Math 207 Section A, Quiz 7 Name: Answer Key

Cheating will not be tolerated. If there is any indication that a student gave or received unauthorized aid on this test, the case will be referred to the ISU Office of Judicial Affairs.

"On my honor as a student I, ______________________________, have neither given nor received unauthorized aid on this quiz." (print name clearly)

Signature: ______________________________ Date: ______________

Multiple choice section. Circle the letter next to the correct answer(s). A question may have more than one correct answer. *Select all that apply.*

1. The **rank** of an $m \times n$ matrix A is equal to

   (a) the row space of A.

   (b) the dimension of the row space A.

   (c) the dimension of the column space of A.

   (d) n – nullity(A).

   (e) dimension of the solution space of $A\mathbf{x} = \mathbf{0}$.

   (f) the relative position of A in the matrix army.

2. If A and B are **row equivalent** matrices then

   (a) B can be derived from A using elementary row operations.

   (b) A and B must have the same number of nonzero rows.

   (c) the row space of A must equal the row space of B.

   (d) the column space of A must equal the column space of B.

   (e) the rank of A must equal the rank of B.

   (f) A and B must row boats up river at equivalent rates.

3. The **nullity** of an $m \times n$ matrix A is equal to

   (a) the row space of A.

   (b) the dimension of the row space A.

   (c) the dimension of the column space of A.

   (d) n – rank(A).

   (e) dimension of the solution space of $A\mathbf{x} = \mathbf{0}$.

   (f) the illegal marriage of A to another matrix of the same dimensions.

4. Consider the following **row equivalent** matrices:

$$A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ -1 & -1 & 1 & 3 & -19 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -2 & 0 & 3 \\ 0 & 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

a. Determine the *rank* and *nullity* of A.

rank$(A) =$ 3 nullity$(A) =$ 2

b. Find a basis for the *nullspace*-of A. $\underline{x} \in$ nullspace$(A) \iff A\underline{x} = \underline{0}$

$A\underline{x} = \underline{0} \iff B\underline{x} = \underline{0} \iff \left\{ \begin{array}{l} x_1 + x_3 + x_5 = 0 \\ x_2 - 2x_3 + 3x_5 = 0 \\ x_4 - 5x_5 = 0 \end{array} \right. \iff \begin{array}{l} x_1 = -x_3 - x_5 \\ x_2 = 2x_3 - 3x_5 \\ x_4 = 5x_5 \end{array}$

Let $x_3 = s$, $x_5 = t$

So a basis for nullspace is $\left\{ \begin{pmatrix} -1 \\ 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ -3 \\ 0 \\ 5 \\ 1 \end{pmatrix} \right\}$

So a soln to $A\underline{x} = \underline{0}$ has the form: $\underline{x} = s\begin{pmatrix} -1 \\ 2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} -1 \\ -3 \\ 0 \\ 5 \\ 1 \end{pmatrix}$

c. Find a basis for the *row space* of A. (Use vectors appearing in matrices above!)

$\{(1,0,1,0,1), (0,1,-2,0,3), (0,0,0,1,-5)\}$ are the non zero rows of the rref matrix B, so this set is a basis for the row space of A.

d. Find a basis for the *column space* of A. (Use vectors appearing in matrices above!)

$\left\{ \begin{pmatrix} -2 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -5 \\ 3 \\ -1 \\ 7 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 3 \\ 5 \end{pmatrix} \right\}$ are the columns of A corresponding to the "leading" columns in the rref matrix B.

e. The rows of A are linearly

(a) dependent (b) independent (c) libertarian

(circle the letter next to the correct answer)