

Name:

Math 207 Section A, Quiz 4

1(a) (4 points) Suppose people are loyal to either Coke or Pepsi. During any month, there is a 10% chance that a Coke drinker switches to Pepsi and a 20% chance that a Pepsi drinker switches to Coke. Find the matrix of transition probabilities P .

$$\begin{array}{cc} & \begin{matrix} C & P \end{matrix} \\ \begin{bmatrix} .9 & .2 \\ .1 & .8 \end{bmatrix} & \begin{matrix} C \\ P \end{matrix} \end{array}$$

(b) (6 points) Suppose there are initially 65 Coke drinkers and 85 Pepsi drinkers. Find the steady state matrix for P .

Know $P\bar{x} = \bar{x}$

$$P\bar{x} - \bar{x} = \vec{0}$$

~~(P-I)~~ $(P-I)\bar{x} = \vec{0}$, solve for \bar{x}

$$\left[\begin{array}{cc|c} -.1 & .2 & 0 \\ .1 & -.2 & 0 \end{array} \right] \xrightarrow{R_1 + R_2} \left[\begin{array}{cc|c} -.1 & .2 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

x_2 free, let $x_2 = s$

~~-0.1~~ $-0.1x_1 + 0.2s = 0$

$$x_1 = 2s$$

Initial Pop is $65 + 85 = 150$, so

$$x_1 + x_2 = 150$$

$$2s + s = 150$$

$$s = 50, \text{ so}$$

$$\bar{x} = \begin{bmatrix} 100 \\ 50 \end{bmatrix}$$

2. (10 points) Find the determinant of

$$\begin{bmatrix} -1 & 2 & 0 & 0 \\ -3 & 4 & 3 & 2 \\ 0 & 2 & 0 & 0 \\ 1 & 1 & -1 & 1 \end{bmatrix}$$

3rd row expansion

$$0 - 2 \begin{vmatrix} -1 & 0 & 0 \\ -3 & 3 & 2 \\ 1 & -1 & 1 \end{vmatrix} + 0 + 0$$

1st row expansion

$$-2 \left(-1 \begin{vmatrix} 3 & 2 \\ -1 & 1 \end{vmatrix} + 0 + 0 \right)$$

$$(-2)(-1)(3 - (-2)) = 10$$