AP&PM

Post-graduation in Data Science for Finance

Asset Pricing and Portfolio Management

Group Project

Investigating financial market returns and testing portfolio strategies

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1. ABSTRACT

This research delves into the empirical characteristics of financial market returns and evaluates the performance of various portfolio investment strategies. By analyzing daily, weekly, and monthly log-returns of a diversified portfolio of securities, the study explores stylized facts such as the absence of autocorrelation, non-normality, skewness, volatility clustering, leverage effects, and conditional non-normality.

To assess the effectiveness of different strategies, a walk-forward backtesting approach is employed. This involves generating 100 randomized 2-year datasets from historical market data and evaluating the performance of equally weighted, mean-variance, global minimum variance, maximum Sharpe ratio, inverse volatility, risk parity, most diversified, and maximum decorrelation portfolios.

The study concludes by critically discussing the results using various performance metrics, like returns, Sharpe ratios, Sterling ratios, and Drawdowns, providing insights into the strengths and weaknesses of each strategy under different market conditions.

KEYWORDS:
Financial Markets' Returns; Investment Strategies; Markowitz Modern Portfolio Theory; Backtesting.

2. INTRODUCTION

In today's dynamic financial landscape, the integration of diverse asset classes is essential for informed decision-making and strategic investment. As markets become increasingly complex and interrelated, understanding the performance and interactions of various financial instruments is crucial for investors looking to optimize their portfolios. This project harnesses the potential of a wide array of financial instruments, including stocks, cryptocurrencies, indexes, and commodities, to create a holistic investment strategy.

Our primary objective is to analyze selected assets across different sectors to uncover valuable insights that drive robust investment strategies and enhance portfolio performance. Specifically, our team aims to identify trends, assess risk levels, and evaluate the correlation between asset classes, allowing for more informed investment decisions in an ever-evolving market.

2.1 ASSETS TO BE TESTED

For our analysis, our team has selected a comprehensive list of assets across different sectors and classes, which includes:

Asset Class	Ticker	Asset Name	Additional Information
Stocks	AAPL	Apple	Tech Company
	JPM	JPMorgan Chase	Financial Company
	MSFT	Microsoft	Tech Company
	TSLA	Tesla	Automotive & Energy Company
Indexes	QQQ	NASDAQ	NASDAQ-100 index's ETF
	^GSPC	S&P 500	500 large-cap U.S. companies' performance index
Commodities	GC=F	Gold	Gold Futures traded on the COMEX exchange
	CL=F	Oil	Crude Oil Futures traded on the NYMEX
Cryptocurrencies	BTC-USD	Bitcoin	Proof-of-Work consensus Cryptocurrency
	ETH-USD	Etherium	Proof-of-Stake consensus Cryptocurrency

3. IMPORTANCE OF VOLATILITY AND CORRELATION

3.1 VOLATILITY

The volatility is essential for assessing risk and variability in the returns of financial assets. It serves as a gauge for price fluctuations, which is crucial in shaping investment choices and portfolio strategies, as assets with higher volatility typically imply greater risk.

The accompanying graph captures the volatility patterns of various assets across different timeframes: daily, weekly, and monthly log returns. The visualization reveals that volatility levels vary significantly between asset types, underscoring how different investments can experience distinct levels of price stability or instability over time.

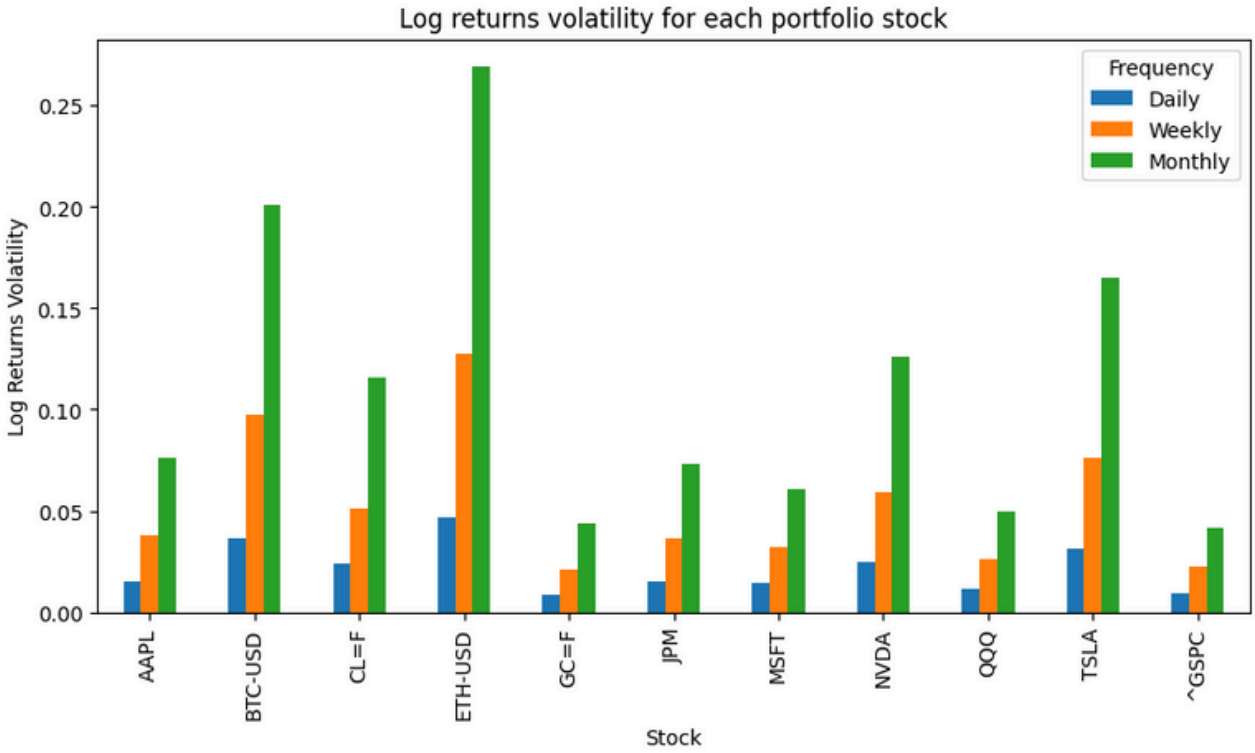

Image 1 - Volatility per Asset

It is evident that there is a stark difference in volatility between market indices and cryptocurrencies. The indices, represented by **S&P 500** (^GSPC) and **Nasdaq 100** (QQQ), exhibit significantly lower volatility compared to cryptocurrencies like **Bitcoin** (BTC-USD) and **Ethereum** (ETH-USD). This suggests that cryptocurrency investments are generally riskier due to their price fluctuations. It is also interesting to see that commodities, like **Gold** (GC=F), can be an interesting choice for an investor that is looking for more stable assets.

3.1 VOLATILITY

Low Volatility Assets	Medium Volatility Assets	HIgh Volatility Assets
Gold (CG=F)	Apple (AAPL)	Tesla (TSLA)
S&P 500 (^GSPC)	JPMorgan Chase (JPM)	Bitcoin (BTC-USD)
NASDAQ 100 (QQQ)	Nvidia (NVDA)	Etherium (ETH-USD)
Microsoft (MSFT)	Oil (CL=F)	-

In summary, this volatility analysis provides a comprehensive look at the fluctuation levels across a range of assets, enabling investors to better understand the relative risk associated with each of them. These insights can guide them in structuring portfolios that align with their risk tolerance and investment objectives.

3.2 CORRELATION

In the realm of investment analysis, understanding the relationships between different financial assets is crucial for an effective portfolio management. Correlation, which measures the degree to which two assets move in relation to each other, provides valuable insights that can significantly impact investment strategies.

By analyzing correlation, investors can better navigate market dynamics, assess risk, and optimize their portfolio allocations. This understanding is especially important in today's interconnected financial markets, where the behavior of one asset can influence others.

For instance, if two assets have a high correlation and one wants to minimize the risk associated with one of the assets, it should be considered reducing the investment in the other asset as well. The S&P 500 and Nasdaq 100, being both indexes, tend to move together.

As a result, investing in both may not provide sufficient diversification, as they often exhibit similar performance patterns. Therefore, if one wishes to mitigate risk, it may be more effective to choose a less correlated asset rather than allocating funds to both indexes.

3.2 CORRELATION

For further analysis, our team analysed the correlation between different assets expressed in the heatmap below:

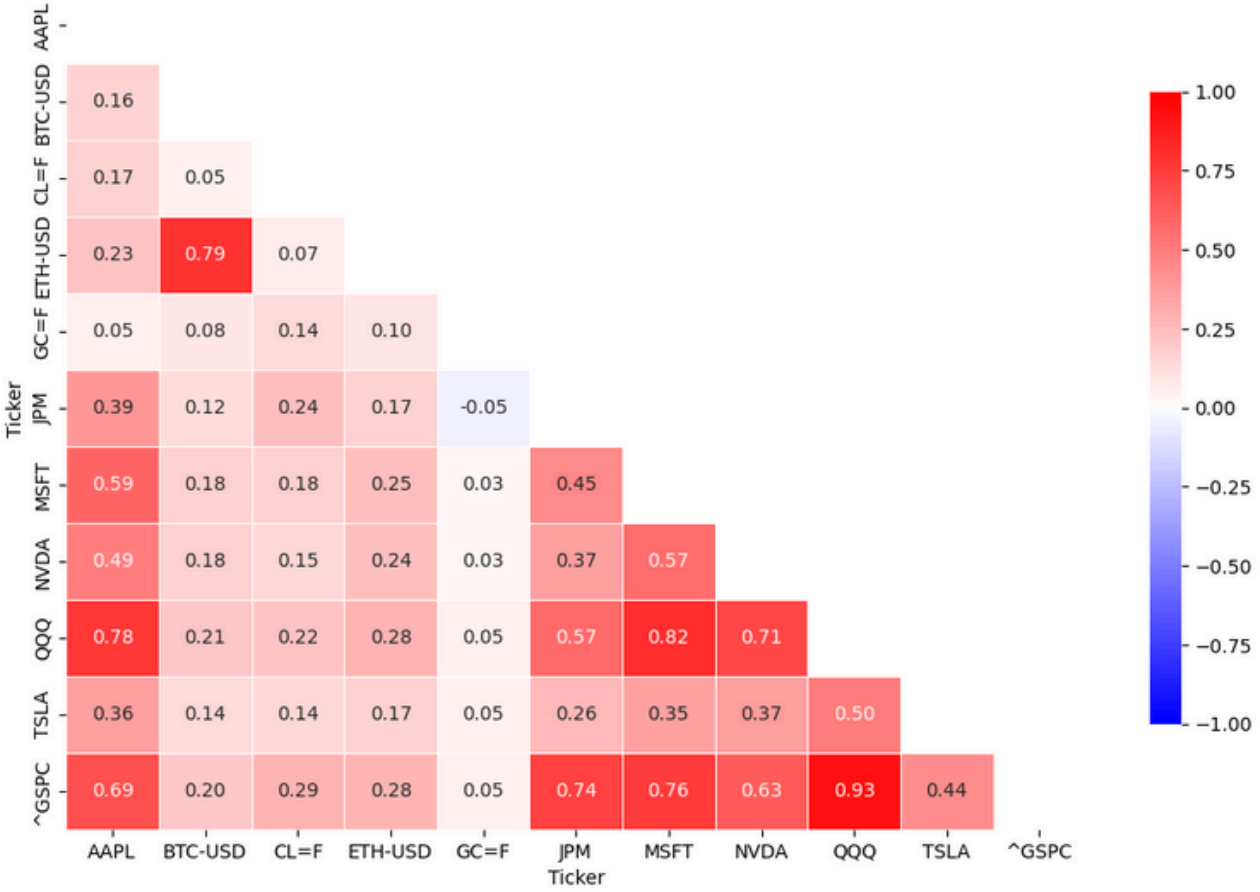

Image Title: Correlation Heatmap

The analysis above suggests a positive correlation between the market and most other assets, indicating a lack of significant negative relationships. Several asset pairs exhibited a strong positive correlation, exceeding 0.65.

These pairs included:

- **Bitcoin** (Crypto) with **Ethereum** (Crypto)
- **S&P 500** (Index of Companies) with **Apple** (AAPL), **JPMorgan Chase** (JPM), **Microsoft** (MSFT) (Individual Companies)
- **Nasdaq** (Index of Tech) with **Apple** (AAPL) and **Microsoft** (MSFT) (Technology Companies)
- **S&P 500** with **Nasdaq 100** (Both Indexes)

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PART 1

FINANCIAL MARKETS RETURNS' EMPIRICAL INVESTIGATION

1. **AUTOCORRELATION ABSENCE**
2. **DEVIATION FROM A NORMAL DISTRIBUTION AND PRESENCE OF FAT TAILS**
3. **ASYMMETRY AND SKEWNESS OF RETURN'S DISTRIBUTION**
4. **VOLATILITY CLUSTERING**
5. **LEVERAGE EFFECTS**
6. **CONDITIONAL NON-NORMALITY**

3. FINANCIAL MARKETS RETURNS' EMPIRICAL INVESTIGATION

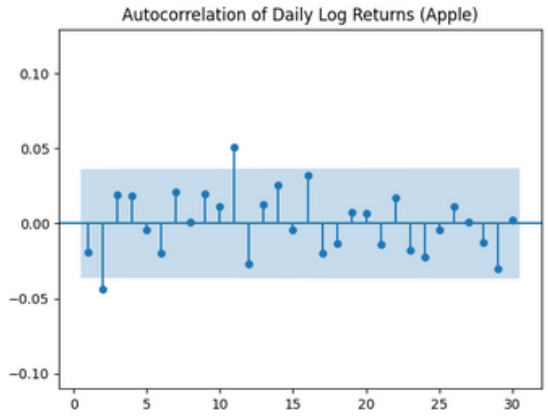
3.1 AUTOCORRELATION ABSENCE

Autocorrelation is a statistical metric that evaluates the correlation between a time series and its own past values, revealing temporal connections within the data. Autocorrelation, which analyzes how current observations relate to prior ones, aids in the identification of patterns, trends, and cyclic behaviors, making it a vital tool in fields such as finance, economics and signal processing.

Understanding autocorrelation not only helps to identify repeating structures in data, but it also improves the prediction potential of models by identifying underlying linkages that may not be immediately obvious.

$$Autocorrelation = \frac{\sum_{t=1}^{N}(X_t - \bar{X})(X_{t-k} - \bar{X})}{\sum_{t=1}^{N}(X_t - \bar{X})^2}$$

X_t - The value of the data at time t.
$\bar{X}$ - The average (mean) of all data values.
X_{t-k} - The value of the data k steps before t (lagged value).
N - The total number of data points.
k - The lag, or how far back in time we're looking.



The Autocorrelation Function (ACF) graph is a visual tool used to identify patterns or dependencies within a time series.

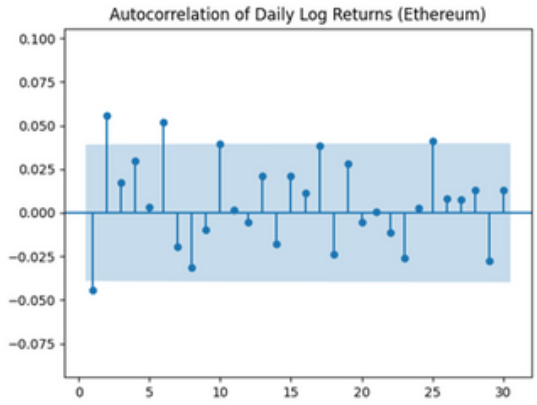
- **X-axis**: Represents the number of lags (time periods).
- **Y-axis**: Represents the autocorrelation coefficient, which measures the correlation between the current value and past values at different lags (difference in time, lag 1 = 1 day before and so on).

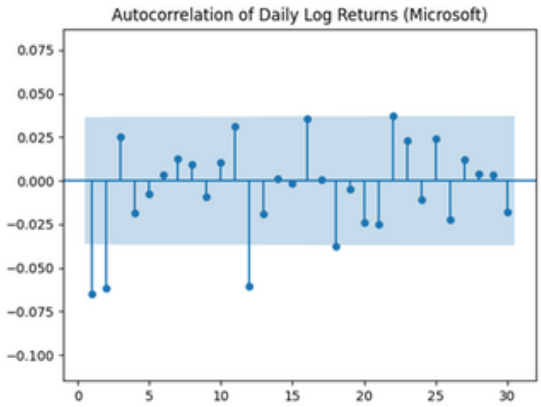
As for example, in this ACF graph, it can be observed that the majority of the autocorrelation coefficients are within the shaded confidence bands, and many of them are close to zero. This suggests that there is **no significant autocorrelation** between the daily returns of **Apple** stock. However, lag 2 and lag 11 tend to be significant as they are outside the confidence interval.

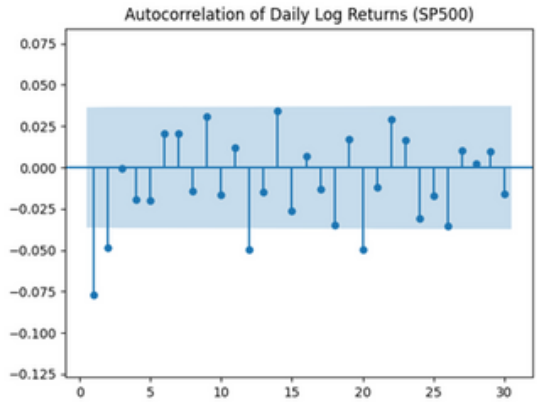
3.1 AUTOCORRELATION ABSENCE

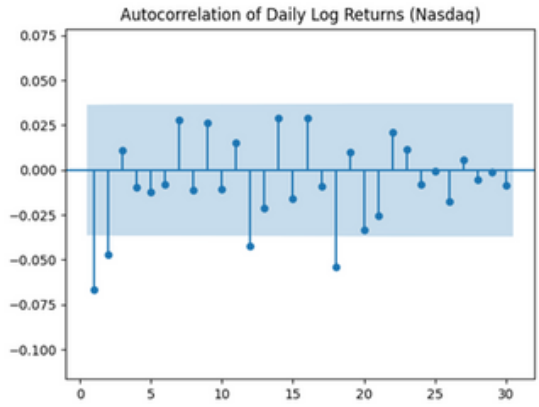
3.1.1 Daily Analysis

In assets like Ethereum, Microsoft , Nasdaq, and the S&P 500, the first two lags often exhibit significant autocorrelation. When these lags are negative, it indicates a negative correlation with values from the previous two periods, suggesting that the current value is likely to decrease following positive values in those periods. This behavior implies a reversing or mean-reverting trend. Additionally, lags such as lag 12 and lag 18 may also significantly influence today's price, indicating that patterns from these periods can affect current market behavior.





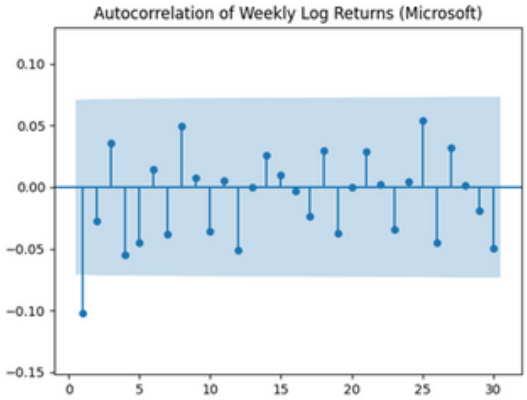


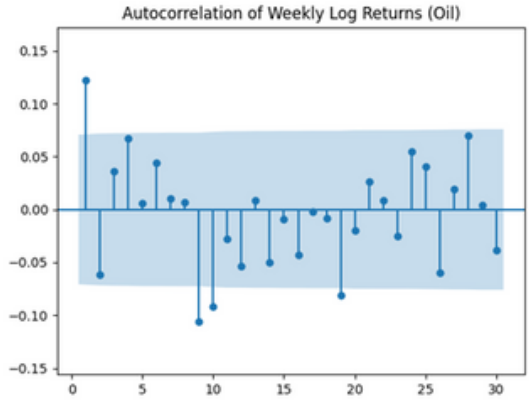


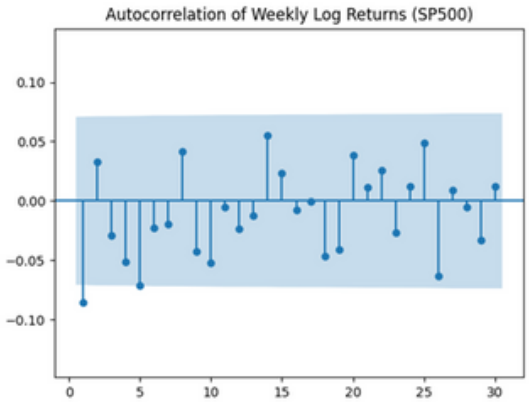
3.1 AUTOCORRELATION ABSENCE

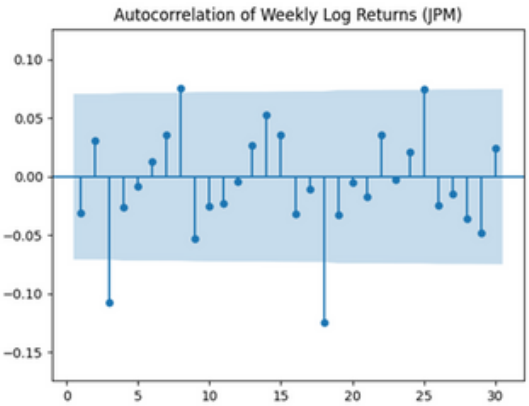
3.1.2 Weekly Analysis

For the weekly data, there were fewer significant lags, but they still existed. Among the assets shown below, lag 1 was almost always significant, indicating that the price from the previous week influenced the current week's price. In the case of oil, additional significant lags, such as 9 and 10, suggest a connection between prices from roughly two and a half months prior. In JPMorgan (JPM), autocorrelation is observed at lag 3 and lag 18 (representing 3 weeks and approximately 4 and a half months), which may correspond to quarterly cycles. The other assets exhibited little to no autocorrelation.





