AP&PM

Post-graduation in Data Science for Finance

Asset Pricing and Portfolio Management

Individual Project

Comparative Analysis of Portfolio Strategies: Risk and Performance Evaluation

November 2024

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1. ABSTRACT

This project examines the performance of various portfolio management strategies by analyzing daily log returns from a diverse set of assets, including equities, indices, commodities, and cryptocurrencies.

A holdout partition method is employed, where the training set is used to optimize the portfolio weights for each strategy, and the test set is used for performance evaluation.

The strategies explored include the Mean-Downside Risk, Hierarchical Risk Parity, Mean-CVaR, Minimum CDaR, Mean Absolute Deviation, Maximum Omega Ratio, Tactical Dual Momentum, Adaptive Asset Allocation, Black-Litterman Model and Minimum Tail Dependence.

Performance is assessed using multiple risk-adjusted metrics, such as returns, volatility, Sharpe ratio, and maximum drawdown. The analysis aims to provide insights into the effectiveness of these strategies under different market conditions, highlighting their risk-return characteristics and comparative advantages.

KEYWORDS:

Financial Markets' Returns; Investment Strategies; Portfolio Management; Risk-Adjusted Performance; Log Returns; Asset Allocation; Mean-Downside Risk; Hierarchical Risk Parity; Mean-CVaR; Minimum CDaR; Maximum Omega Ratio; Tactical Dual Momentum; Black-Litterman Model; Adaptive Asset Allocation; Minimum Tail Dependence; Holdout; Sharpe Ratio; Drawdown; Volatility; Comparative Analysis;

2. INTRODUCTION

2.1 PROJECT SCOPE

The scope of this project involves a straightforward evaluation of multiple portfolio management strategies using daily log returns data from a diverse set of assets, including equities, indices, commodities, and cryptocurrencies. The primary goal is to explore the risk-return characteristics of each strategy using a simple framework. This includes employing a holdout partition where the training dataset is used to optimize portfolio weights, and the test dataset serves for performance assessment.

The analysis focuses on strategies such as Mean-Downside Risk, Hierarchical Risk Parity, Mean-CVaR, Minimum CDaR, Mean Absolute Deviation, Maximum Omega Ratio, Tactical Dual Momentum, Adaptive Asset Allocation, the Black-Litterman Model and also the Minimum Tail Dependence strategy.

The project emphasizes the use of alternative risk-adjusted performance metric, such as, the annualized returns, the annualized volatility, Sharpe ratio and maximum drawdown in order to provide an accessible yet meaningful comparison of these approaches.

Rather than implementing complex backtesting evaluations, this project maintains a practical and simplified approach, offering insights into the relative effectiveness of these strategies under varying market conditions.

2.2 CONTEXT AND RELEVANCE

In an increasingly complex and volatile financial environment, effective portfolio management strategies are essential for balancing risk and return. Traditional methods of portfolio optimization often fail to capture the nuances of financial markets, particularly when dealing with assets that have different risk profiles and market behaviors. This has led to the development and adoption of more sophisticated strategies that account, for example, for downside risk, tail dependencies, and asset correlations in a dynamic and evolving manner.

This project is set within this context, where potential investors seek to maximize returns while mitigating the risk. It leverages daily log returns from a selected set of assets, including equities like AAPL and MSFT, indices such as ^GSPC, and alternative investments like cryptocurrencies (BTC-USD and ETH-USD). The assets chosen reflect the broad spectrum of investment opportunities available, each contributing unique characteristics to the overall risk-return profile.

Moreover, the use of a holdout partition methodology, where portfolio weights are optimized with a training set and evaluated on a test set, ensures a systematic and practical approach to performance assessment.
By applying a variety of advanced strategies, from Hierarchical Risk Parity to the Black-Litterman Model, the project aims to explore how some of these adaptive investment frameworks can effectively manage risks and maximize returns across different market conditions.

This approach provides valuable insights into the practical applications of these strategies for achieving more resilient and well-optimized portfolios.

3. DATA and FRAMEWORK

The analysis in this project uses the same set of assets as in the Part1 of the group project. These assets provide a well-rounded foundation for evaluating the performance of various portfolio management strategies, as each asset class exhibits unique risk and return characteristics.

This selection of assets captures a broad spectrum of market opportunities, from tech giants like Apple and Microsoft, to major commodities and leading cryptocurrencies.

Asset Class	Ticker	Asset Name	Additional Information
Stocks	AAPL	Apple	Tech Company
	JPM	JPMorgan Chase	Financial Company
	MSFT	Microsoft	Tech Company
	TSLA	Tesla	Automotive & Energy Company
Indexes	QQQ	NASDAQ	NASDAQ-100 index's ETF
	^GSPC	S&P 500	500 large-cap U.S. companies' performance index
Commodities	GC=F	Gold	Gold Futures traded on the COMEX exchange
	CL=F	Oil	Crude Oil Futures traded on the NYMEX
Cryptocurrencies	BTC-USD	Bitcoin	Proof-of-Work consensus Cryptocurrency
	ETH-USD	Etherium	Proof-of-Stake consensus Cryptocurrency

Table 1: Selected Assets for the Analysis

3.1 DATASET DESCRIPTION

As previously stated, the dataset used in this project originates from Part 1 of the group project. It comprises daily log returns for each selected asset, originally spanning from January 1, 2010, to September 26, 2024. However, the inclusion of cryptocurrency data, particularly Ethereum, introduced null values in the earlier years when cryptocurrency data was unavailable.

To address this, rows containing null values were removed, resulting in a refined dataset covering the period from November 10, 2017, to September 26, 2024. This adjustment ensures that the dataset remains complete for the performance evaluation of the different portfolio management strategies applied. Besides that, the chosen time frame still ensures a wide range of market conditions, providing a robust basis for assessing the strategies effectiveness.

	AAPL	BTC-USD	CL=F	ETH-USD	GC=F	JPM	MSFT	QQQ	TSLA	^GSPC
2017-11-10 00:00:00+00:00	-0.003315	-0.076400	-0.007550	-0.069790	-0.010321	-0.001230	-0.002619	-0.000065	0.000000	-0.000898
2017-11-14 00:00:00+00:00	-0.015233	0.011559	-0.018852	0.063948	0.003283	-0.006047	0.001429	-0.003646	-0.021472	-0.002312
2017-11-15 00:00:00+00:00	-0.013278	0.097529	-0.006665	-0.012740	-0.003909	0.009414	-0.007803	-0.004708	0.008387	-0.005541
2017-11-16 00:00:00+00:00	0.011876	0.073272	-0.003440	-0.007325	0.000705	0.002848	0.002648	0.012698	0.003847	0.008163
2017-11-17 00:00:00+00:00	-0.005568	-0.020886	0.025250	0.004432	0.014302	-0.003357	-0.009662	-0.003825	0.008127	-0.002629
...	...	...	...	...	...	...	...	...	...	...
2024-09-19 00:00:00+00:00	0.036395	0.020721	0.014560	0.039316	0.006707	0.014115	0.018126	0.024970	0.071010	0.016834
2024-09-20 00:00:00+00:00	-0.002932	0.004004	-0.000417	0.038335	0.012251	0.002894	-0.007827	-0.001905	-0.023520	-0.001943
2024-09-24 00:00:00+00:00	0.003966	0.015234	0.016769	0.002191	0.009360	0.000709	-0.010062	0.004812	0.016936	0.002508
2024-09-25 00:00:00+00:00	-0.004408	-0.018186	-0.026479	-0.028649	0.003013	-0.006639	0.006827	0.000927	0.010757	-0.001863
2024-09-26 00:00:00+00:00	0.005067	0.031764	-0.029414	0.020268	0.004016	-0.001953	-0.001853	0.007485	-0.010954	0.004031

1351 rows × 10 columns

Image 1: Final Dataset Structure Exemplification

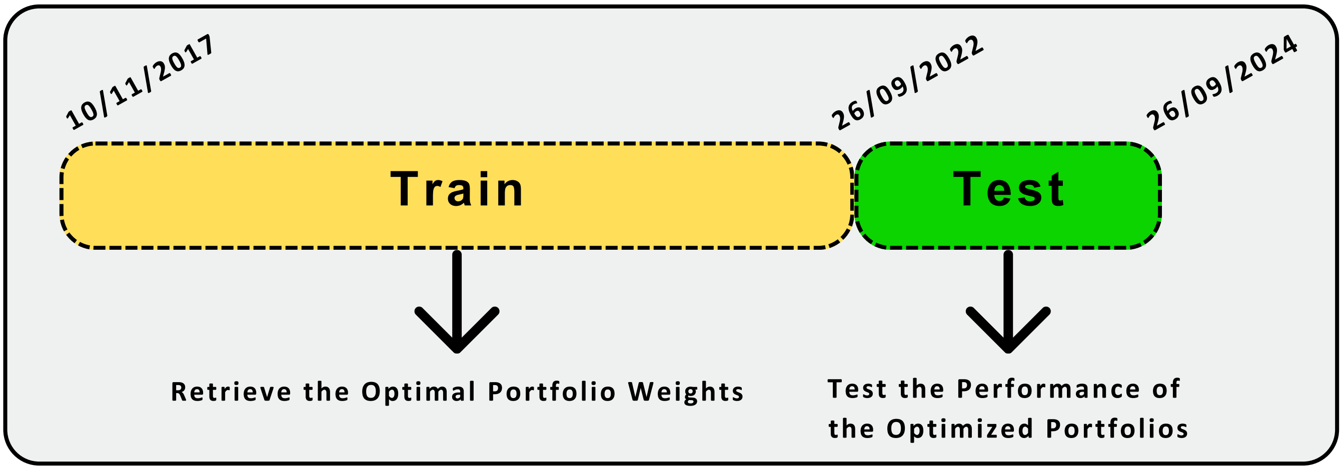
3.2 OVERVIEW OF THE PROJECT FRAMEWORK

This project builds on the foundation established during the group project, where a comprehensive exploratory analysis of the log returns data, including assessments of correlation, volatility, normality, and other statistical properties, was already conducted. As a result, no additional exploratory investigation was carried out, allowing the focus to remain on the technical implementation and evaluation of the portfolio management strategies.

