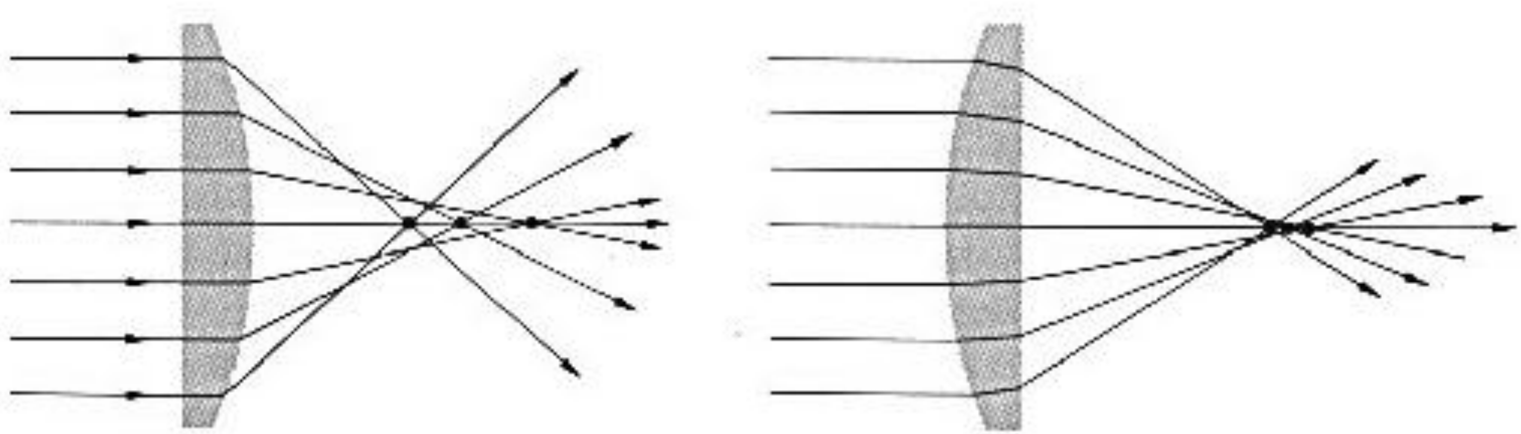
- Aberrations are caused by imperfect manufacturing and by our approximate models
  - Lenses typically have a spherical surface
    - Aspherical lenses would better compensate for refraction but are more difficult to manufacture
  - Typically 1st order approximations are used
    - Remember $\sin \Omega = \Omega - \Omega^3/3! + \Omega^5/5! - \ldots$
    - Thus, thin-lens equations only valid iff $\sin \Omega \approx \Omega$

Aberrations

- Most common aberrations:
  - Spherical aberration
  - Coma
  - Astigmatism
  - Curvature of field
  - Chromatic aberration
  - **Distortion**