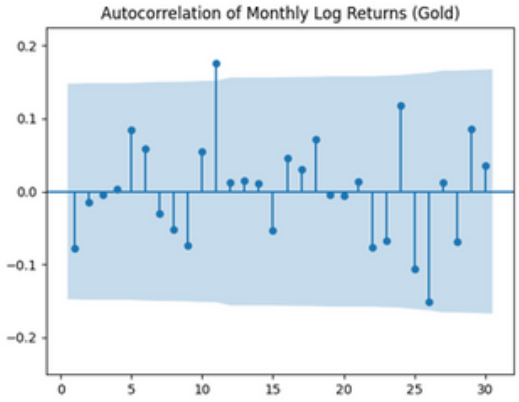


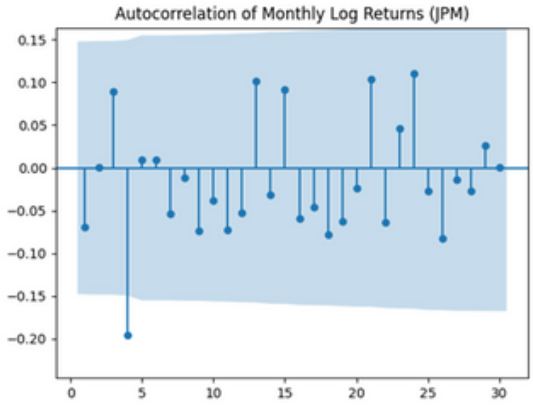


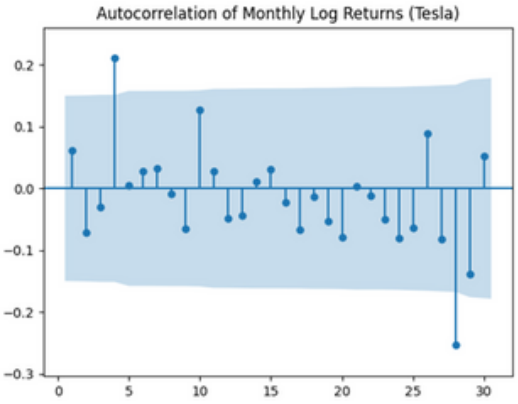
3.1 AUTOCORRELATION ABSENCE

3.1.3 Monthly Analysis

In the monthly data, there are even fewer assets with lags that pass the significance threshold. Lag 4, present in both Tesla and JPMorgan assets, indicates that the price from four months prior is affecting the current month's price. This may also relate to quarterly cycles (3 months), as data can sometimes not align closely with theoretical expectations regarding behavior. The same applies to lag 28 in Tesla. In the case of Gold, a significant lag at 12 suggests that the prices from each month are homologous, indicating a relationship between the current month and the same month in the previous year.



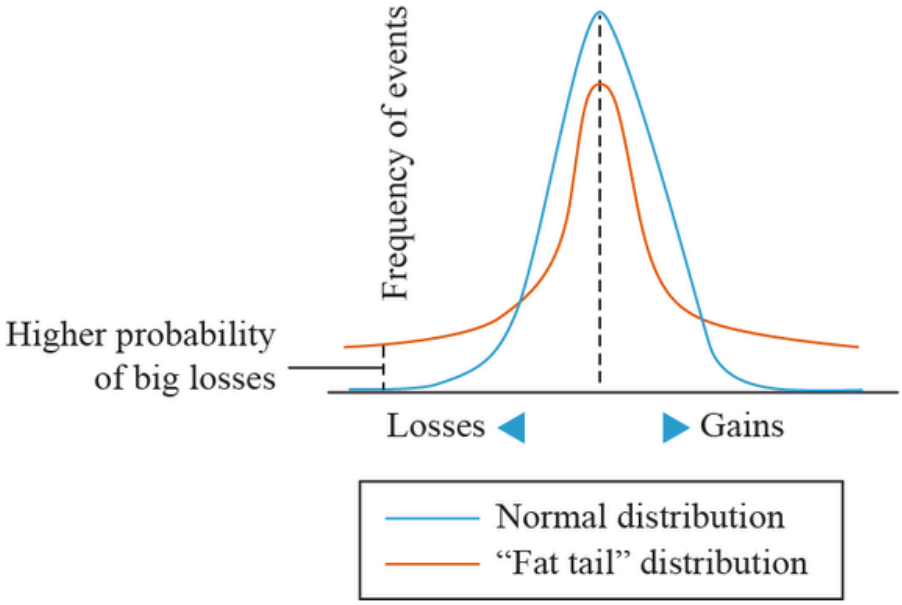




3.2 DEVIATION FROM A NORMAL DISTRIBUTION/ PRESENCE OF FAT TAILS

The unconditional distribution of daily returns in financial markets often deviates from the normal distribution, exhibiting what is commonly referred to as fat tails. Fat tails describe the phenomenon where extreme events, such as large price changes or market shocks, occur more frequently than a normal distribution would predict. In essence, this means that the probability of observing returns that are far from the mean is higher than what would be expected under the assumption of normality.

This characteristic of fat tails is a crucial consideration in finance and economics, as it highlights the increased likelihood of extreme volatility and outlier events in financial markets. Traditional models, assuming normal distribution, often underestimate the risk of extreme movements such as crashes or sudden surges. Recognizing fat tails provides a more realistic understanding of market behaviors, making it essential for more effective risk management and portfolio optimization strategies. With this approach, investors can better account for rare but high-impact events that traditional models might fail to capture, thereby improving decision-making and prediction accuracy.



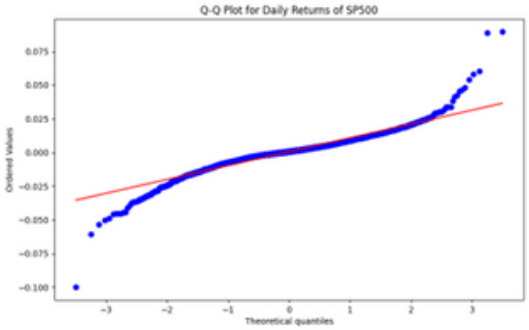
In conclusion, recognizing fat tails in financial markets is vital for better risk assessment and decision-making. Unlike normal distributions, fat-tailed models account for the higher likelihood of extreme events, such as market crashes or surges. If an asset exhibits more negative extreme events than positive ones, it may indicate that the asset is more vulnerable to market crashes, either alone or alongside other assets. Incorporating fat tails into financial strategies helps improve risk management and prepares investors for rare, high-impact events that traditional models might overlook.

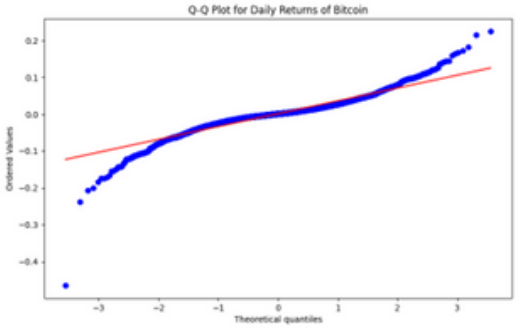
3.2 DEVIATION FROM A NORMAL DISTRIBUTION/ PRESENCE OF FAT TAILS

3.2.1 Daily Analysis

In the case of daily returns, the distribution clearly exhibits fat tails. From the graph, our team could observe that returns falling below the -2 quantile and above the +2 quantile deviate significantly from the normal distribution line. Specifically, values below the -2 quantile are notably lower than predicted by the normal distribution, while values above the +2 quantile are higher. This suggests that both extreme negative and positive returns occur far more frequently than the normal distribution would anticipate.

In a normal distribution, extreme events are rare and would occur with much lower frequency, but the data shows larger-than-expected losses and gains. This behavior is typical in financial markets, where abrupt price swings, crashes, and surges happen more often than traditional models, which assume normality, would predict. Understanding this pattern is critical for risk management, as it reveals that financial markets are far more prone to significant volatility than traditional models would suggest.





3.2 DEVIATION FROM A NORMAL DISTRIBUTION/ PRESENCE OF FAT TAILS

3.2.1 Weekly Analysis

When analyzing weekly returns, the pattern of fat tails is still present, although the frequency of extreme outliers is lower compared to daily returns. In these graphs, one can see a few observations that deviate significantly from the normal distribution, though they are less frequent.

The global market shock caused by the COVID-19 pandemic in 2020 is a notable example, with some weekly returns dropping by more than 15%, and in the case of Bitcoin, even exceeding 30%. This highlights the presence of factor risk, where extreme events can lead to significant deviations from typical market behavior. Despite such dramatic losses, markets often recover with periods of rapid gains, such as the subsequent weeks of 30% surges following a -40% drop in asset values.

If it is excluded the extreme drop in Apple's price during this period, the weekly return distribution for Apple appears to follow a nearly normal distribution. This demonstrates that while fat tails remain a feature in financial returns, the frequency and intensity of extreme events are less pronounced on a weekly basis than on a daily scale. Nonetheless, the impact of such extreme events is significant and cannot be ignored.

3.2 DEVIATION FROM A NORMAL DISTRIBUTION/ PRESENCE OF FAT TAILS

3.2.1 Monthly Analysis

Moving to monthly returns, the behavior of different assets begins to diverge. Apple's returns, for example, show an almost perfect normal distribution, similar to Bitcoin. This suggests that, over a longer time horizon, the returns of these assets tend to resemble a normal distribution, with fewer extreme deviations.

However, the returns of JPMorgan Chase (JPM) show some significant low values in the lower quantiles, indicating a slight deviation from normality. Still, without these extreme observations, JPM's return distribution could be considered nearly normal.

Similarly, the S&P 500 index shows that, if the lowest observation is removed, it closely approximates to a normal distribution. This supports the idea that, over longer time periods like monthly returns, the market can behave in a way that aligns more closely with traditional models of normality, though extreme outliers can still influence the overall distribution.

3.3 ASYMMETRY AND SKEWNESS OF RETURN'S DISTRIBUTION

Skewness, a measure of asymmetry in a distribution, is a crucial factor to consider in portfolio optimization. A positively skewed distribution indicates a longer tail to the right, suggesting a higher probability of large positive returns. Conversely, a negatively skewed distribution implies a longer tail to the left, indicating a higher likelihood of large negative returns.

In the context of portfolio optimization, understanding skewness is important because it allows investors to assess the potential upside and downside risks associated with different assets. By incorporating skewness into portfolio construction, investors can potentially construct portfolios with higher expected returns and lower downside risk.

The skewness of daily, weekly or monthly returns can provide valuable insights into the risk profile of an asset. Here's a breakdown of the different scenarios:

- **Negative Skewness**: This indicates a longer tail to the left of the distribution, suggesting a higher probability of large negative returns. Such assets might be considered riskier, as they have a greater potential for downside losses.

- **Positive Skewness**: This implies a longer tail to the right, indicating a higher probability of large positive returns. While this might seem attractive, it's important to note that positive skewness can also be associated with higher volatility.

- **Neutral Skewness**: This suggests a more symmetrical distribution, closer to a normal distribution. Such assets might be considered less risky, as the probability of extreme returns, both positive and negative, is lower.

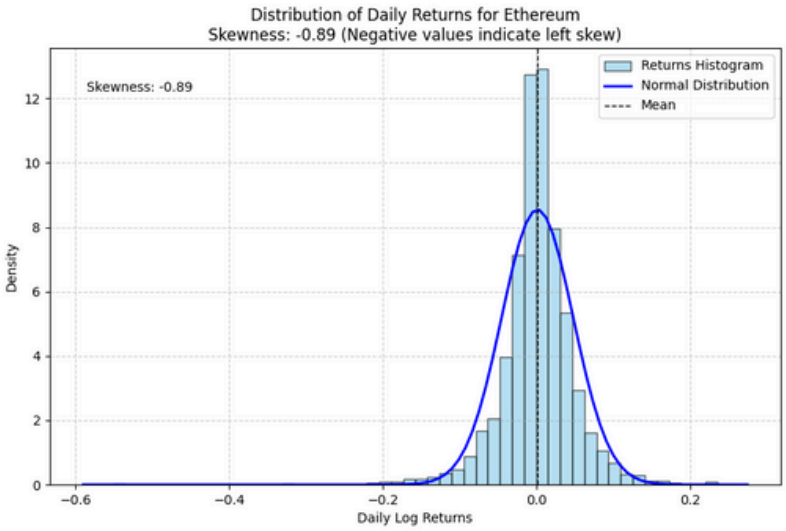
3.3 ASYMMETRY AND SKEWNESS OF RETURN'S DISTRIBUTION

3.3.1 DAILY ANALYSIS

By looking at the skewness histograms, the observed phenomena is the following:

Negative Skewed Distribution (Skewness < - 0.2)	Neutral Skewed Distribution (- 0.2 < Skewness < 0.2)	Positive Skewed Distribution (Skewness > 0.2)
Etherium	Microsoft	Oil
Bitcoin	Tesla	JPMorgan
Gold	Apple	-
NASDAQ 100	-	-
S&P 500	-	-

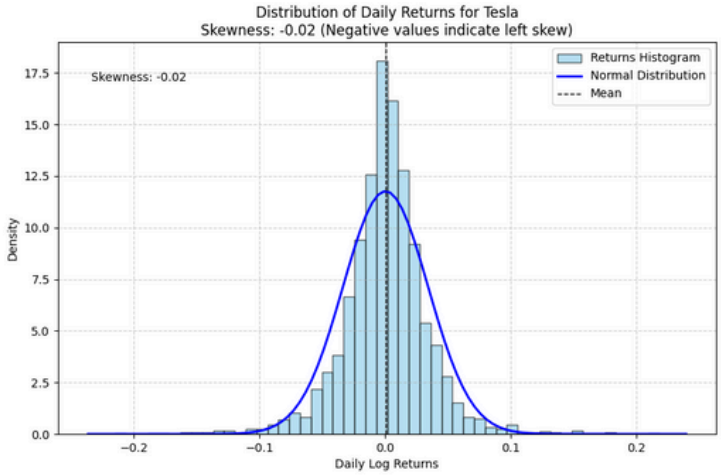
Ethereum's daily return (ETH-USD) distribution exhibits significant **negative skewness (-0.89)**, deviating notably from a normal distribution. This indicates a higher probability of experiencing substantial negative returns compared to positive ones. While the majority of daily returns cluster around the mean, the skewed distribution suggests increased downside risk.



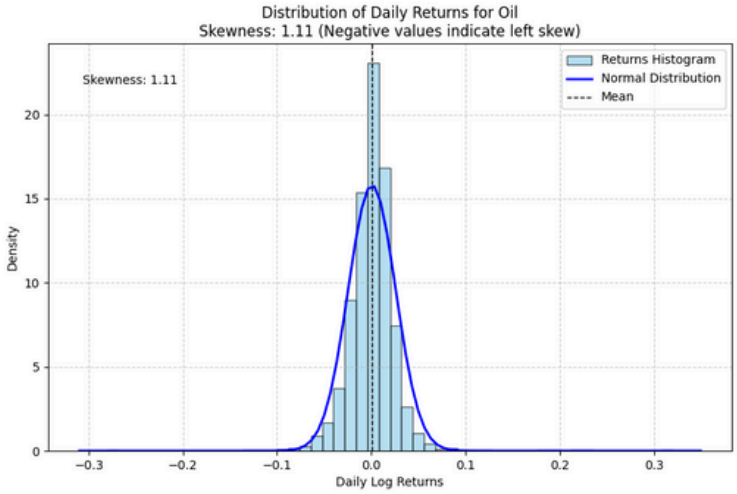
3.3 ASYMMETRY AND SKEWNESS OF RETURN'S DISTRIBUTION

3.3.1 DAILY ANALYSIS

Tesla's daily return (TSLA) distribution exhibits a near-neutral skewness (-0.02), deviating deviating only slightly from a normal distribution. This suggests that the probability of large positive and negative returns is relatively balanced. While the histogram indicates some volatility in daily returns, the overall distribution suggests a moderate risk profile.



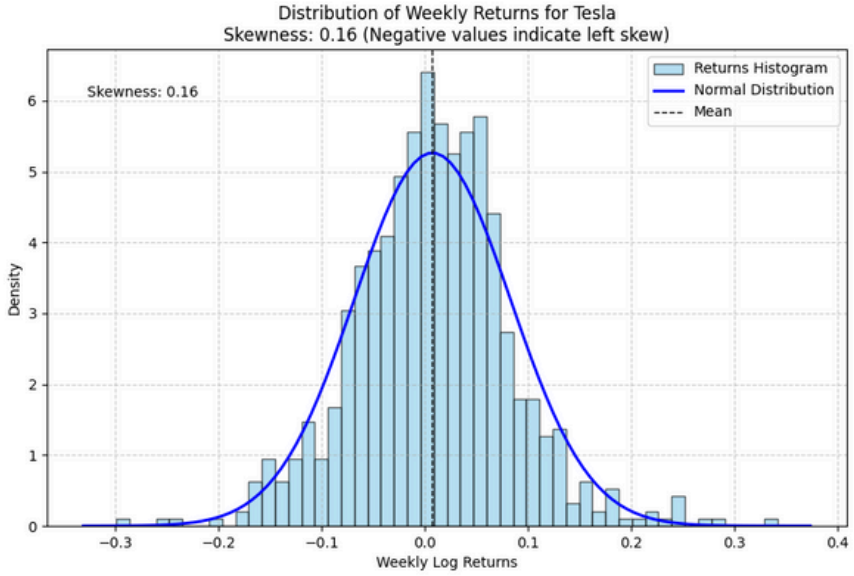
Oil's daily return distribution exhibits significant **positive skewness (1.11)**, deviating substantially from a normal distribution. This indicates a higher probability of experiencing large positive returns compared to negative ones. While this suggests a favorable risk-reward profile, it's important to note that this higher upside potential is often accompanied by increased volatility. The histogram reveals that Oil's returns can fluctuate significantly, highlighting the need for careful risk management when investing in this commodity. It's crucial to consider the impact of factors such as geopolitical events, economic conditions, and supply-demand dynamics on Oil's future performance.



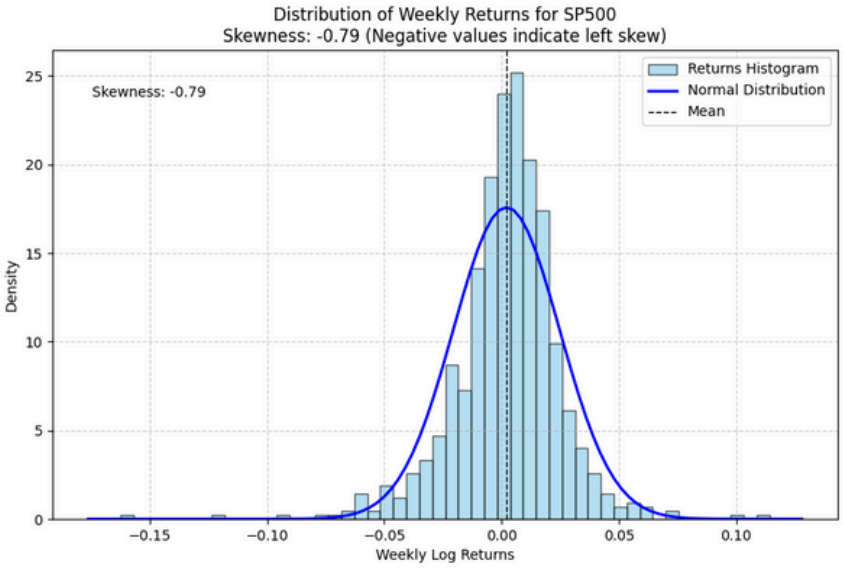
3.3 ASYMMETRY AND SKEWNESS OF RETURN'S DISTRIBUTION

3.3.2 WEEKLY ANALYSIS

By looking at the weekly return distribution histograms, we see that the only asset with **positive skewness** is **Tesla**. Which indicates a higher probability of experiencing large positive returns compared to negative ones.



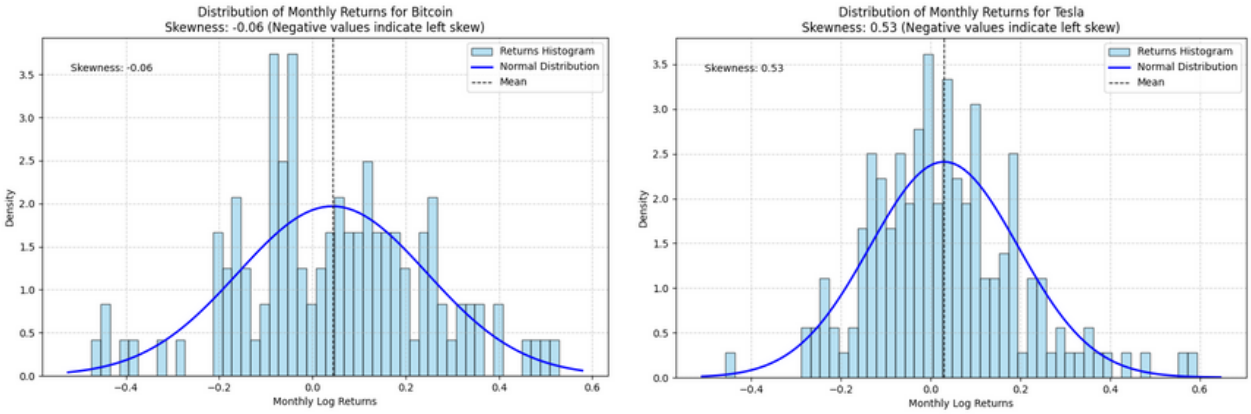
Surprisingly, **S&P 500**'s weekly return distribution exhibits significant **negative skewness (-0.79)**, deviating notably from a normal distribution. This indicates a higher probability of experiencing large negative returns compared to positive ones. While the majority of weekly returns cluster around the mean, the skewed distribution suggests increased downside risk.



