Name:

Math 207 Section A, Quiz 3

1. (10 points) Find the inverse of

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & -3/2 \\ 1 & 0 & 1/2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ -3 & 1 & -3/2 & 0 & 1 & 0 \\ 1 & 0 & 1/2 & 0 & 0 & 1 \end{array}\right] \xrightarrow[-R_1+R_3]{3R_1+R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -3/2 & 3 & 1 & 0 \\ 0 & 0 & 1/2 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{3R_3+R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 3 \\ 0 & 0 & 1/2 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{2R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & -2 & 0 & 2 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ -2 & 0 & 2 \end{bmatrix}$$

2. (10 points) Find an LU factorization for

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 2 & 1 \\ 0 & -4 & 1 \end{bmatrix},$$

$\xrightarrow{-2R_1+R_2}$ $\begin{bmatrix} 1 & -1 & 3 \\ 0 & 4 & -5 \\ 0 & -4 & 1 \end{bmatrix}$, $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\xrightarrow{R_2+R_3}$ $\begin{bmatrix} 1 & -1 & 3 \\ 0 & 4 & -5 \\ 0 & 0 & -4 \end{bmatrix}$, $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$E_2 E_1 A = U$, so $A = \underbrace{E_1^{-1} E_2^{-1}}_{L} U$

$L = E_1^{-1} E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 4 & -5 \\ 0 & 0 & -4 \end{bmatrix}$$