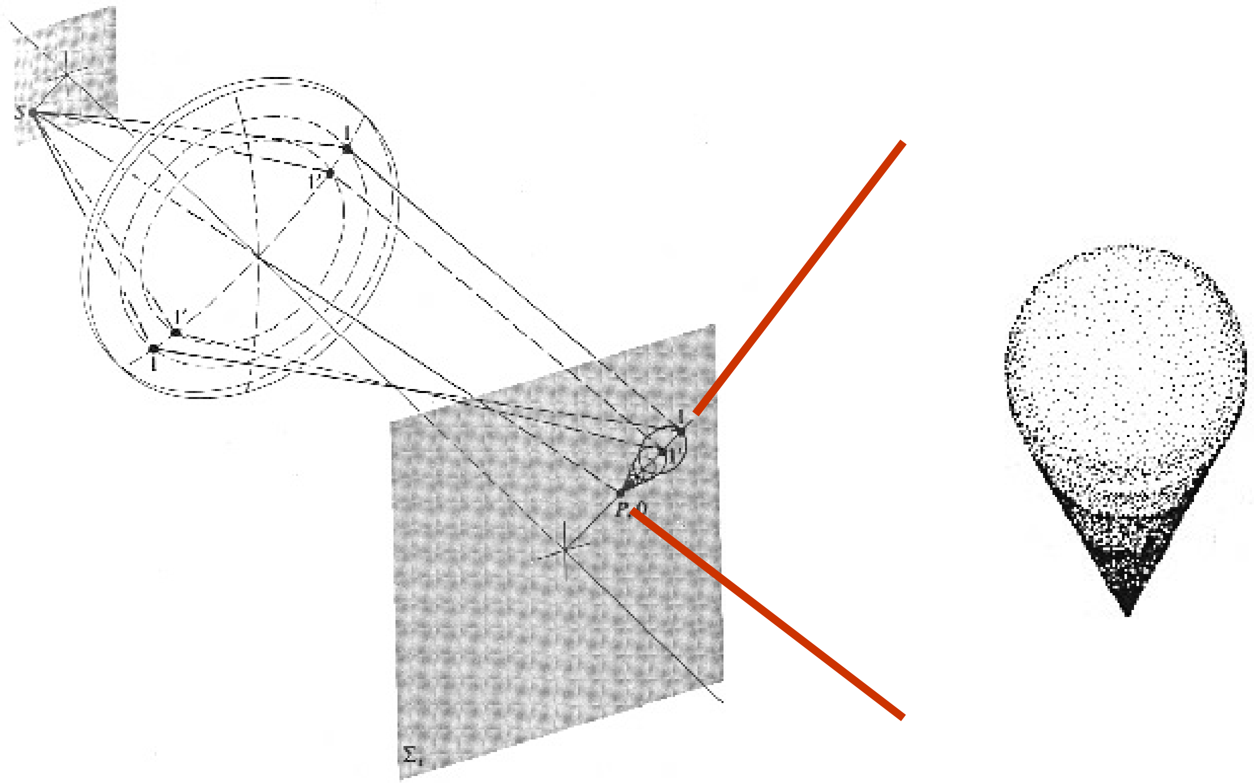
Spherical Aberration

- Deteriorates axial image



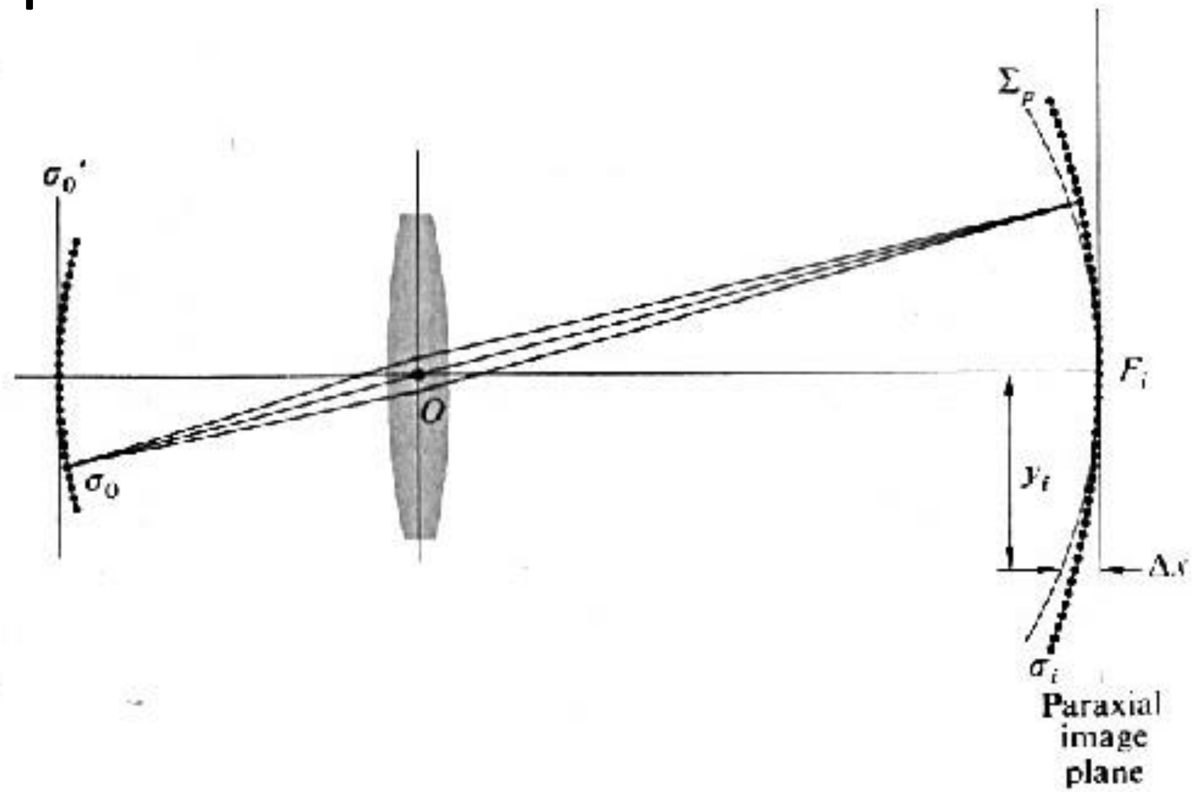
Coma

- Deteriorates off-axial bundles of rays



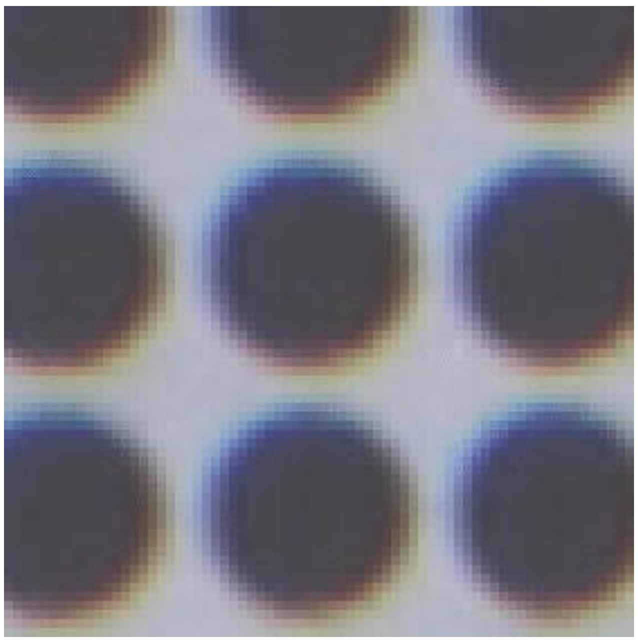
Astigmatism and Curvature of Field

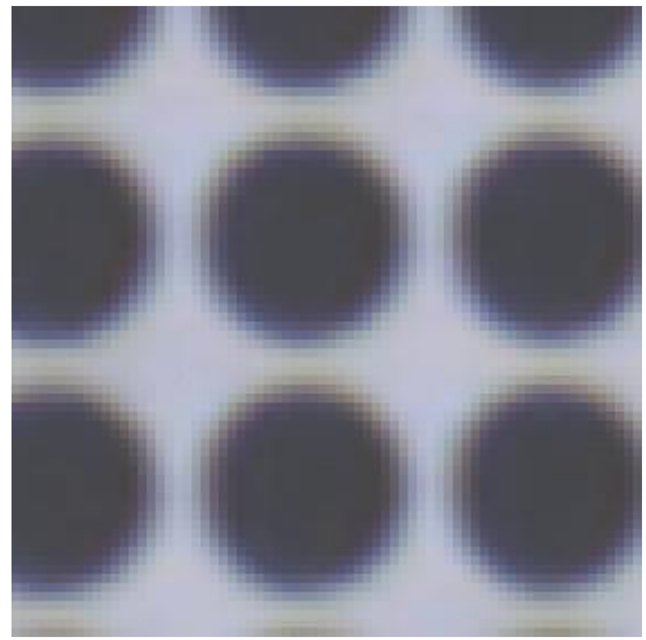
- Produces multiple (two) images of a single object point