The core of this project revolves around a structured framework that involves performing a **holdout split** on the daily log returns data. The data was divided into two distinct subsets: the **training set** and the **testing set**. The training data, spanning from November 10, 2017, to September 26, 2022, was used to optimize the portfolio weights for each strategy. Subsequently, the testing data, from September 27, 2022, to September 26, 2024, was used to evaluate the performance metrics of these optimized portfolios.

While it is acknowledged that a more robust backtesting framework, such as a resampling approach, would likely yield more reliable and generalizable results, similar to the methodology developed in Part 2 of the group project, this project intentionally opts for a simpler approach. By using a single train-test split, the analysis remains straightforward and easier to interpret, which was deemed appropriate given the project's scope and the need to prioritize clarity over complexity.

The performance evaluation focused on key metrics such as annualized returns, annualized volatility, Sharpe ratio and maximum drawdown. After calculating these metrics for each portfolio strategy, the results were compared and analyzed to extract meaningful insights into the relative effectiveness and risk-return trade-offs of the different strategies under consideration.



Schema 1: Representation of the Data Partition Process

4. PORTFOLIO MANAGEMENT STRATEGIES

This section outlines the portfolio management strategies implemented in this project. Each strategy is designed to optimize the risk-return trade-off using different approaches, from traditional risk measures to more advanced techniques that account for downside risk and hierarchical dependencies. The aim is to compare these strategies to understand how they perform under varying market conditions.

The portfolios analyzed in this project include:

1. **Mean-Downside Risk Portfolio**
2. **Hierarchical Risk Parity Portfolio**
3. **Mean-CVaR (Conditional Value at Risk) Portfolio**
4. **Minimum CDaR (Conditional Drawdown at Risk) Portfolio**
5. **Mean Absolute Deviation Portfolio**
6. **Maximum Omega Ratio Portfolio**
7. **Tactical Dual Momentum Strategy**
8. **Adaptive Asset Allocation Strategy**
9. **Black-Litterman Model Portfolio**
10. **Minimum Tail Dependence Portfolio**

- An **Equally Weighted Portfolio** is also included as a benchmark index. This portfolio equally allocates weights to all assets and serves as a reference point to assess the relative performance of the other strategies. By comparing each optimized strategy against this baseline, the analysis aims to highlight the benefits and trade-offs associated with more sophisticated portfolio optimization techniques.

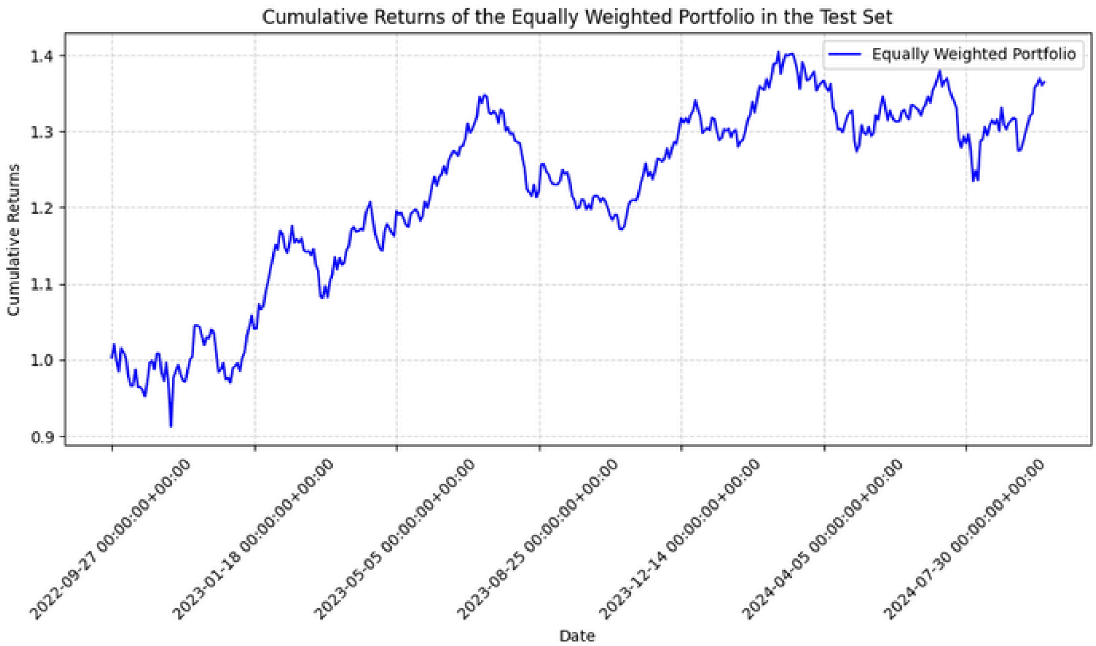


Image 2: Log Returns For the EWP on the test set

4.1 Mean-Downside Risk Portfolio

THEORETICAL BACKGROUND

The Mean-Downside Risk Portfolio focuses on minimizing downside risk, specifically measured by the **semivariance**. Unlike traditional risk measures that consider both upward and downward deviations, semivariance captures only the negative deviations from the specified target return. This approach is particularly useful for risk-averse investors who are more concerned about potential losses rather than the volatility of returns overall.

Semivariance Formula:

$$Semivariance = n1 \times rt < Average \sum n(Average - rt)2$$

$n =$ The total number of observations below the mean
$r_t =$ The observed value
$\text{Average} =$ The mean or target value of the dataset

TECHNICAL IMPLEMENTATION

- **Defining Downside Risk (Semivariance) function:**
  - Semivariance is computed by calculating the squared deviations of returns below a target level (set to 0 in this case).
  - The function identifies negative deviations and computes their squared average.
- **Set Up the Portfolio Optimization Problem:**
  - Using the training data to define the optimization problem, where the objective is to minimize the portfolio's semivariance.
  - Constraints are applied to ensure:
    - Non-Negativity: Asset weights are non-negative (no short-selling).
    - Full Investment: The sum of all weights equals 1.
- **Optimize the Portfolio:**
  - Leverage numerical optimization methods to find the weight allocation that minimizes semivariance.

4.1 Mean-Downside Risk Portfolio

PORTFOLIO PERFORMANCE VISUALIZATION

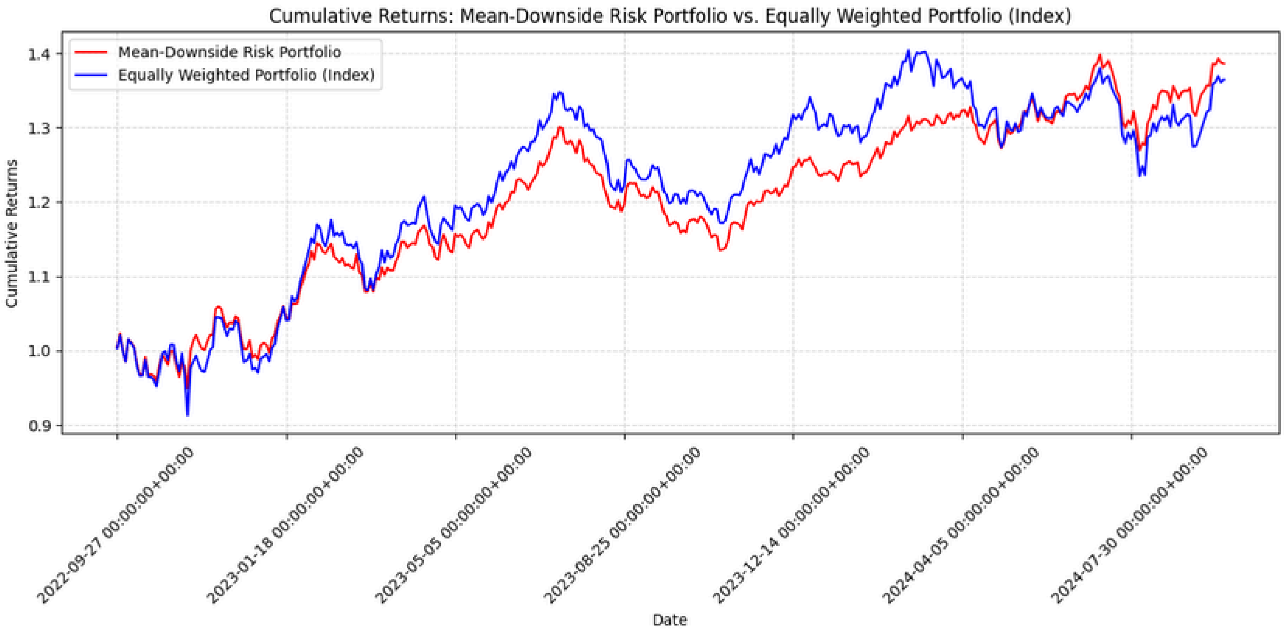
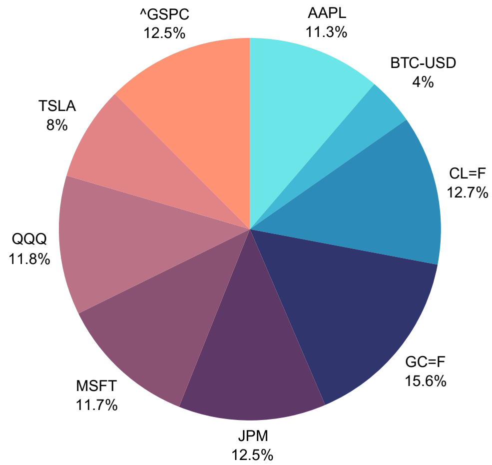


Image 3: Index Performance vs Mean-Downside Risk Portfolio

OPTIMAL WEIGHTS ANALYSIS

The optimal weights reveal a higher allocation to Gold Futures (15.6%) and Crude Oil Futures (12.7%), with no allocation to Ethereum (ETH-USD). This suggests a conservative approach, favoring assets less susceptible to significant losses.



