Optimization-based Control of Robotic Systems

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January 16, 2023

This course will introduce students to modern optimization-based methods for robot control. Robot models will be described first. Then, unconstrained and constrained optimization problems will be introduced. The special case of convex optimization will be presented and used to formulate stabilizing and safety-ensuring controllers for robotic systems. Finally, two lectures will be dedicated to the optimization-based control of manipulators and mobile robots, respectively. By the end of the course, students should be able to formulate and solve robot control problems arising in their research projects by means of optimization-based control techniques.

Contents

Lecture 1 — Monday, 16 January 2023

Introduction to Feedback Control

Try to balance a stick with your eyes closed.

Solution: impossible.

Figure 1: Balancing a stick.

Opening your eyes closes the loop.

And now balancing the stick becomes possible.

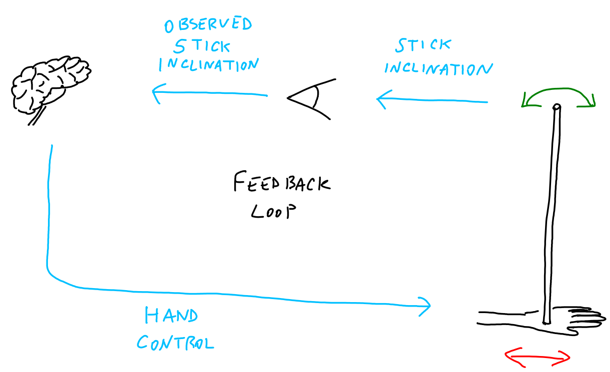

Figure 2: Closing the loop to balance a stick.

Closed-loop, or feedback, control is a powerful tool to make systems (e.g. sticks on our hand) behave as we wish (e.g. stay upright).

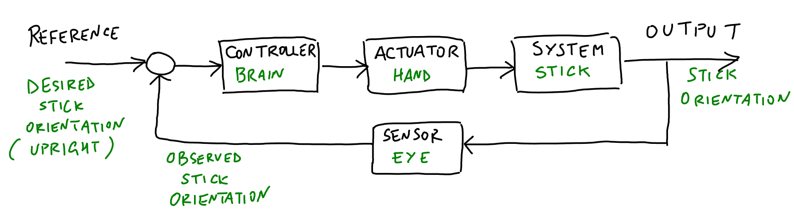

Figure 3: Block diagram of a closed-loop stick inclination control.

Feedback Control of Robotic Systems

Robotics is commonly defined as the science studying the intelligent connection between perception and action[1]—which does not sound too different from what feedback control is. With the tremendous developments that artificial intelligence and machine learning had in the last few decades, and the application of these disciplines to robotic systems, the definition given above probably does not encompass everything that nowadays we would recognize to be a robot. For the sake of controlling robots, however, the above is still an accurate definition.

[1] Bruno Siciliano, Lorenzo Sciavicco, Luigi Villani, and Giuseppe Oriolo. *Robotics: modelling, planning and control*. Springer Science & Business Media, 2010

In these lectures, we will focus on robotic manipulators and mobile robots. The former are comprised of multiple rigid bodies interconnected to each other by different types of joints, and are typically anchored to a fix point in space. The latter can (more or less) freely move in space.

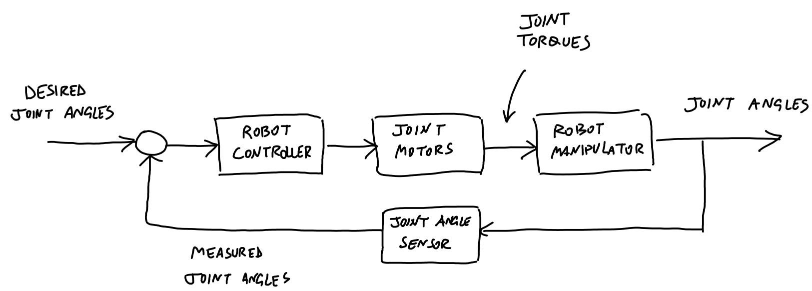

Figure 4: Feedback control of a robotic manipulator.

Figure 4 shows the feedback loop used to control a robotic manipulator, where joint torques are used to regulate joint angles to desired values.

