CHE 324: Diffusion Problem Solution 2

Problem Statement

In leaching substance A from solid particles by a solvent B, the rate-controlling step is the diffusion of A through a stagnant liquid film (B) of thickness δ out into the main stream.

- The molar solubility of A in B at $z = 0$ is x_{A0}. - The concentration in the main stream at $z = \delta$ is $x_{A\delta}$. - Assume D_{AB} is constant. - Species A is only slightly soluble in B. - Neglect the curvature of the particle.

Using the Equation of Continuity for Species A, determine:

(i) The steady-state concentration profile $x_A(z)$.

(ii) A sketch of the concentration profile.

(iii) The molar flux of A at $z = 0$.

Solution

(i) Steady-State Concentration Profile

At steady state and under one-dimensional diffusion with no reaction, the equation of continuity becomes:

$$\frac{d^2 x_A}{dz^2} = 0$$

Integrating twice:

$$\frac{dx_A}{dz} = C_1 \quad \text{and} \quad x_A = C_1 z + C_2$$

Applying boundary conditions:

- At $z = 0$: $x_A = x_{A0} \Rightarrow C_2 = x_{A0}$
- At $z = \delta$: $x_A = x_{A\delta} \Rightarrow C_1 = \dfrac{x_{A\delta} - x_{A0}}{\delta}$

Hence, the steady-state concentration profile is:

$$x_A(z) = x_{A0} + \frac{x_{A\delta} - x_{A0}}{\delta} z$$

(ii) Sketch of the Concentration Profile

The concentration profile is linear between $z = 0$ and $z = \delta$:

```
x_A

|      • (x_{A0}, z=0)
|     /
|    /
|   /
|  •  (x_{A\delta}, z=\delta)
------- z
```

(You can manually sketch this graph or use a plotting tool. It's a straight line from $(0, x_{A0})$ *to* $(\delta, x_{A\delta})$*.)*

(iii) Molar Flux at $z = 0$ Using the Continuity Equation (Appendix B.11)

According to Appendix B.11, the steady-state species continuity equation in Cartesian coordinates (in terms of mass fraction ω_A) is:

$$\frac{d}{dz}\left(\rho D_{AB}\frac{d\omega_A}{dz}\right) = 0$$

Assuming:

- Constant density ρ and diffusivity D_{AB}
- One-dimensional diffusion (in z direction)
- No convection ($\vec{v} = 0$)
- Steady-state ($\partial/\partial t = 0$)

The equation simplifies to:

$$\frac{d^2\omega_A}{dz^2} = 0$$

This implies a linear concentration profile in terms of ω_A, similar to that of mole fraction x_A under dilute solution assumptions. Using the concentration gradient:

$$\frac{dx_A}{dz} = \frac{x_{A\delta} - x_{A0}}{\delta}$$

The molar flux of species A is given by Fick's First Law:

$$N_A = -D_{AB}\frac{dx_A}{dz}$$

Substituting:

$$N_A = -D_{AB} \cdot \frac{x_{A\delta} - x_{A0}}{\delta} = \frac{D_{AB}(x_{A0} - x_{A\delta})}{\delta}$$

Final expression for molar flux:

$$\boxed{N_A = \frac{D_{AB}(x_{A0} - x_{A\delta})}{\delta}}$$