Chromatic Aberration

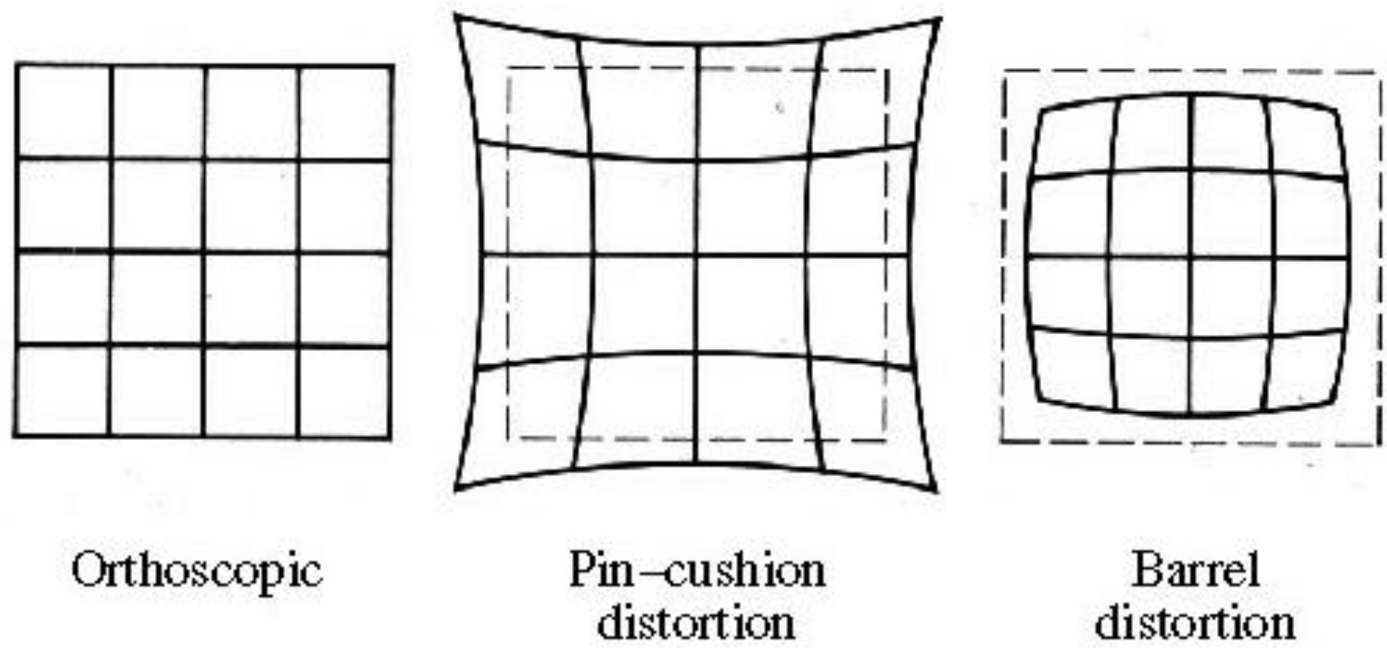
- Caused by wavelength dependent refraction
  - Apochromatic lenses (e.g., RGB) can help



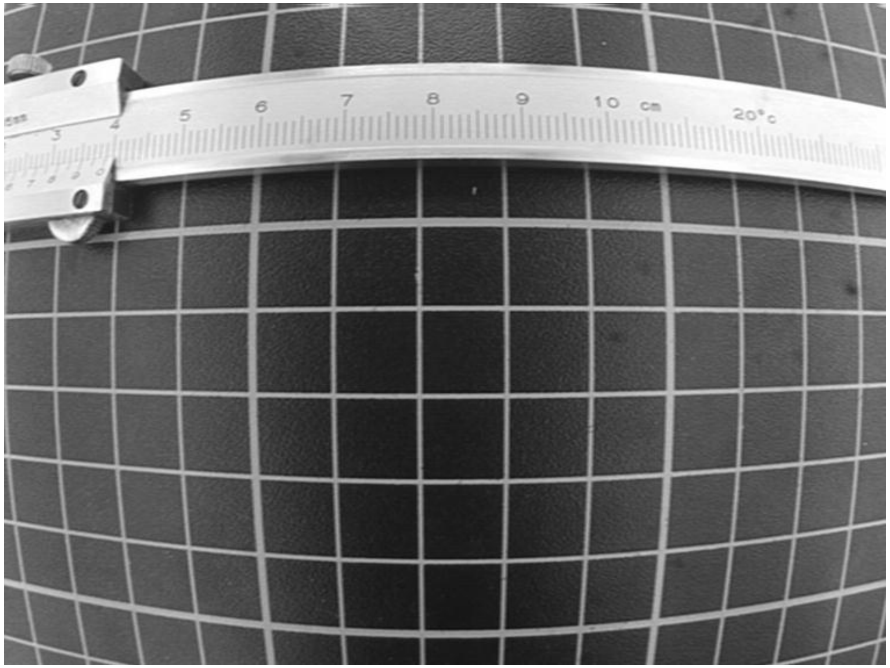


Distortion

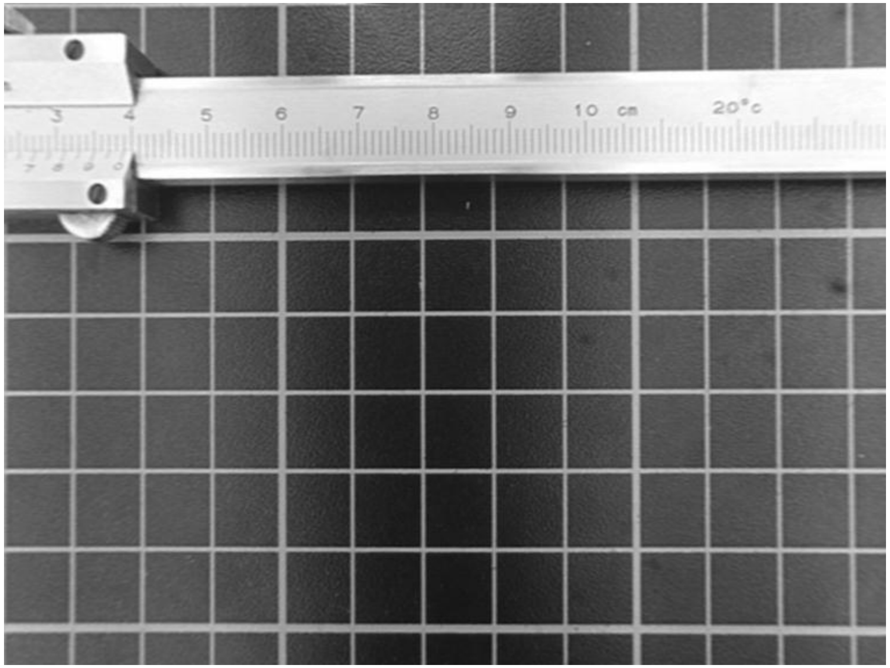
- Radial (and tangential) image distortions



Radial Distortion



before



after

Radial Distortion

- (x, y) pixel before distortion correction
- (x', y') pixel after distortion correction
- Let $r = (x^2 + y^2)^{-1}$

- Then
  - $x' = x(1 - \Delta r/r)$
  - $y' = y(1 - \Delta r/r)$
  - where $\Delta r = k_0 r + k_1 r^3 + k_2 r^5 + \ldots$

