Problem :1 Proof that $\mathbf{J}_A + \mathbf{J}_B = 0$ for a Binary System

Consider a binary mixture consisting of species A and B. The diffusive mass flux of species i is given by Fick's law:

$$\mathbf{J}_i = -\rho D_{AB} \nabla w_i$$

where:

- $\mathbf{J}_i$ is the diffusive mass flux of species i,
- ρ is the total mass density of the mixture (assumed constant),
- D_{AB} is the binary mass diffusivity,
- w_i is the mass fraction of species i.

Since the system is binary:

$$w_A + w_B = 1 \Rightarrow \nabla w_A + \nabla w_B = 0$$

Now, summing the diffusive mass fluxes of species A and B:

$$\begin{aligned} \mathbf{J}_A + \mathbf{J}_B &= -\rho D_{AB} \nabla w_A - \rho D_{AB} \nabla w_B \\ &= -\rho D_{AB} (\nabla w_A + \nabla w_B) \\ &= -\rho D_{AB} \cdot 0 \\ &= \mathbf{0} \end{aligned}$$

Therefore,

$$\boxed{\mathbf{J}_A + \mathbf{J}_B = 0}$$

This indicates that the total diffusive mass flux in a binary mixture is zero, ensuring mass conservation during diffusion in the absence of bulk motion.