3.3 ASYMMETRY AND SKEWNESS OF RETURN'S DISTRIBUTION

3.3.2 MOTHLY ANALYSIS

By looking at the monthly return distribution histograms, our team observes that most assets have a **negatively skewed** distribution. However, there are four assets that stand out: **Bitcoin**, **Gold**, and **Microsoft** with nearly **neutral skewed** distributions, and **Tesla** with a positively skewed distribution.



Our team concluded that there are assets that are more suitable as a long term investment, where the monthly return distribution histograms showed neutral or positively skewed distributions, meaning that there is a lower probability of experiencing large negative returns. On the other hand, there were some securities that showed their strength in shorter periods, i.e., the ones with positive skewed distributions in daily and weekly return distribution histograms.

3.4 VOLATILITY CLUSTERING

Volatility clustering refers to the tendency of periods of high volatility in financial markets to be followed by more high volatility, while periods of low volatility are followed by continued low volatility. This phenomenon highlights the non-constant nature of asset price movements, where periods of calm are interspersed with periods of heightened market activity. Recognizing and understanding volatility clustering is crucial for portfolio optimization as it allows for a more nuanced assessment of risk.

By incorporating volatility clustering into portfolio models, investors can better anticipate and manage periods of market turbulence, potentially leading to more robust and resilient investment strategies.

- **High volatility clustering** can indicate periods of significant market turbulence, which can be both a risk and an opportunity. It might suggest that certain assets are more sensitive to market shocks.

- **Low volatility clustering** might suggest a more stable market environment, but it could also indicate less potential for significant returns.

Ultimately, the ideal level of volatility clustering depends on the specific investment goals and risk tolerance. A more conservative investor might prefer lower volatility clustering, while a more aggressive investor might be willing to tolerate higher levels in pursuit of higher returns.

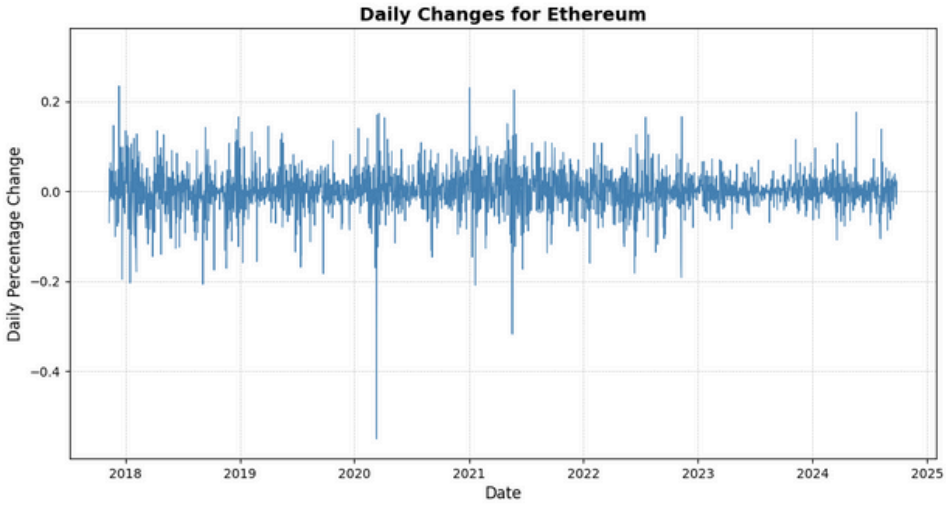
3.4 VOLATILITY CLUSTERING

3.4.1 DAILY ANALYSIS

After analyzing the daily volatility clustering charts of the chosen assets, our team concluded the following:

High Volatility Clustering	Moderate Volatility Clustering	Low Volatility Clustering
Etherium	JPMorgan	Gold
Bitcoin	Oil	S&P 500
Tesla	Nasdaq 100	Apple
-	-	Microsoft

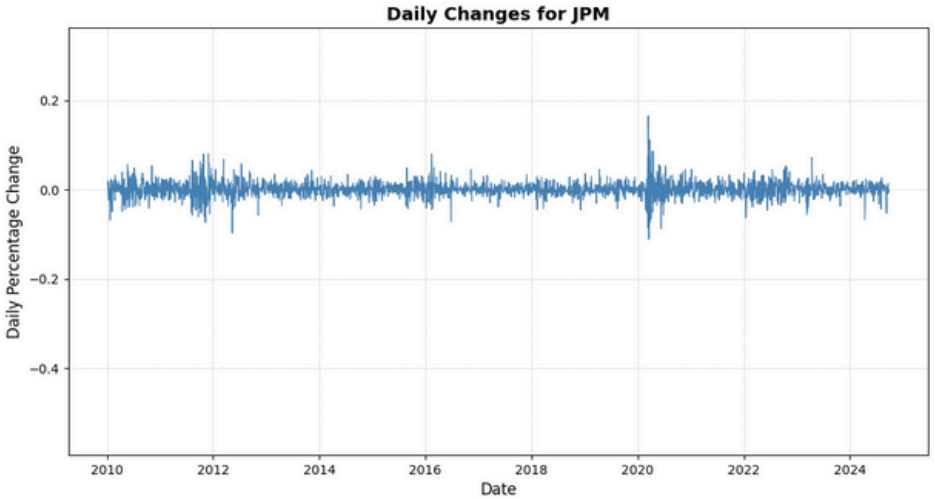
Ethereum shows **significant volatility**, i.e., high volatility with clustered periods of high daily changes, particularly around major market movements in cryptocurrencies. Clustering is prominent, as expected for crypto assets, which experience heightened volatility and price swings.



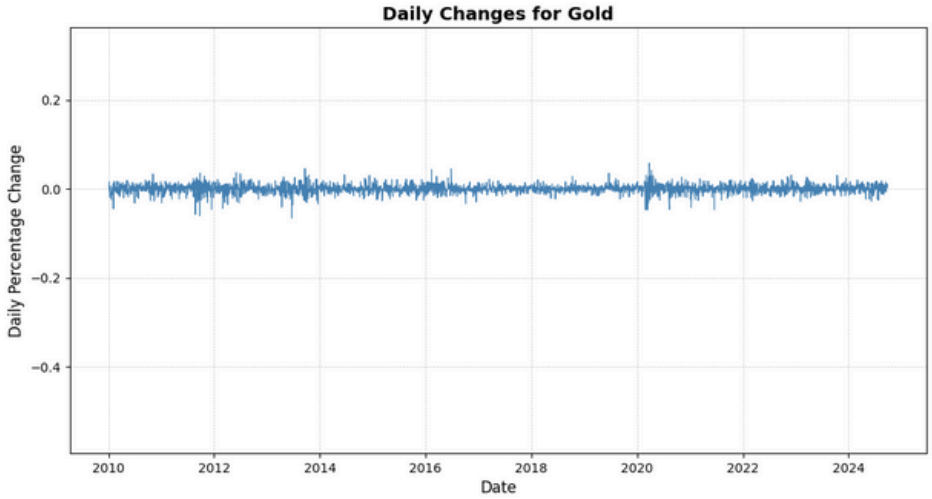
3.4 VOLATILITY CLUSTERING

3.4.1 DAILY ANALYSIS

The JPM chart displays occasional spikes, particularly around events like the COVID-19 pandemic in 2020. Some clustering of higher volatility seems to be present around that period, which aligns with the financial sector's sensitivity to macroeconomic events.



Gold's daily changes are **generally consistent**, with very little deviation over time. This stability aligns with gold's reputation as a safe-haven asset, showing minimal clustering or volatility spikes.



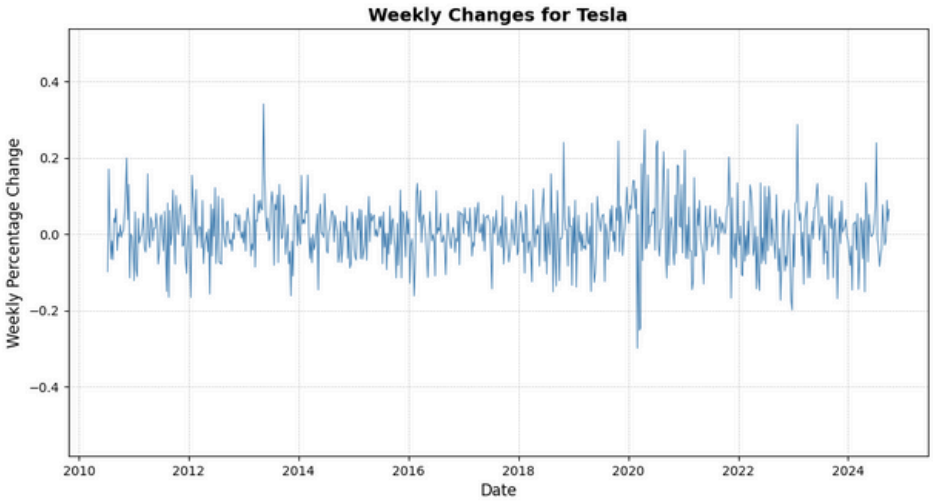
3.4 VOLATILITY CLUSTERING

3.4.2 WEEKLY ANALYSIS

After analyzing the weekly volatility clustering charts of the chosen assets, our team concluded the following:

High Volatility Clustering	Moderate Volatility Clustering	Low Volatility Clustering
Etherium	Oil	Gold
Bitcoin	Apple	S&P 500
Tesla	JPMorgan	Nasdaq 100
-	-	Microsoft

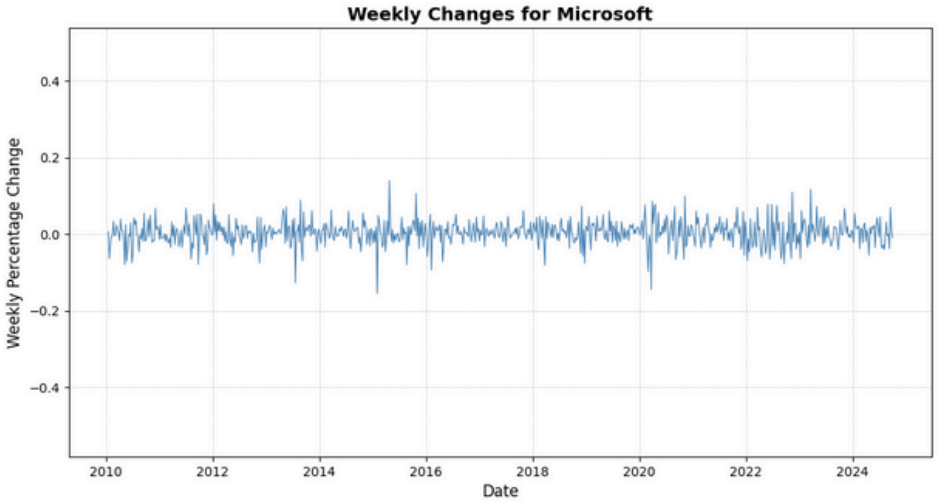
Tesla's weekly returns exhibit **pronounced volatility clustering**. Periods of heightened volatility occur frequently, likely reflecting the stock's sensitivity to news, innovation cycles, and market speculation.



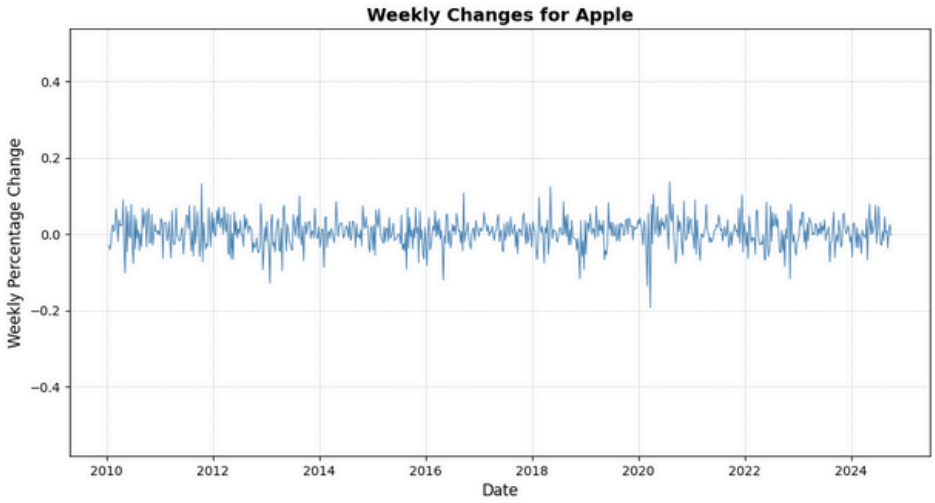
3.4 VOLATILITY CLUSTERING

3.4.2 WEEKLY ANALYSIS

The **Microsoft** weekly returns are **relatively stable**, with minor volatility clustering. Some slight increases are noticeable around periods of economic stress (e.g., 2020), but overall, it's stable with minimal clustering.



Apple shows slightly **more volatility** than **Microsoft**, with a few clusters of higher volatility during market events, such as the COVID-19 period. However, it generally remains stable.



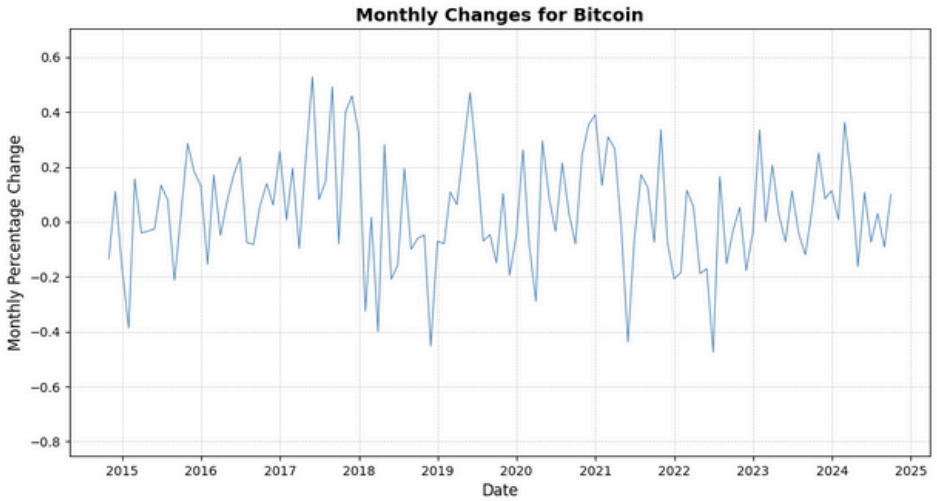
3.4 VOLATILITY CLUSTERING

3.4.3 MONTHLY ANALYSIS

After analyzing the monthly volatility clustering charts of the chosen assets, our team concluded the following:

High Volatility Clustering	Moderate Volatility Clustering	Low Volatility Clustering
Etherium	JPMorgan	Gold
Bitcoin	Oil	S&P 500
Tesla	Apple	Nasdaq 100
-	-	Microsoft

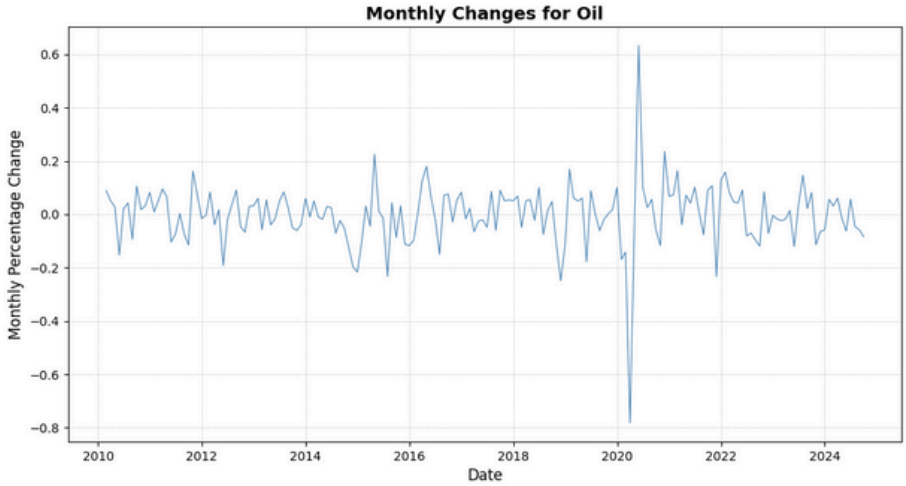
Bitcoin's monthly changes show **significant clustering**. There are distinct periods of high volatility, corresponding to market bubbles or crashes, emphasizing the speculative nature of Bitcoin.



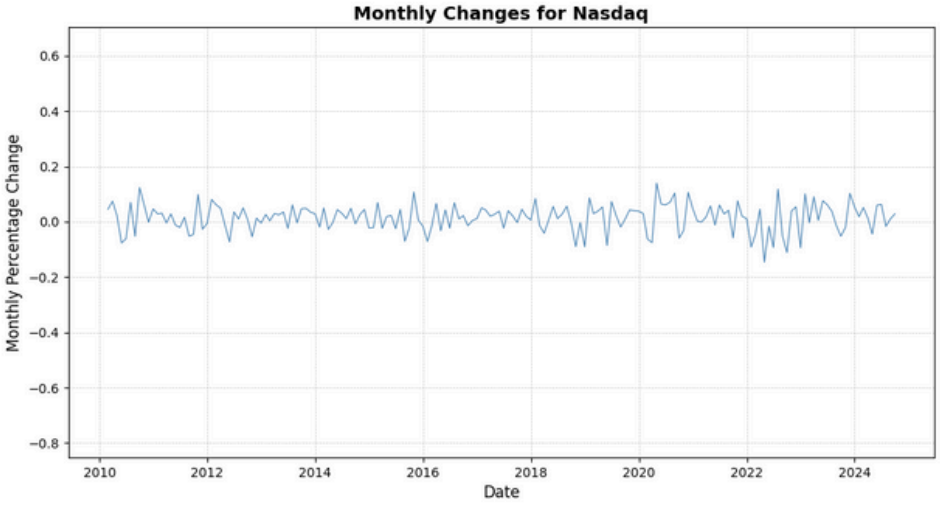
3.4 VOLATILITY CLUSTERING

3.4.3 MONTHLY ANALYSIS

Oil shows **some volatility** clustering, with clear spikes during geopolitical events or economic disruptions. The monthly changes are sensitive to global supply and demand factors, making oil more volatile compared to other commodities like gold.



The **Nasdaq** exhibits **moderate monthly changes** with slight clustering during economic downturns, though it is generally more stable than, for example, cryptocurrencies. The index's diversification contributes to its lower volatility.



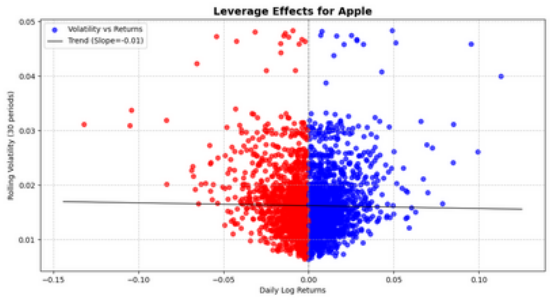
3.5 LEVERAGE EFFECTS

Leverage effects refers to the phenomenon in financial markets where an asset's volatility is negatively correlated with its returns. In simple terms, when a specific asset has negative returns, its volatility tends to increase, while positive returns generally come with lower volatility. This effect is often associated with equities and is linked to the asymmetric response of volatility to market news or shocks.

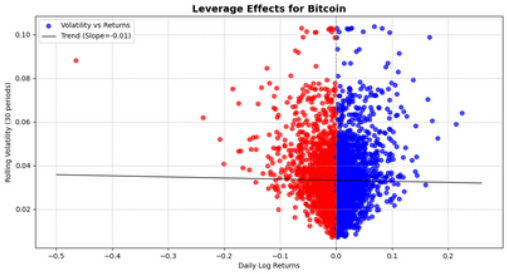
Understanding leverage effects is important for risk management, especially for portfolios that include leverage assets. Higher volatility following negative returns usually implies a greater risk of further larger moves, which can exponentiate losses.

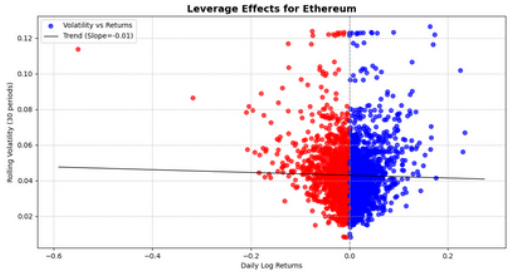
3.5.1 Daily Analysis

For **Apple**, there is a mild negative slope of -0.01, indicating that volatility increases modestly with negative returns. This is characteristic of stable, blue-chip tech stocks that see greater market anxiety when facing adverse news. Despite the leverage effect, the clustering of points near zero reflects Apple's relatively steady performance.



Bitcoin and **Ethereum** display a more pronounced leverage effect, with significant increases in volatility as returns become negative. Their slopes, though modest at -0.01, carry more weight due to the wide dispersion of returns, reflecting the extreme risk and sensitivity of cryptocurrencies to market sentiment and regulatory news.

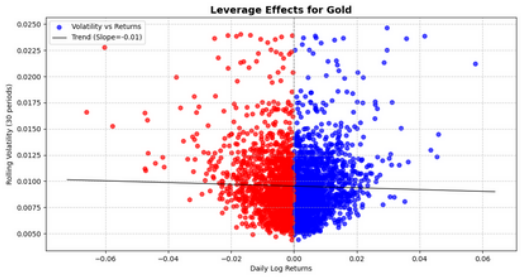




3.5 LEVERAGE EFFECTS

3.5.1 Daily Analysis

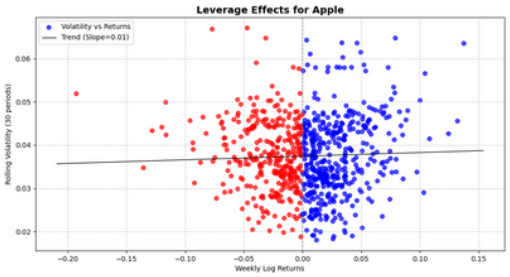
Gold exhibits a relatively stable behavior with a mild negative slope. The leverage effect is present but less pronounced, underscoring Gold's role as a safe-haven asset. It maintains a tight cluster of returns, which reflects its historical use in hedging against economic downturns.