Pie Chart 1: Optimal Returns for Mean-Downside Risk

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4.2 HIERARCHICAL RISK PARITY (HRP) PORTFOLIO

THEORETICAL BACKGROUND

The Hierarchical Risk Parity (HRP) Portfolio is an advanced method designed to overcome the limitations of traditional risk parity approaches. Rather than relying on the covariance matrix directly, HRP uses hierarchical clustering to build a risk-balanced portfolio. This strategy is especially useful when dealing with highly correlated assets, as it clusters assets based on their similarities and allocates capital in a manner that seeks to minimize overall risk.

The HRP method consists of three main steps:

- **Hierarchical Clustering**: Assets are grouped into clusters based on their correlation, using methods such as single linkage.
- **Recursive Bisection**: The capital is allocated hierarchically, dividing it between clusters and further subdividing it until individual assets are assigned weights.
- **Risk-Based Allocation**: Weights are determined to ensure that risk contributions are balanced across the hierarchy.

TECHNICAL IMPLEMENTATION

The implementation of the HRP Portfolio follows a structured approach:

- **Calculating the Distance Matrix**: Based on asset correlations, converted into a distance metric.
- **Perform Hierarchical Clustering**: Using the single linkage method to group assets.
- **Allocate Weights Using Recursive Bisection**: Distributing capital across clusters to achieve risk parity.

4.2 Hierarchical Risk Parity (HRP) Portfolio

PORTFOLIO PERFORMANCE VISUALIZATION

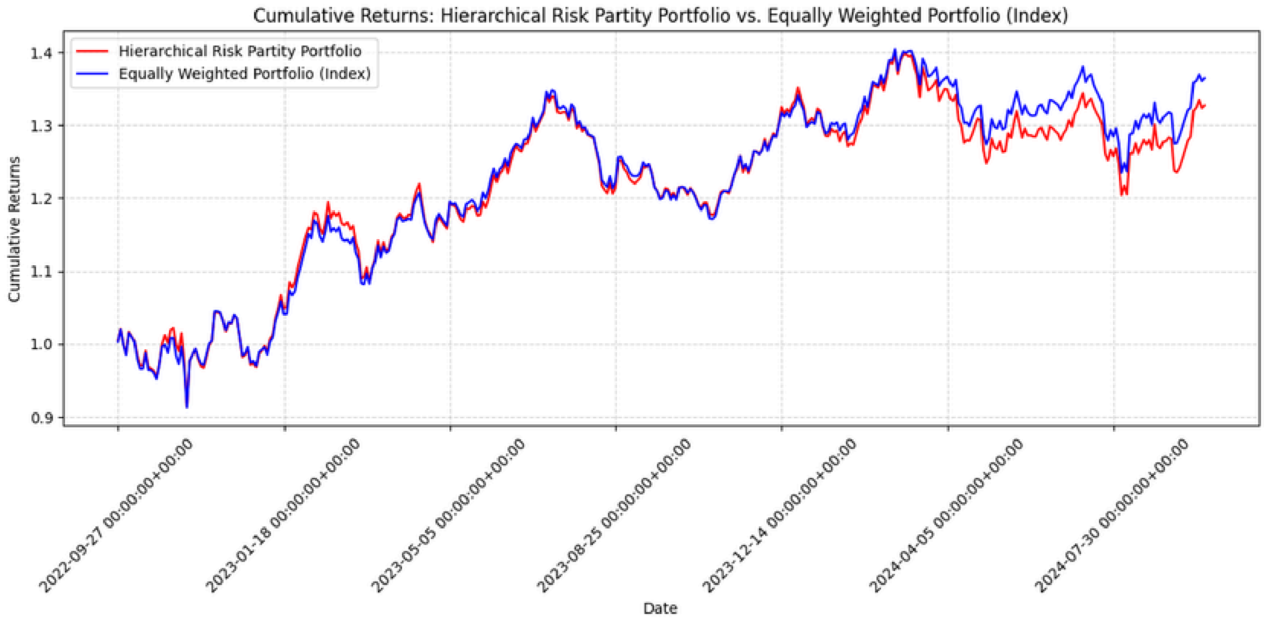
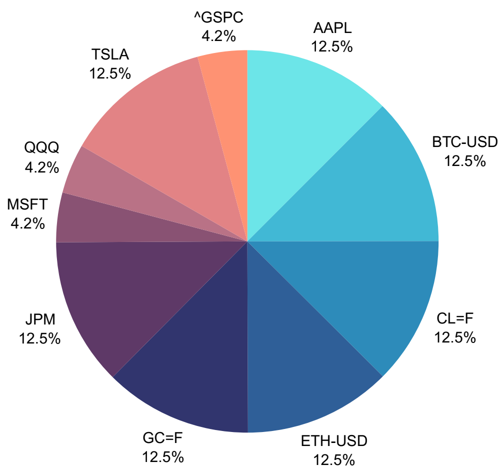


Image 4: Index Performance vs Hierarchical Risk Parity Portfolio

OPTIMAL WEIGHTS ANALYSIS

The table of optimal weights reveals that the HRP Portfolio allocates equal weights (12.5%) to most assets, with reduced exposure to Microsoft (MSFT), QQQ, and ^GSPC (each at 4.2%). This distribution reflects the clustering process, where certain assets are grouped and allocated proportionally based on risk contribution.



Pie Chart 2: Optimal Returns for Hierarchical Risk Partity

4.3 Mean-CVaR (Conditional Value at Risk) Portfolio

THEORETICAL BACKGROUND

The Mean-CVaR (Conditional Value at Risk) Portfolio strategy focuses on minimizing the potential losses of the worst-case scenarios beyond a specified confidence level. CVaR, also known as expected shortfall, measures the average loss that occurs beyond the Value at Risk (VaR) threshold, providing a more comprehensive view of downside risk. This makes it particularly suitable for risk-averse investors who wish to mitigate extreme losses rather than simply managing average risk.

The CVaR calculation involves:

- Computing VaR at a chosen confidence level (e.g., 95% or 99%).
- Calculating the expected loss of returns that fall below the VaR threshold.

TECHNICAL IMPLEMENTATION

The implementation involves the following steps:

- **Define CVaR Calculation**: A function to compute the CVaR for a given alpha level (in this case, 0.05).
- **Portfolio Optimization:** Using an optimization routine to minimize CVaR while imposing constraints such as no short-selling and a total weight sum of one.

4.3 Mean-CVaR (Conditional Value at Risk) Portfolio

PORTFOLIO PERFORMANCE VISUALIZATION

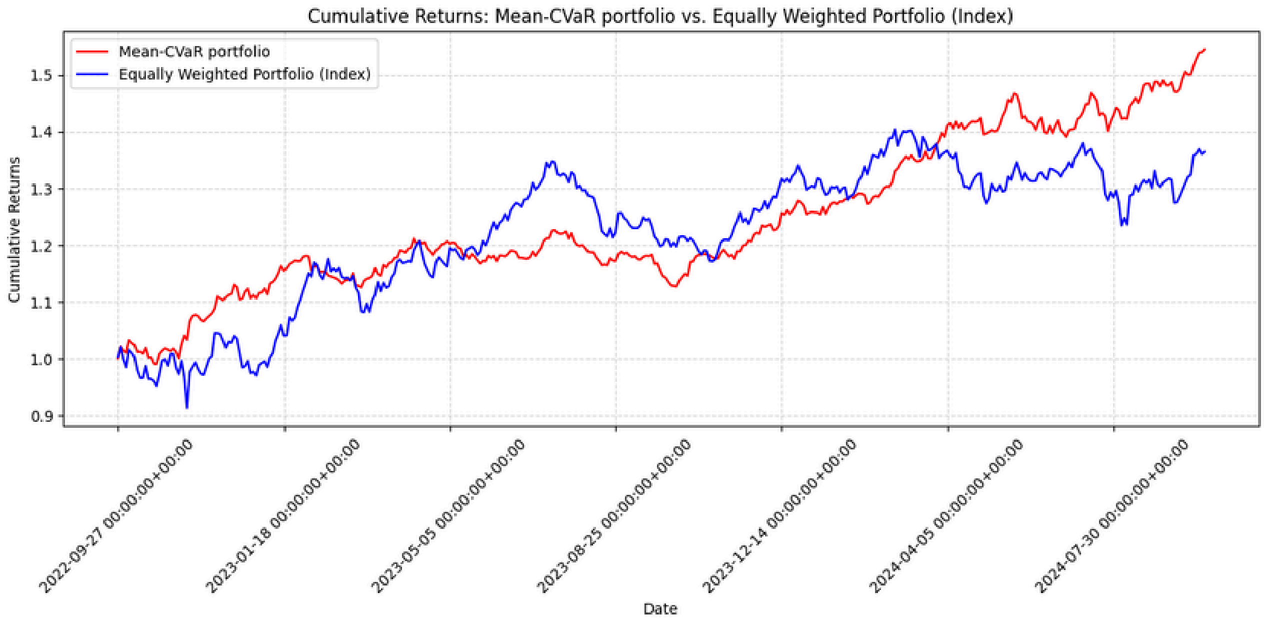
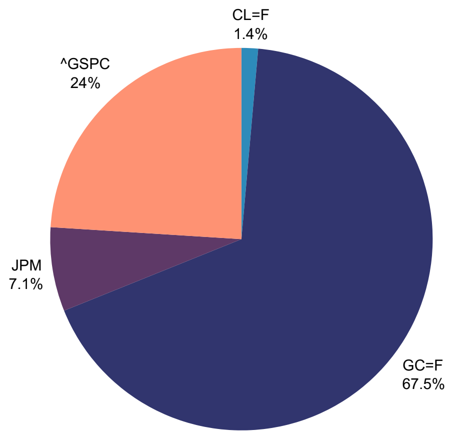


