## DYNAMIC PROGRAMMING ALGS

**longest increasing subsequence:** $O(n^2)$: $L(j)$=len of longest subsequence ending @ j. $L(j) = 1 + \max\{L(i)\}$ where $i < j, a_i < a_j \rightarrow (i,j)$ is an edge. Return $\max\{L(i) for i \leq n\}$, and loop from $i = n \rightarrow 0$

**edit distance:** $O(m \cdot n)$: $E[m,n]$ =min num of edits to change $x[1 \cdots m]$ to $y[1 \cdots n]$. E[m,n] =

$$\min \begin{cases} \cdots x[m], \cdots - \\ \text{delete x[m]: } E[m,n] = 1 + E[m-1,n] \\ \cdots -, \cdots y[n] \\ \quad \text{insert y[n]: } E[m,n] = 1 + E[m,n-1] \\ \cdots x[m], \cdots y[n] \\ \quad \text{change x[m] to y[n]:} \\ \ E[m,n] = E[m-1,n-1] + \text{ (1 if diff, 0 else)} \end{cases}$$

Initialize $i = 0 \cdots m : E[i,0] = i \quad j = 0 \cdots n : E[0,j] = j$. Loop $i = 1 \cdots m, j = 1 \cdots n, E[i,j]$

**Knapsack:** $O(W_{\max} \cdot n)$: $K(w)$ =max value of weight $w \leq W_{\max}$. $K(w) = \max_{w_i \leq w}\{v_i + K(w - w_i)\}$ $K(0) = 0$. Loop $w = 1 \cdots W_{\max}$

**Chain Matrix Multiply:** $O(n^3) : C(i,j) =$ cost of best solution to multiplying $A_i \cdots A_j$.

$$C(i,j) = \min_{i \leq k \leq j}\{C(i,k) + C(k+1,j) + m_{i-1} \cdot m_k \cdot m_j\}$$

Solve in order of increasing subproblem length: for $i = 1 \cdots n-1 : C(i, i = 0)$. for $s = 1 \cdots n-1$, for $i = 1 \cdots n-s : j = i+s$; update. Return $C(1,n)$.

**Shortest path (all pairs of vertices):** $O(|V| \cdot (|V| + |E|))$: $dist(v,i) =$ dist from $s$ to $v$ using $i$ edges. $dist(v,i) = \min_{e=(w,v)}\{len(e) + dist(w,i-1)\}$. For all $v \in V \quad dist(v,0) = \infty, dist(s,0) = 0$. for $i = 1 \cdot |V|$, for all $v \in V$, update.

**Independent Sets:** $O(|V| + |E|)$: $I(u)$ =size of largest independent set in subtree rooted at $u$.

$$I(u) = \max \begin{cases} 1 + \sum_{grandchildren} I(g_i) & \text{if } u \in I(n) \\ \sum_{children} I(c_i) & \text{u isn't} \end{cases}$$

DFS traversal: postvisit[u] = update (does $u$ after children of $u$ are done)

**Travelling Salesman:** $O(2^n n^2)$ Subset $S \leq V$, $\{1\} \in S$.
for $j \in S, C(S,j)$ =length of shortest path that starts at 1, ends at $j$, and visits each vertex in S once. If $|S| > 1, C(S,1) = \infty$.

update: $C(S,j) = \min_{i \in S, i \neq j, (i,j) \in E}\{C(S - \{j\}, i) + len(i,j)\}$
For $S = 2 \cdots n$ : for all sizes of S, for all subsets $S \leq \{1 \cdots n\}$ of size $s$ (including $\{i\}$): $C(S,1) = \infty$ for all $j \in S, j \neq 1, update$