

Homework 4

Due: 1:00 pm, 10/02/2023

Question 1: Government Spending

Consider an economy with a representative, infinitely lived consumer who has the utility function

$$U = \sum_{t=0}^{\infty} \beta^t U(c_t, \ell_t, g_t)$$

where c_t is private consumption, ℓ_t is labor supply, and g_t is the government spending. The utility function is

$$U(c, \ell, g) = \log c + \log(1 - \ell) + \log g$$

Suppose that the production function is $F(k, \ell) = k^\alpha \ell^{1-\alpha}$, and the capital depreciation rate is δ .

The government can only use labor income tax to finance the government spending g_t and the government has to run a balanced budget every period. Denote the tax rate in period t as τ_t and the total labor income tax is $\tau_t w_t \ell_t$. Assume that the initial capital k_0 is predetermined.

(1) Define a tax-distorted Arrow-Debreu equilibrium for this economy, where consumers take $\{\tau_t, g_t\}$ as given.

From now on, assume that $\delta = 1$ and the government always chooses a constant tax rate $\tau \in (0, 1)$.

(2) Consider the competitive equilibrium defined in part (a). First guess that the capital-output ratio $\frac{k_{t+1}}{y_t}$ is always a constant s . Given this guess, show that the consumption-output ratio $\frac{c_t}{y_t}$, the government spending-output ratio $\frac{g_t}{y_t}$, and the labor supply are all constants. Second, verify this guess and derive the value for the capital-output ratio s .

(3) Display the Bellman equation for the social planner in this economy. What is the private consumption to government spending ratio in the solution to the planner's problem?

(4) Based on your results in part (2), find the private consumption to government spending ratio in the competitive equilibrium. Briefly explain why this ratio is different from that in the social planner's solution.

(5) Use economic intuition (no equations required) to explain whether the steady state level of government spending is monotonically increasing in τ or not.

Question 2: Endogenous Labor

There is an economy with two groups of consumers. The preferences of all consumers are identical and are given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$$

where n_t is the labor supply.

The measure of consumers in group A is μ , and their labor efficiency is ϵ_A . Similarly, the measure of consumers in group B is $1 - \mu$, and their labor efficiency is ϵ_B .

Suppose the production function in this economy is given by

$$Y = \theta K^\alpha N^{1-\alpha}$$

and the capital depreciation rate is δ .

- (1) Define a recursive equilibrium for this economy.
- (2) What is the aggregate capital level in the steady state? Write down the system of equations that characterizes the consumption, saving, and labor supply in the steady state.
- (3) Suppose the preference of consumers in group A becomes

$$\sum_{t=0}^{\infty} \beta^t u(c_t, n_t, n_{t-1})$$

Define a recursive equilibrium for this economy. Is the allocation in this equilibrium efficient or not?

Question 3: Heterogeneous Wealth

There is an economy with two groups of consumers. The measure of consumers in group A is μ , and the measure of consumers in group B is $1 - \mu$. The initial capital holding for these two groups of consumers are different, which are K_0^1 and K_0^2 respectively. Their preference is the same, given by

$$\sum_{t=0}^{\infty} \beta^t \log(c_t)$$

Suppose the production function in this economy is given by

$$Y = \theta K^\alpha N^{1-\alpha}$$

and the capital depreciation rate is δ .

Assume that individual asset holding has to be greater than or equal to \underline{D} .

(1) Define a recursive equilibrium for this economy. Characterize the steady state capital holding K^* of the parameters.

(2) Write a computer code to solve the aggregate law of motion for the recursive equilibrium. Suppose that $K_0^1 = 0.5K^*$, $K_0^2 = 1.2K^*$, and $\mu = 0.5$. Plot the transition path for the saving of these two groups in the first 200 periods.