SVM

Given the data $DATA_1 = [X_1\ X_2\ ...\ X_{10}]$ and $DATA_2 = [X_{11}\ X_{12}\ ...\ X_{20}]$

$$DATA_1 = \begin{bmatrix} 1.08 & 0.75 & 0.85 & 0.94 & 0.40 & 1.25 & 1.19 & 0.99 & 0.69 & 1.32 \\ 0.08 & -0.19 & -0.11 & 0.01 & -0.09 & -0.21 & 0.07 & 0.04 & -0.02 & 0.02 \end{bmatrix}$$

$$DATA_2 = \begin{bmatrix} 0.01 & -0.01 & 0.09 & -0.05 & -0.45 & 0.07 & -0.33 & -0.06 & -0.33 & -0.24 \\ 0.85 & 1.05 & 0.93 & 1.41 & 1.45 & 1.20 & 0.88 & 1.08 & 1.10 & 1.01 \end{bmatrix}$$

1. Assign $t_i = 1\ \forall\ i = 1, \ldots, 10$, $t_i = -1\ \forall\ i = 11, \ldots, 20$
2. Construct the linear equation

$$\begin{bmatrix} K(X_1,X_1)t_1^2 & K(X_1,X_2)t_1t_2 & \ldots & K(X_1,X_{20})t_1t_{20} \\ K(X_2,X_1)t_2t_1 & K(X_2,X_2)t_2^2 & \ldots & K(X_2,X_{20})t_2t_{20} \\ \vdots & \vdots & \ddots & \vdots \\ K(X_{20},X_1)t_{20}t_1 & K(X_{20},X_2)t_{20}t_2 & \ldots & K(X_{20},X_{20})t_{20}^2 \\ t_1 & t_2 & \ldots & t_{20} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{20} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 0 \end{bmatrix}$$

   where $K(X_i, X_j) = X_i^T.X_j$ with $\sigma^2 = 1$
3. Solve for $a_1, \ldots, a_{20}$.
4. Choose the values $0 \leq a_i \leq C$. C is the box variable. Choose it as 10. Let $n(M_1) = N_1$. Let the set be M_1.

5. Choose the values $0 \leq a_i \leq C$. Let $n(M_2) = N_2$. Let the set be M_2.
6. Compute $\mathbf{W}$ matrix as follows

$$\underline{\mathbf{W}} = \sum_{n=1, n\in M_2}^{20} a_n t_n \underline{x}_n$$

$\underline{\mathbf{W}} =$

7. Obtain $\mathbf{b}$ using the following:

$$\mathbf{b} = \frac{1}{N_1} \sum_{m\in M_1} \left[t_m - \sum_{n\in M_2} a_n t_n K(X_n, X_m) \right]$$

$\underline{\mathbf{b}} =$

8. Plot the data.
   Plot the line $\mathbf{W}^T.\underline{X} + \underline{\mathbf{b}}$

   Inference: | See that the obtained line partitions $DATA_1$ and $DATA_2$ |

   Use the kernel function used is $K(X_i, X_j) = exp\big(-\frac{(X_i - X_j)^T(X_i - X_j)}{2\sigma^2}\big)$, with $\sigma^2 = 1$.
   Obtain the expression for the linear separation line/plane/hyperplane that partitions the classes.

   Obtain the index obtained for the following test data using the trained SVM.

Data	$\begin{bmatrix} 0.0083 \\ 0.85 \end{bmatrix}$	$\begin{bmatrix} 0.08 \\ 0.93 \end{bmatrix}$	$\begin{bmatrix} 0.06 \\ 1.2 \end{bmatrix}$	$\begin{bmatrix} 1.13 \\ 0.07 \end{bmatrix}$	$\begin{bmatrix} 1.21 \\ 0.20 \end{bmatrix}$
Index					

(OR) Demonstrate soft-margin SVM classifier using Image.mat (dataset).