- Finally,
  - $x' = x(1 - k_0 - k_1 r^2 - k_2 r^4 - \ldots)$
  - $y' = y(1 - k_0 - k_1 r^2 - k_2 r^4 - \ldots)$

Camera Calibration

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- **Perspective Projection**
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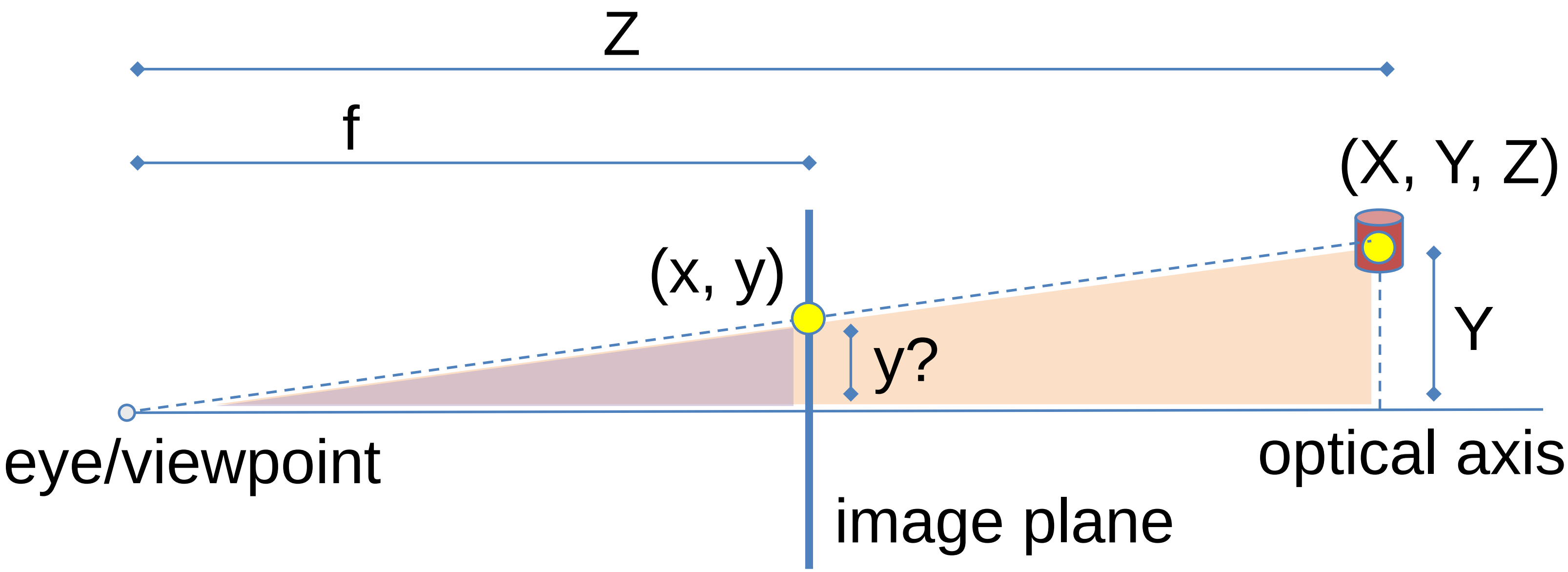
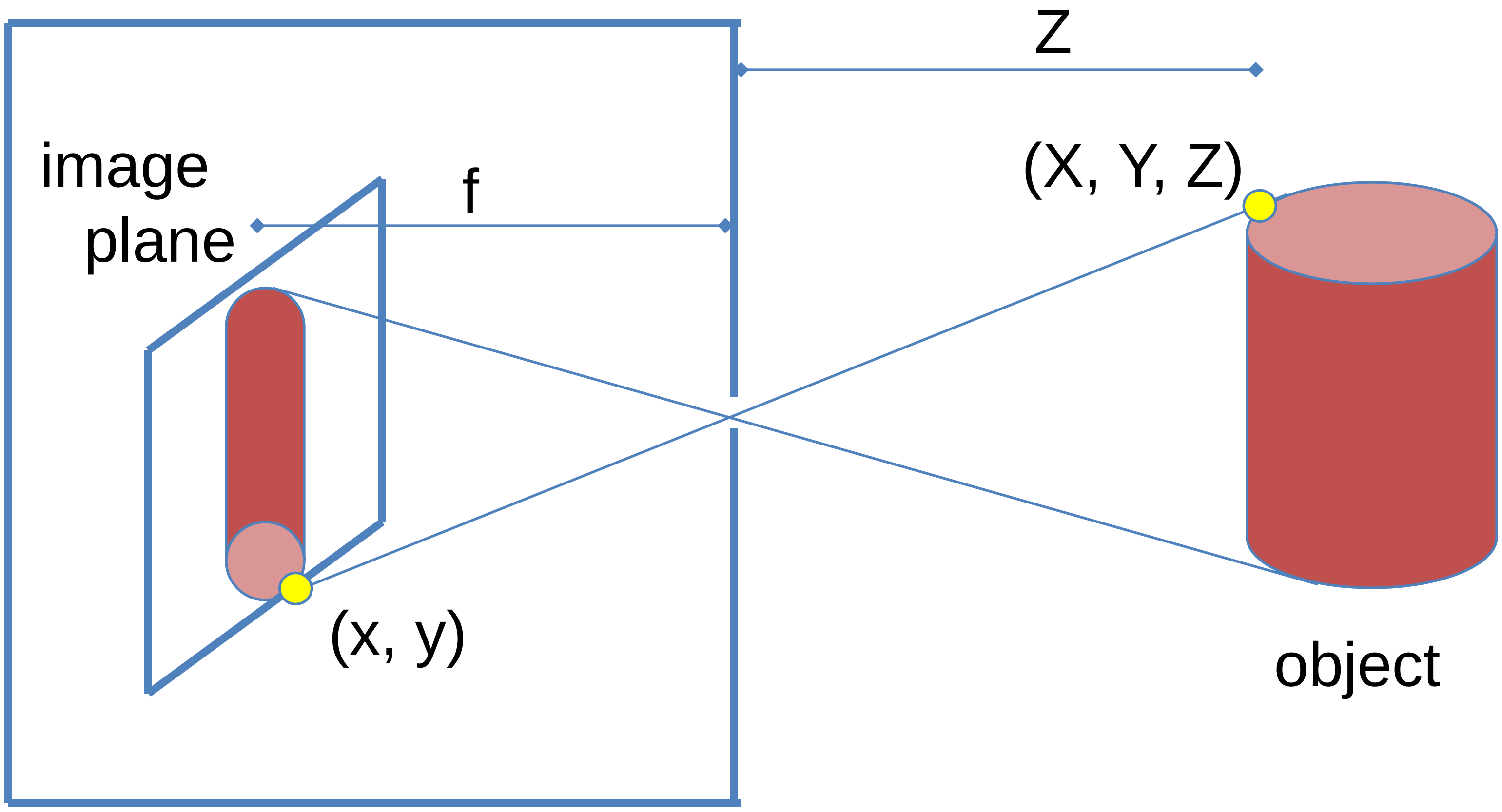
Perspective Projection



$$\frac{y}{f} = \frac{Y}{Z} \quad \Rightarrow \quad y = f\,\frac{Y}{Z} \quad \& \quad x = f\,\frac{X}{Z}$$

