Solution: PFR with Co-current Heat Exchanger

Governing Equations

1. Mass Balance for PFR: For a first-order reaction $A \to B + C$:

$$\frac{dX}{dV} = \frac{-r_A}{F_{A0}}$$

with:

$$-r_A = k(T)C_A, \quad C_A = C_{A0}(1 - X).$$

Hence:

$$\frac{dX}{dV} = \frac{k(T)C_{A0}(1 - X)}{F_{A0}}.$$

2. Temperature Dependence of Rate Constant:

$$k(T) = k(T_0)\exp\left[\frac{E}{R}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right].$$

3. Energy Balance for the Reactor (Co-current Cooling): Assume constant C_p and density ρ:

$$\frac{dT}{dV} = \frac{-\Delta H_R\,(-r_A) - Ua(T - T_c)}{\rho C_p\,v}.$$

Here:

- U = overall heat transfer coefficient (cal/min·m^2·K),
- a = heat transfer area per reactor volume (m^2/m^3),
- v = volumetric flow rate.

4. Energy Balance for the Coolant (Co-current):

$$\frac{dT_c}{dV} = \frac{Ua(T - T_c)}{\rho_c C_{p,c}\,v_c}.$$

Here:

- ρ_c = coolant density,
- $C_{p,c}$ = coolant heat capacity,
- v_c = coolant volumetric flow rate.

5. Initial Conditions (Co-current):

$$X(0) = 0, \quad T(0) = T_f, \quad T_c(0) = T_{c,\text{in}}.$$

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Modification for Counter-current Heat Exchanger

For counter-current operation, the coolant inlet is at the reactor outlet end. Let V increase from reactor inlet to outlet. The coolant temperature equation becomes:

$$\frac{dT_c}{dV} = -\frac{Ua(T - T_c)}{\rho_c C_{p,c}\, v_c}.$$

The boundary condition for T_c is now:

$$T_c(V_{\text{total}}) = T_{c,\text{in}}.$$

This requires solving the system of ODEs as a boundary-value problem or integrating the coolant equation in the opposite direction.