

# Bit-level Binaries and Generalized Comprehensions in Erlang

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### Binaries as we know them

Introduced in 1992 as a container for object code

Used in applications that do I/O, networking or protocol programming

A proposal for a binary datatype and a syntax was made in 1999 and a revised version was adopted in 2000

Since then, binaries have been used extensively, often providing innovative solutions to common telecom programming tasks



### Binaries are not so flexible

#### Some limitations:

- Binaries are byte streams, not bit streams
- Segment sizes cannot be arbitrary arithmetic expressions

Both undermine the use of the binary syntax for writing high level specifications

#### This work:

We show how to lift these limitations while maintaining backward compatibility



### Make binaries as flexible as lists

#### - In lists:

- deconstructing a list always yields valid terms
- can be constructed using list comprehensions
- In binaries:
  - deconstructing a binary sometimes yields terms which cannot be represented as Erlang binaries
  - no binary comprehensions are available

#### This work:

- allows binaries to represent bit streams
- introduces binary comprehensions
- introduces extended comprehensions to make conversions between lists and binaries simpler



## Flexible bit-level binaries

• The multiple-of-eight size restriction is lifted

• The size field of a segment can contain an arbitrary arithmetic expression

 No type specifier is needed in binary construction



### Pros and cons of bit-level binaries

- + Allows natural representation of bit fields
  - <<BitSize:8, BitField:BitSize/binary, ...
- + Helps avoid padding calculations
  - Pad = (8 ((X + Y) rem 8)) rem 8,
- + Makes binary matching as easy for bit streams as it was for byte streams
- Introduces a speed trade-off



# Pattern Matching

- byte streams vs bit streams

```
keep_0XX(<<0:8,X:16,Rest/binary>>) ->
        <<0:8,X:16,keep_0XX(Rest)/binary>>;
keep_0XX(<<_:24,Rest/binary>>) ->
        keep_0XX(Rest);
keep_0XX(Rest);
keep_0XX(<<>>) ->
        <<>>.
```

This function only keeps the <u>byte</u> triples whose first byte is 0.

But what if me want to keep the <u>bit</u> triples whose first bit is 0?



## Pattern Matching

- byte streams vs bit streams

```
keep_0XX(<<0:1,X:2,Rest/binary>>) ->
     <<0:1,X:2,keep_0XX(Rest)/binary>>;
keep_0XX(<<_:3,Rest/binary>>) ->
     keep_0XX(Rest);
keep_0XX(Rest);
keep_0XX(<<>>) ->
     <<>>.
```

This is how it ought to look!



# Pattern Matching

- byte streams vs bit streams

```
keep_0XX(Bin) -> keep_0XX(Bin, 0, 0, <<>>).
keep 0XX(Bin, N1, N2, Acc) ->
  Pad1 = (8 - ((N1+3) rem 8)) rem 8,
  Pad2 = (8 - ((N2+3) rem 8)) rem 8,
  case Bin of
    <<_:N1, 0:1, X:2, _:Pad1, _/binary>> ->
      NewAcc =
      <<Acc:N2/binary-unit:1, 0:1, X:2, 0:Pad2>>,
      keep 0XX(Bin, N1+3, N2+3, NewAcc);
    <<_:N1, _:3, _:Pad1, _/binary>> ->
      keep 0XX(Bin, N1+3, N2, Acc);
    << :N1>> -> Acc
  end.
```

This is how you have to write it today!