Perspective Projection



$$x = f\,\frac{X}{Z} \qquad y = f\,\frac{Y}{Z}$$

Camera Calibration

- Digital Cameras and CCDs
- Aberrations
- Perspective Projection
- **Calibration**

Tsai’s Camera Calibration

- A widely used camera model to calibrate conventional cameras based on a pinhole camera

- Reference
  - “A Versatile Camera Calibration Technique for High-Accuracy 3D Machine Vision Metrology Using Off-the-Shelf TV Cameras and Lenses”, Roger Y. Tsai, IEEE Journal of Robotics and Automation, Vol. 3, No. 4, August 1987

Zhang's Camera Calibration

- Another widely used camera model to calibrate conventional cameras based on a pinhole camera
- Many implementations are floating around!
- Reference
  - "A Flexible New Technique for Camera Calibration", Zhengyou Zhang, IEEE Trans. on PAMI, 22(11):1330-1334, 2000

Bouguet's Camera Calibration

- Another widely used camera model to calibrate conventional cameras based on a pinhole camera
- Many implementations are floating around!
- Reference:
  http://www.vision.caltech.edu/bouguetj

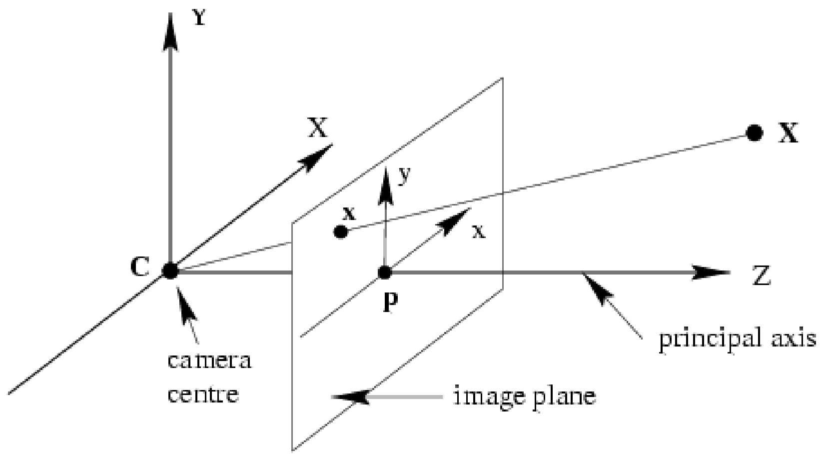
Calibration Goal

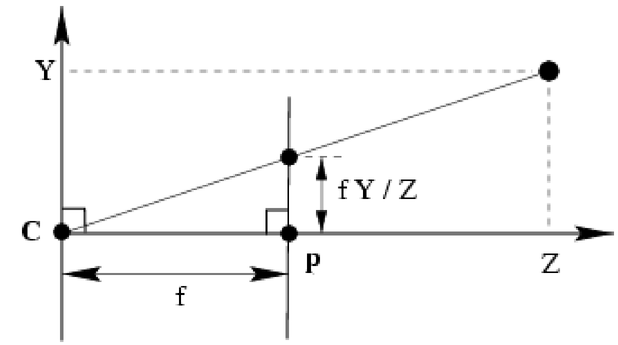
- Determine the intrinsic and extrinsic parameters of a camera (with lens)

Camera Parameters

- Intrinsic/Internal
  - Focal length f
  - Principal point (center) p_x, p_y
  - Pixel size s_x, s_y
  - (Distortion coefficients) $k_1, \ldots$
- Extrinsic/External
  - Rotation ϕ, φ, ψ
  - Translation t_x, t_y, t_z

Focal Length

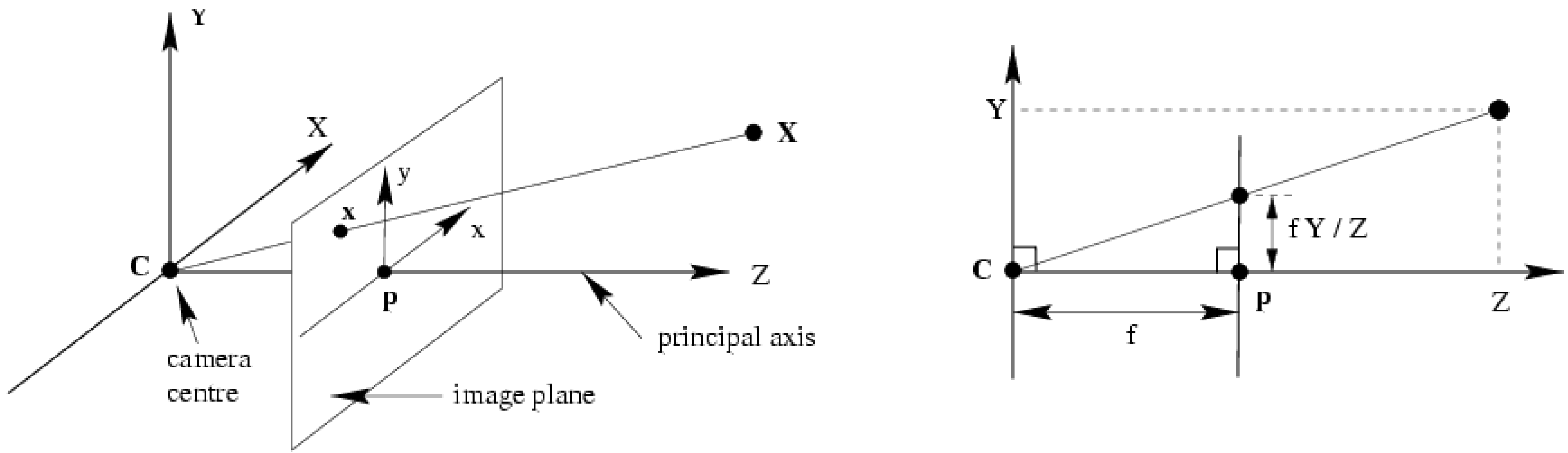




$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} fX/Z \\ fY/Z \end{pmatrix}$$

