

Econ 510 Part I: Homework 1

Due: 11:00 am, 09/11/2023

Question 1

Consider an economy with two groups of infinitely lived consumers, all of whom share the same preference

$$U(c_0, c_1, \dots) = \sum_{t=0}^{\infty} \beta^t \log(c_t)$$

where $0 < \beta < 1$. Suppose the endowment processes for the two groups are given by

$$w^1 = \{\delta, 4 - \delta, \delta, 4 - \delta, \dots\}$$

$$w^2 = \{4 - \delta, \delta, 4 - \delta, \delta, \dots\}$$

where $\delta \in (2, 4)$.

- (1) Describe the structure of an Arrow-Debreu market in this economy. Explain when the markets are open, what are traded, who trades with whom and so on.
- (2) Define Pareto efficient allocation in this economy. Calculate the Pareto efficient allocation in this economy as a function of the weight (α_1, α_2) in the planner's problem.
- (3) Calculate the Arrow-Debreu equilibrium in this economy. Show that the allocation in this equilibrium is efficient. Explain how the allocation varies with δ and why.
- (4) Describe the structure of a sequential market. Define a sequential markets equilibrium. Calculate the sequential markets equilibrium. How is the sequential market equilibrium related to the Arrow-Debreu market equilibrium?
- (5) Define an Arrow-Debreu equilibrium with transfers. Find the transfer payments necessary to implement the allocation $(c_t^1, c_t^2) = (2, 2)$.

Question 2

Consider an economy with two groups of infinitely lived consumers, all of whom share the same preference

$$U(c_0, c_1, \dots) = \sum_{t=0}^{\infty} \beta^t \log(c_t)$$

where $0 < \beta < 1$. Suppose the endowment process is given by

$$\begin{aligned} w^1 &= \{w_t^1\}_{t=0}^{\infty} \\ w^2 &= \{w_t^2\}_{t=0}^{\infty} \end{aligned}$$

- (1) Calculate the Arrow-Debreu equilibrium in this economy. Comment on the properties of the allocation.
- (2) Calculate the consumers' continuation welfare in each period under autarky.
- (3) Supposing the contract in the Arrow-Debreu economy cannot be enforced, do you think the consumers will respect the contract in part (1) or not? Suppose the endowment process is the same as in Question 1, under what condition, consumers will stay in the Arrow-Debreu contract?
- (4) Suppose the markets are incomplete, and only goods in period $t = 1, 3, 5, 7, \dots$ can be traded. Assuming the endowment process is the same as in Question 1, what is the allocation in the Arrow-Debreu equilibrium in this case?

Question 3

Consider an economy with N groups of infinitely lived consumers. The preference for group $i \in \{1, \dots, N\}$ is

$$U_i(c_0, c_1, \dots) = \sum_{t=0}^{\infty} \beta^t u_i(c_t)$$

where $0 < \beta < 1$ and $u'_i(c)$ is monotonically decreasing. Suppose the endowment process for group i is given by $w^i = \{w_t^i\}_{t=0}^{\infty}$.

- (1) In an Arrow-Debreu equilibrium, what are the properties of the allocation?
- (2) Does the first welfare theorem still hold?