

DYNAMIC PROGRAMMING ALGS

longest increasing subsequence: $O(n^2)$: $L(j)$ =len of longest subsequence ending @ j. $L(j) = 1 + \max\{L(i)\}$ where $i < j, a_i < a_j \rightarrow (i, j)$ is an edge. Return $\max\{L(i) \text{ for } i \leq n\}$, and loop from $i = n \rightarrow 0$
edit distance: $O(m \cdot n)$: $E[m, n]$ = min num of edits to change $x[1 \dots m]$ to $y[1 \dots n]$.

$$E[m, n] = \min \begin{cases} \dots x[m], \dots - & \text{delete } x[m]: E[m, n] = 1 + E[m-1, n] \\ \dots -, \dots y[n] & \text{insert } y[n]: E[m, n] = 1 + E[m, n-1] \\ \dots x[m], \dots y[n] & \text{change } x[m] \text{ to } y[n]: E[m, n] = E[m-1, n-1] + (1 \text{ if diff, } 0 \text{ else}) \end{cases}$$

Initialize $i = 0 \dots m : E[i, 0] = i$ $j = 0 \dots n : E[0, j] = j$. Loop $i = 1 \dots m, j = 1 \dots n, E[i, j]$

Knapsack: $O(W_{\max} \cdot n)$: $K(w)$ = max value of weight $w \leq W_{\max}$. $K(w) = \max_{w_i \leq w} \{v_i + K(w - w_i)\}$ $K(0) = 0$.

Loop $w = 1 \dots W_{\max}$

Chain Matrix Multiply: $O(n^3)$: $C(i, j)$ = cost of best solution to multiplying $A_i \dots A_j$.

$$C(i, j) = \min_{i \leq k \leq j} \{C(i, k) + C(k+1, j) + m_{i-1} \cdot m_k \cdot m_j\}$$

Solve in order of increasing subproblem length: for $i = 1 \dots n-1 : C(i, i) = 0$. for $s = 1 \dots n-1$, for $i = 1 \dots n-s : j = i+s$; update. Return $C(1, n)$.

Shortest path (all pairs of vertices): $O(|V| \cdot (|V| + |E|))$: $dist(v, i)$ = dist from s to v using i edges. $dist(v, i) = \min_{e=(w,v)} \{len(e) + dist(w, i-1)\}$. For all $v \in V$ $dist(v, 0) = \infty, dist(s, 0) = 0$. for $i = 1 \dots |V|$, for all $v \in V$, update.

Independent Sets: $O(|V| + |E|)$: $I(u)$ = size of largest independent set in subtree rooted at u .

$$I(u) = \max \begin{cases} 1 + \sum_{grandchildren} I(g_i) & \text{if } u \text{ in largest independent set} \\ \sum_{children} I(c_i) & \text{u isn't} \end{cases}$$

DFS traversal: postvisit[u] = update (does u after children of u are done)

Travelling Salesman: $O(2^n n^2)$