# Homework 3

Due: 1:00 pm, 09/25/2023

## Question 1

(1) Prove Theorem 3.2 and 3.3 in SLP.

(2) Exercise 4.4 in SLP.

## Question 2: Guess-and-Verify

Consider the following dynamic programming problem

$$V(k) = \max_{k'} \log(\theta k^{\alpha} - k') + \beta V(k')$$

where $\beta \in (0, 1)$.

(1) Use a guess-and-verify approach to obtain an closed-form solution of the value function $V(k)$ and policy function $g(k)$.

(2) Use your results from part (1) to show that for any initial capital $k_0 > 0$, the economy will converge to a unique steady state.

(3) Suppose the agent also values leisure $(1 - \ell)$, and the dynamic programming problem becomes

$$V(k) = \max_{c,\ell,k'} \log c + \log(1 - \ell) + \beta V(k')$$

subject to

$$c + k' = \theta k^{\alpha} \ell^{1-\alpha}$$

Can you still obtain a closed-form solution of the value function $V(k)$, policy function $k' = g(k)$ and $\ell = h(k)$?

## Question 3: Growth Model in Transition Again

Consider the same growth model in Homework 2 Question 3.

(1) Write down the dynamic programming problem for the social planner in this economy.

(2) With the same parameters as you used in Homework 2 Question 3, compute the policy function for capital, $k' = g(k)$, using both value function iteration and policy function iteration.

(3) Use the policy function to find the steady state capital level $\overline{k}$. Is this steady state capital level the same as your analytical result?

(4) Compute the transition paths for this economy when $k_0 = 0.8\overline{k}$ and $k_0 = 1.2\overline{k}$. Are these transition paths the same as you obtained in Homework 2 Question 3?