A natural extension of degree centrality is eigenvector centrality. In-degree centrality awards one centrality point for every link a node receives. But not all vertices are equivalent: some are more relevant than others, and, reasonably, endorsements from important nodes count more. The eigenvector centrality thesis reads:

A node is important if it is linked to by other important nodes.

Eigenvector centrality differs from in-degree centrality: a node receiving many links does not necessarily have a high eigenvector centrality (it might be that all linkers have low or null eigenvector centrality). Moreover, a node with high eigenvector centrality is not necessarily highly linked (the node might have few but important linkers).

Eigenvector centrality, regarded as a ranking measure, is a remarkably old method. Early pioneers of this technique are Wassily W. Leontief (The Structure of American Economy, 1919-1929. Harvard University Press, 1941) and John R. Seeley (The net of reciprocal influence: A problem in treating sociometric data. The Canadian Journal of Psychology, 1949). Math Let $A = (a_{i,j})$ be the adjacency matrix of a graph. The eigenvector centrality x_i of node i is given by:

$$x_i = \frac{1}{\lambda} \sum_k a_{k,i}\, x_k$$

where $\lambda \neq 0$ is a constant. In matrix form we have:

$$\lambda x = xA$$

Hence the centrality vector x is the left-hand eigenvector of the adjacency matrix A associated with the eigenvalue λ. It is wise to choose λ as the largest eigenvalue in absolute value of matrix A. By virtue of Perron-Frobenius theorem, this choice guarantees the following desirable property: if matrix A is irreducible, or equivalently if the graph is (strongly) connected, then the eigenvector solution x is both unique and positive.

The power method can be used to solve the eigenvector centrality problem. Let $m(v)$ denote the signed component of maximal magnitude of vector v. If there is more than one maximal component, let $m(v)$ be the first one. For instance, $m(-3, 3, 2) = -3$. Let $x^{(0)}$ be an arbitrary vector. For $k \geq 1$:

repeatedly compute $x^{(k)} = x^{(k-1)}A$; normalize $x^{(k)} = x^{(k)}/m(x^{(k)})$;

until the desired precision is achieved. It follows that $x^{(k)}$ converges to the dominant eigenvector of A and $m(x^{(k)})$ converges to the dominant eigenvalue of A. If matrix A is sparse, each vector-matrix product can be performed in linear time in the size of the graph.

The method converges when the dominant (largest) and the sub-dominant (second largest) eigenvalues of A, respectively denoted by λ_1 and λ_2, are separated, that is they are different in absolute value, hence when $|\lambda_1| > |\lambda_2|$. The rate of convergence is the rate at which $(\lambda_2/\lambda_1)^k$ goes to 0. Hence, if the sub-dominant eigenvalue is small compared to the dominant one, then the method quickly converges.