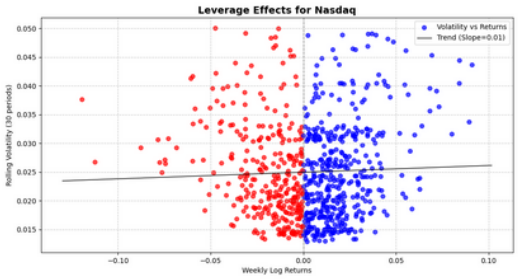


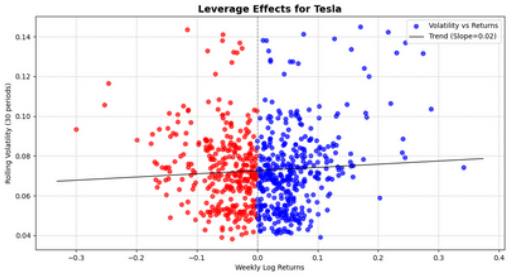
JPM, **Microsoft**, **Nasdaq**, **S&P500**, and **Tesla** show neutral to mildly positive slopes, suggesting minimal volatility change in response to negative returns. Their returns cluster tightly, reflecting market stability, strong fundamentals, and confidence in these assets.

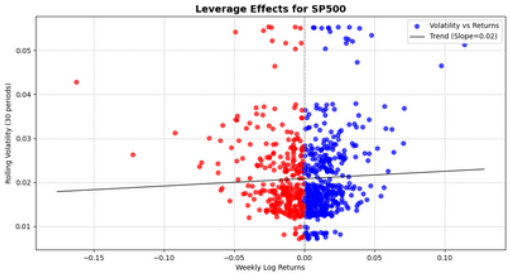
3.5.2 Weekly Analysis

Apple, **Nasdaq**, **Tesla** and **S&P500** all show slightly positive slopes (0.01 to 0.02), indicating that volatility increases modestly with positive weekly returns. This could suggest a market-driven momentum effect, especially for tech and index-related assets. The data spread for these assets is broader compared to daily plots, capturing more significant weekly fluctuations.





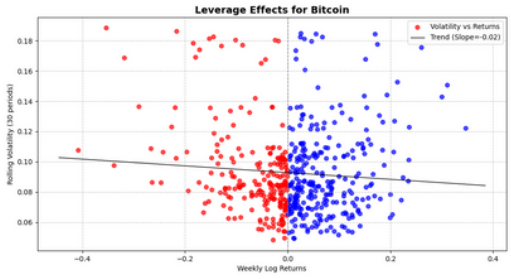


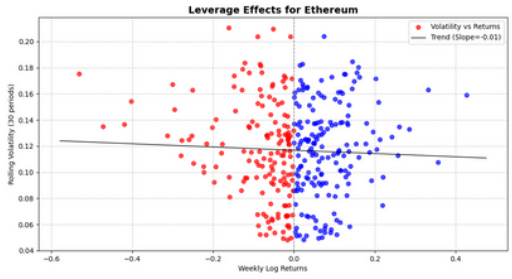


3.5 LEVERAGE EFFECTS

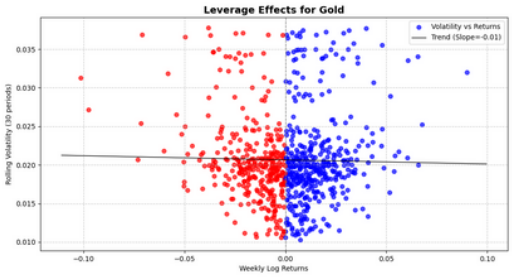
3.5.2 Weekly Analysis

Bitcoin and **Ethereum** continue to exhibit negative slopes (-0.02 and -0.01, respectively), signifying stronger leverage effects compared to the daily analysis. Negative weekly returns lead to noticeable increases in volatility, reinforcing the high-risk profile of cryptocurrencies. The extensive dispersion highlights their susceptibility to market sentiment and news, emphasizing the need for caution over weekly horizons.

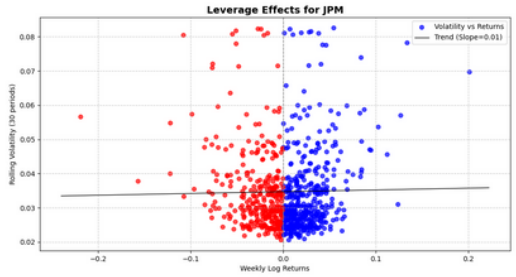


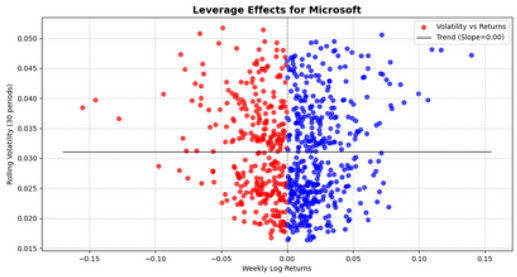


Gold maintains a minor negative slope (-0.01), showing stable behavior with a subdued leverage effect. Its tight clustering reflects limited reaction to weekly return changes, consistent with its role as a hedge against broader market volatility.

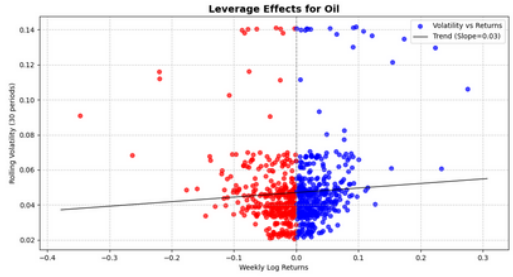


JPM and **Microsoft** have near-neutral slopes (0.00 to 0.01), indicating stability. Their volatility remains relatively unchanged in response to weekly returns, underscoring the resilience of these large, well-established companies.





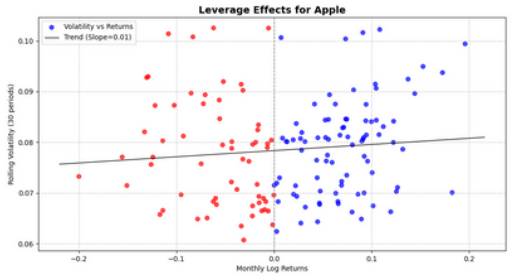
Oil stands out with a more pronounced positive slope (0.03), suggesting that volatility increases with positive weekly returns. This reinforces Oil's sensitivity to geopolitical and economic factors, making it a distinct asset class with substantial weekly risk.

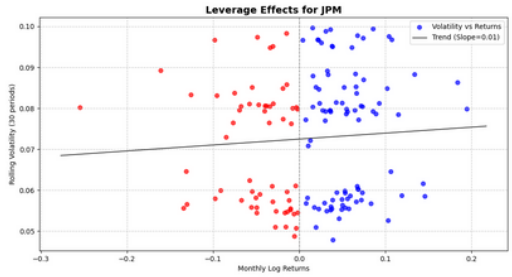


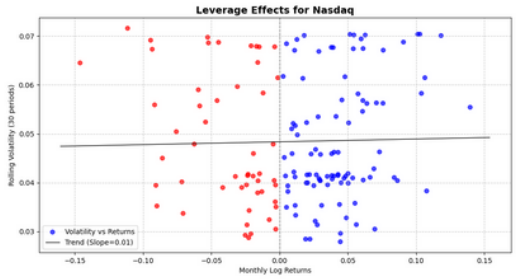
3.5 LEVERAGE EFFECTS

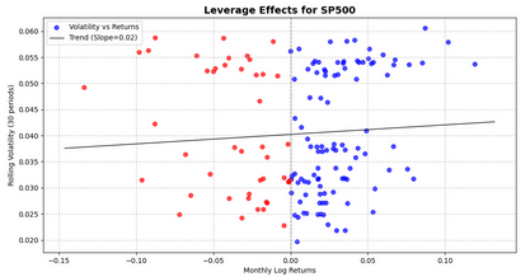
3.5.3 Monthly Analysis

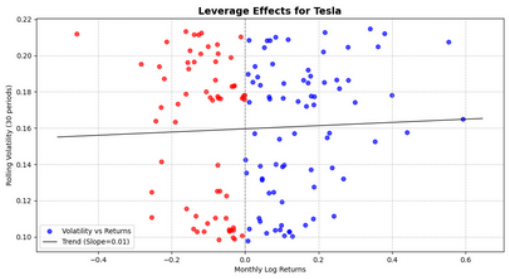
Apple, **JPM**, **Nasdaq**, **SP500**, and **Tesla** exhibit a positive leverage effect, indicated by a slightly increasing slope in volatility as returns become more positive. The trend suggests that these assets tend to display increasing volatility alongside positive returns, implying a moderate leverage effect. For example, **Apple**, **SP500**, and **Nasdaq** all have slopes near 0.01 to 0.02, showing a consistent but subtle pattern of rising volatility with gains. Similarly, **Tesla** and **JPM's** slopes are slightly positive, reinforcing this observation as shown.







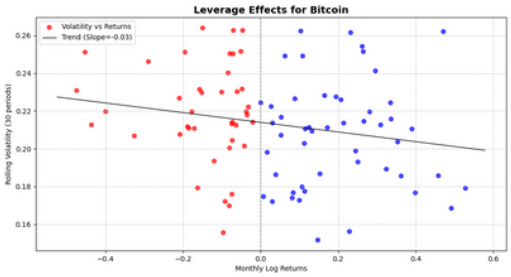


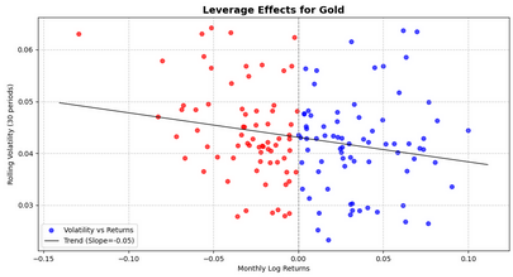


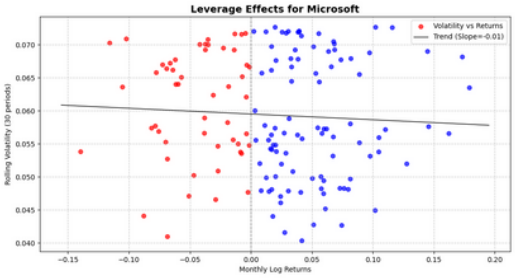
3.5 LEVERAGE EFFECTS

3.5.3 Monthly Analysis

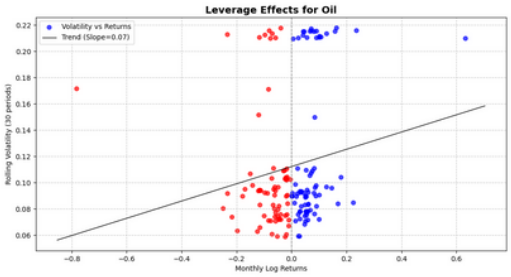
Bitcoin, **Gold**, and **Microsoft** show a negative leverage effect, meaning that as returns become more negative, volatility increases. Bitcoin and Gold, with more pronounced negative slopes of -0.03 and -0.05, respectively, highlight a more significant negative leverage effect, implying increased market anxiety during negative return periods. Microsoft has a milder negative slope, indicating a subtler response to declining returns.







Oil stands out with a highly pronounced positive leverage effect, indicated by a steep slope of 0.07. This suggests substantial volatility increases as returns grow, likely reflecting oil's sensitivity to market conditions and external factors.



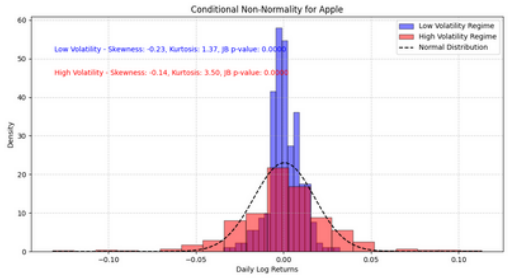
3.6 CONDITIONAL NON-NORMALITY

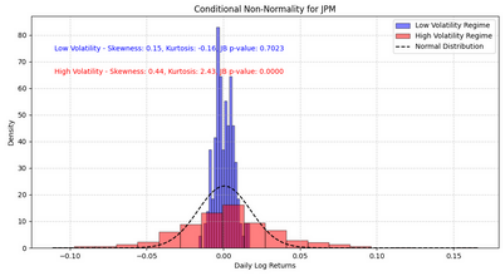
Conditional non-normality refers to the observation that the distribution of returns deviates from normality when conditioned on certain states, such as periods of high or low volatility. In financial time series, while unconditional returns (all returns over time) often exhibit non-normal characteristics, this non-normality can become even more pronounced under certain conditions (e.g., during market turmoil or calm).

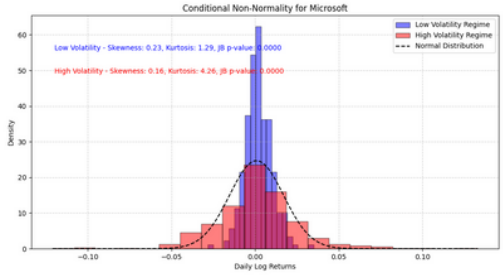
Understanding conditional non-normality is crucial in risk management, as models that assume normally distributed returns can underestimate risk, especially during turbulent periods.

3.6.1 DAILY ANALYSIS

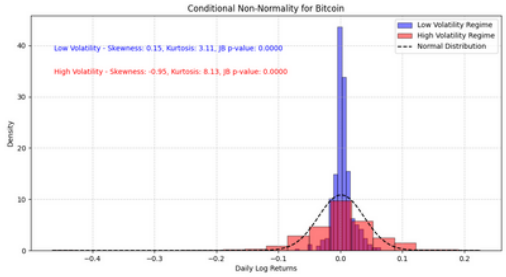
Apple, **Microsoft**, and **JPM** exhibit modest skewness and kurtosis in low volatility, with slight deviations from normality. Their metrics increase significantly in high volatility, indicating greater tail risk and departures from normal distribution.

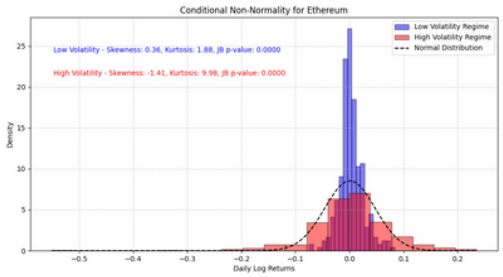






Bitcoin and **Ethereum** are more extreme. In low volatility, both show signs of non-normality, with Bitcoin's kurtosis at 3.11 and Ethereum's at 1.88. During high volatility, these deviations become severe, with Bitcoin's kurtosis at 8.13 and skewness at -0.95, and Ethereum's kurtosis at 9.98 and skewness at -1.41, highlighting significant tail risk.

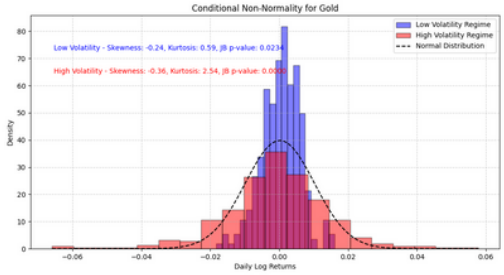


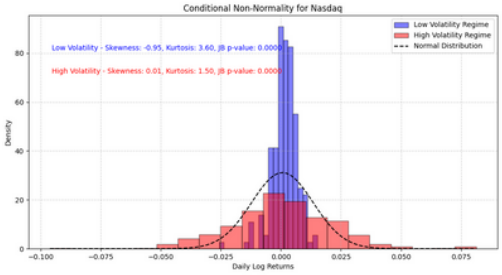


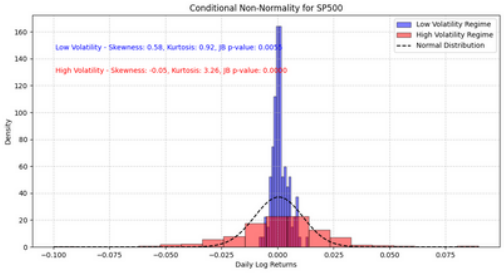
3.6 CONDITIONAL NON-NORMALITY

3.6.1 DAILY ANALYSIS

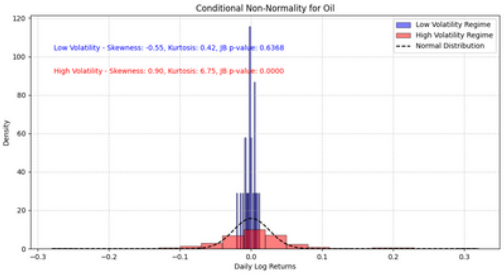
Gold, **Nasdaq**, and **S&P 500** display relatively moderate behavior. Gold is closest to normality in low volatility, while Nasdaq and S&P 500 show non-normality. All three experience increased kurtosis in high volatility, though less extreme than digital assets.

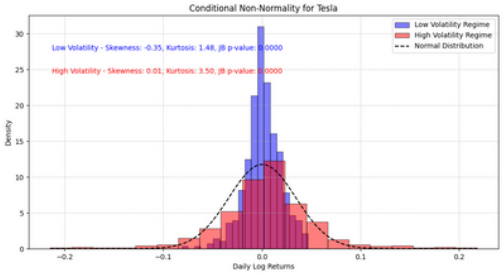






Oil and **Tesla** have unique patterns. Oil shows near-normality in calm periods but shifts drastically with high kurtosis and positive skewness in volatility. Tesla has moderate non-normality that intensifies in turbulent conditions, underscoring significant risk in extreme scenarios.

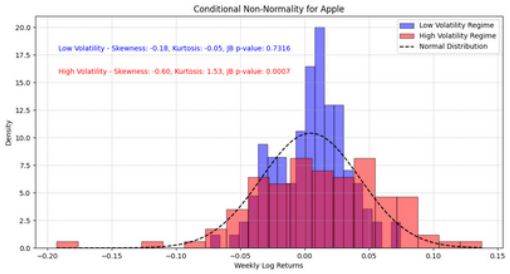


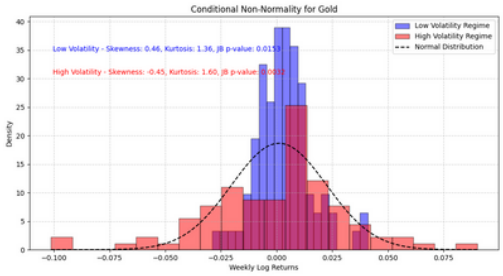


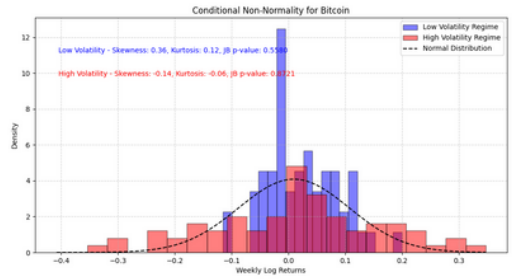
3.6 CONDITIONAL NON-NORMALITY

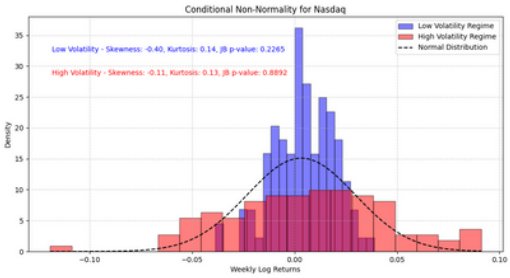
3.6.2 WEEKLY ANALYSIS

Apple and **Gold** both experience increased negative skewness and higher tail risk under high volatility, transitioning from nearly normal distributions to pronounced risk profiles. In contrast, **Bitcoin** and **Nasdaq** show minimal changes in distribution, with stable skewness and kurtosis even as volatility rises.

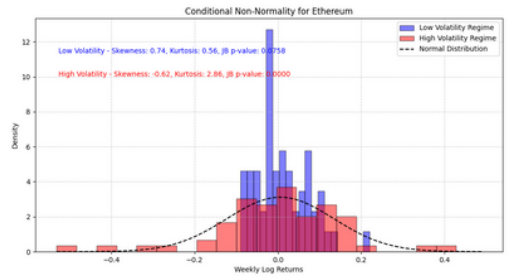


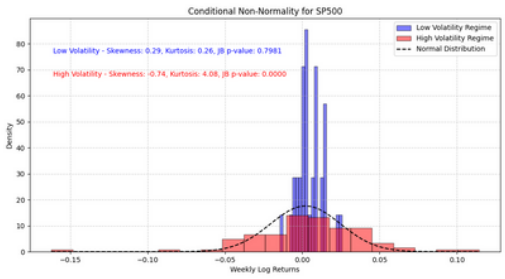


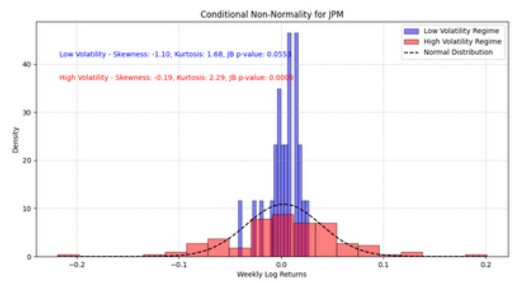


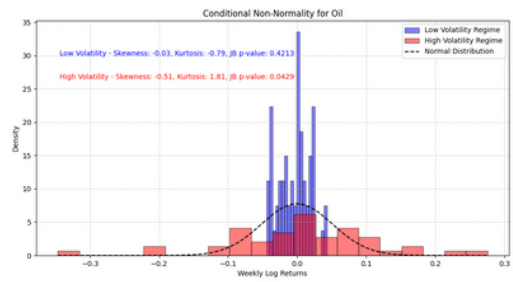


Ethereum and **SP500** exhibit significant shifts: both start with a relatively normal distribution but become heavily negatively skewed with more pronounced tails in volatile conditions. **JPM** and **Oil** display asymmetry, with JPM's heavy negative skew reducing under high volatility but increasing tail risk, while Oil shifts from nearly normal to more negative skew and tail risk.





