Example: $\boxed{\text{Math 207}}$ [2015/04/06] 1

Show $S_1 = \{(1,2,-1), (0,1,1), (2,5,-1)\}$
& $S_2 = \{(-2,-6,0), (1,1,-2)\}$
span the same subspace of $\mathbb{R}^3$

Soln: To find the subsp. spanned by S_1, do RREF on

$$\begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 5 \\ -1 & 1 & -1 \end{pmatrix} \xrightarrow{R_1+R_3 \to R_3} \begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 5 \\ 0 & 1 & 1 \end{pmatrix}$$

$$\xrightarrow{-2R_1+R_2 \to R_2} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \xrightarrow{-R_2+R_3 \to R_3} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

So $B = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$ is a basis for the subspace of $\mathbb{R}^3$ spanned by S_1.

Similarly for S_2:

$$\begin{pmatrix} -2 & 1 \\ 6 & 1 \\ 0 & -2 \end{pmatrix} \xrightarrow{3R_1+R_2 \to R_2} \begin{pmatrix} -2 & 1 \\ 0 & 4 \\ 0 & -2 \end{pmatrix} \xrightarrow{2R_3+R_2 \to R_3} \begin{pmatrix} -2 & 1 \\ 0 & 4 \\ 0 & 0 \end{pmatrix}$$

$$\xrightarrow{\frac{1}{4}R_2 \to R_2} \begin{pmatrix} -2 & 1 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \xrightarrow{-R_2+R_1 \to R_1} \begin{pmatrix} -2 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \xrightarrow{-\frac{1}{2}R_1 \to R_1} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

So $B = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$ is a basis for the subspace of $\mathbb{R}^3$ spanned by S_2.