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Solutions

Name:

# Math 207 Section A, Quiz 5

(6.5 pts) 1. Consider the set $\{(x, y) : x \geq 0, y \geq 0\}$ with the standard operations in $R^2$. Determine whether the set is a vector space. Justify your answer.

The set $S = \{(x,y) : x \geq 0, y \geq 0\} \subset \mathbb{R}^2$ with the standard operations is not a vector space. It is not closed under the unary operation

$$(x,y) \longmapsto (-x,-y).$$

(Also acceptable: note that $S$ is not closed under scalar multiplication since, if $c < 0$, and if $(x,y) \in S$, then $c(x,y) = (cx, cy) \notin S$.)

(6.5 pts) 2. Let $W = \{(x_1, x_2, x_1x_2) : x_1$ and $x_2$ are real numbers$\}$. Determine whether W is a subspace of $R^3$ with the standard operations. Justify your answer.

$W = \{(x_1, x_2, x_1x_2) : x_1, x_2 \in \mathbb{R}\}$ is not a subspace of $\mathbb{R}^3$. It is not closed under the standard operations. For example it's not closed under $+$:

$$(x_1, x_2, x_1x_2) + (x_1', x_2', x_1'x_2') = (x_1+x_1', x_2+x_2', x_1x_2+x_1'x_2')$$

If this sum is to belong to $W$, then we must have

$$x_1x_2 + x_1'x_2' = (x_1+x_1')(x_2+x_2')$$

which certainly does not hold for every $x_1, x_2 \in \mathbb{R}$.

(Also acceptable: Note that $W$ is not closed under scalar multiplication since $c(x_1, x_2, x_1x_2) = (cx_1, cx_2, cx_1x_2) \neq (cx_1, cx_2, c^2x_1x_2$

3. Let W be the set of all $3 \times 3$ diagonal matrices. Determine whether W is a subspace of $M_{3,3}$ with the standard operations of matrix addition and scalar multiplication. Justify your answer.

Claim: The set $W = \left\{ \begin{pmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{pmatrix} : d_i \in \mathbb{R} \right\}$ is a subspace of $M_{3,3}$.

Proof: We must show that if $D = \begin{pmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{pmatrix}$ and $D' = \begin{pmatrix} d_1' & 0 & 0 \\ 0 & d_2' & 0 \\ 0 & 0 & d_3' \end{pmatrix}$ are two diagonal matrices and if $c$ is a scalar (real number), then

1. $D' + D \in W$ (Recall "$\in$" means "belongs to.")
2. $-D \in W$
3. $cD \in W$.

Indeed,

1. $D' + D = \begin{pmatrix} d_1+d_1' & 0 & 0 \\ 0 & d_2+d_2' & 0 \\ 0 & 0 & d_3+d_3' \end{pmatrix}$, which is diagonal.
2. $-D = \begin{pmatrix} -d_1 & 0 & 0 \\ 0 & -d_2 & 0 \\ 0 & 0 & -d_3 \end{pmatrix}$ is diagonal.
3. $cD = \begin{pmatrix} cd_1 & 0 & 0 \\ 0 & cd_2 & 0 \\ 0 & 0 & cd_3 \end{pmatrix}$ is diagonal.

By verifying properties 1., 2., and 3., we have shown that W is closed under the standard operations of $M_{3,3}$.