Image 5: Index Performance vs Mean-CVaR Portfolio

OPTIMAL WEIGHTS ANALYSIS

The optimal weights exhibit a highly concentrated allocation: 67.5% in Gold Futures (GC=F) and 24% in the S&P 500 index (^GSPC), with negligible or zero weights assigned to assets such as AAPL, BTC-USD, and MSFT. This allocation pattern underscores the strategy's defensive posture, favoring assets with lower risk profiles.



Pie Chart 3: Optimal Returns for Mean-CVaR

4.4 Minimum CDar (Conditional Drawdown At Risk) portfolio

THEORETICAL BACKGROUND

The Minimum CDaR (Conditional Drawdown at Risk) Portfolio aims to minimize the risk of significant drawdowns. CDaR measures the average drawdown that exceeds a certain threshold, focusing on the potential severity of peak-to-trough (strategy used to identify market tops and bottoms) declines in a portfolio's value. This approach is especially relevant for investors who are highly sensitive to large and prolonged losses, as it provides a more targeted risk management framework than traditional volatility-based measures.

The CDaR is calculated as follows:

- **Drawdown Calculation**: The drawdown at each point in time is the percentage decline from the historical peak.
- **Conditional Drawdown**: The CDaR is the expected drawdown beyond the chosen confidence level, reflecting the worst-case drawdown scenarios.

TECHNICAL IMPLEMENTATION

The implementation of the Minimum CDaR Portfolio includes:

- **Calculate Drawdown:** The drawdown function computes the drawdown at each point as the percentage decline from the historical peak.
- **Defining CDaR Function:** The CDaR is calculated based on the 95% confidence level (α=0.05), which reflects the average of the most extreme 5% of drawdowns.
- **Optimization Setup**: The portfolio optimization aims to minimize CDaR using the training data, subject to constraints:
  - No Short-Selling: All weights are constrained to be non-negative.
  - Weights Sum to 1: The sum of all asset weights equals 1.
- **Optimal Weights Extraction**: The optimized weights are then extracted and used to evaluate performance on the test data.

4.4 Minimum CDaR (Conditional Drawdown At Risk) portfolio

PORTFOLIO PERFORMANCE VISUALIZATION

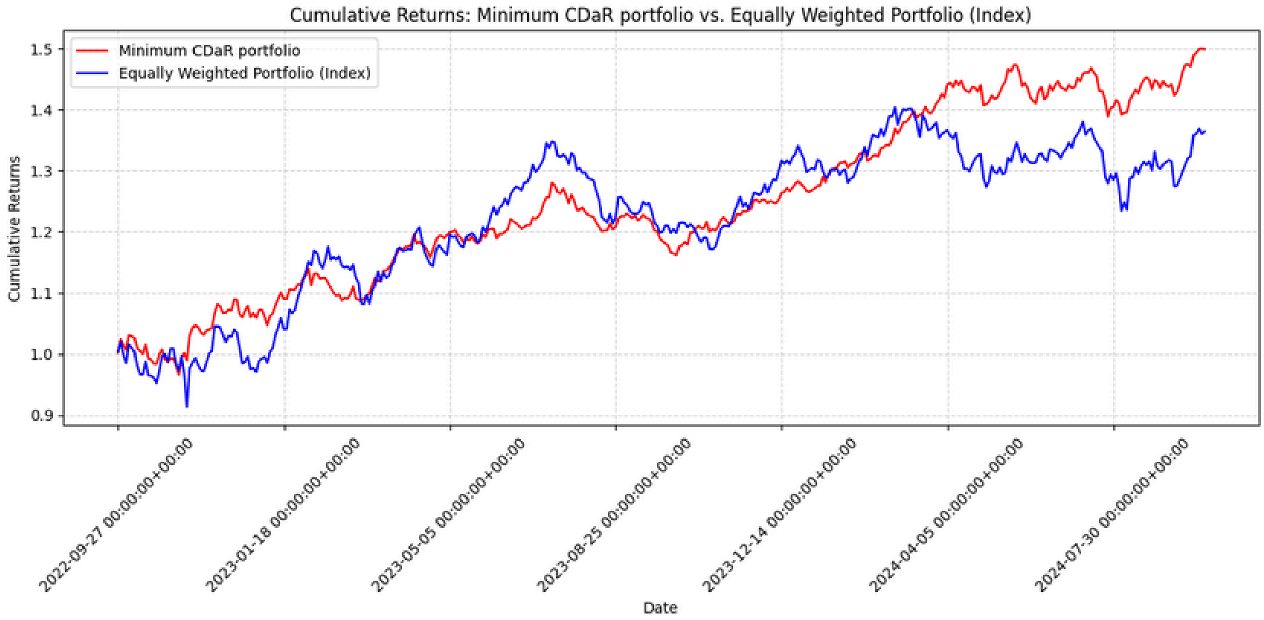
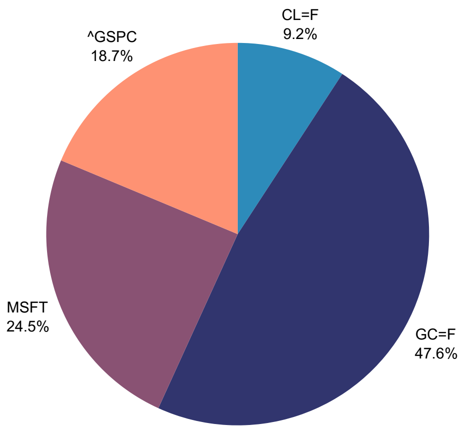


Image 6: Index Performance vs Minimum CDaR Portfolio

OPTIMAL WEIGHTS ANALYSIS

The optimal weights for the Minimum CDaR Portfolio highlight a concentrated allocation: 47.6% in Gold Futures (GC=F) and 24.5% in Microsoft (MSFT), with a significant portion (18.7%) in the S&P 500 index (^GSPC). Notably, several assets, such as AAPL, BTC-USD, and TSLA, receive zero weight, indicating their potential to contribute to large drawdowns.



Pie Chart 4: Optimal Returns for Minimum CDaR

4.5 Mean Absolute Deviation portfolio (MAD)

THEORETICAL BACKGROUND

The Mean Absolute Deviation (MAD) Portfolio seeks to minimize the average absolute deviation of returns from the mean return. Unlike variance, which emphasizes larger deviations due to squaring, MAD is a linear measure of dispersion that treats all deviations equally. This approach is especially useful for investors who are concerned with consistent performance and prefer a simpler risk measure that does not disproportionately penalize extreme values.

The MAD is calculated as:

$$MAD = n1i = 1 \sum n \mid ri - mean(r) \mid$$

where r_i represents the individual asset returns

TECHNICAL IMPLEMENTATION

The implementation of the MAD Portfolio follows these steps:

- **Calculate MAD:** A function is defined to calculate the MAD of portfolio returns.
- **Set Up Optimization Problem:** The optimization minimizes MAD, subject to constraints:
  - No Short-Selling: All weights are constrained to be non-negative.
  - Weights Sum to 1: The total weight allocation must equal 100%.
- **Optimize Weights:** The optimal weights are derived using the training data and applied to the test data for performance evaluation.

