Solution: CSTR with Cooling Coil

Given Data

$$
\begin{aligned}
&\text{Reaction:} \quad A \to C \quad \text{(exothermic, first-order)}\\
&V = 50\ \text{dm}^3, \quad v_0 = 5\ \text{dm}^3/\text{min}, \quad C_{A0} = 2.0\ \text{mol/dm}^3,\\
&T_f = 300\ \text{K}, \quad \Delta H_R = -50,000\ \text{cal/mol},\\
&T_c = 270\ \text{K}, \quad \dot{m}_c = 1500\ \text{g/min}, \quad C_{p,c} = 1\ \text{cal/(g}\cdot\text{K)},\\
&U = 50\ \text{cal/(min}\cdot\text{m}^2\cdot\text{K)},\\
&k(300\,\text{K}) = 0.20\ \text{dm}^3/(\text{mol}\cdot\text{min}), \quad E = 20,000\ \text{cal/mol},\\
&R = 1.987\ \text{cal/(mol}\cdot\text{K)}.
\end{aligned}
$$

(a) Minimum Conversion without Cooling Limitations

Mass Balance: For a steady-state CSTR:

$$F_{A0} - F_A = -r_A V$$

with

$$F_A = C_A v_0 = C_{A0}(1-X)v_0, \quad -r_A = k(T)C_A.$$

Substitute:

$$C_{A0} X v_0 = k(T) C_{A0}(1-X)V.$$

Cancelling C_{A0}:

$$X = \frac{k(T)V/v_0}{1 + k(T)V/v_0}.$$

Temperature Dependence:

$$k(T) = k(300)\ \exp\left[\frac{E}{R}\left(\frac{1}{300} - \frac{1}{T}\right)\right].$$

Energy Balance (no coil): Only sensible heat removal by outlet stream:

$$\rho C_p v_0 (T_f - T) + (-\Delta H_R) v_0 C_{A0} X = 0.$$

Hence:

$$T = T_f + \frac{(-\Delta H_R) C_{A0} X}{\rho C_p}.$$

Coupling Equations: The above mass and energy balance equations are solved simultaneously for X and T.

(b) Cooling Coil Surface Area to Maintain Conversion from (a)

With a coil, the steady-state energy balance is:

$$\rho C_p v_0(T_f - T) + UA(T_c - T) + (-\Delta H_R)v_0 C_{A0} X = 0.$$

Rearranging for A:

$$A = \frac{(-\Delta H_R)v_0 C_{A0} X - \rho C_p v_0 (T - T_f)}{U(T - T_c)}.$$

Here, X and T are taken from part (a) to maintain the same conversion.

(c) Minimum Conversion in the System

The minimum conversion corresponds to the thermal runaway limit, obtained by simultaneously solving:

$$\frac{dT}{dX} \text{ from energy balance} \quad \text{and} \quad X(T) \text{ from mass balance.}$$

This requires checking the intersection points of the X–T curves.