1) $\left[\begin{array}{cccc|c} 1 & 2 & -3 & 3 & -5 \\ -2 & -4 & 8 & -6 & 14 \end{array}\right]$

$\xrightarrow{2R_1+R_2} \left[\begin{array}{cccc|c} 1 & 2 & -3 & 3 & -5 \\ 0 & 0 & 2 & 0 & 4 \end{array}\right]$

$\xrightarrow{\frac{3}{2}R_2+R_1} \left[\begin{array}{cccc|c} 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 2 & 0 & 4 \end{array}\right]$

$x_1 + 2x_2 + 3x_4 = 1$

$2x_3 = 4,\ x_3 = 2$

$x_1 = 1 - 2x_2 - 3x_4,\ x_2, x_4$ free, let $x_2 = s$, $x_4 = t$

$= 1 - 2s - 3t$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1-2s-3t \\ s \\ 2 \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix} + s\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} -3 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

2) $\left[\begin{array}{cc|c} 2 & 1 & h \\ 6 & 3 & -6 \end{array}\right] \xrightarrow{-3R_1+R_2} \left[\begin{array}{cc|c} 2 & 1 & h \\ 0 & 0 & -3h-6 \end{array}\right]$

a) Need $-3h-6=0$ to be consistent

$h = -2$

b) The matrix is not invertible because $\det(A) = 0$

3) $p(x) = a_0 + a_1 x + a_2 x^2$

$p(1) = a_0 + a_1 + a_2 = -1$

$p(2) = a_0 + 2a_1 + 4a_2 = -2$

$p(3) = a_0 + 3a_1 + 9a_2 = 1$,

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 1 & 2 & 4 & -2 \\ 1 & 3 & 9 & 1 \end{array}\right]$$

$$\xrightarrow[-R_1+R_3]{-R_1+R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 0 & 1 & 3 & -1 \\ 0 & 2 & 8 & 2 \end{array}\right] \xrightarrow{-2R_2+R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 2 & 4 \end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right] \xrightarrow[-R_3+R_1]{-3R_3+R_2} \left[\begin{array}{ccc|c} 1 & 1 & 0 & -3 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\xrightarrow{-R_2+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$a_0 = 4$

$a_1 = -7$

$a_2 = 2$

$p(x) = 4 - 7x + 2x^2$

4a) $3A-B$

$$3\begin{bmatrix}0 & 2 & 1\\ -1 & -2 & 0\end{bmatrix} - \begin{bmatrix}3 & 0 & 1\\ 0 & 2 & -1\end{bmatrix} = \begin{bmatrix}-3 & 6 & 2\\ -3 & -8 & 4\end{bmatrix}$$

b) $\underset{2\times3}{A}\ \underset{2\times3}{B}$

columns of $A \neq$ rows of B, so operation is not possible

c) $$\begin{bmatrix}0 & 2 & 1\\ -1 & -2 & 0\end{bmatrix}\begin{bmatrix}3 & 0\\ 0 & 2\\ 1 & -1\end{bmatrix} = \begin{bmatrix}1 & 3\\ -3 & -4\end{bmatrix}$$

d) A is not a square matrix, so cannot compute $\det(A)$

5) $A = \begin{bmatrix} 2 & 2 & 0 \\ -4 & 1 & 3 \\ 4 & 4 & 3 \end{bmatrix}$

$\xrightarrow{2R_1+R_2} \begin{bmatrix} 2 & 2 & 0 \\ 0 & 5 & 3 \\ 4 & 4 & 3 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\xrightarrow{-2R_1+R_3} \begin{bmatrix} 2 & 2 & 0 \\ 0 & 5 & 3 \\ 0 & 0 & 3 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

$= U$

$E_2 E_1 A = U$, so $A = \underbrace{E_1^{-1} E_2^{-1}}_{L} U$

$$L = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_1^{-1}} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}}_{E_2^{-1}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}}_{L}$$

6) $A = \begin{bmatrix} \cancel{-1} & \cancel{2} & \cancel{8} & \cancel{1} & \cancel{1} \\ 3 & 0 & 1 & -2 & 1 \\ 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 1 & -4 & 1 \end{bmatrix}$

$|A| = 2(-1) \begin{vmatrix} 3 & 1 & -2 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 1 & -4 & 1 \end{vmatrix}$

$= -2(3) \begin{vmatrix} 2 & 1 & 0 \\ -1 & 2 & 0 \\ 1 & -4 & 1 \end{vmatrix}$

$= -6(1) \begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} = -6(4+1) = -30$

7a) $R_1 \longleftrightarrow R_2$, so determinant $= -4$

b) $2R_1 + R_2$, determinant stays the same

$3R_2$, determinant $= 12$

8) $A = \begin{bmatrix} -1 & -3 \\ 2 & 4 \end{bmatrix}$, $b = \begin{bmatrix} 7 \\ -8 \end{bmatrix}$

$|A| = -4 - (-6) = 2$

$A_1 = \begin{bmatrix} 7 & -3 \\ -8 & 4 \end{bmatrix}$, $|A_1| = 28 - 24 = 4$

$A_2 = \begin{bmatrix} -1 & 7 \\ 2 & -8 \end{bmatrix}$, $|A_2| = 8 - 14 = -6$

$$x_1 = \frac{|A_1|}{|A|} = \frac{4}{2} = 2$$

$$x_2 = \frac{|A_2|}{|A|} = \frac{-6}{2} = -3$$