Solution

Given Data

$$\begin{aligned}
&\text{Reaction:} \quad A + B \to C + D \quad \text{(isothermal, elementary, irreversible)}\\
&C_{A0} = 1 \text{ mol}\,\text{dm}^{-3}, \quad C_{B0} = 2 \text{ mol}\,\text{dm}^{-3} \quad (\theta_B = 2)\\
&v_0 = 5 \text{ dm}^3/\text{min}, \quad k_a(400\,\text{K}) = 0.25 \text{ dm}^3/(\text{kg}\cdot\text{min})\\
&\alpha = 0.03 \text{ kg}^{-1}, \quad W = 30 \text{ kg}\\
&\epsilon = 0 \ \text{ (no change in total moles)}
\end{aligned}$$

Rate Expression

For an isothermal ideal gas:

$$C_A = C_{A0}\, y\, (1 - X), \quad C_B = C_{A0}\, y\, (\theta_B - X), \quad y \equiv \frac{P}{P_0}.$$

The rate per kg catalyst is:

$$-r'_A = k_a\, C_A\, C_B = k_a (C_{A0} y)^2 (1 - X)(\theta_B - X).$$

The packed-bed design equation is:

$$\frac{dX}{dW} = \frac{-r'_A}{F_{A0}}, \quad F_{A0} = C_{A0} v_0.$$

Thus:

$$\frac{dX}{dW} = \frac{k_a C_{A0}}{v_0}\, y^2\, (1 - X)(\theta_B - X).$$

Numerically:

$$\frac{k_a C_{A0}}{v_0} = \frac{0.25 \times 1}{5} = 0.05.$$

(a) With Pressure Drop

For $\epsilon = 0$:

$$y(W) = (1 - \alpha W)^2 \quad \Rightarrow \quad y^2 = (1 - \alpha W)^4.$$

The design equation becomes:

$$\frac{dX}{(1-X)(2-X)} = 0.05\,(1-\alpha W)^4\,dW.$$

Integrating from $W=0$ to $W=30$:

$$\int_0^X \frac{dX}{(1-X)(2-X)} = \left[\ln\frac{2-X}{1-X} - \ln 2\right]_0^X,$$

$$\int_0^{30} 0.05\,(1-\alpha W)^4\,dW = \frac{0.01}{\alpha}\left[1-(1-\alpha W)^5\right]_0^{30}.$$

With $\alpha = 0.03$, $(1-\alpha W) = 0.1$:

$$\ln\frac{2-X}{1-X} - \ln 2 = \frac{0.01}{0.03}\left[1-0.1^5\right] \approx 0.3333.$$

Thus:

$$\ln\frac{2-X}{1-X} \approx 0.6931 + 0.3333 = 1.0264,$$

$$\frac{2-X}{1-X} \approx e^{1.0264} \approx 2.791 \quad \Rightarrow \quad X \approx 0.442.$$

$$\boxed{X_{\text{with drop}} \approx 0.44}$$

(b) No Pressure Drop ($\alpha = 0$)

Then $y = 1$ and:

$$\frac{dX}{(1-X)(2-X)} = 0.05\,dW.$$

Integrating:

$$\ln\frac{2-X}{1-X} - \ln 2 = 0.05 \times 30 = 1.5,$$

$$\ln\frac{2-X}{1-X} = 0.6931 + 1.5 = 2.1931,$$

$$\frac{2-X}{1-X} \approx e^{2.1931} \approx 8.96 \quad \Rightarrow \quad X \approx 0.874.$$

$$\boxed{X_{\text{no drop}} \approx 0.87}$$

(c) Explanation of Difference

With pressure drop, $P/P_0 = (1-\alpha W)^2$ falls to 0.01 at the bed exit. Since the rate is second order in concentration, $-r'_A \propto y^2$, the reaction slows sharply downstream, lowering the overall conversion compared to the no-drop case.