# Allowing arithmetic expressions in the size field

Consider this classic example of the bit syntax:

```
case IP_Packet of
    <<4:4, Hlen:4, SrvcType:8, TotLen:16,
        ID:16, Flgs:3, FragOff:13, TTL:8, Proto:8,
        SrcIP:32, DestIP:32,
        RestDgrm/binary>> ->
        OptsLen = Hlen - 5,
        <<Opts:OptsLen/binary-unit:32,
        Data/binary>> = RestDgrm,
        ...
end
```



# Allowing arithmetic expressions in the size field

Using flexible binaries it could be written in the following manner:

```
case IP_Packet of
  <<4:4, Hlen:4, SrvcType:8, TotLen:16,
    ID:16, Flgs:3, FragOff:13, TTL:8,
    Proto:8, SrcIP:32, DestIP:32,
    Opts:((Hlen - 5)*32)/binary,
    Data/binary>> -> ...
end,
```



# No need for a type-specifier in binary construction

Consider the following code:

$$X = <<1, 2, 3>>, B = <>$$

It causes a runtime exception. To avoid this you must explicitly specify the type

We want to lift this restriction, the type should default to the type of the variable.



# Binary Comprehensions

Analogous to List Comprehensions

List Comprehensions represent a combination of map and filter

Comprehensions require a notion of an element

For binary comprehensions the user must specify what they consider as an element



# Binary Comprehensions:

Introductory Example, invert

Using list comprehension:

```
invert(ListOfBits) ->
  [bnot(X) | X <- ListOfBits]</pre>
```

Using binary comprehension:

```
invert(Binary) ->
  <<bnot(X):1 | | X:1 <- Binary>>
```

If your binary is byte-sized:

```
invert(Binary) ->
  <<bnot(X):8 || X:8 <- Binary>>
```



# Binary Comprehensions:

**UU-decode** 

Using a binary comprehension UU-decode basically becomes a one-liner in Erlang

```
uudecode(UUBin) -> <<(X-32):6 | X:8 <- UUBin, 32=<X, X=<95 >>
```

Note the filter expressions which make sure that inserted characters such as line-breaks are dropped



# Extended comprehensions

Can we use list generators in binary comprehensions?

```
convert_to_binary(ListofWords) ->
     <<X:32 || X <- ListofWords>>.
```

YES!



## Extended comprehensions

Can we use binary generators in list comprehensions?

```
convert_to_listofwords(Binary) ->
  [X || X:32 <- Binary].</pre>
```

YES!



#### Generators

Note that we need to be able to separate list generators from binary generators.

List generators:

Binary generators:

$$S_1 \ldots S_n \leftarrow E_B$$

P – a pattern

 $E_L$  – an Erlang expression which evaluates to a list

S, - a binary segment

 $E_L$  – an Erlang expression which evaluates to a binary



# Implementation of extended binary comprehensions

- We present a simple translation of extended comprehensions into Erlang in the form of rewrite rules in the paper
- Using these simple rules the cost of building the resulting binary is quadratic in the number of segments
- We present another set of rewrite rules which gives linear complexity, but the rules are slightly less straight-forward



# Implementation of extended binary comprehensions

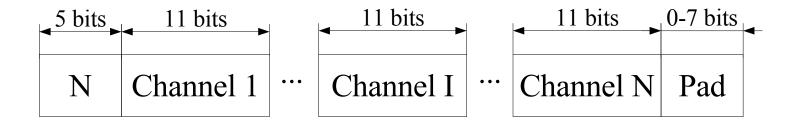
When the size of the resulting binary can be calculated as a function of a generator binary, the translation can be very efficient

This allows us to preallocate the memory that is needed for the resulting binary



## Example: IS-683 PRL

#### **Data Structure**



Task: Create a list of Channels



# First "Padding" Solution:

```
decode(<<NumChans:5, _Pad:3, _Rest/binary>> = Bin) ->
    decode(Bin, NumChans, NumChans, []).

decode(_, _, 0, Acc) ->
    Acc;
decode(Bin, NumChans, N, Acc) ->
    SkipBefore = (N - 1) * 11,
    SkipAfter = (NumChans - N) * 11,

Pad = 8 - ((NumChans * 11 + 5) rem 8),
    <<_:5, _:SkipBefore, Chan:11,
    _:SkipAfter,_:Pad>> = Bin,
    decode(Bin, NumChans, N - 1, [Chan | Acc]).
```