4.5 Mean Absolute Deviation portfolio (MAD)

PORTFOLIO PERFORMANCE VISUALIZATION

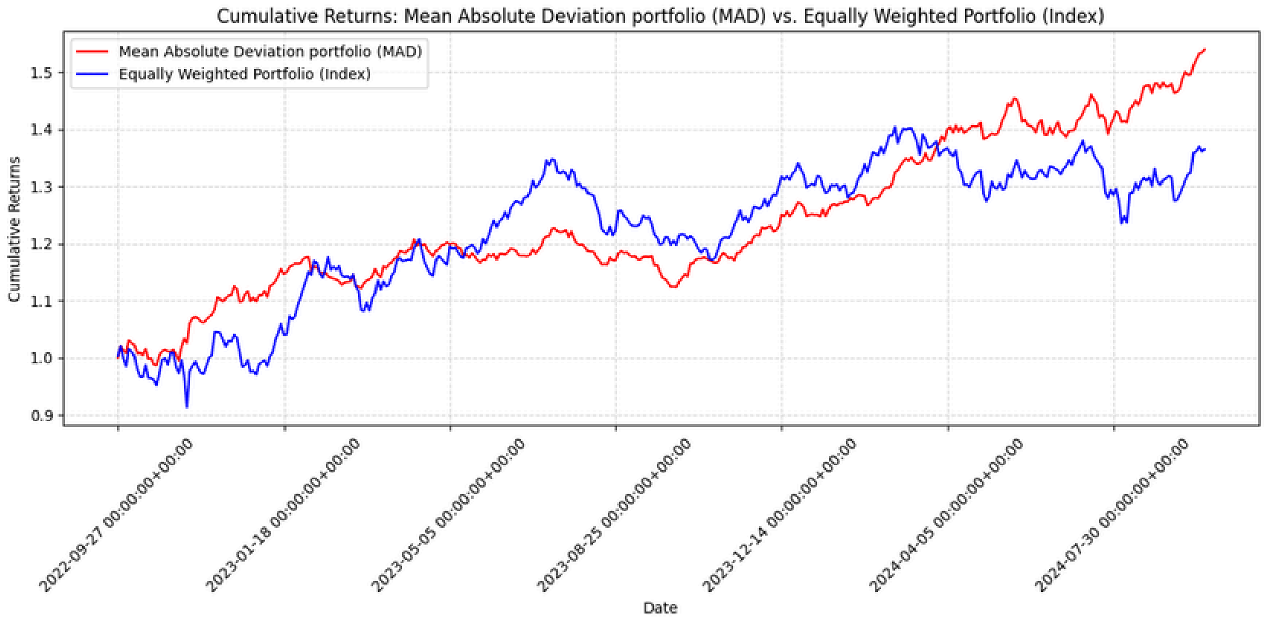
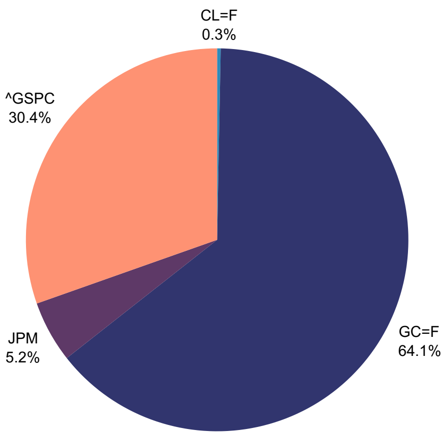


Image 7: Index Performance vs Mean Absolute Deviation Risk Portfolio

OPTIMAL WEIGHTS ANALYSIS

The optimal weights for the MAD Portfolio are heavily skewed, with 64.1% allocated to Gold Futures (GC=F) and 30.4% to the S&P 500 index (^GSPC). Other assets, such as AAPL, BTC-USD, and TSLA, receive zero weight, reflecting their potential to introduce higher deviations from the mean.



Pie Chart 5: Optimal Returns for Mean Absolute Deviation

4.6 Maximum Omega Ratio (MOR)

THEORETICAL BACKGROUND

The Maximum Omega Ratio Portfolio aims to optimize the Omega ratio, which is a performance measure that considers both the probability of achieving returns above a specified threshold and the magnitude of potential gains relative to losses. The Omega ratio is calculated as the ratio of the sum of returns above a given threshold to the sum of returns below it, making it an attractive metric for assessing the likelihood of exceeding a target return.

The Omega ratio is given by:

$$\Omega(returns, threshold) = \sum(returns < threshold) / \sum(returns > threshold)$$

where returns are divided into positive and negative sums relative to the threshold.

TECHNICAL IMPLEMENTATION

The implementation of the Maximum Omega Ratio Portfolio involves:

- **Define Omega Ratio Calculation:** A function calculates the Omega ratio, using a threshold of 0 (representing a risk-free rate of 0).
- **Set Up Optimization Problem:** The portfolio weights are optimized to maximize the Omega ratio, subject to constraints:
  - No Short-Selling: Weights are constrained to be non-negative.
  - Weights Sum to 1: The total weight allocation must equal 100%.
- **Optimize Weights:** The optimal weights are determined using the training data and then evaluated on the test data.

4.6 Maximum Omega Ratio (MOR)

PORTFOLIO PERFORMANCE VISUALIZATION

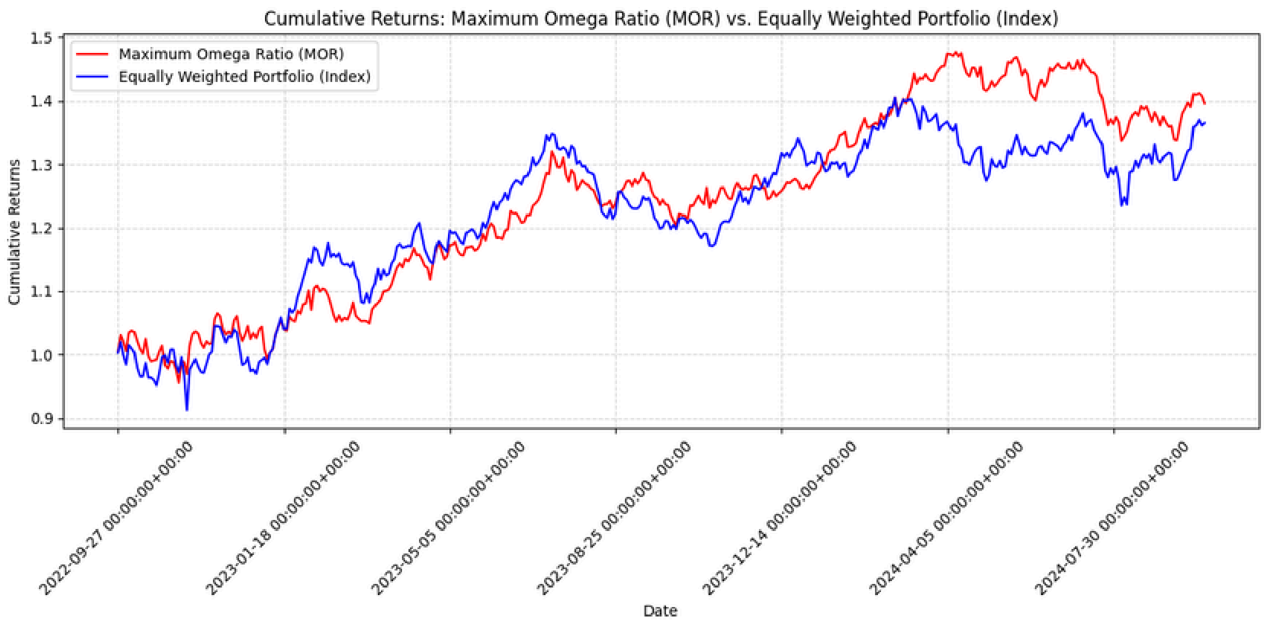
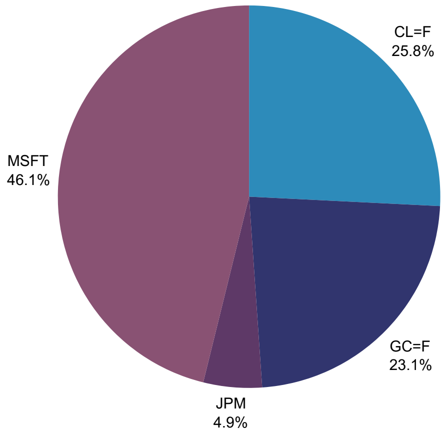


Image 8: Index Performance vs Maximum Omega Ratio (MOR) Portfolio

OPTIMAL WEIGHTS ANALYSIS

The optimal weights for the Maximum Omega Ratio Portfolio indicate a strong allocation to Microsoft (48.5%), followed by Crude Oil Futures (27.1%) and Gold Futures (24.3%). Notably, several assets, such as AAPL, BTC-USD, and TSLA, are excluded, reflecting the portfolio's emphasis on maximizing the Omega ratio by focusing on assets that offer favorable return characteristics relative to risk.



Pie Chart 6: Optimal Returns for Maximum Omega Ratio (MOR)

4.7 Tactical Dual Momentum strategy

THEORETICAL BACKGROUND

The Tactical Dual Momentum Strategy is a momentum-based investment approach that combines both relative and absolute momentum principles. Relative momentum compares the performance of different assets to identify which one has the best momentum, while absolute momentum evaluates whether the asset with the strongest momentum is outperforming a risk-free rate. If the best-performing asset has positive absolute momentum, the strategy invests in that asset, otherwise, it stays in cash to preserve capital.

TECHNICAL IMPLEMENTATION

The implementation of the Tactical Dual Momentum Strategy involves the following steps:

- **Calculate Momentum:** Momentum is calculated using a 1-year lookback period (252 trading days). The rolling product of returns over the lookback period is computed to assess past performance.
- **Implement Dual Momentum Rules:**
  - Identifying the asset with the highest relative momentum (the best-performing asset over the lookback period).
  - Checking if this asset’s absolute momentum (cumulative return) exceeds the risk-free rate. If it does, invest in this asset; otherwise, allocate the portfolio to cash (or a risk-free asset).