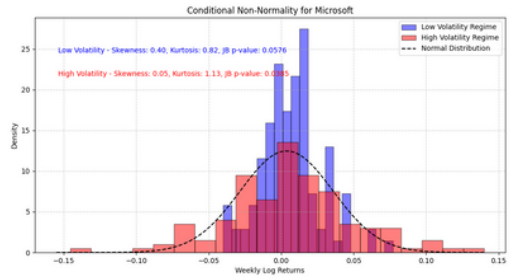


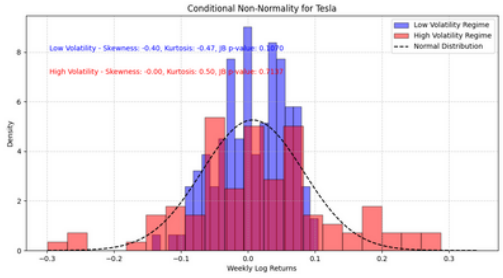


3.6 CONDITIONAL NON-NORMALITY

3.6.2 WEEKLY ANALYSIS

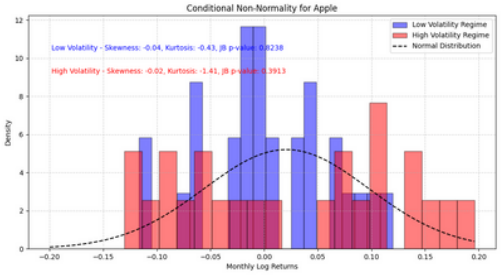
Microsoft and **Tesla** show symmetry under high volatility. Microsoft's skewness diminishes, and Tesla becomes more balanced, with reduced distributional risk.

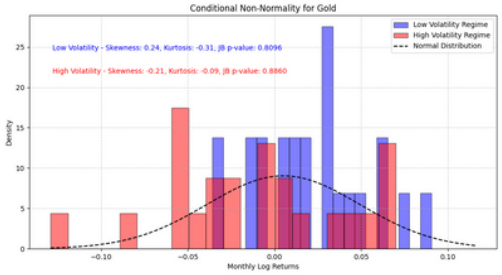




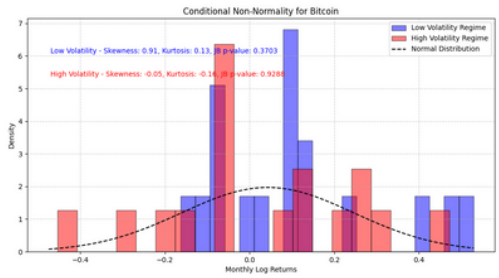
3.6.3 MONTHLY ANALYSIS

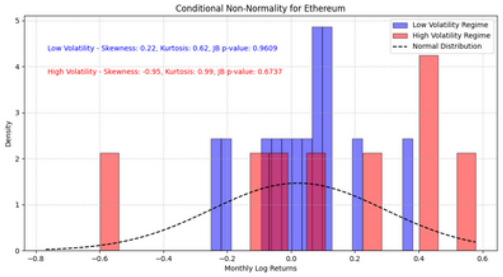
Apple and **Gold** exhibit minor changes between volatility states, with skewness close to zero and low kurtosis. Their distributions align relatively well with the normal distribution, shown by high p-values, indicating no significant deviations from normality.





Bitcoin and **Ethereum** are notably volatile. Bitcoin's skewness decreases with volatility, while Ethereum shows a stark shift with negative skewness and higher kurtosis in the high volatility state, though both have high p-values, suggesting non-significance in JB tests. These assets share a tendency for pronounced tails and wider spreads in high volatility periods.

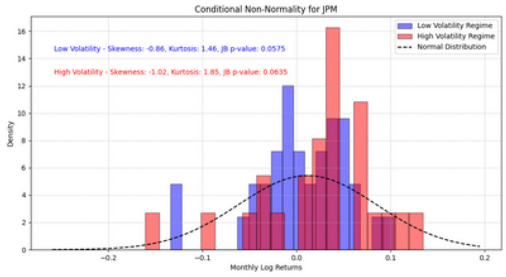


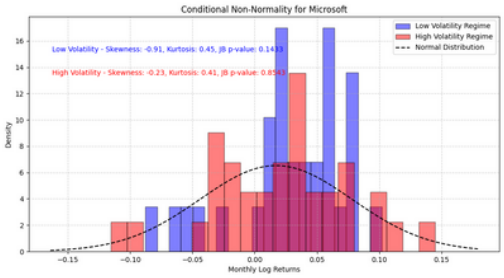


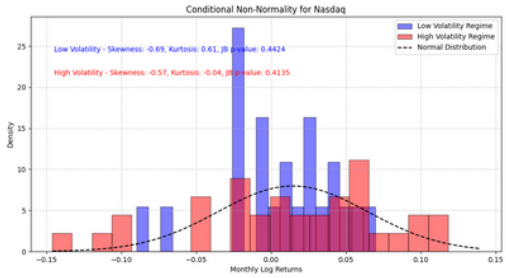
3.6 CONDITIONAL NON-NORMALITY

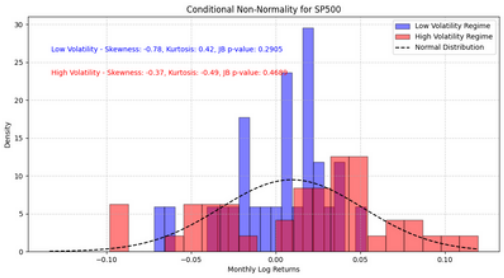
3.6.3 MONTHLY ANALYSIS

JPM and **Microsoft** show negative skewness in both regimes but differ in kurtosis. JPM's shift to higher kurtosis indicates fatter tails, while Microsoft has consistent behavior, implying moderate deviations from normality only in extreme market movements. **Nasdaq** resembles **SP500**, with skewness and kurtosis becoming more pronounced under volatility. Both indices show non-normality tendencies, particularly evident in high volatility periods where tail risks increase.

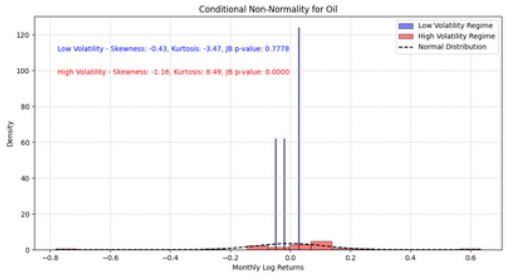




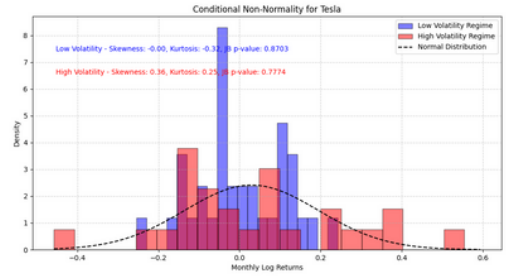




Oil stands out for its extreme behavior. Low volatility yields heavy kurtosis, with dramatic skewness and fat tails in high volatility. It highlights significant deviations from normality, confirmed by low p-values.



Tesla behaves uniquely, shifting from zero skewness to mild positive skewness in high volatility. It remains relatively well-aligned with normal distributions, despite changes in regime, suggesting less extreme deviation.



NOVA IMS Information Management School

PART 2

INVESTMENT STRATEGIES' PERFORMANCE INVESTIGATION

1. **EQUALLY WEIGHTED PORTFOLIO (EWP)**
2. **MARKOWITZ'S MEAN-VARIANCE PORTFOLIO (MVP)**
3. **GLOBAL MINIMUM VARIANCE PORTFOLIO (GMVP)**
4. **MAXIMUM SHARPE RATIO PORTFOLIO (MSRP)**
5. **INVERSE VOLATILITY PORTFOLIO (IVP)**
6. **RISK PARITY PORTFOLIO (RPP)**
7. **MOST DIVERSIFIED PORTFOLIO (MDP)**
8. **MAXIMUM DECORRELATION PORTFOLIO (MDC)**

4. INVESTMENTS STRATEGIES' PERFORMANCE INVESTIGATION

4.0 INTRODUCTION

While in the first part, *Python* was used for the asset analysis, in the second, we decided to use *R* as our main tool, although we had to make a few changes.

For the second part of the project, we slightly narrowed the quantity of assets used: we removed **JPMorgan**, and **Ethereum** from our portfolio, and added **NVIDIA**, which had been having a very good performance during the last months. Apart from that, we changed few tickers of a number of assets, as we could not get the information of some tickers that were used above.

4.0.1 ASSETS THAT COMPOSE THE PORTFOLIO

For portfolio strategy testing we used the following group of assets:

Asset Class	Ticker	Asset Name	Additional Information
Stocks	AAPL	Apple	Information Technology Company
	NVDA	NVIDIA	Information Technology Company
	MSFT	Microsoft	Information Technology Company
	TSLA	Tesla	Automotive & Energy Company
Indexes	QQQ	NASDAQ 100 ETF	ETF that tracks the performance of the NASDAQ-100
	SPY	S&P 500 ETF*	ETF that tracks the performance of the S&P 500
Commodities	GLD	Gold ETF	SPDR Gold Shares ETF
	BNO	Oil ETF	ETF that tracks the price of Brent crude oil
Cryptocurrencies	GBTC	Bitcoin	Grayscale Bitcoin Trust

* used as a benchmark

Before testing the portfolio strategies, we conducted several preparatory steps, including defining benchmarks and resampling our dataset.

As benchmarks we selected the **SPY**, the ETF that tracks the performance of the S&P 500, as it represents the overall market performance and a passive investment strategy, and the **Equally Weighted Portfolio**, as it is one of the most simple portfolio methods, and represents a more active, diversified strategy that can outperform the market under certain conditions.

We selected data ranging from 1st January of 2010, to 27th September of 2024, and then resampled this data to create multiple smaller datasets.

Each resampled dataset included:

- **Five** randomly selected **assets** from the original dataset.
- A time frame of **two years** (252 trading days per year) for each selected asset.

In total, we generated **100 resampled datasets** to facilitate robust testing of the portfolio strategies.

Resampling is a valuable technique for assessing the variability of statistical estimates.

By generating multiple resampled datasets, we are aiming to achieve several goals:

- **Evaluate the robustness of portfolio strategies**: Resampling enables testing strategies across different historical periods, allowing us to observe how they perform under varying market conditions.
- **Reduce the impact of data snooping bias**: By working with multiple datasets, we reduce the likelihood of overfitting our models to specific historical data, making the results more generalizable.
- **Enhance the statistical significance of results**: Increasing the number of observations improves the reliability and power of statistical tests.

In the context of portfolio strategy testing, resampling enables simulation of diverse market scenarios, providing insight into how well strategies perform across a range of conditions. This helps identify robust strategies capable of enduring market volatility and generating consistent returns over time.

We applied backtesting to evaluate our portfolio strategies on historical asset data, enabling us to assess how each strategy might have performed.

Backtesting is essential in portfolio development, offering insights into potential performance, risk, and the overall suitability of a strategy for live investment.

In our backtesting strategy we will be comparing the performances of the portfolio models to two benchmarks: the **SPY**, an ETF of S&P500, and the **Equally Weighted** portfolio (1/N). The portfolios will be rebalanced and optimized every 63 trading days (approximately every three months), allowing for updates to asset weights based on the latest market data, with each optimization considering a 126-day lookback window (about half a trading year). When we backtested, we did not allow short-selling in our portfolios, and the transaction costs associated with buying, selling and leveraging were set close to zero, as we wanted to focus on the pure performance of the strategies without accounting for fees.

To provide a clearer and more focused view of our findings, we present the portfolio performance for each strategy using a different, randomly selected dataset from the 100 used in our analysis. By doing so, we offer an unbiased and representative demonstration of each strategy's practical outcomes, highlighting the variability and adaptability across diverse market conditions before delving into the detailed results.

Worth noticing that some of the portfolios do not correspond to their potencial since, in some periods, the strategy could not have the performance it would have if it was analysed in a longer period (or starting in a different period).

4.1 EQUAL WEIGHT PORTFOLIO (EWP)

The Equal Weight Portfolio is an asset allocation strategy where each asset in the portfolio is assigned the same weight, regardless of its market capitalization, volatility, or risk characteristics. Unlike traditional approaches that may favor larger, more established companies or assets based on their market cap or performance potential, the Equal Weight approach treats every asset equally, ensuring each has an identical impact on the portfolio's performance.

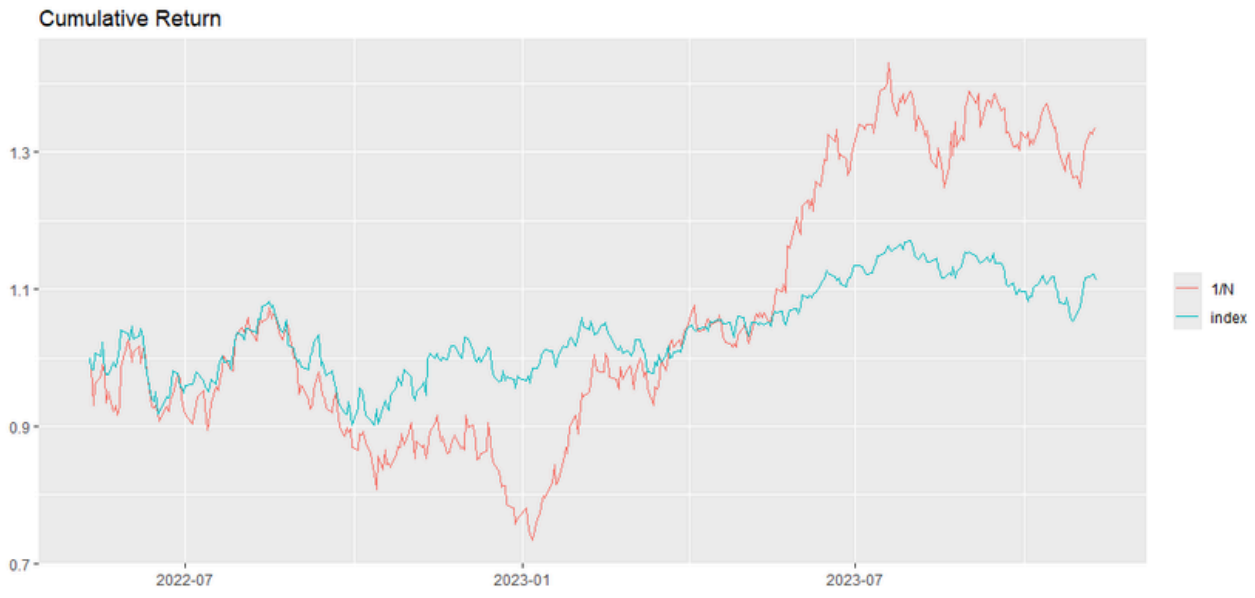
The idea behind the Equal Weight Portfolio is to avoid overexposure to any single asset or asset class, which could lead to concentration risk. By giving all assets the same weight, the strategy reduces the influence of dominant stocks or sectors, creating a more diversified portfolio.

HOW DOES IT WORK?

In an Equal Weight Portfolio, each asset is assigned the same percentage of the total portfolio, regardless of its market size. For example, in a portfolio with 10 assets, each would make up 10%. This differs from market-capitalization-weighted strategies, where larger assets receive a higher weight.

The main benefit of this approach is its simplicity and diversification, as it avoids concentration risk by treating each asset equally. However, it can be more volatile since smaller, higher-growth assets are given equal importance alongside larger ones. This strategy helps reduce overconcentration in dominant sectors or assets.

4.1.1 CUMULATIVE RETURNS (PORTFOLIO 8)



4.1.1 CUMULATIVE RETURNS

Initially, the EWP's cumulative returns move closely with the index, but it starts to underperform from mid-2022 to early 2023, due to worse performance across the asset holdings in this specific portfolio. Given that it holds an equal allocation in each asset rather than weighting them by market cap as an index would.

From mid-2023 onward, the EWP begins to outperform the index, due to a strong rally in some assets, particularly in tech. **Nvidia** and **Tesla**, for instance, had periods of great growth, which would benefit an equally weighted approach by giving these stocks a bigger opportunity to impact portfolio returns. Despite this outperformance, the EWP exhibits greater volatility than the index, reflecting the higher risk associated with an equal-weighted approach to these sectors.

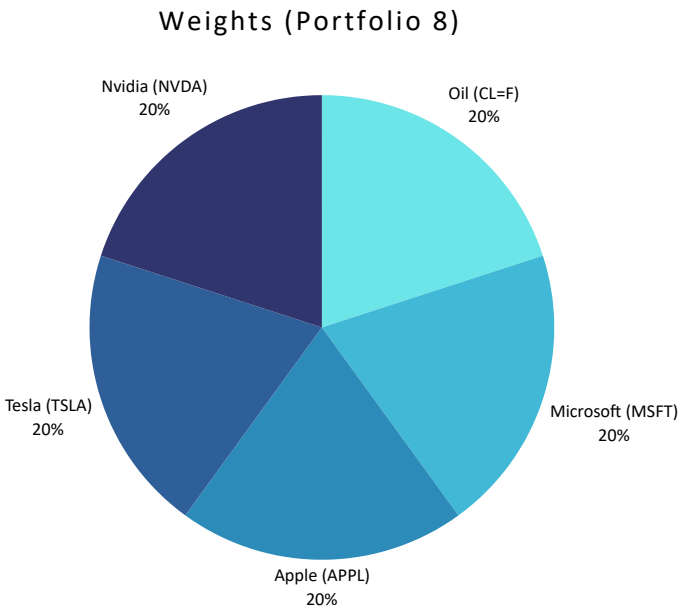
This suggests the potential advantages of equal weighting in favorable market conditions but also shows the higher volatility of individual assets.

4.1.2 PIE CHART

From the pie chart below, we can observe that the random portfolio is evenly distributed across five assets, with each comprising **20%** of the total weight. The portfolio includes four equities (**AAPL**, **MSFT**, **TSLA, NVDA**) and one commodity (**CL=F**).

Equities make up **80%** of the portfolio, which may reflect a growth-oriented approach. The **20%** allocation to commodities (**Oil**) may serve as a hedge against inflation and economic uncertainty, adding diversification and stability.

This specific portfolio suggests a not so balanced approach, only seeking growth which in market downturns can be very negative.



4.2 MARKOWITZ'S MEAN-VARIANCE PORTFOLIO (MVP)

The Markowitz Mean-Variance Portfolio is an asset allocation strategy that focuses on maximizing the expected return of a portfolio for a given level of risk, or alternatively, minimizing risk for a target level of return. Developed by Harry Markowitz in the 1950s, this approach is one of the most foundational concepts in modern portfolio theory.

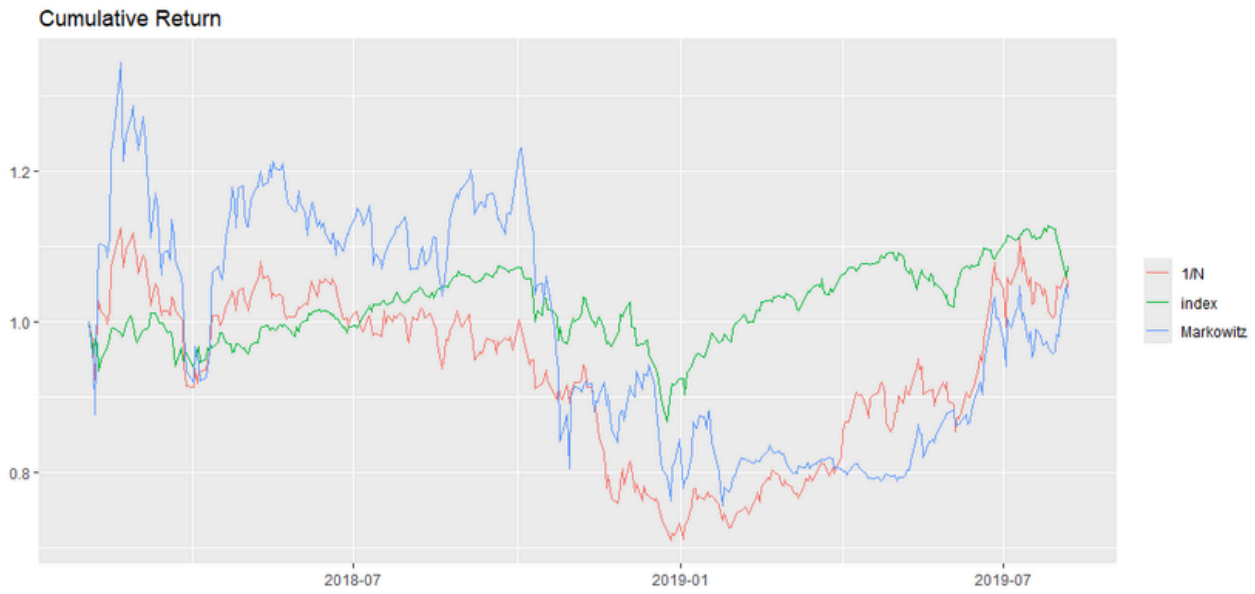
The key principle behind the MVP is the idea of constructing a portfolio that optimally balances risk and return. In contrast to strategies that focus solely on maximizing returns or reducing risk, the Markowitz approach combines both factors by considering not just the individual returns and risks of assets, but also how those assets interact with each other through their correlations.

HOW DOES IT WORK?

