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Aptitude Made Simple

Power Cycle ($x^n -$ Unit Place Digit)

Each of the aptitude question exam paper contains 2 0r more questions on identifying unit place digit for some number(x) to the power of some other number (n).

Let us look at 1 of the example :

What will be unit′s place digit for 77^{123} ?

- ➔ Do you really feel that we are supposed to multiply 77 number 123 times during exam?
- ➔ **Obviously Not!** We need to work with the way so that without doing that much multiplication we should be capable of answering.

Power cycle will help us to solve this problem.

Let us take real life example to understand power cycle

Let us assume **Sun** as number for which index **n** is to be calculated.

Sun follows below 2 steps:

Step 1: Sunrise
Step 2: Sunset
So let us look at power and respective values for Sun to the power n

Sun^1	Sunrise
Sun^2	Sunset
Sun^3	Sunrise
Sun^4	Sunset
Sun^5	Sunrise

Looking at above table we can see that Sun follow pattern Sunrise, Sunset.

(Sunrise, Sunset) is Power cycle for Sun

We will identify power cycles for all numbers 0 to 9 and you would be able to solve any problem asked in exam easily and post practice orally as well.

If you can remember power cycle values that would be great however even if you are unable to remember it, we will look at technique to **calculate power cycle of number in less than 1 minute during exam** as well and you can solve problem.

Power Cycles for all Numbers (Focus on Unit place digit only)

Number 0 :

Let us calculate values for 0^1 to 0^5

Power of 0	Value
0^1	0
0^2	0
0^3	0
0^4	0
0^5	0

So as you can observe:
Value of 0^1 to 0^5 is 0 only. So Unit place digit is 0 for any power of 0

Power Cycle for 0 : (0)

Problem 1

What will be unit′s place digit for 250^{123} ?

Solution :

Look at the unit place of number 25**0**.

Unit's place digit is **0**.

Power Cycle of 0 : (0)

Answer is unit′s place digit for 250^{123} will be 0

Problem 2

What will be unit′s place digit for 670^{4123} ?

Solution :

Look at the unit place of number 67**0**.

Unit’s place digit is **0**.

Power Cycle of 0 : (0)

$\mathbf{Answer\ is\ unit's\ place\ digit\ for\ 670^{4123}\ will\ be\ 0}$

Number 1 :

$\mathbf{Let\ us\ calculate\ values\ for\ 1^1\ to\ 1^5}$

Power of 1	Value
$\mathbf{1^1}$	1
$\mathbf{1^2}$	1
$\mathbf{1^3}$	1
$\mathbf{1^4}$	1
$\mathbf{1^5}$	1

So as you can observe:

$\mathbf{Value\ of\ 1^1\ to\ 1^5\ is\ 1\ only.\ So\ Unit\ place\ digit\ is\ 1\ for\ any\ power\ of\ 1}$

Power Cycle for 1 : (1)

Problem 1

What will be unit′s place digit for 121^{53} ?

Solution :

Look at the unit place of number 12**1**.

Unit’s place digit is 1.

Power Cycle of 1 : (1)

$\mathbf{Answer\ is\ unit's\ place\ digit\ for\ 121^{53}\ will\ be\ 1}$

Problem 2

What will be unit′s place digit for 791^{5643} ?

Solution :

Look at the unit place of number 79**1**.

Unit’s place digit is 1.

Power Cycle of 1 : (1)

$\mathbf{Answer\ is\ unit's\ place\ digit\ for\ 791^{5643}\ will\ be\ 1}$

<u>Number 2 :</u>

$\mathbf{Let\ us\ calculate\ values\ for\ 2^1\ to\ 2^5}$

Power of 2	Value
$\mathbf{2^1}$	**2**
$\mathbf{2^2}$	**4**
$\mathbf{2^3}$	**8**
$\mathbf{2^4}$	1**6**
$\mathbf{2^5}$	3**2**

So as you can observe:

Unit place digit of 2^1 to 2^5 is in order $2, 4, 8, 6$ and it will keep repeating as $2, 4, 8, 6$

Power Cycle for 2 : (2, 4, 8, 6)

Problem 1

What will be unit′s place digit for 2^{33} ?

Solution :

Look at the unit place of number **2**.

Unit's place digit is **2**.

Power Cycle of 2 : (2, 4, 8, 6)

There are total 4 values which keep repeating always for power of 2.

Now look at index which is to be identified: 33

As 4 numbers keep on repeating for power cycle of 2, we need to divide 33 by 4 and identify remainder of it so that we can understand what can be unit place number.