4.7 Tactical Dual Momentum Strategy

PORTFOLIO PERFORMANCE VISUALIZATION

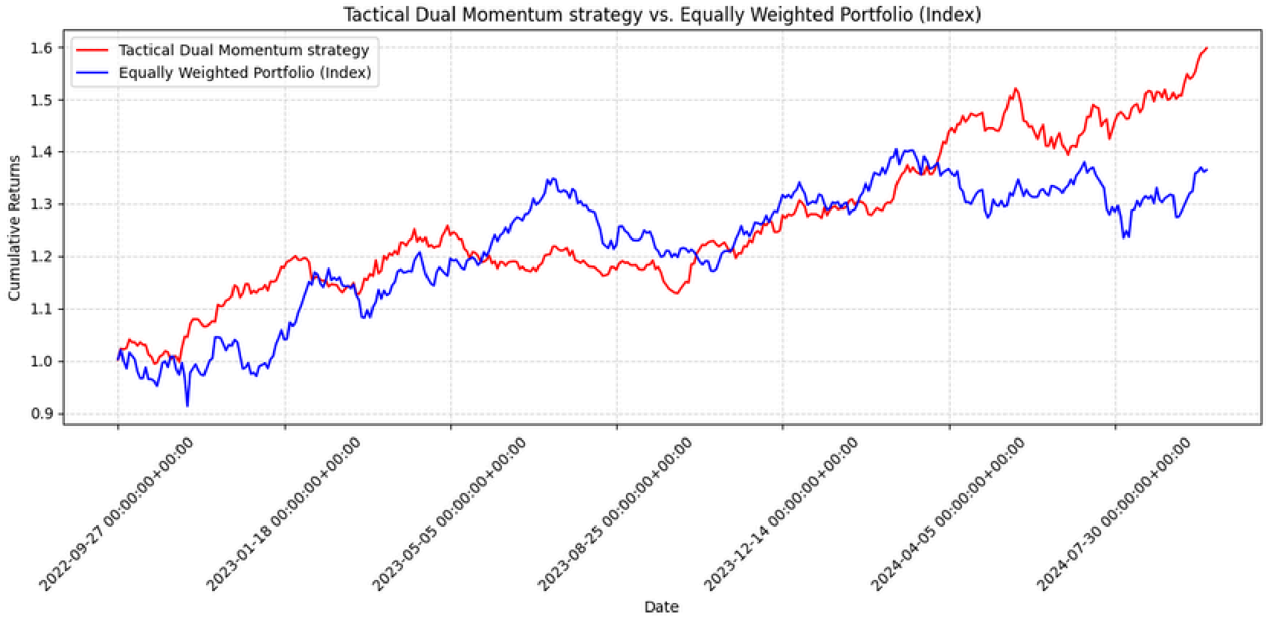
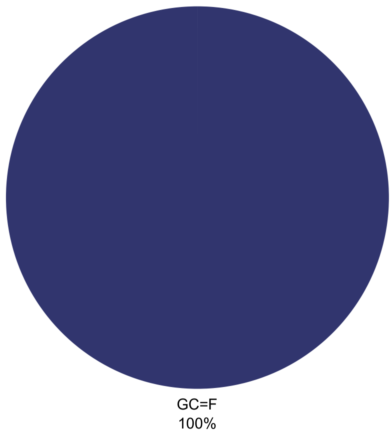


Image 9: Index Performance vs Tactical Dual Momentum Strategy Portfolio

OPTIMAL WEIGHTS ANALYSIS

The optimal weights for the Tactical Dual Momentum Strategy reveal a decisive allocation to Gold Futures (GC=F), with a weight of 100%. This indicates that Gold had the strongest momentum relative to other assets and satisfied the absolute momentum condition. All other assets, including equities and cryptocurrencies, are excluded, reflecting the strategy's strict adherence to momentum signals.



Pie Chart 7: Optimal Returns for Tactical Dual Momentum Strategy

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4.8 Adaptive Asset Allocation strategy

THEORETICAL BACKGROUND

The Adaptive Asset Allocation (AAA) Strategy dynamically adjusts portfolio weights based on changing market conditions. This strategy incorporates both momentum and volatility to maximize risk-adjusted returns, typically optimizing the Sharpe ratio. By using historical data to estimate future returns and risks, the strategy seeks to invest in assets with high momentum and low volatility, thus aiming to achieve a favorable risk-return balance.

TECHNICAL IMPLEMENTATION

The implementation of the Adaptive Asset Allocation Strategy consists of several key steps:

- **Calculate Momentum and Volatility:** Momentum and volatility are computed using a rolling window over a 1-year lookback period (252 trading days). Momentum is assessed as the rolling product of returns, while volatility is measured as the annualized standard deviation.
- **Optimize for Risk-Adjusted Return:** The objective function is defined to maximize the Sharpe ratio, which is calculated as the excess return over the risk-free rate divided by the portfolio's volatility. The optimization problem is solved subject to constraints:
  - No Short-Selling: Weights are constrained to be non-negative.
  - Weights Sum to 1: The total weight allocation must equal 100%.

4.8 Adaptive Asset Allocation strategy

PORTFOLIO PERFORMANCE VISUALIZATION

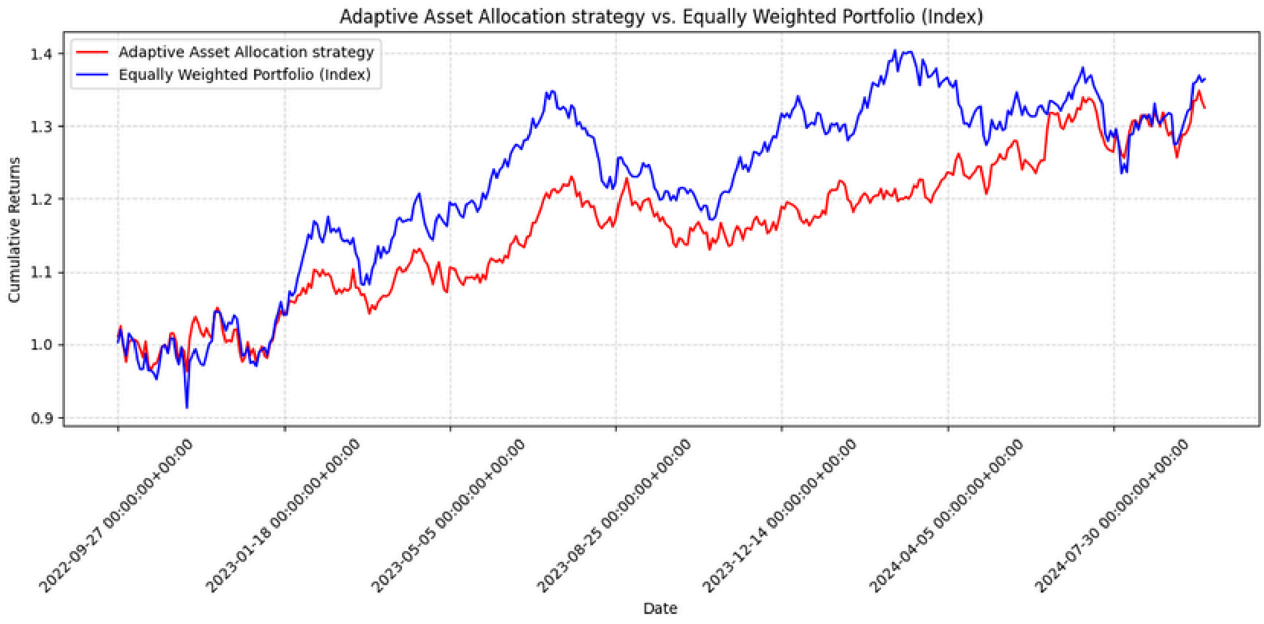
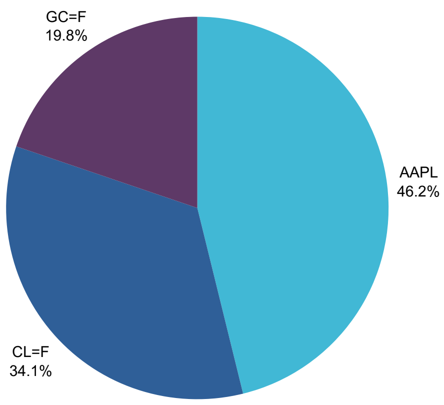


Image 10: Index Performance vs Adaptive Asset Allocation Portfolio

OPTIMAL WEIGHTS ANALYSIS

The optimal weights for the Adaptive Asset Allocation Strategy reveal a significant allocation to AAPL (46.2%) and Crude Oil Futures (CL=F) at 34.1%, with a smaller allocation to Gold Futures (GC=F) at 19.8%. Other assets, including cryptocurrencies and other equities, are excluded from the portfolio, reflecting the emphasis on assets with favorable momentum and risk characteristics.



Pie Chart 8: Optimal Returns for Adaptive Asset Allocation

4.9 The Black-Litterman Model Portfolio

THEORETICAL BACKGROUND

The Black-Litterman Model is an advanced portfolio optimization framework that combines modern portfolio theory with subjective investor views. It improves upon the traditional mean-variance optimization by incorporating views about expected returns in a more intuitive way, merging market equilibrium returns with investor's unique views on asset performance. The model adjusts the expected returns using a parameter that reflects the confidence level in these views.

Key Components:

- **Implied Equilibrium Returns**: Calculated using market weights and a risk aversion coefficient.
- **Investor Views**: Specified as beliefs about the expected returns of certain assets, with an associated level of confidence.
- **Adjusted Expected Returns**: Obtained using a blend of implied equilibrium returns and views, adjusted for uncertainty.

TECHNICAL IMPLEMENTATION

The Black-Litterman Model is implemented as follows:

- **Calculate Mean and Covariance:** Compute the mean returns and covariance matrix from the training data.
- **Set Market Weights:** Use equal weights as a proxy for market weights. This serves as a neutral starting point for the model.
- **Implied Equilibrium Returns**: Calculate the implied equilibrium returns using the formula: $ImpliedReturns = \tau \times CovarianceMatrix \times MarketWeights$
  - Here, τ=0.25, reflecting a moderate level of uncertainty in the equilibrium returns.
  - The risk aversion coefficient was set to λ=3, meaning the investor requires three units of excess return for each unit of risk.
- **Incorporate Investor Views**:
  - Asset 9 (Ticker: ^GSPC) will outperform Asset 1 (Ticker: AAPL) by **4%**.
  - Asset 4 (Ticker: ETH-USD) will outperform Asset 2 (Ticker: BTC-USD) by **8%**.
  - Asset 1 (Ticker: AAPL) will outperform Asset 5 (Ticker: GC=F) by **2%**.
- **Calculate Adjusted Returns**: Use the Black-Litterman formula to adjust returns.
- **Optimize Portfolio**: Maximize the Sharpe ratio using the adjusted returns.