The Markowitz Mean-Variance Portfolio aims to balance risk and return by optimizing the asset allocation in a portfolio. It works by considering three key factors: the expected returns of assets, their volatility (risk), and how assets correlate with each other.

The strategy seeks to find the optimal mix of assets that either maximizes returns for a given level of risk or minimizes risk for a target return. By considering not just individual asset risks but also how assets move together, it achieves diversification and reduces overall portfolio risk. This results in a portfolio that offers the best possible tradeoff between risk and return.

4.2.1 CUMULATIVE RETURNS (PORTFOLIO 21)



4.2.1 CUMULATIVE RETURNS

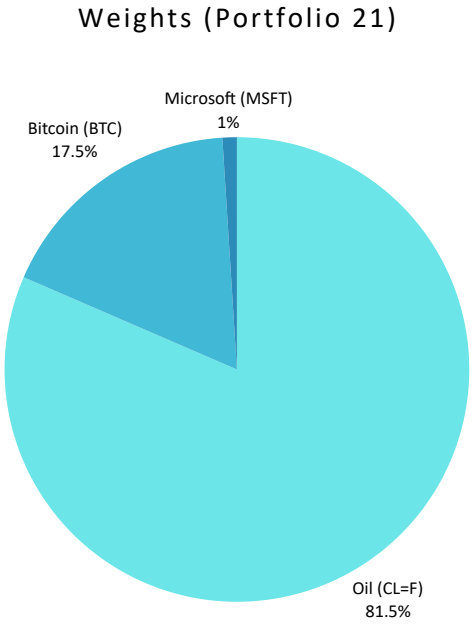
The Market Index generally outperformed both the EWP and Markowitz strategies over the entire period. The Markowitz portfolio, designed to optimize the portfolio's risk-return trade-off, experienced periods of outperformance, particularly in the latter part of 2018. However, it also encountered significant drawdowns. The EWP strategy, while simple, demonstrated a more stable performance, avoiding extreme fluctuations.

Overall, the graph suggests that the Market Index, despite its simplicity, achieved higher cumulative returns and potentially lower risk compared to both the Markowitz portfolio and the EWP strategy during the analyzed period. However, it's important to note that past performance is not indicative of future results.

4.2.2 PIE CHART

In this particular case, most of the portfolio's weight is concentrated in **BNO** (**82%**) and **GBTC** (**17.9%**), with nearly zero or even slightly negative weights assigned to **GLD**, **TSLA**, and **MSFT** (**1%**). The near-zero weights for these assets suggest that the optimizer found little to no benefit from including them in the portfolio, possibly due to their correlations with other assets or their individual risk-return profiles not adding substantial value to the overall risk minimization.

The portfolio's heavy weighting in **Oil** implies it offers a strong return relative to its risk and low correlation with other assets, making it a key diversifier. This result is useful for understanding where to focus investment in order to achieve the lowest variance.



4.3 GLOBAL MINIMUM VARIANCE PORTFOLIO (GMVP)

The Global Minimum Variance Portfolio (GMVP) is an asset allocation strategy that focuses on minimizing the risk of a portfolio regardless of its expected return. It is a specific application of the Markowitz Mean-Variance Portfolio, and it was developed as part of Harry Markowitz's foundational work in the 1950s, which is one of the cornerstones of modern portfolio theory.

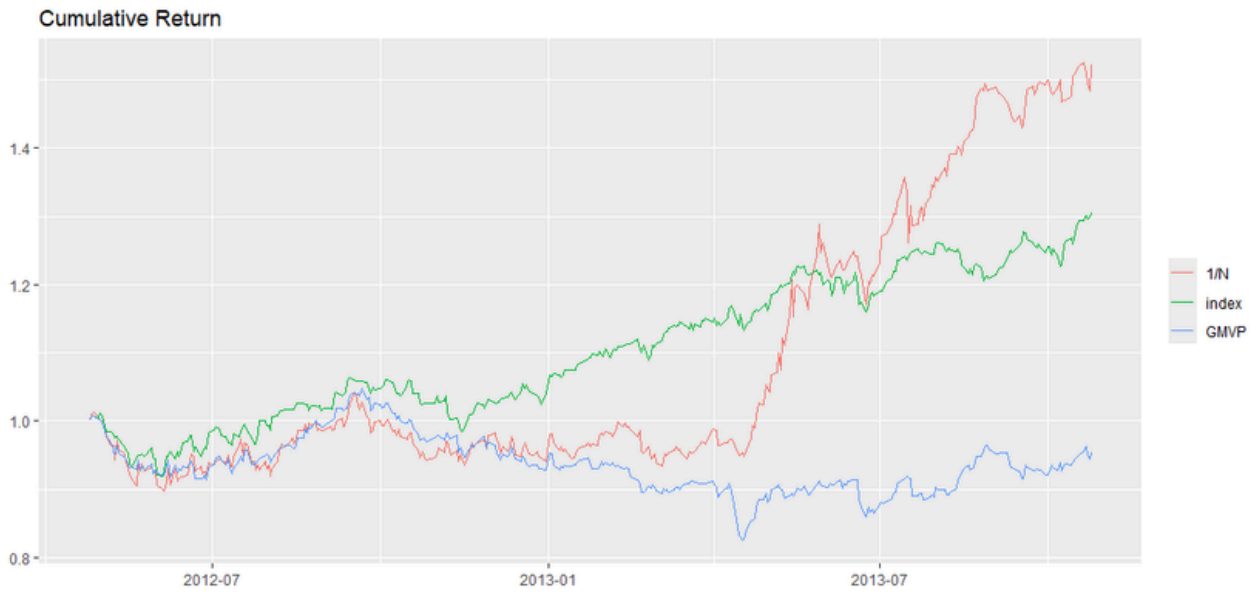
The key principle behind the GMVP is the idea of constructing a portfolio that achieves the lowest possible risk through diversification. Unlike strategies that focus on maximizing returns or balancing risk and return, the GMVP prioritizes risk reduction by considering not just the individual risks of assets but also how those assets interact through their correlations.

HOW DOES IT WORK?

The Global Minimum Variance Portfolio aims to minimize risk by optimizing the asset allocation in a portfolio. It works by considering three key factors: the volatility (risk) of each asset, the correlations between assets, and their weights within the portfolio.

The strategy seeks to find the optimal mix of assets that results in the lowest possible variance. By focusing on both individual asset risks and how assets move together, the GMVP achieves diversification and reduces overall portfolio risk. This results in a portfolio that offers the minimum achievable level of risk for any given combination of assets, making it ideal for highly risk-averse investors.

4.3.1 CUMULATIVE RETURNS (PORTFOLIO 36)



4.3.1 CUMULATIVE RETURNS

The Market Index generally outperformed both the EWP and GMVP strategies over the entire period. The GMVP, designed to minimize portfolio risk, demonstrated a more stable performance, avoiding significant drawdowns. However, this stability came at the cost of lower returns compared to the Market Index and EWP strategy. The EWP strategy, while simple, experienced periods of outperformance but also faced significant volatility.

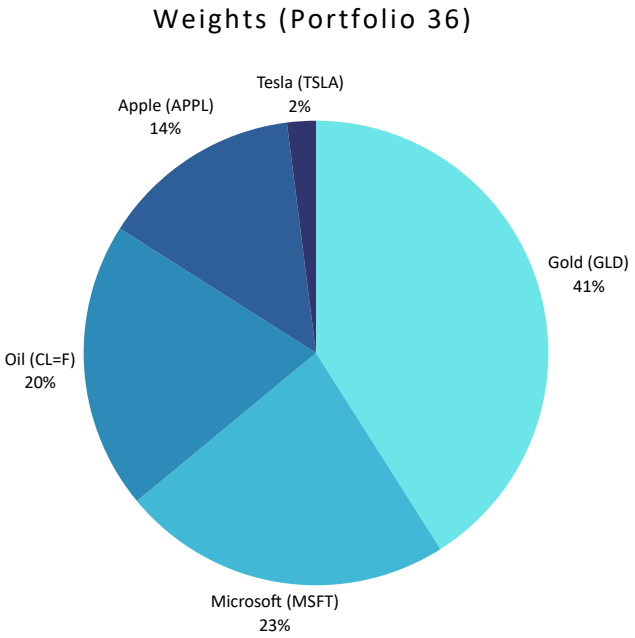
Overall, the graph suggests that the Market Index, despite its simplicity, achieved higher cumulative returns. The GMVP, while effective in risk mitigation, may not be the optimal choice for maximizing returns. The EWP strategy, though volatile, offered a middle ground between risk and return.

4.3.2 PIE CHART

A GMVP aims to achieve the lowest possible risk or variance. The weights in this portfolio likely result from the relation of each asset's volatility and correlation with the others. Commodities like **Gold** and **Oil** show low or even negative correlations with stocks making them valuable for reducing portfolio risk. These commodities, though they may have lower expected returns, act as stabilizers due to their risk-dampening diversification effect.

On the other hand, **TSLA's** small allocation (**1.42%**) reflects its high volatility, which could significantly increase the portfolio's variance despite potential gains, thus making it less desirable in this low-risk-focused strategy.

MSFT and **AAPL** offer a more stable risk-return profile, earning a balanced allocation in the portfolio. Ultimately, these weights reflect an optimization where assets are chosen not for return potential alone but for their contribution to reducing the overall portfolio risk.



4.4 MAXIMUM SHARPE RATIO PORTFOLIO (MSRP)

The Maximum Sharpe Ratio Portfolio (MSRP) is an asset allocation strategy that aims to maximize the risk-adjusted return of a portfolio. This is achieved by finding the optimal combination of assets that yields the highest **Sharpe ratio**.

$$S_a = \frac{E[R_a - R_b]}{\sigma_a}$$

where:

- **Sa**: Sharpe Ratio;
- **E**: Expected Value;
- **Ra**: Return on the asset;
- **Rb**: Risk-free return;
- **σa**: Standard deviation of the excess return on the asset

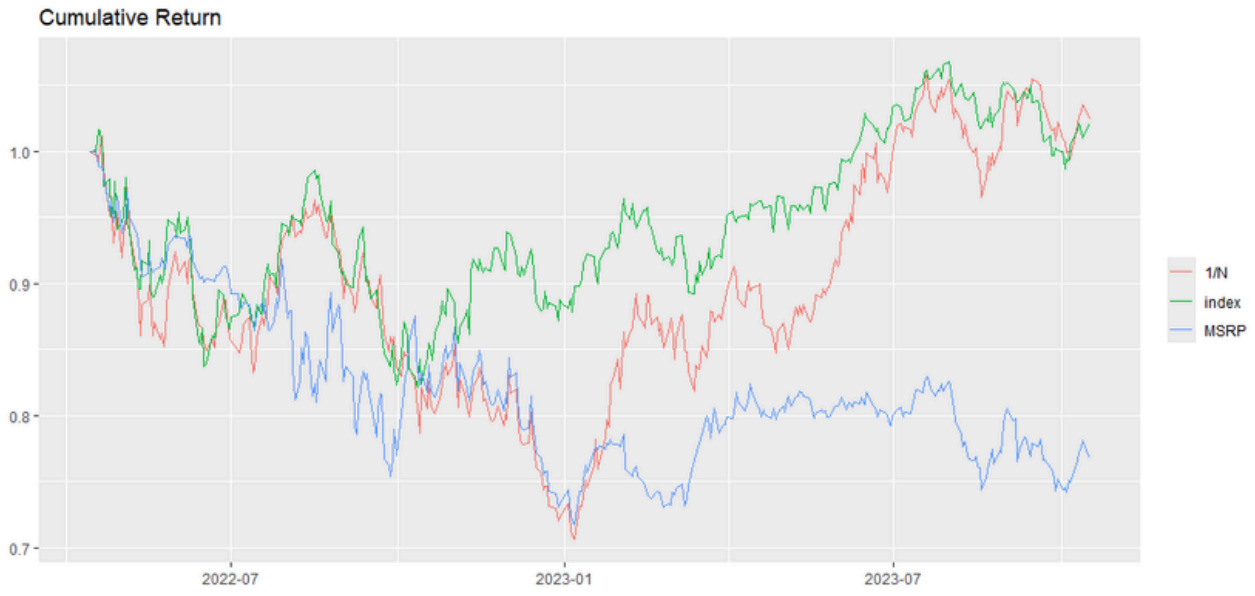
Developed by Nobel laureate William Sharpe, the Sharpe ratio is a measure of a portfolio's risk-adjusted performance, considering both its excess return over a risk-free rate and its volatility.

Unlike strategies that solely focus on maximizing returns or minimizing risk, the MSRP prioritizes the efficiency of the portfolio. A higher Sharpe ratio indicates a better risk-adjusted performance, meaning the portfolio generates higher returns for a given level of risk.

HOW DOES IT WORK?

The Maximum Sharpe Ratio Portfolio (MSRP) aims to optimize the asset allocation to maximize risk-adjusted returns. It considers the expected returns of each asset, their volatility, and how they correlate with each other. By carefully analyzing these factors, the MSRP identifies the optimal portfolio that **maximizes the Sharpe ratio**. This involves finding the right balance of assets to achieve the highest reward per unit of risk. In sum, the MSRP seeks to construct a portfolio that offers the best possible trade-off between risk and return.

4.4.1 CUMULATIVE RETURNS (PORTFOLIO 40)



4.4.1 CUMULATIVE RETURNS

The graph compares three investment strategies from mid-2022 to mid-2023: MSRP, equal-weight (EWP), and the Market index. The MSRP portfolio, shown in blue, consistently underperformed throughout the period, displaying a notable downward trend and never recovering to the levels of its counterparts. The EWP, represented by the red line, demonstrated better performance than MSRP but still faced challenges, initially tracking closer to the index before experiencing a significant downward divergence before 2023.

The index, depicted in green, emerged as the clear winner among the three approaches. While all strategies experienced a decline towards the end of 2022, the index showed remarkable resilience and recovery starting from early 2023, leading to superior cumulative returns by mid-2023. This performance disparity suggests that the MSRP strategy might need refinement to better compete with traditional benchmarks, particularly during market recovery phases, while also highlighting the challenges of beating a simple index strategy during certain market conditions.

4.4.2 PIE CHART

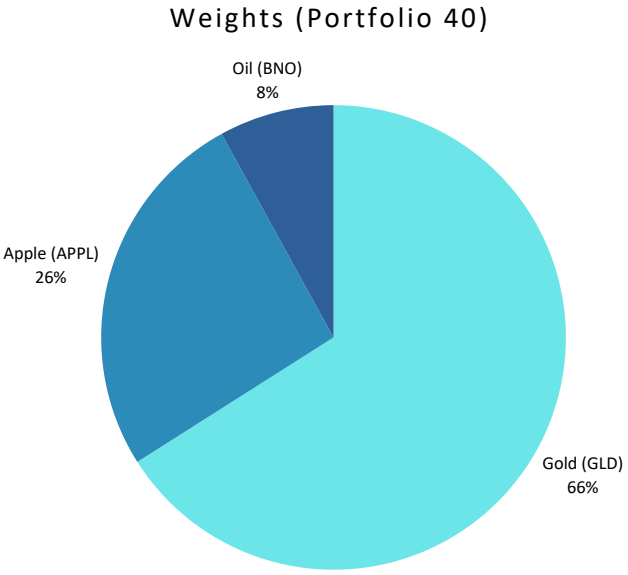
The MSRP seeks to balance risk and return by allocating capital to assets in a way that maximizes the risk-adjusted return. In this case, **Gold** has the highest allocation (**66%**), likely because it is a stable and lower-risk asset with low or negative correlation to the other assets. This reduces the overall volatility of the portfolio, contributing to a higher Sharpe ratio.

Apple, with its potential for strong returns, also has a significant weight (**26%**), adding growth potential while balancing the portfolio's risk. **Oil** is given a small allocation (around **7%**), possibly for diversification benefits, as oil prices tend to behave differently from tech stocks or gold.

On the other hand, **Nasdaq** and **Tesla** have near-zero weights, which suggests that they either add too much volatility or are highly correlated with other assets, like **AAPL**.

This redundancy reduces their utility in maximizing the Sharpe ratio, as they do not significantly enhance returns relative to the additional risk they introduce.

By excluding or minimizing exposure to these assets, the portfolio becomes more efficient in terms of risk-adjusted returns, focusing on assets that provide the best risk-return tradeoff.



4.5 INVERSE VOLATILITY PORTFOLIO (IVP)

The Inverse Volatility Portfolio is an asset allocation strategy that emphasizes allocating more weight to less volatile assets and less weight to more volatile ones. The underlying principle is straightforward: assets with higher volatility are riskier, so the strategy assigns them a lower allocation, while less volatile assets receive a higher allocation. This approach aims to reduce overall portfolio risk while achieving a balanced risk exposure across all included assets.

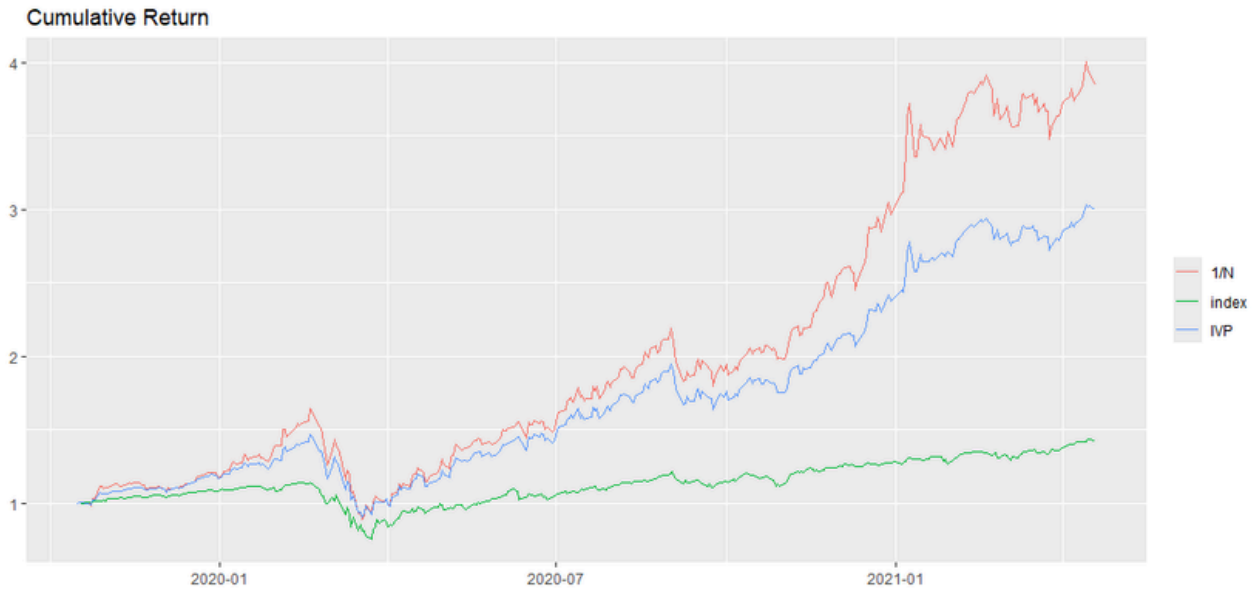
By prioritizing assets based on their inverse volatility, the Inverse Volatility Portfolio strategy seeks to minimize exposure to riskier assets, resulting in a more stable investment structure compared to conventional equal-weighted or return-maximization strategies.

HOW DOES IT WORK?

In an Inverse Volatility Portfolio, the weight assigned to each asset is inversely proportional to its volatility. This means that assets with higher levels of price fluctuations, such as stocks or commodities, are given a smaller weight, reducing the impact of these riskier assets on the overall portfolio. Conversely, assets with more stable price movements, such as bonds or certain indexes, receive a larger weight, helping to anchor the portfolio and enhance stability.

The calculation process involves determining the volatility of each asset, typically using the standard deviation of historical returns, then taking the inverse of these values. Finally, these values are normalized so that the weights sum to one. This approach results in a portfolio structure that de-emphasizes riskier assets and emphasizes stability, aligning the portfolio with lower-volatility assets for a more balanced risk profile.

4.5.1 CUMULATIVE RETURNS (PORTFOLIO 55)



4.5.1 CUMULATIVE RETURNS

The IVP strategy, shown in blue, performs moderately well, tracking closely with the EWP strategy initially but diverges slightly while maintaining a positive trend throughout the period. The EWP, represented by the red line, shows strong performance, especially from mid-2020 onwards. It consistently outperforms the other strategies, with a significant upward trend and peaks around early 2021. The index, depicted in green, shows the least growth among the three strategies, remaining relatively stable with minimal fluctuations, indicating lower volatility but also lower returns.

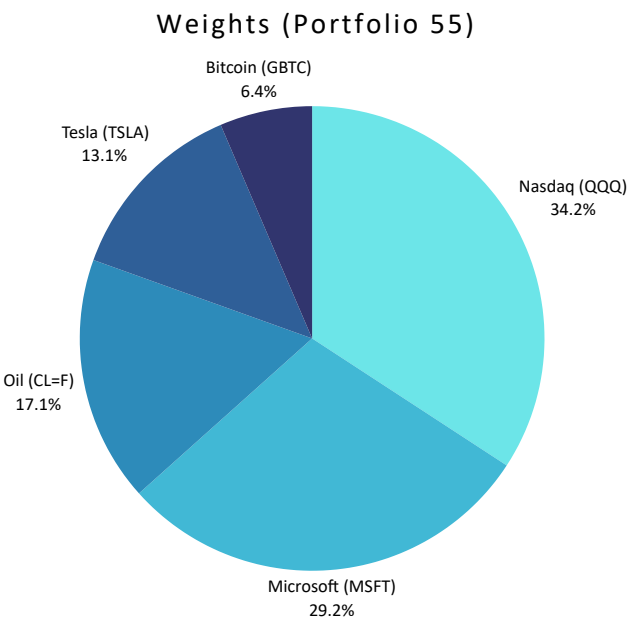
The performance disparity between these strategies is particularly notable in the latter half of the period, where the EWP demonstrates superior returns compared to both IVP and the index. This suggests that during this specific timeframe, a simple EWP allocation proved more effective than both the IVP and the market index. The consistent outperformance of the EWP might indicate that during this period, diversification benefits were particularly strong, possibly due to the market recovery phase following the early 2020 market downturn (COVID-19).