Buggy calculation of padding



# Correct "Padding" Solution:

```
decode(<<NumChans:5, _Pad:3, _Rest/binary>> = Bin) ->
    decode(Bin, NumChans, NumChans, []).

decode(_, _, 0, Acc) ->
    Acc;
decode(Bin, NumChans, N, Acc) ->
    SkipBefore = (N - 1) * 11,
    SkipAfter = (NumChans - N) * 11,
    Pad = (8 - ((NumChans * 11 + 5) rem 8)) rem 8,
    <<_:5, _:SkipBefore, Chan:11,
    _:SkipAfter,_:Pad>> = Bin,
    decode(Bin, NumChans, N - 1, [Chan | Acc]).
```



decode(Channels) ->

case Channels of

## Expanded solution:

```
<<0:5, :3>>->
<<1:5.X1:11. :0>>->
 <<2:5,X1:11,X2:11, :5>>->
   [X1,X2]:
 <<3:5,X1:11,X2:11,X3:11, :2>>->
[X1,X2,X3];
<<4:5.X1:11.X2:11.X3:11.X4:11. :7>>->
    [X1,X2,X3,X4];
 <<5:5,X1:11,X2:11,X3:11,X4:11,X5:11,_:4>>->
    [X1,X2,X3,X4,X5];
 <<6:5,X1:11,X2:11,X3:11,X4:11,X5:11,X6:11,_:1>>->
                                                                                                                                                                                                                                                                                                                                                                                                                                                                       <<3:5,X1:11,X2:11,X3:11,_:2>> ->
   [X1,X2,X3,X4,X5,X6];
 <<7:5,X1:11,X2:11,X3:11,X4:11,X5:11,X6:11,X7:11,_:6>>->
    [X1,X2,X3,X4,X5,X6,X7];
 <<8:5,X1:11,X2:11,X3:11,X4:11,X5:11,X6:11,X7:11,X8:11,_:3>>->
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  [X1,X2,X3];
   [X1.X2,X3,X4,X5,X6,X7,X8];
 <<9:5,X1:11,X2:11,X3:11,X4:11,X5:11,X6:11,X7:11,X8:11,X9:11,_:0>>->
   [X1,X2,X3,X4,X5,X6,X7,X8,X9];
<<10:5,X1:11,X2:11,X3:11,X4:11,X5:11,X6:11,X7:11,X8:11,X9:11,X10:11,_:5>>->
    [X1,X2,X3,X4,X5,X6,X7,X8,X9,X10];
 <<11:5,X1:11,X2:11,X3:11,X4:11,X5:11,X6:11,X7:11,X8:11,X9:11,X10:11,X11:11, :2>>>
    [X1,X2,X3,X4,X5,X6,X7,X8,X9,X10,X11];
 <<12:5,X1:11,X2:11,X3:11,X4:11,X5:11,X6:11,X7:11,X8:11,X9:11,X10:11,X11:11,X12:11,_:7>>>
    [X1,X2,X3,X4,X5,X6,X7,X8,X9,X10,X11,X12];
 <<13:5,X1:11,X2:11,X3:11,X4:11,X5:11,X6:11,X7:11,X8:11,X9:11,X10:11,X11:11,X12:11,X13:11,\\ \cdot 4>>>
    [X1,X2,X3,X4,X5,X6,X7,X8,X9,X10,X11,X12,X13];
 <<\!\!14\!:\!5,\!X1\!:\!11,\!X2\!:\!11,\!X3\!:\!11,\!X4\!:\!11,\!X5\!:\!11,\!X6\!:\!11,\!X7\!:\!11,\!X8\!:\!11,\!X9\!:\!11,\!X10\!:\!11,\!X11\!:\!11,\!X12\!:\!11,\!X13\!:\!11,\!X14\!:\!11,\!\ldots\!!>>>
    [X1,X2,X3,X4,X5,X6,X7,X8,X9,X10,X11,X12,X13,X14];
 <<15:5,X1:11,X2:11,X3:11,X4:11,X5:11,X6:11,X7:11,X8:11,X9:11,X10:11,X11:11,X12:11,X13:11,X14:11,X15:11,\\ \cdot 6>>>
