Name:

Math 207 Section A, Quiz 1

1. (12 points) Solve the system of linear equations

$$\begin{aligned} x_1 + 2x_2 - x_3 + 2x_4 &= 0 \\ -2x_1 - 4x_2 + 3x_3 - 3x_4 &= 0 \\ x_1 + 2x_2 + x_3 + 4x_4 &= 0 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 2 & 0 \\ -2 & -4 & 3 & -3 & 0 \\ 1 & 2 & 1 & 4 & 0 \end{array}\right] \xrightarrow[-R_1+R_3]{2R_1+R_2} \left[\begin{array}{cccc|c} 1 & 2 & -1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 2 & 0 \end{array}\right]$$

$$\xrightarrow{-2R_2+R_3} \left[\begin{array}{cccc|c} 1 & 2 & -1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{R_2+R_3} \left[\begin{array}{cccc|c} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

x_2, x_4 free

$x_1 + 2x_2 + 3x_4 = 0, \quad x_1 = -2x_2 - 3x_4$

$x_3 + x_4 = 0, \quad x_3 = -x_4$

Let $x_2 = s$, $x_4 = t$

$x_1 = -2s - 3t$

$x_2 = s$

$x_3 = -t$

$x_4 = t$

2. (8 points) Solve the system of linear equations

$$\begin{aligned} x_1 + x_3 &= 1 \\ 4x_2 + x_3 &= 15 \\ -3x_1 + 2x_2 - 3x_3 &= 1 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 4 & 1 & 15 \\ -3 & 2 & -3 & 1 \end{array}\right] \xrightarrow{3R_1+R_3} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 4 & 1 & 15 \\ 0 & 2 & 0 & 4 \end{array}\right]$$

$$\xrightarrow{-2R_3+R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 7 \\ 0 & 2 & 0 & 4 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 1 & 7 \end{array}\right]$$

$$\xrightarrow[\frac{1}{2}R_2]{-R_3+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 7 \end{array}\right]$$

$x_1 = -6$

$x_2 = 2$

$x_3 = 7$