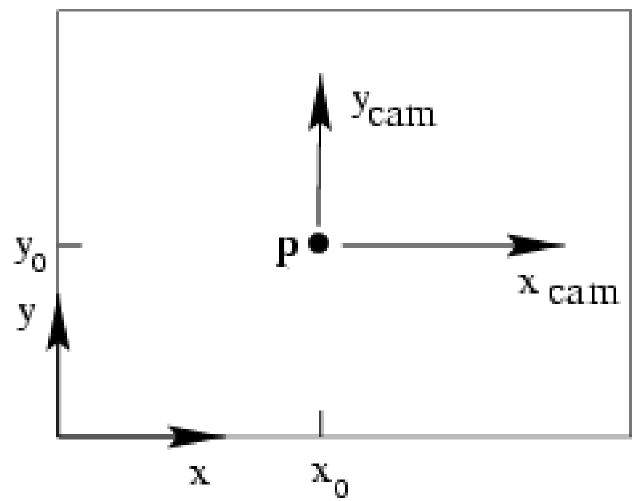
$$\begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} \quad & \quad & \quad & \quad \\ \quad & \quad & \quad & \quad \\ \quad & \quad & \quad & \quad \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Focal Length



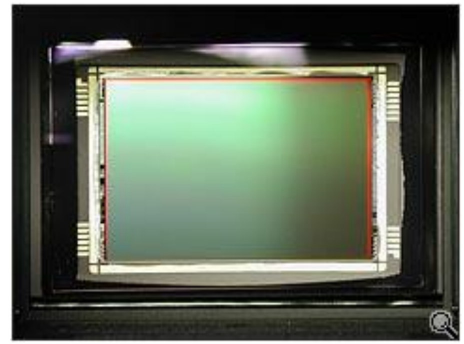
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} fX/Z \\ fY/Z \end{pmatrix} \Leftarrow \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Principal Point



$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \Leftarrow \begin{pmatrix} fX + Zp_x \\ fY + Zp_y \\ Z \end{pmatrix} = \begin{bmatrix} f & 0 & p_x & 0 \\ 0 & f & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

CCD Camera: Pixel Size

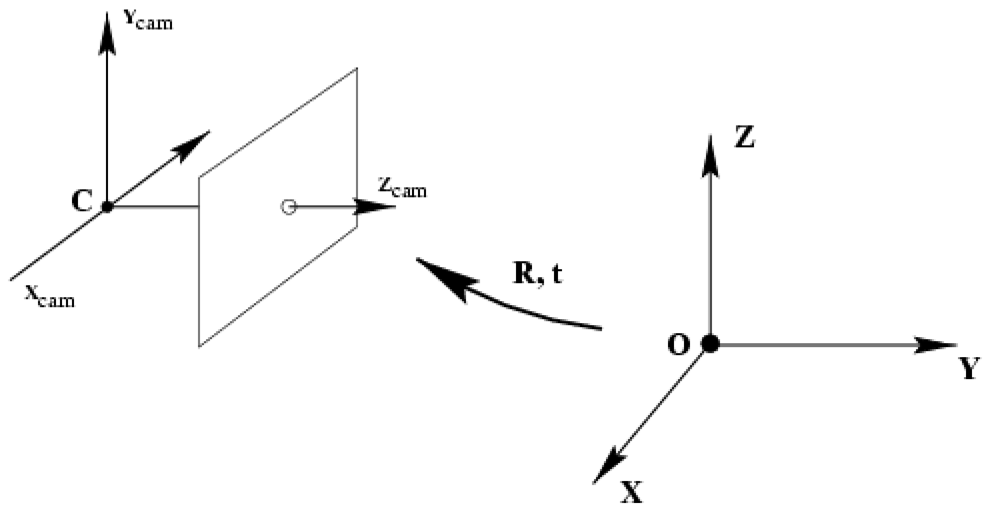


$$K = \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix} \begin{bmatrix} f & 0 & p_x & 0 \\ 0 & f & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$K = \begin{bmatrix} \alpha_x & 0 & p_x & 0 \\ 0 & \alpha_y & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(intrinsic) calibration matrix

Translation & Rotation



$$\tilde{x}_{cam} = R(\tilde{X} - C)$$

$$\tilde{x}_{cam} = R\tilde{X} - \underbrace{RC}_{-t}$$

$$\tilde{x}_{cam} = \underbrace{\begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}}_{P} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$\boxed{\text{(extrinsic) calibration matrix}}$ P

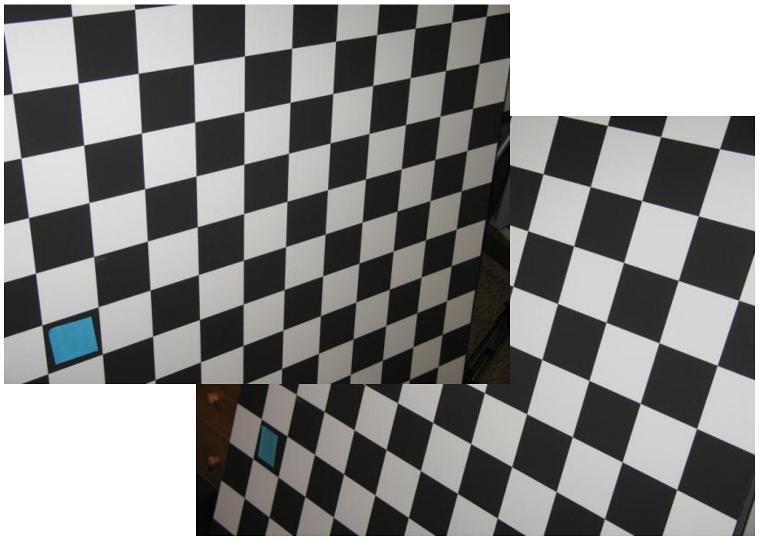
$$R = R_\phi R_\varphi R_\psi$$

3x3 rotation matrices

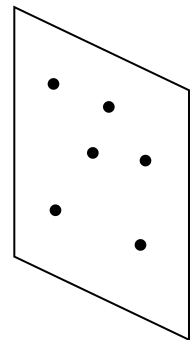
$$t = \begin{bmatrix} t_x & t_y & t_z \end{bmatrix}^T$$