    [X1,X2,X3,X4,X5,X6,X7,X8,X9,X10,X11,X12,X13,X14,X15];
<<\!16:5,\!X1:\!11,\!X2:\!11,\!X3:\!11,\!X4:\!11,\!X5:\!11,\!X6:\!11,\!X7:\!11,\!X8:\!11,\!X9:\!11,\!X10:\!11,\!X11:\!11,\!X12:\!11,\!X13:\!11,\!X14:\!11,\!X15:\!11,\!X16:\!11,\!.:\!13>>>>
    [X1,X2,X3,X4,X5,X6,X7,X8,X9,X10,X11,X12,X13,X14,X15,X16];
 <<17:5,X1:11,X2:11,X3:11,X4:11,X5:11,X6:11,X7:11,X8:11,X9:11,X10:11,X11:11,X12:11,X13:11,X14:11,X15:11,X16:11,X17:11,\_:0>>>
    [X1,X2,X3,X4,X5,X6,X7,X8,X9,X10,X11,X12,X13,X14,X15,X16,X17];
 <<18:5.X1:11.X2:11.X3:11.X4:11.X5:11.X6:11.X7:11.X8:11.X9:11.X10:11.X11:11.X12:11.X13:11.X14:11.X15:11.X16:11.X17:11.X18:11, \\ :5>>>>
    [X1,X2,X3,X4,X5,X6,X7,X8,X9,X10,X11,X12,X13,X14,X15,X16,X17,X18];
 <<19:5.X1:11.X2:11.X3:11.X4:11.X5:11.X6:11.X7:11.X8:11.X9:11.X10:11.X11:11.X12:11.X13:11.X14:11.X15:11.X16:11.X17:11.X18:11.X19:11.:2>>>
    [X1,X2,X3,X4,X5,X6,X7,X8,X9,X10,X11,X12,X13,X14,X15,X16,X17,X18,X19];
 <\!<\!20:5,\!X1:11,\!X2:11,\!X3:11,\!X4:11,\!X5:11,\!X6:11,\!X7:11,\!X8:11,\!X9:11,\!X10:11,\!X11:11,\!X12:11,\!X13:11,\!X14:11,\!X15:11,\!X16:11,\!X17:11,\!X18:11,\!X19:11,\!X20:11,\!_:7>\!>>\!>
    [X1,X2,X3,X4,X5,X6,X7,X8,X9,X10,X11,X12,X13,X14,X15,X16,X17,X18,X19,X20];
 <<21:5,X1:11,X2:11,X3:11,X4:11,X5:11,X6:11,X7:11,X8:11,X9:11,X10:11,X11:11,X12:11,X13:11,X14:11,X15:11,X16:11,X17:11,X18:11,X19:11,X20:11,X21:11,\\ :4>>>
   [X1,X2,X3,X4,X5,X6,X7,X8,X9,X10,X11,X12,X13,X14,X15,X16,X17,X18,X19,X20,X21];
 [X1,X2,X3,X4,X5,X6,X7,X8,X9,X10,X11,X12,X13,X14,X15,X16,X17,X18,X19,X20,X21,X22];
 <\!<\!23:5,\!X1:\!11,\!X2:\!11,\!X3:\!11,\!X4:\!11,\!X5:\!11,\!X6:\!11,\!X7:\!11,\!X8:\!11,\!X9:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11,\!X1:\!11
    [X1,X2,X3,X4,X5,X6,X7,X8,X9,X10,X11,X12,X13,X14,X15,X16,X17,X18,X19,X20,X21,X22,X23];
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 <\!<\!29:5,\!X1:11,\!X2:11,\!X3:11,\!X4:11,\!X5:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:1
    [X1, X2, X3, X4, X5, X6, X7, X8, X9, X10, X11, X12, X13, X14, X15, X16, X17, X18, X19, X20, X21, X22, X23, X24, X25, X26, X27, X28, X29];
 <<30:5,X1:11,X2:11,X3:11,X4:11,X5:11,X6:11,X7:11,X8:11,X9:11,X10:11,X1:11,X12:11,X13:11,X14:11,X15:11,X10:11,X12:11,X13:11,X12:11,X13:11,X12:11,X13:11,X12:11,X13:11,X12:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13:11,X13
    <\!<\!31:5,\!X1:11,\!X2:11,\!X3:11,\!X2:11,\!X3:11,\!X2:11,\!X3:11,\!X2:11,\!X3:11,\!X2:11,\!X3:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:11,\!X2:1
   [X1,X2,X3,X4,X5,X6,X7,X8,X9,X10,X11,X12,X13,X14,X15,X16,X17,X18,X19,X20,X21,X22,X23,X24,X25,X26,X27,X28,X29,X30,X31]
```



