Enzyme Kinetics from Lineweaver–Burk Plot

The Lineweaver–Burk relation is

$$\frac{1}{v} = \frac{K_m}{V_{\max}}\frac{1}{[S]} + \frac{1}{V_{\max}},$$

so the y-intercept is $1/V_{\max}$, the slope is $K_m/V_{\max}$, and the x-intercept is $-1/K_m$.

(a) Identify the lines (no inhibitor and inhibitor types)

From the plotted families:

- Line **c**: intersects the y-axis at the lowest value and serves as the baseline. **No inhibitor**.
- Line **a**: shares (approximately) the same y-intercept as line c but has a larger slope (steeper). **Competitive inhibitor** (same $1/V_{\max}$, increased K_m).
- Line **d**: shares (approximately) the same x-intercept as line c but has a larger y-intercept. **Noncompetitive inhibitor** (same $-1/K_m$, decreased $V_{\max}$).
- Line **b**: roughly parallel to line c with both intercepts shifted (higher y-intercept and more negative x-intercept). **Uncompetitive inhibitor** (both K_m and $V_{\max}$ decrease by the same factor, so slope $K_m/V_{\max}$ is unchanged).

(e) Estimate K_m and $V_{\max}$ (no inhibitor)

Using line c (no inhibitor) and the graph scales:

$$\left.\frac{1}{v}\right|_{1/[S]=0} \approx 0.02 \quad \Rightarrow \quad V_{\max} \approx \frac{1}{0.02} = 50$$

Units follow the plot: if $1/v$ is $\mathrm{min\,L\,mmol^{-1}}$ then $V_{\max}$ is $\mathrm{mmol\,L^{-1}\,min^{-1}}$.

For the x-intercept, line c crosses near -0.05 on the $1/[S]$ axis:

$$\left.\frac{1}{[S]}\right|_{1/v=0} \approx -0.05 \quad \Rightarrow \quad K_m \approx \frac{1}{0.05} = 20 \text{ mmol L}^{-1}.$$

$$\boxed{K_m \approx 20\,\mathrm{mmol\,L^{-1}}, \qquad V_{\max} \approx 50\,\mathrm{mmol\,L^{-1}\,min^{-1}}}$$

(f) Batch time to 95% conversion, no inhibitor

For a batch system with Michaelis–Menten rate,

$$-\frac{dS}{dt} = v = \frac{V_{\max}S}{K_m + S}.$$

Integrating from S_0 to S gives

$$t = \frac{1}{V_{\max}}\left[(S_0 - S) + K_m \ln\left(\frac{S_0}{S}\right)\right].$$

Given $S_0 = 30\,\mathrm{mmol\,L^{-1}}$ and $X = 0.95 \Rightarrow S = (1-X)S_0 = 1.5\,\mathrm{mmol\,L^{-1}}$,

$$t = \frac{1}{50}\left[(30 - 1.5) + 20\ \ln\left(\frac{30}{1.5}\right)\right] = \frac{1}{50}\left[28.5 + 20\ln(20)\right] \approx \frac{1}{50}\left(28.5 + 59.915\right) \approx 1.77\,\mathrm{min}.$$

$$\boxed{t \approx 1.77\,\mathrm{min}\text{ to reach }95\%\text{ conversion}}$$