4.9 The Black-Litterman Model Portfolio

PORTFOLIO PERFORMANCE VISUALIZATION

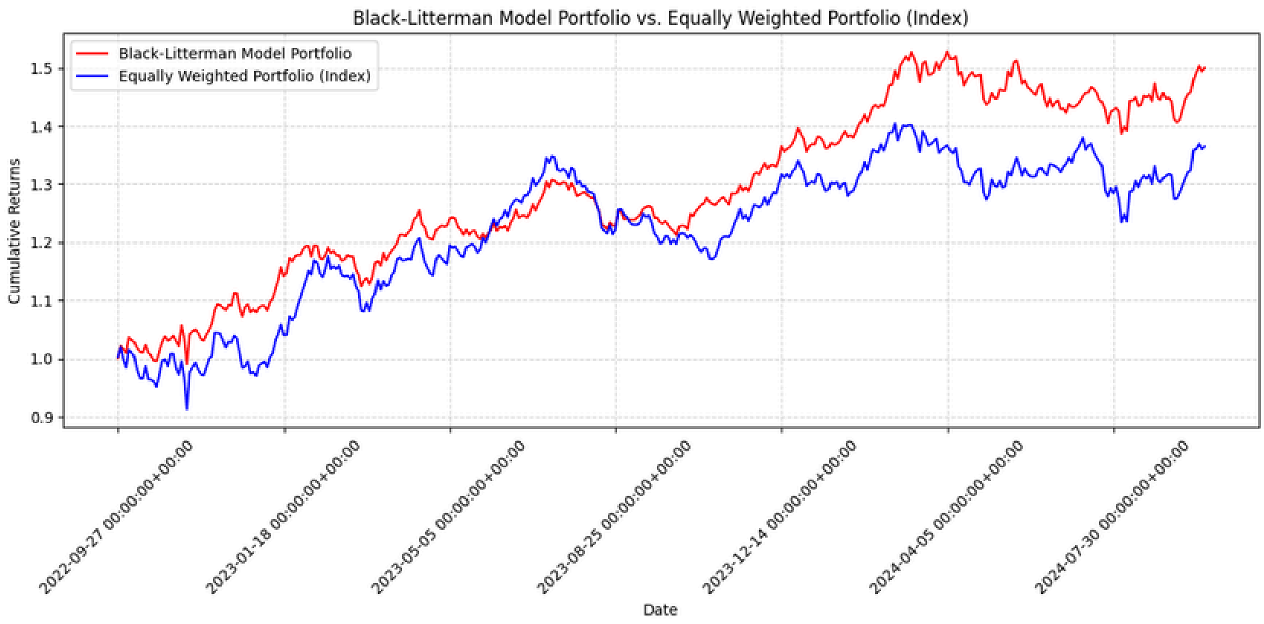
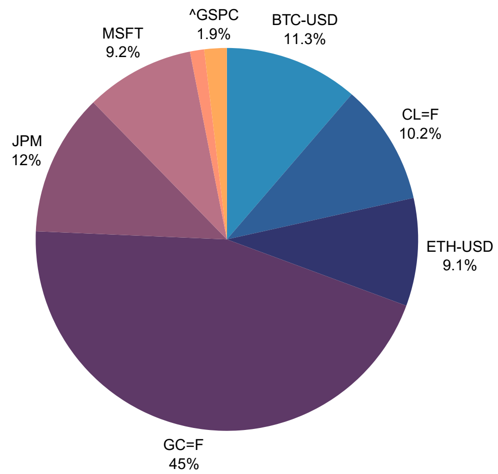


Image 11: Index Performance vs Black-Litterman Model Portfolio

OPTIMAL WEIGHTS ANALYSIS

The optimal weights from the Black-Litterman Model show a diversified allocation, with a significant focus on Gold Futures (45%), followed by JPM (12%) and BTC-USD (11.3%). The model adjusts weights based on the implied equilibrium returns and investor views, creating a portfolio that reflects both market expectations and subjective insights.



Pie Chart 9: Optimal Returns for Black-Litterman Model

4.10 Minimum Tail Dependence Portfolio

THEORETICAL BACKGROUND

The Minimum Tail Dependence Portfolio focuses on minimizing the tail dependence between assets. Tail dependence measures the likelihood that extreme losses occur simultaneously across multiple assets. By minimizing this metric, the strategy aims to reduce the risk of severe portfolio drawdowns during market downturns. Kendall's Tau is commonly used to quantify tail dependence, as it measures the strength of the relationship between pairs of asset's extreme values.

TECHNICAL IMPLEMENTATION

The implementation steps for the Minimum Tail Dependence Portfolio are as follows:

- **Calculate Tail Dependence:** Using Kendall's Tau to estimate the tail dependence between each pair of assets.
- **Formulate Tail Dependence Matrix:** The matrix captures the strength of tail dependencies across the assets, with higher values indicating stronger tail relationships.
- **Implied Equilibrium Returns**: Compute these using the risk aversion coefficient and covariance matrix.
- **Optimize for Minimal Tail Dependence**: The objective function minimizes a weighted sum of tail dependence coefficients, subject to constraints:
  - No Short-Selling: Weights are constrained to be non-negative.
  - Weights Sum to 1: The total allocation must equal 100%.

4.10 Minimum Tail Dependence Portfolio

PORTFOLIO PERFORMANCE VISUALIZATION

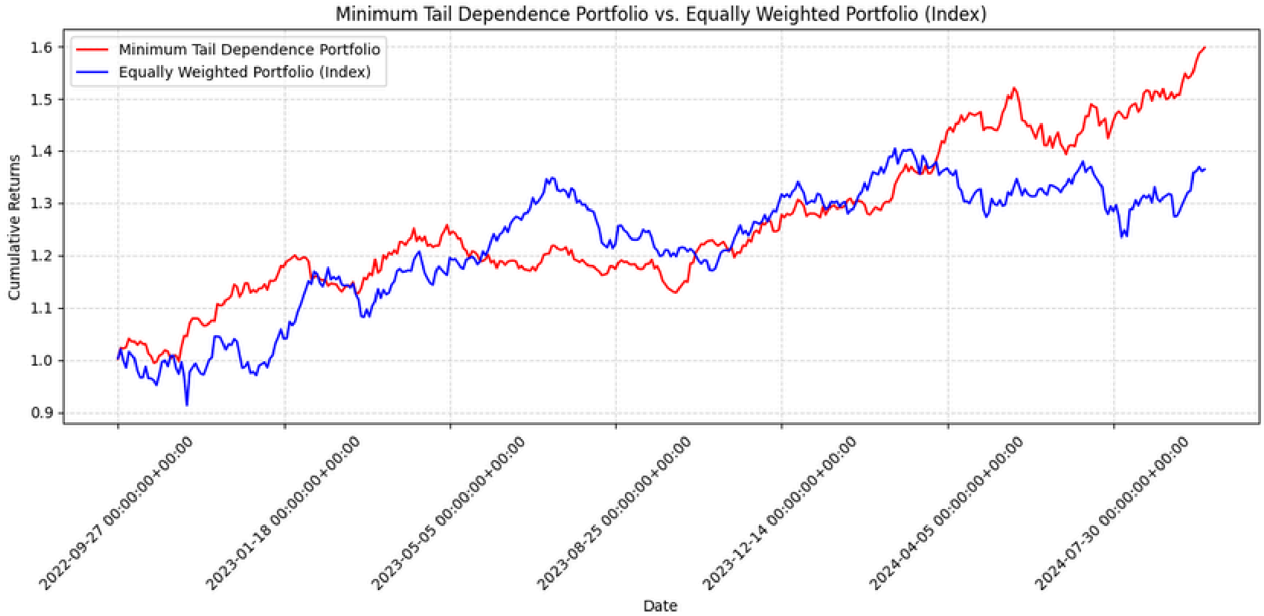
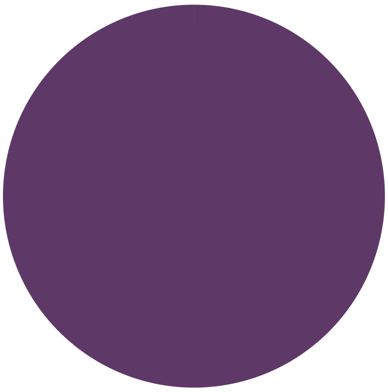


Image 12: Index Performance vs Minimum Tail Dependence Portfolio

OPTIMAL WEIGHTS ANALYSIS

The optimized portfolio weights show a 100% allocation to Gold Futures (GC=F), with no allocation to other assets. This heavy concentration suggests that Gold Futures had the least tail dependence with the rest of the assets, making it the optimal choice for minimizing extreme downside risk.



Pie Chart 10: Optimal Returns for Minimum Tail Dependence

5. FINAL RESULTS AND COMPARATIVE ANALYSIS

5.1 Comparative Performance Metrics

To evaluate the performance of the different portfolio allocation strategies employed in this project, several risk-adjuted metrics were used. Given the project's context and methodology, where the data was splitted into training and test sets to derive and assess the optimal portfolio weights, these metrics are essential for an objective comparison. The evaluation is conducted on the test data, which spans from 27-09-2022 until 26-09-2024, ensuring that the performance of the portfolio strategies is assessed in an out-of-sample setting.

