Homework 5

Due: 11:00 am, Oct 12, 2023

Question 1: Arrow-Debreu Endowment Economy

Consider the following endowment economy. There are two groups of consumers, A and B. The endowment for group i in period t is ω_t^i. In period 0, the endowment is $\omega_0^A = 1$ and $\omega_0^B = 3$. For $t \geq 1$, the endowment process satisfies

$$\omega_t^A + \omega_t^B = 4$$

where $\omega_t^A = 3$ and $\omega_t^B = 1$ with probability p and $\omega_t^A = 1$ and $\omega_t^B = 3$ with probability $1 - p$. The period utility function is $u(c) = \log(c)$ and the discount rate is β.

(1) Define the history s^t for this economy and derive $\pi(s^t)$.

(2) Define an Arrow-Debreu equilibrium for this economy.

(3) Derive the price $p(s^t)$ for the equilibrium where $p(s^0)$ is normalized to 1.

(4) Solve for the allocation $c^i(s^t)$ for $i = A$ and $i = B$.

Question 2: Production Economy with Shocks

Consider the production economy we studied in the class. The production function is given by

$$y = zk^{\alpha}\ell^{1-\alpha}.$$

Assume that the technology level z can take two values: $z_H = 1 + \epsilon$ and $z_L = 1 - \epsilon$, and it follows a Markov process given by $\Gamma(z'|z)$ given by

$$\Gamma(z'|z) = \begin{bmatrix} p & 1-p \\ 1-p & p \end{bmatrix}$$

The capital evolves according to

$$k' = (1-\delta)k + i.$$

The preference of the representative household is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t).$$

and the labor supply cannot be larger than 1.

(1) Define an Arrow-Debreu equilibrium for this economy.

(2) Define a social planner's problem for this economy. Show that the allocation in part (1) and (2) satisfy the same set of system of equations.

(3) Write down the value function for the social planner's problem.

(4) Assume that $u(c) = \log(c)$, $\alpha = 0.3$, $\delta = 0.07$, $\beta = 0.99$, $\epsilon = 0.05$, and $p = 0.5$. Solve for the policy function for k' in part (3) numerically. Now set $p = 0.8$, compare the policy function with that when $p = 0.5$, and explain your findings.

Question 3: Individual Saving and Consumption Problem

Suppose there is an individual consumer who faces stochastic income over time. Assume that the income level z can take two values: $z_H = 1 + \epsilon$ and $z_L = 1 - \epsilon$, and it follows a Markov process given by $\Gamma(z'|z)$ given by

$$\Gamma(z'|z) = \begin{bmatrix} p & 1-p \\ 1-p & p \end{bmatrix}$$

The consumer can borrow and save at a constant interest rate $1 + r$, but there is a borrowing limit $\underline{b}$.

The preference of the consumer is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t).$$

(1) Write down the value function for the consumer's problem.

(2) Assume that $1 + r = 1.02$, $\beta = 0.96$, $\underline{b} = -0.1$, $\epsilon = 0.1$ and $p = 0.8$. Solve for the saving policy functions, and compare them with the 45 degree line. Explain your findings.

(3) Vary the value of p, ϵ, and $\underline{b}$. Explain how these parameters affect consumers' saving behavior and why.