translation vector

Calibration Task

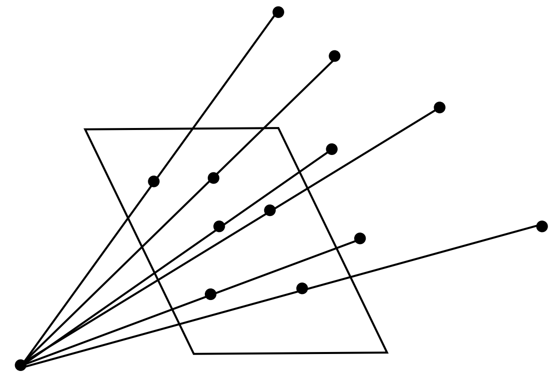


physical arrangement
(calibration pad)



observation
(camera with initial parameters)

points in space



calibration result
(camera with calibrated parameters)

Given $\tilde{X}_i \leftrightarrow \tilde{x}_i$ What is K? P ?

Camera Calibration: Conics

Conic is degree 2 curve on a plane:

$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$

or

$$x^T C x = 0$$

$$C = \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix}$$

Camera Calibration: Conics

- A point transformation on an image is

$$x' = Hx$$

(for homography H)

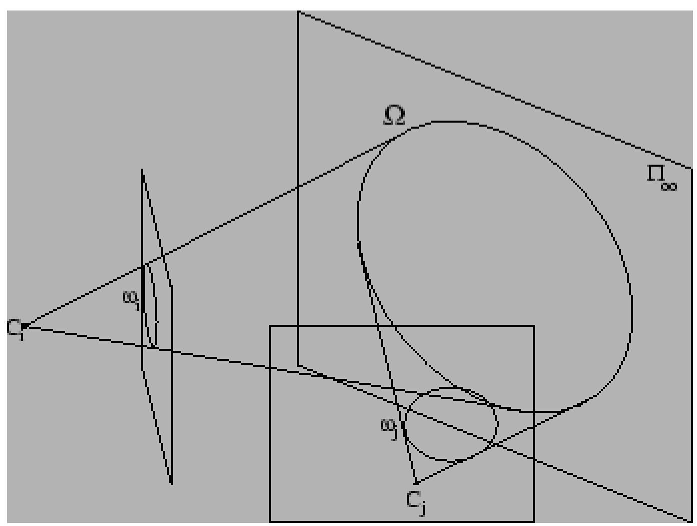
- A conic transformation on an image is

$$C' = H^{-T} C H^{-1}$$

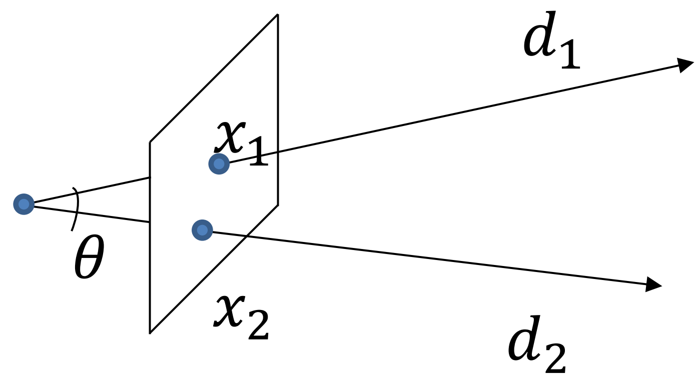
Camera Calibration: Absolute Conic

- The Absolute Conic Ω is invariant under Euclidean transformations and critical to camera calibration

(conic = degree 2 curve on a plane)



Angle between two rays



$$x_i = K d_i$$

$$\cos\theta = \frac{d_1^T d_2}{\|d_1\|\|d_2\|} = \frac{(K^{-1}x_1)^T (K^{-1}x_2)}{\|K^{-1}x_1\|\|K^{-1}x_2\|}$$

$$= \frac{x_1^T (K^{-T} K^{-1}) x_2}{\|x_1^T (K^{-T} K^{-1}) x_1\|\|x_2^T (K^{-T} K^{-1}) x_2\|}$$

Absolute Conic

- Given point on Ω called $x_\infty = [d^T \ 0]^T$, its image on a general camera is

$$x = KRd$$

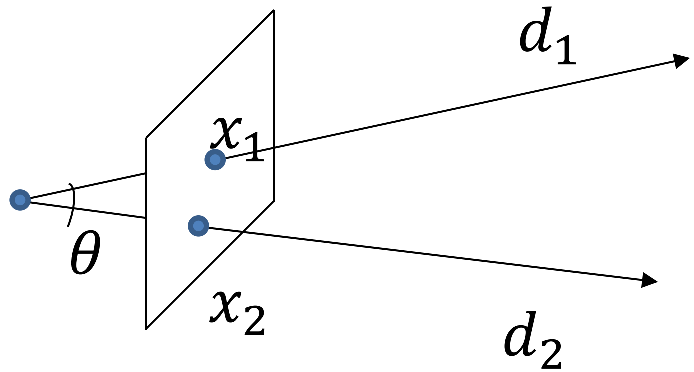
- Recall $x' = Hx$
- Thus image of the conic is

$$\omega = H^{-T} C H^{-1} = (KR)^{-T} C (KR)^{-1} = (KK^T)^{-1}$$

or

$$\omega = K^{-T} K^{-1}$$

Angle between two rays

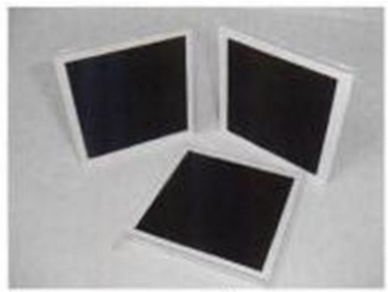


$$x_i = K d_i$$

$$\cos\theta = \frac{x_1^T \omega x_2}{\|x_1^T \omega x_1\| \|x_2^T \omega x_2\|}$$