4.5.2 PIE CHART

The random portfolio is heavily weighted towards technology stocks, with a significant allocation to the **Nasdaq** (**QQQ**). This exposes the portfolio to tech-specific risks and market volatility.

A substantial portion is also invested in **Microsoft** (**MSFT**), a stable tech giant, contributing to overall stability. However, a smaller allocation to **Tesla** (**TSLA**) adds risk due to its high volatility. The portfolio also includes exposure to commodities through **Oil** (**BNO**) and cryptocurrencies through a **Bitcoin** trust (**GBTC**). While these add diversification, they also introduce significant volatility and risk.

The portfolio aims to balance risk and return, but its heavy reliance on technology and volatile assets requires careful monitoring and rebalancing.



4.6 RISK PARITY PORTFOLIO (RPP)

The Risk Parity Portfolio is an asset allocation strategy that focuses on balancing the risk contributions of each asset within the portfolio. Unlike traditional approaches that aim to maximize returns or simply allocate capital equally, the Risk Parity approach distributes risk equally across assets, creating a more resilient and stable portfolio structure.

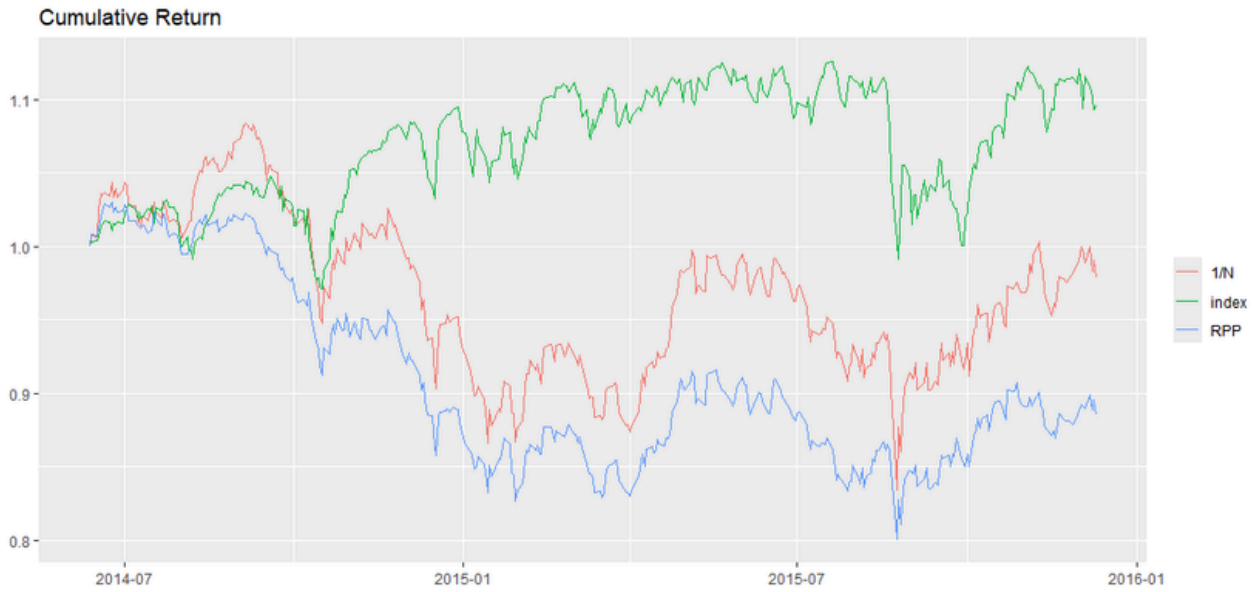
The idea is to ensure that each asset or asset class contributes an equal portion to the portfolio's overall risk, rather than letting riskier assets (stocks/crypto) dominate the risk profile. By equalizing risk contributions, the Risk Parity Portfolio seeks to limit the influence of any single asset on the portfolio's total volatility.

HOW DOES IT WORK?

In a Risk Parity Portfolio, assets with higher volatility (stocks/crypto) generally receive a lower weight, while assets with lower volatility (indexes) receive a higher weight. However, unlike a simple Inverse Volatility Portfolio (where weights are directly proportional to the inverse of each asset's volatility), the Risk Parity Portfolio considers not only each asset's volatility but also its correlation with other assets in the portfolio.

This means that asset weights are adjusted to achieve a balance where each asset's risk contribution—a combination of its own volatility and its relationship with other assets—matches the target risk allocation.

4.6.1 CUMULATIVE RETURNS (PORTFOLIO 96)



4.6.1 CUMULATIVE RETURNS

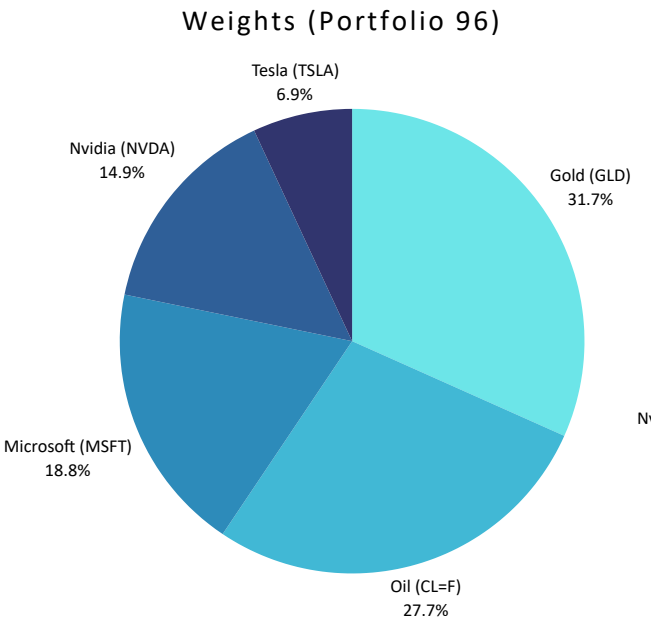
The graph displays the cumulative returns of the Risk Parity Portfolio over a period from mid-2014 to the end of 2015. The RPP, represented by the blue line, shows a more stable performance compared to the other two portfolios. It experiences less volatility, which is a characteristic feature of risk parity strategies that aim to balance risk across different assets. This stability is evident as the RPP maintains a relatively consistent trajectory with fewer sharp declines, suggesting effective diversification and risk management.

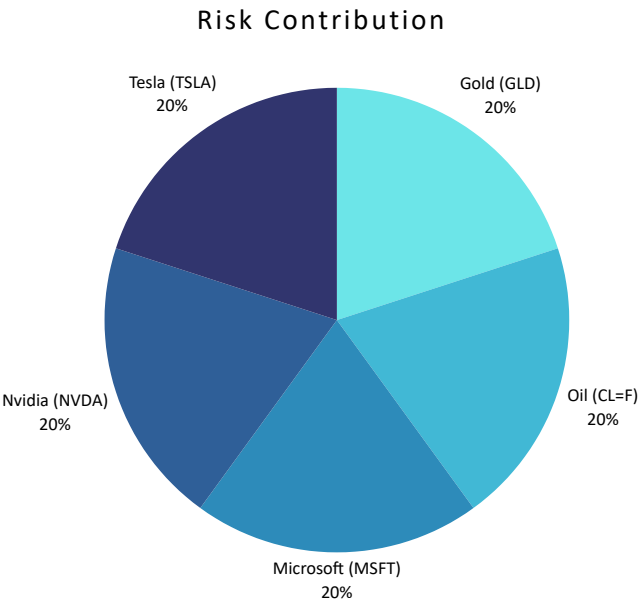
By the end of the period, the RPP ends slightly below its starting point, indicating a small loss, whereas the S&P Index ends above its starting point, showing a gain. The EWP portfolio also ends with a gain but with more volatility than the RPP. Overall, the RPP demonstrates its strength in risk management by providing a smoother return path, albeit at the cost of potentially lower returns compared to the index.

4.6.2 PIE CHARTS

The provided portfolio weights indicate a risk parity approach, aiming to allocate capital based on risk contribution. **Gold** (**GLD**) and **Oil** (**BNO**) are included to diversify the portfolio and mitigate risk. However, the significant allocation to technology stocks, particularly **Microsoft** (**MSFT**) and **Nvidia** (**NVDA**), could potentially skew the risk profile. While these tech stocks offer growth potential, they are also subject to market volatility.

It's crucial to ensure that the portfolio's risk is truly balanced across different asset classes. A regular review and rebalancing of the portfolio may be required to maintain the desired risk profile over time.





4.7 MOST DIVERSIFIED PORTFOLIO (MDP)

The Most Diversified Portfolio is an asset allocation strategy focused on maximizing the diversification benefit across different assets in a portfolio. Unlike traditional approaches that aim to maximize returns or allocate capital equally, the Most Diversified Portfolio seeks to maximize the diversification ratio, aiming to spread exposure across a wide range of uncorrelated assets.

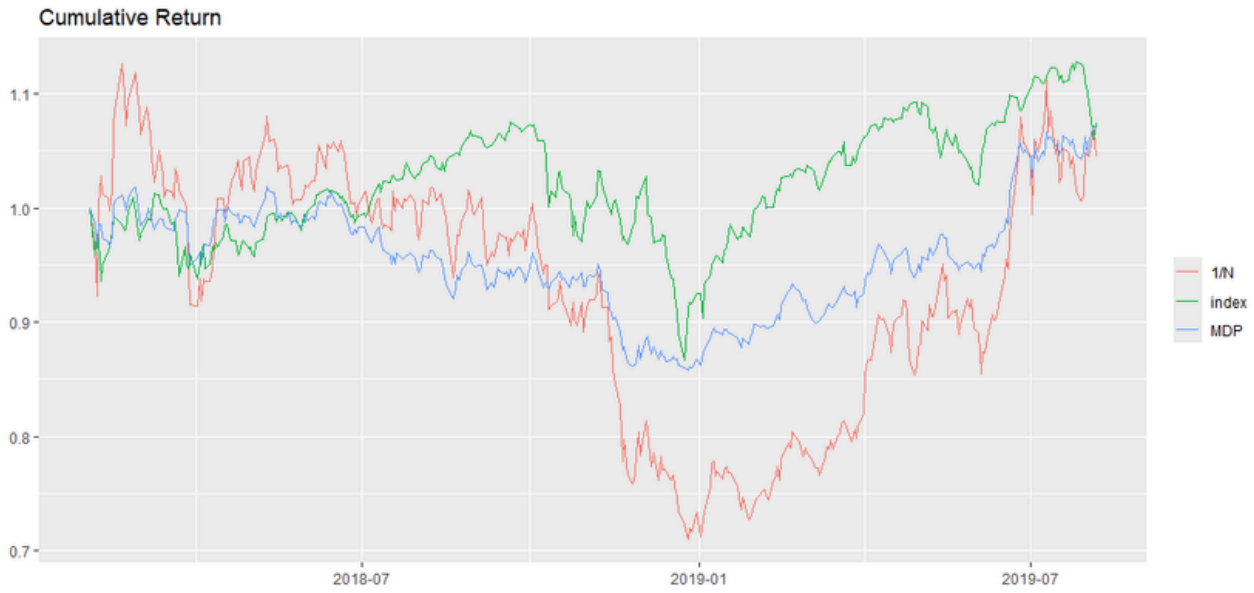
The concept centers on achieving the highest level of diversification by selecting assets that provide the most diverse sources of return, reducing the impact of any single asset or asset class on the portfolio's performance. By optimizing for diversification, the Most Diversified Portfolio aims to reduce volatility and achieve more stable, consistent returns over time.

HOW DOES IT WORK?

In a Most Diversified Portfolio, assets are weighted to maximize diversification, with a focus on reducing correlations between them. Unlike traditional strategies that prioritize returns or volatility, this approach aims to achieve the highest diversification ratio, which compares the weighted sum of individual asset volatilities to the overall portfolio volatility.

Assets with low correlations (e.g., stocks, bonds, commodities) are selected to enhance diversification, and their weights are adjusted to maximize this benefit. This creates a portfolio that balances risk and aims for greater stability by minimizing the impact of any single asset on overall performance. Asset weights are determined by their contribution to portfolio diversification, rather than by their volatility or expected return alone.

4.7.1 CUMULATIVE RETURNS (PORTFOLIO 77)



4.7.1 CUMULATIVE RETURNS

Over the displayed period, the MDP generally outperforms both benchmarks, showcasing its ability to generate superior returns. This outperformance is particularly pronounced during periods of market volatility, where the MDP demonstrates its resilience and ability to capture opportunities.

While the MDP exhibits a higher degree of fluctuation than the Index, its overall trajectory remains upward. This suggests that the MDP's diversification strategy, which aims to mitigate risk by spreading investments across a broad range of assets, effectively balances risk and return.

This diversification allows the MDP to capture upside potential from various asset classes while mitigating downside risk. In contrast, the EWP portfolio consistently underperforms the MDP and the Index, highlighting the potential drawbacks of a simple EWP approach. This approach may not adequately account for differences in risk and return characteristics across various assets, leading to suboptimal performance.

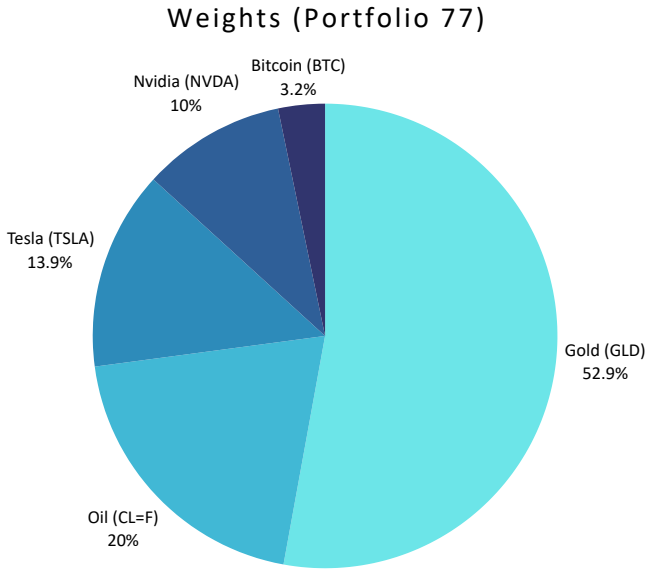
4.7.2 PIE CHART

The provided portfolio weights indicate a focus on **Gold** (**GLD**) and **Oil** (**BNO**), which together account for over **70%** of the portfolio. This significant allocation to commodities can provide diversification benefits and hedge against market volatility. However, it also exposes the portfolio to commodity-specific risks, such as fluctuations in oil prices and geopolitical events.

The inclusion of technology stocks, **Nvidia** (**NVDA**) and **Tesla** (**TSLA**), adds growth potential but also introduces higher volatility. The allocation to **Bitcoin** (**GBTC**) further enhances diversification but also exposes the portfolio to the risks associated with cryptocurrencies.

While this portfolio may not strictly adhere to a most diversified portfolio (MDP) approach, it does incorporate elements of diversification by including different asset classes.

However, **Gold** having more than half of the weight might not be as beficial as one would expect since it keeps the portfolio with lower risk which can influence returns.



4.8 MAXIMUM DECORRELATION PORTFOLIO (MDC)

The Maximum Decorrelation Portfolio (MDC) is a relatively recent concept in portfolio theory, gaining prominence in the early 2000s. It was popularized by Yves Choueifaty and Yves Robert in their 2000 paper "Optimal Portfolio Selection with Maximum Decorrelation."

MDC diverges from traditional mean-variance optimization, which focuses on maximizing returns for a given level of risk. Instead, MDP prioritizes minimizing the correlation between assets within a portfolio.

$$\rho_{X,Y} = \frac{\mathbb{E}[(X - \mu_X)(Y - \mu_Y)]}{\sigma_X \sigma_Y}$$

where:

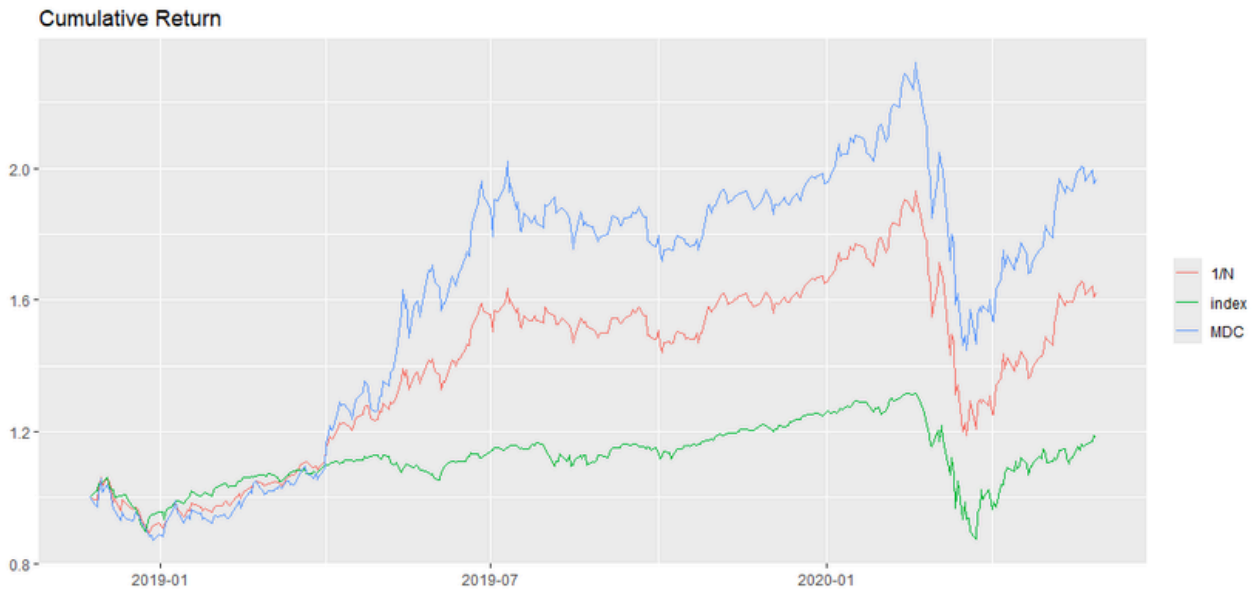
- **ρ**: Correlation;
- **E**: Expected Value;
- **μX**: mean of X;
- **μY**: mean of Y;
- **σX**: standard deviation of X;
- **σY:** standard deviation of Y

HOW DOES IT WORK?

The Maximum Decorrelation Portfolio works by constructing a portfolio that maximizes the sum of the pairwise absolute correlations between assets. By doing so, it aims to create a portfolio that is highly diversified, reducing the overall risk.

The process involves calculating the correlation matrix of the assets, which measures the degree to which the returns of two assets move in relation to each other. The MDP algorithm then assigns weights to each asset in the portfolio, with the goal of minimizing the overall correlation. This approach can lead to portfolios with lower volatility and more stable returns compared to traditional portfolios.

4.8.1 CUMULATIVE RETURNS (PORTFOLIO 18)



4.8.1 CUMULATIVE RETURNS

From the graph, we observe that the MDC generally outperforms the index and EWP portfolios over the majority of the period. This indicates that the decorrelation strategy employed in constructing the Most Decorrelation Portfolio has been successful in generating superior returns compared to a simple equal-weighting approach or the benchmark index.

The MDC ability to outperform despite lower volatility suggests that the decorrelation strategy has also helped to mitigate risk. This is a significant advantage, as it demonstrates that the MDC can generate attractive returns while taking on less risk compared to the benchmark index.

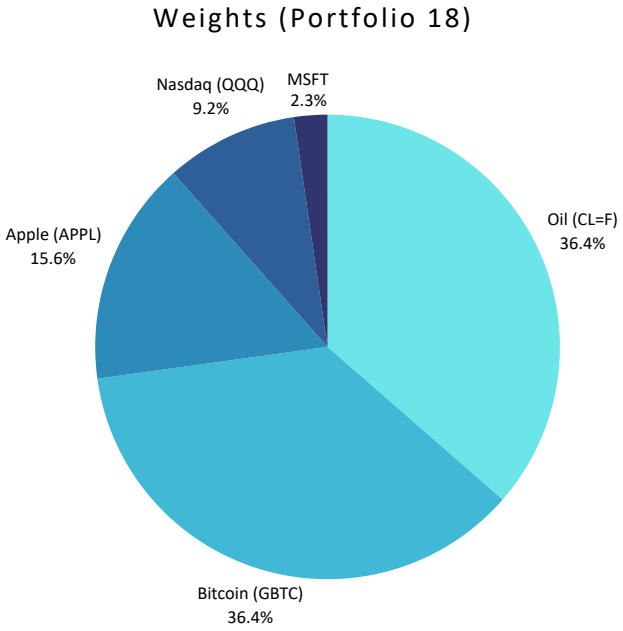
However, it's important to consider that past performance is not indicative of future results, and the effectiveness of the decorrelation strategy may vary depending on market conditions.

4.8.2 PIE CHART

The portfolio weights reflect an optimization strategy aimed at minimizing the correlation between the assets, thus improving diversification and potentially reducing risk. The assets chosen include a mix of commodities (**BNO**), cryptocurrency (**GBTC**), and technology stocks (**MSFT**, **QQQ**, **AAPL**).

Commodities and cryptocurrencies have lower or negative correlations with traditional equities, which makes them attractive for a decorrelated portfolio. By assigning higher weights to **BNO** and **GBTC** (around **36%** each), the portfolio achieves greater diversification, as these assets behave differently from each other and from equities.

On the other hand, **MSFT**, **QQQ**, and **AAPL** are all technology-focused assets, which we've seen to have higher correlations with one another, as they are influenced by similar market factors. As a result, they are assigned smaller weights to avoid overconcentration in highly correlated assets.



