

$$\begin{aligned}
& (x_1 \bmod m + x_2 \bmod m + \dots + x_i \bmod m) \bmod m \\
&= \left[(x_1 \bmod m + x_2 \bmod m + \dots + x_{i-1} \bmod m) \bmod m + x_i \bmod m \right] \bmod m \\
&= \left[\left[(x_1 \bmod m + x_2 \bmod m + \dots + x_{i-2} \bmod m) \bmod m + x_{i-1} \bmod m \right] \bmod m + x_i \bmod m \right] \bmod m \\
&= \left[\left[\left[\left[(x_1 + x_2) \bmod m + x_3 \bmod m \right] \bmod m \right] \dots \right] \bmod m + x_{i-1} \bmod m \right] \bmod m + x_i \bmod m \right] \bmod m
\end{aligned}$$

$$\begin{aligned}
& (x_1 + x_2 + \dots + x_i) \bmod m \\
&= \left[(x_1 + x_2 + \dots + x_{i-1}) \bmod m + x_i \bmod m \right] \bmod m \\
&= \left[\left[(x_1 + x_2 + \dots + x_{i-2}) \bmod m + x_{i-1} \bmod m \right] \bmod m + x_i \bmod m \right] \bmod m \\
&= \left[\left[\left[\left[(x_1 + x_2) \bmod m + x_3 \bmod m \right] \bmod m \right] \dots \right] \bmod m + x_{i-1} \bmod m \right] \bmod m + x_i \bmod m \right] \bmod m
\end{aligned}$$

$$\therefore (x_1 + x_2 + \dots + x_i) \bmod m = (x_1 \bmod m + x_2 \bmod m + \dots + x_i \bmod m) \bmod m$$