1.

A	B	O	Ω	Θ
n^3	$n^2\log n$		✓	
2^n	3^n	✓		
$\log(n!)$	$n\log n$	✓		
$\sqrt{n}$	$n^{0.5}$	✓	✓	✓
$(\frac{1}{2})^n$	1		✓	
$n^{1.5}$	$n(\log n)^2$	✓		
$\log_2 n$	$\ln n$	✓	✓	✓
4^n	2^{2n}	✓	✓	✓
n	$\log n(n)$	✓		
$100n+50$	n	✓	✓	✓

2.

$f_2(n) \le f_3(n) \le f_6(n) \le f_1(n) \le f_4(n) \le f_5(n)$.

so we compare the consecutive pair.

① $f_2(n) \le O(f_3(n)) \Leftrightarrow \sqrt{2n} \le C\cdot(n+10)$

$\Leftrightarrow \underline{C\cdot(n+10)} - \sqrt{2n} \ge 0$

$\Leftrightarrow (C-1)(n+10) + (n-\sqrt{2n}) \ge 0.$

when $n>2$. $C>1$.

$(C-1)(n+10) + (n-\sqrt{2n}) \ge 0$ is correct.

② $f_3(n) \le O(f_6(n)) \Leftrightarrow n+10 \le C\cdot n^2\log n$.

when $n>5$ $C>1$

$n+10 \le n^2 \le n\log n \le C\cdot n^2\log n$.

$\therefore f_3(n) \le O(f_6(n))$

③ $f_6(n) \le C\cdot O(f_1(n))$

$\Leftrightarrow n^2\log n \le C\cdot n^{2.5}$

$\Leftrightarrow \log n \le C\sqrt{n}$.

we have proved it before.

④ $f_1(n) \le C\cdot O(f_4(n))$

$\Leftrightarrow n^{2.5} \le C\cdot 10^n$.

we also have proved it before.

⑤ $f_4(n) \le C\cdot O(f_5(n))$

$\Leftrightarrow 10^n \le C\cdot 100^n$.

let $x = 10^n$.

$\Leftrightarrow x \le C\cdot x^2$.

Obviously it is correct.

3.

(there is two solution and I chose the less time complexity one) code.

```
def. find_two_sum(nums, target)
    num_map = {}
    for i, num in enumerate(nums):
        temp = target - num.
        if temp in num_map:
            return [num_map[temp], i]
        num_map[num] = i.
```

C_1	1
C_2	n
C_3	n
C_4	n.
C_5	1
C_6	n.

$T = C_1 + C_5 + (C_2+C_3+C_4)\,n = \cancel{\Theta(n)}$ $O(n)$.

4.

① $2^{n+1} = 2 \cdot 2^n \le C \cdot 2^n = O(2^n)$ when $C>2$.

so it is correcte. True.

② $2^{2n} = (2^n)^2$ let $2^n = x$

$\therefore x^2 \le C \cdot x$ Obviouly it is incorrece.

False.

5. $n^2 = O(2^n) \Leftrightarrow n^2 \le C \cdot 2^n$

$\Leftrightarrow \frac{n^2}{2^n} \le C$

we all know that.

$\lim_{n\to\infty} \frac{n^2}{2^n} = 0$. it means that.

$\forall C>0$ $\exists n_0>0$ s.t. when $n>n_0$ $\frac{n_0^2}{2^{n_0}} \le C$.

so $\frac{n^2}{2^n} \le C$ is true.

~~6 Simply we can let~~ $C = 1$,

$$\cancel{\frac{\frac{n^2}{2^n}}{\frac{(n+1)^2}{2^{n+1}}} = \frac{2n^2}{(n+1)^2}}$$

~~when $n>1$~~ $\frac{2n^2}{(n+1)^2}$