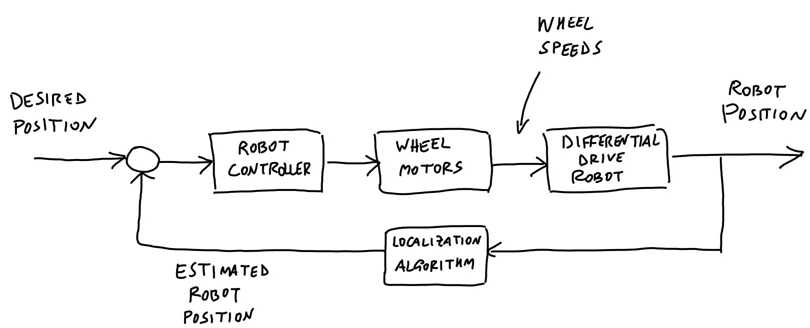

Figure 5: Feedback control of a differential drive mobile robot.

Figure 5 shows the feedback loop used to control a differential-drive mobile robot, where the speeds of its left and right wheels can be independently controlled to move forward and backward, and to turn left and right. This allows the robot to move to a desired position.

Controlling a robot means defining functions to evaluate the control inputs (e.g. joint torques, wheel speeds) to be supplied to the robot for it to achieve a desired behavior. In this course, we will look at optimal ways to define such functions. In particular, the controller synthesis will involve solving optimization problems in the feedback loop.

Kinematic Model of Robotic Systems

The robot kinematics are mathematical relations describing how a robot moves without considering the forces and torques that caused the motion.

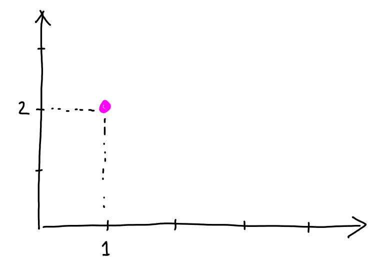

Figure 6: Planar robot at configuration $q = [1, 2] \in \mathbb{R}^2$.

The configuration q of a robot is a complete description of the location of every point of the robot. The set of all configurations is the configuration space and it is denoted by $\mathcal{C}$.

Example 1 *The configuration of a mobile robot translating on a plane can be described using a 2-dimensional vector whose components are the coordinates of the robot in a reference frame defined on the plane. Therefore,* $q = [x, y]^T \in \mathbb{R}^2 = \mathcal{C}$ *(see Fig. 6).*

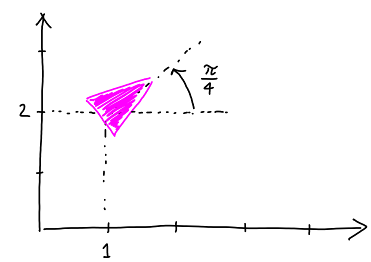

Figure 7: Planar robot at configuration $q = [1, 2, \pi/4] \in \mathbb{R}^2 \times SO(2)$.

Example 2 *The configuration of a mobile robot translating and rotating on a plane can be described using a 3-dimensional vector whose components are the coordinates of the robot in a reference frame defined on the plane and its orientation with respect to a fixed axis in the plane. Therefore,* $q = [x, y, \theta]^T \in \mathbb{R}^2 \times SO(2) = \mathcal{C}$ *(see Fig. 7).*

Example 3 *The configuration of a manipulator with n revolute joints can be described using a n-dimensional vector whose components are the angles of the n revolute joints of the robot. Therefore,* $q = [\theta_1, \ldots, \theta_n]^T \in \mathbb{T}^n = \underbrace{S^1 \times \ldots \times S^1}_{n} = \mathcal{C}$ *(see Fig. 8).*

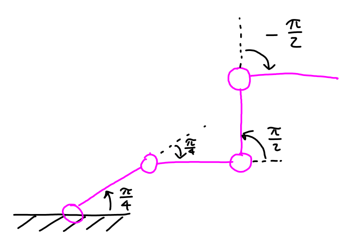

Figure 8: Manipulator robot at configuration $q = [\pi/4, -\pi/4, \pi/2, -\pi/2] \in \mathbb{T}^4$.

The forward kinematics consists in determining the pose (position and orientation) of the end effector, x_e, as a function of the configuration (angles of the joints) of the robot, q:

$$x_e = f(q), \tag{1}$$

where $f : \mathcal{C} \rightarrow \mathcal{T} : q \mapsto x_e$ maps from the configuration space to the *task space* $\mathcal{T}$, to which the pose of the end effector belongs[2].