## Smart, but inefficient solution

```
decode(<<N_channels:5, Alignment_bits:3, Tail/binary>>) ->
  decode2(N_channels, <<Alignment_bits:3, Tail/binary, 0:5>>).

decode2(0, _) ->
  [];

decode2(N, <<C:11, A:5, T/binary>>) ->
  [C|decode2(N-1, <<A:5, T/binary, 0:3>>)].
```

Avoids complicated padding calculations, at the cost of recreating the binary in each iteration.

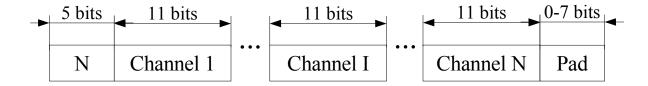


# Using Flexible binaries

Since flexible binaries can represent bit streams properly and leads to a natural solution

```
decode(<<N:5, Channels:(11*N)/binary,_/binary>>) ->
  decode2(Channels).

decode2(<<C:11, T/binary>>) ->
  [C|decode2(T)];
decode2(<<>>>) ->
  [].
```





# Extended comprehensions

Using extended comprehensions and flexible binaries we can solve the problem in two lines:

```
decode(<<N:5, Channels:(11*N)/binary,_/binary>>) ->
  [X | X:11 <= Channels].</pre>
```





### Succintness of flexible binaries

- as measured in line counts

Program in	С	Java	Erlang (R10B)	Erlang (this)
keep 0XX	51	33	14	2
μ-law encode	30	25	25	13
UU-decode	19	14	10	2

μ-law encode - Compresses sound
 keep 0XX - Keeps bit-triples that start with 0
 UU-decode - Decodes UU-encoded binaries



## Conclusion

- Introducing bit-level binaries makes it easy to represent bit streams as binaries
- This makes it possible to write high level specifications of operations on bit streams
- Extended comprehensions allow for powerful manipulation of binaries
- Together these extensions make binaries as easy to use as other datatypes in Erlang such as tuples and lists
- The extensions we propose are backwards compatible
- They will probably be included in the R11 release of Erlang/OTP



## **Future Work**

- A standard library for dealing with binaries
- A better representation of binaries to avoid quadratic complexity when appending binaries
- New compilation techniques which allow for inplace updates of binaries



# Adapting BIF:s to bit-level binaries

```
size(Bin)
```

• should return the minimal number of bytes needed to represent the binary.

```
bitsize(Bin)
```

new bif which returns the size in bits

```
binary_to_list(Bin)
```

> the following should hold:

```
Bin == list_to_binary(binary_to_list(Bin))
```



# binary\_to\_list(Bin)

#### Desired property:

```
Bin == list_to_binary(binary_to_list(Bin))
```

```
binary_to_list(<<X:8,Rest/binary>>) ->
    [X|binary_to_list(Rest)];
binary_to_list(<<>>) ->
    [];
binary_to_list(Bin) when is_binary(Bin) ->
    [Bin].
```

#### gives:

```
[0,0,<<0:4>>] == binary_to_list(<<0:20>>)
```