ANNUALIZED RETURNS:

- The annualized returns represents the compounded rate of return that a portfolio would achieve over a year, assuming the returns are reinvested.
- Since investors are generally concerned with long-term performance, this metric provides a standardized way to compare how well each strategy performs over time.

ANNUALIZED VOLATILITY:

- Annualized volatility is a measure of the standard deviation of a portfolio's returns, scaled to a yearly basis. It quantifies the degree of variation or dispersion in returns over time.
- This metric acts as a proxy for risk in financial markets. By annualizing it, volatility is then comparable across different strategies and suitable for long-term investment opportunities.

5.1 Comparative Performance Metrics

SHARPE RATIO:

- The Sharpio Ratio measures the risk-adjusted return of a portfolio. It is calculated by dividing the portfolio's excess return (over the risk-free rate) by the annualized volatility.
- The Sharpe ratio is important for comparing different strategies because it emphasizes how efficiently a portfolio compensates investors for taking risk. In this project, it's crucial for assessing which strategies are most effective in generating returns per unit of risk.

$$SharpeRatio = (Rx – Rf) / StdDevRx$$

where:

- Rx = Expected Portfolio Return
- RF = Risk-Free Rate of the Return
- $StdDevRx$ = Standard Deviation of Portfolio Return

MAXIMUM DRAWDOWN:

- Maximum Drawdown is the largest peak-to-trough decline in a portfolio's value during the evaluation stage and it represents the biggest loss an investor would have experienced.
- This performance metric is equally important for risk-averse investors who are concerned with potential large losses. Since some of the applied strategies try to minimize downside risk like, for example, Minimum CDaR and Mean-CVaR portfolios, this metric provides insight into the effectiveness of these risk management approaches

5.2 Critical Analysis and Insights

COMBINED PORTFOLIO PERFORMANCE VISUALIZATION

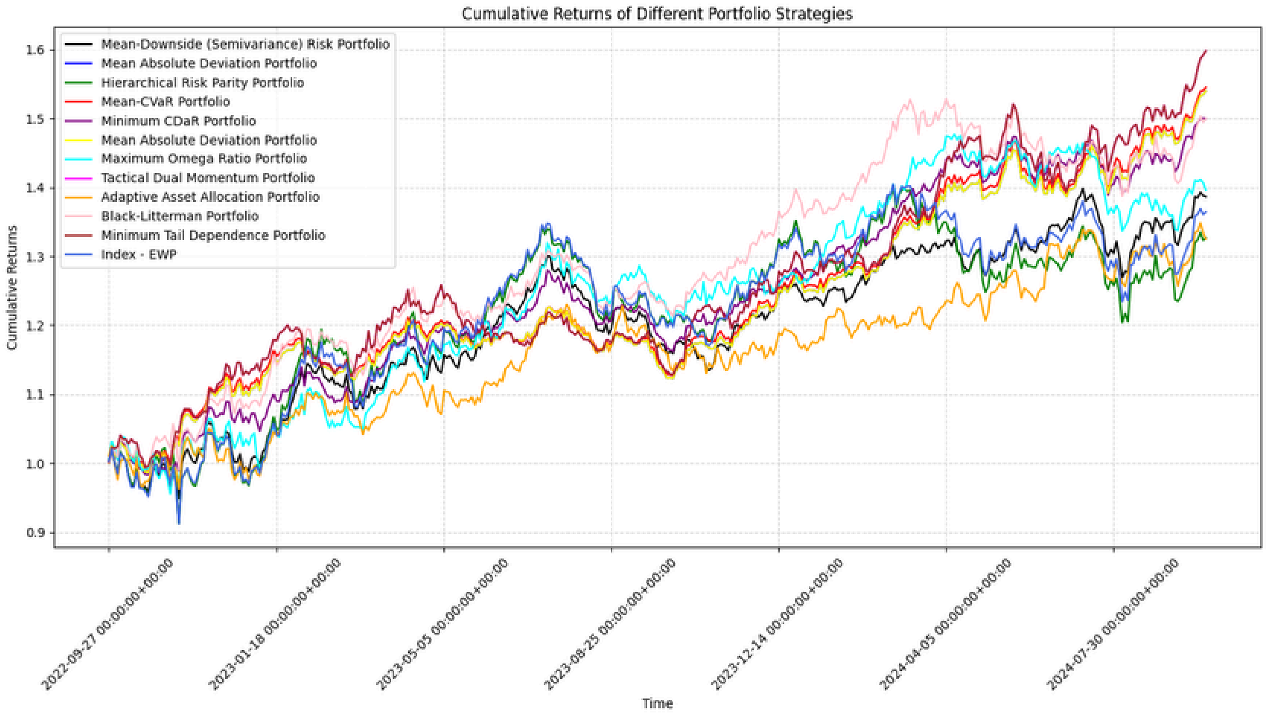


Image 13: Index Performance vs All the Portfolios

Portfolio	Annualized Return	Annualized Volatility	Sharpe Ratio	Maximum Drawdown
Mean-Downside (Semivariance) Risk Portfolio	0.248	0.1596	1.5541	-0.1274
Mean Absolute Deviation Portfolio	0.3267	0.1157	2.8226	-0.0842
Hierarchical Risk Parity Portfolio	0.2248	0.209	1.0754	-0.1398
Mean-CVaR Portfolio	0.3298	0.1165	2.8303	-0.081
Minimum CDaR Portfolio	0.3064	0.1279	2.3964	-0.0926
Maximum Omega Ratio Portfolio	0.2554	0.1685	1.5159	-0.0947
Tactical Dual Momentum Portfolio	0.3637	0.1444	2.5181	-0.1033
Adaptive Asset Allocation Portfolio	0.2148	0.1705	1.2594	-0.0813
Black-Litterman Portfolio	0.3118	0.1562	1.9969	-0.0924
Minimum Tail Dependence Portfolio	0.3637	0.1444	2.5181	-0.1032
Index	0.2436	0.1964	1.2403	-0.131

Image 14: Performance Metric Results

5.2 Critical Analysis and Insights

COMPARISON TO THE INDEX:

- The Index has an annualized return of **0.2436**, annualized volatility of **0.1964**, and a Sharpe Ratio of **1.2403**.
- Many of the portfolio strategies outperform the Index in terms of risk-adjusted returns. For example, the Mean-CVaR Portfolio and Mean Absolute Deviation Portfolio have significantly higher Sharpe Ratios of **2.8303** and **2.8226**, respectively, showcasing their superior risk-return profiles.
- However, the Hierarchical Risk Parity Portfolio underperforms the Index, with both lower returns (**0.2248**) and a lower Sharpe Ratio (**1.0754**), despite having higher volatility.

HIGH-RETURN STRATEGIES:

- Tactical Dual Momentum and Minimum Tail Dependence Portfolios both achieve the highest annualized return of **0.3637**. This impressive return is primarily due to both strategies allocating all capital to the GC=F asset (Gold Futures), which had good performance in the test set. However, relying on a single asset could imply high concentration risk.
- The Mean-CVaR Portfolio shows a strong annualized return of **0.3298**, indicating that its focus on minimizing extreme downside risk has resulted in favorable performance during the test period.

RISK MANAGEMENT AND VOLATILITY:

- The Mean Absolute Deviation Portfolio continues to exhibit the lowest annualized volatility at **0.1157**, making it an excellent choice for risk-averse investors who seek stability.
- The Hierarchical Risk Parity Portfolio again appears as the most volatile strategy, with a volatility of **0.209**, indicating that this risk-parity approach may not be as effective in controlling volatility as expected.

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5.2 Critical Analysis and Insights

SHARPE RATIO ANALYSIS:

- The Mean-CVaR Portfolio has the highest Sharpe Ratio of **2.8303**, indicating the best performance relative to risk. This is a significant improvement over the Index, emphasizing the benefits of focusing on downside risk.
- The Mean Absolute Deviation Portfolio has a nearly equivalent Sharpe Ratio of **2.8226**, showing that reducing overall volatility while maintaining strong returns is highly effective.
- The Tactical Dual Momentum and Minimum Tail Dependence Portfolios share a Sharpe Ratio of **2.5181**, showcasing high efficiency in generating returns relative to risk.

MAXIMUM DRAWDOWN:

- The Adaptive Asset Allocation Portfolio and Mean-CVaR Portfolio show good drawdown management, with the smallest maximum drawdowns of **-0.0813** and **-0.081**, respectively. This indicates strong downside protection and is a positive feature for conservative investors.
- The Index experiences a maximum drawdown of **-0.131**, which is worse than most strategies, except for the Hierarchical Risk Parity Portfolio and Mean-Downside (Semivariance) Risk Portfolio, which have drawdowns of **-0.1398** and **-0.1274**, respectively.

6. CONCLUSION

6.1 Summary of key Findings

- The **Mean-CVaR Portfolio** and **Mean Absolute Deviation Portfolio** provide the **highest Sharpe Ratios**, suggesting these are the best strategies for maximizing returns relative to risk.

- The **Tactical Dual Momentum** and **Minimum Tail Dependence** Portfolios achieved **high returns** but did so by fully allocating capital to GC=F. This concentration could be risky if market conditions change unfavorably for this asset.

- The **Black-Litterman Portfolio** shows a well-rounded performance with a Sharpe Ratio of **1.9969** and moderate drawdown protection, benefiting from incorporating market equilibrium and investor views.

- For investors prioritizing stability, the **Mean Absolute Deviation Portfolio** is appealing due to its **low volatility and high Sharpe Ratio**.

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