`Link to Google Colab for the forward kinematics of manipulators.`

[2] For robots comprised of a single rigid body, the function f is trivial, while it may be quite complicated for robotic systems comprised of multiple connected rigid bodies, such as robotic manipulators and articulated mobile robots

THE DIFFERENTIAL (OR VELOCITY) KINEMATICS express the relation between the velocities in the task space, $\dot{x}_e$, and the velocities in the configuration space, $\dot{q}$. Since the forward kinematics map q to x_e, the mathematical expression of the differential kinematics can be determined by differentiating (1):

$$\dot{x}_e = J(q)\dot{q}, \tag{2}$$

where

$$J(q) = \frac{\partial f}{\partial q}(q) \tag{3}$$

is the Jacobian of f, which plays an important role in the analysis of the motion of robotic systems[3].

In the case of mobile robots, the kinematic model expresses the relation between velocities in the configuration space, $\dot{q}$, and control inputs, generally denoted by $u \in \mathbb{R}^m$, for some m. A full treatment of how to derive the kinematic model of mobile robots can be found in several robotics books[4]. In the following, an important example of mobile robot is reported.

[3] The Jacobian is also used in algorithms to solve the inverse kinematics problem, i. e., finding the configuration $\bar{q}$ to achieve a given pose of the end effector $\bar{x}_e$. Using optimization-based controllers we will not need to deal with this problem explicitly.

[4] Mark W. Spong, Seth Hutchinson, and Mathukumalli Vidyasagar. *Robot modeling and control*. John Wiley & Sons, 2020; and Alessandro De Luca and Giuseppe Oriolo. Modelling and control of nonholonomic mechanical systems. In *Kinematics and dynamics of multi-body systems*, pages 277–342. Springer, 1995

Example 4 (Unicycle) *Unicycles are used to model a large variety of mobile robotic systems: ground, marine, and even aerial robots are very often abstracted using a rigid body that can roll without slipping on a planar surface as a coin (see Fig. 9). The kinematic model of the unicycle is given by:*

$$\begin{cases} \dot{x} = v\cos\theta \\ \dot{y} = v\sin\theta \\ \dot{\theta} = \omega, \end{cases} \tag{4}$$

where x and y are the coordinates of the position of the system in a reference frame defined on the plane where the robot moves, θ is its orientation, and v and ω are the linear and angular velocity control inputs. Therefore, defining the configuration of the robot $q = [q_1, q_2, q_3]^T = [x, y, \theta]^T$ and the control input vector $u = [u_1, u_2]^T = [v, \omega]^T$, we can write (4) as follows:

$$\dot{q} = \begin{bmatrix} \cos q_3 \\ \sin q_3 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_2. \tag{5}$$

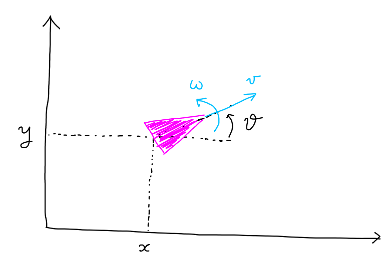

Figure 9: Unicycle.

`Link to Google Colab for the differential kinematics of manipulators and mobile robots.`

Dynamic Model of Robotic Systems

While the kinematic description of a robot is purely geometric, the dynamics of a robot consist in the mathematical relation describing the effect that generalized forces (forces and torques) acting on the generalized coordinates (components of the robot configuration) of the robot have on the motion of the robot. In other words, the dynamics of a robot tell us, for instance, how joint torques, τ, of a manipulator generate joint accelerations, $\ddot{q}$. Mathematically, this can be expressed as follows:

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau, \tag{6}$$

where the terms on the left-hand side represent the torques acting on the joints due to inertial ($D(q)\ddot{q}$), centrifugal and Coriolis ($C(q, \dot{q})\dot{q}$), and gravitational ($g(q)$) effects.

In this course, we will focus on kinematic models of robotic systems. The same formulation, however, can be applied to dynamic models as well.

References

Alessandro De Luca and Giuseppe Oriolo. Modelling and control of nonholonomic mechanical systems. In *Kinematics and dynamics of multi-body systems*, pages 277–342. Springer, 1995.

Bruno Siciliano, Lorenzo Sciavicco, Luigi Villani, and Giuseppe Oriolo. *Robotics: modelling, planning and control*. Springer Science & Business Media, 2010.

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