$\frac{\text{Index to be found}}{\text{Size of power cycle}} = \frac{33}{4}$, Quotient = 8 and **Remainder = 1**

Remainder	Unit Place digit
1	**2**
2	4
3	8
0	6

You don't need to remember this table you just need to make sure as you know pattern of power cycle you have to reach till index number.

Like in this case:

To reach 33 and you have size of 4

4, 8, 12…..32 so 32^{nd} index would be last number in power cycle that is 6

33^{rd} index would have 1^{st} number in power cycle that is 2

Answer is unit's place digit for 2^{33} will be 2

Problem 2

What will be unit's place digit for 1222^{438}

Solution :

Look at the unit place of number 122**2**.

Unit's place digit is **2**.

Power Cycle of 2 : (2, 4, 8, 6)

There are total 4 values which keep repeating always for power of 2.

Now look at index which is to be identified: 438

As 4 numbers keep on repeating for power cycle of 2, we need to divide 438 by 4 and identify remainder of it so that we can understand what can be unit place number.

$\frac{\text{Index to be found}}{\text{Size of power cycle}} = \frac{438}{4}$, Quotient = 109 and **Remainder = 2**

Remainder	Unit Place digit
1	2
2	**4**
3	8
0	6

Answer is unit's place digit for 1222^{438} will be 4

Number 3 :

Let us calculate values for 3^1 to 3^5

Power of 3	Value
3^1	**3**
3^2	**9**
3^3	2**7**
3^4	8**1**
3^5	24**3**

So as you can observe:

Unit place digit of 3^1 to 3^5 is in order $3, 9, 7, 1$ and it will keep repeating as $3, 9, 7, 1$

Power Cycle for 3 : (3, 9, 7, 1)

Problem 1

What will be unit′s place digit for 3^{36} ?

Solution :

Look at the unit place of number **3**.

Unit's place digit is **3**.

Power Cycle of 3 : (3, 9, 7, 1)

There are total 4 values which keep repeating always for power of 3.

Now look at index which is to be identified: 36

As 4 numbers keep on repeating for power cycle of 3, we need to divide 36 by 4 and identify remainder of it so that we can understand what can be unit place number.

$\frac{\text{Index to be found}}{\text{Size of power cycle}} = \frac{36}{4}$, Quotient = 9 and **Remainder = 0**

Whenever remainder is 0 it is last digit in power cycle.

Remainder	Unit Place digit
1	3
2	9
3	7
0	**1**

Answer is unit′s place digit for 3^{36} will be 1

Problem 2

What will be unit′s place digit for 123^{498} ? ?

Solution :

Look at the unit place of number 12**3**.

Unit's place digit is **3**.

Power Cycle of 3 : (3, 9, 7, 1)

There are total 4 values which keep repeating always for power of 3.

Now look at index which is to be identified: 498

As 4 numbers keep on repeating for power cycle of 3, we need to divide 498 by 4 and identify remainder of it so that we can understand what can be unit place number.

$\frac{\text{Index to be found}}{\text{Size of power cycle}} = \frac{498}{4}$, Quotient = 124 and **Remainder = 2**

Remainder	Unit Place digit
1	3
2	**9**
3	7
0	1

Answer is unit′s place digit for 123^{498} will be 9

Number 4 :

Let us calculate values for 4^1 to 4^5

Power of 4	Value
$\mathbf{4^1}$	**4**
$\mathbf{4^2}$	1**6**
$\mathbf{4^3}$	6**4**
$\mathbf{4^4}$	25**6**
$\mathbf{4^5}$	102**4**

So as you can observe:

Unit place digit of 4^1 to 4^5 is in order 4, 6 and it will keep repeating as 4, 6

Power Cycle for 4 : (4, 6)

Problem 1

What will be unit′s place digit for 4^{360} ?

Solution :

Look at the unit place of number **4**.

Unit's place digit is **4**.

Power Cycle of 4 : (4, 6)

There are total 2 values which keep repeating always for power of 4.

Now look at index which is to be identified: 360

As 2 numbers keep on repeating for power cycle of 4, we need to divide 360 by 2 and identify remainder of it so that we can understand what can be unit place number.

$\frac{\text{Index to be found}}{\text{Size of power cycle}} = \frac{360}{2}$, Quotient = 180 and **Remainder = 0**

Whenever remainder is 0 it is last digit in power cycle.

Remainder	Unit Place digit
1	4
0	**6**

$$\textbf{Answer is unit's place digit for } \mathbf{4^{360}} \textbf{ will be 6}$$

Problem 2

What will be unit's place digit for 1234^{6987} ?

Solution :

Look at the unit place of number 123**4**.