The overall goal is to reduce the portfolio's overall volatility by minimizing the risk of assets moving in the same direction, which is why the portfolio weights are skewed toward commodities and cryptocurrency rather than traditional equities.

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RESULTS

INVESTMENTS STRATEGIES' PERFORMANCE INVESTIGATION

1. GENERAL METRICS
2. RETURNS
3. SHARP RATIO
4. STERLING RATIO
5. DRAWDOWN
6. FINAL CONCLUSIONS

5. RESULTS

5.0 GENERAL METRICS

The table below offers a comprehensive overview of the various investment strategies, highlighting their performance across multiple dimensions. While most strategies demonstrate positive annual returns, there is a clear trade-off between risk and reward. Strategies like **MSRP** and **Markowitz**, for instance, stand out with significantly higher returns, but they also exhibit higher volatility and maximum drawdown.

In contrast, strategies such as **IVP**, **RPP**, and **MDP** offer a more balanced approach, combining solid returns with lower risk. Their high Sharpe and Sortino ratios, coupled with low downside deviation, indicate strong risk-adjusted performance. The Sterling ratio further supports the efficiency of **IVP** and **RPP** in generating returns relative to the benchmark index.

Most strategies exhibit favorable Omega ratios, suggesting their ability to generate positive returns in up markets while limiting losses in down markets. However, a deeper dive into individual metrics, such as returns, Sharpe ratio, Sterling ratio, and maximum drawdown, is necessary to fully understand the nuances of each strategy's performance.

It is also important not to desconsider the time horizon over which these metrics are calculated, as long-term performance may differ from short-term results.

Performance Table

	Sharpe ratio	max drawdown	annual return	annual volatility	Sortino ratio	downside deviation	Sterling ratio	Omega ratio	VaR (0.95)	CVaR (0.95)
IVP	1.16	16.0%	20.0%	18.0%	1.72	0.12	1.20	1.22	2.0%	2.0%
RPP	1.15	15.0%	18.0%	17.0%	1.67	0.12	1.17	1.22	2.0%	2.0%
MSRP	1.13	23.0%	28.0%	27.0%	1.70	0.18	1.51	1.22	2.0%	4.0%
MDP	1.12	16.0%	18.0%	17.0%	1.73	0.12	1.19	1.21	2.0%	2.0%
1/N	**1.10**	**22.0%**	**24.0%**	**25.0%**	**1.62**	**0.17**	**1.15**	**1.21**	**2.0%**	**3.0%**
GMVP	1.09	13.0%	17.0%	15.0%	1.58	0.11	1.11	1.20	2.0%	2.0%
MDC	1.04	21.0%	22.0%	24.0%	1.57	0.16	1.10	1.20	2.0%	3.0%
Markowitz	1.01	38.0%	45.0%	45.0%	1.56	0.27	1.33	1.22	4.0%	6.0%
index	**0.79**	**13.0%**	**14.0%**	**15.0%**	**1.08**	**0.11**	**0.72**	**1.16**	**2.0%**	**2.0%**

5. RESULTS

5.1 RETURNS

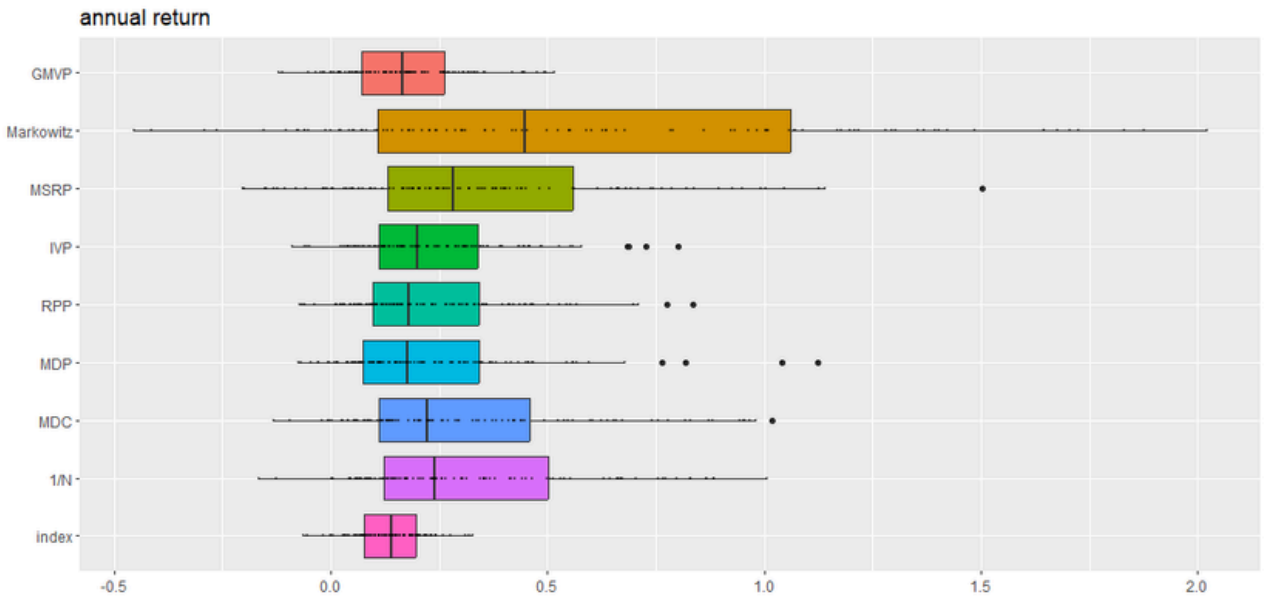
The annual returns for the strategies in the backtest vary considerably, with **Markowitz** delivering the highest **annual return** of **44.79%**. This reflects the higher return potential of the MVP approach, which actively adjusts the portfolio weights to exploit expected asset returns. However, such returns come at the cost of **increased volatility** and **drawdown**, as seen in its risk measures.

In contrast, the **GMVP** (Global Minimum Variance Portfolio) shows a **lower return** of **16.64%**, but with much **less volatility**, emphasizing the trade-off between return and risk. Other strategies like **MSRP, MDC** and **IVP** offer more **moderate returns** (**28.38%**, **22%** and **20.03%**, respectively), but they balance risk more effectively, with these approaches showing lower drawdowns and volatility than Markowitz. The **EWP** strategy, as expected, gives a relatively **balanced return** of **24.10%**, but with moderate risk.

Overall, this results highlight the diverse return-risk profiles of the tested strategies. **Higher returns** are generally associated with **higher volatility**, while **conservative strategies** like GMVP provide **lower** but **more stable returns**.

	Markowitz	MSRP	**EWP (1/N)**	MDC	IVP	RPP	MDP	GMVP	**index**
Annual Return	45%	28%	**24%**	22%	20%	18%	18%	17%	**14%**

The boxplots visualize the **Annual Return** distributions of diverse portfolio strategies. The **MVP** exhibits the widest return range and highest median return, contrasting with the **GMVP**'s narrowest range and lowest median return. Overall, the portfolio strategies outperform the **SPY** in terms of median return, though the index displays a tighter distribution.



5.2 SHARPE RATIO

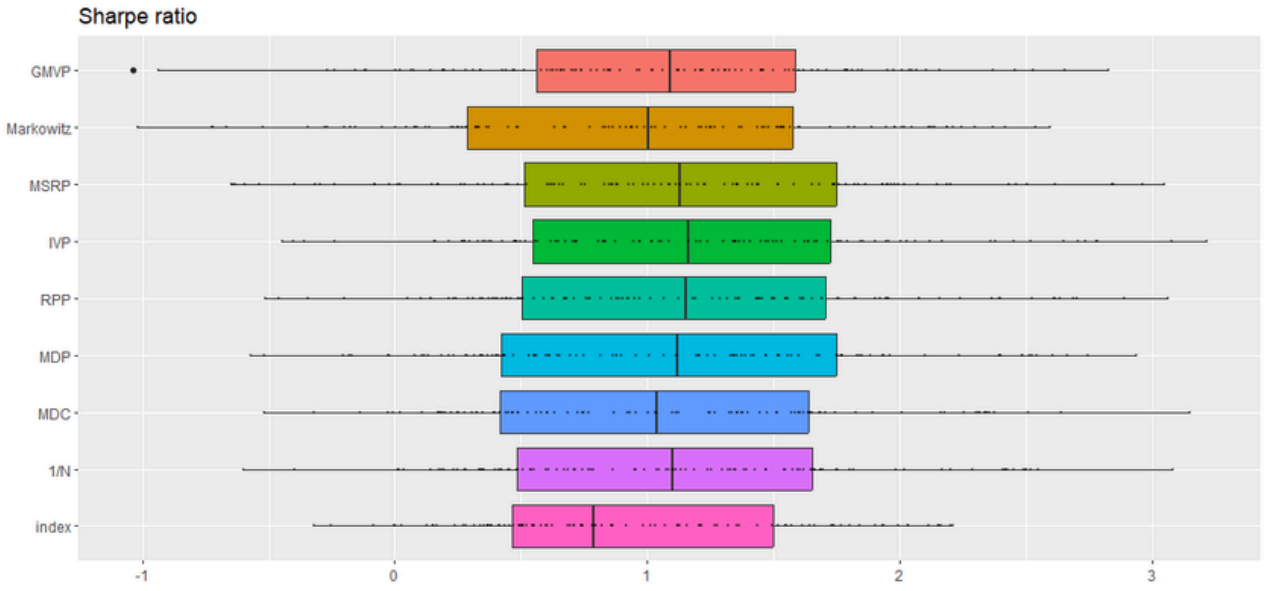
The Sharpe ratio, a key measure of risk-adjusted returns, reveals critical insights into the effectiveness of each portfolio strategy in balancing return and risk. It quantifies the excess return an investment generates per unit of risk. A **positive** Sharpe Ratio indicates that the investment strategy has generated a return higher than the risk-free rate. A higher Sharpe Ratio signifies a better risk-adjusted performance, meaning the strategy has generated higher returns for a given level of risk.

Strategies like **IVP**, **RPP**, **MSRP**, and **MDP** (**>1.1**) exhibit significantly higher Sharpe Ratios compared to the benchmark index, suggesting superior performance. They have been more effective in generating excess returns while managing risk. On the other hand, **GMVP**, **MDC** and **Markowitz** have lower Sharpe Ratios (**<1.1**), indicating less favorable risk-adjusted returns. The index scores even lower with a Sharpe Ratio of 0.79 describing a lower return over time since the volatility of the index is minimal.

These strategies may have experienced higher volatility or lower returns, leading to a less favorable risk-adjusted performance. **EWP** falls somewhere in the middle, with a Sharpe Ratio higher than the index but lower than the top-performing strategies. It has generated some excess return but may not be as efficient as the top-performing strategies in terms of risk-adjusted performance.

	IVP	RPP	MSRP	MDP	**EWP (1/n)**	GMVP	MDC	Markowitz	**index**
Sharpe Ratio	1,16	1,15	1,13	1,12	**1,1**	1,09	1,04	1,01	**0,79**

The boxplots illustrate **Sharpe Ratio** distributions across various portfolio strategies. The **IVP** and **RPP** strategies, with the highest median Sharpe ratios, offer the best risk-adjusted returns. On the other hand, the **MVP** and **MDC** strategies exhibit the lowest median Sharpe ratios. The benchmark **SPY**, with the lowest median Sharpe ratio, underperforms all portfolio strategies in terms of risk-adjusted return.



5.3 STERLING RATIO

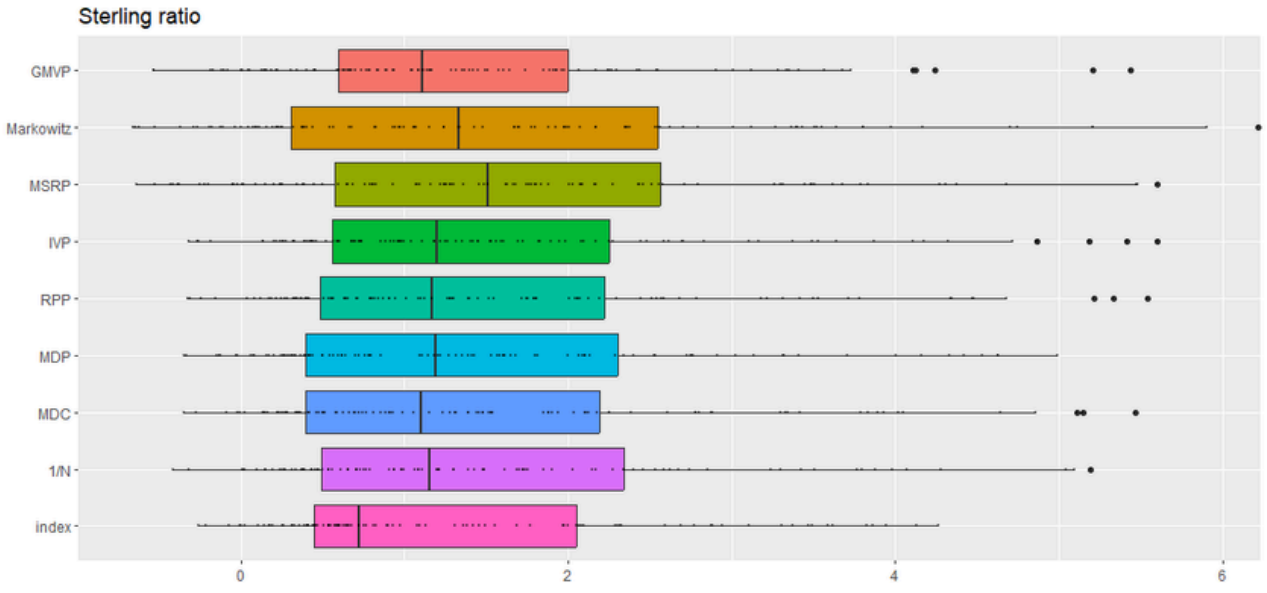
The Sterling Ratio is another risk-adjusted performance measure, focusing specifically on the relationship between returns and drawdowns. A higher Sterling ratio indicates a portfolio's ability to generate returns relative to its maximum drawdown.

MSRP and **Markowitz** exhibit significantly higher Sterling Ratios (**>1.3**), suggesting superior risk-adjusted performance. They have been more effective in generating returns while managing downside risk. **IVP**, **MDP** and the **RPP** have medium Sterling Ratios (**1.15 < SR < 1.3**), suggesting reasonable risk-adjusted performance. **GMVP**, **MDC**, and the **index** have lower Sterling Ratios (**<1.15**), indicating less favorable risk-adjusted returns. These strategies may have experienced higher downside risk or lower returns, leading to a less favorable risk-adjusted performance. The **EWP** strategy, while demonstrating good risk-adjusted performance overall, has a relatively modest Sterling ratio of **1.15**, suggesting that its returns are moderate when considering the drawdown risk.

The Sterling ratio results emphasize the importance of evaluating strategies based on both return generation and the risk of large losses, with Markowitz and MSRP standing out as the most favorable in this regard.

	MSRP	Markowitz	IVP	MDP	RPP	**EWP (1/N)**	GMVP	MDC	**index**
Annual Return	28%	45%	20%	18%	18%	**24%**	17%	22%	**14%**
Max Drawdow	23%	38%	16%	16%	15%	**22%**	13%	21%	**13%**
Sterling Ratio	1,51	1,33	1,2	1,19	1,17	**1,15**	1,11	1,1	**0,72**

The boxplots visualize **Sterling Ratio** distributions for different portfolio strategies. The **MSRP** strategy offers the highest risk-adjusted return, as indicated by its highest median Sterling Ratio. The remaining strategies, while exhibiting slightly lower median Sterling ratios, still outperform the **SP500 ETF** in terms of risk-adjusted performance.



5.4 DRAWDOWN

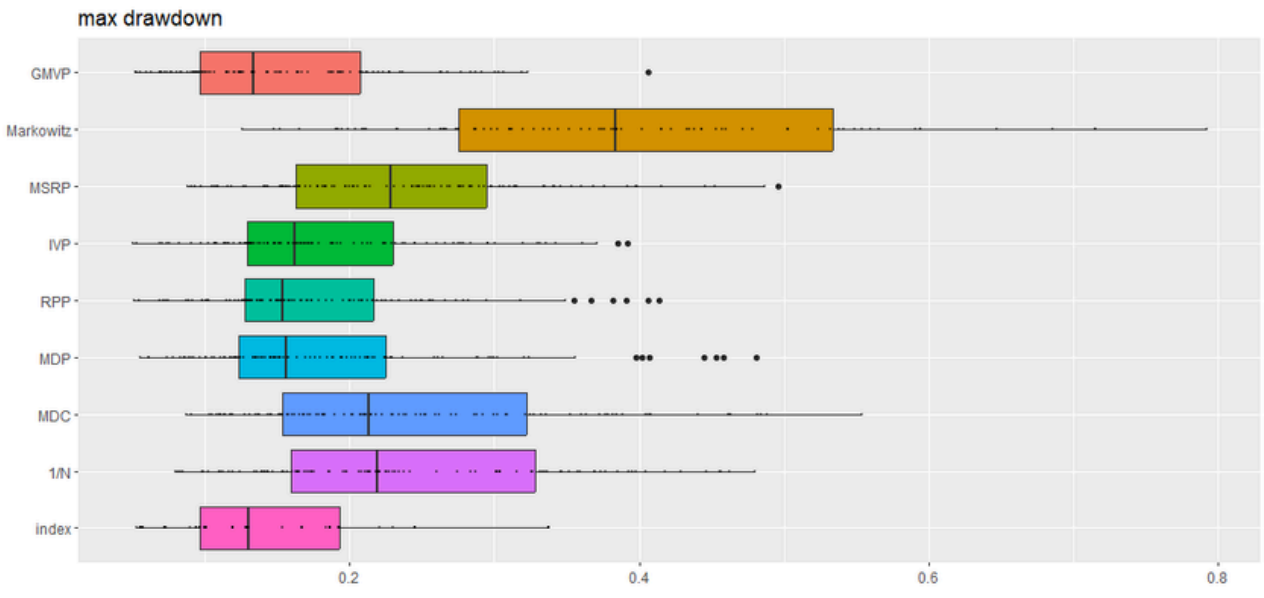
Drawdown represents the peak-to-trough decline in the value of a portfolio, and it is a crucial metric for understanding the potential losses an investor may face during periods of market stress.

The table below shows a clear relationship between annual volatility and max drawdown across the different strategies. The **Markowitz** portfolio, with the highest annual volatility at **45%**, also has a high max drawdown of **38%**, which suggests a **high-risk, high-return** profile with significant exposure to **large losses**. **MSRP**, with a lower volatility of 27%, also has a more moderate max drawdown of **23%**, indicating a more **balanced risk-return** profile. The **EWP** further reduces volatility and max drawdown to **25%** and **22%**, respectively, representing a more diversified and somewhat conservative option.

As volatility decreases across strategies (from **MDC**, **IVP**, **MDP**, to **RPP**), we observe a corresponding decrease in max drawdown, illustrating that lower volatility generally leads to lower risk of substantial losses. The **GMVP** and the **index** stand out as the least volatile portfolios, each with **15%** volatility and a **13%** max drawdown, making them the most conservative and least likely to experience extreme losses.

	Markowitz	MSRP	**EWP (1/N)**	MDC	IVP	MDP	RPP	GMVP	**index**
Annual Volatility	45%	27%	**25%**	24%	18%	17%	17%	15%	**15%**
Max Drawdow	38%	23%	**22%**	21%	16%	16%	15%	13%	**13%**

The boxplots illustrate **Maximum Drawdown** distributions across various portfolio strategies. The **MVP** strategy exhibits the highest peak-to-trough decline, while the **GMVP** strategy the lowest, coming very close to the benchmark. Overall, most portfolio strategies underperform the **GMVP** strategy in terms of maximum drawdown, as well as, the index.



6. FINAL CONCLUSIONS

To sum up, the outcomes of this backtest offer important information about the risk and performance traits of different portfolio allocation techniques. Metrics like the Sharpe ratio, Sterling ratio, and maximum drawdown offer insight into each portfolio's risk-adjusted performance, which is often sought after by investors who want to strike a balance between maximizing returns and limiting risk.

For investors focused on maximizing returns with acceptable risk, the **MSRP** (Most Sharpe Ratio Portfolio) stands out. It has the highest annual return at **28%** and a relatively strong Sharpe ratio of **1.13**, indicating good risk-adjusted returns. However, with a max drawdown of **23%** and a higher Sterling ratio (**1.51**), **MSRP** carries more downside risk than lower-volatility options like **IVP** or **RPP**. This makes it suitable for moderately aggressive investors.

For investors seeking a balanced approach with lower risk, **IVP** (Inverse Volatility Portfolio) and **RPP** (Risk Parity Portfolio) are appealing options. **IVP** offers a **20%** annual return with a **1.16** Sharpe ratio and a lower max drawdown of **16%**. **RPP** also has low max drawdown (**15%**) and moderate annual return (**18%**) with a Sharpe ratio of **1.15**. Both **IVP** and **RPP** have relatively low Sterling ratios, indicating controlled downside risk, making them suitable for investors with moderate risk tolerance who still desire decent returns.

Finally, for highly conservative investors who prioritize low risk, the **GMVP** (Global Minimum Variance Portfolio) is ideal. With the lowest max drawdown of **13%** and low volatility, it offers a modest **17%** annual return. Though it has a lower Sharpe ratio (**1.09**), the low Sterling ratio (**1.11**) and controlled drawdown make **GMVP** a very stable choice for risk-averse investors who prioritize capital preservation.

In our analysis, the **index** has a relatively low annual volatility of **15%** and a max drawdown of **13%**, indicating controlled risk compared to many of the actively managed portfolios. Although the annual return in this dataset is shown as **14%**, which is lower than some of the other portfolios, the index's Sharpe ratio of **0.79** and Sterling ratio of **0.72** show its efficiency in balancing return with risk. This steady performance, coupled with historical data, demonstrates why the S&P 500 index is considered a reliable, low-cost investment choice for long-term allocation.

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