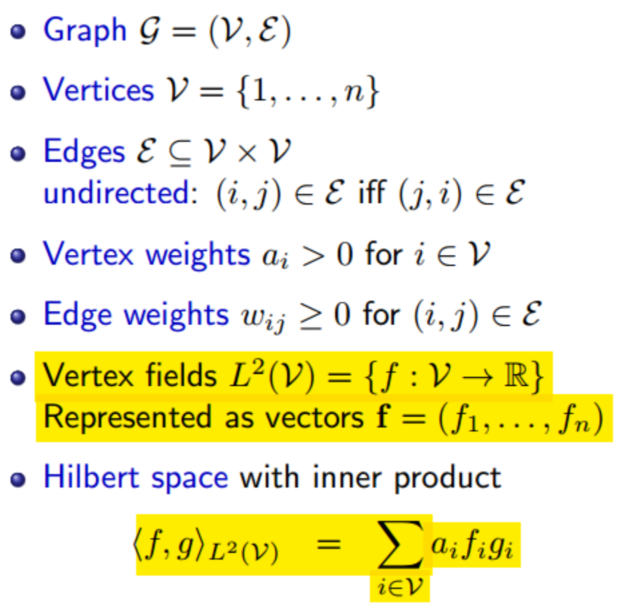
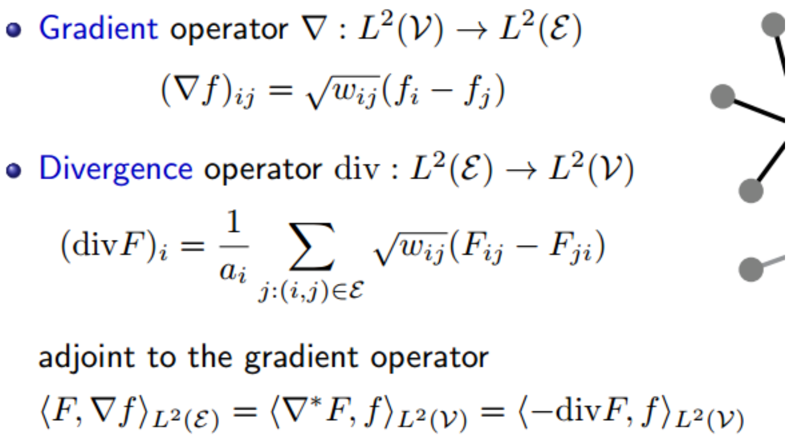
What are vertex fields, gradient and divergence on graphs?





I have a few questions these two slides on the topic of calculus on graphs:

1. What are the vertex fields defined here? My understanding is that it is a set of functions that takes in a vertex and gives a real number output. And because each vertex may need to undergo different transformation, each vertex v_i has its corresponding function f_i. Is that right?
2. What does the inner product here means?
3. Why is there a square root of weight in gradient and divergence operator? Is it necessary? My understanding is that multiplying by weight and not square root of weight is sufficient.
4. What is F in divergence operator?

These are all the slides that I have and I am having a lot of trouble understanding it. Is it that it is badly written? If not, can someone kindly explain to me please? Thanks.

(calculus) (graph-theory) (intuition) (laplacian) (divergence)

edited Nov 6 '17 at 22:12
Alex Ravsky
36.6k 3 20 73

asked Nov 6 '17 at 10:56
Aha
953 8 15

1 Answer

1. A vertex field is a square-summable function from the set of vertices into $\mathbb{R}$. (If there are only finitely many vertices, saying "square-summable" is unnecessary.) Imagine a graph, say the triangle K_3. Put a number next to each vertex, say $3, 6, -2$. You have a vertex field.
2. The concept of inner product is explained on Wikipedia. Here the inner product of two vertex field f, g means: multiply each value of f by the corresponding value of g and by the weight of that vertex. Add the

results.

3. The reason for having a square root in $\sqrt{w_{ij}}$ will become apparent on a later slide, where the graph Laplacian is defined as the divergence of gradient. Since both the gradient and the divergence involve multiplying by $\sqrt{w_{ij}}$, the Laplacian will have w_{ij}. The author would rather have a simpler formula for Laplacian, because it will be used often in the future.
4. F is a square-summable function defined on the edges. (If there are only finitely many edges, saying "square-summable" is unnecessary.) This is what the first line of definition "$\operatorname{div} : L^2(\mathcal{E}) \to L^2(\mathcal{V})$" is for, to state what are the domain and codomain of this map.

answered Nov 11 '17 at 22:02

user357151
12.7k 3 9 38

why must vertex field and F be square-summable function? – user136266 Nov 12 '17 at 14:54

with regards to 4, isn't the domain for gradient operator an edge and the codomain a real number? – user136266 Nov 12 '17 at 14:58

(a) because the slides tell you so. L^2 means square summable. The person who defines these concepts gets to make those calls. (b) No. The slides specify the domain of gradient operator as $L^2(\mathcal{V})$ and the codomain as $L^2(\mathcal{E})$. The domain consists of square-summable functions on vertices, the codomain is square-summable functions on edges. – user357151 Nov 12 '17 at 15:52

The form of the gradient operator clearly shows that the result is a real number and since this function takes in an edge and spits out a real number, how is it true that the domain and codomain are $L^2(V)$ and $L^2(E)$?. The same applies to divergence. The domain and codomain defined doesnt seem right. – Aha Nov 13 '17 at 2:09

For (3), how is Laplacian related to gradient and divergence? I understand the relationship in vector calculus, but the author's definition in this slides, they don't seem to be related – Aha Nov 13 '17 at 2:12