Unit's place digit is **4**.

Power Cycle of 4 : (4, 6)

There are total 2 values which keep repeating always for power of 4.

Now look at index which is to be identified: 6987

As 2 numbers keep on repeating for power cycle of 3, we need to divide 6987 by 2 and identify remainder of it so that we can understand what can be unit place number.

$\frac{\text{Index to be found}}{\text{Size of power cycle}} = \frac{6987}{2}$, Quotient = 3493 and **Remainder = 1**

Remainder	Unit Place digit
1	**4**
0	6

$$\textbf{Answer is unit's place digit for } \mathbf{1234^{6987}} \textbf{ will be 4}$$

Number 5:

Let us calculate values for 5^1 to 5^5

Power of 1	Value
$\mathbf{5^1}$	**5**
$\mathbf{5^2}$	2**5**
$\mathbf{5^3}$	12**5**
$\mathbf{5^4}$	62**5**
$\mathbf{5^5}$	312**5**

So as you can observe:

Unit place digit of 5^1 to 5^5 is 5 only. So Unit place digit is 5 for any power of 5

Power Cycle for 5 : (5)

Problem 1

What will be unit′s place digit for 5^{12}

Solution :

Look at the unit place of number **5**.

Unit's place digit is 5.

Power Cycle of 5 : (5)

Answer is unit′s place digit for 5^{12} will be 5

Problem 2

What will be unit′s place digit for 25^{56} ?

Solution :

Look at the unit place of number 2**5**.

Unit’s place digit is 5.

Power Cycle of 5 : (5)

$\textbf{Answer is unit's place digit for } \mathbf{25^{56}} \textbf{ will be 5}$

Number 6:

$\textbf{Let us calculate values for } \mathbf{6^1} \textbf{ to } \mathbf{6^5}$

Power of 6	Value
$\mathbf{6^1}$	**6**
$\mathbf{6^2}$	3**6**
$\mathbf{6^3}$	21**6**
$\mathbf{6^4}$	129**6**
$\mathbf{6^5}$	777**6**

So as you can observe:

$\textbf{Unit place digit of } \mathbf{6^1} \textbf{ to } \mathbf{6^5} \textbf{ is 6 only. So Unit place digit is 6 for any power of 6}$

Power Cycle for 6 : (6)

Problem 1

What will be unit′s place digit for 56^{142} ?

Solution :

Look at the unit place of number 5**6**.

Unit’s place digit is **6**.

Power Cycle of 6 : (6)

$\textbf{Answer is unit's place digit for } \mathbf{56^{142}} \textbf{ will be 6}$

Problem 2

What will be unit′s place digit for 286^{56} ?

Solution :

Look at the unit place of number 28**6**.

Unit's place digit is 6.

Power Cycle of 6 : (6)

Answer is unit′s place digit for 286^{56} will be 6

<u>Number 7 :</u>

Let us calculate values for 7^1 to 7^5

Power of 7	Value
7^1	**7**
7^2	4**9**
7^3	34**3**
7^4	240**1**
7^5	168**07**

So as you can observe:

Unit place digit of 7^1 to 7^5 is in order 7, 9, 3, 1 and it will keep repeating as 7, 9, 3, 1

Power Cycle for 7 : (7, 9, 3, 1)

Problem 1

What will be unit′s place digit for 7^{77} ?

Solution :

Look at the unit place of number **7**.

Unit’s place digit is **7**.

Power Cycle of 7 : $(\mathbf{7}, \mathbf{9}, \mathbf{3}, \mathbf{1})$

There are total 4 values which keep repeating always for power of 7.

Now look at index which is to be identified: 77

As 4 numbers keep on repeating for power cycle of 7, we need to divide 77 by 4 and identify remainder of it so that we can understand what can be unit place number.

$\frac{\text{Index to be found}}{\text{Size of power cycle}} = \frac{77}{4}$, Quotient = 19 and **Remainder = 1**

Remainder	Unit Place digit
1	**7**
2	9
3	3
0	1

Answer is unit′s place digit for 7^{77} will be 7

Problem 2

What will be unit′s place digit for 1237^{496} ?

Solution :

Look at the unit place of number 123**7**.

Unit’s place digit is **7**.

Power Cycle of 7 : (7, 9, 3, 1)

There are total 4 values which keep repeating always for power of 7.

Now look at index which is to be identified: 496

As 4 numbers keep on repeating for power cycle of 7, we need to divide 496 by 4 and identify remainder of it so that we can understand what can be unit place number.