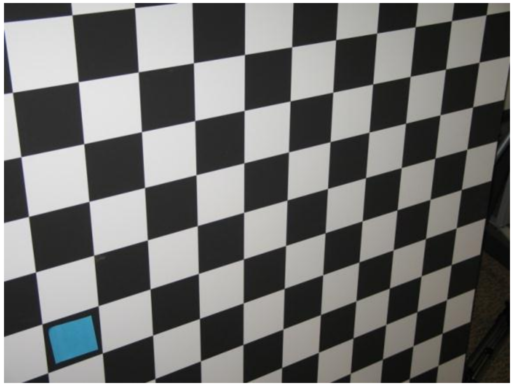
Simple Calibration Device

- Observe these 3 planes, forming 3 homographies



- Each $H = [h_1\ h_2\ h_3]$ gives constraints
  $h_1^T \omega h_2 = 0$ and $h_1^T w h_1 = h_2^T h_2$
- Conic ω is determined from 5 or more such equations, up to a scale
- Compute K from $\omega = (KK^T)^{-1}$ using Cholesky factorization, for example

Zhang's Camera Calibration



- **1. Detect corners**
- **2. Estimate matrix P**
- **3. Recover instrinsic/extrinsic parameters**
- 4. Refine: bundle adjustment

Typical Formulation

Let $M = KP$

$$\tilde{x}_{cam} = M\tilde{X}$$

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x'/w' \\ y'/w' \end{pmatrix}$$

$$x = (m_1 \cdot \tilde{X})/(m_3 \cdot \tilde{X})$$

$$y = (m_2 \cdot \tilde{X})/(m_3 \cdot \tilde{X})$$

A Linear Formulation

$$x = (m_1 \cdot \tilde{X})/(m_3 \cdot \tilde{X})$$
$$y = (m_2 \cdot \tilde{X})/(m_3 \cdot \tilde{X})$$

for $i = 1..n$ observations

$$(m_1 - x_i m_3) \cdot \tilde{X}_i = 0$$
$$(m_2 - y_i m_3) \cdot \tilde{X}_i = 0$$

$2n$ homogeneous linear equations and 12 unknowns (coefficients of M)

Thus, given n≥6 can solve for M; namely $Qm = 0$

$$Q = \begin{bmatrix} \tilde{X}_1{}^T & 0^T & -x_1 \tilde{X}_1{}^T \\ 0^T & \tilde{X}_1{}^T & -y_1 \tilde{X}_1{}^T \\ \ldots & \ldots & \\ \tilde{X}_n{}^T & 0^T & -x_n \tilde{X}_n{}^T \\ 0^T & \tilde{X}n^T & -y_n \tilde{X}_n{}^T \end{bmatrix} \qquad m = \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}$$

A Linear Formulation

Goal: $\min \|Qm\|$ subject to $\|m\| = 1$

Solution: eigenvector of $Q^T Q$ associated with the smallest eigenvalue. Use m to make matrix M.

Decomposing M into Camera Parameters

$$M = \rho[A\ b] = K[R\ t]$$

$$K = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

(often $\gamma = \pi/2$ which means no skew))

Decomposing M into Camera Parameters

$$M = \rho[A\ b] = K[R\ t]$$

$$K = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$ (often $\gamma = 0$ which means no skew))

$B = KR$ and $b = Kt$ (so B is first 3x3 of M)

Let $A = BB^T = KK^T$

$$A = \begin{bmatrix} \alpha^2 + \gamma^2 + u_0^2 & u_0 v_0 + c\beta & u_0 \\ u_0 v_0 + c\alpha & \alpha_v^2 + v_0^2 & v_0 \\ u_0 & v_0 & 1 \end{bmatrix}$$

Decomposing M into Camera Parameters

$$A = \begin{bmatrix} \alpha^2 + \gamma^2 + u_0^2 & u_0 v_0 + c\beta & u_0 \\ u_0 v_0 + c\alpha & \alpha_v^2 + v_0^2 & v_0 \\ u_0 & v_0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} k_u & k_c & u_0 \\ k_c & k_v & v_0 \\ u_0 & v_0 & 1 \end{bmatrix}$$ (assumes square pixels and equal focal length in x and y)

Decomposing M into Camera Parameters

$$\mathrm{A} = \begin{bmatrix} k_u & k_c & u_0 \\ k_c & k_v & v_0 \\ u_0 & v_0 & 1 \end{bmatrix}$$

$$u_0 = A_{13}$$

$$v_0 = A_{23}$$

$$\beta = \sqrt{k_v - v_0^2}$$

$$\gamma = \frac{k_c - u_0 v_0}{\beta}$$

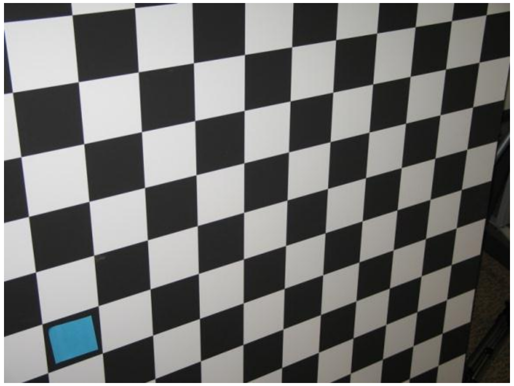
$$\alpha = \sqrt{k_u - u_0^2 - \gamma^2}$$

$$K = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = K^{-1} B$$

$$t = K^{-1} b$$

Zhang's Camera Calibration



- 1. Detect corners
- 2. Estimate matrix P
- 3. Recover instrinsic/extrinsic parameters
- **4. Refine: bundle adjustment**

Bundle Adjustment

- Given initial guesses, use nonlinear least squares to refine/compute the calibration parameters
- Simple but good convergence depends on accuracy of initial guess

Bundle Adjustment

Recall

$$x = (m_1 \cdot \tilde{X}) / (m_3 \cdot \tilde{X})$$

$$y = (m_2 \cdot \tilde{X}) / (m_3 \cdot \tilde{X})$$

$$E = \frac{1}{mn} \sum_{ij} \left[(x_{ij} - \frac{m_{i1} \cdot \tilde{X}_j}{m_{i3} \cdot \tilde{X}_j})^2 + (y_{ij} - \frac{m_{i2} \cdot \tilde{X}_j}{m_{i3} \cdot \tilde{X}_j})^2 \right]$$

Goal is $E \to 0$

Bundle Adjustment

Option A:

Define M as a matrix of 11 unknowns (i.e., $m_{34} = 1$)

And solve for m_{ij}

⇒ Can be made very efficient, especially for sparse matrices

Option B:

Define M as function of intrinsic and extrinsic parameters so that it is "recomputed" during each loop of the optimization