$\frac{\text{Index to be found}}{\text{Size of power cycle}} = \frac{496}{4}$, Quotient = 124 and **Remainder = 0**

Whenever remainder is 0 that is last number of power cycle.

Remainder	Unit Place digit
1	7
2	9
3	3
0	**1**

$\mathbf{Answer\ is\ unit's\ place\ digit\ for\ 1237^{496}\ will\ be\ 1}$

<u>Number 8 :</u>

$\mathbf{Let\ us\ calculate\ values\ for\ 8^1\ to\ 8^5}$

Power of 8	Value
$\mathbf{8^1}$	**8**
$\mathbf{8^2}$	6**4**
$\mathbf{8^3}$	51**2**
$\mathbf{8^4}$	409**6**
$\mathbf{8^5}$	3276**8**

So as you can observe:

Unit place digit of 8^1 to 8^5 is in order $8, 4, 2, 6$ and it will keep repeating as $8, 4, 2, 6$

Power Cycle for 8 : $(8, 4, 2, 6)$

Problem 1

What will be unit′s place digit for 8^{67} ?

Solution :

Look at the unit place of number **8**.

Unit's place digit is **8**.

Power Cycle of 7 : $(8, 4, 2, 6)$

There are total 4 values which keep repeating always for power of 8.

Now look at index which is to be identified: 67

As 4 numbers keep on repeating for power cycle of 8, we need to divide 67 by 4 and identify remainder of it so that we can understand what can be unit place number.

$\frac{\text{Index to be found}}{\text{Size of power cycle}} = \frac{67}{4}$, Quotient = 14 and **Remainder = 3**

Remainder	Unit Place digit
1	8
2	4
3	**2**
0	6

Answer is unit′s place digit for 8^{67} will be 2

Problem 2

What will be unit′s place digit for 128^{6802} ?

Solution :

Look at the unit place of number 12**8**.

Unit's place digit is **8**.

Power Cycle of 8 : (8, 4, 2, 6)

There are total 4 values which keep repeating always for power of 8.

Now look at index which is to be identified: 6802

As 4 numbers keep on repeating for power cycle of 8, we need to divide 6802 by 4 and identify remainder of it so that we can understand what can be unit place number.

$\frac{\text{Index to be found}}{\text{Size of power cycle}} = \frac{6802}{4}$, Quotient = 1700 and **Remainder = 2**

Remainder	Unit Place digit
1	8
2	**4**
3	2
0	6

Answer is unit′s place digit for 128^{6802} will be 4

Number 9 :

Let us calculate values for 9^1 to 9^5

Power of 9	Value
9^1	**9**
9^2	8**1**
9^3	72**9**
9^4	665**1**
9^5	5985**9**

So as you can observe:

Unit place digit of 9^1 to 9^5 is in order 9, 1 and it will keep repeating as 9, 1

Power Cycle for 9 : (9, 1)

Problem 1

What will be unit's place digit for 9^{99} ?

Solution :

Look at the unit place of number **9**.

Unit's place digit is **9**.

Power Cycle of 9 : (9, 1)

There are total 2 values which keep repeating always for power of 9.

Now look at index which is to be identified: 99

As 2 numbers keep on repeating for power cycle of 9, we need to divide 99 by 2 and identify remainder of it so that we can understand what can be unit place number.

$$\frac{\text{Index to be found}}{\text{Size of power cycle}} = \frac{99}{2}$$, Quotient = 49 and **Remainder = 1**

Remainder	Unit Place digit
1	**9**
0	1

Answer is unit′s place digit for 9^{99} will be 9

Problem 2

What will be unit′s place digit for 999^{1000} ?

Solution :

Look at the unit place of number 99**9**.

Unit’s place digit is **9**.

Power Cycle of 9 : (9, 1)

There are total 2 values which keep repeating always for power of 9.

Now look at index which is to be identified: 1000

As 2 numbers keep on repeating for power cycle of 9, we need to divide 1000 by 2 and identify remainder of it so that we can understand what can be unit place number.

$$\frac{\text{Index to be found}}{\text{Size of power cycle}} = \frac{1000}{2}$$, Quotient = 500 and **Remainder = 0**

Remainder	Unit Place digit
1	9
0	**1**

Whenever remainder is 0 it is last digit in power cycle.

Answer is unit′s place digit for 999^{1000} will be 1

Summary of Power Cycle (Unit place Digit)

Number	Power Cycle	Size of power Cycle
0	0	1
1	1	1
2	2, 4, 6, 8	4
3	3, 9, 7, 1	4
4	4, 6	2
5	5	1
6	6	1
7	7, 9, 3, 1	4
8	8, 4, 2,6	4
9	9, 1	2